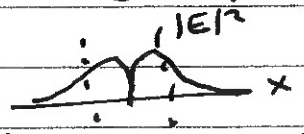
EE4-06 Optical Communications
SOLUTIONS

1 a) Loss $= 10\log(I_2/I_1) = 20$ dB
thus, dividing by $L = 50$ km, $\alpha_{dB} = \underline{0.4 \text{ dB/km}}$

b) Odd numbered modes are anti-symmetric, so have zero intensity at the centre, so intensity is higher at the boundary.



c) The number of modes supported is inversely proportional to the wavelength. Since the number is quite high, there will certainly be fewer modes at 1550 nm.

d) The propagation time equivalent to 600 m will be $\tau = L/v_g \approx n'L/c$. We can estimate $n' = 1.5$ so $\tau \approx 1.5 \times 600/3\times10^8 = 3\,\mu s$
Then $B = 3750/3\times10^{-6} = \underline{1.25 \text{ Gbit/s}}$

e) Silicon has a bandgap energy corresponding to a wavelength $\sim 0.9\,\mu m$. At 1550 nm silicon is transparent, so cannot be used for detection.

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f) laser slope efficiency is based on $\eta \times$ one photon per electron, thus

$$S = \eta \frac{hc}{e\lambda} = \frac{0.88 \times 6.63\times10^{-34} \times 3\times10^{8}}{1.6\times10^{-19} \times 1330\times10^{-9}}$$

$$= \underline{0.823 \ \mathrm{W/A}}$$

g) Thermal noise does not depend on received power. Shot noise will drop as received power drops with increased attenuation, so for the longer length thermal noise will dominate.

h) A reasonable approximation for LED spectral width in energy terms is $2kT \cong 50\,\mathrm{meV}$

Since $\left|\frac{\Delta\lambda}{\lambda}\right| \cong \left|\frac{\Delta E}{E}\right|$, then $\Delta\lambda \cong \frac{\Delta E \cdot \lambda^2}{hc}$

$$\Delta\lambda = \frac{50\times10^{-3} \times 1.6\times10^{-19} \times (0.58\times10^{-6})^2}{6.63\times10^{-34} \times 3\times10^{8}} = \underline{13.5\ \mathrm{nm}}$$

i) Since $N_A = N_D$, then $w_p = w_n = w/2$

$|E_{max}| = \frac{e}{\varepsilon} w_p N_A \qquad V = \frac{1}{2}(w_p + w_n)|E_{max}|$

$= w_p |E_{max}|$

$$\therefore\ V = \frac{\varepsilon}{e}\frac{|E_{max}|^2}{N_A} = \frac{12\times8.85\times10^{-12}}{1.6\times10^{-19}} \times \frac{10^{10}}{4\times10^{20}}$$

$$= \underline{\underline{16.6\ \mathrm{mV}}}$$

j) The wave propagates as $E_0 \exp j(nk_0 z)$

Taking $\mathrm{Im}\{n\} = n_x$, $|E| \propto E_0 \exp(-n_x k_0 z)$

so intensity $\propto |E|^2 \propto \exp(-2n_x(\frac{2\pi}{\lambda_0})z)$

$$\alpha = \frac{2\times10^{-6}\times2\pi}{0.6\times10^{-6}} = \underline{21\ \mathrm{m^{-1}}}$$

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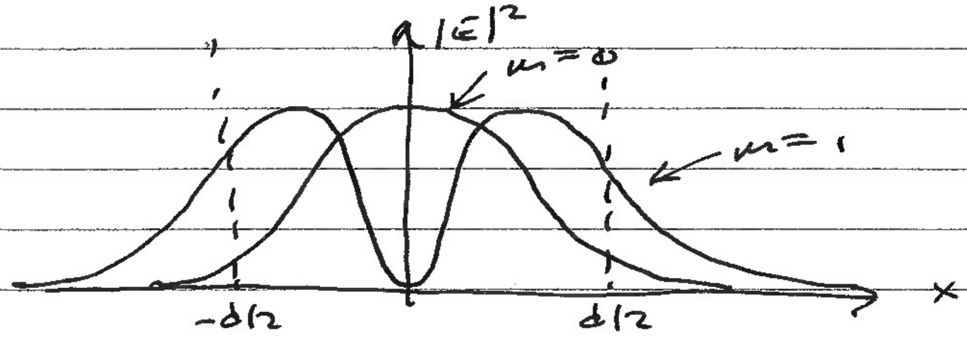
(2) a) A mode m is supported if $d > \frac{m\lambda_0}{2NA}$

Here $2NA \cdot d/\lambda_0 = \frac{2\sqrt{1.51^2 - 1.50^2} \cdot 4}{0.92} = 1.51$

Two modes are supported, $m=0$ & $m=1$

b)



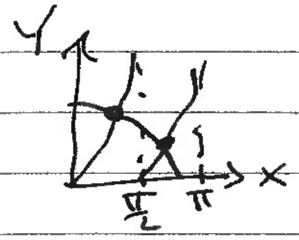
c) For $m=0$ (even), using $X = k_x d/2$, $Y = \kappa d/2$

$Y = X \tan X$

$X^2 + Y^2 = R^2 \qquad R = NA \cdot k_0 d/2$

$X^2(1+\tan^2 X) = R^2 \qquad \frac{\cos X}{X} = \pm\frac{1}{R}$

$R = \sqrt{1.51^2 - 1.5^2} \cdot \pi \cdot 4/(0.92 \times 2) = \frac{5.303}{2} = 2.651$



starting approximation $X_0 = 1$, successive approx gives $X = 1.13$

$k_{1x} = \frac{2X}{d} = 0.565\ \mu m^{-1}$

$n_0' = \sqrt{n_1^2 - (k_{1x}/k_0)^2}$

$= \sqrt{1.51^2 - \left(\frac{.565 \times .92}{2\pi}\right)^2}$

$= \underline{\underline{1.508}}$

For $m=1$ use $\frac{\sin X}{X} = \frac{\pm 1}{R} \qquad X = 2.175$

$n_1' = \sqrt{n_1^2 - (2X/k_0 d)^2} = \sqrt{1.51^2 - \left(\frac{2 \times 2.175 \times .92}{2\pi \cdot 4}\right)^2}$

$= \underline{\underline{1.502}}$

(2)

d) For an $m=1$ mode, $E(x) = E_0 \sin(k_{1x} x)$

This peaks at $x_m = \frac{\pi/2}{k_{1x}}$

Since $(n_1 k_0)^2 = (n' k_0)^2 + k_{1x}^2$

then $k_{1x} = \sqrt{(n_1)^2 - (n')^2}\, k_0$

Then $x_m = \frac{\frac{\pi}{2} \cdot \lambda_0}{2\pi \sqrt{n_1^2 - n'^2}} = \frac{\lambda_0}{4\sqrt{n_1^2 - n'^2}}$

(3)

a) $V_p = \omega / k \qquad V_g = d\omega / dk$

V_g describes the speed that pulses (or information) propagates.

b) $V_g = \omega/k = \frac{1}{dk/d\omega} \qquad k = n k_0 \qquad \omega = c k_0$

$dk/d\omega = k_0 \frac{dn}{d\omega} + n \cdot \frac{1}{c}$

$\frac{1}{V_g} = \frac{2\pi}{\lambda_0} \frac{dn}{d\lambda_0} \frac{d\lambda_0}{d\omega} + \frac{n}{c} \qquad \frac{d\lambda_0}{d\omega} = \frac{-\lambda_0}{\omega} = \frac{-\lambda_0}{c k_0}$

$= \frac{1}{c}\left[n - \lambda_0 \frac{dn}{d\lambda_0} \right]$

$\frac{dn}{d\lambda_0} = \frac{-2D_1}{\lambda_0^3} - 2D_2 \lambda_0 \quad \therefore \frac{1}{V_g} = \frac{1}{c}\left\{ D_0 + \frac{3D_1}{\lambda_0^2} + 3 D_2 \lambda_0^2 \right\}$

$\frac{1}{V_p} = \frac{n}{c} = \frac{1}{c}\left[D_0 + \frac{D_1}{\lambda_0^2} - D_2 \lambda_0^2 \right]$

$\therefore \frac{1}{V_g} = \frac{1}{V_p} + \frac{1}{c}\left(\frac{2D_1}{\lambda_0^2} + 2D_2 \lambda_0^2 \right)$

Since everything in the brackets is positive, $\frac{1}{V_g} > \frac{1}{V_p}$

so $V_g < V_p$

3 b) continued

Required expressions are

$$v_p = \frac{c}{D_0 + D_1\lambda_0^{-2} - D_2\lambda_0^2} \qquad v_g = \frac{c}{D_0 + 3D_1\lambda_0^{-2} + D_2\lambda_0^2}$$

An alternative solution to compare them:

$$dn/d\lambda_0 = \frac{-2D_1}{\lambda_0^3} - 2D_2\lambda_0 . \quad \text{Since } D_1, D_2 > 0 \quad dn/d\lambda_0 < 0$$

$$\therefore \; n - \lambda_0 dn/d\lambda_0 > n \quad \text{so} \quad \frac{1}{v_g} > \frac{1}{v_p} \quad v_g < v_p$$

c) $$\frac{d^2n}{d\lambda_0^2} = \frac{d}{d\lambda_0}\left(\frac{-2D_1}{\lambda_0^3} - 2D_2\lambda_0\right) = \frac{6D_1}{\lambda_0^4} - 2D_2$$

Zero dispersion is at $d^2n/d\lambda_0^2 = 0$

giving $\lambda_0^4 = 3D_1/D_2$ $\quad \lambda_0 = \sqrt[4]{\frac{.003 \times 3}{.0032}}$

$= 1.295 \; \mu m$

d) Phase change experienced by a wave of wavelength λ_0 propagating L is just

$$\Delta\varphi = \frac{2\pi L}{\lambda} = \frac{2\pi n L}{\lambda_0}$$

In this case $\Delta\varphi = 2\pi L\left(D_0\lambda_0^{-1} + D_1\lambda_0^{-3} - D_2\lambda_0\right)$

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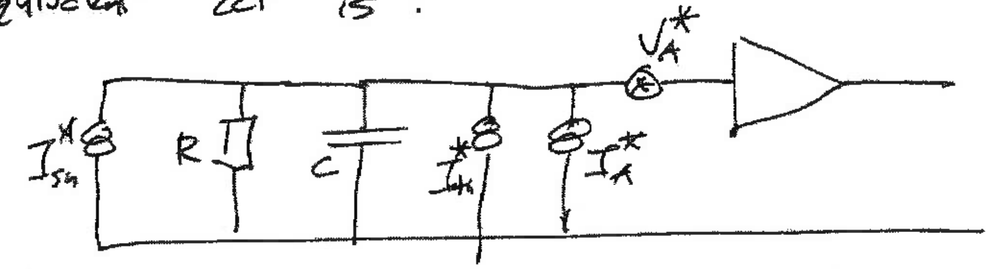
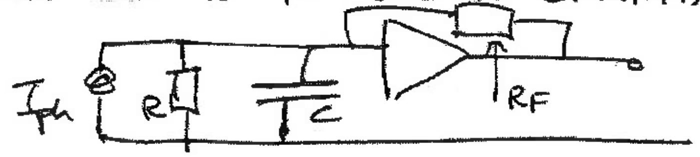
④ a) Discussion should include UV and IR absorption, Rayleigh scattering, and the underlying mechanisms of each of these impurity absorption, particularly water. A plot of these should be sketched over a sensible wavelength range. Bending loss should be mentioned. The λ dependence of Rayleigh scattering ($\propto \lambda^{-4}$) should be indicated.

b) Noise sources are shot noise, thermal noise, and amplifier voltage and current noise. Noise equivalent cct is:



They should explain that R & C are combined components from :- load and amplifier input resistance
- diode and amplifier input capacitance

An alternative configuration is the transimpedance amplifier. In this case the photodiode is effectively grounded, so the frequency dependence of gain due to R & C is eliminated.



This also means the frequency dependence of SNR is largely eliminated.

c)

(4)

c) i) To compare the noise sources, they must be both expressed in terms of optical power of photo current. Taking the latter:

$$SNR_{opt} = \frac{I_{ph}}{\left[\Sigma I_*^{*2}\right]^{1/2} \Delta f^{1/2}}$$

For shot noise, $I_{sh}^{*2} = 2eI_{ph}$

For receiver noise:

$$SNR_{opt} = \frac{\Phi_R}{NEP\ \Delta f^{1/2}} = \frac{R\Phi_R}{R\,NEP\ \Delta f^{1/2}} = \frac{I_{ph}}{\left[R^2 NEP^2\right]^{1/2} \Delta f^{1/2}}$$

So we need to compare

$$I_R^{*2} = R^2 NEP^2$$

with $R = \frac{\eta e \lambda}{hc} = \frac{1 \cdot 1.6\times10^{-19} \times 1.31\times10^{-6}}{6.63\times10^{-34} \times 3\times10^{8}} = 1.05\ \frac{A}{W}$

$$I_R^{*2} = 1.05^2 (8\times10^{-12})^2 = 7.06\times10^{-23}$$

Shot noise $\Phi_R = \Phi_T - \text{loss}$

$4\ mW = +6\ dBm$, $\text{loss} = 100 \times .35 = 35\ dB$

$\Phi_R = -29\ dBm$

or $4 \times 10^{-3.5} = 1.26\ \mu W$

Then $I_{ph} = R\Phi_R = 1.05 \times 1.26 = 1.32\ \mu A$

$I_{sh}^{*2} = 2eI_{ph} = 2\times1.6\times10^{-19}\times 1.32\times10^{-6}$

$= 4.2\times10^{-25} \ll I_R^{*2}$

So receiver noise dominates.

ii) $\Delta f^{1/2} = \frac{\Phi_R}{SNR \times NEP} = \frac{1.26\times10^{-6}}{12\times8\times10^{-12}} = 13.1\times10^{3}$

$B = 2\Delta f = 2(13\times10^{3})^2 = \underline{343\ Mbit/s}$

iii) We can approximate this limit as $BLD\sigma_\lambda = 0.2$

Then $D_{max} = \frac{0.2}{100 \times 343\times10^{6} \times 10^{-9}} = .06\ \frac{s}{m \cdot km} = 60\ \frac{ps}{nm \cdot km}$

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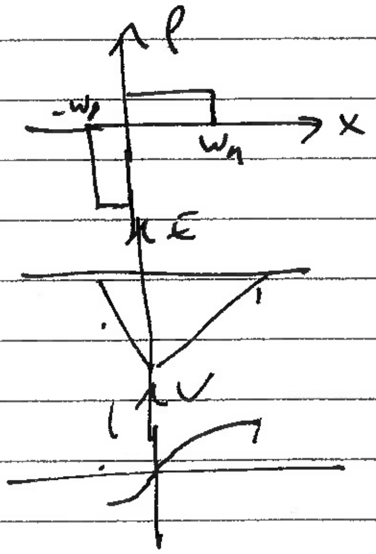
(5) a)

$$w_n = \frac{N_A w_p}{N_D} = 5 w_p$$



$$w = w_n + w_p = 6 w_p = (6\ \mu m)$$

$$|E_{max}| = \frac{e N_A w_p}{\varepsilon} = \frac{e N_A w}{6 \varepsilon}$$

$$V_b = \frac{1}{2} |E_{max}| w$$

$$V_b = \frac{e N_A w^2}{12 \varepsilon_r \varepsilon_0}$$

so $w = \sqrt{\dfrac{12 \varepsilon_r \varepsilon_0 V_b}{e N_A}}$

for $w = 6\ \mu m$ $\quad V_b = \dfrac{1.6 \times 10^{-19} \times 5 \times 10^{20} \times (6 \times 10^{-6})^2}{12 \times 12 \times 8.85 \times 10^{-12}}$

$$= \underline{\underline{2.26\ V}}$$

b) $\eta = e^{-\alpha x_1} - e^{-\alpha x_2} = e^{-\alpha h}\left(e^{\alpha w_p} - e^{-5 \alpha w_p}\right)$

$$\alpha h = .85 \times 10^5 \times 6 \times 10^{-6} = 0.51$$

$$e(-\alpha h) = 0.600$$

Define $\theta = \alpha w_p$ then $\eta = 0.6(e^{\theta} - e^{-5\theta}) = 0.8$

Solve this by successive approximation:

$e^{\theta} - e^{-5\theta} = 1.33$ gives $\theta = 0.39$

Then $w_p = \dfrac{0.39}{.85 \times 10^5} = 4.59\ \mu m$

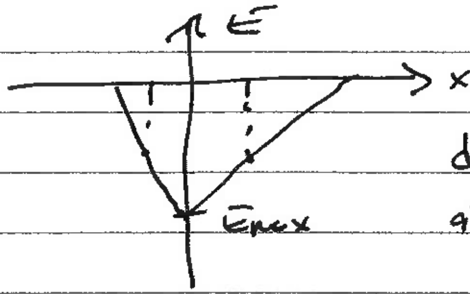
$w = 6 w_p = 27.5\ \mu m$

From (a),

$$V_b = \frac{e N_A w^2}{12 \varepsilon_r \varepsilon_0} = \frac{1.6 \times 10^{-19} \times 5 \times 10^{20} \times (27.5 \times 10^{-6})^2}{12 \times 12 \times 8.85 \times 10^{-12}}$$

$$= \underline{\underline{47.5\ V}}$$

c) A typical $v_{dsat} \cong 10^5$ m/s
(this has to be remembered, can't be derived).
In a reverse biased p-n junction,
the electric field varies linearly:



Thus half the depletion width has $\mathcal{E}$ at half $\mathcal{E}_{max}$ or more.

For $v_d = v_{dsat}$ we need $\mathcal{E} = \dfrac{10^5 \text{ m/s}}{0.12 \text{ m}^2/\text{V-s}} = 8.33\times10^5$ V/m

Thus need $|\mathcal{E}_{max}| = 1.67\times10^6$ V/m

From (a), $|\mathcal{E}_{max}| = \dfrac{eN_A w}{6\varepsilon}$ and $V_b = \frac{1}{2}|\mathcal{E}_{max}| w$

Then $V_b = \frac{1}{2}\left(\dfrac{6\varepsilon}{eN_A}\right)|\mathcal{E}_{max}|^2 = \dfrac{3\times12\times8.85\times10^{-12}\times(1.67\times10^6)^2}{1.6\times10^{-19}\times5\times10^{20}}$

$\underline{\underline{V_b = 11.1\text{V}}}$