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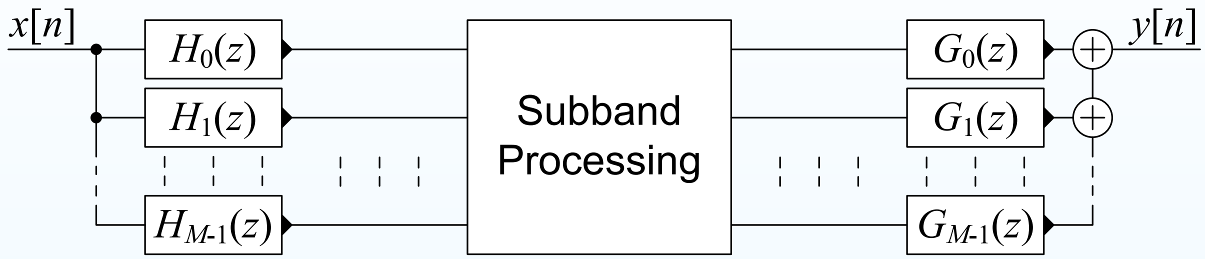

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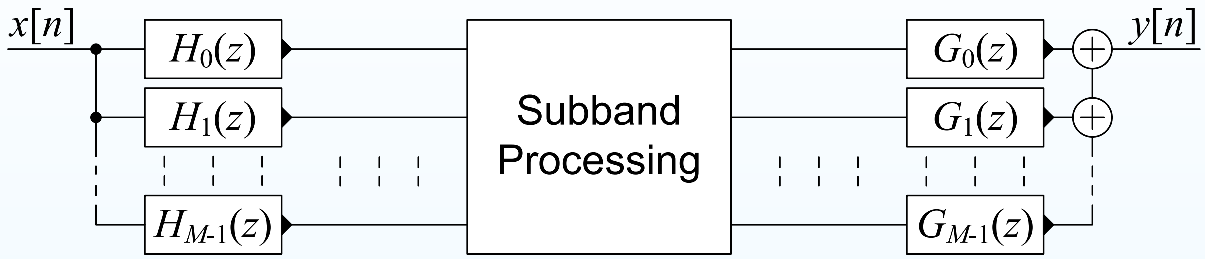


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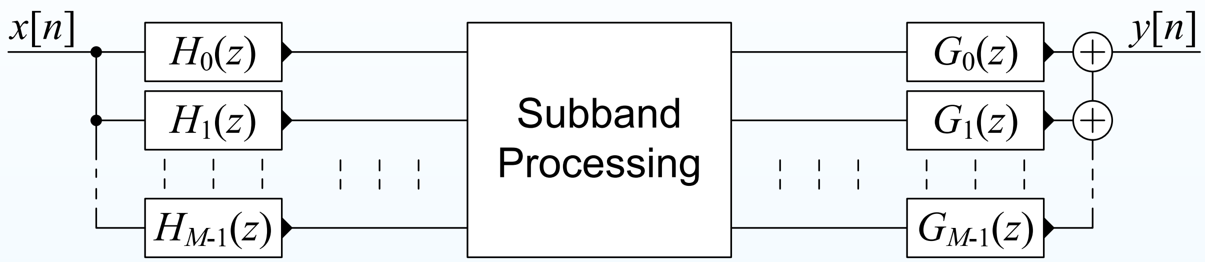


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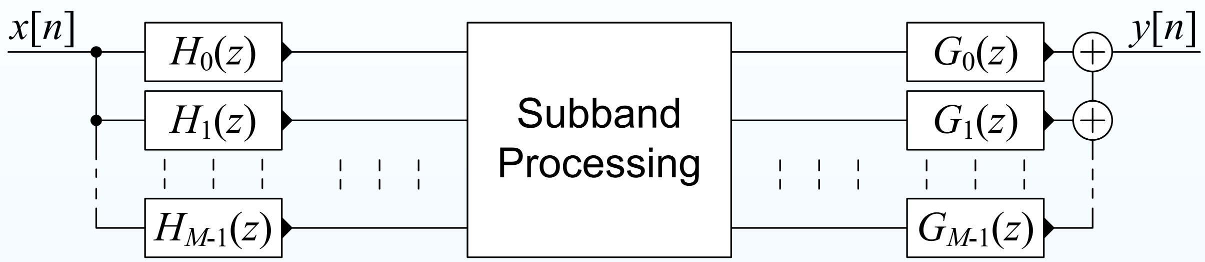


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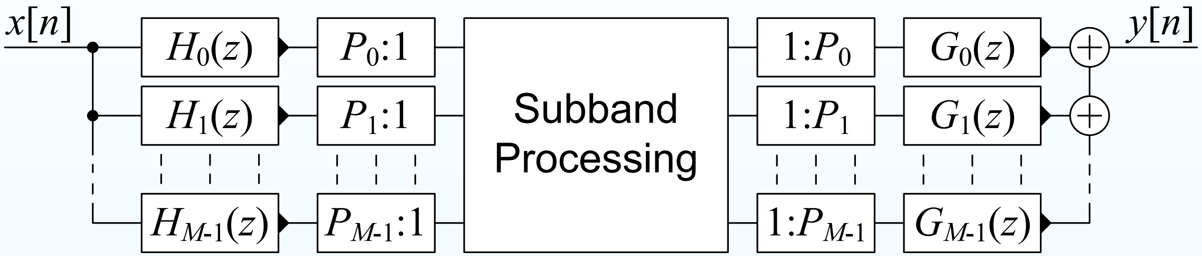


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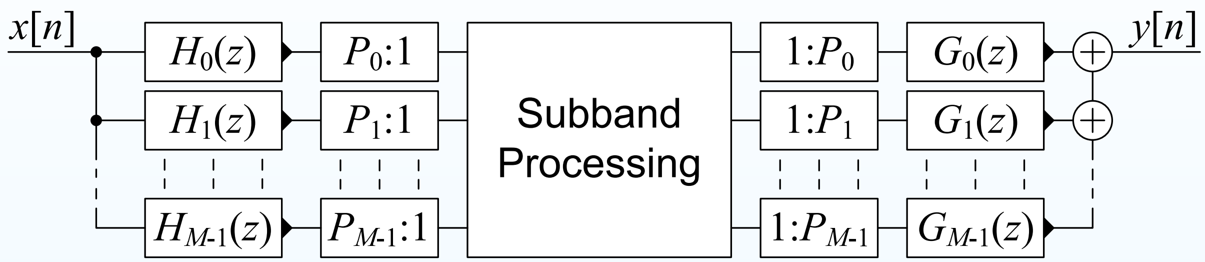


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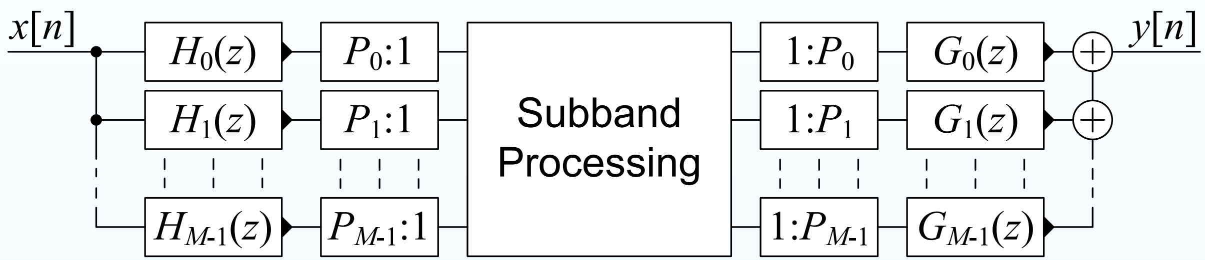


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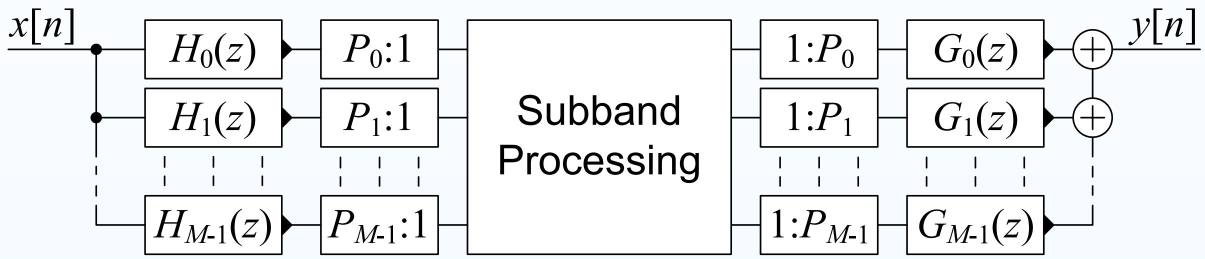


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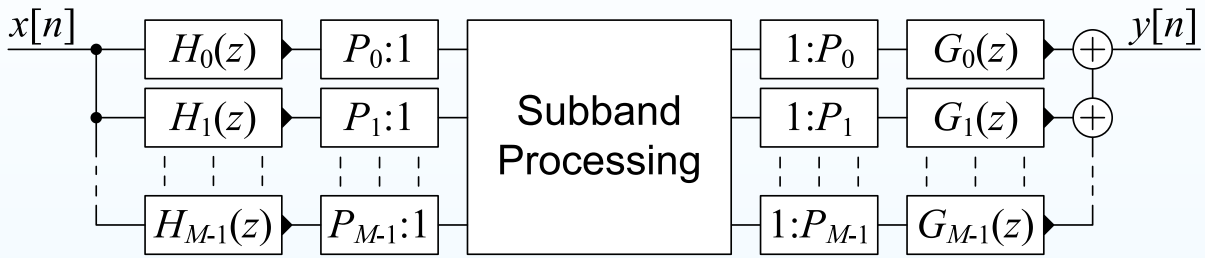


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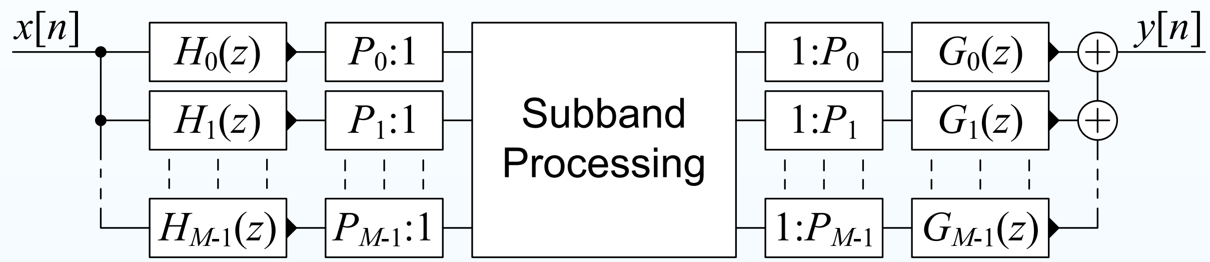


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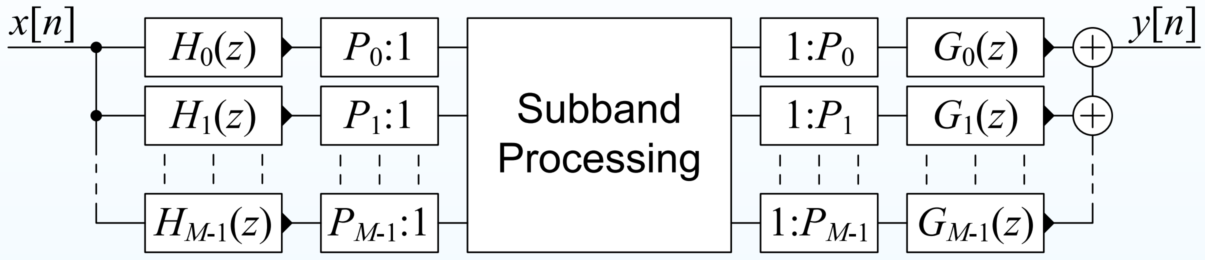


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- Goals:

  (a) good frequency selectivity in $H_m(z)$

  (b) *perfect reconstruction*: $y[n] = x[n-d]$ if no processing
- Benefits: Lower computation, faster convergence if adaptive

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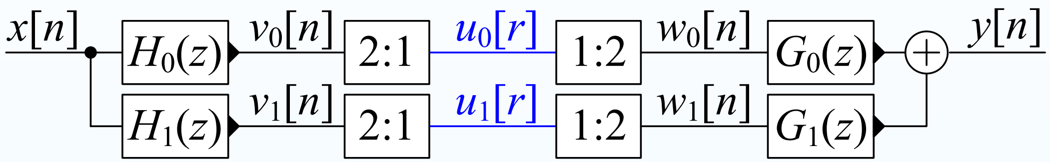

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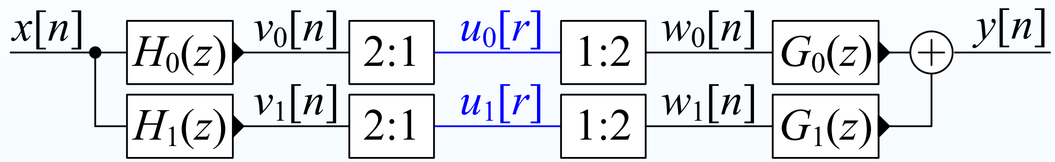


$$V_m(z) = H_m(z)X(z) \qquad [m \in \{0, 1\}]$$

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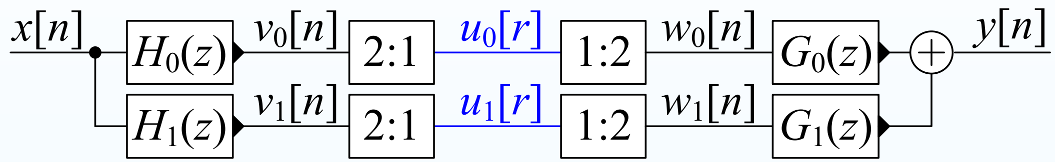


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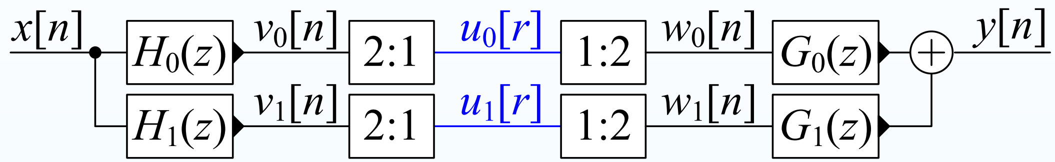


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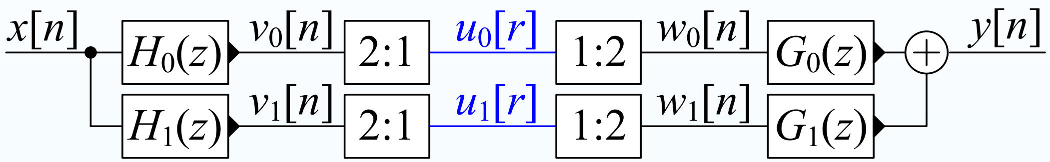


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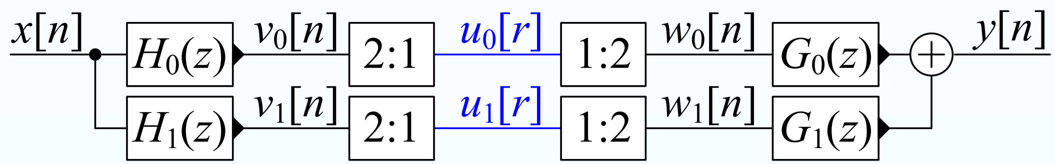


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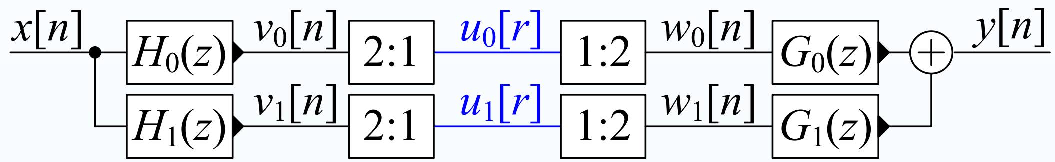


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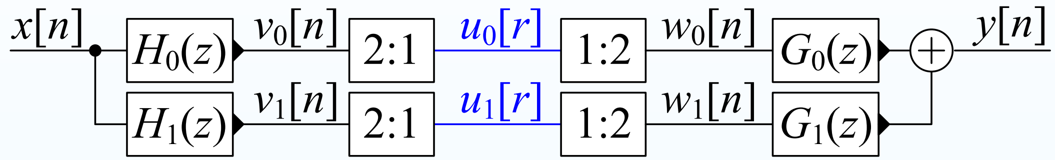


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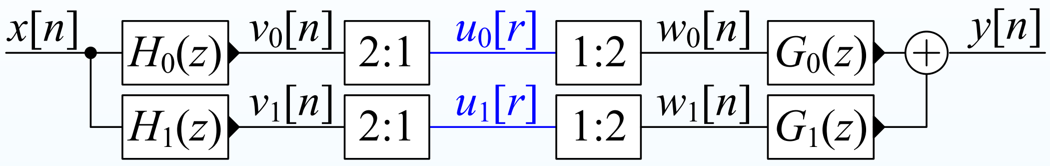


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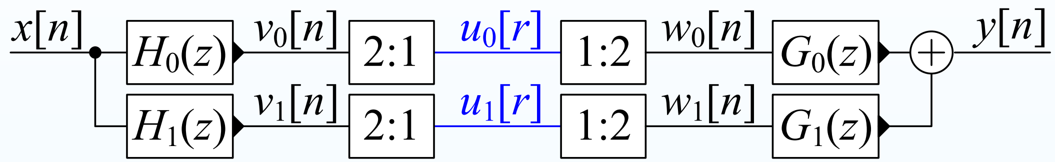


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We want (a) $T(z) = \frac{1}{2}\left\{H_0(z)G_0(z) + H_1(z)G_1(z)\right\} = z^{-d}$

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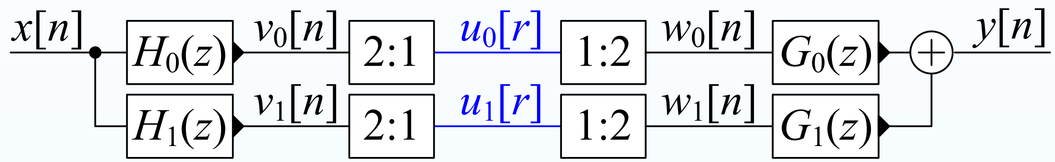


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$$= \begin{bmatrix} X(z) & X(-z) \end{bmatrix} \begin{bmatrix} T(z) \\ A(z) \end{bmatrix}$$

[$X(-z)A(z)$ is "aliased" term]

We want (a) $T(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\} = z^{-d}$

and (b) $A(z) = \frac{1}{2}\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\} = 0$

# Perfect Reconstruction

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For perfect reconstruction without aliasing, we require

$$\frac{1}{2}\begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix}\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \begin{bmatrix} z^{-d} \\ 0 \end{bmatrix}$$

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For all filters to be FIR, we need the denominator to be

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = cz^{-k}$$

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Note: $c$ just scales $H_i(z)$ by $c^{\frac{1}{2}}$ and $G_i(z)$ by $c^{-\frac{1}{2}}$.

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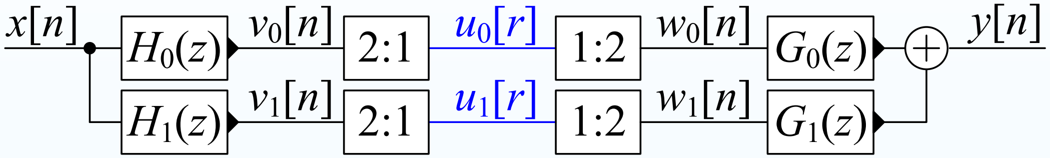

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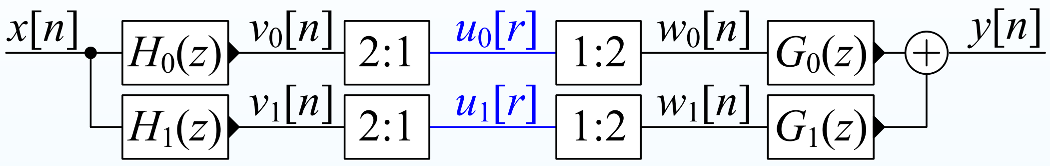


QMF satisfies:

(a) $H_0(z)$ is causal and real

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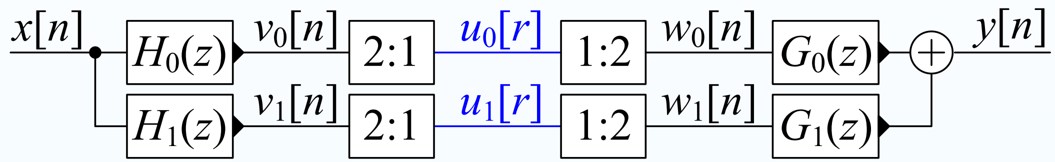


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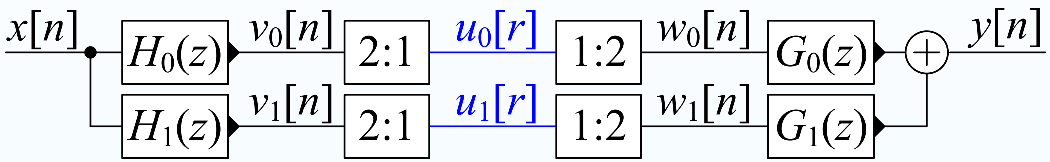


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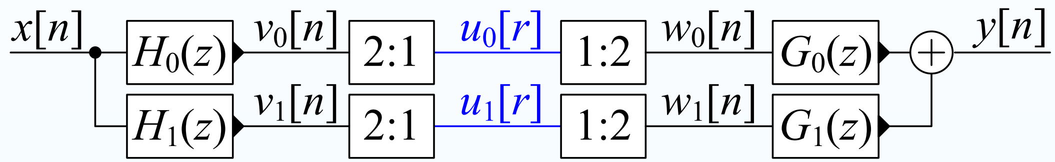


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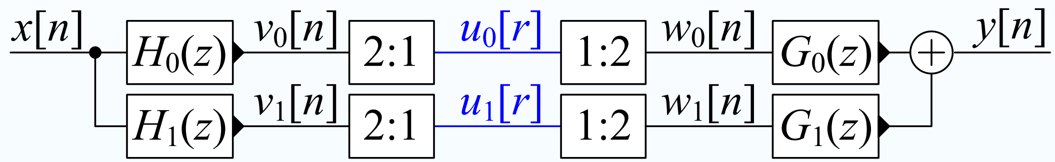


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$$A(z) = \frac{1}{2}\left\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\right\}$$

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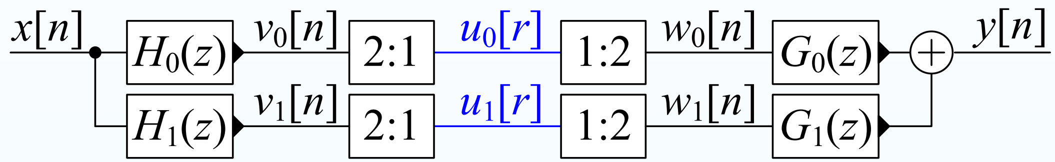


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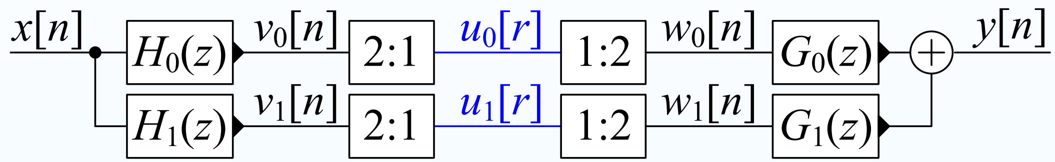


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**QMF Transfer Function:**

$$T(z) = \tfrac{1}{2}\left\{H_0(z)G_0(z) + H_1(z)G_1(z)\right\}$$

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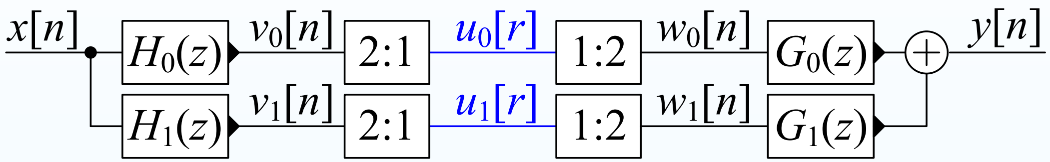


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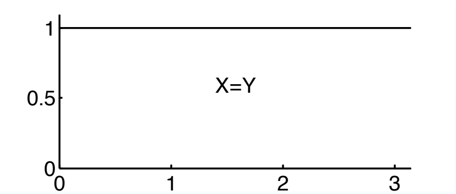
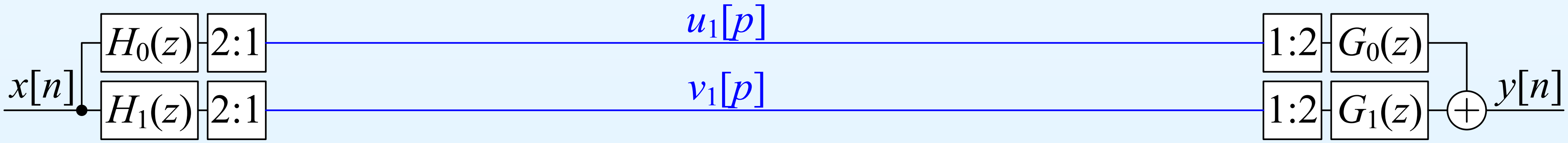
# Tree-structured filterbanks

A *half-band filterbank* divides the full band into two equal halves.



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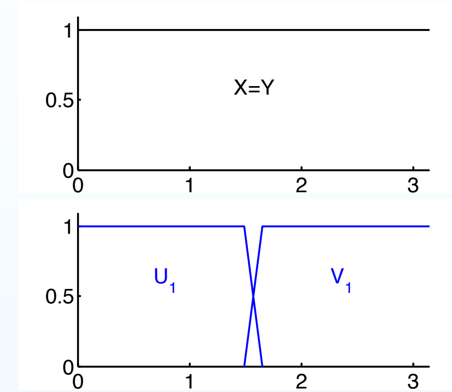


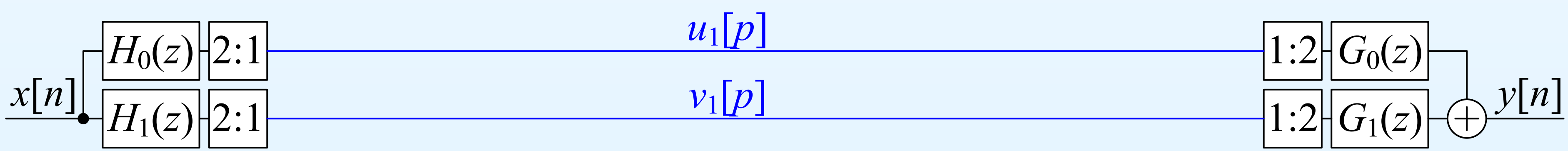

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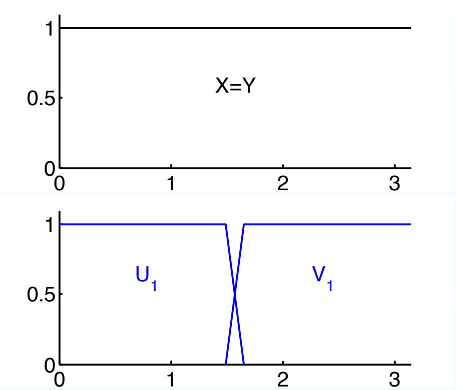
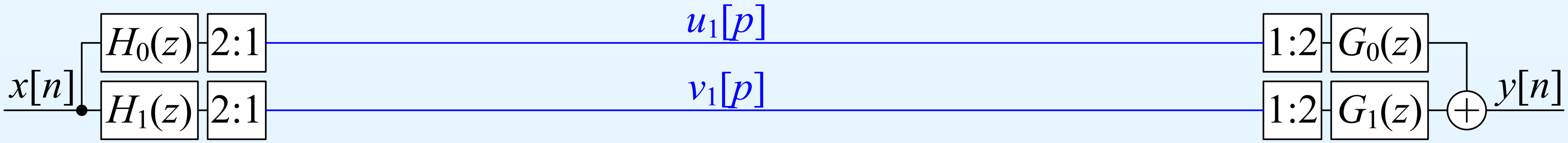
A *half-band filterbank* divides the full band into two equal halves.

You can repeat the process on either or both of the signals $u_1[p]$ and $v_1[p]$.



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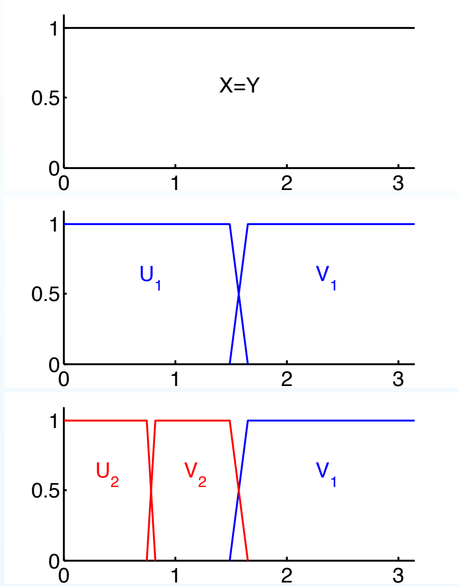


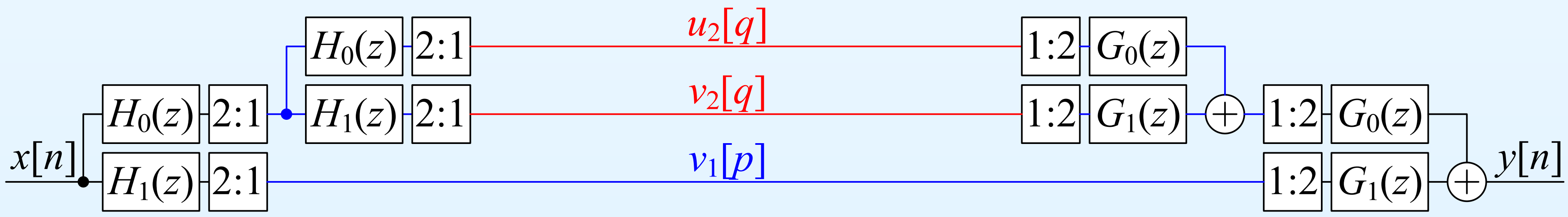

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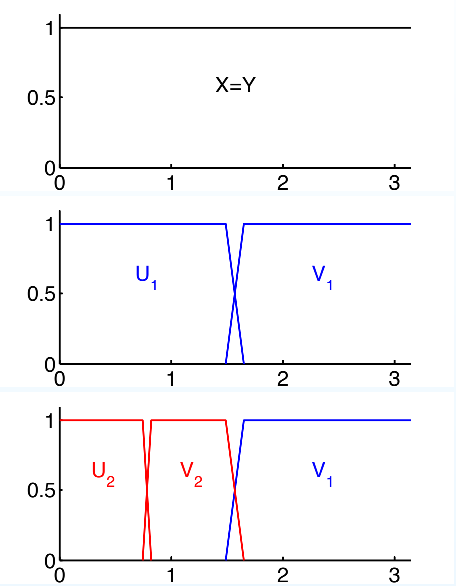
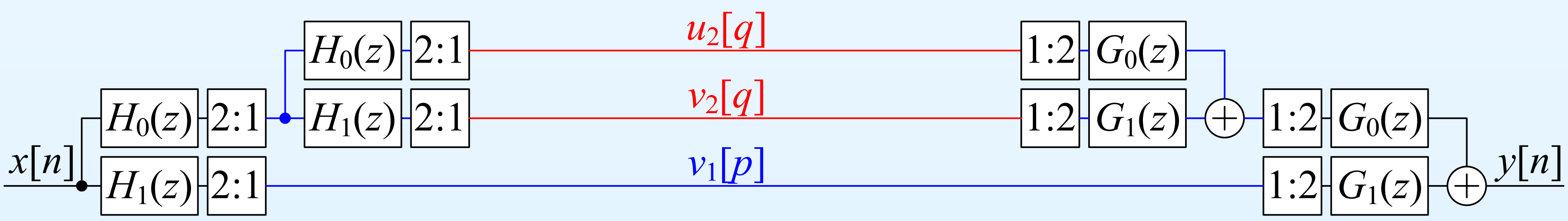
Dividing the lower band in half repeatedly results in an *octave band filterbank*.



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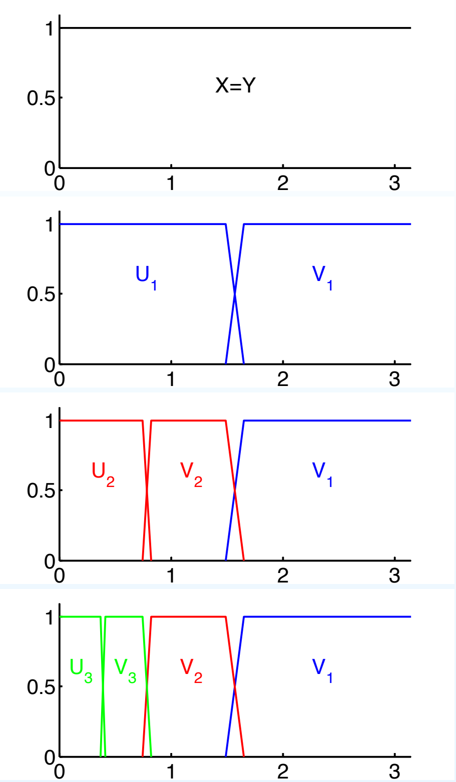
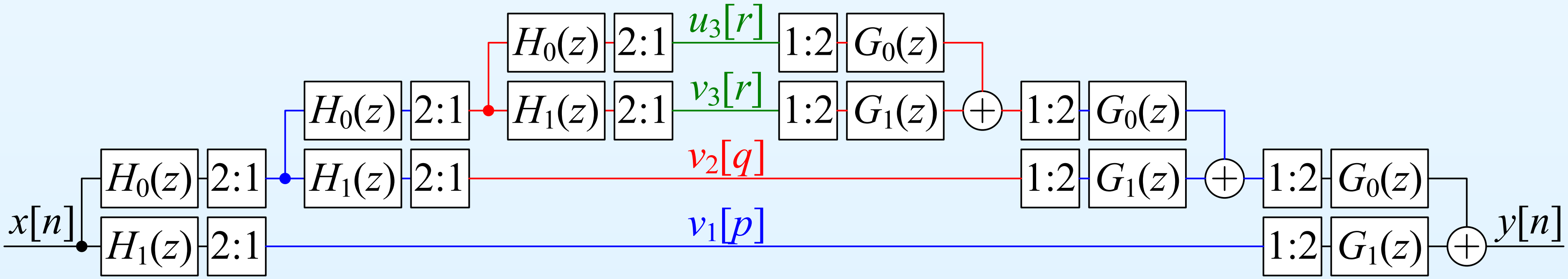


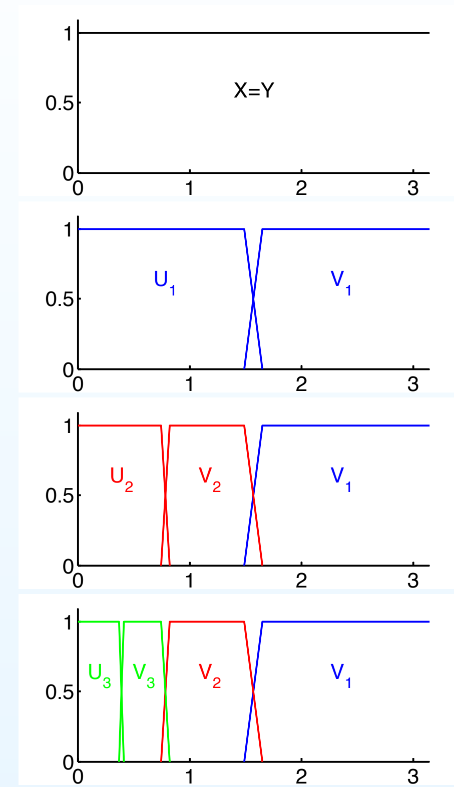



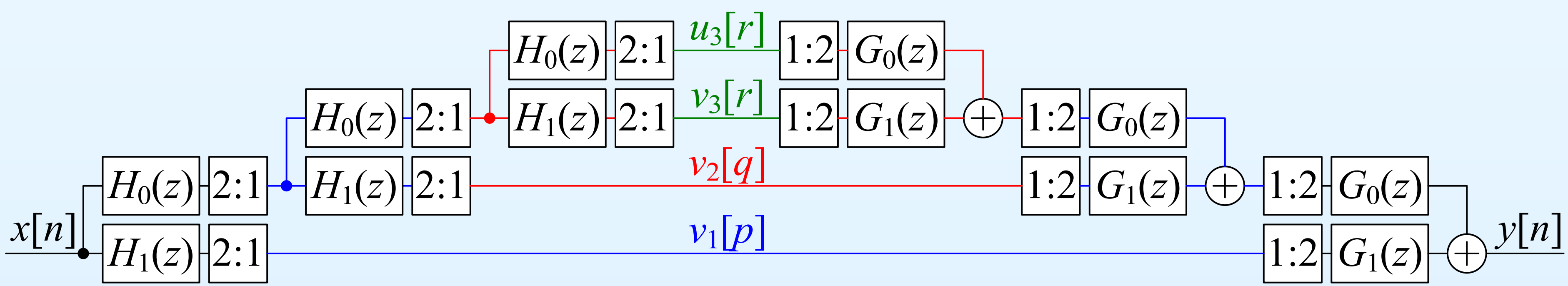
# Tree-structured filterbanks

A *half-band filterbank* divides the full band into two equal halves.

You can repeat the process on either or both of the signals $u_1[p]$ and $v_1[p]$.

Dividing the lower band in half repeatedly results in an *octave band filterbank*. Each subband occupies one octave (= a factor of 2 in frequency) except the first subband.



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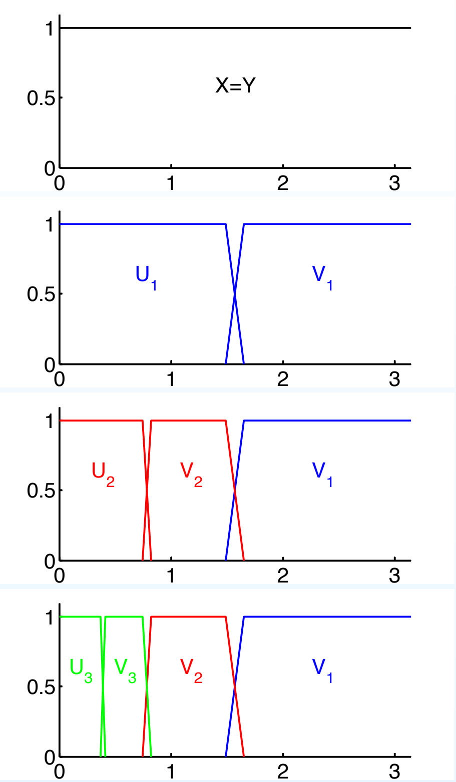
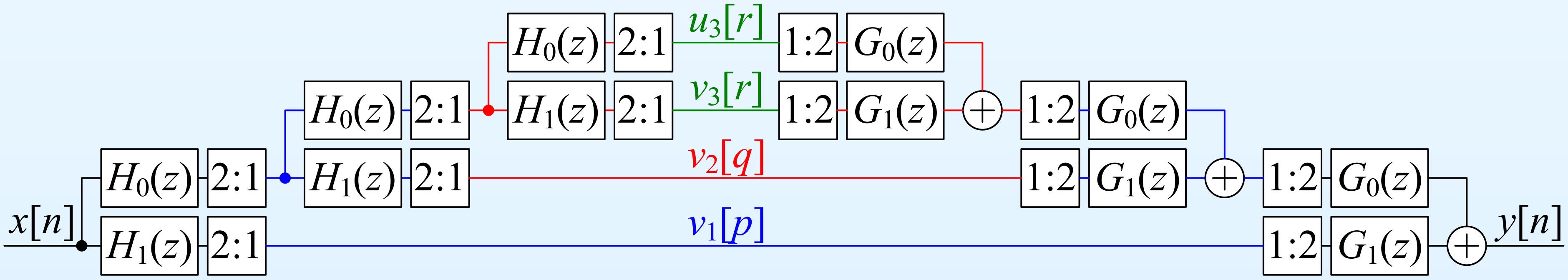
The properties "perfect reconstruction" and "allpass" are preserved by the iteration.



15: Subband Processing
- Subband processing
- 2-band Filterbank
- Perfect Reconstruction
- Quadrature Mirror Filterbank (QMF)
- Polyphase QMF
- QMF Options
- Linear Phase QMF
- IIR Allpass QMF
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- Half-band filterbank:
  - Reconstructed output is $T(z)X(z) + A(z)X(-z)$
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- Quadrature Mirror Filterbank (QMF) adds an additional symmetry constraint $H_1(z) = H_0(-z)$.
  - Perfect reconstruction now impossible except for trivial case.
  - Neat polyphase implementation with $A(z) = 0$
  - Johnston filters: Linear phase with $T(z) \approx 1$
  - Allpass filters: Elliptic or Butterworth with $|T(z)| = 1$

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See Mitra chapter 14 (which also includes some perfect reconstruction designs).