Optical Communication 2018
Solution 5

1. a) Assume the glass has $n \doteq 1.5$, then $R = \left(\frac{1.5-1}{1.5+1}\right)^2 = 0.04$, so 4% is lost

at each of the two surfaces, the effect of multiple reflections can be neglected, then:

$$\text{loss (dB)} = 10 \log\left(\frac{1}{0.92}\right) = \underline{0.36 \text{ dB}}$$

b) We need to assume one of the indices, let us take $n_0 = 1.46$, then $NA = \sqrt{1.05^2 - 1} \times 1.46 = \underline{0.150}$

c) Advantage: can be lower cost, or less susceptible to fracture. Disadvantage: can be higher loss, higher dispersion, less robust to temp. variation or chemical damage.

d) An A.R. coating is $\lambda/4$ with $n = \sqrt{n_1 n_2}$

This gives $n_{AR} = \sqrt{3.6} = \underline{1.9}$

$$t_{AR} = \frac{780\text{nm}}{4 \times 1.9} = \underline{103 \text{ nm}}$$

e) Reflection will be $0.1 \times 5\text{mW} = 0.5\text{mW}$ minus the total loss of $40 \times 0.4 = 16$ dB

so $P = 0.5 \times 10^{-1.6} = \underline{12.6 \text{ mW}}$

1. f) $SNR_{opt} = \dfrac{I_{ph}}{\sqrt{2eI_{ph}}\sqrt{\Delta f}} = \sqrt{\dfrac{I_{ph}}{2e \cdot \Delta f}}$

Taking $B = 2\Delta f$, $SNR^2 = \dfrac{I_{ph}}{eB}$

giving $I_{ph} = 10^2 \times 1.6 \times 10^{-19} \times 2 \times 10^9 = 3.2 \times 10^{-8}$
$= \underline{32\ nA}$

g) We need $\Delta E \simeq \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.3 \times 10^{-6}}$

$\Delta E = 1.53 \times 10^{-19}\ J = \dfrac{1.53 \times 10^{-19}}{1.6 \times 10^{-19}} = \underline{0.96\ eV}$

h) $w_p / w_n = N_D / N_A = 0.25$

$|E_{max}| = \dfrac{e}{\varepsilon} w_p N_A \qquad V = \frac{1}{2} |E_{max}| (w_p + w_n)$

$\therefore V = \frac{1}{2} \left(\frac{e}{\varepsilon} w_p N_A\right)(w_p + 4w_p) = \frac{5}{2} \frac{e}{\varepsilon} N_A w_p^2$

$w_p = \frac{1}{5} w_{dep} = 0.4\ \mu m$

$\therefore V = \dfrac{2.5 \times 1.6 \times 10^{-19} \times 4 \times 10^{20} \times (0.4 \times 10^{-6})^2}{12 \times 8.85 \times 10^{-12}}$

$= \underline{0.24\ V}$

i) The mode with the lowest n' (1.475) is the most weakly guided and so will most easily leak into the cladding.

j) For wavelengths longer than the main telecomms region of 1.3 - 1.6 μm, infra-red absorption, caused by inter-atomic vibration, is the main loss mechanism.

2 a) We need mode $m = 1$ to exist, giving:

$$d \geq \frac{1}{2} \frac{\lambda_0}{NA}$$

but mode $m = 2$ not to exist, $d < \frac{1}{2}\left(\frac{2\lambda_0}{NA}\right)$

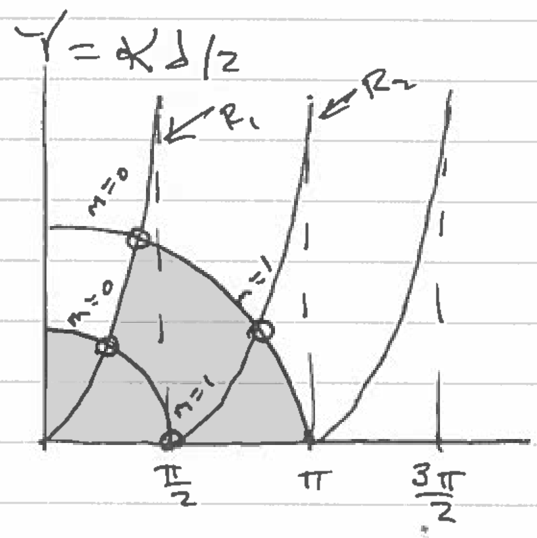
so $\quad NA \cdot d < \lambda_0 \leq 2 NA \cdot d$

$NA = \sqrt{1.48^2 - 1.47^2} = 0.172$

so $\quad 0.86\,\mu m < \lambda_0 \leq 1.72\,\mu m$

(whether it's $<$ or $\leq$ at either end not important)

b)



$R = NA \cdot k_0 d/2 = \frac{NA\pi d}{\lambda_0}$

$R_1 = \frac{NA \cdot \pi d}{\lambda_{0max}}$

$R_2 = NA\,\pi d / \lambda_{0min}$

Modes that are just cut off have $X = R$

$\therefore \quad \frac{k_{1x} d}{2} = NA \frac{k_0 d}{2}, \quad k_{1x} = NA \cdot k_0$

$n' = \frac{\beta}{k_0} = \sqrt{n_1^2 - (k_{1x}/k_0)^2} = \sqrt{n_1^2 - NA^2}$

but $NA^2 = n_1^2 - n_2^2$ so $n' = \sqrt{n_1^2 - (n_1^2 - n_2^2)}$

$$\underline{\underline{n' = n_2}}$$

2 c) For X_{0max} we use $R_1 = \pi/2$

We can see in the plot that $X \approx 1$

which provides a starting point for iterating.

m=0 is an even mode!

$$\cos X / X = \frac{1}{R} = \frac{2}{\pi}$$

$$X = \frac{\pi}{2} \cos X$$

iterating from $X \cong 1$ gives $X = 0.934$

Then $k_{1x} = \frac{2X}{d}$

$$n' = \sqrt{n_1^2 - (k_{1x}/k_0)^2} = \sqrt{n_1^2 - \left(\frac{2X}{d} \cdot \frac{\lambda_{0max}}{2\pi}\right)^2}$$

$$= \sqrt{1.48^2 - \left(\frac{.934}{\pi} \times \frac{1.72}{5}\right)^2} = \underline{\underline{1.476}}$$

3. a) $NEP_A = 9.0\ pW/\sqrt{Hz}$, $NEP_B = 6.5\ pW/\sqrt{Hz}$

Thus receiver A needs to receive 9/6.5 times the optical power to achieve the same SNR. The attenuation difference is 0.01 dB/km. The ratio 9/6.5 is $10 \log(9/6.5) = 1.41$ dB, therefore after 141 km the extra loss in B will cancel the lower NEP.

b) We need to convert the NEP into an equivalent noise current, which we do by:

$$I_R^{*2} = (R \cdot NEP)^2 \quad \text{with } R = e\lambda/hc \quad (\eta=1)$$

Since $I_{sh}^{*2} = 2e I_{ph}$, we get, for $I_{sh} = I_R$:

$$(R \cdot NEP)^2 = 2e \cdot R \Phi_R \quad \text{giving } \Phi_R = \frac{\lambda}{2hc} \cdot NEP^2$$

$$\Phi_R = \frac{1.3\times10^{-6} \times (9\times10^{-12})^2}{2\times6.63\times10^{-34}\times3\times10^8} = 265\ \mu W$$

This implies a loss in the fibre of $10\log\left(\frac{10}{.265}\right) = 15.8$ dB

so $L = \frac{15.8}{.32} = 49.3$ km

c) We can approximate the pulse spreading in time as $\Delta t = L \cdot D \cdot \sigma_\lambda$

and we need to keep this below ~0.25 bits,

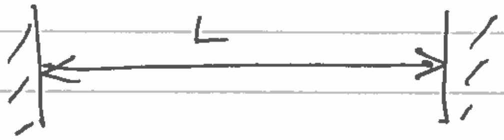
giving $L D \sigma_\lambda < \frac{0.25}{B}$ $\quad B_{max} = \frac{0.25}{10\times10^{-12}\times2\times49.3} = 25.4$ Mbit/s

d) Here $SNR = I_{ph} / \sqrt{2eI_{ph}}\sqrt{B/2}$

$SNR^2 = I_{ph}/eB$ $\quad$ But $\frac{I_{ph}}{e} = \frac{\text{electrons}}{\text{sec}}$ $\quad B = \frac{\text{bits}}{\text{sec}}$

$\therefore SNR^2 = \text{electrons}/\text{bit}$

4. a)



We have $L = \frac{m\lambda}{2} = \frac{m\lambda_{om}}{2n'}$

so $\lambda_{om} = \frac{2n'L}{m}$ $\quad \Delta\lambda_o = 2n'L\left(\frac{1}{m} - \frac{1}{m+1}\right) \cong \frac{2n'L}{m^2}$ $\quad (m>>1)$

$\therefore \Delta\lambda_o = \frac{\lambda_o^2}{2n'L}$ $\quad \lambda_o = \sqrt{2 \times 3.8 \times 450 \times 10^{-6} \times 0.3 \times 10^{-9}} = \underline{\underline{1.01\,\mu m}}$

b) The total wavelength spread is the same as for an LED for which the variation in photon energy is $\approx 2kT = 50\,meV = \Delta E$

Then : $\Delta E = hc\left(\frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}}\right) = \frac{hc(\lambda_{max} - \lambda_{min})}{\lambda_o^2}$

(taking $\lambda_o^2 \cong \lambda_{max}\lambda_{min}$)

so $(\lambda_{max} - \lambda_{min}) = \frac{(1.01 \times 10^{-6})^2 \times 50 \times 10^{-3} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} = 41\,nm$

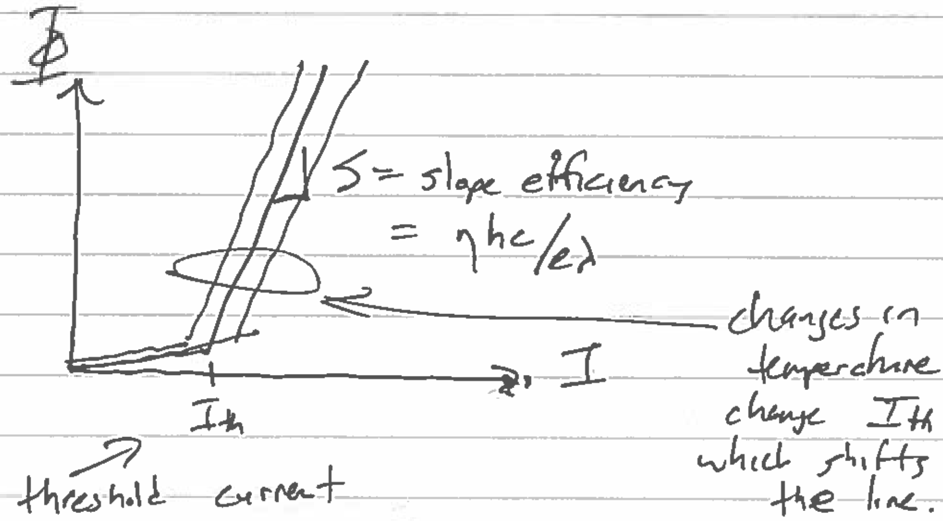
so the number of modes $N = \frac{41}{.3} \cong \underline{\underline{137}}$

c) The grating period Λ should be half the wavelength in the device, ie

$\Lambda = \frac{0.5 \times 1.01\,\mu m}{3.8} = \underline{\underline{0.133\,\mu m}}$

This has the advantage of only reflecting one of the cavity modes, which greatly reduces the spectral width.

4. d)



e) An ideal laser would produce one output photon per input electron. Because of I_{th}, this relation gives only the differential output, i.e. the slope of the steep line in the plot. In practice the output is not ideal because only η photons get produced per electron, η being the quantum efficiency.
Since the slope is optical power Φ / input current,

then $$S = \frac{\Delta\Phi}{\Delta I} = \frac{\text{optical energy}}{\text{input charge}}$$

$$\therefore\ S = \frac{\text{photons}}{\text{electron}} \times \frac{\text{energy/photon}}{\text{charge/electron}}$$

$$= \eta \times \frac{hc/\lambda}{e} = \frac{\eta hc}{e\lambda}$$

5. a) The fraction of photons absorbed in the intrinsic layer is

$$f = e^{-\alpha x_1} - e^{-\alpha x_2}$$

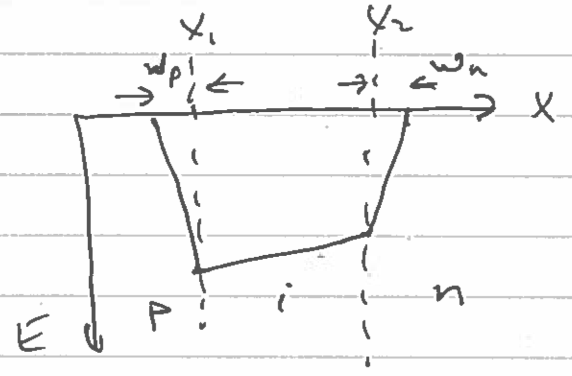
taking $\frac{df}{d\alpha} = -x_1 e^{-\alpha x_1} + x_2 e^{-\alpha x_2} = 0$

gives $x_2/x_1 = e^{-\alpha(x_1 - x_2)}$

$$\alpha_{opt} = \frac{\ln(x_2/x_1)}{x_2 - x_1} = 0.25 \mu m^{-1}$$

$$= \underline{\underline{0.25 \times 10^6 \, m^{-1}}} \quad (\text{not } 10^{-6}!)$$

b)



Since dE/dx is proportional to the doping level, in the intrinsic layer we have $\Delta E = 0.2(E(x_2))$ over $5 \mu m$, in n layer $\Delta E = E(x_2)$ over $1 \mu m$, so $25\times$ the slope.

$\therefore N_D^+ = 25 \times N_D^- = \underline{\underline{25 \times 10^{19} \, m^{-3}}}$

In the p region, the $\Delta E = 1.2 \times E(x_2)$ over $0.5 \, \mu m$, so $N_A = 6 \times 10 \times N_D^- = \underline{\underline{6 \times 10^{20} \, m^{-3}}}$

c) V is just the integral under $E(x)$.

$E(x_1) = 2 \times 10^6 \, V/m \qquad E(x_2) = \frac{2 \times 10^6}{1.2} = 1.67 \times 10^6$

$$\Delta V = \tfrac{1}{2}(2 \times 10^6)(.5 \times 10^{-6}) + \tfrac{1}{2}(2 \times 10^6 + 1.67 \times 10^6)(5 \times 10^{-6}) + \tfrac{1}{2}(1.67 \times 10^6)10^{-6}$$

$$= \underline{\underline{10.51 \, V}}$$

d) This is simply a parallel plate capacitor,

$C/\text{area} = \frac{\varepsilon_r \varepsilon_0}{t} \qquad t = .5 + 5 + 1 = 6.5 \, \mu m$

$\varepsilon_r = 12$

$$\frac{C}{\text{area}} = 12 \times 8.85 \times 10^{-12} \times 6.5 \times 10^{-6} = \underline{\underline{6.9 \times 10^{8} \, \frac{F}{m^2}}}$$