# Chapter 9: Traversable functors and contrafunctors

Sergei Winitzki

Academy by the Bay

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# Motivation for the traverse operation

- Consider data of type List<sup>A</sup> and processing  $f: A \Rightarrow \mathsf{Future}^B$
- Typically, we want to wait until the entire data set is processed
- What we need is  $List^A \Rightarrow (A \Rightarrow Future^B) \Rightarrow Future^{List^B}$
- Generalize:  $L^A \Rightarrow (A \Rightarrow F^B) \Rightarrow F^{L^B}$  for some type constructors F, L
- This operation is called traverse
  - ▶ How to implement it: for example, a 3-element list is  $A \times A \times A$
  - ▶ Consider  $L^A \equiv A \times A \times A$ , apply map f and get  $F^B \times F^B \times F^B$
  - ▶ We will get  $F^{L^B} \equiv F^{B \times B \times B}$  if we can apply zip as  $F^B \times F^B \Rightarrow F^{B \times B}$
- So we need to assume that F is applicative
- ullet In Scala, we have Future.traverse() that assumes L to be a sequence
  - ▶ This is the iconic example that fixes the requirements
- Questions:
  - Which functors L can have this operation?
  - ► Can we express traverse through a simpler operation?
  - ▶ What are the required laws for traverse?
  - What about contrafunctors or profunctors?

### Deriving the sequence operation

- The type signature of traverse is a complicated "lifting"
  - ▶ A "lifting" is always equivalent to a simpler natural transformation
- To derive it, ask: what is missing from fmap to do the job of traverse?

$$\mathsf{fmap}: (A \Rightarrow F^B) \Rightarrow L^A \Rightarrow L^{F^B}$$

- We need  $F^{L^B}$ , but the traverse operation gives us  $L^{F^B}$  instead
  - What's missing is a natural transformation sequence :  $L^{F^B} \Rightarrow F^{L^B}$
- The functions traverse and sequence are computationally equivalent:

$$\operatorname{trav} f^{A \Rightarrow F^B} = \operatorname{fmap} f \circ \operatorname{seq}$$



Here F is an arbitrary applicative functor

- lacktriangle Keep in mind the example Future.sequence : List Future  $\Rightarrow$  Future List  $\Rightarrow$
- Examples: List;
- Non-traversable:  $L^A \equiv R \Rightarrow A$ ; lazy lists ("infinite streams")
  - ★ Note: We cannot have the opposite transformation  $F^{L^B} \Rightarrow L^{F^B}$

## Polynomial functors are traversable

- Generalize from the example  $L^A \equiv A \times A \times A$  to other polynomials
- Polynomial functors have the form

$$L^{A} \equiv Z \times A \times ... \times A + Y \times A \times ... \times A + ... + Q \times A + P$$

- To implement seq :  $L^{F^B} \Rightarrow F^{L^B}$ , consider monomial  $L^A \equiv Z \times A \times ... \times A$
- We have  $L^{F^B} = Z \times F^B \times ... \times F^B$ ; apply zip and get  $Z \times F^{B \times ... \times B}$
- Lift Z into the functor F using  $Z \Rightarrow F^A \Rightarrow F^{Z \times A}$  (or with F.pure)
- The result is  $F^{Z \times B \times ... \times B} \equiv F^{L^B}$ 
  - For a polynomial  $L^A$ , apply this procedure to each monomial
  - Note that we could apply zip in various different orders
- The traversal order is arbitrary, may be application-specific
- Non-polynomial functors are not traversable (see Bird et al., 2013)
  - ▶ Example:  $L^A \equiv E \Rightarrow A$ ,  $F^A \equiv R \Rightarrow A$ ; can't have seq :  $L^{F^B} \Rightarrow F^{L^B}$
- All polynomial functors are traversable, and usually in several ways
  - ▶ Therefore it is useful to have a type class for traversable functors

### Motivation for the laws of the traverse operation

- The "law of traversals" paper (2012) argues that traverse should "visit each element" of the container  $L^A$  exactly once, and evaluate each corresponding "effect"  $F^B$  exactly once; then they formulate the laws
- To derive the laws, use the "lifting" intuition for traverse,

trav : 
$$(A \Rightarrow F^B) \Rightarrow L^A \Rightarrow F^{L^B}$$

Look for "identity" and "composition" laws:

- "Identity" as pure :  $A \Rightarrow F^A$  must be lifted to pure :  $L^A \Rightarrow F^{L^A}$
- ② "Identity" as  $id^{A\Rightarrow A}$  with  $F^A\equiv A$  (identity functor) lifted to  $id^{L^A\Rightarrow L^A}$
- **③** "Compose"  $f: A \Rightarrow F^B$  and  $g: B \Rightarrow G^C$  to get  $h: A \Rightarrow F^{G^C}$ , where F, G are applicative; a traversal with h maps  $L^A$  to  $F^{G^{L^C}}$  and must be somehow equal to the composition of traversals with f and then with g

#### Questions:

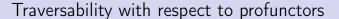
- Are the laws for the sequence operation simpler?
- Are all these laws independent?
- What functors L satisfy these laws for all applicative functors F?

### Formulation of the laws for traverse

# Derivation of the laws for sequence

### Constructions of traversable functors

# Traversable profunctors



"Folding" a traversable functor into a monoid

#### Exercises

Show that any traversable functor L admits a method

consume : 
$$(L^A \Rightarrow B) \Rightarrow L^{F^A} \Rightarrow F^B$$

for any applicative functor F. Show that traverse and consume are equivalent.