

What everyone will need: \* A way to view this worksheet (printed, laptop, tablet, etc) \* A way to take notes (paper or a device)

## While and If

Learning to use `if` and `while` is an essential skill. During this discussion, focus on what we've studied in the first three lectures: `if`, `while`, assignment (`=`), comparison (`<`, `>`, `==`, ...), and arithmetic. Please don't use features of Python that we haven't discussed in class yet, such as `for`, `range`, and lists. We'll have plenty of time for those later in the course, but now is the time to practice the use of `if` (textbook section 1.5.4) and `while` (textbook section 1.5.5).

### Q1: Race

The `race` function below sometimes returns the wrong value and sometimes runs forever.

```
def race(x, y):
    """The tortoise always walks x feet per minute, while the hare repeatedly
    runs y feet per minute for 5 minutes, then rests for 5 minutes. Return how
    many minutes pass until the tortoise first catches up to the hare.

    >>> race(5, 7) # After 7 minutes, both have gone 35 steps
    7
    >>> race(2, 4) # After 10 minutes, both have gone 20 steps
    10
    """
    assert y > x and y <= 2 * x, 'the hare must be fast but not too fast'
    tortoise, hare, minutes = 0, 0, 0
    while minutes == 0 or tortoise - hare:
        tortoise += x
        if minutes % 10 < 5:
            hare += y
        minutes += 1
    return minutes
```

Find positive integers `x` and `y` (with `y` larger than `x` but not larger than `2 * x`) for which either: - `race(x, y)` returns the wrong value or - `race(x, y)` runs forever

You just need to find one pair of numbers that satisfies either of these conditions to finish the question, but if you want to think of more you can.

Notes: - `x += 1` is the same as `x = x + 1` when `x` is assigned to a number. - `0` is a false value and all other numbers are true values.

The value of `race(x, y)` is incorrect when it is not the **first** time the tortoise passes the hare. Try some small numbers (below 5) to see if you can find a case where `tortoise` has become larger than `hare`, but the expression

## 2 Control

`tortoise - hare` was not zero when it happened.

The return value is incorrect when the time that the tortoise first passes the hare is not an integer number of minutes (e.g., for `race(2, 3)` the tortoise passes the hare after 7.5 minutes), but there is some (larger) integer number of minutes after which both animals have gone the same distance.

The `race` function will run forever if the only times that the tortoise and hare have gone the same distance are not integers (e.g., for `race(4, 5)` the tortoise passes the hare after 6.2 minutes, and the hare never catches up).

**Q2: Fizzbuzz**

Implement the classic *Fizz Buzz* sequence. The `fizzbuzz` function takes a positive integer `n` and prints out a *single line* for each integer from 1 to `n`. For each `i`:

- If `i` is divisible by both 3 and 5, print `fizzbuzz`.
- If `i` is divisible by 3 (but not 5), print `fizz`.
- If `i` is divisible by 5 (but not 3), print `buzz`.
- Otherwise, print the number `i`.

Try to make your implementation of `fizzbuzz` concise.

```
def fizzbuzz(n):
    """
    >>> result = fizzbuzz(16)
    1
    2
    fizz
    4
    buzz
    fizz
    7
    8
    fizz
    buzz
    11
    fizz
    13
    14
    fizzbuzz
    16
    >>> print(result)
    None
    """
    i = 1
    while i <= n:
        if i % 3 == 0 and i % 5 == 0:
            print('fizzbuzz')
        elif i % 3 == 0:
            print('fizz')
        elif i % 5 == 0:
            print('buzz')
        else:
            print(i)
        i += 1
```

Be careful about the order of your `if` and `elif` clauses: try first checking if the current number is divisible by both 3 and 5, then check for just divisibility by 3 and just divisibility by 5.

## 4 *Control*

[Video walkthrough](#)

# Problem-Solving

A useful approach to implementing a function is to: 1. Pick an example input and corresponding output. 2. Describe a process (in English) that computes the output from the input using simple steps. 3. Figure out what additional names you'll need to carry out this process. 4. Implement the process in code using those additional names. 5. Determine whether the implementation really works on your original example. 6. Determine whether the implementation really works on other examples. (If not, you might need to revise step 2.)

Importantly, this approach doesn't go straight from reading a question to writing code.

For example, in the `is_prime` problem below, you could: 1. Pick `n` is 9 as the input and `False` as the output. 2. Here's a process: Check that 9 (`n`) is not a multiple of any integers between 1 and 9 (`n`). 3. Introduce `i` to represent each number between 1 and 9 (`n`). 4. Implement `is_prime` (you get to do this part with your group). 5. Check that `is_prime(9)` will return `False` by thinking through the execution of the code. 6. Check that `is_prime(3)` will return `True` and `is_prime(1)` will return `False`.

Try this approach together on the next two problems.

**Important:** It's highly recommended that you **don't** check your work using a computer right away. Instead, talk to your group and think to try to figure out if an answer is correct. On exams, you won't be able to guess and check because you won't have a Python interpreter. Now is a great time to practice checking your work by thinking through examples. You could even draw an environment diagram!

If you're not sure about how something works or get stuck, ask for help from the course staff.

## Q3: Is Prime?

Write a function that returns `True` if a positive integer `n` is a prime number and `False` otherwise.

A prime number `n` is a number that is not divisible by any numbers other than 1 and `n` itself. For example, 13 is prime, since it is only divisible by 1 and 13, but 14 is not, since it is divisible by 1, 2, 7, and 14.

Use the `%` operator: `x % y` returns the remainder of `x` when divided by `y`.

Here's a `while` statement that goes through all numbers above 1 and below `n`:

```
i = 2
while i < n:
    ...
    i = i + 1
```

You can use `n % i == 0` to check whether `i` is a factor of `n`. If it is, `return False`.

```
def is_prime(n):  
    """  
    >>> is_prime(10)  
    False  
    >>> is_prime(7)  
    True  
    >>> is_prime(1) # one is not a prime number!!  
    False  
    """  
    if n == 1:  
        return False  
    k = 2  
    while k < n:  
        if n % k == 0:  
            return False  
        k += 1  
    return True
```

**Presentation Time:** Come up with a **one-sentence description** of the process you implemented to solve `is_prime` that you think someone could understand without looking at your code. Present your solution to your group / present your group's solution to the class.

**Q4: Unique Digits**

Write a function that returns the number of unique digits in a positive integer.

**Hints:** You can use `//` and `%` to separate a positive integer into its one's digit and the rest of its digits.

You may find it helpful to first define a function `has_digit(n, k)`, which determines whether a number `n` has digit `k`.

```

def unique_digits(n):
    """Return the number of unique digits in positive integer n.

    >>> unique_digits(8675309) # All are unique
    7
    >>> unique_digits(13173131) # 1, 3, and 7
    3
    >>> unique_digits(101) # 0 and 1
    2
    """
    unique = 0
    while n > 0:
        last = n % 10
        n = n // 10
        if not has_digit(n, last):
            unique += 1
    return unique

# Alternate solution
def unique_digits_alt(n):
    unique = 0
    i = 0
    while i < 10:
        if has_digit(n, i):
            unique += 1
        i += 1
    return unique

def has_digit(n, k):
    """Returns whether k is a digit in n.

    >>> has_digit(10, 1)
    True
    >>> has_digit(12, 7)
    False
    """
    assert k >= 0 and k < 10
    while n > 0:
        last = n % 10
        n = n // 10
        if last == k:
            return True
    return False

```

One approach is to loop through every digit from 0 to 9 and check whether `n` has the digit. Count up the ones it has.



We have provided two solutions: - In one solution, we look at the current digit, and check if the rest of the number contains that digit or not. We only say it's unique if the digit doesn't exist in the rest. We do this for every digit. - In the other, we loop through the numbers 0-9 and just call `has_digit` on each one. If it returns true then we know the entire number contains that digit and we can one to our unique count.

### Q5: Ordered Digits

Implement the function `ordered_digits`, which takes as input a positive integer and returns `True` if its digits, read left to right, are in non-decreasing order, and `False` otherwise. For example, the digits of 5, 11, 127, 1357 are ordered, but not those of 21 or 1375.

```
def ordered_digits(x):
    """Return True if the (base 10) digits of X>0 are in non-decreasing
    order, and False otherwise.

    >>> ordered_digits(5)
    True
    >>> ordered_digits(11)
    True
    >>> ordered_digits(127)
    True
    >>> ordered_digits(1357)
    True
    >>> ordered_digits(21)
    False
    >>> result = ordered_digits(1375) # Return, don't print
    >>> result
    False

    >>> cases = [(1, True), (9, True), (10, False), (11, True), (32, False),
    ...         (23, True), (99, True), (111, True), (122, True), (223, True),
    ...         (232, False), (999, True),
    ...         (13334566666889, True), (987654321, False)]
    >>> [ordered_digits(s) == t for s, t in cases].count(False)
    0

    """
    last = x % 10
    x = x // 10
    while x > 0 and last >= x % 10:
        last = x % 10
        x = x // 10
    return x == 0
```

We split off each digit in turn from the right, comparing it to the previous digit we split off, which was the one immediately to its right. We stop when we run out of digits or we find an out-of-order digit.

def ordered\_digits(x): """Return True if the (base 10) digits of X>0 are in non-decreasing order, and False otherwise.

```
>>> ordered_digits(5)
True
>>> ordered_digits(11)
True
>>> ordered_digits(127)
True
>>> ordered_digits(1357)
True
>>> ordered_digits(21)
False
>>> result = ordered_digits(1375) # Return, don't print
>>> result
False

>>> cases = [(1, True), (9, True), (10, False), (11, True), (32, False),
...          (23, True), (99, True), (111, True), (122, True), (223, True),
...          (232, False), (999, True),
...          (13334566666889, True), (987654321, False)]
>>> [ordered_digits(s) == t for s, t in cases].count(False)
0

"""
last = x % 10
x = x // 10
while x > 0 and last >= x % 10:
    last = x % 10
    x = x // 10
return x == 0
```

## Submit Attendance

You're done! Excellent work this week. Please be sure to ask your section TA for the attendance form link and fill it out for credit. (one submission per person per section).