Chapter 1: Values, types, expressions, functions

Sergei Winitzki

Academy by the Bay

October 21, 2017

What is "Functional Programming"?

Functional programming...

- treats programs as mathematical expressions
- uses age-old mathematical intuition to design software
- is natural and effective in OCaml, Haskell, F#, Scala, Swift, etc.
- ... but not in C, C++, JavaScript, Java (before version 8), or Python!

We will be using Scala for all examples...

- ...but the same material looks very similar in the other languages

Examples of functional programs

Compute the factorial of a natural number:

$$n! = \prod_{k=1}^{n} k$$

Check whether a natural number is a prime:

$$prime(n) = \forall i \text{ such that } 2 \leq i < n \ : \ n \mod i \neq 0$$

Count how many even numbers there are in a given set S of integers:

$$count\_even = \sum_{k \in S} is\_even(k)$$

$$\text{where we defined } is\_even(k) = \begin{cases} 1 & \text{if } k \mod 2 = 0 \\ 0 & \text{otherwise} \end{cases}$$

- Scala programs implementing this are similar to the math
  - Programs in Java or Python are *not* similar to the math!

What exactly is "math-like" in that code?

- The code represents a *mathematical expression* that we want to compute
- Each value is immutable and has a fixed *type* (integer, set, function, etc.)
- The code can define new names or new functions *within an expression*
- There is no "goto" or "repeat"
  - Have you ever seen a math book that says,
    "now change k to $k-1$ and repeat Equation 123 until $k=0$" ?

Adapting math to programming I

Values, expressions, and types

In math, there are two kinds of "variables":

- named constant values,

```
val a = 123
```

- function arguments,

```
def f(x: Int, y: Int) = x + y - 1
```

Math texts never try to "modify" a value, so...

- "`val`"s and function arguments are immutable

Each value has a fixed type (`Int`, `Boolean`, `Set[Int]`, etc.)

- Type represents the set of possible values of the function argument
- Type is automatically assigned to named constants

Adapting math to programming II

Anonymous functions vs. named functions

There are two ways of defining a function in Scala:

- **named** function – using `def` with a name

```
def merge(x: Int, y: Int): Int = { x + y - 1 }
```

- **anonymous** function – in math notation, $x \mapsto f(x)$:

```
(x: Int, y: Int) => x + y - 1
```

Anonymous functions are *values*:

- they are immutable and have a fixed type, e.g. `(Int, Int) => Int`
- they can be assigned a name and used later in an expression:

```
val double: (Int => Int) = { x => x * 2 }; double(y)
```

- or they can be used directly as arguments of other functions:

```
(1 to 100).map(x => x * x)
```

Some collections in Scala

What is (1 to 100)? What type does it have?

```
scala> (1 to 100)
res0: scala.collection.immutable.Range.Inclusive = Range 1 to 100
```

- Sequence, `Seq` and its subtypes: `List`, `IndexedSeq` etc.

```
val a: Seq[Int] = Seq(2, 4, 6, 8)
val b = a(0) // now b: Int == 2
```

- Set: `Set`

```
val b: Set[String] = Set("x", "y", "z")
```

- Dictionary: `Map`

```
val c: Map[String, Int] = Map("x" -> 5, "y" -> 3, "z" -> 1)
```

Note the **parameterized** types `Seq[Int]` and `Map[String, Int]`

Adapting math to programming III

Encoding sums, products, quantifiers using anonymous functions

The methods map, filter, forall, exists are defined on all collections
The methods sum, product are defined on collections of *numbers*

Mathematical notation	*Scala code*
$x \mapsto \sqrt{x^2 + 1}$	`x => math.sqrt(x * x + 1)`
$[f(1), \ ..., \ f(k)]$	`(1 to n).map(k => f(k))`
$\sum_{k=1}^{n} k^2$	`(1 to n).map(k => k*k).sum`
$\prod_{k=1}^{n} f(k)$	`(1 to n).map(f).product`
$\forall k$ such that $1 \leq k \leq n : p(k)$ holds	`(1 to n).forall(k => p(k))`
$\exists k, \ 1 \leq k \leq n$ such that $p(k)$ holds	`(1 to n).exists(k => p(k))`
$\sum_{k \in S,\, p(k) \text{ holds}} f(k)$	`s.filter(p).map(f).sum`

Adapting math to programming IV

Higher-order functions

Derivatives and integrals could be implemented as functions:

```
def deriv(f: Double => Double): (Double => Double) = { ??? }
def integ(f: Double => Double, range: (Double, Double)): Double = ???
```

Higher-order functions take function arguments and/or return functions

- Many computations with sequences, sets, dictionaries can be done using higher-order functions, *without loops*, concisely and error-free
- The Scala standard library has many more higher-order methods for collections
  - `reduce`, `zip`, `flatMap`, `foldLeft`, `foldRight`, `scan`, `collect`, `groupBy`, ...
- Write code by formulating the problem as a mathematical expression