Digital Signal Processing and Digital Filters

Imperial College London

Practice Sheet 3

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The purpose of the practice sheet is to enhance the understanding of the course materials. The practice sheet does not constitute towards the final grade. Students are welcome to discuss the problems amongst themselves. The questions will be discussed in the Q&A sessions with the instructor.

1) Which of the following are true?

   (a) Antiperiodic signals are not periodic
   (b) Antiperiodic signals are periodic
   (c) The DCT does not preserve input energy but the orthogonal DCT does
   (d) The orthogonal DCT does not preserve input energy but the DCT does

2) The DCT is computed by applying the DFT to a modified version of the original signal; the modified signal is generated by repeating the original signal in reverse order, and inserting a zero in between every pair of samples. Based on this knowledge, why is the DFT always real?

3) Let $x[n] = [1, 1, 1, 1]$.

   (a) Compute the DCT $X[k]$.
   (b) What is the minimal number of samples required to encode the DCT accurately?
   (c) Compute the energy of $x[k]$ and $X[k]$. By what amount is the energy of the DCT increased/decreased?
   (d) Calculate the orthogonal DCT, which we denote $X_{\perp}[k]$, and show that it preserves the energy of the original singal.

4) Let $x[n] = [x_1, x_2, x_3, x_4]$. Find all possible values for $x_i, i = 1, \ldots, 4$, that guarantee that $X[k] = 0, k > 0$.

Answers.

1) Correct options: $(b), (c)$.

2) Because symmetry and real values in the time domain guarantee that the DFT is also real-valued.

3) (a) $X[k] = 4 \cdot \delta_{k0}$.

(b) One sample.

(c) $\sum_{n=0}^{N-1} (x[n])^2 = 4$, $\sum_{k=0}^{N-1} (X[k])^2 = 16$. The energy is increased by 4.

(d) $X_{\perp}[k] = 2 \cdot \delta_{k0}$, $\sum_{k=0}^{N-1} (X_{\perp}[k])^2 = 4$.

4) $x[n] = \alpha [1, 1, 1, 1], \forall \alpha \in \mathbb{R}$.