

Functional Reactive Programming and Elm

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Part 1. Functional reactive programming and Elm

FRP has little to do with...

- multithreading, message-passing concurrency, “actors”
- distributed computing on massively parallel, load-balanced clusters
- map/reduce, “reactive extensions”, the “reactive manifesto”

FRP means (in my definition)...

- **pure functions** using **temporal types** as primitives
 - ▶ (temporal type \approx lazy stream of values)

FRP is probably most useful for:

- GUI and event-driven programming

Elm is...

- a viable implementation of FRP geared for Web GUI apps

Difficulties in reactive programming

Imperative implementation is one problem among many...

- Input signals may come at unpredictable times
 - ▶ Imperative updates are difficult to keep in the correct order
 - ▶ Flow of events becomes difficult to understand
- Asynchronous (out-of-order) callback logic becomes opaque
 - ▶ “callback hell”: deeply nested callbacks, all mutating data
- Inverted control (“the system will call you”) obscures the flow of data
- Some concurrency is usually required (e.g. background tasks)
 - ▶ Explicit multithreaded code is hard to write and debug

FRP basics

- Reactive programs work on **infinite streams** of input/output values
- Main idea: make streams **implicit**, as a new “temporal” type
 - ▶ $\Sigma\alpha$ — an infinite stream of values of type α
 - ▶ alternatively, $\Sigma\alpha$ is a value of type α that “changes with time”
- Reactive programs are viewed as **pure functions**
 - ▶ a GUI is a pure function of type $\Sigma \text{Inputs} \rightarrow \Sigma \text{View}$
 - ▶ a Web server is a pure function $\Sigma \text{Request} \rightarrow \Sigma \text{Response}$
 - ▶ all mutation is **implicit** in the program
 - ★ instead of updating an $x:\text{Int}$, we define a value of type ΣInt
 - ★ our code is 100% immutable, no side effects, no IO monads
 - ▶ asynchronous behavior is **implicit**: our code has no callbacks
 - ▶ concurrency / parallelism is **implicit**
 - ★ the FRP runtime will provide the required scheduling of events

Czaplicki's original Elm 2012 in a nutshell

- Elm is a pure polymorphic λ -calculus with products and sums
- **Temporal type** $\Sigma\alpha$ — a lazy sequence of values of type α
- Temporal **combinators** in core Elm:

constant: $\alpha \rightarrow \Sigma\alpha$

map2: $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \Sigma\alpha \rightarrow \Sigma\beta \rightarrow \Sigma\gamma$

scan: $(\alpha \rightarrow \beta \rightarrow \beta) \rightarrow \beta \rightarrow \Sigma\alpha \rightarrow \Sigma\beta$

drop: $(\alpha \rightarrow \text{Bool}) \rightarrow \Sigma\alpha \rightarrow \Sigma\alpha$

async: $\Sigma\alpha \rightarrow \Sigma\alpha$

- **No nested** temporal types: constant (constant x) is ill-typed!
- Domain-specific primitive types: Bool, Int, Float, String, View
- Standard library with data structures, HTML, HTTP, JSON, ...
 - ▶ ...and signals Time.every, Mouse.position, Window.dimensions, ...
 - ▶ ...and some utility functions: map, merge, drop, ...

Details: Elm type judgments [Czaplicki 2012]

- Polymorphically typed λ -calculus (also with temporal types)

$$\frac{\Gamma, (x : \alpha) \vdash e : \beta}{\Gamma \vdash (\lambda x. e) : \alpha \rightarrow \beta} \text{Lambda} \quad \frac{\Gamma \vdash e_1 : \alpha \rightarrow \beta \quad \Gamma \vdash e_2 : \alpha}{\Gamma \vdash (e_1 \ e_2) : \beta} \text{Apply}$$

- Temporal types are denoted by $\Sigma\tau$
 - In these rules, type variables α, β, γ **cannot** involve Σ :

$$\frac{\Gamma \vdash e : \alpha}{\Gamma \vdash (\text{constant } e) : \Sigma\alpha} \text{Const}$$
$$\frac{\Gamma \vdash m : \alpha \rightarrow \beta \rightarrow \gamma \quad \Gamma \vdash p : \Sigma\alpha \quad \Gamma \vdash q : \Sigma\beta}{\Gamma \vdash (\text{map2 } m \ p \ q) : \Sigma\gamma} \text{Map2}$$
$$\frac{\Gamma \vdash u : \alpha \rightarrow \beta \rightarrow \beta \quad \Gamma \vdash e : \beta \quad \Gamma \vdash q : \Sigma\alpha}{\Gamma \vdash (\text{foldp } u \ e \ q) : \Sigma\beta} \text{FoldP}$$

- A value of type $\Sigma\Sigma\alpha$ is impossible in a well-typed expression!

Elm operational semantics 1: Current values

- Temporal expressions are built from **input** signals and combinators
 - It is not possible to “consume” a signal ($\Sigma\alpha \rightarrow \beta$)!
- Every temporal expression has a **current value** denoted by $e^{[c]}$

$$\frac{\Gamma \vdash e : \Sigma\alpha \quad \Gamma \vdash c : \alpha}{\Gamma \vdash e^{[c]} : \Sigma\alpha} \text{CurVal}$$

- Every predefined **input** signal $i : \Sigma\alpha, i \in \mathcal{I}$ has an initial value: $i^{[a]}$
- Initial** current values for all expressions are derived:

$$\frac{\Gamma \vdash (\text{constant } c) : \Sigma\alpha}{\Gamma \vdash (\text{constant } c)^{[c]} : \Sigma\alpha} \text{ConstInit}$$

$$\frac{\Gamma \vdash (\text{map2 } m \ p^{[a]} \ q^{[b]}) : \Sigma\gamma}{\Gamma \vdash (\text{map2 } m \ p \ q)^{[m \ a \ b]} : \Sigma\gamma} \text{Map2Init}$$

$$\frac{\Gamma \vdash (\text{foldp } u \ e \ q) : \Sigma\beta}{\Gamma \vdash (\text{foldp } u \ e \ q)^{[a]} : \Sigma\beta} \text{FoldPInit}$$

Elm operational semantics 2: Update steps

- Update steps happen only to **input signals** $s \in \mathcal{I}$ and **one at a time**
- Update steps $\mathbf{U}_{s \leftarrow a} \{ \dots \}$ are applied to the **whole program** at once:

$$\frac{\Gamma \vdash s : \Sigma \alpha \quad s \in \mathcal{I} \quad \Gamma \vdash a : \alpha \quad \Gamma \vdash e^{[c]} : \Sigma \beta \quad \Gamma \vdash c' : \beta}{\Gamma \vdash \mathbf{U}_{s \leftarrow a} \{ e^{[c]} \} \Rightarrow e^{[c]}}$$

- An update step on s will leave all other **input** signals unchanged:

$$\forall s \neq s' \in \mathcal{I} : \quad \mathbf{U}_{s \leftarrow b} \{ s^{[a]} \} \Rightarrow s^{[b]} \quad \mathbf{U}_{s \leftarrow b} \{ s'^{[c]} \} \Rightarrow s'^{[c]}$$

- Efficient implementation:
 - ▶ The instances of input signals within expressions are not duplicated
 - ▶ Unchanged current values are cached and not recomputed

Elm operational semantics 3: Updating combinators

- Operational semantics does not reduce temporal expressions:
 - The whole program **remains** a static temporal expression tree
 - Only the current values are updated in all subexpressions

$$\mathbf{U}_{s \leftarrow a} \left\{ (\text{constant } c)^{[c]} \right\} \Rightarrow (\text{constant } c)^{[c]} \quad \text{ConstUpd}$$

$$\begin{aligned} \mathbf{U}_{s \leftarrow a} \{ \text{map2 } m \ p \ q \} \\ \Rightarrow \left(\text{map2 } m \ \mathbf{U}_{s \leftarrow a} \{ p \}^{[v]} \ \mathbf{U}_{s \leftarrow a} \{ q \}^{[w]} \right)^{[m \ v \ w]} \end{aligned} \quad \text{Map2Upd}$$

$$\mathbf{U}_{s \leftarrow a} \left\{ (\text{foldp } u \ e \ q)^{[b]} \right\} \Rightarrow \left(\text{foldp } u \ e \ \mathbf{U}_{s \leftarrow a} \{ q \}^{[c]} \right)^{[u \ c \ b]} \quad \text{FoldPUpd}$$

- All computations during an update step are **synchronous**
 - The expression $\mathbf{U}_{s \leftarrow b} \{ e^{[c]} \}$ is reduced **after** all subexpressions of e
 - Current values are non-temporal and are evaluated **eagerly**

GUI building: “Hello, world” in Elm

- The value called `main` will be visualized by the runtime

```
import Graphics.Element (..)
import Text (..)
import Signal (..)

text : Element
text = plainText "Hello, World!"

main : Signal Element
main = constant text
```

- Try Elm online at <http://elm-lang.org/try>

Example of using foldp

- Specification:

- ▶ *I work only after the boss comes by and unless the phone rings*

- Implementation:

```
after_unless : (Bool, Bool) -> Bool -> Bool
```

```
after_unless (b,p) w = (w || b) && not p
```

```
boss_and_phone : Signal (Bool,Bool)
```

```
i_work : Signal Bool
```

```
i_work = foldp after_unless False (boss_and_phone)
```

- Demo ([boss_phone_work.elm](#))

Typical GUI boilerplate in Elm

- A state machine with stepwise update:

`update : Command → State → State`

- A rendering function (View is either Element or Html):

`draw : State → View`

- A manager that merges the required input signals into one:

- ▶ may use Mouse, Keyboard, Time, HTML stuff, etc.

`merge_inputs : Signal Command`

- Main boilerplate:

`init_state : State`

`main : Signal View`

`main = map draw (foldp update init_state merge_inputs)`

Asynchrony and concurrency in Elm

- Long-running computations will delay signal updates!
 - ▶ Example: `foldp f e s` where $f : \alpha \rightarrow \beta \rightarrow \beta$ takes a long time
- Elm's solution is to use `async` : $\Sigma\alpha \rightarrow \Sigma\alpha$
- Operational semantics: (i is a **new** input signal for each `async`)

$$\frac{\Gamma \vdash e^{[c]} : \Sigma\alpha}{\Gamma, (i : \Sigma\alpha) \vdash (\text{async}_i e)^{[c]} : \Sigma\alpha} \text{ AsyncInit}$$
$$\mathbf{U}_{s \leftarrow a} \left\{ (\text{async}_i e)^{[c]} \right\} \Rightarrow \mathbf{U}_{i \leftarrow c'}^\dagger \left(\text{async}_i \mathbf{U}_{s \leftarrow a}^\dagger \{e\}^{[c']} \right)^{[c]} \text{ AsyncSched}$$
$$\mathbf{U}_{i \leftarrow c'} \left\{ (\text{async}_i e)^{[c]} \right\} \Rightarrow (\text{async}_i e)^{[c']} \text{ AsyncUpd}$$

- The update computation $\mathbf{U}_{s \leftarrow a}^\dagger \{e\}$ runs on another thread...
 - ▶ ...while the current value c remains unchanged
 - ▶ Another update $\mathbf{U}_{i \leftarrow c'}^\dagger$ is **scheduled** but not yet triggered
 - ▶ When c' is ready, $\mathbf{U}_{i \leftarrow c'} \{...\}$ runs and sets the current value to c'

Example of using async

- UI that shows results of some long computations:

```
draw : Int -> Int -> View
draw x y = ...
```

```
s : Signal Int -- input values
```

```
fSlow : Int -> Int
res1 = async (map fSlow s)
fFast : Int -> Int
res2 = map fFast s
```

```
main : Signal View = map2 draw res1 res2
```

- Both results are updated as soon as they are computed

Some limitations of Elm-style FRP

- No higher-order signals: $\Sigma(\Sigma\alpha)$ is disallowed by the type system
- No distinction between continuous time and discrete time
- The signal processing logic is fully specified statically
- No constructors for user-defined signals
- No recursion possible in signal definition!
- Incomplete semantics for `async` : $\Sigma\alpha \rightarrow \Sigma\alpha$
 - ▶ Example: `async (map f s)` where `f` takes a long time
 - ▶ The initial value of this signal will not be available at initial time!
 - ▶ Need `async' : $\alpha \rightarrow \Sigma\alpha \rightarrow \Sigma\alpha$` to specify initial value?
- No full concurrency (e.g., “dining philosophers”)

Elm cannot simulate “dining philosophers”

- A philosopher thinks for a random time, then eats for a random time
 - ▶ Can a signal value $p : \text{Signal Unit}$ update itself at random times?
- No! There is no way to delay the update times of a signal **at runtime**
- $\text{Time.delay} : \text{Int} \rightarrow \Sigma \alpha \rightarrow \Sigma \alpha$ cannot use a time-varying delay value
- $\text{Time.every} : \text{Int} \rightarrow \Sigma \text{Int}$ also requires a fixed delay value
- Cannot lift Time.every into $\Sigma \text{Int} \rightarrow \Sigma \Sigma \text{Int}$ to achieve variable delay

The JavaScript backend for Elm (2015)

Features:

- Good support for HTML/CSS, HTTP requests, JSON
- Good performance of caching HTML views
- Support for Canvas and HTML-free UI building

Limitations:

- No implementation for `async` (JavaScript lacks concurrency)
- The lack of recursive signals is compensated by *ad hoc* primitives
- Ordinary recursion may generate invalid JavaScript!

Elm-style FRP: the good parts

- Transparent, declarative modeling of data through ADTs
- Immutable and safe data structures (Array, Dict, ...)
- No runtime errors or exceptions!
- Space/time leaks are impossible!
- Language is Haskell-like but simpler for beginners
- Full type inference
- Easy deployment and interop in Web applications

Some conservative extensions of Elm

- Fix initial value semantics for `async'` : $\alpha \rightarrow \Sigma\alpha \rightarrow \Sigma\alpha$
- Allow recursive definitions for signals
 - ▶ generate updates as `s0`, `f(s0)`, `f(f(s0))`, ... are being computed:
`s = async' s0 (map f s)`
- Add monadic signal combinator, `bind` : $(\alpha \rightarrow \Sigma\beta) \rightarrow \Sigma\alpha \rightarrow \Sigma\beta$
 - ▶ use input signals from dynamically created UI:
`viewS = map draw stateS`
`stateS = foldp update_on_click (bind get_clicks viewS)`
- Allow user-defined signals constructed from asynchronous APIs
 - ▶ Generate signal updates whenever callback is called:
`type C $\alpha\beta$ = $\alpha \rightarrow (\beta \rightarrow \perp) \rightarrow \perp$`
`chain : C $\alpha\beta$ $\rightarrow \Sigma\alpha \rightarrow \Sigma\beta$`
`some_async_api : C $\alpha\beta$`
`values_of_b = chain some_async_api values_of_a`

Part 2. Temporal logic and FRP

- Reminder (Curry-Howard): logical expressions will be types
 - ▶ ...and the axioms will be primitive terms
- We only need to control the **order** of events: no “hard real-time”
- How to understand temporal logic:
 - ▶ classical propositional logic \approx Boolean arithmetic
 - ▶ intuitionistic propositional logic \approx same but without **true** / **false** dichotomy
 - ▶ (linear-time) temporal logic LTL \approx Boolean arithmetic for *infinite sequences*
 - ▶ intuitionistic temporal logic ITL \approx same but without **true** / **false** dichotomy
- In other words:
 - ▶ an ITL type represents a **single infinite sequence** of values

Boolean arithmetic: notation

- Classical propositional (Boolean) logic: $T, F, a \vee b, a \wedge b, \neg a, a \rightarrow b$
- A notation better adapted to school-level arithmetic: $1, 0, a + b, ab, a'$
- The only “new rule” is $1 + 1 = 1$
- Define $a \rightarrow b = a' + b$
- Some identities:

$$\begin{aligned}0a &= 0, & 1a &= a, & a + 0 &= a, & a + 1 &= 1, \\a + a &= a, & aa &= a, & a + a' &= 1, & aa' &= 0, \\(a + b)' &= a'b', & (ab)' &= a' + b', & (a')' &= a \\a(b + c) &= ab + ac, & (a + b)(a + c) &= a + bc\end{aligned}$$

Boolean arithmetic: example

Of the three suspects A, B, C, only one is guilty of a crime.

Suspect A says: "B did it". Suspect B says: "C is innocent."

The guilty one is lying, the innocent ones tell the truth.

$$\phi = (ab'c' + a'bc' + a'b'c) (a'b + ab') (b'c' + bc)$$

Simplify: expand the brackets, omit aa' , bb' , cc' , replace $aa = a$ etc.:

$$\phi = ab'c' + 0 + 0 = ab'c'$$

The guilty one is A.

Propositional linear-time temporal logic (LTL)

- We work with *infinite boolean sequences* (“linear time”)

Boolean operations:

$$a = [a_0, a_1, a_2, \dots]; \quad b = [b_0, b_1, b_2, \dots];$$

$$a + b = [a_0 + b_0, a_1 + b_1, \dots]; \quad a' = [a'_0, a'_1, \dots]; \quad ab = [a_0 b_0, a_1 b_1, \dots]$$

Temporal operations:

$$\text{(Next)} \quad \mathbf{N}a = [a_1, a_2, \dots]$$

$$\text{(Sometimes)} \quad \mathbf{F}a = [a_0 + a_1 + a_2 + \dots, a_1 + a_2 + \dots, \dots]$$

$$\text{(Always)} \quad \mathbf{G}a = [a_0 a_1 a_2 a_3 \dots, a_1 a_2 a_3 \dots, a_2 a_3 \dots, \dots]$$

Other notation (from modal logic):

$$\mathbf{N}a \equiv \bigcirc a; \quad \mathbf{F}a \equiv \Diamond a; \quad \mathbf{G}a \equiv \Box a$$

- Weak Until: $p\mathbf{U}q = “p \text{ holds from now on until } q \text{ first becomes true}”$

$$p\mathbf{U}q = q + p\mathbf{N}(q + p\mathbf{N}(q + \dots))$$

Temporal logic redux

Designers of FRP languages must face some choices:

- LTL as type theory: do we use $\mathbf{N}\alpha$, $\mathbf{F}\alpha$, $\mathbf{G}\alpha$ as new types?
- Are they to be functors, monads, ...?
- Which temporal axioms to use as language primitives?
- What is the operational semantics? (I.e., how to compile this?)

A sophisticated example: [Krishnaswamy 2013]

- uses full LTL with higher-order temporal types and fixpoints
- uses linear types to control space/time leaks

Interpreting values typed by LTL

- What does it mean to have a value x of type, say, $\mathbf{G}(\alpha \rightarrow \alpha \mathbf{U} \beta)$??
 - ▶ $x : \mathbf{N}\alpha$ means that $x : \alpha$ will be available *only* at the *next* time tick (x is a **deferred value** of type α)
 - ▶ $x : \mathbf{F}\alpha$ means that $x : \alpha$ will be available at *some* future tick(s) (x is an **event** of type α)
 - ▶ $x : \mathbf{G}\alpha$ means that a (different) value $x : \alpha$ is available at *every* tick (x is an **infinite stream** of type α)
 - ▶ $x : \alpha \mathbf{U} \beta$ means a **finite stream** of α that may end with a β
- Some temporal **axioms** of intuitionistic LTL:

(deferred apply) $\mathbf{N}(\alpha \rightarrow \beta) \rightarrow (\mathbf{N}\alpha \rightarrow \mathbf{N}\beta)$;

(streamed apply) $\mathbf{G}(\alpha \rightarrow \beta) \rightarrow (\mathbf{G}\alpha \rightarrow \mathbf{G}\beta)$;

(generate a stream) $\mathbf{G}(\alpha \rightarrow \mathbf{N}\alpha) \rightarrow (\alpha \rightarrow \mathbf{G}\alpha)$;

(read infinite stream) $\mathbf{G}\alpha \rightarrow \alpha \mathbf{N}(\mathbf{G}\alpha)$

(read finite stream) $\alpha \mathbf{U} \beta \rightarrow \beta + \alpha \mathbf{N}(\alpha \mathbf{U} \beta)$

Elm as an FRP language

- λ -calculus with type $\mathbf{G}\alpha$, primitives `map2`, `foldp`, `async`

`map2` : $(\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow \mathbf{G}\alpha \rightarrow \mathbf{G}\beta \rightarrow \mathbf{G}\gamma$

`foldp` : $(\alpha \rightarrow \beta \rightarrow \beta) \rightarrow \beta \rightarrow \mathbf{G}\alpha \rightarrow \mathbf{G}\beta$

`async` : $\mathbf{G}\alpha \rightarrow \mathbf{G}\alpha$

- (`map2` makes \mathbf{G} an applicative functor)
- `async` is a special *scheduling instruction*
- Limitations:
 - ▶ Cannot have a type $\mathbf{G}(\mathbf{G}\alpha)$, also not using \mathbf{N} or \mathbf{F}
 - ▶ Cannot construct temporal values by hand
 - ▶ This language is an *incomplete* Curry-Howard image of LTL!

Conclusions

- There are some languages that implement FRP in various *ad hoc* ways
- The ideal is not (yet) reached
- Elm-style FRP is a promising step in the right direction

Abstract

In my day job, most bugs come from implementing reactive programs imperatively. FRP is a declarative approach that promises to solve these problems.

FRP can be defined as a λ -calculus that admits temporal types, i.e. types given by a propositional intuitionistic linear-time temporal logic (LTL). Although the Elm language uses only a subset of LTL, it achieves high expressivity for GUI programming. I will formally define the operational semantics of Elm. I discuss the current limitations of Elm and outline possible extensions. I also review the connections between temporal logic, FRP, and Elm.

My talk will be understandable to anyone familiar with Curry-Howard and functional programming. The first part of the talk is a self-contained presentation of Elm that does not rely on temporal logic or Curry-Howard. The second part of the talk will explain the basic intuitions behind temporal logic and its connection with FRP.

Suggested reading

E. Czaplicki, S. Chong. [Asynchronous FRP for GUIs](#). (2013)

E. Czaplicki. [Concurrent FRP for functional GUI](#) (2012).

N. R. Krishnaswamy.

<https://www.mpi-sws.org/~neelk/simple-frp.pdf> Higher-order functional reactive programming without spacetime leaks (2013).

M. F. Dam. Lectures on temporal logic. Slides: [Syntax and semantics of LTL](#), [A Hilbert-style proof system for LTL](#)

E. Bainomugisha, et al. [A survey of reactive programming](#) (2013).

W. Jeltsch. [Temporal logic with Until, Functional Reactive Programming with processes, and concrete process categories](#). (2013).

A. Jeffrey. [LTL types FRP](#). (2012).

D. Marchignoli. [Natural deduction systems for temporal logic](#). (2002). – See Chapter 2 for a natural deduction system for modal and temporal logics.

- What Evan calls “syntax-directed initialization semantics” is perhaps an important optimization, making sure that no signal values are copied.
- Either we update the signal tree bottom-up (as suggested in Evan’s paper, even concurrently, with message passing), or we evaluate it top-down (as I presented). Synchronization does not pose problems, except for the `async` primitive. It remains to be seen which semantics is easier to formulate. The bottom-up semantics might be easier if we figure out how to denote the cascading update of expressions that depend on lower-level subexpressions. (The cascading update is easy to present in the top-down formulation of updates.)
- The semantics of `async` still requires refinement. First, we need an initial value explicitly assigned to `async` expressions. (What are the typical ways these initial values can be determined?) Second, we need to denote the facts that the first update is scheduled on a background thread, the second update is scheduled but not run, and - more importantly - that the second update is scheduled to run on the entire Elm program being evaluated, not only on the local `async` expression being evaluated. This is perhaps easier to express with bottom-up

evaluation?

- The semantics of updates needs refinement to specify when current values remain unchanged but an update is scheduled, and when current values remain unchanged while no update is performed on that subexpression. This has implications only for `foldp`, which depends on the absence of “stuttering” updates.
- Need to present typical FRP primitives, such as those used in Rx; promises; futures; `async.js` library primitives; `bacon.js` library primitives, etc., through Elm primitives
- It is necessary to make `chain` implicitly asynchronous, just like `bind` and recursion. But the `drop` primitive still cannot be expressed through `chain` - by omitting the callback invocation when the predicate returns `false` we will approximate `drop` but, when the predicate returns `true`, we will generate an asynchronous update of the second signal, while what we presumably wanted is a synchronous update.
- However, `async` itself can be expressed through `chain`. Can `bind` be expressed through `chain`? I think it can't because we can't convert

$\alpha \rightarrow \Sigma\beta$ into $\mathbf{C}\alpha\beta$. And what if we could? Consider `react` : $\Sigma\alpha \rightarrow (\alpha \rightarrow \perp) \rightarrow \perp$ and `signal` : $((\alpha \rightarrow \perp) \rightarrow \perp) \rightarrow \Sigma\alpha$. These two terms can build operational equivalence of $\Sigma\alpha$ and $(\alpha \rightarrow \perp) \rightarrow \perp$, since a callback can be called zero times or multiple times (simulating a signal update) - and then we would reimplement Elm's runtime by hand. It is not clear that this is desirable. The implementation in terms of $\mathbf{C}\alpha\beta$ definitely seems to be too low-level, and there is some advantage in restricting the signal primitives so that safety and clarity are maintained.

- Perhaps the easiest and clearest way of presenting the semantics of Elm is to translate Elm into callback types?
- What are the axioms of temporal logic in Ewald's old paper, and how do they translate to Elm primitives? What are the missing primitives?