Simultaneous Transmission of Information and Power

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I. Introduction

The problem of communication is usually cast as one of transmitting a message generated at one point to another point. Therefore, electricity in the wires became merely a carrier of messages, not a source of power. This understanding of signals independently from their physical embodiments led to modern communication theory, but it also blocked other possible directions. This led to the division of electrical engineering into two distinct subfields, electric power engineering and communication engineering. Some have argued that the greatest inventions of civilization either transform, store, and transmit energy or they transform, store, and transmit information. Although quite reasonable, many engineering systems actually deal with both energy and information. Representation of signals requires the modulation of energy, matter, or some such thing.

Are there scenarios where one would want to transmit energy and information simultaneously over a single line? If there is a power-limited receiver that can harvest received energy, then one should want both things. Modern communication systems that operate under severe energy constraints may also benefit from harvesting received energy. A powerful base station, may effectively be used to recharge mobile devices. In RFID systems, the energy provided through the forward channel is used to transmit over the backward channel.

This experiment deals with the tradeoff between transferring energy and transmitting information over a single noisy line. A characterization of communication systems that simultaneously meet two goals:

1) large transferred energy per unit time, and
2) large transmitted information per unit time,

is provided.

II. Aim and Outline

In this experiment, the problem of wireless information and power transfer over a noisy frequency selective channel will be studied. It is assumed that the communication link is point-to-point with an Additive White Gaussian Noise (AWGN).

The main goal of the experiment is to familiarize the students with the emerging concept of simultaneous wireless information and power transfer and the existing tradeoff between the information rate and transferred energy at the receiver.

Through this experiment, students will be exposed to basics of communication theory, probability and optimization.

In Section III of the experiment, the point-to-point frequency selective channel is introduced. Later, in Section IV, Karush Kuhn Tucker (KKT) optimality condition is introduced via an example. In Section V, the main problem of the experiment is considered. Using the methods introduced in Sections III and IV, the analytical solution is to be obtained and Matlab implementation is required.

For references, students can refer to [1]–[3].

III. Frequency-Selective Channels

In the following, we provide a summary on frequency-selective channels. For more details on discrete-time baseband model in Section III-A, refer to [1, Sections 2.4.1, 2.4.2]. For more details on frequency-selective channels in Section III-B, refer to [1, Sections 3.4.4], and for more details on Orthogonal Frequency Division Multiplexing (OFDM) in Section III-C, refer to [1, Sections 5.3.3].

A. A discrete-time baseband model

A useful channel model is a one, which converts the continuous-time channel to a discrete-time channel. We adopt the usual approach of the sampling theorem[1]. Assume that the input waveform is band-limited to W. The baseband equivalent is then limited to $W/2$ and can be represented as

$$x_b(t) = \sum_n x[n]\mathrm{sinc}(Wt - n), \tag{1}$$

where $x[n] = x_b(n/W)$ and $\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$.

Note that this representation follows from the sampling theorem, which says that any waveform band-limited to $W/2$ can be expanded in terms of the orthogonal basis $\{\mathrm{sinc}(Wt-n)\}_n$, with coefficients given by the samples (taken uniformly at integer multiples of $1/W$).

The complex baseband equivalent channel is given as

$$y_b(t) = \sum_i a_i^b(t) x_b(t - \tau_i(t)) + w(t), \tag{2}$$

where $a_i^b(t) = a_i(t)e^{-j2\pi f_c \tau_i(t)}$ is the complex channel response of the i^{th} path, with $a_i(t)$ denoting the attenuation factor of path i and $\tau_i(t)$ denoting the corresponding delay of path i[2]. This representation is easy to interpret in the time domain, where the effect of the carrier frequency f_c can be seen explicitly[3]. The baseband output is the sum, over each path, of the delayed replicas of the baseband input.

In the following exercise, follow the steps to obtain the output of the baseband channel in terms of the samples taken each $1/W$ seconds.

Exercise 1:

step 1) Obtain the substitution of (1) in (2)

step 2) Obtain the sampled output (denoted by $y[m]$) at multiples of $1/W$ (,i.e., $y[m] = y_b(m/W)$) by simply substituting $t = m/W$

step 3) Obtain the following equation by using the transformation $l \triangleq m - n$

$$y[m] = \sum_l x[m-l] \sum_i a_i^b(m/W)\mathrm{sinc}(l - \tau_i(m/W)W) + w[m]. \tag{3}$$

Defining

$$h_l[m] = \sum_i a_i^b(m/W)\mathrm{sinc}(l - \tau_i(m/W)W), \tag{4}$$

[1]Due to Nyquist sampling theorem, the sampling frequency must be equal or greater than the twice of the greatest frequency in the signal.

[2]Note that the attenuation factor $a_i(t)$ and delay $\tau_i(t)$ of any path can be in general function of time t.

[3]Note that, the carrier frequency f_c only affects the baseband equivalent channel in (2) via the phase of the complex gain, i.e., $a_i^b(t) = a_i(t)e^{-j2\pi f_c \tau_i(t)}$.

the equation (3) can be written as

$$y[m] = \sum_{l} h_l[m]x[m-l] + w[m]. \tag{5}$$

Note that $h_l[m]$ is known as the l^{th} complex channel filter tap at time m. In the special case where the gains $a_i^b(t)$ and the delays $\tau_i(t)$ of the different paths are time-invariant (independent of time), (4) simplifies to

$$h_l[m] = \sum_{i} a_i^b \text{sinc}(l - \tau_i W). \tag{6}$$

B. *Frequency-Selective channels*

In this subsection, the concept of frequency-selective channels are introduced. An important general parameter of a wireless system is the multipath delay spread, T_d, defined as

$$T_d \triangleq \max_{i,j} |\tau_i(t) - \tau_j(t)| \tag{7}$$

The delay spread of the channel dictates its frequency coherence. Wireless channels change both in time and frequency. The *coherence bandwidth* W_c shows how quickly it changes in frequency. The coherence bandwidth, Wc, is given by

$$W_c \triangleq \frac{1}{2T_d}. \tag{8}$$

When the bandwidth W of the input is considerably less than W_c, the channel is usually referred to as flat fading. When the bandwidth W is much larger than W_c, the channel is said to be frequency-selective, and it has to be represented by multiple taps (see equation (5)).

C. *Orthogonal Frequency Division Multiplexing (OFDM)*

If the channel is linear time-invariant, sinusoidal functions can be utilized in order to transform the system into a particularly simple one. In real-world the communication channels can be approximately time-invariant. Therefore, transformation into the frequency domain can be a fruitful approach to communication over frequency-selective channels. This is the basic idea behind OFDM.

We begin with the discrete-time baseband model in (5), with this assumption that different taps are independent of time, i.e., $h_l[m] = h_l$ and there are finite number of nonzero taps, namely, L. Therefore, (5) can be rewritten as

$$y[m] = \sum_{l=0}^{L-1} h_l x[m-l] + w[m]. \tag{9}$$

Assume that there is an average power constraint P on the channel input. In [1, Section 3.4.4], it is shown that the frequency-selective channel can be converted into N_c independent sub-carriers by adding a cyclic prefix (CP) of length $L-1$ ($L \ll N_c$) to a data vector of length N_c. Suppose this operation is repeated over blocks of data symbols (of length N_c each, along with the corresponding CP of length $L-1$). Then communication over the i^{th} OFDM block can be written as

$$y_n[i] = h_n[i]d_n[i] + w_n[i], \;\; n = 0, ..., N_c - 1, \tag{10}$$

where

$$\boldsymbol{d}[i] = [d_0[i], ..., d_{N_c-1}[i]], \tag{11}$$

$$\boldsymbol{w}[i] = [w_0[i], ..., w_{N_c-1}[i]], \tag{12}$$

$$\boldsymbol{y}[i] = [y_0[i], ..., y_{N_c-1}[i]], \tag{13}$$

are the DFTs of the input, the noise and the output of the i^{th} OFDM block, respectively. $\boldsymbol{h}$ is the DFT of the channel. The transformed channel in (10) can be viewed as a collection of sub-channels, one for each sub-carrier n. Each of the sub-channels is an AWGN channel. The transformed noise is distributed as a complex circularly symmetric Gaussian $\mathcal{CN}(0, N_0)$ in each of the sub-channels and, moreover, the noise is independent across sub-channels. The power constraint on the input symbols in time translates to one on the data symbols on the sub-channels (Parseval theorem for DFTs) given as

$$E[||\boldsymbol{d}[i]||^2] \leq P. \tag{14}$$

In information theory, a channel which consists of a set of noninterfering sub-channels, each of which is corrupted by independent noise, is called a parallel channel. Thus, the transformed channel here is a parallel AWGN channel, with a total power constraint P across the sub-channels. We allocate power to each sub-channel, P_n to the n^{th} sub-channel, such that the total power constraint is met, i.e., $\sum_n P_n = P$. Then, a separate capacity-achieving AWGN code is used to communicate over each of the subchannels. The maximum rate of reliable communication is

$$\sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|h_n|^2}{N_0}\right), \text{ bits per OFDM symbol.} \tag{15}$$

Further, the power allocation (choosing appropriate values for different P_n, such that $\sum_n P_n = P$) can be chosen appropriately, so as to maximize the rate in (15). The "optimal power allocation", thus, is the solution to the following optimization problem:

$$\begin{aligned} \max_{P_0,\ldots,P_{N_c-1}} \quad & \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|h_n|^2}{N_0}\right) \\ \text{s.t.} \quad & \begin{cases} \sum_0^{N_c-1} P_n \leq P \\ P_n \geq 0, \;\; n = 0, ..., N_c - 1 \end{cases}, \end{aligned} \tag{16}$$

IV. Water filling solution using KKT optimality conditions

The optimal power allocation in (16) can be explicitly found. This optimization problem can be solved by Lagrangian methods. Consider the following Lagrangian

$$L(\lambda, P_0, ..., P_{N_c-1}) = \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n|h_n|^2}{N_0}\right) - \lambda\left(\sum_{0}^{N_c-1} P_n - P\right), \tag{17}$$

where λ is the Lagrange multiplier. The Karush-Kuhn-Tucker condition for the optimality of a power allocation is as

$$\frac{\partial L}{\partial P_n} = \begin{cases} 0 & \text{if } P_n > 0 \\ \leq 0 & \text{if } P_n = 0 \end{cases}, \tag{18}$$

such that we have

$$\sum_{0}^{N_c-1} P_n = P. \tag{19}$$

Exercise 2:

step 1) Differentiate in (17), with respect to P_n and obtain the condition where (18) is satisfied.

step 2) Write the MATLAB code to obtain the optimal power allocation for $N_c = 10$, $N_0 = 1$ and $\lambda = 0.28$. Use the following vector as your $\boldsymbol{h}$.

$$\boldsymbol{h} = [.1 + .1i, .2 + .8i, .01 + .2i, .1 + .9, .3 + .1i, .1 + .7i, .09 + .02i, .1 + .8i, .4 + .8i, .1 + .3i]$$

step 3) Plot the obtained optimal power allocations $P_n,\ n = 0, ..., N_c - 1$ with respect to the absolute value of the corresponding subchannel.

step 4) Explain and interpret the results.

V. Wireless information and power transfer

In this section, we consider a problem similar to the one in Section IV, however, with an additional power constraint at the receiver. The observed output y_n for the discrete model is given as (we removed the index i for brevity)

$$y_n = h_n x_n + w_n,\ n = 0, ..., N_c - 1. \tag{20}$$

The transmitter has a fixed available power budget P, which restricts the use of power at the receiver. Namely, we have

$$\sum_{0}^{N_c-1} P_n \leq P \tag{21}$$

At the same time, the total power delivered at the receiver is required to be more than a given threshold, namely, P_d. We therefore have

$$\sum_{0}^{N_c-1} \mathbb{E}[|y_n|^2] = \sum_{0}^{N_c-1} |h_n|^2 P_n + N_0 \geq P_d. \tag{22}$$

The goal is to maximize the information rate (see equation (15)) given that the average power constraints at the transmitter (21) and the receiver (22) are satisfied.

Exercise 3:

step 1) Noting that $\mathbb{E}[|x_n|^2] = P_n$ and $\mathbb{E}[|y_n|^2] = |h_n|^2 P_n + N_0$, write the Lagrangian for the following problem (Note that you will need to consider two Lagrange multiplier. One (denote it by λ) for average power constraint at the transmitter and the other (denote it by μ) for energy constraint at the receiver.

$$\begin{aligned} \max_{P_0,\ldots,P_{N_c-1}} \quad & \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \\ \text{s.t.} \quad & \begin{cases} \sum_{0}^{N_c-1} P_n \leq P \\ P_n \geq 0,\ n = 0, ..., N_c - 1 \\ -\sum_{n=0}^{N_c-1} \mathbb{E}[|y_n|^2] \leq -P_d \end{cases}, \end{aligned} \tag{23}$$

step 2) Using the KKT condition similar to (17), obtain the optimal power allocation.

step 3) Write the MATLAB code to obtains the optimal power allocation for $N_c = 10$, $N_0 = 1$ and $\lambda = 0.2$ and $\mu = 0.2$. Use the following vector as your $\boldsymbol{h}$.

$$\boldsymbol{h} = [.1 + .1i, .2 + .8i, .01 + .2i, .1 + .9, .3 + .1i, .1 + .7i, .09 + .02i, .1 + .8i, .4 + .8i, .1 + .3i]$$

step 4) Plot the obtained optimal power allocations $P_n,\ n = 0, ..., N_c - 1$ with respect to the absolute value of the corresponding subchannel for different values of P_d. Finally interpret and explain the results.

References

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[2] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge Univ Pr, 2004.

[3] P. Grover and A. Sahai, "Shannon meets tesla: Wireless information and power transfer," in *IEEE Int. Sym. Inf. Theory*, Jun. 2010, pp. 2363–2367.