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Simultaneous Transmission of Information and Power

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Aims – Based on the experiment handout [1], learn the wireless transmission of information and power in the frequency selective channel, with the assumption of point-to-point communication and additive white Gaussian noise (AWGN). The tradeoff between information rate and power at the receiver will be analyzed using the optimization method.

I. INTRODUCTION

In modern communication theory, the electricity is only employed as the carrier of information, because what the communication engineering cares about is the transmission of information from the transmitter to the receiver. However, in electric power engineering, electricity is the source of power, which is capable of providing energy at the receiver. Therefore, in the communication network, if the receiver is capable of harvesting the energy of the received signal, this energy can be exploited to support some functions of the receiver, especially the communication functions with power constraints. For example, in RFID systems, the reivers use the harvested energy from received signals to send information back.

When the information and power are transmitted simultaneously in a noisy channel, there is a tradeoff between the information rate and power at the receiver. In this case, this experiment is designed to investigating the optimization schemes which is capable of achieving the largest transferred energy and the largest transmitted information rate at the receiver.

In this experiment, the point-to-point communication with Orthogonal Frequency Division Multiplexing (OFDM) in frequency selective channel is assumed. The water filling solution with Karush-Kuhn-Tucker (KKT) conditions is employed as the optimization method in the scenario where the information rate is maximized without the power constraint at the receiver. For the scenario where there is a power constraint at the receiver, a similar optimization method is used to maximize the information rate and achieve the power constraint at the same time. The results indicate that the transmitted power spreads over the sub-channels in OFDM if there is no power constraint at the receiver, while transmitted power is wholly allocated to the sub-channel with the best channel condition if the received power is maximized.

II. BACKGROUND

a) *Discrete-time Baseband Model [2]*

The discrete-time baseband model is widely using in the wireless communication theory, which can be obtained by sampling the continuous baseband model $y_b(t)$. Assuming the equivalent baseband input signal $x_b(t)$ is band-limited to $W/2$, according to the sampling theory, it can be represented as

$$x_b(t) = \sum_n x[n] sinc(Wt - n), \tag{1}$$

where $x[n] = x_b(n/W)$.

The equivalent the baseband received signal has the expression of

$$y_b(t) = \sum_i a_i^b(t) x_b\big(t - \tau_i(t)\big) + w(t), \tag{2}$$

where $a_i^b(t) = a_i(t) e^{-j2\pi f_c \tau_i(t)}$ is the baseband channel response of the i^{th} path and $w(t)$ is Additive White Gaussian Noise (AWGN). By substituting $x_b(t)$ in $y_b(t)$, the $y_b(t)$ can be represented with

$$y_b(t) = \sum_n x[n] \sum_i a_i^b(t) sinc(Wt - W\tau_i(t) - n) + w(t). \tag{3}$$

The discrete-time baseband model can be obtained by sampling the $y_b(t)$ with the sampling period $T_s = 1/W$, which has the expression of

$$y[m] = y_b(m/W) = \sum_n x[n] \sum_i a_i^b(m/W) sinc\big(m - n - W\tau_i(m/W)\big) + w[m]. \tag{4}$$

Using the transformation $l \triangleq m - n$, $y[m]$ can be expressed as

$$y[m] = \sum_l x[m-l] \sum_i a_i^b\left(\frac{m}{W}\right) sinc\left(l - W\tau_i\left(\frac{m}{W}\right)\right) + w[m]. \tag{5}$$

Defining

$$h_l[m] = \sum_i a_i^b(m/W) sinc\big(l - W\tau_i(m/W)\big), \tag{6}$$

The equation (5) can be written as

$$y[m] = \sum_l h_l[m] x[m-l] + w[m], \tag{7}$$

where $h_l[m]$ is the l^{th} channel filter tap and $w[m]$ is complex Gaussian noise distributed as $\mathcal{CN}(0, N_0)$. The equation (7) is the discrete-time baseband model.

b) Frequency Selective Channel & Orthogonal Frequency Division Multiplexing [2]

The fading channels can be classified into flat fading channels and frequency-selective channels based on the multipath delay spread T_d, which is defined as

$$T_d \triangleq \max_{i,j} \left|\tau_i(t) - \tau_j(t)\right|. \tag{8}$$

The coherence bandwidth W_c is defined by

$$W_c \triangleq \frac{1}{2T_d}. \tag{9}$$

The signal strength is not changed too much within the coherence bandwidth W_c. If the bandwidth W of the input is much smaller than the coherence bandwidth W_c, the fading channel is a flat fading channel, where the received signal $y[m]$ can be represented with only one channel filter tap.

$$y[m] = h_0[m]x[m] + w[m]. \tag{10}$$

While if the bandwidth W of the input is much larger than the coherence bandwidth W_c, the fading channel is a frequency-selective fading channel, where the received signal $y[m]$ must have the component of several channel filter taps. In the frequency-selective channel, the received signal $y[m]$ has the same equation with equation (7), where $l \geq 1$.

The frequency-selective channel can be divided into several independent sub-carriers, which are capable of achieving the OFDM. It can be proved that the capacity of OFDM is

$$\sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_o}\right) \text{ bits per OFDM symbol,} \tag{11}$$

where N_c is the number of sub-channels in OFDM, $|h_n|^2$ is the channel gain of the n^{th} sub-channel, P_n is the power allocated to the n^{th} sub-channel, and N_0 is the noise power.

c) Lagrangian Methods & Karush-Kuhn-Tucker Conditions [3]

Lagrangian methods with KKT conditions are capable of solving the optimization with inequality constraints. Considering the following optimization problem.

$$\max_x f_0(x) \tag{12}$$

$$s.t.\begin{cases} f_i(x) \le 0, & i = 1, \cdots, m \\ h_i(x) = 0, & i = 1, \cdots, p \end{cases}$$

Its Lagrangian is defined as

$$L(x, \lambda, \nu) = f_0(x) - \sum_{i=1}^{m} \lambda_i f_i(x) - \sum_{i=1}^{p} \nu_i h_i(x). \tag{13}$$

The KKT conditions of the Lagrangian are

$$f_i(x) \le 0, \ \ i = 1, \cdots, m, \tag{14}$$

$$h_i(x) = 0, \ \ i = 1, \cdots, p, \tag{15}$$

$$\lambda_i \ge 0, \ \ i = 1, \cdots, m, \tag{16}$$

$$\lambda_i f_i(x), \ \ i = 1, \cdots, m, \tag{17}$$

$$\nabla f_0(x) - \sum_{i=1}^{m} \lambda_i \nabla f_i(x) - \sum_{i=1}^{p} \nu_i \nabla h_i(x) = 0. \tag{18}$$

If the optimization problem (12) is convex, KKT conditions can be used to address it.

III. OPTIMIZATION FOR TRANSMISSION OF INFORMATION AND POWER

a) Transmission of Information without Power Constraint at Receiver

Assuming the power constraint of OFDM at the transmitter is

$$\sum_{n=0}^{N_c-1} P_n \le P. \tag{19}$$

where P_n is the power allocated to the n^{th} sub-channel is equation (11) and P is maximal power that can be used at the transmitter. By observing the equation (11), all the power P should be used in order to achieve the maximal information rate at the receiver. In this case, the maximization problem of the information rate in equation (11) is given by

$$\max_{P_{1,\cdots P_{N_c-1}}} \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_o}\right)$$
$$s.t.\begin{cases} \sum_{n=0}^{N_c-1} P_n - P = 0 \\ -P_n \le 0, \ \ n = 0, \cdots N_c - 1 \end{cases} \tag{20}$$

Theorem 1. The optimization problem (20) can be solved using the following Lagrangian

$$L(\lambda, P_1, \cdots, P_{N_c-1}) = \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_o}\right) - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right), \tag{21}$$

with the following KKT conditions

$$\frac{\partial L}{\partial P_n} = \begin{cases} 0, & if\ P_n > 0 \\ \leq 0, & if\ P_n = 0 \end{cases} \tag{22}$$

Proof: Please refer to Appendix A ■

Corollary 1. According to the KKT conditions (22), The maximum information rate under transmitted power constraint can be achieved by allocating the n^{th} sub-channel with the power

$$P_n = \begin{cases} \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2}, & if\ \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2} > 0 \\ 0, & if\ \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2} \leq 0 \end{cases} \tag{23}$$

λ can be calculated by

$$\lambda = \frac{N_c}{\left(P + N_0 \sum_{n \in M} \frac{1}{|h_n|^2}\right) ln2} \tag{24}$$

where $M = \{m \mid P_m \neq 0\}$.

Proof: Please refer to Appendix B ■

Remark 1. *The results of (23) demonstrate that for a specific* λ *and* N_0*, the sub-channels with the better channel states (i.e. lager value of* $|h_n|^2$*) will be allocated with more power. If the channel state of a sub-channel is too bad such that* $1/ln2 - N_0/|h_n|^2 < 0$*, this sub-channel is allocated with no power.*

Remark 2. *In the results of (23),* $N_0/|h_n|^2$ *is the noise-to-carrier ratio, which is capable of indicating the channel state of sub-channels. For those sub-channels which are allocated with power, the sum of allocated power* P_n *and the noise-to-carrier ratio is a constant value* $1/(\lambda ln2)$*, which can be represented with a water filling diagram.*

b) Transmission of Information with Power Constraint at Receiver

The communication signal can not only transmit information but also provide power for some functions at the receiver. For received signal y_n, it has a power of

$$\sum_{n=0}^{N_c-1} \mathbb{E}[|y_n|^2] = \sum_{n=0}^{N_c-1} |h_n|^2 P_n + N_0. \tag{25}$$

If the power required by the functions at the receiver is P_d, the optimization problem is given by

$$\max_{P_1,\cdots P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_o}\right)$$

$$s.t.\begin{cases}\sum_{n=0}^{N_c-1} P_n - P = 0 \\ \sum_{n=0}^{N_c-1} \mathbb{E}[|y_n|^2] - P_d = 0 \\ -P_n \le 0,\ \ n=0,\cdots N_c-1\end{cases} \tag{26}$$

Theorem 2. The optimization problem (26) can be solved using the following Lagrangian

$$\begin{aligned} L(\lambda,\mu,P_1,\cdots,P_{N_c-1}) &= \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_o}\right) - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) - \mu\left(\sum_{n=0}^{N_c-1} \mathbb{E}[|y_n|^2] - P_d\right) \\ &= \sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_o}\right) - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) - \mu\left(-\sum_{n=0}^{N_c-1} |h_n|^2 P_n - N_0 - P_d\right) \end{aligned} \tag{27}$$

with the following KKT conditions

$$\frac{\partial L}{\partial P_n} = \begin{cases} 0, & if\ P_n > 0 \\ \le 0, & if\ P_n = 0 \end{cases} \tag{28}$$

Proof: Theorem 2 can be proved using a similar method as Theorem 1 ■

Corollary 2. According to the KKT conditions (28), The maximum information rate under transmitted power constraint and received power constraint can be achieved by allocating the n^{th} sub-channel with the power

$$P_n = \begin{cases} \frac{1}{(\lambda-\mu|h_n|^2)ln2} - \frac{N_0}{|h_n|^2}, & if\ \frac{1}{(\lambda-\mu|h_n|^2)ln2} - \frac{N_0}{|h_n|^2} > 0 \\ 0, & if\ \frac{1}{(\lambda-\mu|h_n|^2)ln2} - \frac{N_0}{|h_n|^2} \le 0 \end{cases} \tag{29}$$

where λ and μ can be calculated according to the two constraints at transmitter and receiver

$$\begin{cases} \sum_{n=0}^{N_c-1} P_n = P \\ \sum_{n=0}^{N_c-1} |h_n|^2 P_n + N_0 = P_d \end{cases} \tag{30}$$

Proof: Corollary 2 can be proved using a similar method as Corollary 1 ■

Remark 3. *If $\mu = 0$, which means there is no received power constraint in the Lagrangian (27), the results of (29) are the same as the results of (23).*

Remark 4. *It is difficult to obtain a closed-form expression of* λ *and* μ *based on the constraints (30). However, there is still rough relationships of* λ, μ, *and* P_d. *By observing (29) and (30), it can be concluded that for a fixed* λ, *if the value of* P_d *increases, the value of* μ *also increases, but* $(\lambda - \mu|h_n|^2) > 0$ *should be guaranteed. Therefore, the impact of* P_d *on the power allocation can be investigated by changing the values of* λ *and* μ.

Proposition 1. Without considering the information rate, if P_d gets the maximum achievable value, all the power is allocated to the sub-channel with the best channel state.

Proof: Assuming the k^{th} *sub-channel has the largest channel gain* $|h_k|^2$, *we have*

$$P_d = \sum_{n=0}^{N_c-1} |h_n|^2 P_n + N_0 \leq |h_k|^2 \sum_{n=0}^{N_c-1} P_n + N_0 = |h_k|^2 P + N_0 \tag{31}$$

The inequality (31) indicates that the maximum value of P_d *is achieved when all the power is allocated to* k^{th} *sub-channel, which has the best channel state.* ■

IV. NUMERICAL RESULTS & DISCUSSION

In this section, the numerical results are given for the cases where the information is transmitted with or without received power constraints. The number of subchannels is set to $N_c = 10$ and the noise power is set to $N_0 = 1$. The channel vector $\mathbf{h}$ of OFDM is set to

$$h = [.1 + .1i, .2 + .8i, .01 + .2i, .1 + .9i, .3 + .1i, .1 + .7i, .09 + .02i, .1 + .8i, .4 + .8i, .1 + .3i] \tag{32}$$

1) Without Received Power Constraint: In this case, the value of λ in equation (23) is set to $\lambda = 0.28$ and the power allocated to each sub-channel P_n can be calculated. In Fig. 1, the power allocation P_n is plotted with respect to the noise to channel ratio $N_0/|h_n|^2$. The blue and orange blocks represent P_n and $N_0/|h_n|^2$, respectively. We can see that for the sub-channels with better channel states (i.e. lower $N_0/|h_n|^2$), they are allocated with more power; for some sub-channels with very bad channel states, they are allocated with no power. It validates the **Remark 1**. The green line indicates that for those sub-channels that are allocated with power, the sum of P_n and $N_0/|h_n|^2$ is a constant like filling water (power) to a "bowl" made by the noise to channel ratio $N_0/|h_n|^2$. It validates the **Remark 2**.

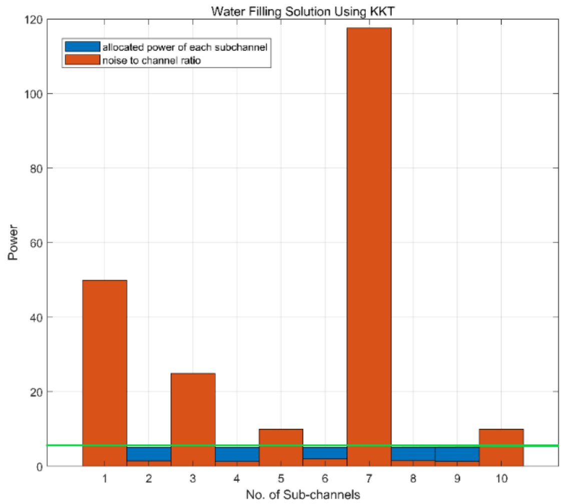

Fig. 1 Power Allocation without Received Power Constraint with respect to Noise to Channel Ratio

2) With Received Power Constraint: In this case, the value of λ in equation (29) is set to $\lambda = 0.2$. According to **Remark 4**, we can change the value of μ to investigate the impact of P_d. In Fig. 2, the power allocations P_n with different P_d are plotted with respect to the noise to channel ratio $N_0/|h_n|^2$. In the condition where $\lambda = 0.2$ and $\mu = 0$, it has a similar water filling figure as the condition without received power constraint, which validates **Remark 3**. With the increase of P_d, we can see that more and more power is concentrated to the 4^{th} sub-channel, which has the best channel state. In the final figure at the right bottom corner, almost all the power is allocated to the 4^{th} sub-channel, which shows the trend indicated by **Proposition 1**.

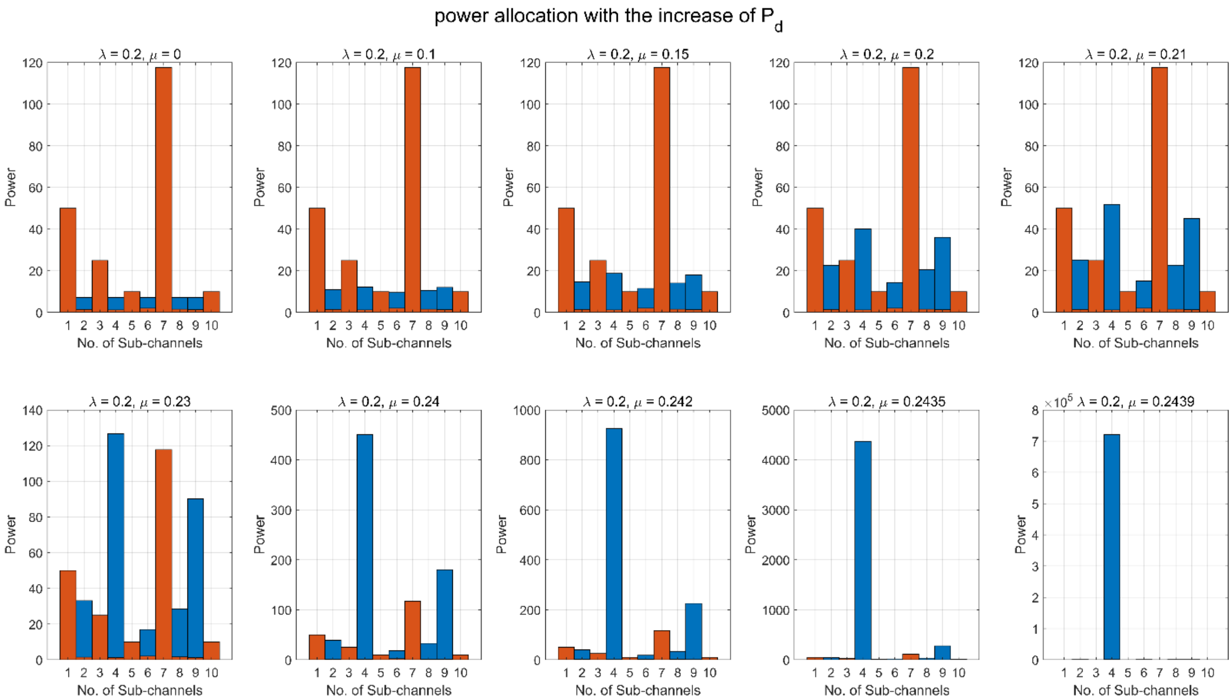

Fig. 2 Power Allocation with Received Power Constraint with respect to Noise to Channel Ratio

V. CONCLUSION

In practice, the received signal is not only the carrier of information but also the source of energy. In this experiment, the optimization methods are investigated to maximize the information rate and achieve the desired power at the receiver. A good method to do this optimization is the Lagrangian method with KKT conditions, which is capable of getting the optimal power allocation to maximize the information rate with the transmitted and received power constraint. This experiment demonstrates that the power spreads over all the sub-channels like water filling if there is no received power constraint, while all the power is concentrated to the best sub-channel if the maximum received power is achieved.

VI. ACKNOWLEDGEMENT

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VII. REFERENCES

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APPENDIX A: PROOF OF THEOREM 1

According to (12) - (18), The Lagrangian of optimization problem (20) is

$$L_o(\lambda, \mu_0, \cdots, \mu_{N_c-1}, P_1, \cdots, P_{N_c-1}) = \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_o}\right) - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) - \sum_{n=0}^{N_c-1} \mu_n(-P_n). \tag{A1}$$

The KKT conditions are

$$\sum_{n=0}^{N_c-1} P_n - P = 0, \tag{A2}$$

$$-P_n \leq 0,\ \ n = 0, \cdots N_c - 1, \tag{A3}$$

$$\mu_n \geq 0,\ \ n = 0, \cdots N_c - 1, \tag{A4}$$

$$\mu_n P_n = 0,\ \ n = 0, \cdots N_c - 1, \tag{A5}$$

$$\nabla_{P_n} L_o = 0, \;\; n = 0, \cdots N_c - 1. \tag{A6}$$

Defining

$$L(\lambda, P_1, \cdots, P_{N_c-1}) = \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|h_n|^2}{N_o}\right) - \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right), \tag{A7}$$

the Lagrangian $L_o(\lambda, \mu_0, \cdots, \mu_{N_c-1}, P_1, \cdots, P_{N_c-1})$ can be expressed as

$$L_o\big(\lambda, \mu_0, \cdots, \mu_{N_c-1}, P_1, \cdots, P_{N_c-1}\big) = L\big(\lambda, P_1, \cdots, P_{N_c-1}\big) - \sum_{n=0}^{N_c-1} \mu_n(-P_n). \tag{A8}$$

Using the equations (A6) and (A8), the following conditions can be obtained.

$$\begin{gathered} \frac{\partial L_0}{\partial P_n} = \frac{\partial L}{\partial P_n} + \mu_n = 0 \\ \frac{\partial L}{\partial P_n} = -\mu_n \end{gathered} \tag{A9}$$

Condition 1. If $P_n = 0$, we have that equation (A5) is always true when $\mu_n \geq 0$. Therefore, according to equation (A9),

$$\frac{\partial L}{\partial P_n} = -\mu_n \leq 0 \tag{A10}$$

Condition 2. If $P_n > 0$, we have that equation (A5) is only true when $\mu_n = 0$. Therefore, according to equation (A9),

$$\frac{\partial L}{\partial P_n} = -\mu_n = 0 \tag{A11}$$

Therefore, based on *Condition 1* and *Condition 2*, the KKT conditions (22) can be derived, and the proof is complete.

APPENDIX B: PROOF OF COROLLARY 1

This is a corollary of Theorem 1. For the condition where $P_n > 0$, we can use equation (A11) to get the expression of P_n. According to (A7) and (A11), we have

$$\begin{gathered} \frac{|h_n|^2}{(N_o + P_n|h_n|^2)ln2} - \lambda = 0 \\ P_n = \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2} \end{gathered} \tag{B1}$$

However, the results of equation (B1) may smaller that zeros. This is the case of (A10), and the actual value is $P_n = 0$. Therefore, we have

$$P_n = \begin{cases} \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2}, & if \;\; \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2} > 0 \\ 0, & if \;\; \frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2} \leq 0 \end{cases} \tag{B2}$$

If all the sub-channels allocated with power are in the set $M = \{m \mid P_m \neq 0\}$, based on the transmitted power constraint $\sum_{n=0}^{N_c-1} P_n = P$ and equation (B1), we have

$$\sum_{n \in M} \left(\frac{1}{\lambda ln2} - \frac{N_0}{|h_n|^2}\right) = P$$

$$\lambda = \frac{N_c}{\left(P + N_0 \sum_{n \in M} \frac{1}{|h_n|^2}\right) ln2} \tag{B3}$$

The proof is complete.