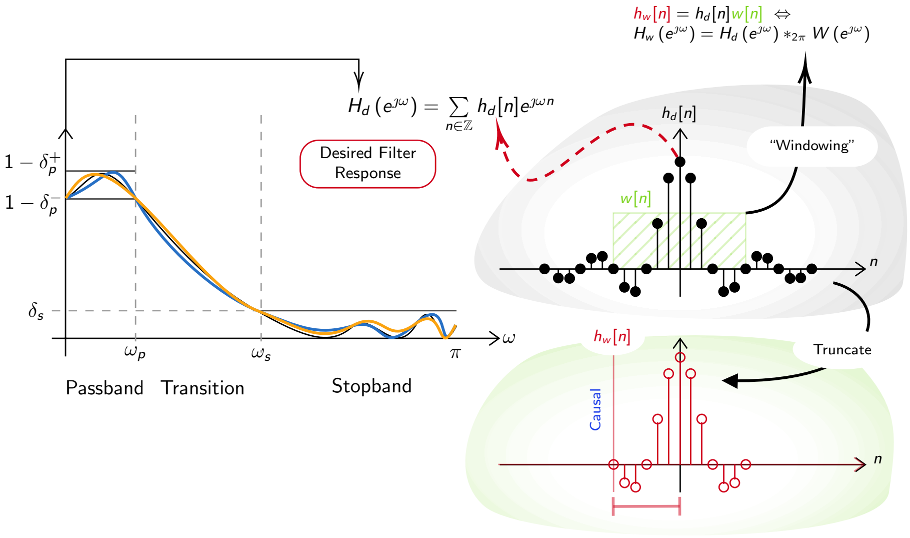
6: Window Filter Design



$h_w[n] = h_d[n]w[n] \Leftrightarrow$
$H_w(e^{\jmath\omega}) = H_d(e^{\jmath\omega}) *_{2\pi} W(e^{\jmath\omega})$
$H_d(e^{\jmath\omega}) = \sum_{n\in\mathbb{Z}} h_d[n]e^{\jmath\omega n}$
$h_d[n]$
$1-\delta_p^+$
$1-\delta_p^-$
Desired Filter Response
"Windowing"
$w[n]$
n
δ_s
ω
ω_p
ω_s
π
$h_w[n]$
Truncate
Passband
Transition
Stopband
Causal
n

Inverse DTFT

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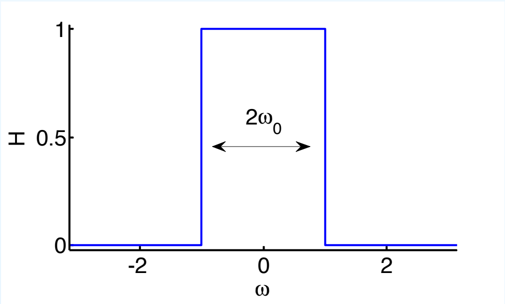

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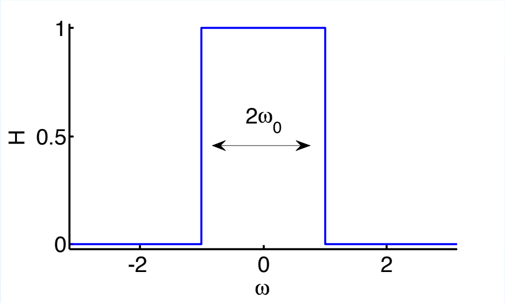

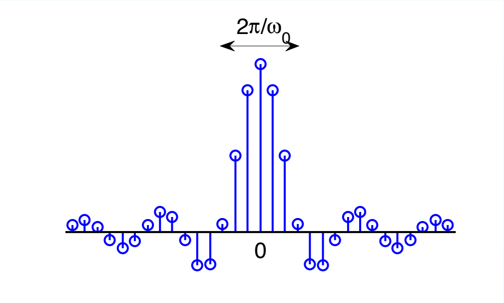

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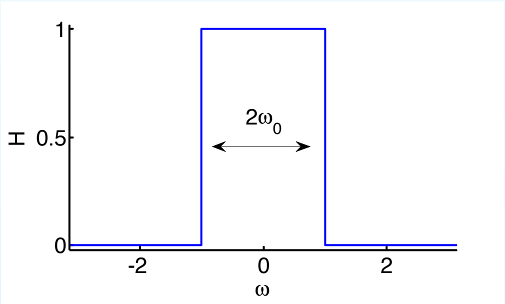

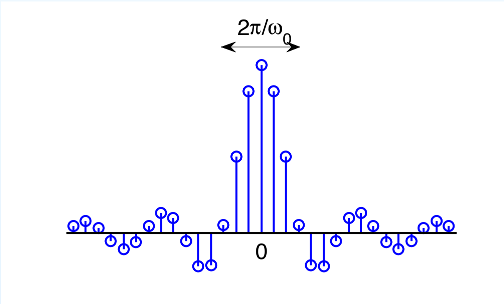

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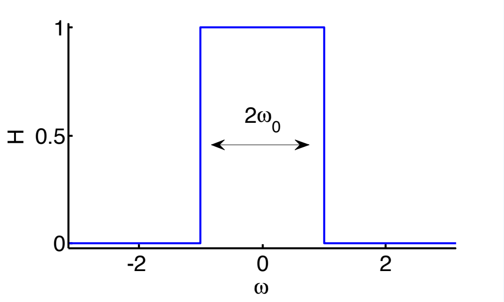

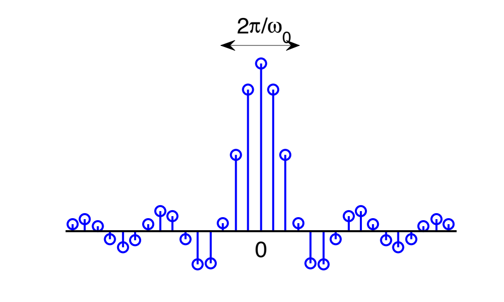

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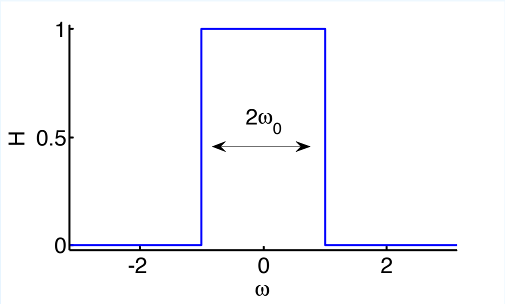

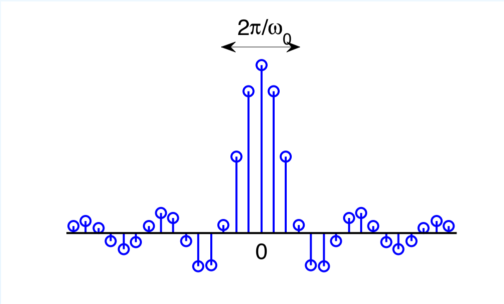

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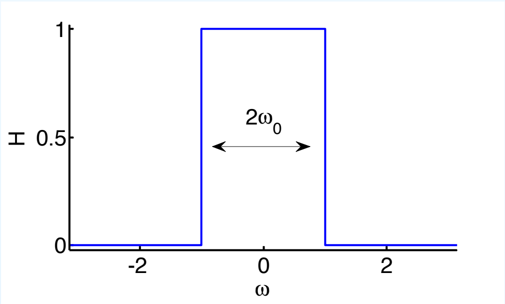

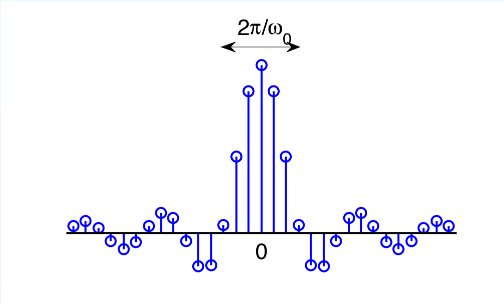

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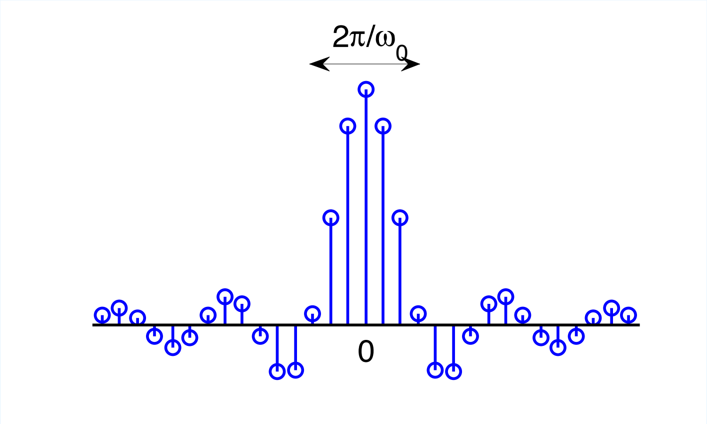
Sadly $h[n]$ is infinite and non-causal. Solution: multiply $h[n]$ by a window

Rectangular window

Truncate to $\pm\frac{M}{2}$ to make finite



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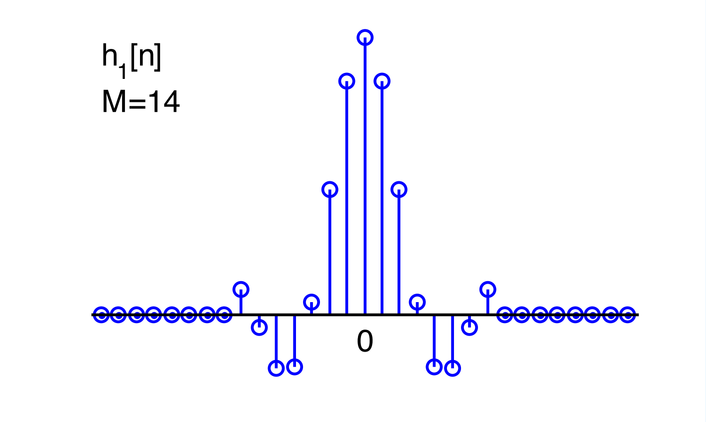

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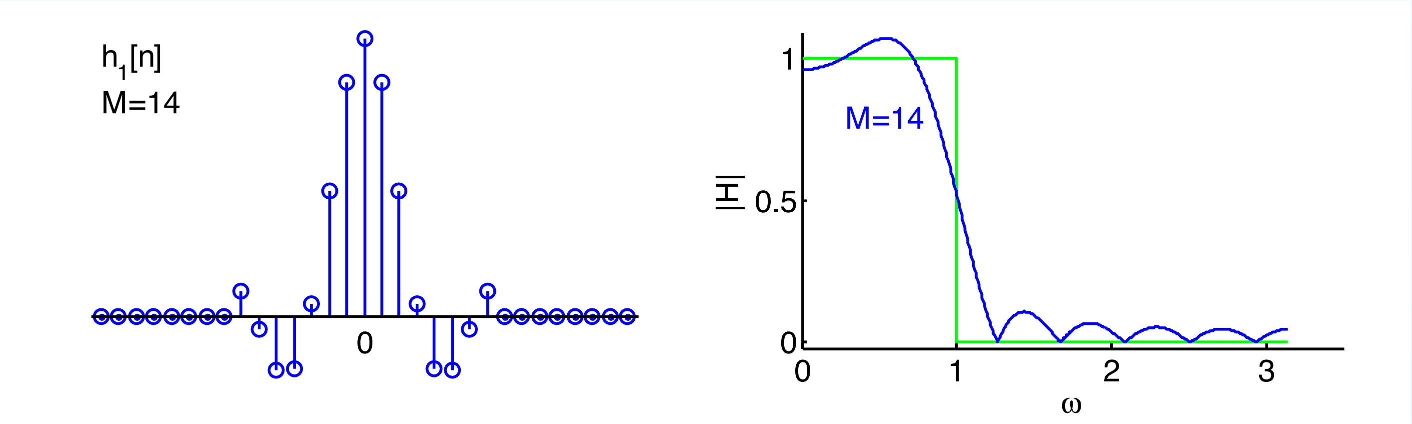
Define mean square error (MSE) in frequency domain

$$E = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega}) - H_1(e^{j\omega})\right|^2 d\omega$$



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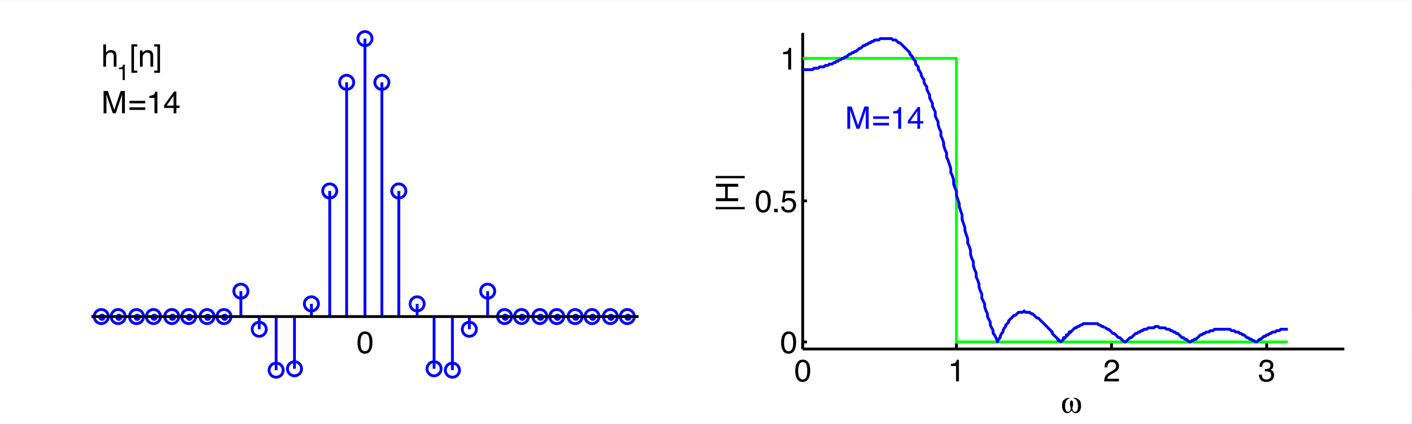

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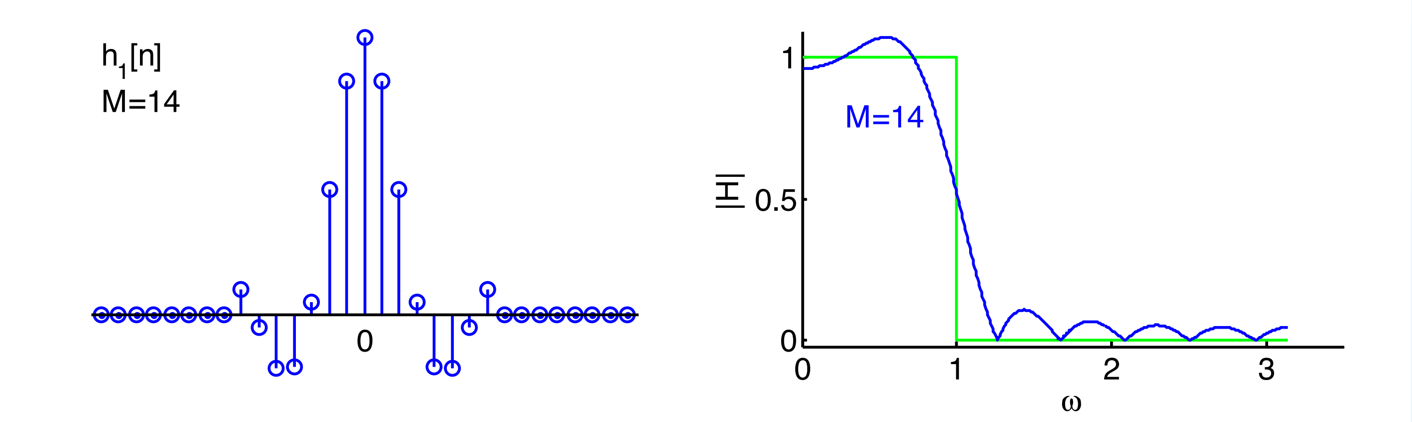

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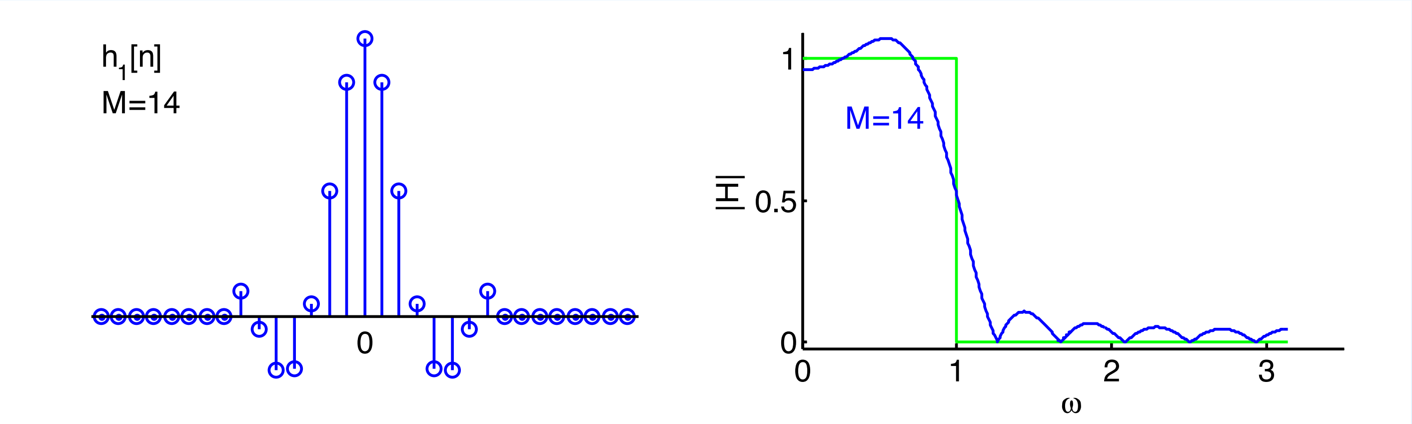

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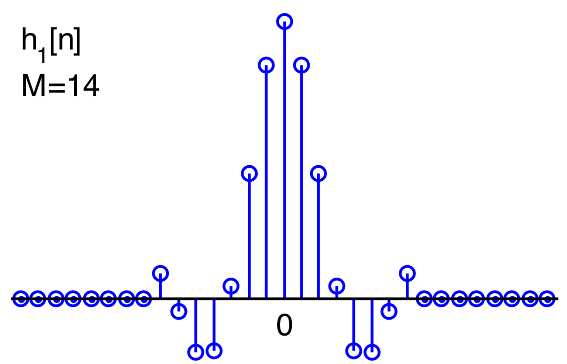

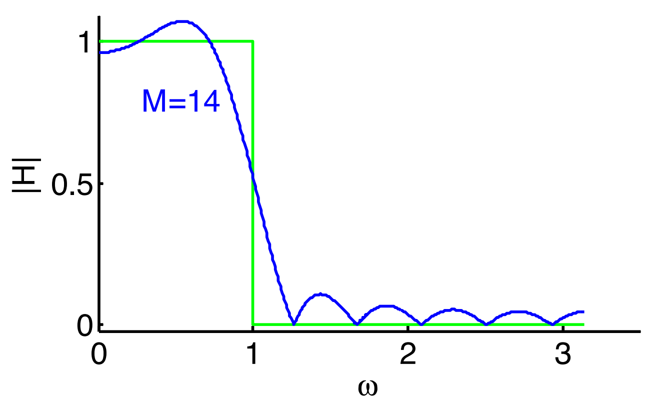

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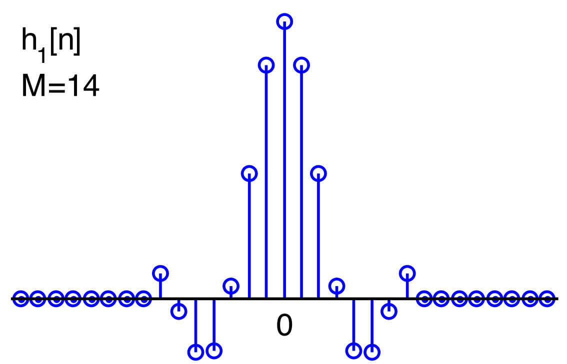

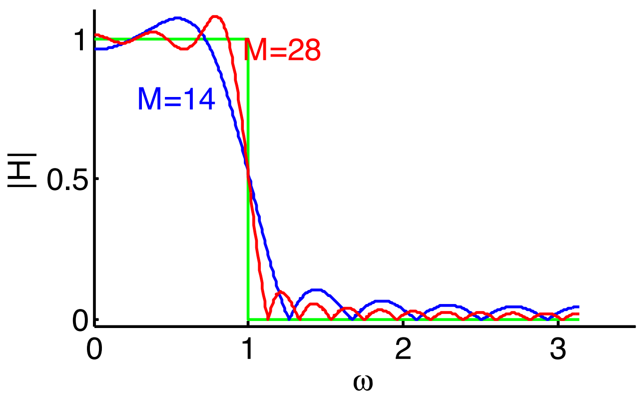

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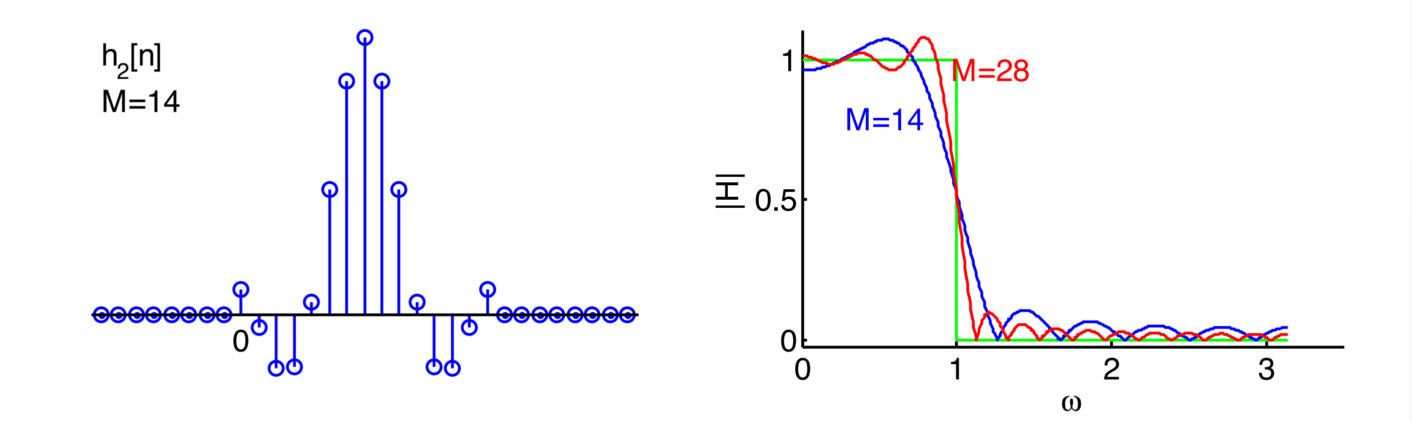

Normal to delay by $\frac{M}{2}$ to make causal. Multiplies $H(e^{j\omega})$ by $e^{-j\frac{M}{2}\omega}$.

Dirichlet Kernel

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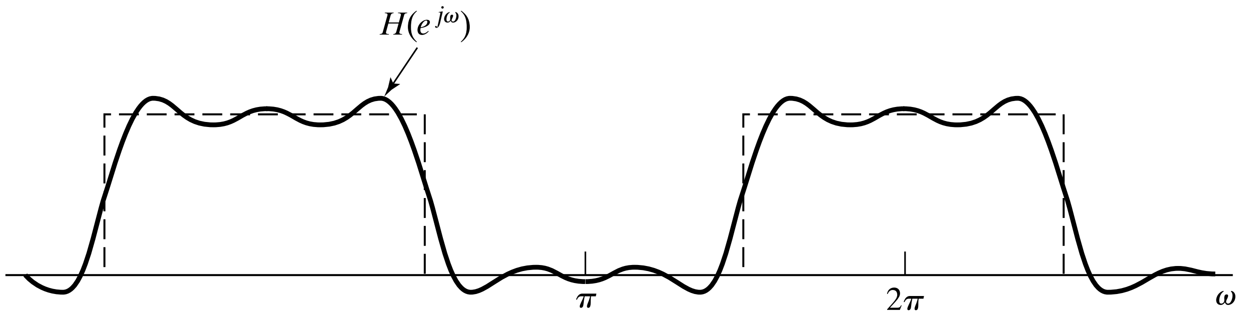
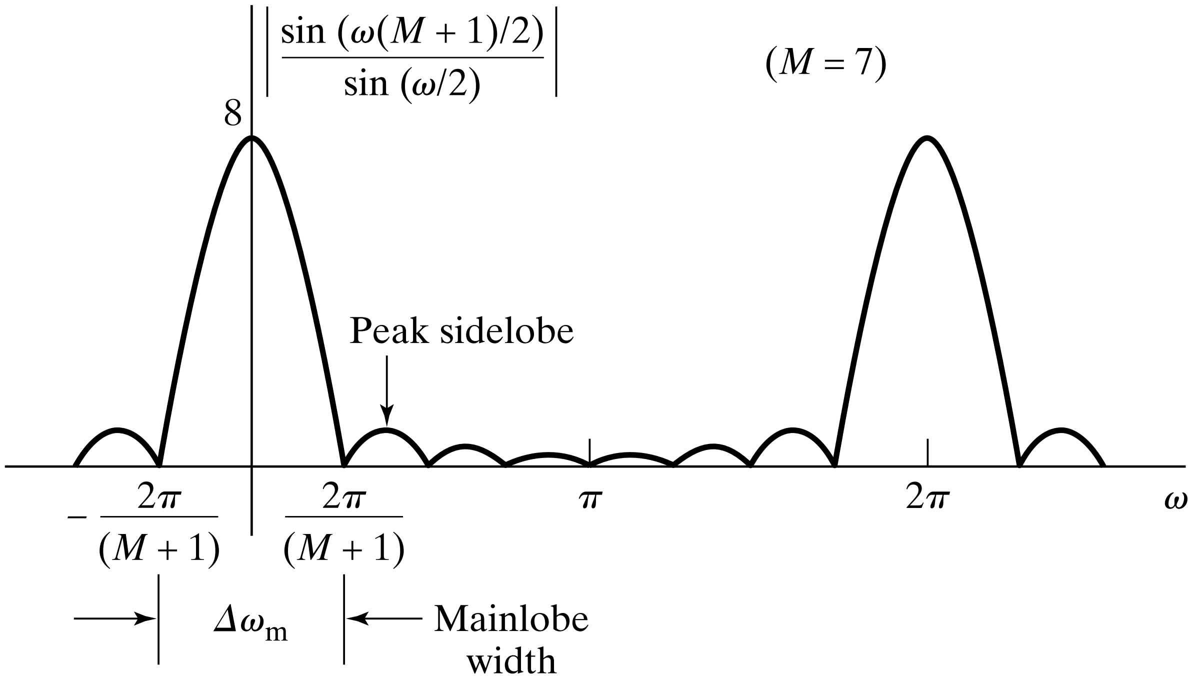


$H(e^{j\omega})$
π
2π
ω



$\left|\frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)}\right|$
$(M = 7)$
8
Peak sidelobe
$-\frac{2\pi}{(M+1)}$
$\frac{2\pi}{(M+1)}$
π
2π
ω
$\Delta\omega_m$
Mainlobe width

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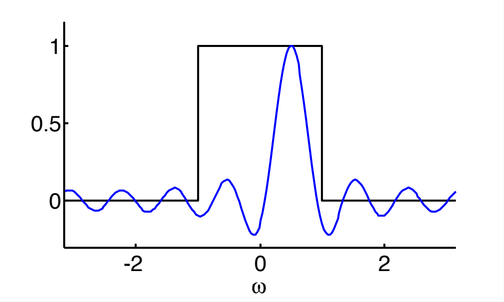

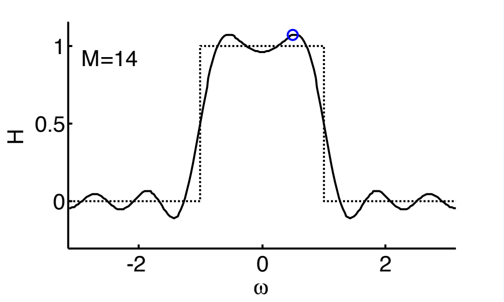

Dirichlet Kernel

Truncation $\Leftrightarrow$ Multiply $h[n]$ by a rectangular window, $w[n] = \delta_{-\frac{M}{2} \le n \le \frac{M}{2}}$

$\Leftrightarrow$ Circular Convolution $H_{M+1}(e^{j\omega}) = \frac{1}{2\pi} H(e^{j\omega}) \circledast W(e^{j\omega})$

$$W(e^{j\omega}) = \sum_{-\frac{M}{2}}^{\frac{M}{2}} e^{-j\omega n} \stackrel{\text{(i)}}{=} 1 + 2\sum_{1}^{0.5M} \cos(n\omega) \stackrel{\text{(ii)}}{=} \frac{\sin 0.5(M+1)\omega}{\sin 0.5\omega}$$

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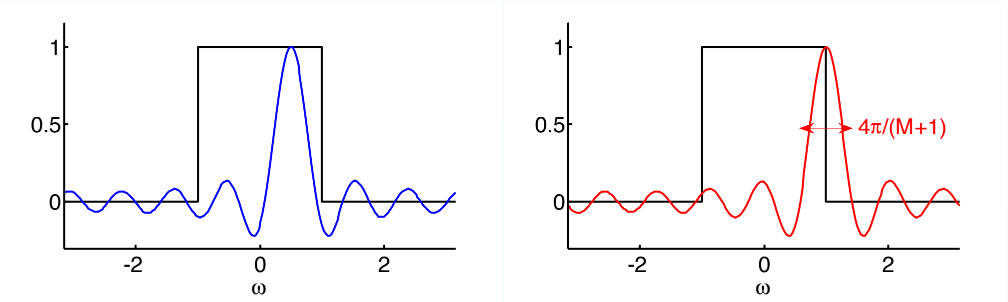

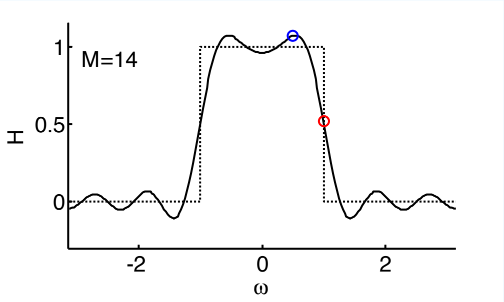

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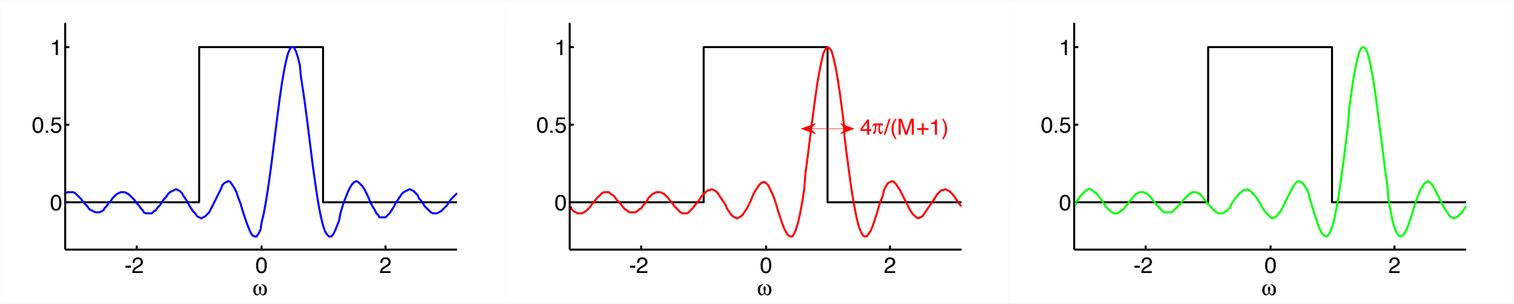

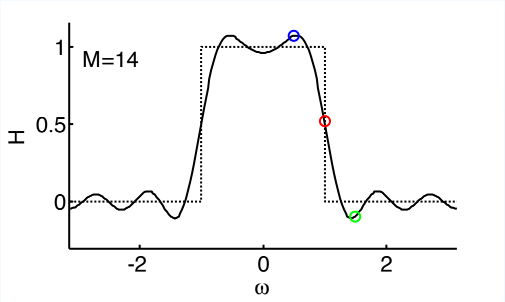

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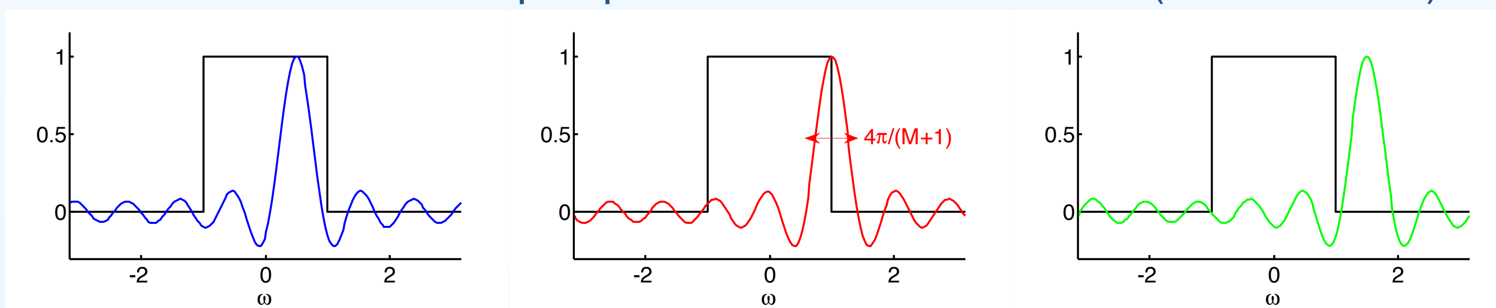

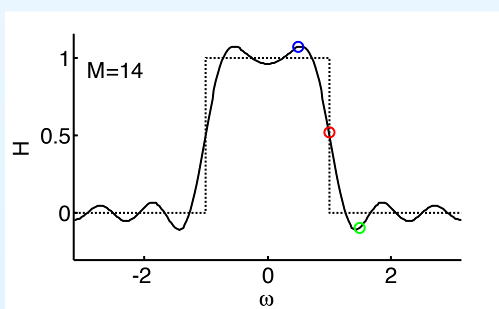

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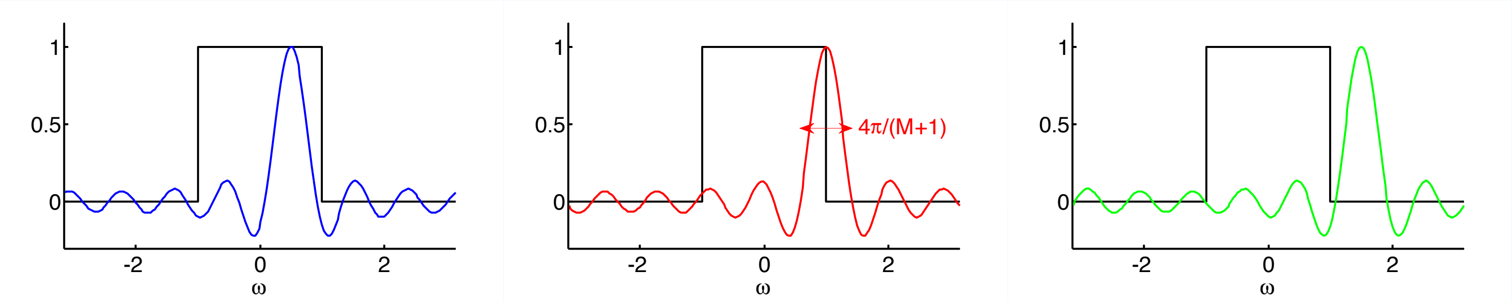

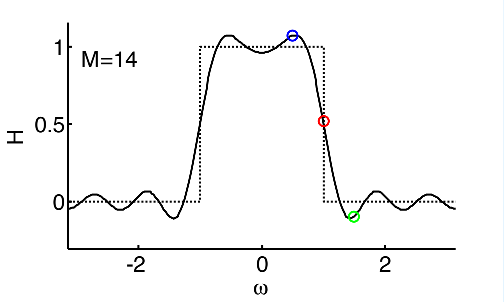

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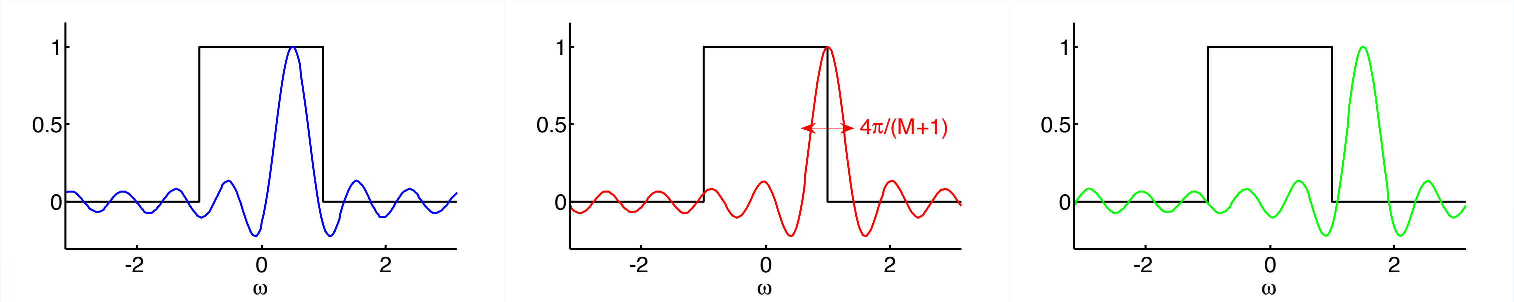

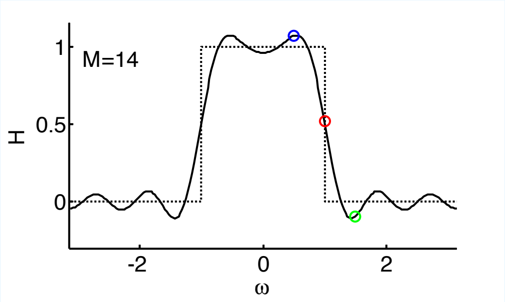

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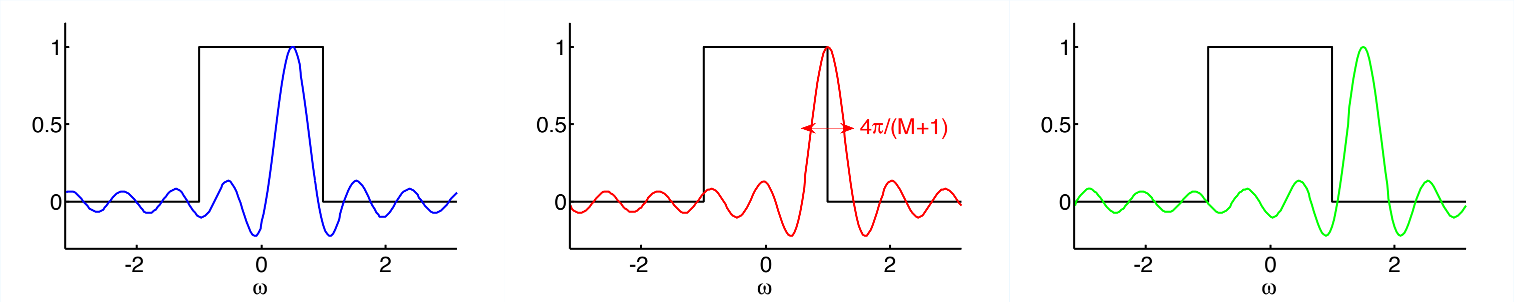

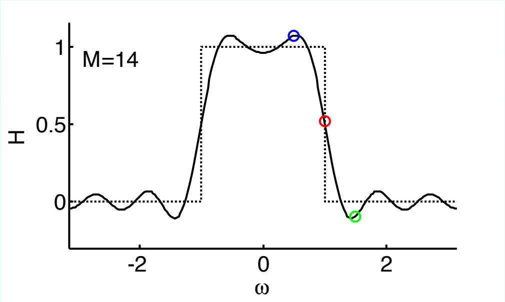

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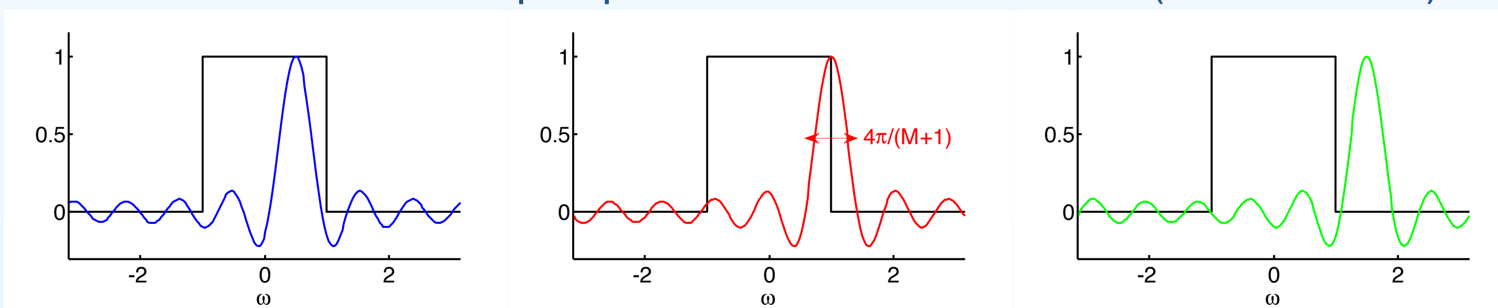

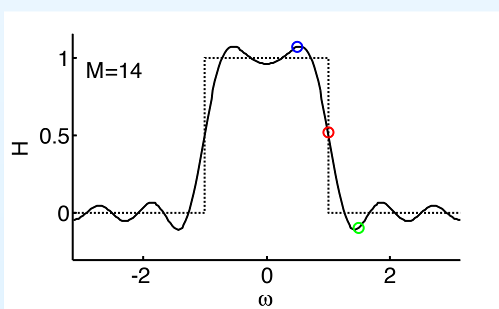

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Transition Gradient: $\left.\frac{d|H|}{d\omega}\right|_{\omega=\omega_0} \approx \frac{M+1}{2\pi}$

Window relationships

When you multiply an impulse response by a window $M+1$ long

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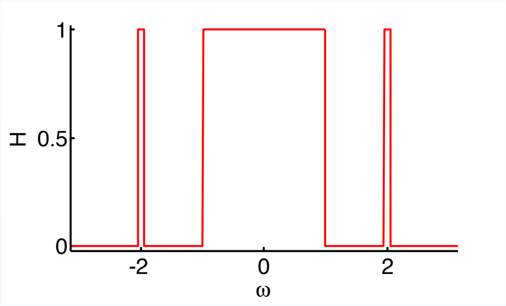

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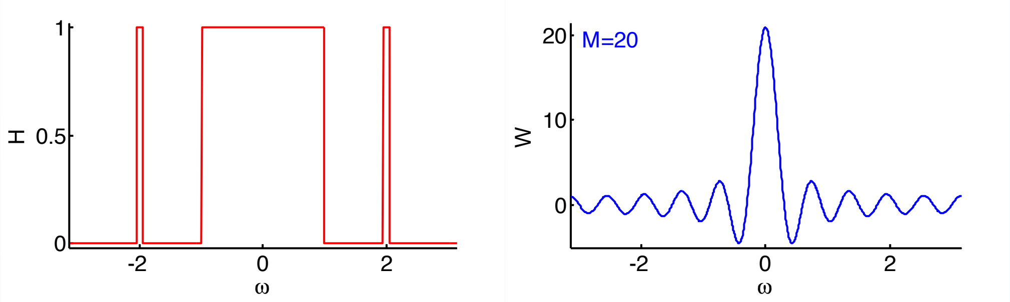

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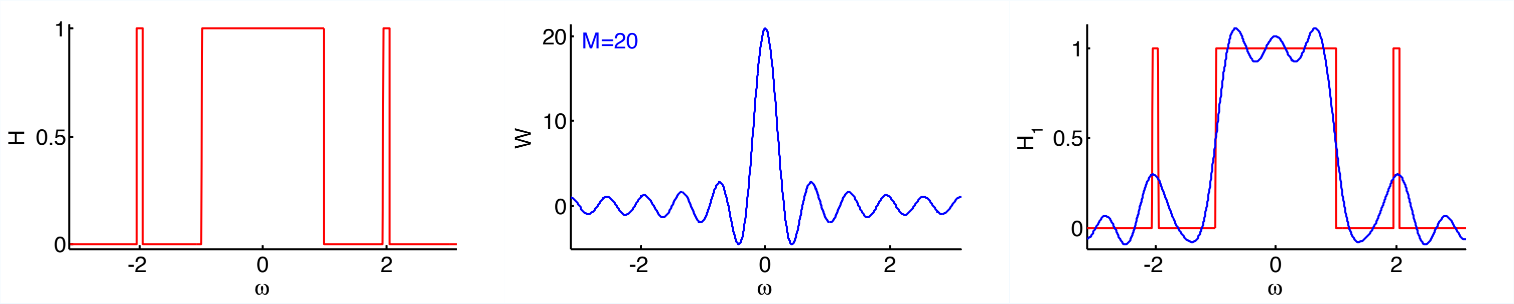

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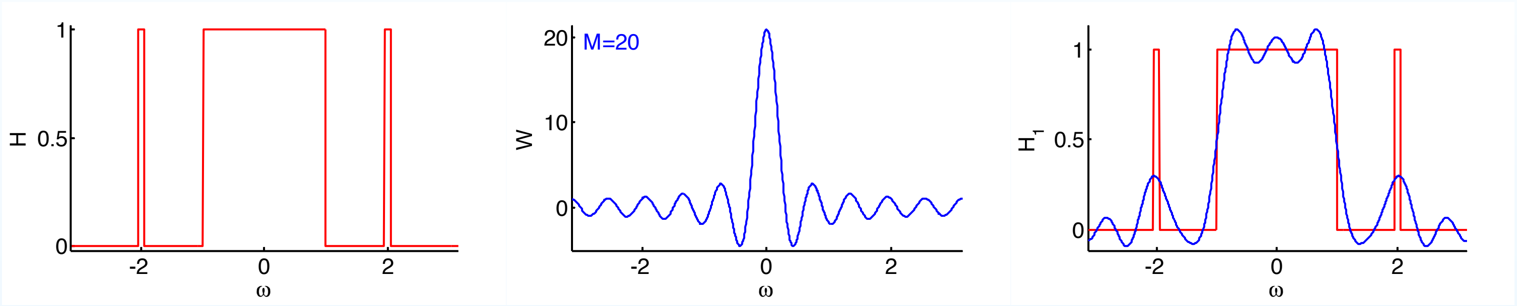

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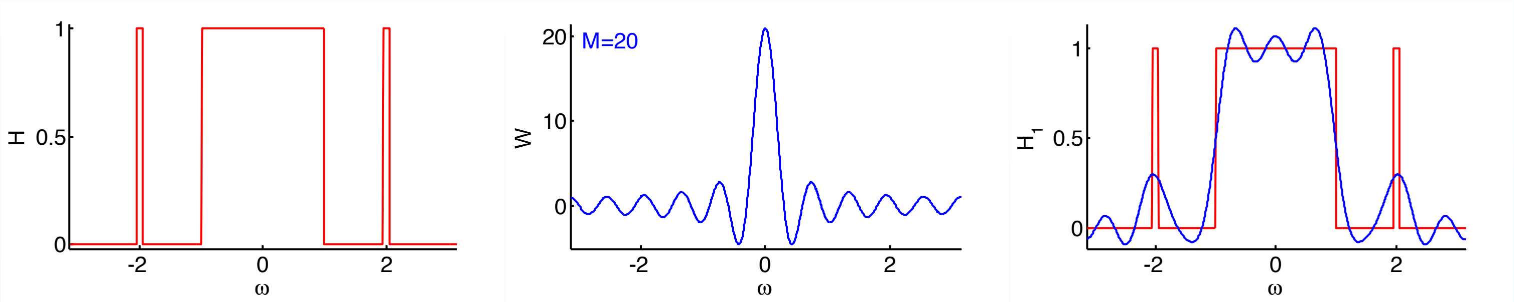

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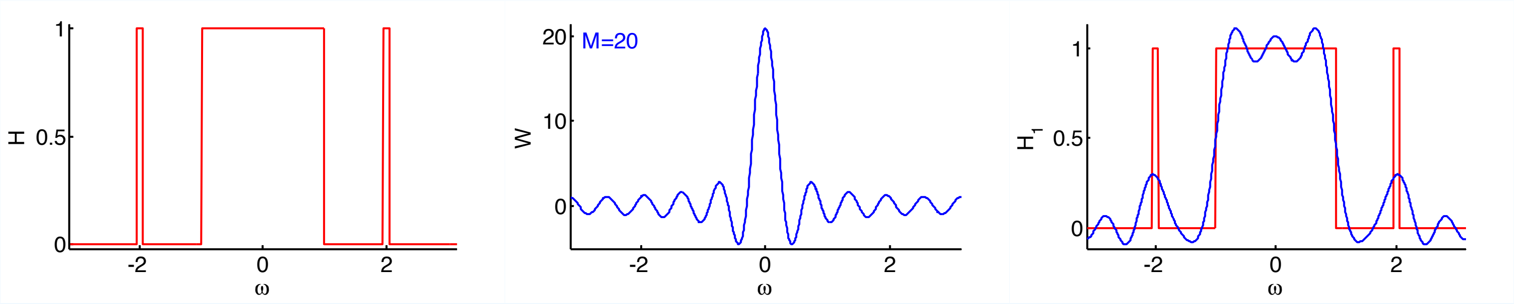

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(b) transition **bandwidth**, $\Delta\omega$ = width of the main lobe

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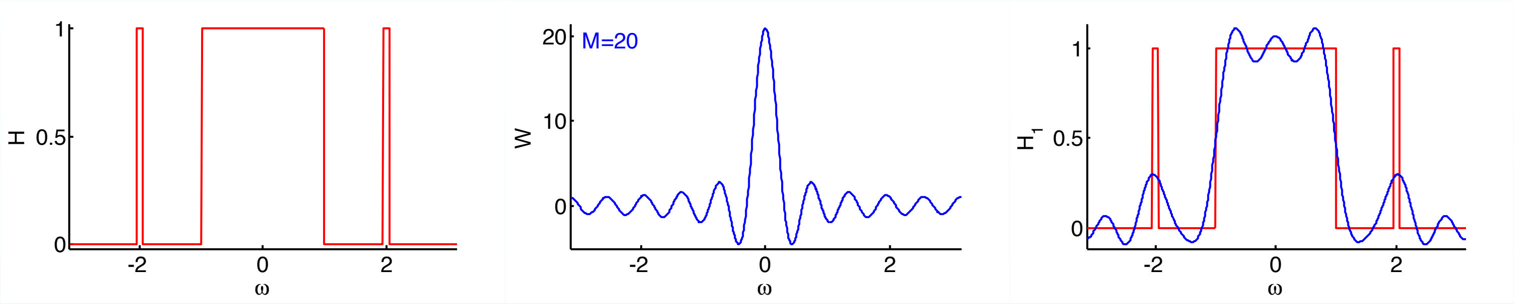

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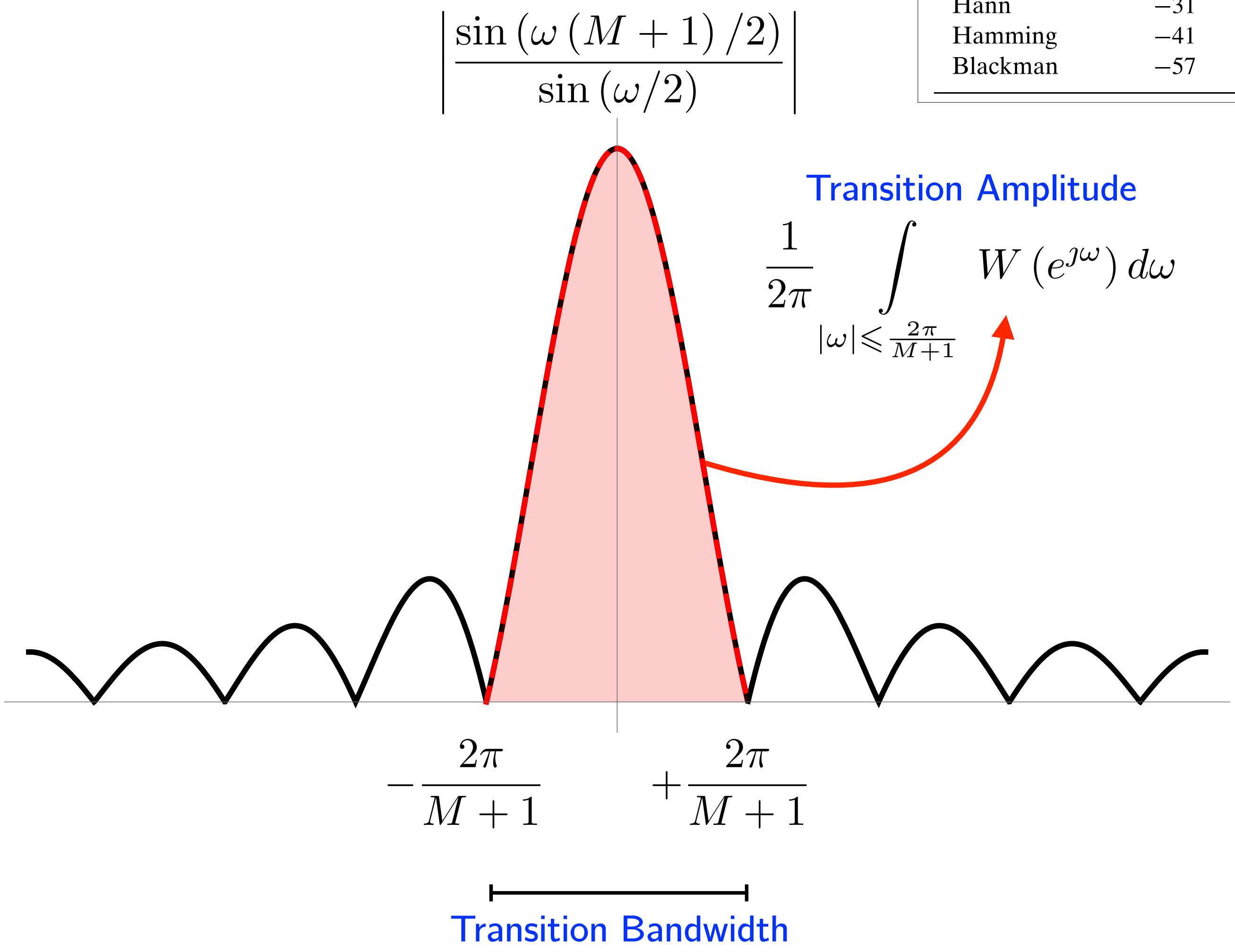
Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	-13	$4\pi/(M+1)$	-21	0	$1.81\pi/M$
Bartlett	-25	$8\pi/M$	-25	1.33	$2.37\pi/M$
Hann	-31	$8\pi/M$	-44	3.86	$5.01\pi/M$
Hamming	-41	$8\pi/M$	-53	4.86	$6.27\pi/M$
Blackman	-57	$12\pi/M$	-74	7.04	$9.19\pi/M$



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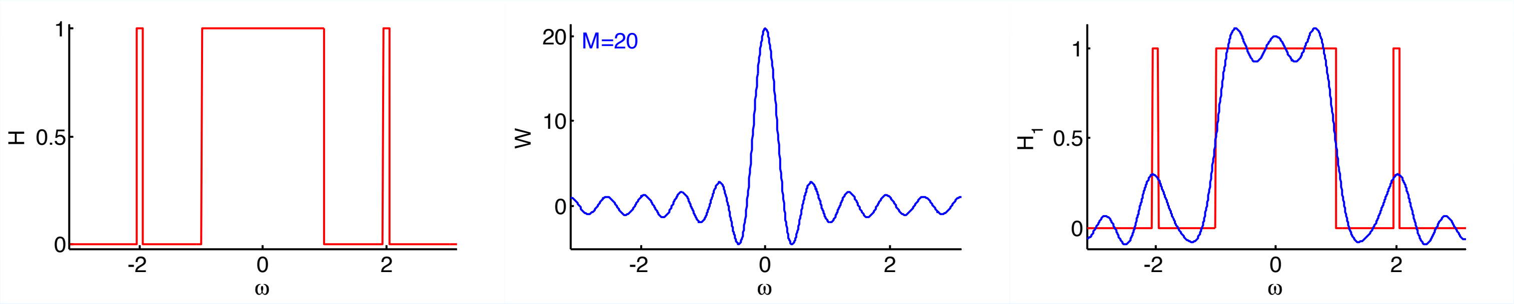

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(c) stopband gain is an integral over oscillating sidelobes of $W(e^{j\omega})$

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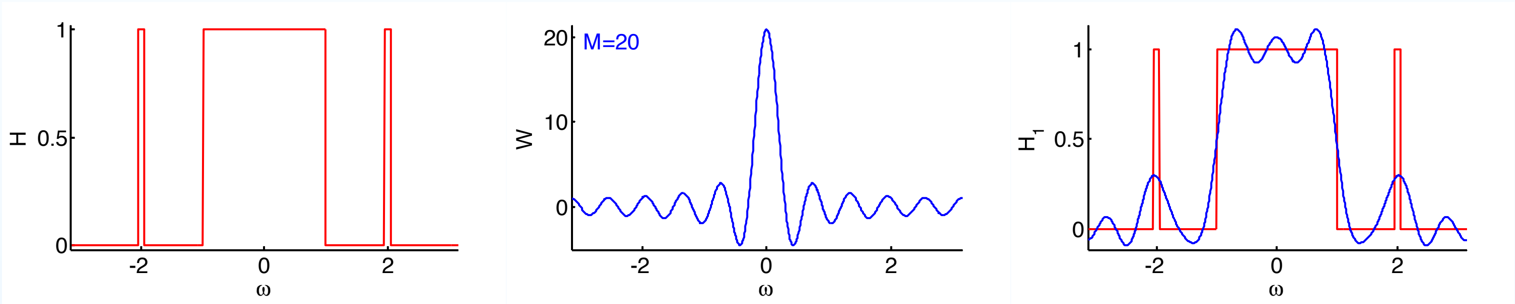

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rect window: $\left|\min H(e^{j\omega})\right| = 0.09 \ll \left|\min W(e^{j\omega})\right| = \frac{M+1}{1.5\pi}$

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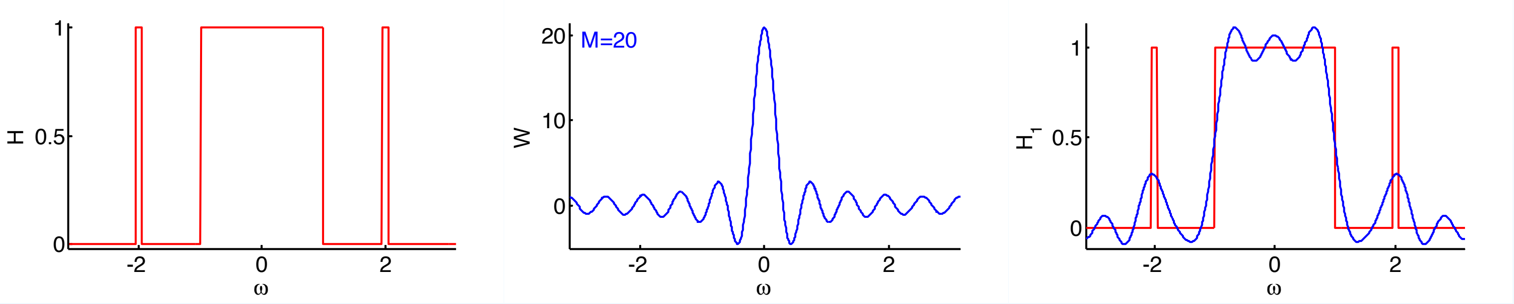

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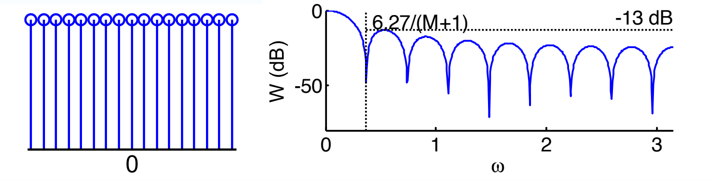
(d) features narrower than the main lobe will be broadened and attenuated

Common Windows

Rectangular: $w[n] \equiv 1$



Common Windows

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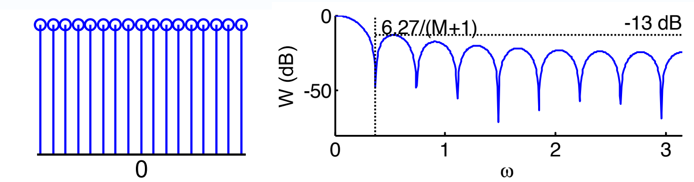


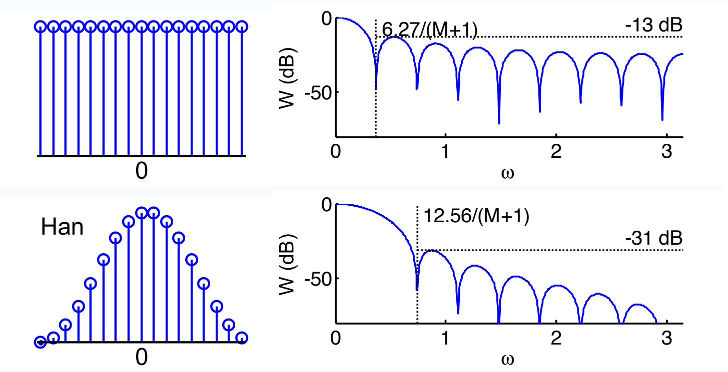
Common Windows

Rectangular: $w[n] \equiv 1$

don't use

Hanning: $0.5 + 0.5c_1$

$c_k = \cos \frac{2\pi kn}{M+1}$



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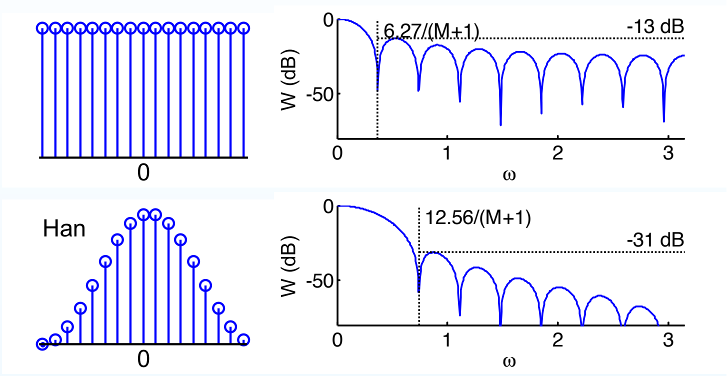
rapid sidelobe decay



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Hanning: $0.5 + 0.5c_1$

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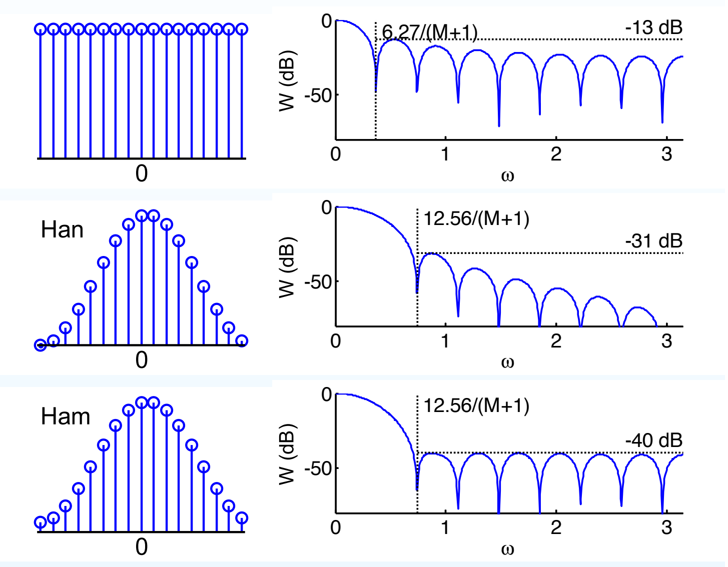
rapid sidelobe decay

Hamming: $0.54 + 0.46c_1$



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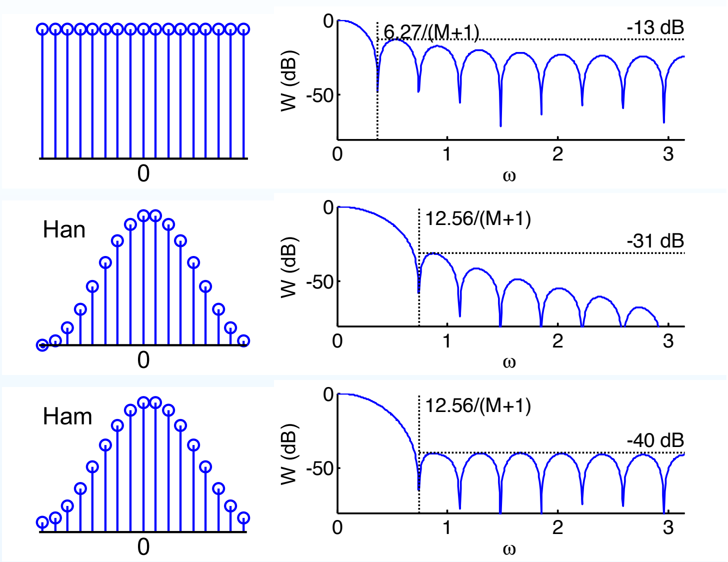
best peak sidelobe



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rapid sidelobe decay

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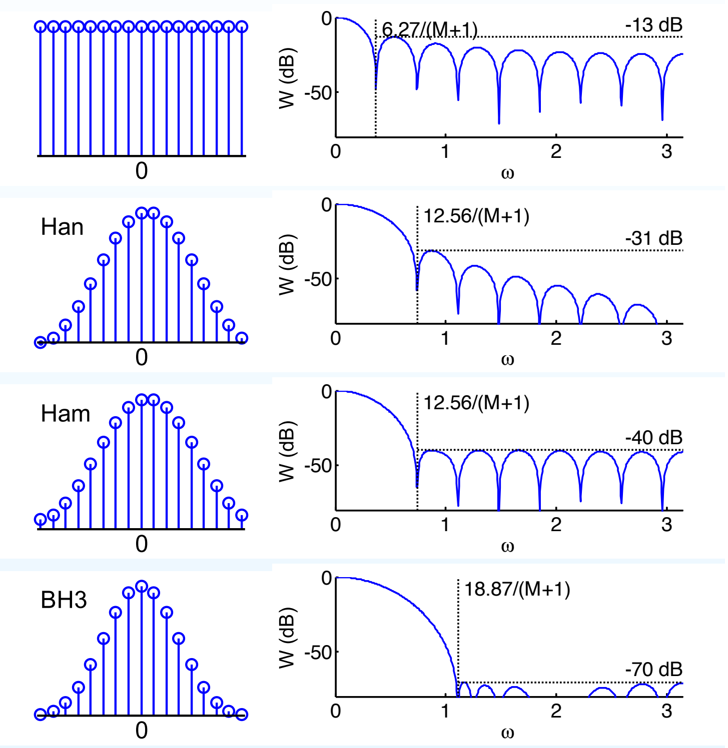
best peak sidelobe

Blackman-Harris 3-term:

$$0.42 + 0.5c_1 + 0.08c_2$$



Common Windows

Rectangular: $w[n] \equiv 1$

don't use

Hanning: $0.5 + 0.5c_1$

$c_k = \cos \frac{2\pi kn}{M+1}$

rapid sidelobe decay

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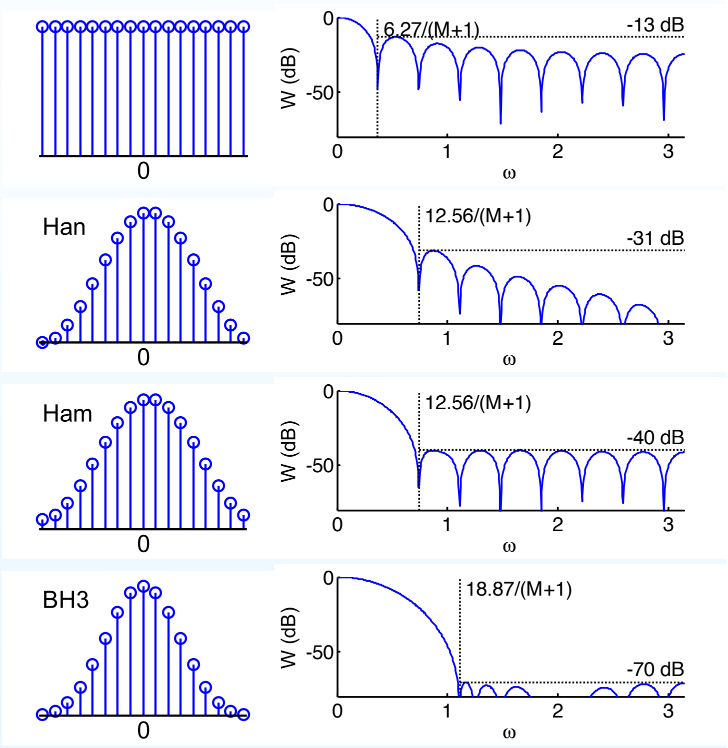

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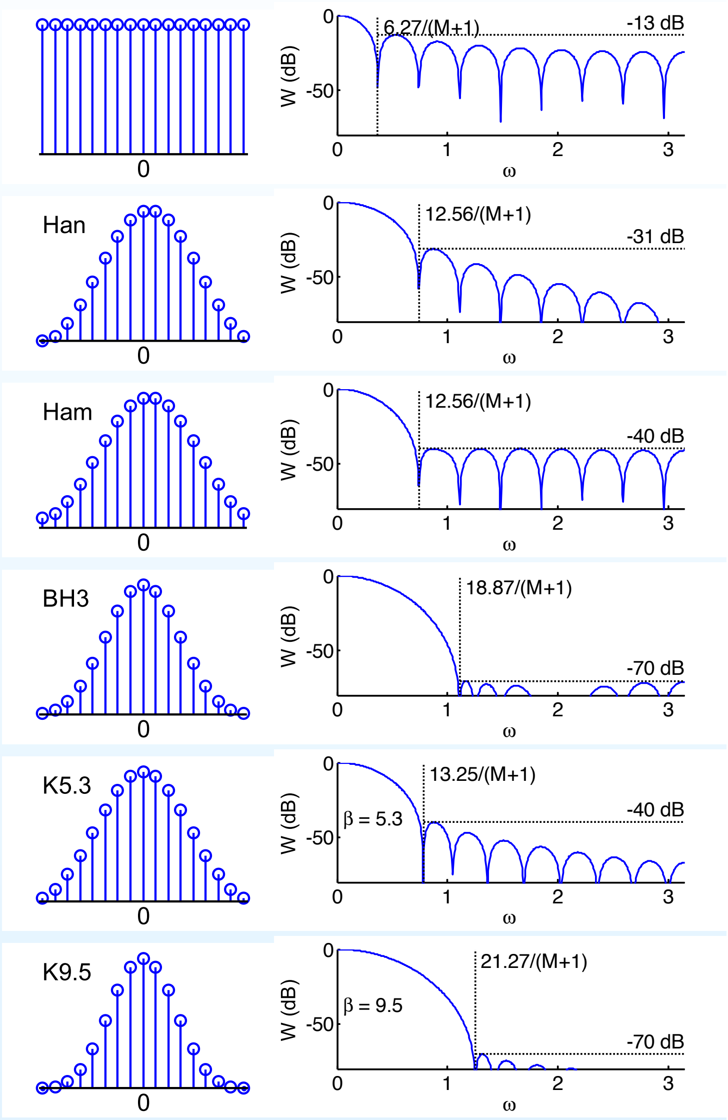

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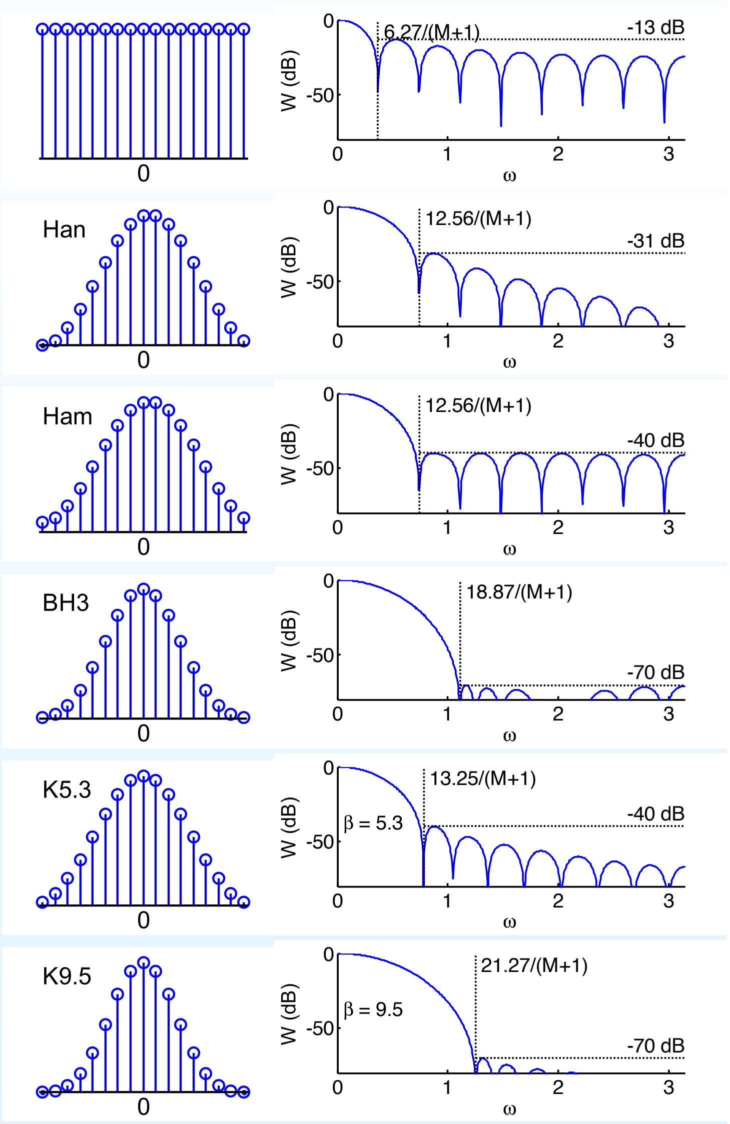
Good compromise:

Width v sidelobe v decay



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Several formulae estimate the required order of a filter, M.

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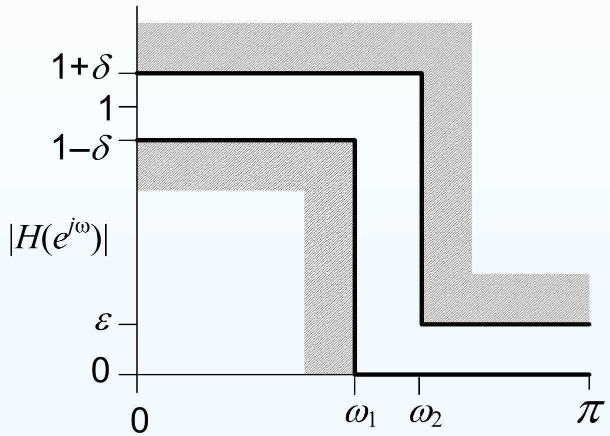

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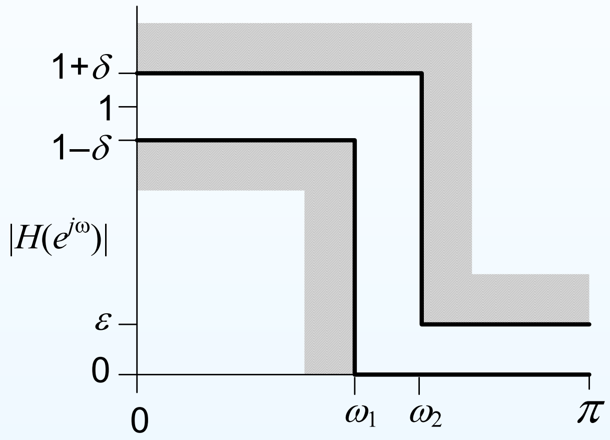

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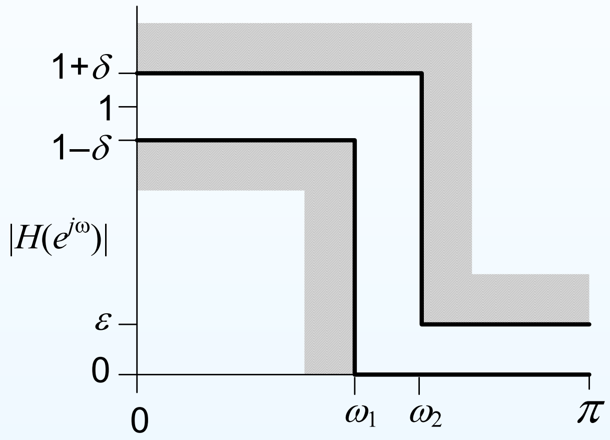

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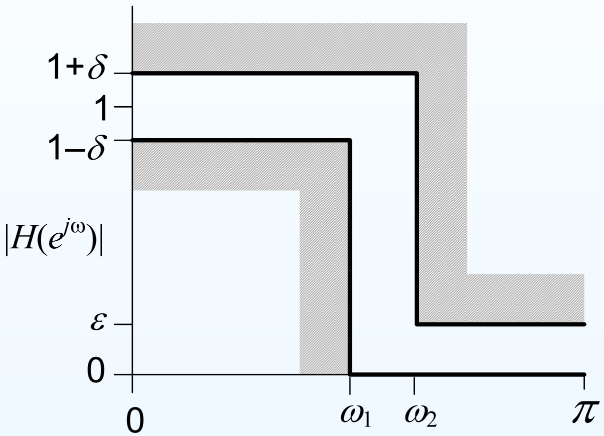

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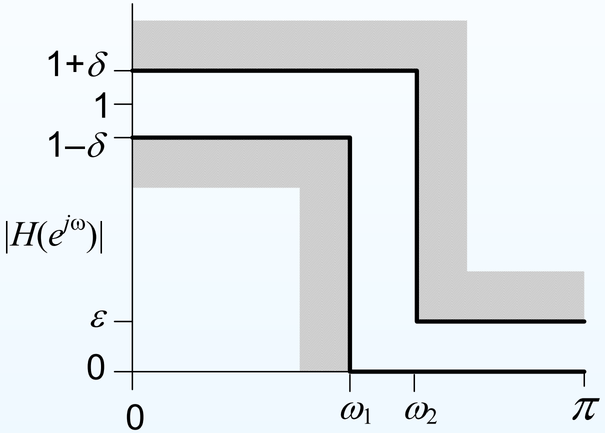

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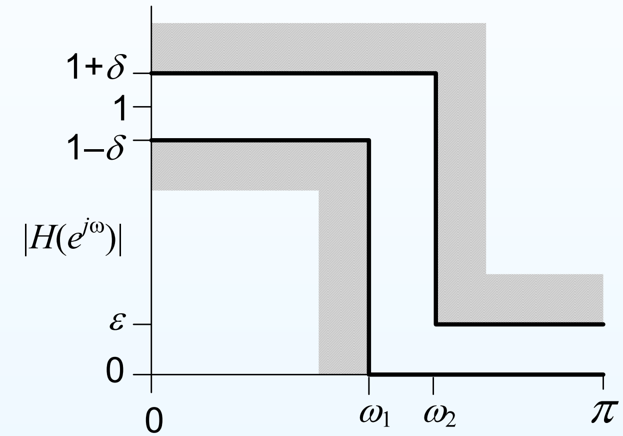

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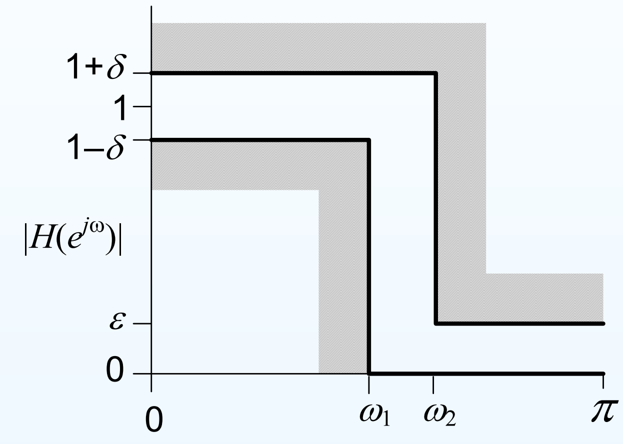

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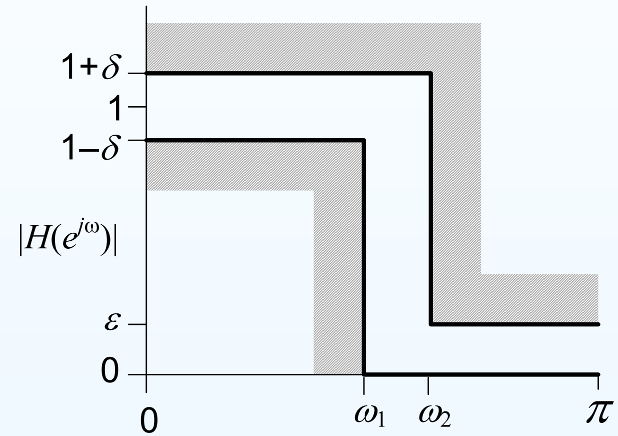

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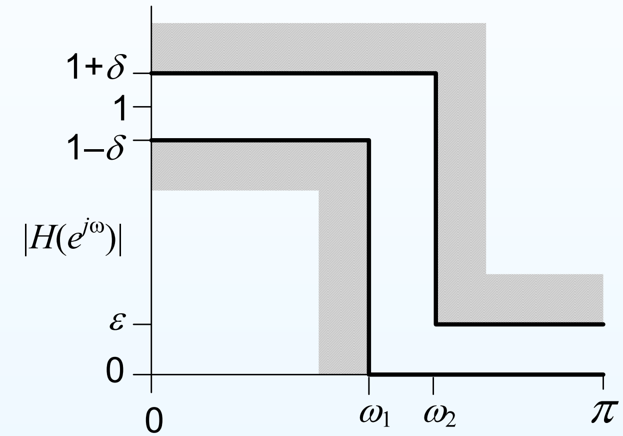

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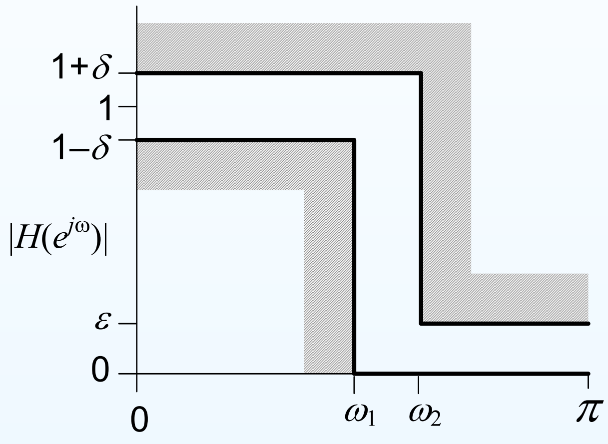

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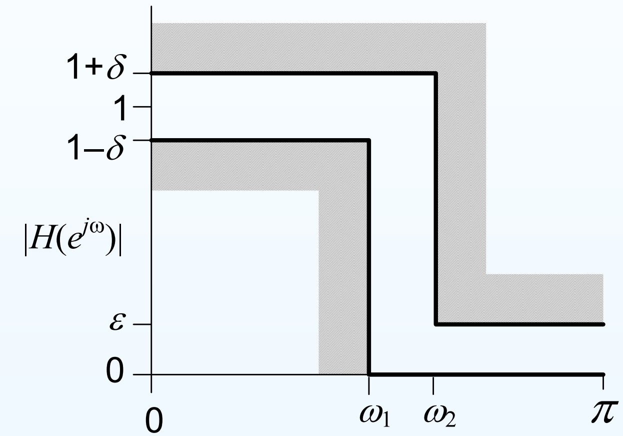

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Only approximate.

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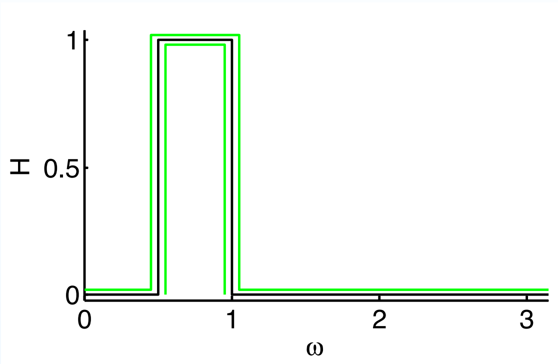
Specifications:

Bandpass: $\omega_1 = 0.5, \omega_2 = 1$



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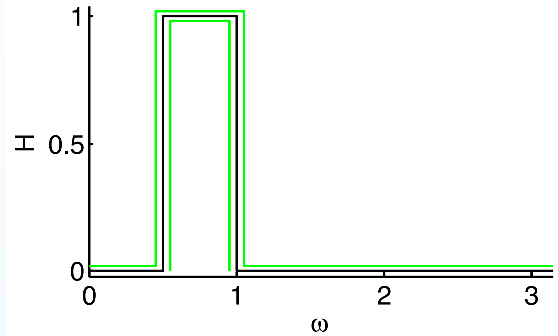

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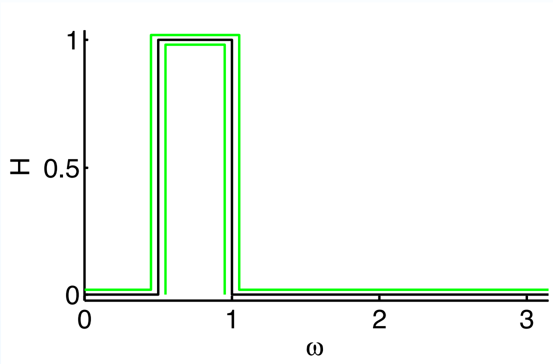

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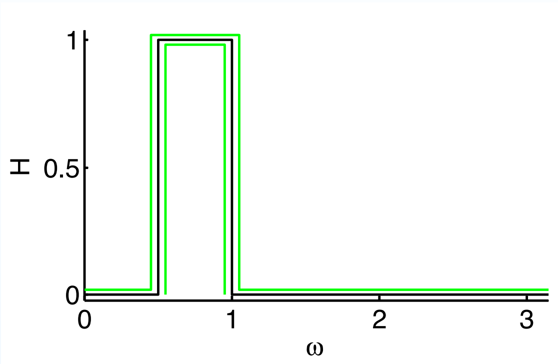

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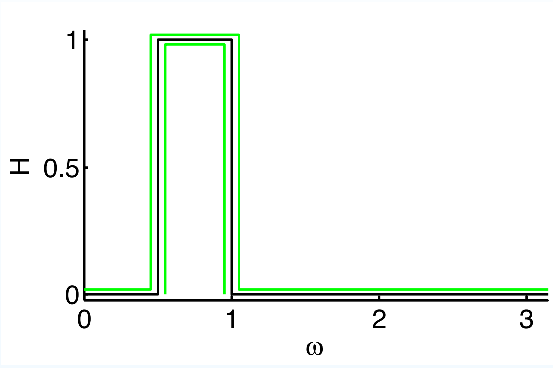

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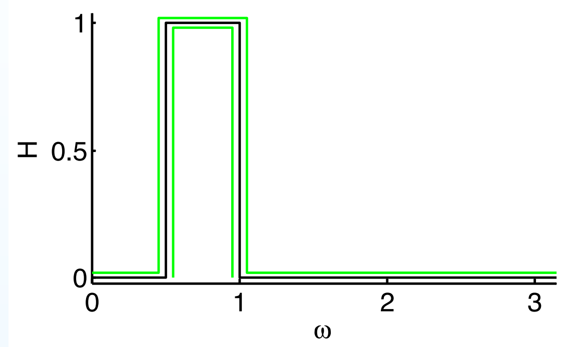

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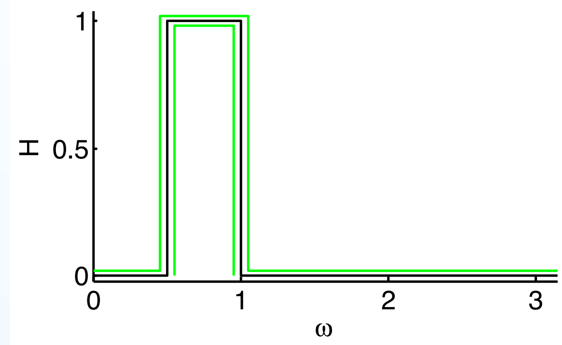

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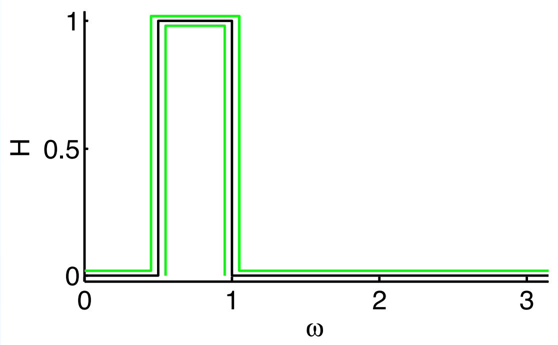

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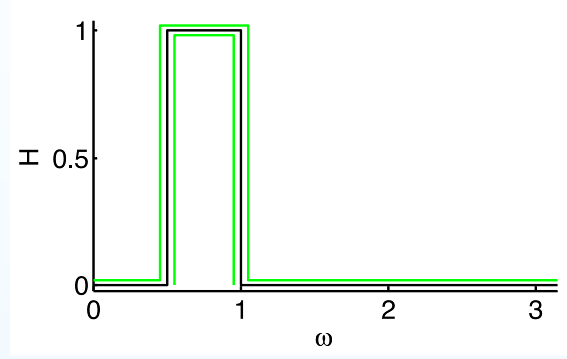

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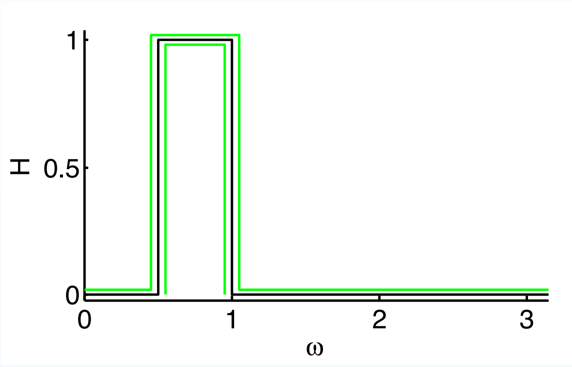

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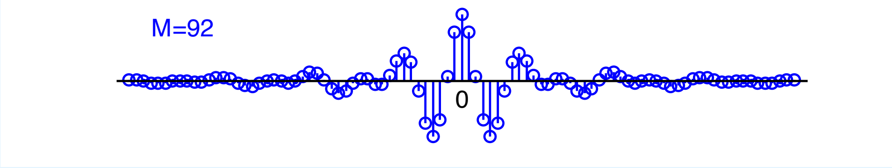

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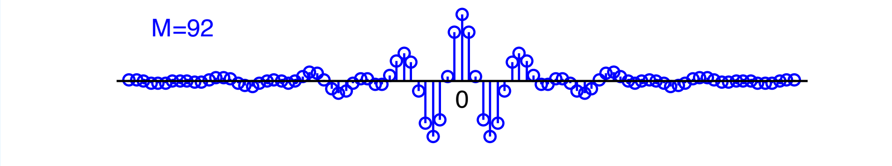

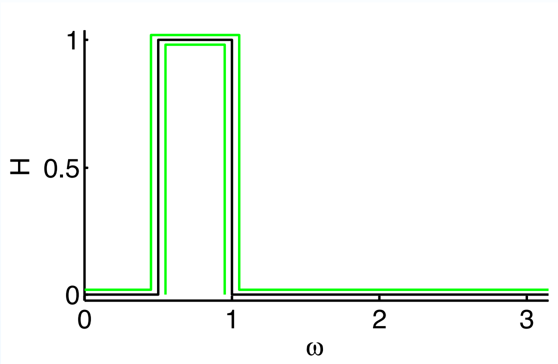

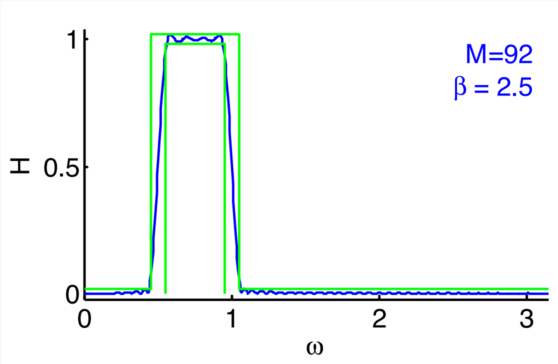

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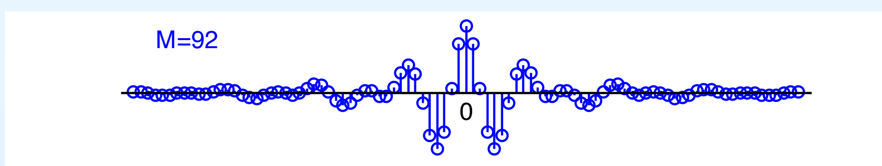

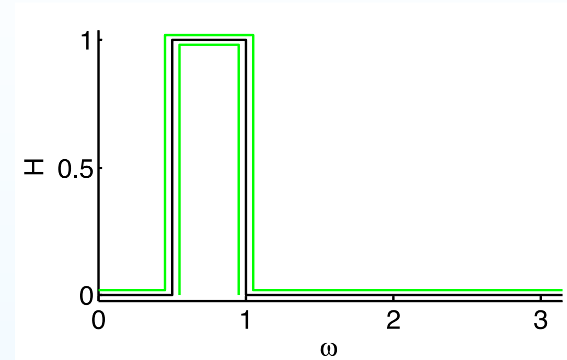

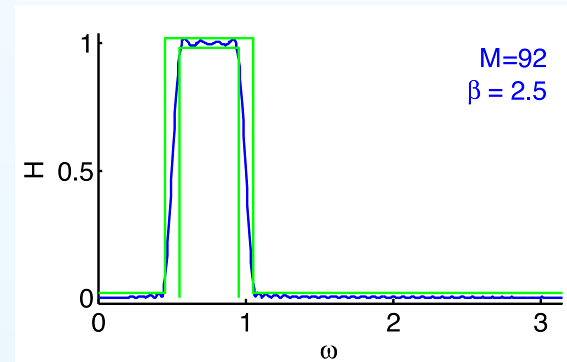

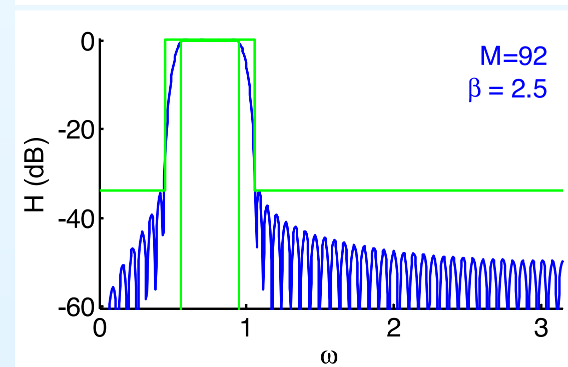

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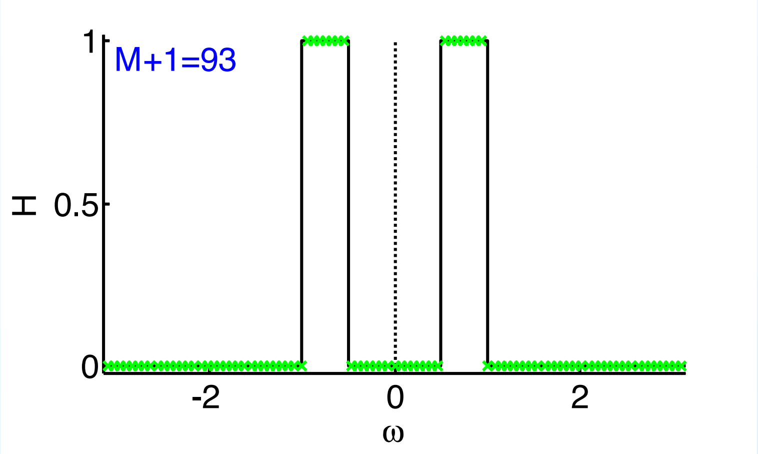
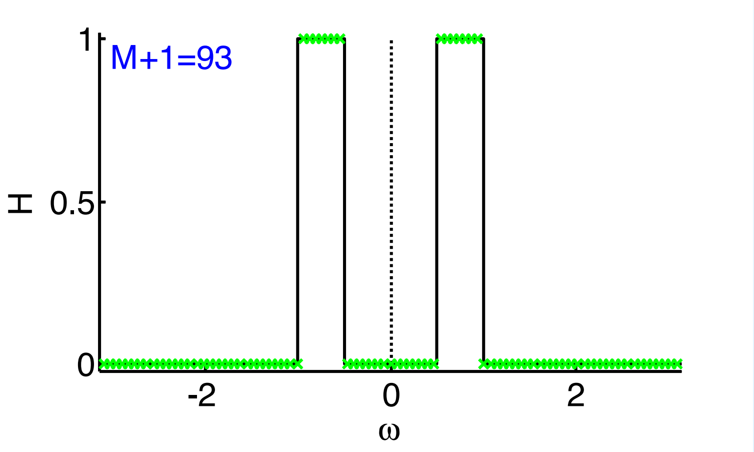


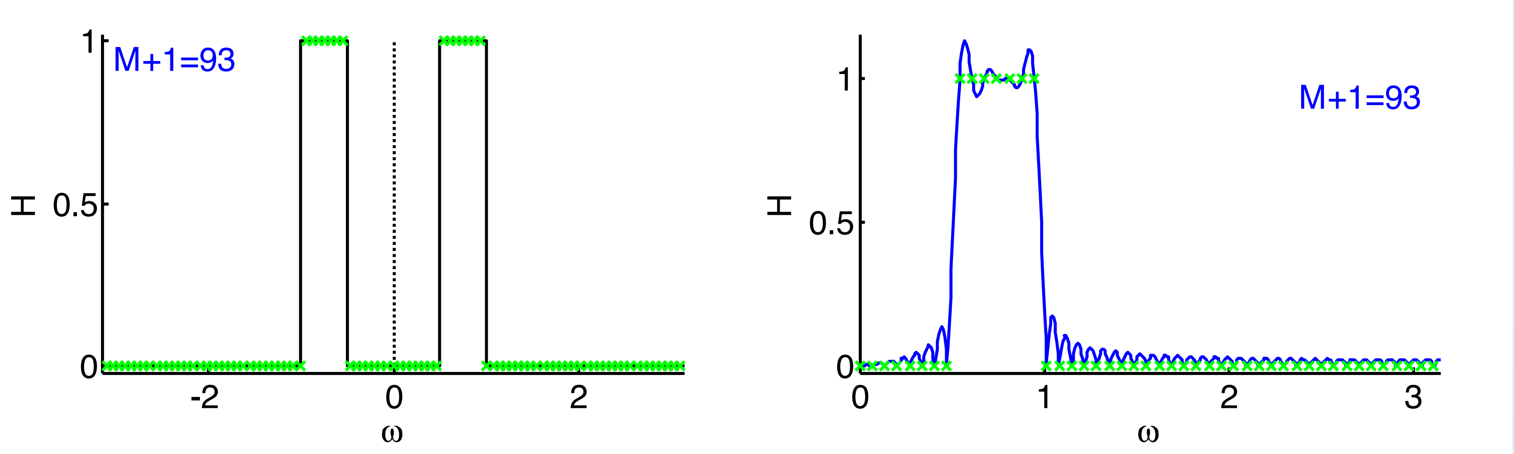
Frequency sampling

Take $M+1$ uniform samples of $H(e^{j\omega})$; take IDFT to obtain $h[n]$



Frequency sampling

Take $M+1$ uniform samples of $H(e^{j\omega})$; take IDFT to obtain $h[n]$



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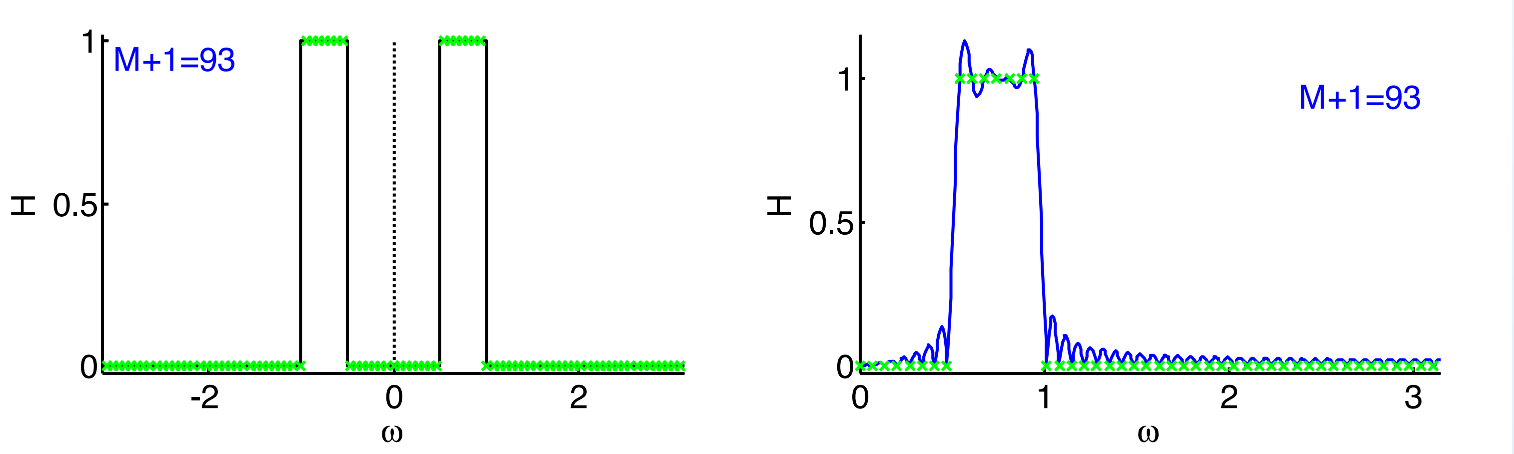

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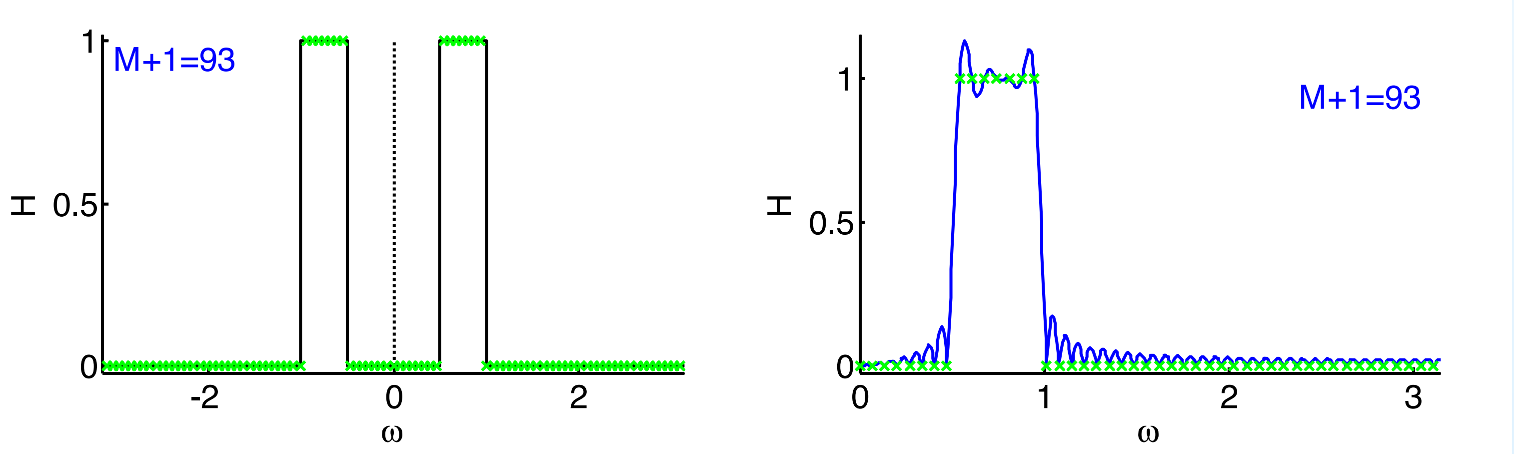

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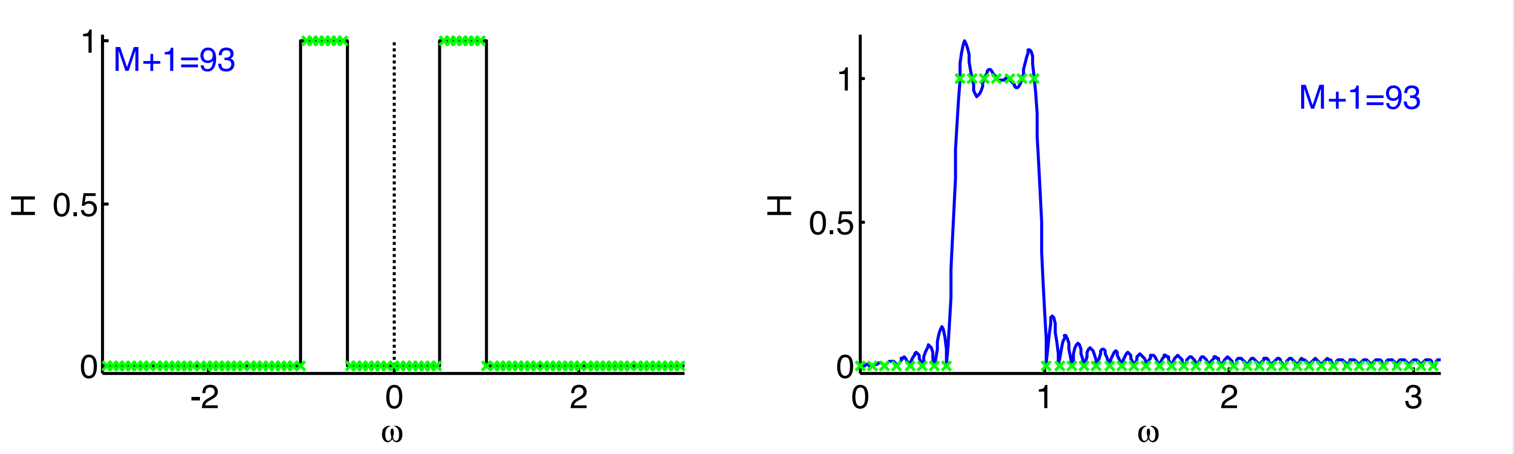

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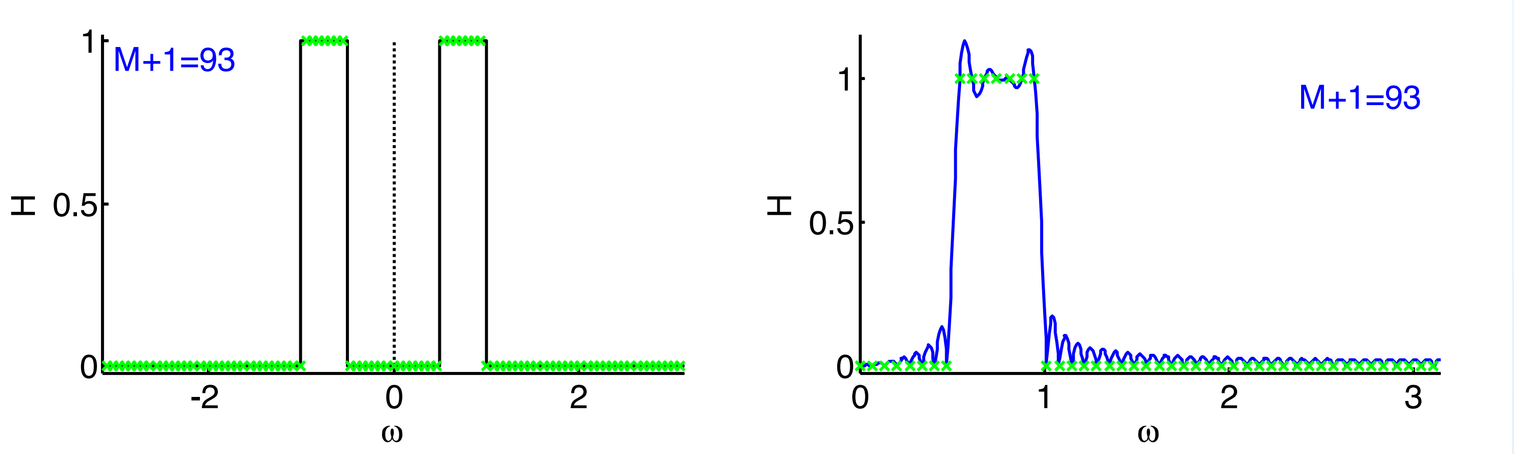

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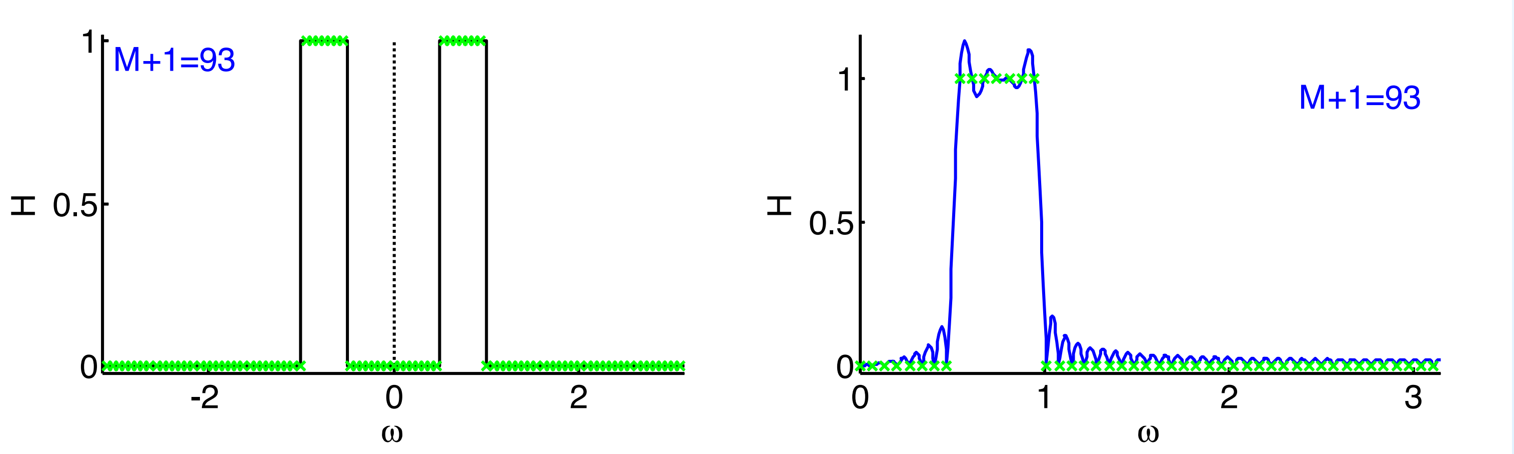
(3) use non-uniform points with more near transition (can't use IDFT)



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For further details see Mitra: 7, 10.

MATLAB routines

diric(x,n)	Dirichlet kernel: $\frac{\sin 0.5nx}{\sin 0.5x}$
hanning hamming kaiser	Window functions (Note 'periodic' option)
kaiserord	Estimate required filter order and β