

# Chapter 8: Applicative and traversable functors

## Part 2: Their laws and structure

Sergei Winitzki

Academy by the Bay

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# Motivation for applicative functors

- Monads are inconvenient for expressing *independent* effects

Monads perform effects *sequentially* even if effects are independent:

```
x ← Future { c1 }
y ← Future { c2 }
z ← Future { c3 }

Future { c1 }.flatMap { x ⇒
  Future { c2 }.flatMap { y ⇒
    Future { c3 }.map { z ⇒ ... }
  } }
```

- We would like to parallelize independent computations
- We would like to accumulate *all* errors, rather than stop at the first one

Changing the order of monad's effects will (generally) change the result:

```
for {
  x ← List(1, 2)
  y ← List(10, 20)
} yield f(x, y)
// f(1, 10), f(1, 20), f(2, 10), f(2, 20)

for {
  y ← List(10, 20)
  x ← List(1, 2)
} yield f(x, y)
// f(1, 10), f(2, 10), f(1, 20), f(2, 20)
```

- We would like to express a computation where effects are unordered
  - This can be done using a method `map2`, *not* defined via `flatMap`: the desired type signature is  $\text{map2} : F^A \times F^B \Rightarrow (A \times B \Rightarrow C) \Rightarrow F^C$
  - An **applicative functor** has `map2` but is not necessarily a monad

## Defining `map2`, `map3`, etc.

Consider 1, 2, 3, ... commutative and independent “effects”

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<code>for { x1 ← c1 } yield f(x1)</code>	<code>c1.map(f)</code>
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<code>for { x1 ← c1   x2 ← c2 } yield f(x1, x2)</code>	<code>(c1, c2).map2(f)</code>
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<code>for { x ← c1   x2 ← c2   x3 ← c3 } yield f(x1, x2, x3)</code>	<code>(c1, c2, c3).map3(f)</code>
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- Generalize to `mapN` from

$$\text{map}_1 : F^A \Rightarrow (A \Rightarrow Z) \Rightarrow F^Z$$

$$\text{map}_2 : F^A \times F^B \Rightarrow (A \times B \Rightarrow Z) \Rightarrow F^Z$$

$$\text{map}_3 : F^A \times F^B \times F^C \Rightarrow (A \times B \times C \Rightarrow Z) \Rightarrow F^Z$$

## Examples of using `mapN`

- $F^A \equiv Z + A$  where  $Z$  is a monoid: collect all errors
- $F^A = Z + A$ : Create a validated case class out of validated parts
- $F^A \equiv \text{Future}[A]$ : perform several computations concurrently
- $F^A \equiv E \Rightarrow A$ : pass arguments to functions automatically
- $F^A \equiv \text{List}^A$ : transposing a matrix uses `map2`
- “fold fusion”: automatically merge several `fold`s into one (`scala-folds`)

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- Can we avoid having to define `mapn` separately for each  $n$ ?

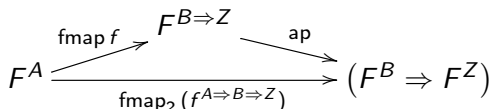
## Deriving the `ap` operation from `map2`

- Use curried arguments,  $\text{fmap}_2 : (A \Rightarrow B \Rightarrow Z) \Rightarrow F^A \Rightarrow F^B \Rightarrow F^Z$
- Set  $A = B \Rightarrow Z$  and apply  $\text{fmap}_2$  to the identity  $\text{id}^{(B \Rightarrow Z) \Rightarrow (B \Rightarrow Z)}$ :  
obtain

$$\text{ap} : F^{B \Rightarrow Z} \Rightarrow F^B \Rightarrow F^Z \equiv \text{fmap}_2 (\text{id})$$

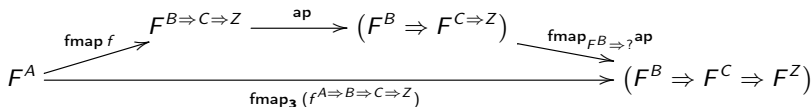
- The functions `fmap2` and `ap` are computationally equivalent:

$$\text{fmap}_2 f^{A \Rightarrow B \Rightarrow Z} = \text{fmap } f \circ \text{ap}$$


$$\begin{array}{ccc} & F^{B \Rightarrow Z} & \\ \text{fmap } f \nearrow & & \searrow \text{ap} \\ F^A & \xrightarrow{\text{fmap}_2 (f^{A \Rightarrow B \Rightarrow Z})} & (F^B \Rightarrow F^Z) \end{array}$$

- The functions `fmap3`, `fmap4` etc. can be defined similarly:

$$\text{fmap}_3 f^{A \Rightarrow B \Rightarrow C \Rightarrow Z} = \text{fmap } f \circ \text{ap} \circ \text{fmap}_{F^B \Rightarrow ?} \text{ap}$$


$$\begin{array}{ccccc} & & F^{B \Rightarrow C \Rightarrow Z} & \xrightarrow{\text{ap}} & (F^B \Rightarrow F^{C \Rightarrow Z}) & \xrightarrow{\text{fmap}_{F^B \Rightarrow ?} \text{ap}} & (F^B \Rightarrow F^C \Rightarrow F^Z) \\ & \text{fmap } f \nearrow & & & & & \\ F^A & \xrightarrow{\text{fmap}_3 (f^{A \Rightarrow B \Rightarrow C \Rightarrow Z})} & & & & & \end{array}$$

## Intuition: the `zip` operation on lists

- Note: Function types  $A \Rightarrow B \Rightarrow C$  and  $A \times B \Rightarrow C$  are equivalent
- Uncurry  $\text{fmap}_2$  to  $\text{fmap2} : (A \times B \Rightarrow C) \Rightarrow F^A \times F^B \Rightarrow F^C$
- Compute  $\text{fmap2}(f)$  with  $f = \text{id}^{A \times B \Rightarrow A \times B}$ , expecting to obtain a simpler natural transformation:

$$\text{zip} : F^A \times F^B \Rightarrow F^{A \times B}$$

- This is quite similar to `zip` for lists:  
`List(1, 2).zip(List(10, 20)) = List((1, 10), (2, 20))`
- The functions `zip` and `fmap2` are computationally equivalent:

$$\begin{array}{ccc} & & F^{A \times B} \\ & \nearrow \text{zip} & \\ F^A \times F^B & \xrightarrow{\quad \quad \quad} & F^C \\ & \searrow \text{fmap2}(f^{A \times B \Rightarrow C}) & \\ & & \end{array}$$

$\text{fmap2}(f^{A \times B \Rightarrow C})$

- The functor  $F$  is “zippable” if such a `zip` exists

## Deriving the `ap` operation from `zip`

- Set  $A \equiv B \Rightarrow C$ , get  $\text{zip}^{[B \Rightarrow C, B]} : F^{B \Rightarrow C} \times F^B \Rightarrow F^{(B \Rightarrow C) \times B}$
- Use `eval` :  $(B \Rightarrow C) \times B \Rightarrow C$  and  $\text{fmap}(\text{eval}) : F^{(B \Rightarrow C) \times B} \Rightarrow F^C$
- Define  $\text{app}^{[B, C]} : F^{B \Rightarrow C} \times F^B \Rightarrow F^C \equiv \text{zip} \circ \text{fmap}(\text{eval})$
- The functions `zip` and `app` are computationally equivalent:
  - ▶ use  $\text{pair} : (A \Rightarrow B \Rightarrow A \times B) = a^A \Rightarrow b^B \Rightarrow a \times b$
  - ▶ use  $\text{fmap}(\text{pair}) \equiv \text{pair}^\uparrow$  on an  $fa^{F^A}$ , get  $(\text{pair}^\uparrow fa) : F^{B \Rightarrow A \times B}$ ; then

$$\text{zip}(fa \times fb) = \text{app}\left((\text{pair}^\uparrow fa) \times fb\right)$$

$$\text{app}^{[B \Rightarrow C, B]} = \text{zip}^{[B \Rightarrow C, B]} \circ \text{fmap}(\text{eval})$$

$$F^{B \Rightarrow C} \times F^B \begin{array}{c} \xrightarrow{\text{zip}} F^{(B \Rightarrow C) \times B} \\ \xrightarrow{\text{app}^{[B \Rightarrow C, B]}} F^C \\ \searrow \text{fmap}(\text{eval}) \end{array}$$

- Rewrite this using curried arguments:  $\text{fzip}^{[A, B]} : F^A \Rightarrow F^B \Rightarrow F^{A \times B}$ ;  $\text{ap}^{[B, C]} : F^{B \Rightarrow C} \Rightarrow F^B \Rightarrow F^C$ ; then  $\text{ap } f = \text{fzip } f \circ \text{fmap}(\text{eval})$ .
- Now  $\text{fzip } p^{F^A} q^{F^B} = \text{ap}(\text{pair}^\uparrow p) q$ , hence we can write as point-free:  $\text{fzip} = \text{pair}^\uparrow \circ \text{ap}$ . With explicit types:  $\text{fzip}^{[A, B]} = \text{pair}^\uparrow \circ \text{ap}^{[B, A \Rightarrow B]}$ .