Elm-style Functional Reactive Programming demystified

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SF Types, Theorems, and Programming Languages

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Part 1. Functional reactive programming in `Elm`

FRP has little to do with...

- multithreading, message-passing concurrency, "actors"
- distributed computing on massively parallel, load-balanced clusters
- map/reduce, the "reactive manifesto"

FRP means...

- **pure functions** using **temporal types** as primitives
  - (temporal type $\approx$ lazy stream of events)

FRP is probably most useful for:

- GUI programming

`Elm` is...

- a viable implementation of FRP geared for Web apps

Transformational vs. reactive programs

Transformational programs	**Reactive** programs
example: `pdflatex elm_talk.tex`	**example**: any GUI program, OS
start, run, then stop	keep running indefinitely
read some input, write some output	wait for signals, send messages
execution: sequential, parallel	"main run loop" + concurrency
difficulty: algorithms	signal/response sequences
specification: classical logic?	classical temporal logic?
verification: proof of correctness?	model checking?
synthesis: extract code from proof?	temporal logic synthesis?
type theory: intuitionistic logic	intuitionistic *temporal* logic

Difficulties in reactive programming

Usually, reactive programs are written imperatively...

- Input signals may come at unpredictable times
  - Imperative updates are difficult to keep in the correct order
  - Flow of events becomes difficult to understand
- Asynchronous (out-of-order) callback logic becomes opaque
  - "callback hell": deeply nested callbacks, all mutating data
- Inverted control ("the system will call you") obscures the flow of data
- Some concurrency is usually required (e.g. background tasks)
  - Explicit multithreaded code is hard to write and debug

Motivation for FRP

- Reactive programs work on **infinite sequences** of input/output values
- Main idea: make infinite sequences implicit, as a new "**temporal**" type
  - (Elm) `Signal` α — an infinite sequence of values of type α
  - alternatively, a value of type α that "changes with time"
- Reactive programs are **pure functions**
  - a GUI is a pure function of type `Signal Inputs` $\to$ `Signal View`
  - a Web server is a pure function `Signal Request` $\to$ `Signal Response`
  - all mutation is **implicit** in `Signal` α; our code is 100% immutable
    - ⋆ instead of updating an `x:Int`, we define a value of type `Signal Int`
  - asynchronous behavior is **implicit**: our code has no callbacks
  - concurrency / parallelism is **implicit**
    - ⋆ the FRP runtime will provide the required scheduling of events

Czaplicki's original `Elm` 2012 in a nutshell

- `Elm` is a pure polymorphic λ-calculus with products and sums
- **Temporal type** $\Sigma\alpha$ — a lazy sequence of values of type α
- Temporal **combinators** in core `Elm`:

  `constant:` $\alpha \to \Sigma\alpha$
  `map2:` $(\alpha \to \beta \to \gamma) \to \Sigma\alpha \to \Sigma\beta \to \Sigma\gamma$
  `foldp:` $(\alpha \to \beta \to \beta) \to \beta \to \Sigma\alpha \to \Sigma\beta$
  `async:` $\Sigma\alpha \to \Sigma\alpha$

- **No nested** temporal types: `constant (constant x)` is ill-typed!
- Domain-specific primitive types: `Bool`, `Int`, `Float`, `String`, `View`
- Standard library with data structures, HTML, HTTP, JSON, ...
  - ...and signals `Time.every`, `Mouse.position`, `Window.dimensions`, ...
  - ...and some utility functions: `map`, `merge`, `drop`, ...

Details: `Elm` type judgments [Czaplicki 2012]

- Polymorphically typed λ-calculus (also with temporal types)

$$\frac{\Gamma, (x : \alpha) \vdash e : \beta}{\Gamma \vdash (\lambda x.e) : \alpha \to \beta}\ \textsc{Lambda} \qquad \frac{\Gamma \vdash e_1 : \alpha \to \beta \quad \Gamma \vdash e_2 : \alpha}{\Gamma \vdash (e_1\, e_2) : \beta}\ \textsc{Apply}$$

- Temporal types are denoted by $\Sigma\tau$
  - In these rules, type variables α, β, γ **cannot** involve Σ:

$$\frac{\Gamma \vdash e : \alpha}{\Gamma \vdash (\text{constant}\ e) : \Sigma\alpha}\ \textsc{Const}$$

$$\frac{\Gamma \vdash m : \alpha \to \beta \to \gamma \quad \Gamma \vdash p : \Sigma\alpha \quad \Gamma \vdash q : \Sigma\beta}{\Gamma \vdash (\text{map2}\ m\ p\ q) : \Sigma\gamma}\ \textsc{Map2}$$

$$\frac{\Gamma \vdash u : \alpha \to \beta \to \beta \quad \Gamma \vdash e : \beta \quad \Gamma \vdash q : \Sigma\alpha}{\Gamma \vdash (\text{foldp}\ u\ e\ q) : \Sigma\beta}\ \textsc{FoldP}$$

- A value of type $\Sigma\Sigma\alpha$ is impossible in a well-typed expression!

`Elm` operational semantics 1: Current values

- Non-temporal expressions are evaluated **eagerly** in pure λ-calculus
- Temporal expressions are built from **input** signals and combinators
  - It is not possible to "consume" a signal $(\Sigma\alpha \to \beta)$!
- Every temporal expression has a **current value** denoted by $e^{[c]}$

$$\frac{\Gamma \vdash e : \Sigma\alpha \qquad \Gamma \vdash c : \alpha}{\Gamma \vdash e^{[c]} : \Sigma\alpha}\ \textsc{CurVal}$$

- Every predefined **input** signal $i : \Sigma\alpha, i \in \mathcal{I}$ has an initial value: $i^{[a]}$
- **Initial** current values for all expressions are derived:

$$\frac{\Gamma \vdash (\text{constant } c) : \Sigma\alpha}{\Gamma \vdash (\text{constant } c)^{[c]} : \Sigma\alpha}\ \textsc{ConstInit}$$

$$\frac{\Gamma \vdash \left(\text{map2 } m\ p^{[a]}\ q^{[b]}\right) : \Sigma\gamma}{\Gamma \vdash (\text{map2 } m\ p\ q)^{[m\ a\ b]} : \Sigma\gamma}\ \textsc{Map2Init}$$

$$\frac{\Gamma \vdash (\text{foldp } u\, e\, q) : \Sigma\beta}{\Gamma \vdash (\text{foldp } u\, e\, q)^{[e]} : \Sigma\beta}\ \textsc{FoldPInit}$$

$\texttt{Elm}$ operational semantics 2: Update steps

- Update steps happen only to **input signals** $s \in \mathcal{I}$ and **one at a time**
- Update steps $\mathbf{U}_{s\leftarrow a}\{...\}$ are applied to the **whole program** at once:

$$\frac{\Gamma \vdash s : \Sigma\alpha \quad s \in \mathcal{I} \quad \Gamma \vdash a : \alpha \quad \Gamma \vdash e^{[c]} : \Sigma\beta \quad \Gamma \vdash e'^{[c']} : \Sigma\beta}{\Gamma \vdash \mathbf{U}_{s\leftarrow a}\left\{e^{[c]}\right\} \Rightarrow e'^{[c']}}$$

- An update step on s will leave all other **input** signals unchanged:

$$\forall s \neq s' \in \mathcal{I}: \qquad \mathbf{U}_{s\leftarrow b}\left\{s^{[a]}\right\} \Rightarrow s^{[b]} \qquad \mathbf{U}_{s\leftarrow b}\left\{s'^{[c]}\right\} \Rightarrow s'^{[c]}$$

- Efficient implementation:
  - The instances of input signals within expressions are not duplicated
  - Unchanged current values are cached and not recomputed

`Elm` operational semantics 3: Updating combinators

- Operational semantics does not reduce temporal expressions
  - The whole program **remains** a static temporal expression tree
  - Only the current values are updated in all subexpressions

$$\mathbf{U}_{s\leftarrow a}\left\{(\text{constant } c)^{[c]}\right\} \Rightarrow (\text{constant } c)^{[c]} \qquad \textsc{ConstUpd}$$

$$\mathbf{U}_{s\leftarrow a}\left\{\text{map2 } m\ p\ q\right\}$$

$$\Rightarrow \left(\text{map2 } m\ \mathbf{U}_{s\leftarrow a}\left\{p\right\}^{[b]}\ \mathbf{U}_{s\leftarrow a}\left\{q\right\}^{[c]}\right)^{[m\ b\ c]} \qquad \textsc{Map2Upd}$$

$$\mathbf{U}_{s\leftarrow a}\left\{(\text{foldp}\ u\ e\ q)^{[b]}\right\} \Rightarrow \left(\text{foldp}\ u\ e\ \mathbf{U}_{s\leftarrow a}\left\{q\right\}^{[c]}\right)^{[u\ c\ b]} \qquad \textsc{FoldPUpd}$$

- All computations during an update step are **synchronous**
  - The expression $\mathbf{U}_{s\leftarrow b}\left\{e^{[c]}\right\}$ is reduced only after all subexpressions of e

GUI building: "Hello, world" in Elm

- The value called main will be visualized by the runtime

```
import Graphics.Element (..)
import Text (..)
import Signal (..)

text : Element
text = plainText "Hello, World!"

main : Signal Element
main = constant text
```

- Try Elm online at http://elm-lang.org/try

Example of using `foldp`

- Specification:
  - *I work only after the boss comes by and unless the phone rings*
- Implementation:

```
after_unless : (Bool, Bool) -> Bool -> Bool
after_unless (b,r) w = (w || b) && not r

boss : Signal Bool
phone: Signal Bool

i_work : Signal Bool
i_work = foldp after_unless false (boss, phone)
```

- Demo

Typical GUI boilerplate in Elm

- A state machine with stepwise update:

```
update : Command → State → State
```

- A rendering function:

```
draw : State → View
```

- A manager that merges the required input signals into one:
  - may use Mouse, Keyboard, Time, HTML stuff, etc.

```
merge_inputs : Signal Command
```

- Main boilerplate:

```
init_state : State
main : Signal View
main = map draw (foldp update init_state merge_inputs)
```

Asynchrony and concurrency in `Elm`

- Long-running computations will delay signal updates!
  - Example: `foldp f e s` where `f`$:\alpha \to \beta \to \beta$ takes a long time
- `Elm`'s solution is to use `async` $: \Sigma\alpha \to \Sigma\alpha$
- Operational semantics: (i is a **new** input signal for each `async`)

$$\frac{\Gamma \vdash e^{[c]} : \Sigma\alpha}{\Gamma, (i : \Sigma\alpha) \vdash (\text{async}_i\ e)^{[c]} : \Sigma\alpha}\ \textsc{AsyncInit}$$

$$\mathbf{U}_{s\leftarrow a}\left\{(\text{async}_i\ e)^{[c]}\right\} \Rightarrow \mathbf{U}^{\dagger}_{i\leftarrow c'}\left(\text{async}_i\ \mathbf{U}^{\dagger}_{s\leftarrow a}\left\{e\right\}^{[c']}\right)^{[c]}\ \textsc{AsyncSched}$$

$$\mathbf{U}_{i\leftarrow c'}\left\{(\text{async}_i\ e)^{[c]}\right\} \Rightarrow (\text{async}_i\ e)^{[c']}\ \textsc{AsyncUpd}$$

- The update computation $\mathbf{U}^{\dagger}_{s\leftarrow a}\left\{e\right\}$ runs on another thread...
  - ...while the current value c remains unchanged
  - Another update $\mathbf{U}^{\dagger}_{i\leftarrow c'}$ is **scheduled** but not yet triggered
  - When c' is ready, $\mathbf{U}_{i\leftarrow c'}\left\{...\right\}$ runs and sets the current value to c'

Example of using async

- UI that shows results of some long computations:

```
draw : Int -> Int -> View
draw x y = ...

s : Signal Int -- input values

fSlow : Int -> Int
res1 = async (map fSlow s)
fFast : Int -> Int
res2 = map fFast s

main : Signal View = map2 draw res1 res2
```

- Both results are updated as soon as they are computed

Some limitations of `Elm`-style FRP

- No higher-order signals: $\Sigma(\Sigma\alpha)$ is disallowed by the type system
- No distinction between continuous time and discrete time
- The signal processing logic is fully specified statically
- No constructors for user-defined signals
- No recursion possible in signal definition!
- Incomplete semantics for `async` : $\Sigma\alpha \to \Sigma\alpha$
  - Example: `async (map f s)` where `f` takes a long time
  - The initial value of this signal will not be available at initial time!
  - Need `async'` : $\alpha \to \Sigma\alpha \to \Sigma\alpha$ to specify initial value?
- No full concurrency (e.g., "dining philosophers")

`Elm` cannot simulate "dining philosophers"

- A philosopher thinks for a random time, then eats for a random time
  - Can a signal value `p : Signal Unit` update itself at random times?
- No! There is no way to delay the update times of a signal **at runtime**
- `Time.delay: Int`$\rightarrow \Sigma\alpha \rightarrow \Sigma\alpha$ cannot use a time-varying delay value
- `Time.every: Int`$\rightarrow \Sigma$`Int` also requires a fixed delay value
- Cannot lift `Time.every` into Σ`Int`$\rightarrow \Sigma\Sigma$`Int` to achieve variable delay

The JavaScript backend for `Elm` (2015)

Features:

- Good support for HTML/CSS, HTTP requests, JSON
- Good performance of caching HTML views
- Support for Canvas and HTML-free UI building

Limitations:

- No implementation for `async` (JavaScript lacks concurrency)
- The lack of recursive signals is compensated by *ad hoc* primitives
- Ordinary recursion may generate invalid JavaScript!

`Elm`-style FRP: the good parts

- Transparent, declarative modeling of data through ADTs
- Immutable and safe data structures (`Array`, `Dict`, ...)
- No runtime errors or exceptions!
- Space/time leaks are impossible!
- Language is Haskell-like but simpler for beginners
- Full type inference
- Easy deployment and interop in Web applications

Some conservative extensions of Elm

- Fix initial value semantics for `async'` : $\alpha \to \Sigma\alpha \to \Sigma\alpha$
- Allow recursive definitions for signals
  - generate updates as `s0`, `f(s0)`, `f(f(s0))`, ... are being computed:

    ```
    s = async' s0 (map f s)
    ```
- Add monadic signal combinator, `bind` : $(\alpha \to \Sigma\beta) \to \Sigma\alpha \to \Sigma\beta$
  - use input signals from dynamically created UI:

    ```
    viewS = map draw stateS
    stateS = foldp update_on_click (bind get_clicks viewS)
    ```
- Allow user-defined signals constructed from asynchronous APIs
  - Generate signal updates whenever callback is called:

    `type` $\mathbf{C}\alpha\beta = \alpha \to (\beta \to \bot) \to \bot$

    `chain :` $\mathbf{C}\alpha\beta \to \Sigma\alpha \to \Sigma\beta$

    `some_async_api :` $\mathbf{C}\alpha\beta$

    `values_of_b = chain some_async_api values_of_a`

Part 2. Temporal logic and FRP

- Reminder (Curry-Howard): logical expressions will be types
  - ...and the axioms will be primitive terms
- We only need to control the **order** of events: no "hard real-time"
- How to understand temporal logic:
  - classical propositional logic ≈ Boolean arithmetic
  - intuitionistic propositional logic ≈ same but without **true** / **false** dichotomy
  - (linear-time) temporal logic LTL≈ Boolean arithmetic for *infinite sequences*
  - intuitionistic temporal logic ITL≈ same but without **true** / **false** dichotomy
- In other words:
  - an ITL type represents a **single infinite sequence** of values

Boolean arithmetic: notation

- Classical propositional (Boolean) logic: T, F, $a \vee b$, $a \wedge b$, $\neg a$, $a \rightarrow b$
- A notation better adapted to school-level arithmetic: 1, 0, $a + b$, ab, a'
- The only "new rule" is $1 + 1 = 1$
- Define $a \rightarrow b = a' + b$
- Some identities:

$$
\begin{gathered}
0a = 0, \quad 1a = a, \quad a + 0 = a, \quad a + 1 = 1, \\
a + a = a, \quad aa = a, \quad a + a' = 1, \quad aa' = 0, \\
(a + b)' = a'b', \quad (ab)' = a' + b', \quad (a')' = a \\
a\,(b + c) = ab + ac, \quad (a + b)\,(a + c) = a + bc
\end{gathered}
$$

Boolean arithmetic: example

Of the three suspects A, B, C, only one is guilty of a crime.
Suspect A says: "B did it". Suspect B says: "C is innocent."
The guilty one is lying, the innocent ones tell the truth.

$$\phi = \left(ab'c' + a'bc' + a'b'c\right)\left(a'b + ab'\right)\left(b'c' + bc\right)$$

Simplify: expand the brackets, omit aa', bb', cc', replace $aa = a$ etc.:

$$\phi = ab'c' + 0 + 0 = ab'c'$$

The guilty one is A.

Propositional linear-time temporal logic (LTL)

- We work with *infinite boolean sequences* ("linear time")
  Boolean operations:

  $$a = [a_0, a_1, a_2, ...] \, ; \quad b = [b_0, b_1, b_2, ...] \, ;$$
  $$a + b = [a_0 + b_0, a_1 + b_1, ...] \, ; \; a' = \left[a'_0, a'_1, ...\right] \, ; \; ab = [a_0 b_0, a_1 b_1, ...]$$

  Temporal operations:

  $$\begin{aligned} \text{(Next)} \quad & \mathbf{N}a = [a_1, a_2, ...] \\ \text{(Sometimes)} \quad & \mathbf{F}a = [a_0 + a_1 + a_2 + ..., \; a_1 + a_2 + ..., \; ...] \\ \text{(Always)} \quad & \mathbf{G}a = [a_0 a_1 a_2 a_3 ..., \; a_1 a_2 a_3 ..., \; a_2 a_3 ..., \; ...] \end{aligned}$$

  Other notation (from modal logic):

  $$\mathbf{N}a \equiv \bigcirc a; \; \mathbf{F}a \equiv \Diamond a; \; \mathbf{G}a \equiv \Box a$$

- Weak Until: $p\mathbf{U}q =$ "p holds from now on until q first becomes true"

  $$p\mathbf{U}q = q + p\mathbf{N}\left(q + p\mathbf{N}\left(q + ...\right)\right)$$

Temporal logic redux

Designers of FRP languages must face some choices:

- LTL as type theory: do we use $\mathbf{N}\alpha$, $\mathbf{F}\alpha$, $\mathbf{G}\alpha$ as new types?
- Are they to be functors, monads, ...?
- Which temporal axioms to use as language primitives?
- What is the operational semantics? (I.e., how to compile this?)

A sophisticated example: [Krishnaswamy 2013]

- uses full LTL with higher-order temporal types and fixpoints
- uses linear types to control space/time leaks

Interpreting values typed by LTL

- What does it mean to have a value x of type, say, $\mathbf{G}(\alpha \to \alpha \mathbf{U} \beta)$??
  - $x : \mathbf{N}\alpha$ means that $x : \alpha$ will be available *only* at the *next* time tick (x is a **deferred value** of type α)
  - $x : \mathbf{F}\alpha$ means that $x : \alpha$ will be available at *some* future tick(s) (x is an **event** of type α)
  - $x : \mathbf{G}\alpha$ means that a (different) value $x : \alpha$ is available at *every* tick (x is an **infinite stream** of type α)
  - $x : \alpha \mathbf{U} \beta$ means a **finite stream** of α that may end with a β
- Some temporal **axioms** of intuitionistic LTL:

$$
\begin{aligned}
\text{(deferred apply)} \quad & \mathbf{N}\,(\alpha \to \beta) \to (\mathbf{N}\alpha \to \mathbf{N}\beta)\,; \\
\text{(streamed apply)} \quad & \mathbf{G}\,(\alpha \to \beta) \to (\mathbf{G}\alpha \to \mathbf{G}\beta)\,; \\
\text{(generate a stream)} \quad & \mathbf{G}\,(\alpha \to \mathbf{N}\alpha) \to (\alpha \to \mathbf{G}\alpha)\,; \\
\text{(read infinite stream)} \quad & \mathbf{G}\alpha \to \alpha \mathbf{N}(\mathbf{G}\alpha) \\
\text{(read finite stream)} \quad & \alpha \mathbf{U} \beta \to \beta + \alpha \mathbf{N}(\alpha \mathbf{U} \beta)
\end{aligned}
$$

`Elm` as an FRP language

- λ-calculus with type $\mathbf{G}\alpha$, primitives `map2`, `foldp`, `async`

$$
\begin{aligned}
\texttt{map2} &: (\alpha \to \beta \to \gamma) \to \mathbf{G}\alpha \to \mathbf{G}\beta \to \mathbf{G}\gamma \\
\texttt{foldp} &: (\alpha \to \beta \to \beta) \to \beta \to \mathbf{G}\alpha \to \mathbf{G}\beta \\
\texttt{async} &: \mathbf{G}\alpha \to \mathbf{G}\alpha
\end{aligned}
$$

- (`map2` makes **G** an applicative functor)
- `async` is a special *scheduling instruction*
- Limitations:
  - Cannot have a type $\mathbf{G}(\mathbf{G}\alpha)$, also not using **N** or **F**
  - Cannot construct temporal values by hand
  - This language is an *incomplete* Curry-Howard image of LTL!

Conclusions

- There are some languages that implement FRP in various *ad hoc* ways
- The ideal is not (yet) reached
- `Elm`-style FRP is a promising step in the right direction

Abstract

In my day job, most bugs come from implementing reactive programs imperatively. FRP is a declarative approach that promises to solve these problems.

FRP can be defined as a λ-calculus that admits temporal types, i.e. types given by a propositional intuitionistic linear-time temporal logic (LTL). Although the `Elm` language uses only a subset of LTL, it achieves high expressivity for GUI programming. I will formally define the operational semantics of `Elm`. I discuss the current limitations of `Elm` and outline possible extensions. I also review the connections between temporal logic, FRP, and `Elm`.

My talk will be understandable to anyone familiar with Curry-Howard and functional programming. The first part of the talk is a self-contained presentation of `Elm` that does not rely on temporal logic or Curry-Howard. The second part of the talk will explain the basic intuitions behind temporal logic and its connection with FRP.

Suggested reading

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