Optical Communication Exam 2020: SOLUTIONS

1.

a) An optical plane wave propagating within a block of glass of refractive index 1.48, surrounded by air, is incident on one of the block's flat surfaces. Calculate the incident angle with respect to normal beyond which the wave will experience total internal reflection.

This is simply $\theta_c = \sin^{-1}(n_a/n_p) = \sin^{-1}\left(\frac{1}{1.48}\right) = \underline{42.5^\circ}$

b) A certain symmetric slab waveguide supports 3 TE modes. State whether the number of TM modes supported is fewer, the same, more, or unknown.

The number of TM modes is always the same as the number of TE modes in a symmetric slab waveguide.

c) A step-index glass optical fibre has a numerical aperture NA = 0.10. Estimate the index difference Δn of the fibre.

$$NA = \sqrt{n_c^2 - n_b^2} \simeq \sqrt{2n \cdot \Delta n} \quad n \simeq 1.5 \quad \therefore NA^2 \simeq 3 \times \Delta n$$

$$\Delta n \simeq \frac{NA^2}{3} \simeq \underline{0.0033}$$

d) A step-index multi-mode glass optical fibre has a core index $n_c = 1.470$, and an index difference $\Delta n = 0.05$. What is the smallest possible value of the effective index n' for a mode propagating in this fibre?

The effective index must lie between the core and cladding indices, so the lowest possible value is simply $n_o = n_c - \Delta n = \underline{1.420}$.

e) An optical signal of power level 5.0 mW is launched into a fibre having attenuation of 0.4 dB/km. Find the power level in mW after the signal has propagated for 50 km.

The total attenuation is 0.4×50 = 20 dB, which is a 100× reduction in power, resulting in a power level of 50 μW.

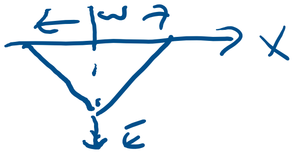
f) A certain laser diode has a slope efficiency at a nominal wavelength $\lambda_o = 0.78$ μm of 1.40 W/A. Calculate the quantum efficiency η.

$$S = \eta hc/\lambda e \quad \therefore \eta = \frac{e\lambda S}{hc} = \frac{1.6\times10^{-19} \times 0.78\times10^{-6} \times 1.4}{6.63\times10^{-34} \times 3\times10^{8}} = \underline{\underline{0.88}}$$

g) An optical link using a p-i-n photodiode has an optical SNR of 10, dominated by receiver noise, for a bit rate of B = 200 Mbit/s and a receiver noise equivalent power of NEP = 7.0 pW/√Hz. Calculate the received optical power Φ_R. You may assume that the quantum efficiency η =1.

$$SNR = \frac{\Phi_R}{NEP\sqrt{\Delta f}} \quad \Delta f = \frac{B}{2} \quad \therefore \Phi_R = 10 \times 7\times10^{-12} \times \sqrt{10^8} = \underline{\underline{0.7\,\mu W}}$$

h) A silicon p-n photodiode has a total depletion layer width of 2.0 µm when a reverse bias of 8.0 V is applied. Calculate the electric field strength *E* at the p-n junction.



The bias voltage is just the area under the curve $V = \frac{1}{2} w E_{max}$ so $E_{max} = 8 \times 10^{6} \frac{V}{m}$

i) Briefly explain why dB/km are not suitable units for the attenuation of a free-space signal caused by beam spreading.

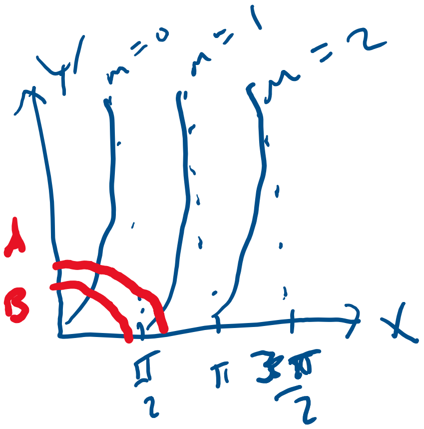
Geometric spreading causes a beam to attenuate as $1/R^2$. This does not produce a constant rate in dB/km. Absorption in a fibre causes the signal to decay exponentially, so that taking the log gives a constant and thus it can be expressed as a constant in dB/km.

j) In a fibre amplifier, briefly explain why the pump wavelength is less than the signal wavelength.

This is simply because the pump photon energy must be enough to pump the beam from the ground to the excited state, which is less than the signal photon energy which corresponds to the energy difference from the metastable state (below the excited state) back to ground. Higher energy means lower wavelength.

2.

a)



Modes are found by drawing a circular arc on this diagram of radius

$$R = NA \frac{d}{\lambda_0} \cdot \pi$$

In this case $R_A = \dfrac{0.1 \times 7.5}{1.49} \pi = 0.503\pi$

so $m=0$ and $m=1$ are supported.

$R_B = \dfrac{0.1 \times 7.5}{1.51} \pi = 0.497\pi$. Only $m=0$ is supported.

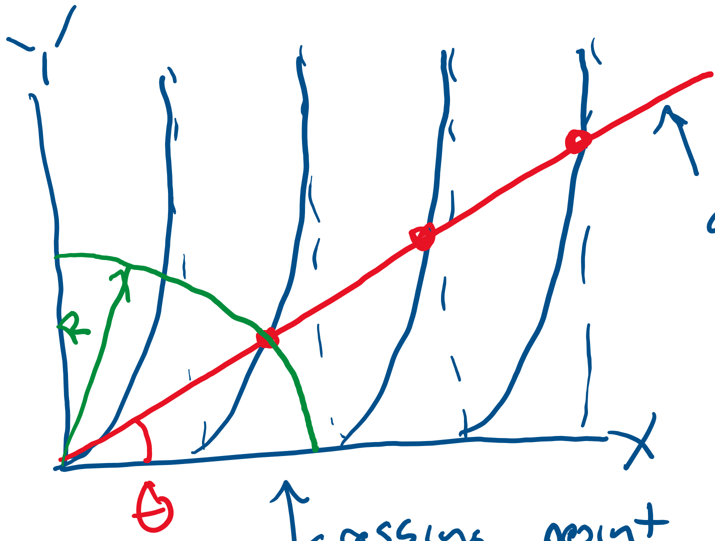
b) For $m=0$ we take the 1st eigenvalue equation $Y = X \tan X$, combine with $X^2 + Y^2 = R^2$ giving $\dfrac{\cos X}{X} = \dfrac{1}{R}$. From the plot we can estimate $X \approx 0.8$, use this as a starting point for successive approximation, giving: $X = 0.936 = \dfrac{k_{1x} d}{2}$ $\therefore$ $k_{1x} = \dfrac{2(.936)}{7.5} = .2496\ \mu m^{-1}$

$$n_0' = \sqrt{n_1^2 - \left[\frac{k_{1x}}{k_0}\right]^2} = \sqrt{1.49^2 - \left(\frac{.2496 \times 1.49}{2\pi}\right)^2} = \underline{1.489}$$

For the $m=1$ mode, use $Y = -X \cot X$, giving $\dfrac{\sin X}{X} = \dfrac{1}{R}$. Clearly X is slightly $> \frac{\pi}{2}$, so start from there and by successive approx: $X = 1.58$

giving $\underline{n_1' = 1.484}$

2 (c)



crossing point is a mode for a circular arc as shown in green.

At this point, $X = R\cos\theta$

we know $n' = \sqrt{n_i^2 - (k_{ix}/k_0)^2}$

$X = \dfrac{k_{1x} d}{2}$ $\quad R = \dfrac{NA\,k_0 d}{2}$ $\quad$ so $\dfrac{k_{1x}}{k_0} = NA\dfrac{X}{R}$

but $\dfrac{X}{R} = \cos\theta$ so

$$n' = \sqrt{n_i^2 - NA^2\cos^2\theta}$$

3 a)

Case A: $SNR = \dfrac{I_{ph}}{[2eI_{ph}]^{1/2}\,\Delta f^{1/2}}$

take $\Delta f = \dfrac{B}{2}$ $\quad SNR^2 = \dfrac{I_{ph}}{eB}$

and $I_{ph} = \Phi_T e^{-\alpha L} \times R$ where $\alpha = \alpha_{dB}/4.54$

Then $\boxed{B_A = \dfrac{R\Phi_T e^{-\alpha L}}{e\,(SNR)^2}}$

Case B: $SNR = \dfrac{I_{ph}}{[4kT/R]^{1/2}\,\Delta f^{1/2}}$, $SNR^2 = \dfrac{I_{ph}^2}{2kTB/R}$

$\boxed{B_B = \dfrac{R^2\Phi_T^2 e^{-2\alpha L}}{(SNR)^2\, 2kT/R}}$

$B_A = B_B$ when $R\Phi_T e^{-\alpha L} = \left(\dfrac{2kT}{e}\right)\dfrac{1}{R} = \dfrac{50\,mV}{10^4\,\Omega} = 5\,\mu A$

$R\Phi_T = 8\,mA \quad \therefore e^{-\alpha L} = \dfrac{5}{8}\times 10^{-3} = 6.25\times10^{-4}$

$L = -\dfrac{\ln(6.25\times10^{-4})}{.4\,dB/km\,/4.54} = \underline{83.7\,km}$

b) Shot Noise:

$\log B = \log Q - \alpha L \log_e$

$= \log Q - \alpha_{dB} L/10$

$\log Q = \log\left(\dfrac{R\Phi_T}{e\,(SNR)^2}\right) = \log\left(\dfrac{0.8\times10^{-2}}{1.6\times10^{-19}\times10^{2}}\right) = 14.7$

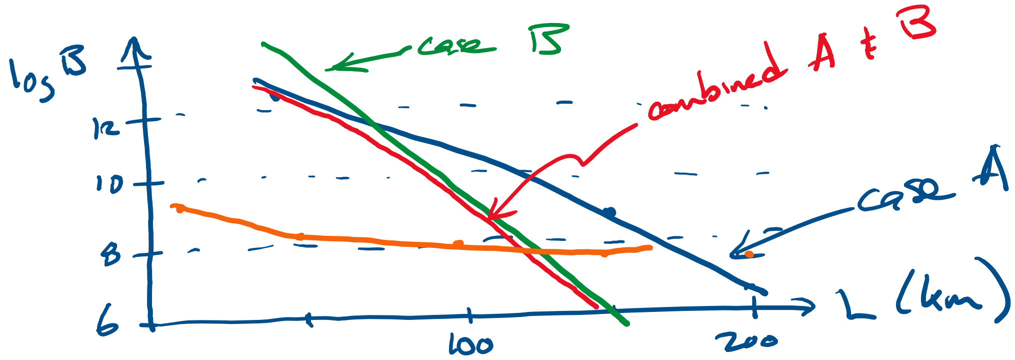
$\log B = 14.7 - .04L$

For thermal noise, $\log B = \log J - \alpha_{dB} L/5$

$$\log J = \log\left(\frac{R^2 \Phi_T^{\,2}}{SNR^2\, 2kT/R}\right) = 17.9$$

$$\log B = 17.9 - .08L$$

For the plot, choose a sensible range of L (e.g. keeping B above 10^6) and a corresponding B range (ignoring unfeasible B e.g. $> 10^{11}$)



c) We need $DL\sigma_\lambda < \frac{0.2}{B}$ giving $B = \frac{0.2}{DL\sigma_\lambda}$

For $D = 10 \frac{ps}{nm\cdot km}$, $\sigma_\lambda = 2.0$ nm, $B = \frac{10^{10}}{L}$, $\log B = 10 - \log L$

added to the plot in orange.

d) The fundamental limitation of loss in a glass is at the minimum sum of Rayleigh scattering and IR absorption. Rayleigh scattering is independent of glass type, but if the IR peak loss is moved to a longer λ, the minimum will occur at lower Rayleigh loss. Thus the minimum will occur at longer λ than in silica.

4 a) The two mechanisms are:

- material dispersion. The refractive indices of the core and cladding glass are both wavelength dependent, making the n' also wavelength dependent.
- waveguide dispersion. The shape of the mode, and therefore how much of its energy lies in each of the two materials, is λ dependent, making the n' wavelength dependent.

b) For a mode, $V_g = \frac{d\omega}{d\beta}$ with $\beta = n' k_0$

Then $\frac{1}{V_g} = \frac{d\beta}{d\omega} = \frac{d\beta}{d\lambda_0} \cdot \frac{d\lambda_0}{d\omega}$ and $\frac{d\lambda_0}{d\omega} = -\frac{\lambda_0^2}{2\pi c}$

$(\omega = c k_0)$

$$\frac{d\beta}{d\lambda_0} = \frac{d(n' k_0)}{d\lambda_0} = k_0 \frac{dn'}{d\lambda_0} + n' \frac{dk_0}{d\lambda_0}$$

$$\frac{dk_0}{d\lambda_0} = -\frac{2\pi}{\lambda_0^2}$$

So
$$\frac{1}{V_g} = \frac{\lambda_0^2}{2\pi c}\left(\frac{2\pi n'}{\lambda_0^2} - \frac{2\pi}{\lambda_0}\frac{dn'}{d\lambda_0}\right)$$
$$= \frac{1}{c}\left(n' - \lambda_0 \, dn'/d\lambda_0\right)$$

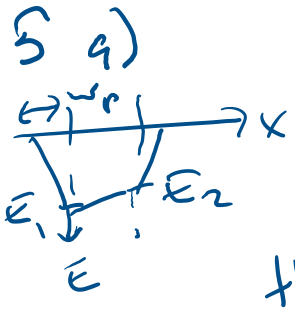
$$\frac{dn'}{d\lambda_0} = D_1 + 2D_2(\lambda_0 - \lambda_{0c})$$

giving
$$V_g = \frac{c}{\left[n_c' - D_1\lambda_{0c} - D_2(\lambda_0^2 - \lambda_{0c}^2)\right]}$$

c) $$\tau_g = \frac{L}{V_g} = \frac{L}{c}\left(n_c' - D_1\lambda_{0c} - D_2(\lambda_0^2 - \lambda_{0c}^2)\right)$$

d) $$\Delta\tau_g = \frac{d\tau_g}{d\lambda_0}\Delta\lambda_0 = \underline{\frac{2L}{c} D_2 \lambda_0 \Delta\lambda_0}$$ (the sign is irrelevant)

5 a)



We have

$$E_1 = -w_p N_A e / \varepsilon$$

then

$$w_p = \frac{-E_1 \varepsilon_r \varepsilon_0}{e N_A}$$

$$= \frac{4.5\times10^5 \times 12 \times 8.85\times10^{-12}}{1.6\times10^{-19} \times 2\times10^{20}}$$

$$= \underline{1.49\ \mu m}$$

$$E_2 = E_1 + w_i N_D^- e/\varepsilon$$

$$= -4.5\times10^5 + \frac{6\times10^{-6} \times 10^{19} \times 1.6\times10^{-19}}{12\times8.85\times10^{-12}}$$

$$= -3.6\times10^5\ V/m$$

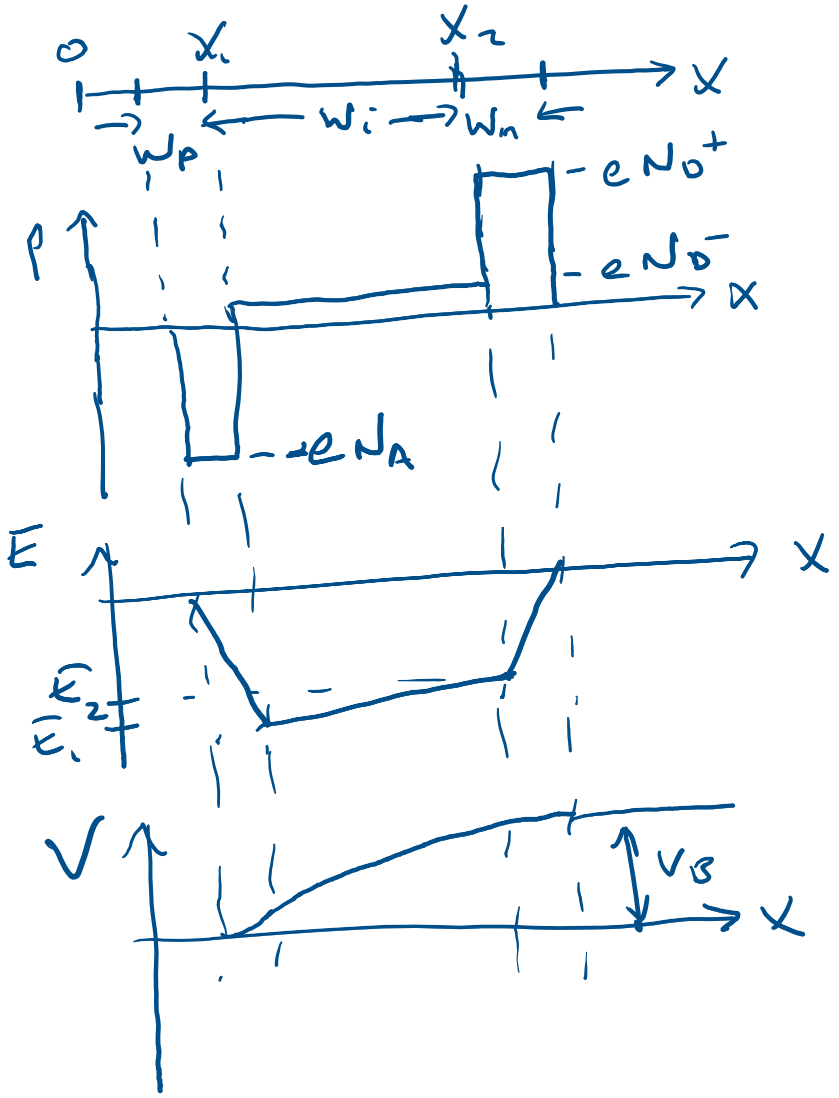
$$w_n = \frac{-E_2 \varepsilon_r \varepsilon_0}{e N_D^+} = \underline{2.38\ \mu m}$$

The bias voltage is the area under the E plot

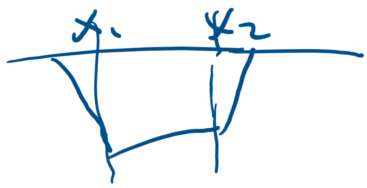
$$V_B = \left| E_1 w_p + E_2 w_n + \frac{E_1 + E_2}{2} w_i \right|$$

$$= 4.5\times10^5 \times 1.49\times10^{-6} + 3.6\times10^5 \times 2.38\times10^{-6} + 4.05\times10^5 \times 6\times10^{-6} = \underline{3.96\ V}$$

b)



5 c)



We can find the time by

$$dt = \frac{dx}{v} \quad \text{and} \quad v = \mu E$$

so

$$\tau = \frac{1}{\mu}\int_{x_2}^{x_1} \frac{dx}{E(x)}$$

In the intrinsic region, taking $u = x_2 - x$, $du = -dx$

$$E(u) = E_2 - \frac{\Delta E}{w_i} u$$

so

$$\tau = \frac{1}{\mu}\int_0^{w_i} \frac{du}{|E_2| + \frac{\Delta E}{w_i} u} \qquad \Delta E = |E_1 - E_2|$$

$$\tau = \frac{1}{\mu}\frac{w_i}{\Delta E} \ln\left(|E_2| + \frac{\Delta E}{w_i}u\right)\Bigg|_0^{w_i} = \frac{1}{\mu}\frac{w_i}{\Delta E}\ln\left(\frac{E_1}{E_2}\right)$$

and $\frac{\Delta E}{w_i} = e N_D^- / \varepsilon$

$$\therefore \tau = \frac{\ln(E_1/E_2)}{\mu_h e N_D^- / \varepsilon_r \varepsilon_0}$$

For the value in this case we'll use $\frac{\Delta E}{w_i} = \frac{9\times10^5}{6\times10^{-6}} = 1.5\times10^{10}\,\frac{V}{m^2}$

$$\tau = \frac{\ln(4.5/3.6)}{0.045\,\frac{m^2}{Vs} \times 1.5\times10^{10}\,\frac{V}{m^2}} = \underline{0.34\text{ ns}}$$