EE401: Advanced Communication Theory

Professor A. Manikas
Chair of Communications and Array Processing

Imperial College London

An Introductory Overview

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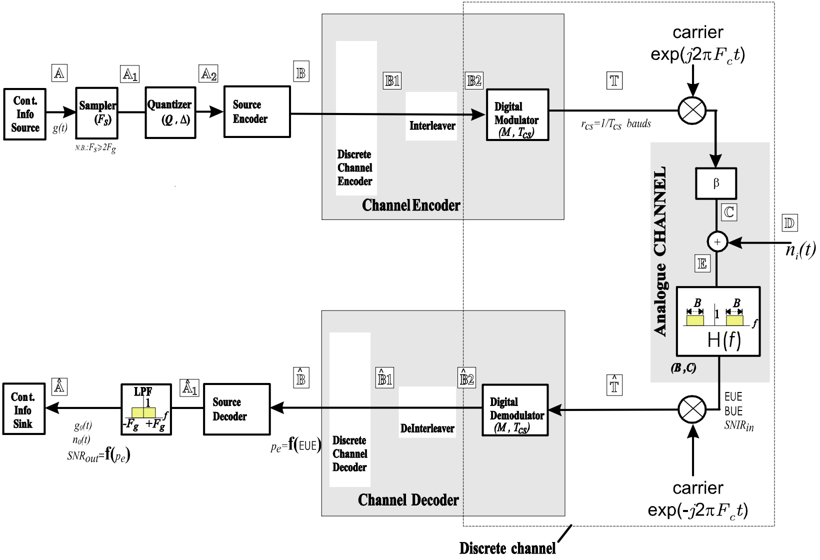
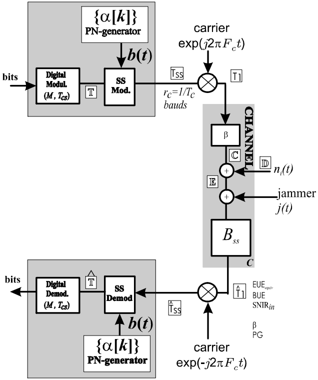
Block Structure of a Basic Digital Comm System

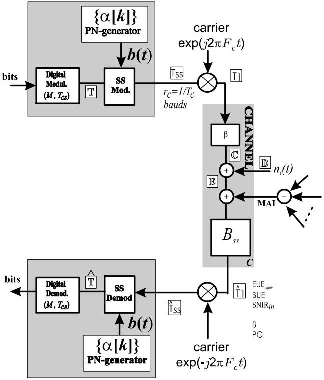


Block Structure of a Spread Spectrum Comm System

SSS:



CDMA:

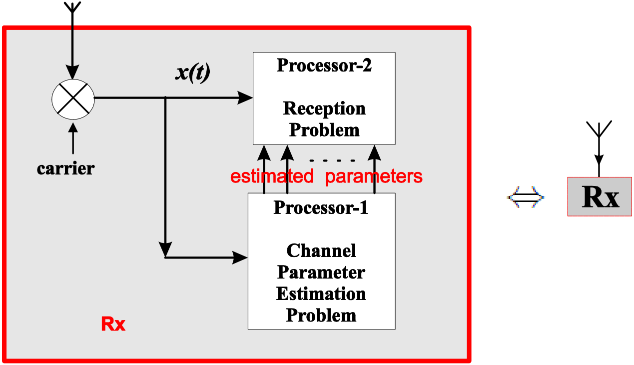


Main Parts (SISO): Tx, Wireless Channel & Rx

- A wireless system can be partitioned into 3 main parts:
  1. **Tx** (a "**source** " that sends/transmits some information using wave propagation)
  2. **Wireless Channel** (the **physical propagation paths**)
  3. **Rx** (a "**sink** " that receives the transmitted waves)

  and the objective in general is
  - to increase the **communication speed** (which is known as channel capacity)
    without sacrificing the **quality of service** (for a given energy + bandwidth)

Generic Rx Architecture

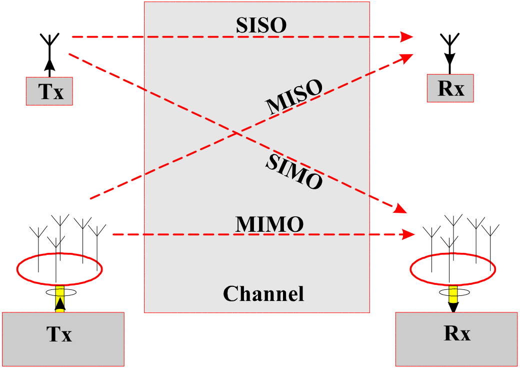


- The **quality of the receiver (Rx)** is a function of the **quality of the estimated channel parameters**
- Note that the receiver is **continuously designed** (based on these estimates) from **time frame** to **time frame.**

Classifications of Wireless Systems

- There are many classifications. For instance:
  1. according to the **bandwidth/carrier**: **narrowband** or **wideband**
  2. according to the **spreading capabilities**: **conventional** or **spread spectrum**
  3. according to the **number of carriers**: **single carrier** or **multicarrier**
  4. according to the **"generation"**: 1G, 2G, **3G** , **3G+**
  5. according to the **"access"**: **TDMA,FDMA** , **CDMA** ,
- The **overall aims:**
  - **speed** $=$ $\uparrow$,
  - **but maintaining** **reliability** (quality of service) & **spectral efficiency** (EUE,BUE)
- The current speed is expected to increase by the utilisation of the new technology of multiple antennas (MIMO) and this gives rise to **a new classification which super-sets** all the above.

- **New** classification: according to the **number of sensors/antennas** used in both Tx and RX



Terminology					
S:	Single		M:	Multiple
I:	Input		O:	Output

- **My Terminology**

Terminology-1 (More Representative)		
1	SISO:	Scalar-Input-Scalar-Output Channel
2	SIVO:	Scalar-Input-Vector-Output Channel
3	VISO:	Vector-Input-Scalar-Output Channel
4	VIVO:	Vector-Input-Vector-Output Channel

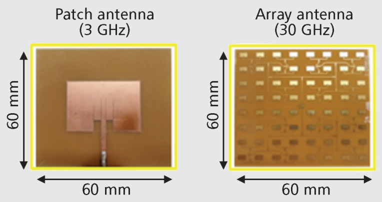
- **Alternative Terminology**

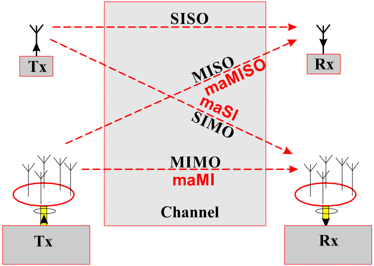
Terminology-2 (Initial)		
1	SESE:	from Single-Element (SE) Tx to Single-Element (SE) Rx
2	SEME:	from Single-Element (SE) Tx to Multiple-Element (ME) Rx
3	MESE:	from Multiple-Element (ME) Tx to Single-Element (SE) Rx
4	MEME:	from Multiple-Element (ME) Tx to Multiple-Element (ME) Rx

Terminology-3 (More Popular)		
1	SISO:	Single-Input-Single-Output
2	SIMO:	Single-Input-Multiple-Output
3	MISO:	Multiple-Input-Single-Output
4	MIMO:	Multiple-Input-Multiple-Output

Massive MIMO

- There is an increased interest in "**massive" MIMO** (i.e. to increase the "degrees-of-freedom" by **increasing the number of antennas)** .





SIMO, MISO, MIMO: Two modelling philosophies

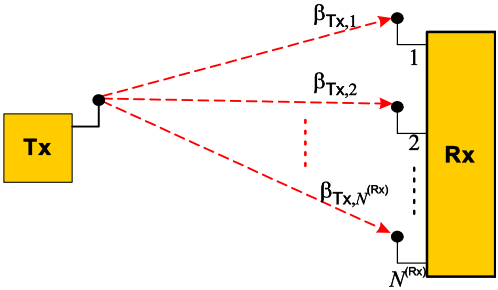
- **Non-Parametric** :

  The antennas are assumed to work independently and thus the various links from antenna-to-antenna are represented by single complex numbers.

- **Parametric:**

  All antennas are "working together" as a single system (an antenna array system) and the various links are mathematically modelled

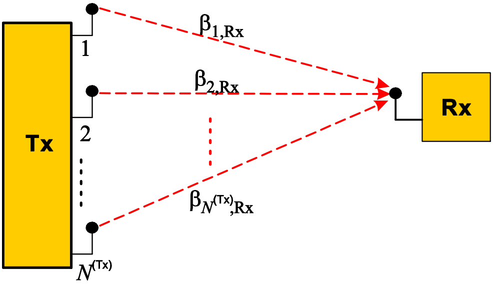
SIMO Wireless Systems (non-parametric)



Single-Input Multiple-Output (SIMO)

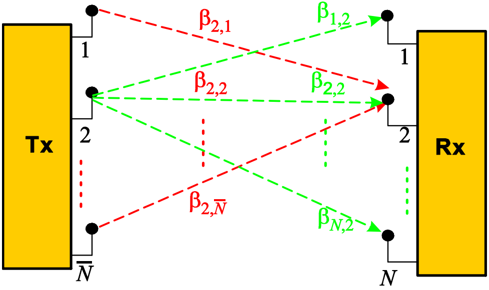
- Remember: SISO - one complex number β per path

MISO Wireless Systems (non-parametric)



Multiple-Input Single-Output (MISO)

MIMO Wireless Systems (non-parametric)



Multiple-Input Multiple-Output (MIMO)

$$\begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,N^{(\mathrm{Tx})}} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,N^{(\mathrm{Tx})}} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{N^{(\mathrm{Rx})},1} & \beta_{N^{(\mathrm{Rx})},2} & \cdots & \beta_{N^{(\mathrm{Rx})},N^{(\mathrm{Tx})}} \end{bmatrix}$$

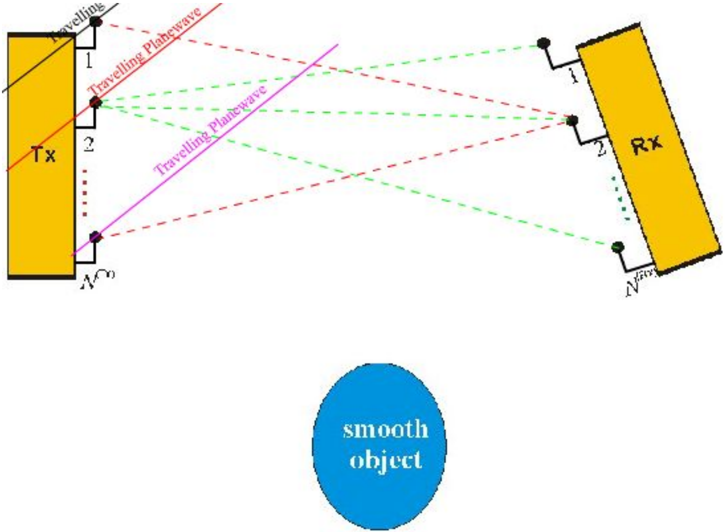
Parametric Approaches (Array Processing)

Introduction

- The above modelling will result into a statistical approach (used in Wiener's estimation theory and Shannon's communication theory). e.g. many MIMO books, papers and tutorials: non parametric

- Although this approach is suitable for single antenna systems (i.e. SISO), it does not properly fit multiple antennas since it
  - ignores the **Cartesian coordinates** and orientations of Tx and Rx (i.e. ignoring the geometry/location of the multiple antennas),
  - ignores the **directions** of the signals,
  - ignores **propagation models** (planewaves or spherical waves),
  - etc.

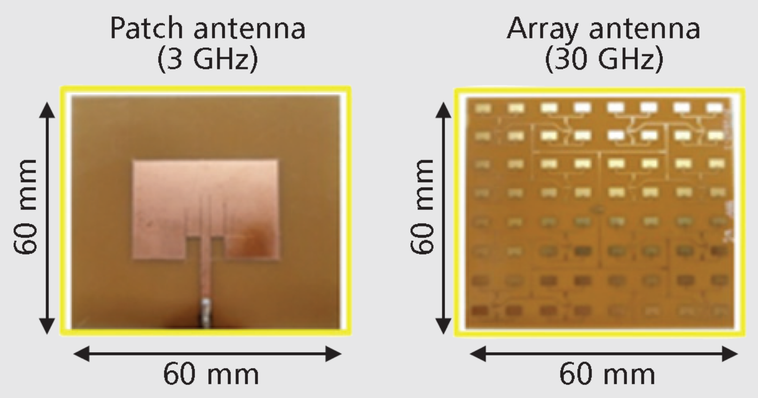
Revisiting Multiple-Input Multiple-Output (MIMO)



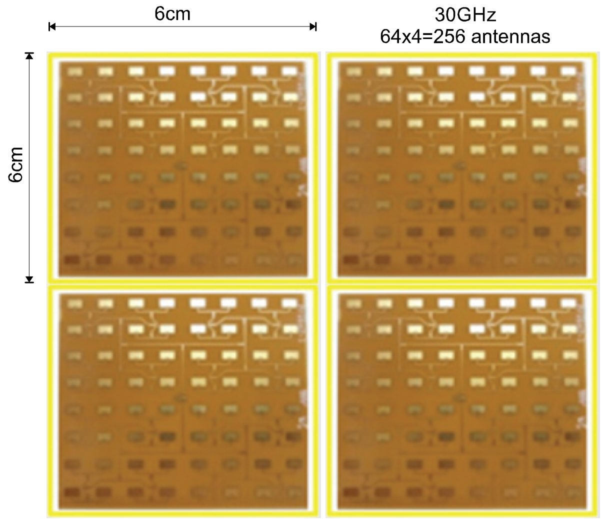
Summary Table of SISO, MISO, SIMO and MIMO

	Non-Parametric	Parametric (Array Processing)
SISO:	β	
SIMO:	$\begin{bmatrix} \beta_{Tx,1} \\ \beta_{Tx,2} \\ \cdots \\ \beta_{Tx,N^{(\mathrm{Rx})}} \end{bmatrix}$	$= \beta \underline{a}^{(\mathrm{Rx})}$
MISO:	$\begin{bmatrix} \beta_{1,\mathrm{Rx}} \\ \beta_{2,\mathrm{Rx}} \\ \cdots\cdot \\ \beta_{N^{(\mathrm{Tx})},\mathrm{Rx}} \end{bmatrix}$	$= \beta \underline{a}^{(\mathrm{Tx})}$
MIMO:	$\begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,N^{(\mathrm{Tx})}} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,N^{(\mathrm{Tx})}} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{N^{(\mathrm{Rx})},1} & \beta_{N^{(\mathrm{Rx})},2} & \cdots & \beta_{N^{(\mathrm{Rx})},N^{(\mathrm{Tx})}} \end{bmatrix}$	$= \beta \underline{a}^{(\mathrm{Rx})} \underline{a}^{(\mathrm{Tx})^H}$ $\Leftrightarrow \beta \underline{a}^{(virtual)}$

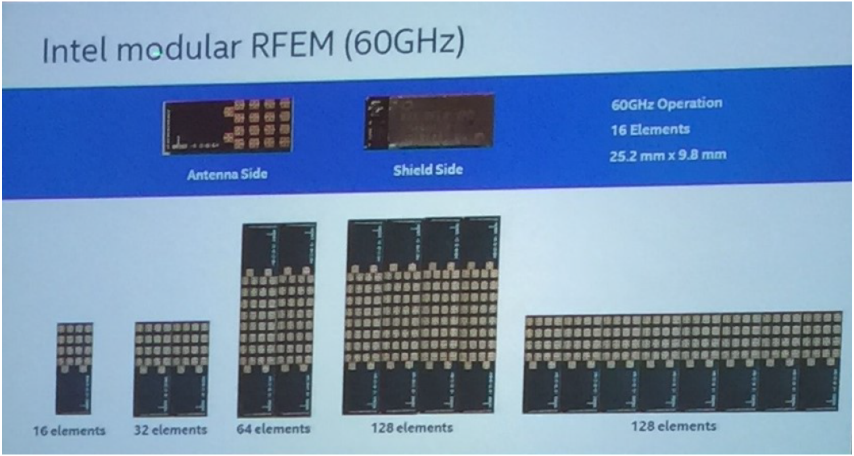
Examples of Antenna Arrays for Modern Wireless Systems



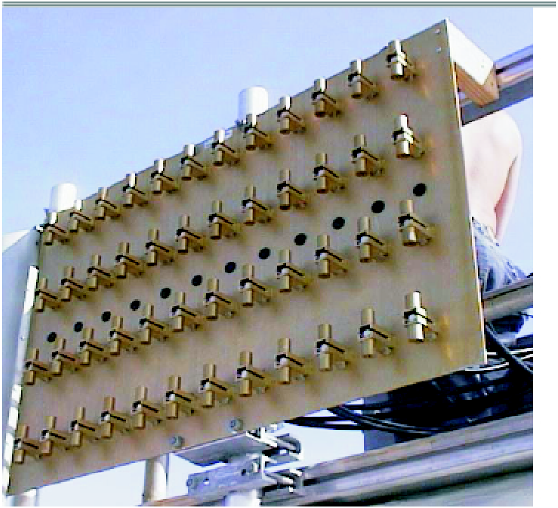
A Planar Array Geometry of 256 antennas at 30GHz



Intel Antenna Arrays - 60GHz

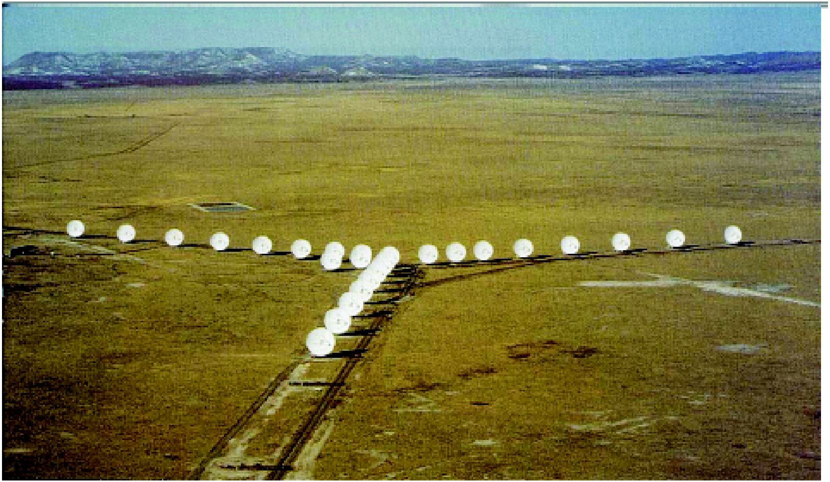


A 2GHz Antenna Array of 48 Elements



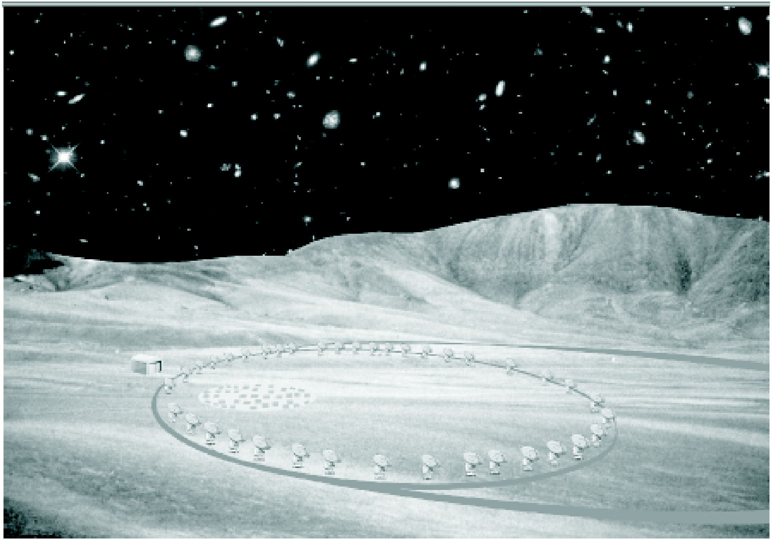
Owens Valley Radio Observatory Array

The New Mexico Very Large Array of 27 Elements

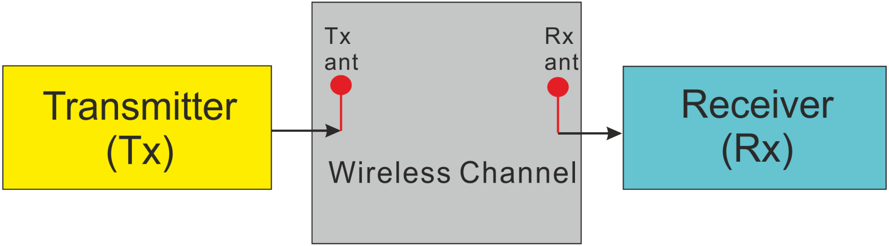


(along rail road tracks - 35km)

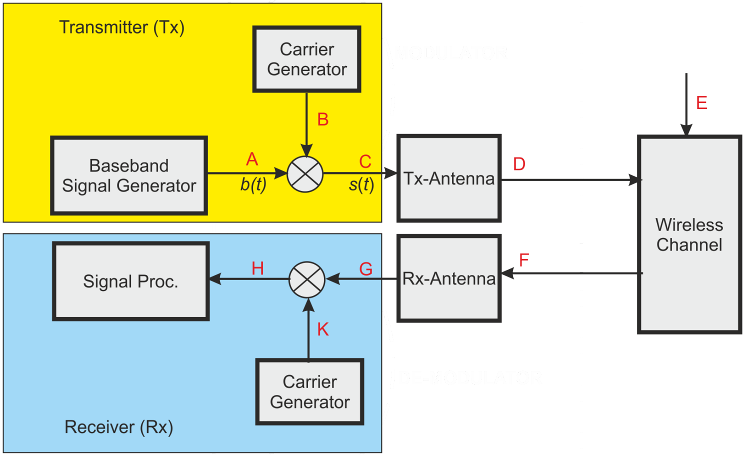
A Large Circular Array



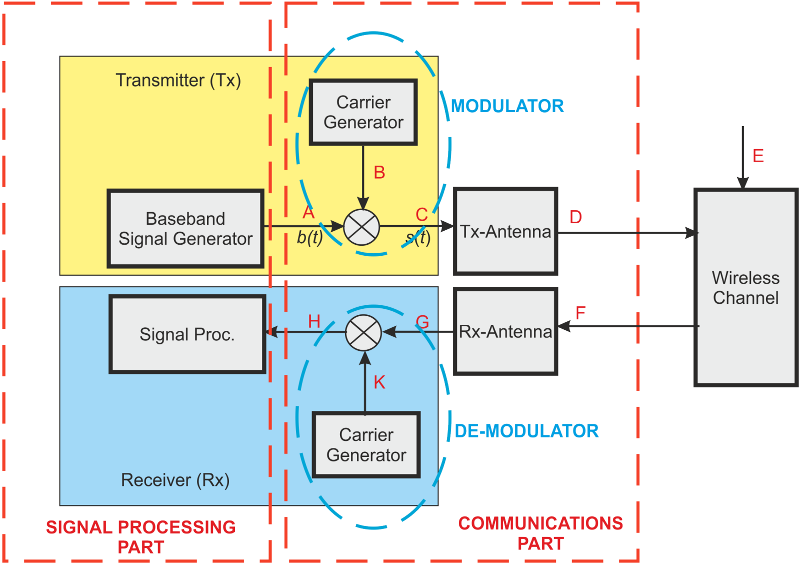
Basic SISO System Architecture



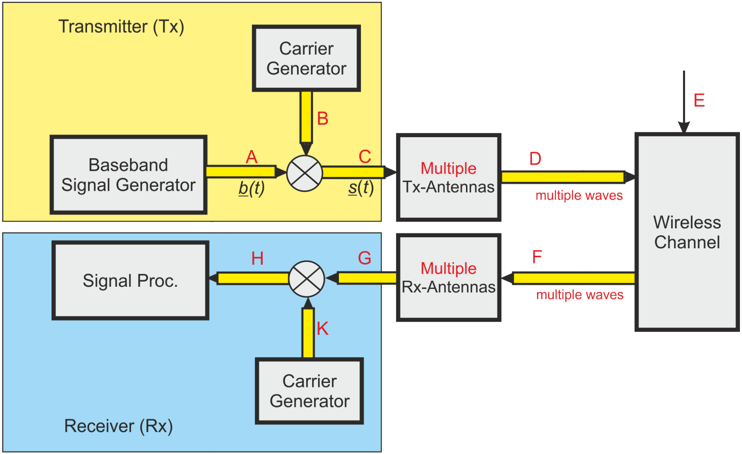
Basic SISO System Architecture



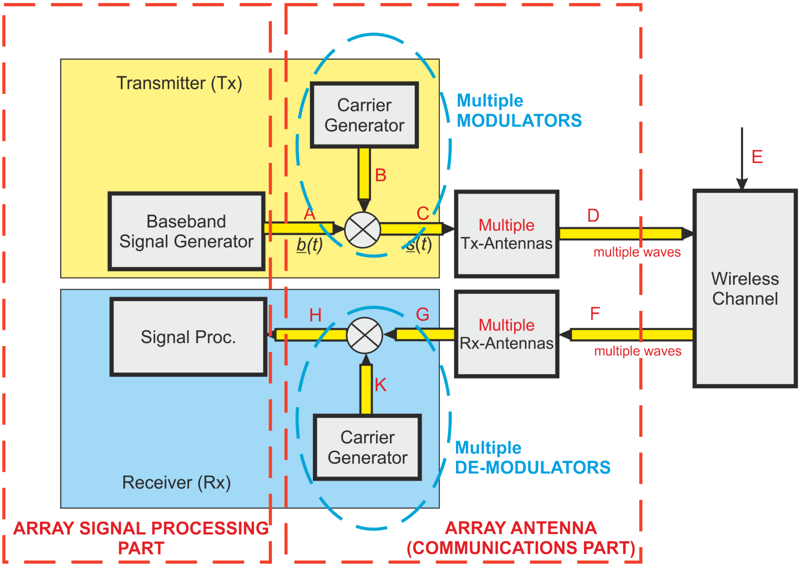
Basic SISO System Architecture



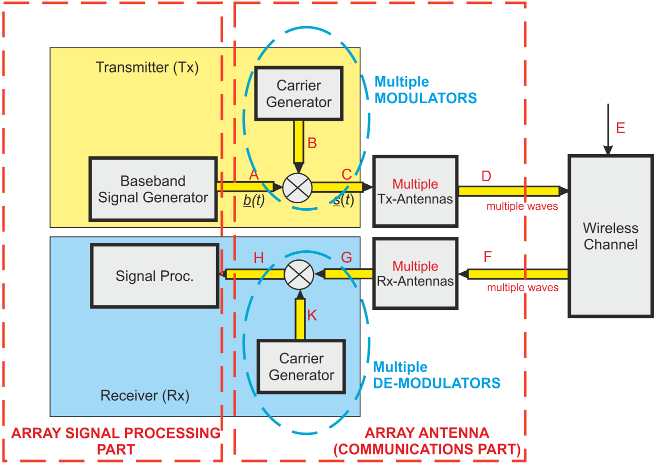
Basic MIMO System Architecture



Basic MIMO System Architecture



Signal at point-H



signal at point $\boxed{\mathrm{H}}$: $\underline{x}(t) =$ function of "array manifold vector" $\underline{S}$

The Structure of the Array Manifold Vector

- The vector $\underline{S}$ is known as
  - Array Manifold Vector or
  - Array Response Vector, or
  - Source Position Vector (SPV)

- The vector $\underline{S}$ has a **profound mathematical structure** and is a function of a number of parameters such as Directions, carrier,etc

$$\underline{S}(\theta, \phi, F_c, c, \underline{r}_1, \underline{r}_2, \underline{r}_3, \ldots, \underline{r}_N)$$

Extended Manifolds

- array manifold: we can add more wireless parameters from the Tx, Rx and channel

- For instance

$\underline{S}(\theta, \phi, F_c, c, \underline{r}_x, \underline{r}_y, \underline{r}_z,$
pseudorandom sequ, delay, polarisation parameters,
No.of subcarriers/carriers, bandwidth, Doppler frequency).

$\Downarrow$

$\Downarrow$

$\underline{h} \triangleq$ spatiotemporal manifold [it is a function of the original $\underline{S}(\theta, \phi, F_c, c, \underline{r}_x, \underline{r}_y, \underline{r}_z)$]

The Structure of the Array Response Vector

- From now on in this presentation the vector $\underline{a}$ will represent all multiple antenna wireless systems, i.e.

$$\underline{a} \triangleq \left\{ \begin{array}{ll} \underline{S}^{(\mathrm{Rx})} & \text{SIMO} \\ \underline{S}^{(\mathrm{Tx})} & \text{MISO} \\ \underline{S}^{(\mathrm{virtual})} = \underline{S}^{(\mathrm{Tx})} \otimes \underline{S}^{(\mathrm{Rx})} & \text{MIMO} \end{array} \right\}$$

- The vector $\underline{a}$ is known as
  - Array Manifold Vector or
  - Array Response Vector (alternative symbol $\underline{S}$)
- The vector $\underline{a}$ has a **profound mathematical structure** and is a function of a number of parameters such as Directions, carrier,etc

$$\underline{a}(\theta, \phi, F_c, c, \underline{r}_1, \underline{r}_2, \underline{r}_3, \ldots, \underline{r}_N)$$

- Various forms of $\underline{a}$ have different dimensions but always a **profound mathematical structure**

  This leads to Differential Geometry which complements the statistical signal processing and Shannon's communication theory.

Differential Geometry

- **Differential geometry** is a branch of mathematics that is concerned with the application of differential calculus for the investigation of the properties of geometric **curves**, **surfaces** and **other objects** known as **'manifolds'**.

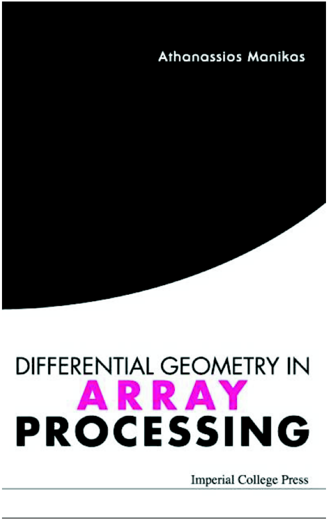
- Manifolds have a **deep** and **profound mathematical structure** and have been **an area of intense pure mathematical analysis**.

$$\underline{p} \longmapsto \text{mathematical object}$$

- In Physics, Albert Einstein (Nobel 1921) used differential geometry to express his **general theory of relativity**
  - where the universe is a **smooth manifold** equipped with pseudo-Riemannian metric (described the curvature of space-time).

Fundamental Questions

- Diff. Geom. helps **answering some fundamental questions** such as:
  - Q1 Is it possible to **express** a wireless system as a space curve or a surface (or a manifold - in general)?
  - Q2 Is it possible to **analyse** a wireless system **by analysing** a curve or a surface?
  - Q3 Is it possible to **design** a wireless system **by designing** a curve or a surface?
  - Q4 What do we **stand to gain** by expressing wireless systems as mathematical objects such as curves or surfaces?

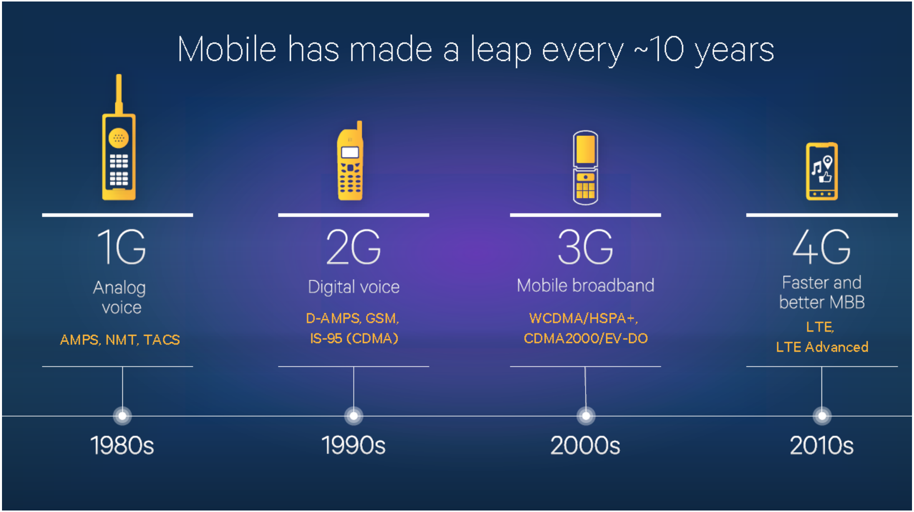


Conclusion

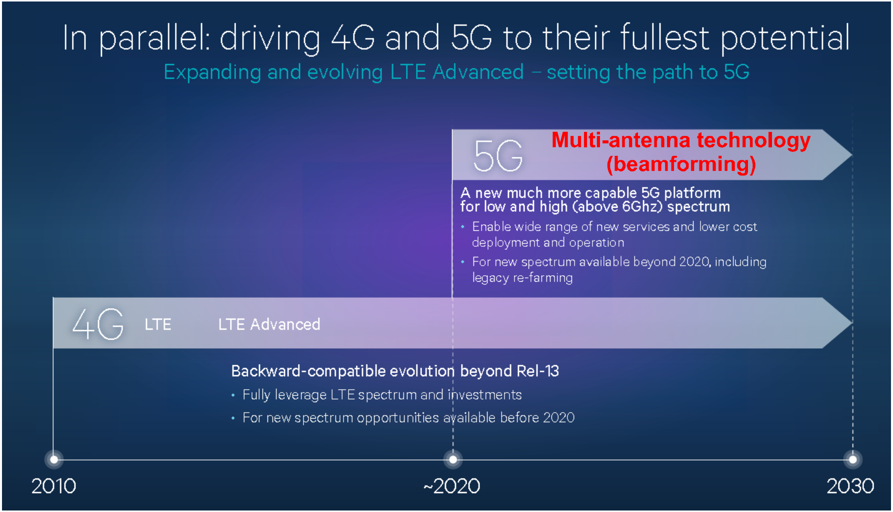
- The overall conclusions that can be drawn are:
  1) Differential Geometry is **hand in hand** with **SIMO** , **MISO** and **MIMO**

  2) Differential Geometry is at **locked horns** with **SISO**

Mobile Evolution

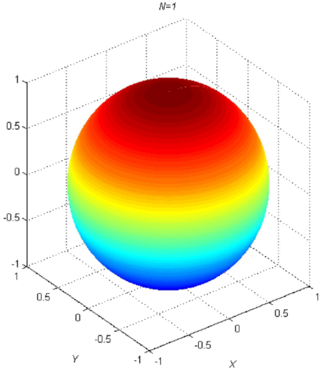


Mobile Evolution (cont.)

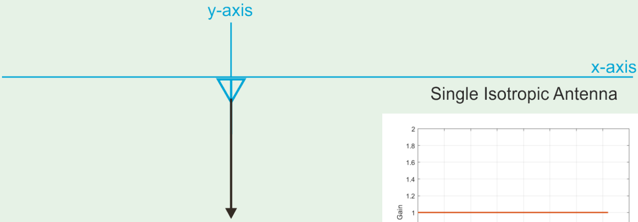


Antenna Array Space Response

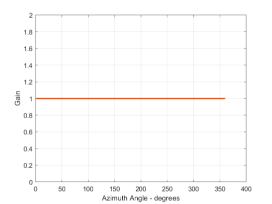
- Isotropic Antenna:

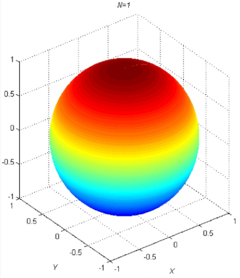


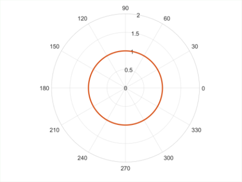
Example (Single Antenna: $N = 1$)



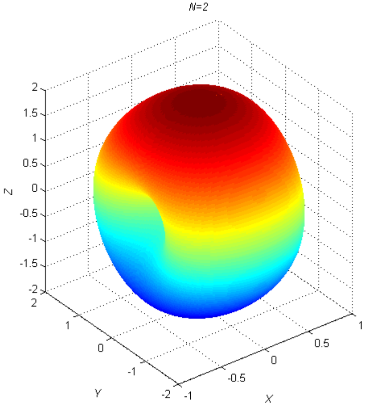
Single Isotropic Antenna

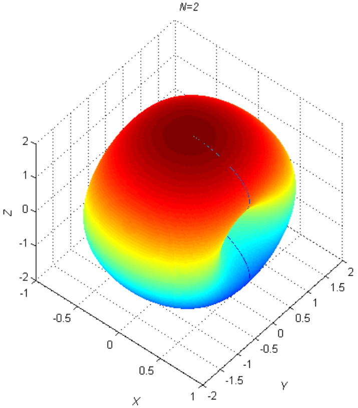






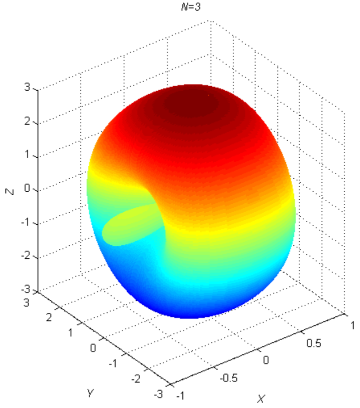
Two Antennas (N=2): Space Response

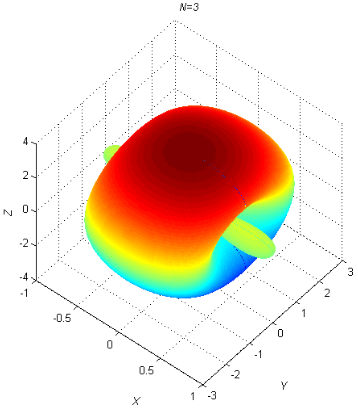




(rotated by 90°)

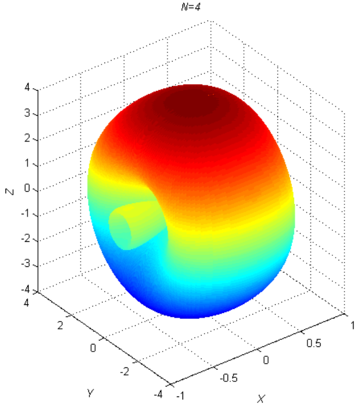
Three Antennas (N=3): Space Response

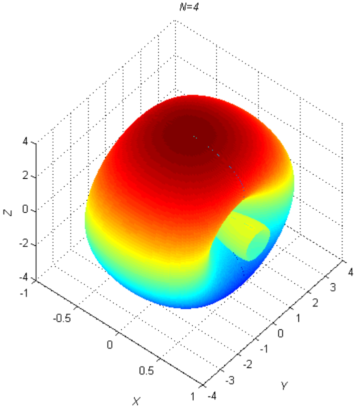




(rotated by 90°)

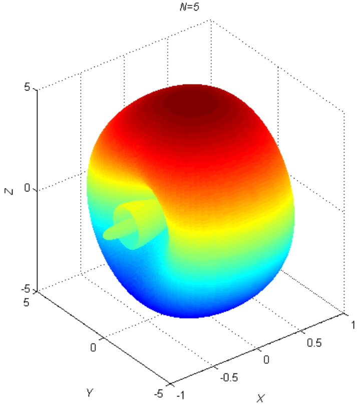
Four Antennas (N=4): Space Response

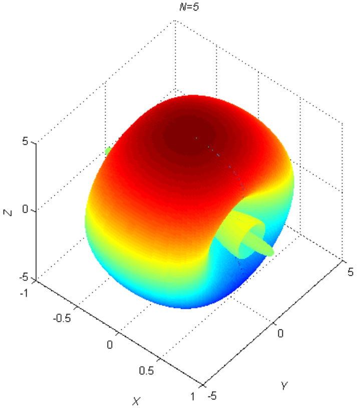




(rotated by 90°)

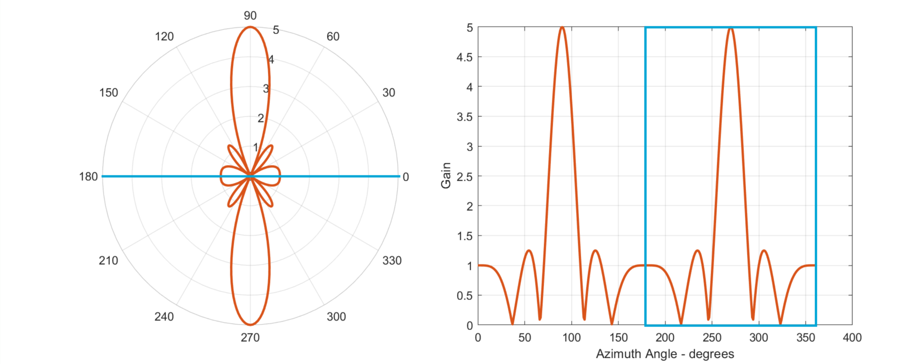
Five Antennas (N=5): Space Response





(rotated by 90°)

Example (Linear Antenna Arrays of 5 elements: $N = 5, d = \lambda/2$)

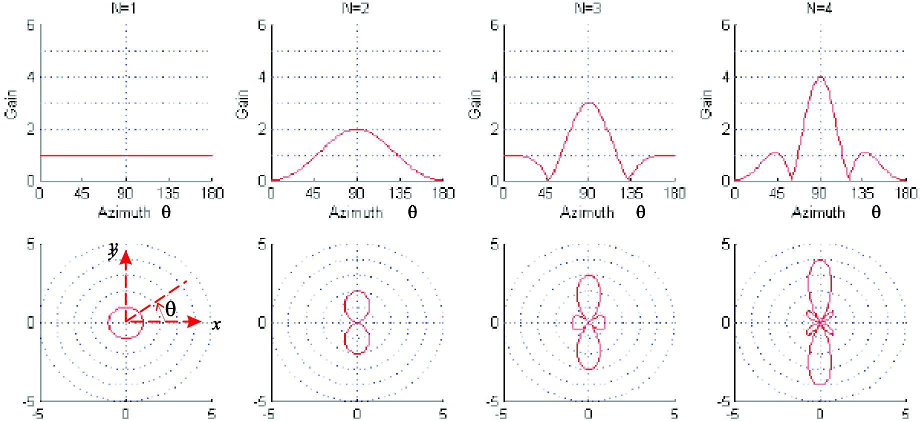


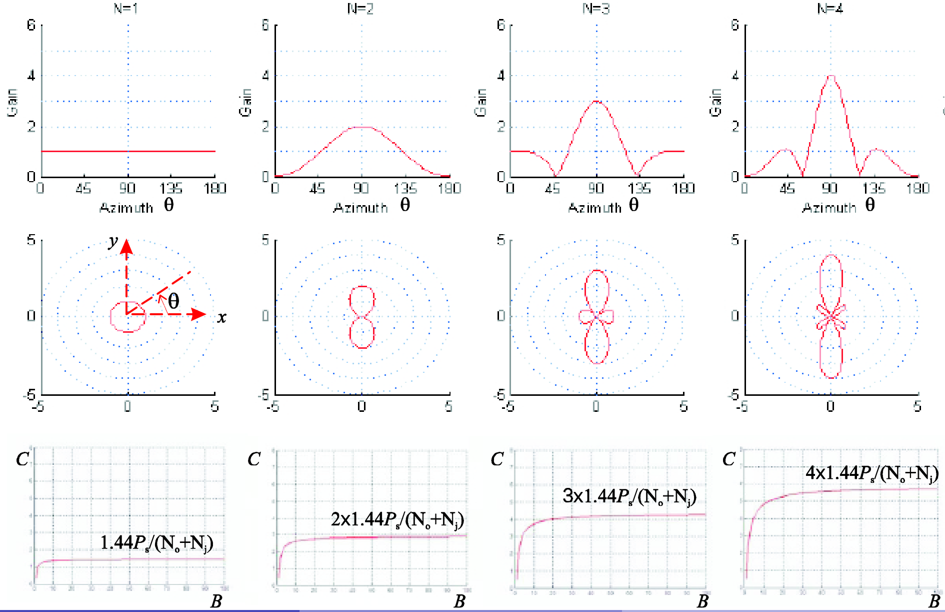
Capacity: Uniform Linear Array (ULA)

- Intersensor spacing$=\lambda/2$;
- N = number of antennas (located on the x-axis).
- Channel Capacity (AWGN):

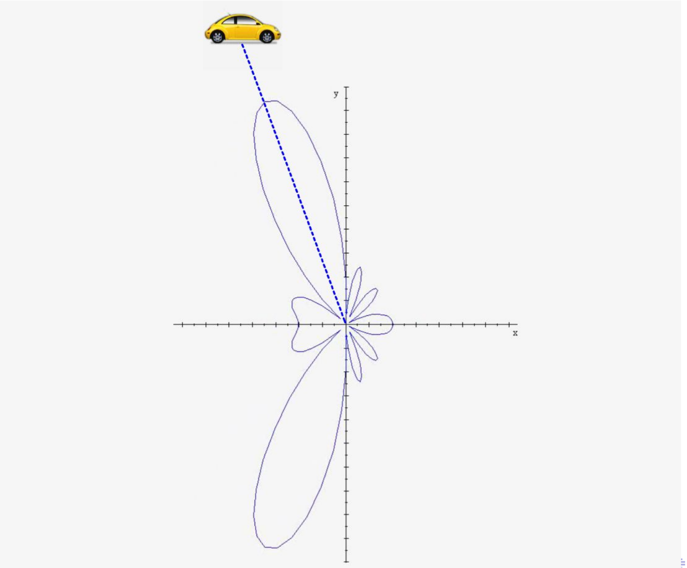
$$C = B\log_2(1 + N \times \mathrm{SNIR}_{in}) \quad (1)$$

$$B \longrightarrow \infty \Rightarrow C \longrightarrow N \times 1.44 \frac{P_s}{N_0 + N_j \downarrow} \quad (2)$$

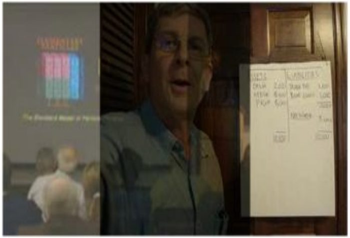

N=1
N=2
N=3
N=4
Gain
Azimuth θ
0
45
90
135
180
0
2
4
6
5
-5
y
x
θ


N=1
N=2
N=3
N=4
Gain
Azimuth θ
y
x
θ
C
B
1.44P_s/(N_o+N_j)
2x1.44P_s/(N_o+N_j)
3x1.44P_s/(N_o+N_j)
4x1.44P_s/(N_o+N_j)

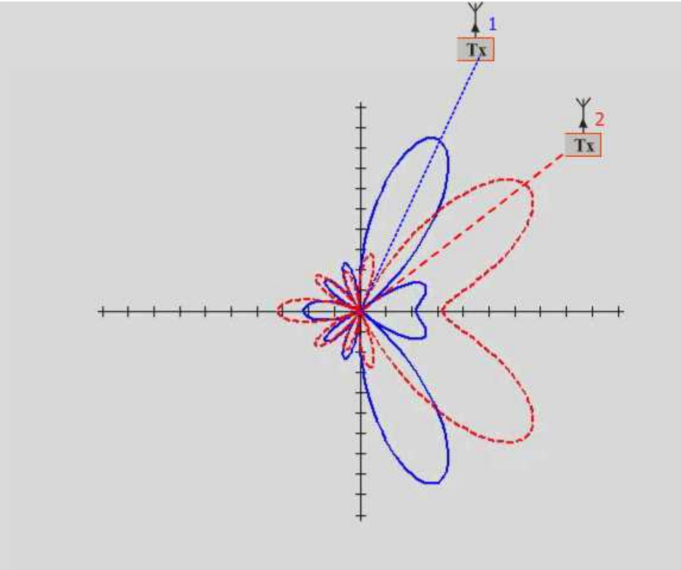
SIMO Wireless Reception and Tracking (ULA, N=5)



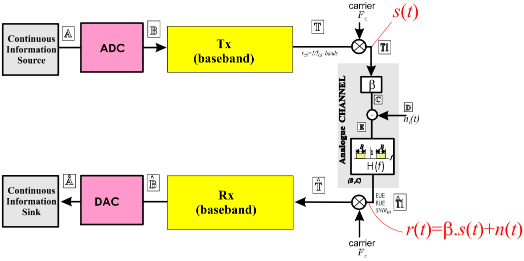
Multiple Access Interference Cancellation (ULA, N=5)



Interference Cancellation with Motion (ULA, N=5)



Appendices - A: Basic Performance Criteria



$$\text{SNR}_{in} = \frac{\text{Power of signal at } \widehat{\text{T}}}{\text{Power of noise at } \widehat{\text{T}}} = \frac{\mathcal{E}\left\{(\beta s(t))^2\right\}}{\mathcal{E}\left\{n(t)^2\right\}} = \frac{\beta^2 P_s}{\underbrace{N_0 B}_{\triangleq P_n}} \quad (3)$$

$$p_e = \text{BER at point } \widehat{\text{B}} \quad (4)$$

$$\text{SNR}_{out} = \frac{\text{Power of signal at } \widehat{\text{A}}}{\text{Power of noise at } \widehat{\text{A}}} = \underbrace{\text{f}\{p_e\}}_{\text{denotes: a function of } p_e} \quad (5)$$

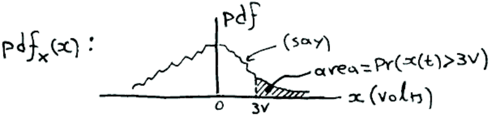
Tail function (or Q-function) for Gaussian Signals

Probablity and Probability-Density-Function (pdf)

- Consider a random signal $x(t)$ with a known amplitude probability density function $\mathrm{pdf}_x(\mathrm{x})$ - not necessarily Gaussian. Then the probability that the amplitude of $x(t)$ is greater than A Volts (say) is given as follows:

$$\Pr(x(t) > A) = \int_A^{\infty} \mathrm{pdf}_x(\mathrm{x}).d\mathrm{x} \tag{6}$$

- e.g.
if $A = 3V \Rightarrow \Pr(x(t) > 3V) = \int_3^{\infty} \mathrm{pdf}_x(\mathrm{x}).d\mathrm{x} = \text{highlighted area}$



Gaussian pdf and Tail function

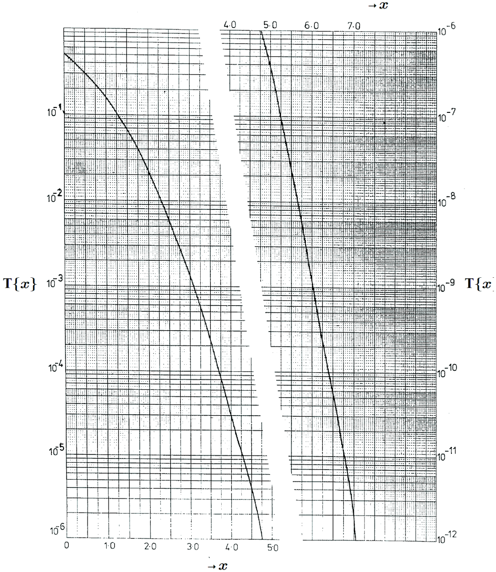
- If $\text{pdf}_x(\mathrm{x}) =$ Gaussian of mean μ_x and standard deviation σ_x (notation used: $\text{pdf}_x(\mathrm{x}) = \mathrm{N}(\mu_x, \sigma_x^2)$), then the above area is defined as the Tail-function (or Q-function)

$$\Pr(x(t) > A) = \int_A^{\infty} \text{pdf}_x(\mathrm{x}).dx \triangleq \mathrm{T}\left\{\frac{|A-\mu_x|}{\sigma_x}\right\} \tag{7}$$

- e.g.
  - if $\text{pdf}_x(\mathrm{x}) = \mathrm{N}(1,4)$ - i.e. $\mu_x = 0, \sigma_x = 2$ - and $A = 3V$ then $\Pr(x(t) > 3V) = \int_3^{\infty} \text{pdf}_x(\mathrm{x}).dx \triangleq \mathrm{T}\left\{\frac{|3-1|}{2}\right\} = \mathrm{T}\left\{1\right\}$
  - if $\text{pdf}_x(\mathrm{x}) = \mathrm{N}(0,1)$ and $A = 3V$ then $\Pr(x(t) > 3V) = \int_3^{\infty} \text{pdf}_x(\mathrm{x}).dx \triangleq \mathrm{T}\left\{3\right\}$
- The Tail function graph is given in the next page

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}.x}.\exp\left\{-\frac{x^2}{2}\right\}$

Fourier Transform Tables

	Description	Function	Transformation
1	Definition	$g(t)$	$G(f)=\int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g\left(\frac{t}{T}\right)$	$\lvert T\rvert .G(fT)$
3	Time shift	$g(t-T)$	$G(f).e^{-j2\pi fT}$
4	Frequency shift	$g(t).e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n}g(t)$	$(j2\pi f)^n.G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n}G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A.g(t)+B.h(t)$	$A.G(f)+B.H(f)$
10	Multiplication	$g(t).h(t)$	$G(f)*H(f)$
11	Convolution	$g(t)*h(t)$	$G(f).H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$
14	Rectangular function	$\mathbf{rect}\{t\} \triangleq \begin{cases} 1 & \text{if } \lvert t\rvert < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$	$\mathbf{sinc}\{f\} \triangleq \frac{\sin(\pi f)}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}\{f\}$
16	Unit step function	$u(t) \triangleq \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$	$\frac{1}{2}\delta(f)-\frac{j}{2\pi f}$
17	Signum function	$\mathrm{sgn}(t) \triangleq \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$	$-\frac{j}{\pi f}$
18	decaying exp (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	decaying exp (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \triangleq \begin{cases} 1-t & \text{if } 0\le t\le 1 \\ 1+t & \text{if } -1\le t\le 0 \end{cases}$	$\mathbf{sinc}^2\{f\}$
22	Repeated function	$\mathbf{rep}_T\{g(t)\}=g(t)*\mathbf{rep}_T\{\delta(t)\}$	$\left\lvert\frac{1}{T}\right\rvert \mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\}=g(t).\mathbf{rep}_T\{\delta(t)\}$	$\left\lvert\frac{1}{T}\right\rvert \mathbf{rep}_{\frac{1}{T}}\{G(f)\}$