IMPERIAL COLLEGE LONDON

EE4-06
EE9-AO9
EE9-SO9

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2020

MSc and EEE/EIE PART IV: MEng and ACGI

OPTICAL COMMUNICATION

Tuesday, 5 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 5

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.M. Yeatman
Second Marker(s) : R.R.A. Syms

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$\kappa = k_{1x}\tan(k_{1x}d/2)$ and $\kappa = -\, k_{1x}\cot(k_{1x}d/2)$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) An optical plane wave propagating within a block of glass of refractive index 1.48, surrounded by air, is incident on one of the block's flat surfaces. Calculate the incident angle with respect to normal beyond which the wave will experience total internal reflection.

b) A certain symmetric slab waveguide supports 3 TE modes. State whether the number of TM modes supported is fewer, the same, more, or unknown.

c) A step-index glass optical fibre has a numerical aperture NA = 0.10. Estimate the index difference Δn of the fibre.

d) A step-index multi-mode glass optical fibre has a core index $n_c = 1.470$, and an index difference $\Delta n = 0.05$. What is the smallest possible value of the effective index n' for a mode propagating in this fibre?

e) An optical signal of power level 5.0 mW is launched into a fibre having attenuation of 0.4 dB/km. Find the power level in mW after the signal has propagated for 50 km.

f) A certain laser diode has a slope efficiency, at a nominal wavelength $\lambda_o = 0.78$ µm, of 1.40 W/A. Calculate the quantum efficiency η.

g) An optical link using a p-i-n photodiode has an optical SNR of 10, dominated by receiver noise, for a bit rate of B = 200 Mbit/s and a receiver noise equivalent power of NEP = 7.0 pW/$\sqrt{\text{Hz}}$. Calculate the received optical power Φ_R. You may assume that the quantum efficiency η =1.

h) A silicon p-n photodiode has a total depletion layer width of 2.0 µm when a reverse bias of 8.0 V is applied. Calculate the electric field strength E at the p-n junction.

i) Briefly explain why dB/km are not suitable units for the attenuation of a free-space signal caused by beam spreading.

j) In a fibre amplifier, briefly explain why the pump wavelength is less than the signal wavelength.

2. a) A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness d = 7.5 µm, a core refractive index n_1 = 1.480, and a numerical aperture NA = 0.1. For each of two nominal wavelengths, λ_{oA} = 1.49 µm and λ_{oB} = 1.51 µm, find the number of TE modes supported by the waveguide. Illustrate your solution by sketching an X-Y plot, where $X = k_{1x}d/2$ and $Y = \kappa d/2$. [8]

b) For the waveguide described in (a), for the wavelength λ_{oA} = 1.49 µm, calculate to 3 decimal places the effective index n′ of each of the supported TE modes. [6]

c) On an X-Y plot for a specific symmetric slab waveguide, a straight line through the origin is drawn, at an angle θ from the X axis. Where this line crosses the lines defined by the eigenvalue equations, modes will exist for specific wavelengths. Show that these modes will all have the same effective index, given by:

$n' = (n_1^2 - NA^2\cos^2\theta)^{1/2}$. [6]

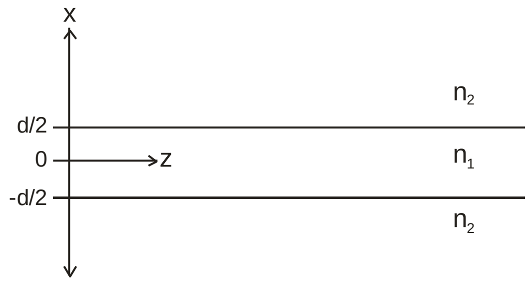

Figure 2.1

3. a) An optical link has a transmitted power Φ_T, a fibre loss coefficient α_{dB}, a receiver responsivity $\Re$ at the operating wavelength, and a receiver input resistance R. Derive expressions for the maximum bit rate B, for a signal-to-noise ratio SNR, as a function of fibre length L, for each of two cases:
A: Considering shot noise as the only noise source; and
B: Considering receiver thermal noise as the only noise source.
State any approximations or assumptions made.

If $\Phi_T = 10$ mW, $\alpha_{dB} = 0.4$ dB/km, $\Re = 0.8$ A/W, R = 10 kΩ, and SNR = 10, find the length L for which shot noise and thermal noise contribute equally. [6]

b) For the link and the parameter values described in (a), sketch a single graph showing log(B) vs L for each of the two cases A and B. Hence, on the same graph, and without further calculations, plot an estimated function log(B) vs L for the case where both noise sources specified are included. [6]

c) Give an approximate equation indicating the maximum bit rate B that ensures that the pulse spreading due to dispersion in a fibre link of length L is not more than 0.2 bits, in terms of the fibre dispersion coefficient D, the transmitter spectral width σ_λ, and L. Add a line indicating this limit to your sketch in (b). Assume that $\sigma_\lambda = 2.0$ nm and $D = 10$ ps/nm·km. [4]

d) It is theoretically possible that an optical fibre could be fabricated having a lower minimum attenuation than the theoretical limit of silica, by using a different type of glass. Would you expect the wavelength of minimum attenuation in such a fibre to be longer or shorter than the value for silica? Briefly explain why. [4]

4. a) Name and briefly describe the two mechanisms that contribute to chromatic dispersion in an optical communications fibre. [4]

b) In the wavelength range of interest, the effective index of the propagating mode in a certain single mode fibre can be approximated by

$$n'(\lambda_o) \cong n'_c + D_1(\lambda_o - \lambda_{oc}) + D_2(\lambda_o - \lambda_{oc})^2 \qquad (4.1)$$

where λ_o is the free space (nominal) wavelength, λ_{oc} is a reference wavelength, and n'_c, D_1 and D_2 are constants. Show that the group velocity v_g can be expressed as:

$$v_g = \frac{c}{n'_c - D_1\lambda_{oc} - D_2(\lambda_o^2 - \lambda_{oc}^2)}$$

in this case. [8]

c) Hence, find the group delay τ_g in this case for a fibre of length L. [4]

d) For a range of nominal wavelengths $\Delta\lambda_o$, for a fibre of length L, what will be the corresponding variation in group delay, $\Delta\tau_g$? [4]

5. A silicon p-i-n photodiode has *p*, *i* and *n* doping levels respectively of $N_A = 2 \times 10^{20}$ m^{-3}, $N_D^- = 10^{19}$ m^{-3} and $N_D^+ = 10^{20}$ m^{-3}. The distances from the diode surface to the top and bottom of the intrinsic layer are $X_1 = 3.0$ µm and $X_2 = 9.0$ µm respectively.

a) If the electric field amplitude at the *p-i* boundary $E_1 = -4.5 \times 10^5$ V/m, calculate the electric field amplitude at the *i-n* boundary E_2, the width of the depleted region in the *p* layer w_p, the width of the depleted region in the *n* layer w_n, and the bias voltage V_B. [8]

b) For the conditions given in (a), sketch plots of the electric field amplitude, the charge density, and the voltage, as functions of the distance from the diode surface *x*. Label and dimension each plot. [6]

c) For a photon which is absorbed at the i-n boundary, show that the time for the resulting hole to propagate back to the p-i boundary is given by:

$$\tau = \frac{ln(E_1/E_2)}{\mu_h e N_D^- / \varepsilon}$$

where μ_h is the hole mobility, e is the electron charge and ε is the permittivity of silicon. Assume that the saturation drift velocity is not reached. Find the value of τ for the parameter values in the case described in (a) using $\mu_h = 450$ cm^2/Vs. [6]