Digital Signal Processing and Digital Filters

Imperial College London

# Practice Sheet 1

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The purpose of the practice sheet is to enhance the understanding of the course materials. The practice sheet does not constitute towards the final grade. Students are welcome to discuss the problems amongst themselves. The questions will be discussed in the Q&A sessions with the instructor.

1) Which of the following statements is true and why?
(Multiple choices may be correct)

*i*) All discrete-time signals are digital signals.
*ii*) All digital signals are discrete-time signals.
*iii*) Some discrete-time signals are digital signals.
*iv*) Some digital signals are discrete-time signals.

2) Let us define a sequence by,

$$x[n] = (r_1)^n u[n] - (r_2)^n u[-(n+1)] \tag{1}$$

where $r_1 = -1/3$ and $r_2 = 1/2$.

- On the z-plane, plot the poles and zeros together with the region-of-convergence.
- In the above sequence in (1), what happens if we exchange $r_1$ and $r_2$?

3) For some linear-time-invariant system, the transfer function is given by,

$$H(z) = \frac{\left(1+z^{-1}\right)^2}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{3}{4}z^{-1}\right)}.$$

Suppose $x[n]$ is the input to the system and $y[n]$ is the output.
Derive the difference equation that is satisfied by $x[n]$ and $y[n]$.

4) **Filter Specification via Z-transform**
Suppose that a function is given by,

$$\phi(t) = \begin{cases} \frac{2}{3} - |t|^2 + \frac{|t|^3}{2} & 0 \leqslant |t| < 1 \\ \frac{(2-|t|)^3}{6} & 1 \leqslant |t| < 2 \\ 0 & 2 \leqslant |t| \end{cases}.$$

- Is $\phi(t)$ a symmetric function? Argue by plotting this function.
- Convert $\phi(t)$ into an FIR filter by sampling it at integer points, that is, $\phi(t), t = k$ where $k = 0, \pm 1, \pm 2, \ldots$.
  Let $\Phi(z)$ be the z-transform of this FIR filter sequence. Write the explicit form of $\Phi(z)$.
- Inverse filter design. Suppose that the FIR filter is defined by

  $$p[k] = \phi(t)|_{t=k}, k = \mathbb{Z} \qquad \text{(that is, } k \text{ takes integer values).}$$

  Then, we say that $p_{\mathsf{inv}}[k]$ is an inverse-filter when,

  $$p_{\mathsf{inv}}[k] * p[k] = \delta[k] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}.$$

  Identify the transfer function of $p_{\mathsf{inv}}[k]$ in terms of $\Phi(z)$.
  Write down the impulse-response of $p_{\mathsf{inv}}[k]$ given the definition of $\phi(t)$. Is $p_{\mathsf{inv}}[k]$ an FIR or IIR filter?
  Plot the impulse response of $p_{\mathsf{inv}}[k]$.

5) **Filter Identification**

The pole-zero plot of a discrete filter is given below.

When the input $x[n] = 1$ for all $n$, the output is exactly the same.

What is the impulse response of such a filter?