MIMO-OFDM Wireless Communications

Introduction to OFDM

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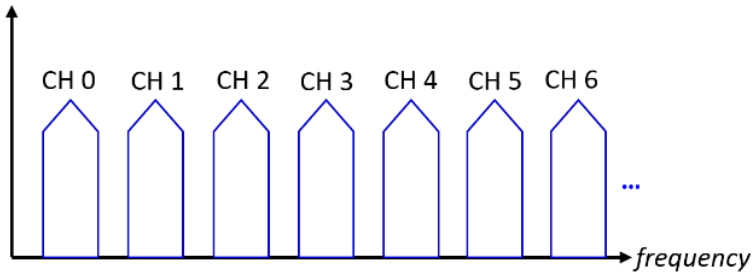
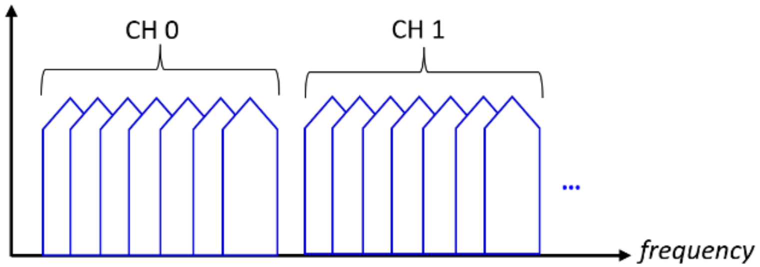
Introduction to OFDM

- Problem with high bandwidth systems → Frequency Selective Fading
- Orthogonal Frequency Division Multiplexing
- Special case of multicarrier FDM transmission →
  - Overlapping subchannels
  - Orthogonal subchannels

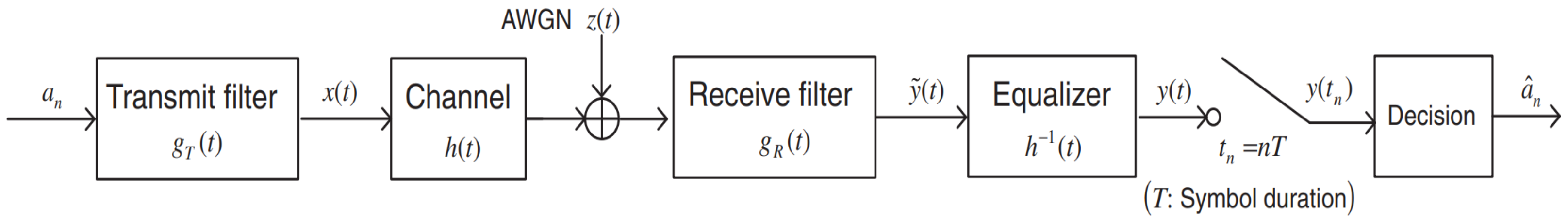


(a) FDM multicarrier system



(b) OFDM system

Single-carrier Transmission



$$y(t) = \sum_{m=-\infty}^{\infty} a_m g(t - mT) + z(t) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

For error-free transmission,

$B_{x(t)} < B_{channel}$

Else, ISI

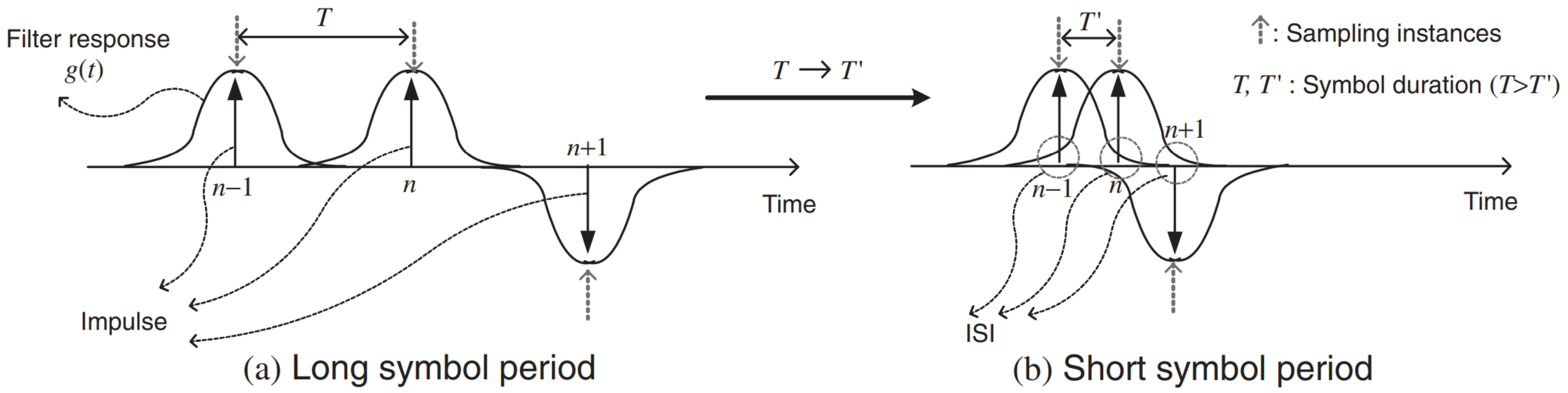
Sampled output of equalizer: $y(t_n) = \sum_{m=-\infty}^{\infty} a_m g((n-m)T) \dots\dots\dots (2),\ with\ t_n = nT$

Isolating $\boldsymbol{n^{th}}$ ***sample to Detect*** $\boldsymbol{a_n}$ **:**

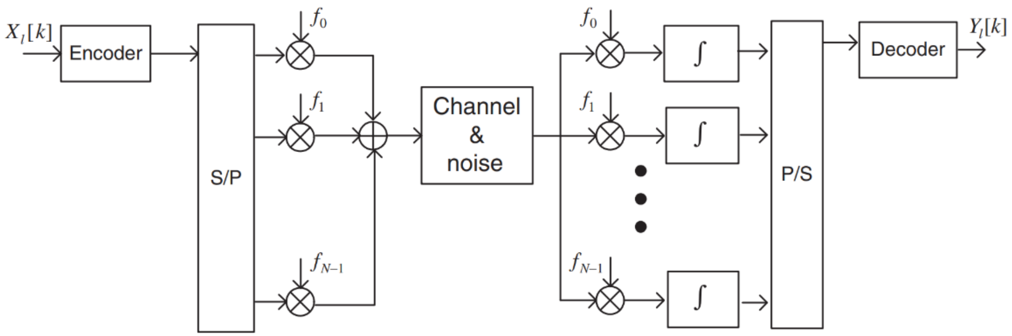
$$y(t_n) = a_n g(0) + \sum_{m=-\infty, m\neq n}^{\infty} a_m g((n-m)T) \dots\dots\dots (3)$$

- The extent of ISI depends on the duration of a symbol period, T

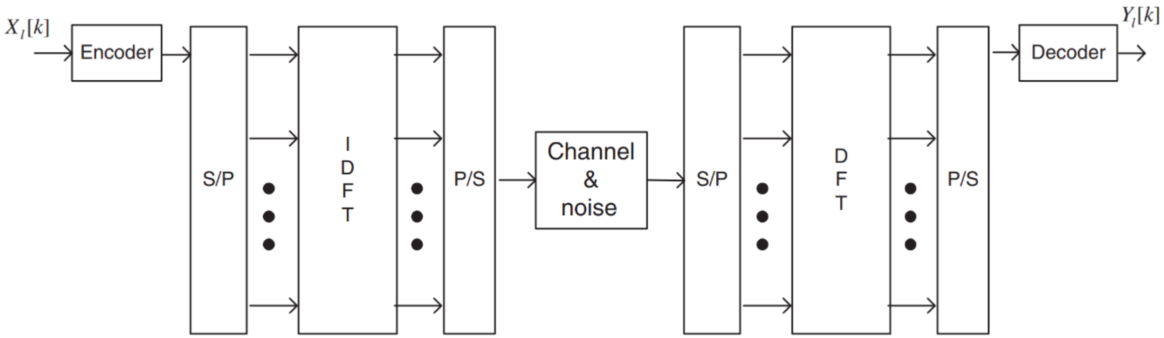
- ISI becomes significant as the data rate increases.



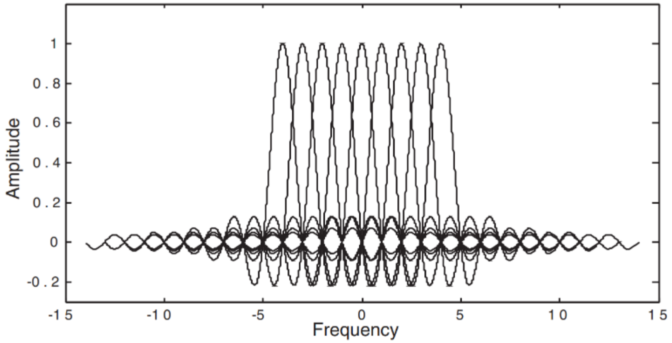
Multi-carrier Transmission



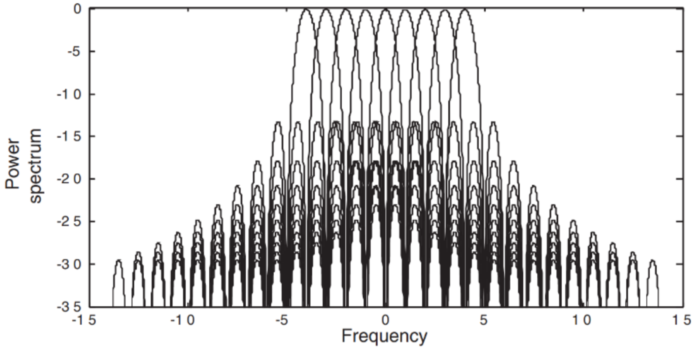
(a) Outline of OFDM transmission scheme



(b) OFDM transmission scheme implemented using IDFT/DFT



(c) The spectrum of OFDM signal (linear scale)



(d) Power spectrum of OFDM signal (dB)

DFT and IDFT are implemented efficiently using FFT and IFFT respectively

- Guard bands reduce out-of-band radiation.
- Guard interval called Cyclic Prefix mitigates ISI

Basic Principles of OFDM
OFDM Modulation and Demodulation

- How is ***orthogonality*** achieved in OFDM?

Consider time-limited complex exponential signal: $\left\{e^{2\pi f_k t}\right\}_{k=0}^{N-1}$

At different subcarriers, $f_k = \dfrac{k}{T_{sym}}$, $\quad where\ 0 \le t \le T_{sym}$

$$\frac{1}{T_{sym}}\int_0^{T_{sym}} e^{j2\pi f_k t} e^{-j2\pi f_i t} dt = \frac{1}{T_{sym}}\int_0^{T_{sym}} e^{j2\pi \frac{k}{T_{sym}} t} e^{-j2\pi \frac{i}{T_{sym}} t} dt$$

$$= \frac{1}{T_{sym}}\int_0^{T_{sym}} e^{j2\pi \frac{(k-i)}{T_{sym}} t} dt$$

$$= \begin{cases} 1, & \forall \text{ integer } k = i \\ 0, & \text{otherwise} \end{cases}$$

Orthogonality condition

- Taking the discrete samples with the sampling instances at $t = nT_s = \frac{nT_{sym}}{N}$, $n = 0,1,2,\dots,N-1$

Discrete-time domain:

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{T_{sym}}\cdot nT_s} e^{-j2\pi\frac{i}{T}\cdot nT_s} = \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{k}{T_{sym}}\cdot\frac{nT}{N}} e^{-j2\pi\frac{i}{T_{sym}}\cdot\frac{nT_{sym}}{N}}$$

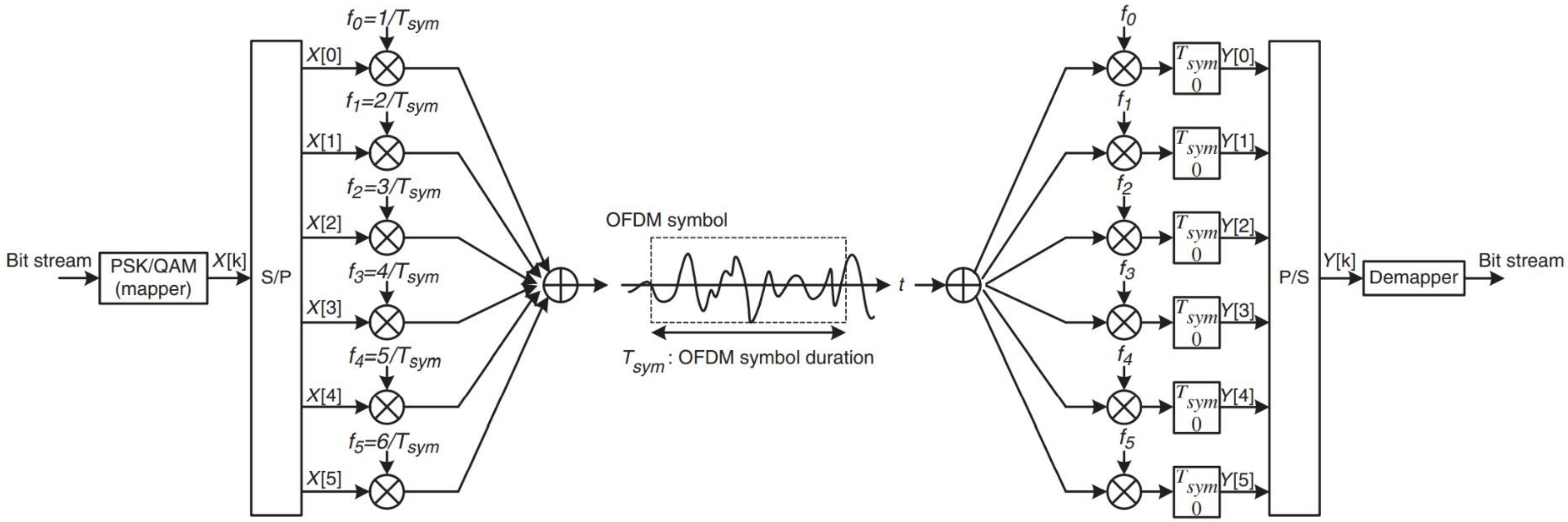
$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi\frac{(k-i)}{N}n}$$

$$= \begin{cases} 1, & \forall \text{ integer } k = i \\ 0, & \text{otherwise} \end{cases}$$

Orthogonality condition

- The above orthogonality is an essential condition for the OFDM signal to be ICI-free

Block diagram of OFDM modulation and demodulation: $N = 6\ symbols$



(a) OFDM modulation/demodulation

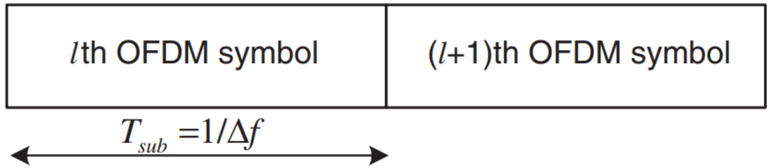
OFDM Guard Interval

Transmitted l^{th} OFDM signal: $x_l(t) = \sum_{k=0}^{N-1} X_l[k] e^{j2\pi f_k (t - lT_{sym})}$ ……………(1)

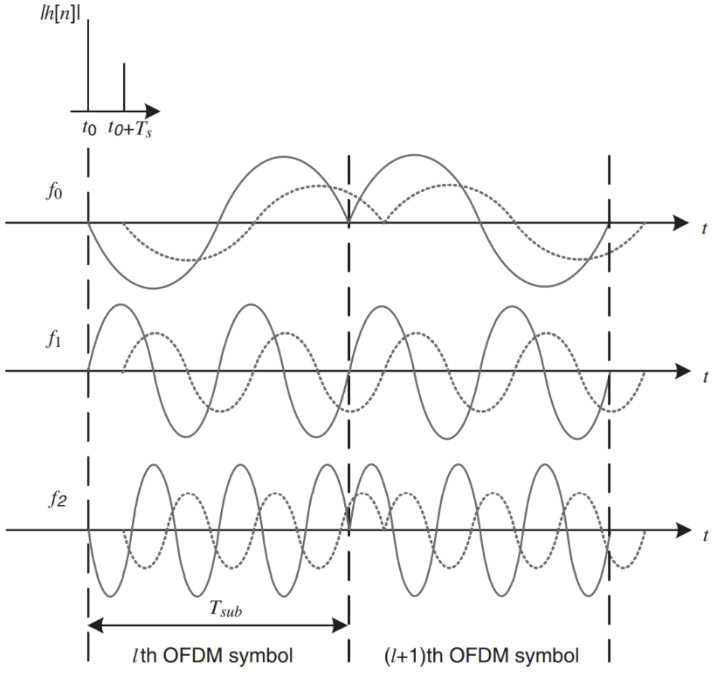
$$lT_{sym} < t \le lT_{sym} + nT_s$$

DT form of sampled received signal:

$$y_l[n] = h_l[n] * x_l[n] + z_l[n] = \sum_{m=0}^{\infty} h_l[m] x[n-m] + z_l[n] \ldots\ldots\ldots(2)$$



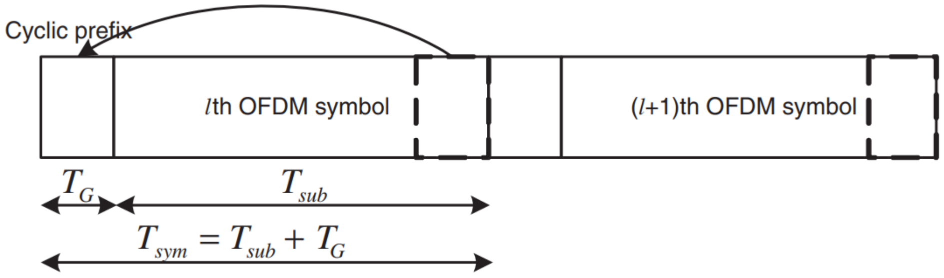
(a) OFDM symbols without guard interval



(b) ISI effect of a multipath channel on the received signal

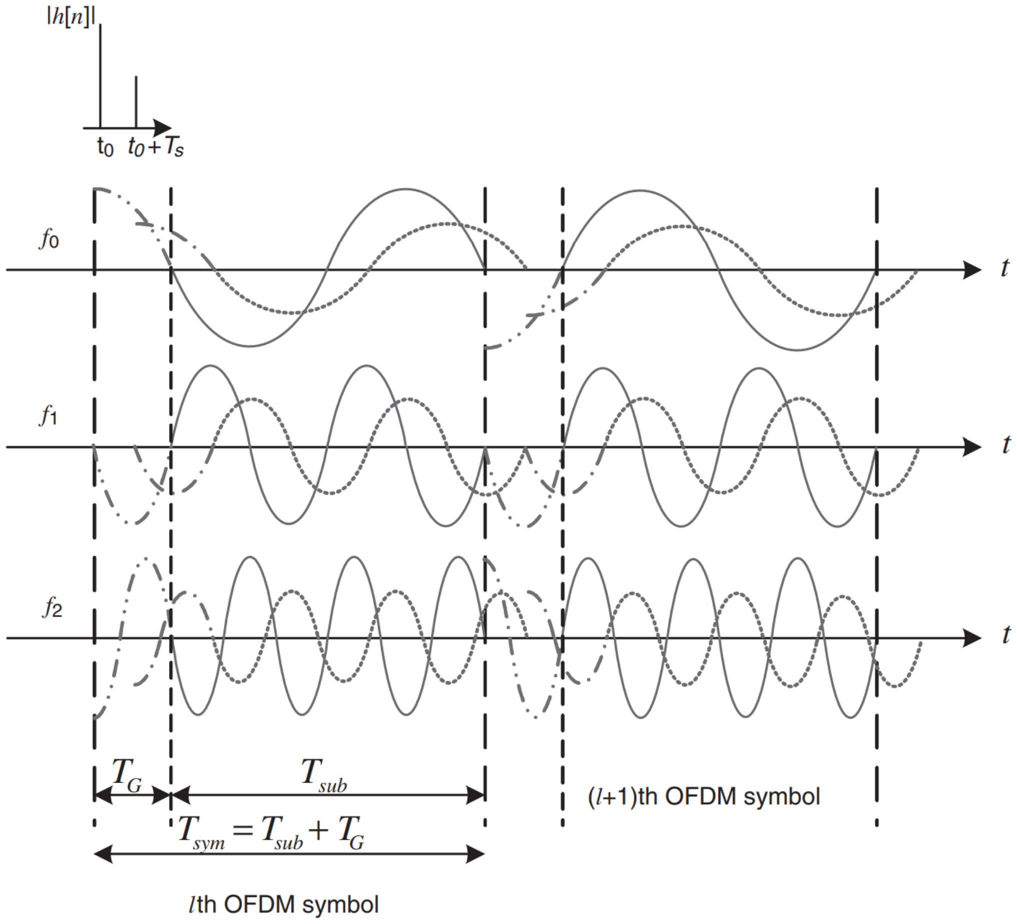
- Let $T_G = length\ of\ CP\ in\ terms\ of\ samples$

The extended OFDM symbols now have the duration of: $T_{sym} = T_{sub} + T_G$



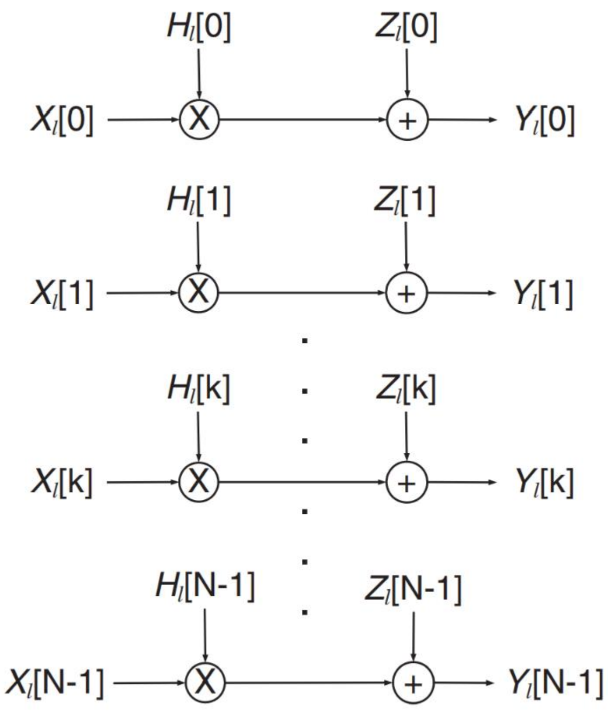
(a) OFDM symbols with CP

If $T_G \geq \sigma_{\tau MAX}$, effect of ISI is confined within the guard interval



(c) ISI effect of a multipath channel for each subcarrier

$$
\begin{aligned}
Y_l[k] &= \sum_{n=0}^{N-1} y_l[n] e^{-j2\pi kn/N} \\
&= \sum_{n=0}^{N-1} \left\{ \sum_{m=0}^{\infty} h_l[m] x_l[n-m] + z_l[n] \right\} e^{-j2\pi kn/N} \\
&= \sum_{n=0}^{N-1} \left\{ \sum_{m=0}^{\infty} h_l[m] \left\{ \frac{1}{N} \sum_{i=0}^{N-1} X_l[i] e^{j2\pi i(n-m)/N} \right\} \right\} e^{-j2\pi kn/N} + Z_l[k] \\
&= \frac{1}{N} \sum_{i=0}^{N-1} \left\{ \left\{ \sum_{m=0}^{\infty} h_l[m] e^{-j2\pi im/N} \right\} X_l[i] \sum_{n=0}^{\infty} e^{-j2\pi(k-i)n/N} \right\} e^{-j2\pi kn/N} + Z_l[k] \\
&= H_l[k] X_l[k] + Z_l[k]
\end{aligned}
$$



Frequency-domain equivalent model of OFDM system

$$
y_l[n] = h_l[n] * x_l[n] + z_l[n] \longrightarrow y_l[n] = h_l[n] \circledast x_l[n] + z_l[n] \quad Y_l[k] = H_l[k] X_l[k] + z_l[k]
$$

BER of OFDM Scheme

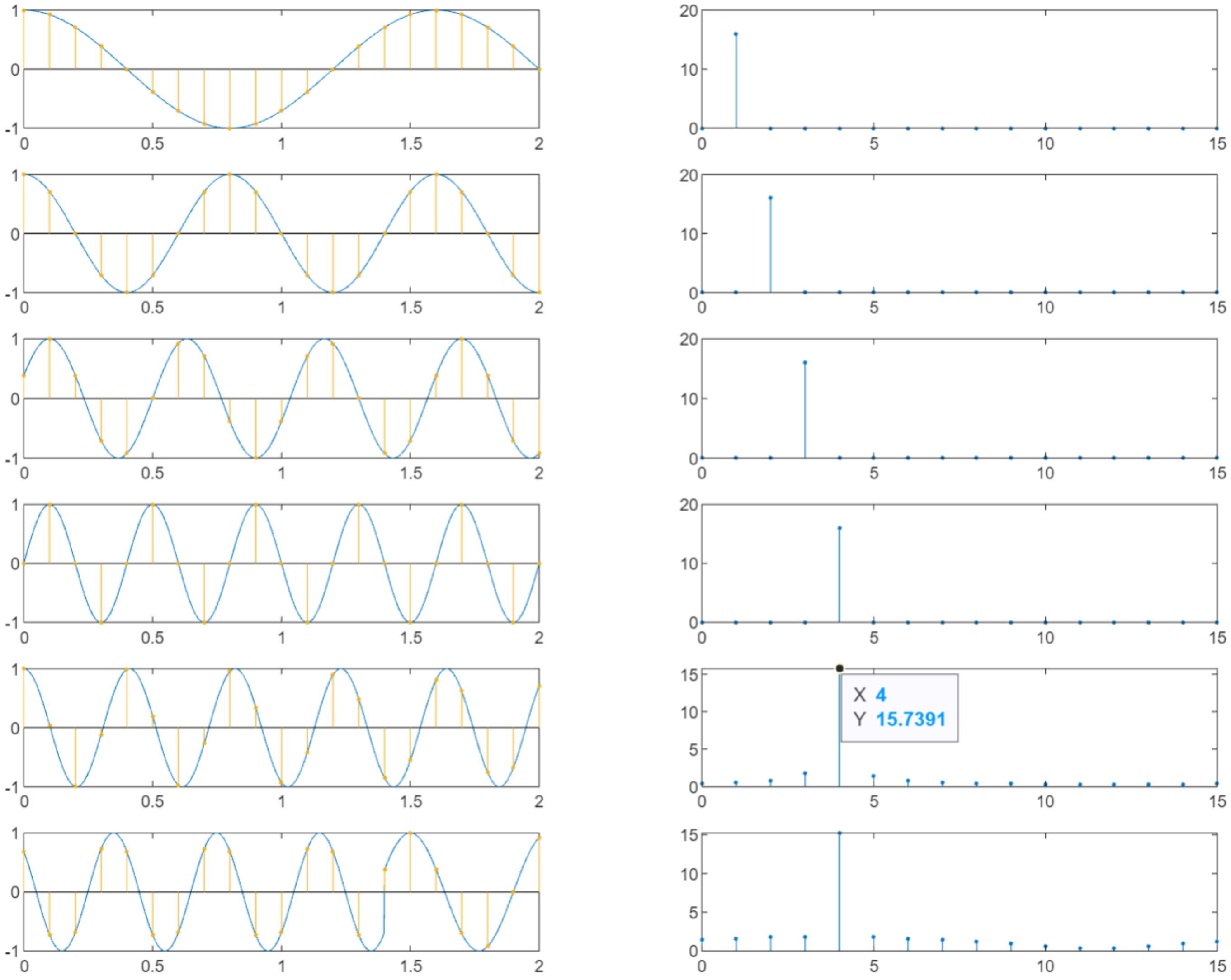
BER for M-*ary* QAM signalling in AWGN and Rayleigh Fading channels

- AWGN: $P_e = \frac{2(M-1)}{Mlog_2M} Q\left(\sqrt{\frac{6E_b}{N_o} \cdot \frac{log_2M}{M^2-1}}\right)$....................(1)

- Rayleigh: $P_e = \frac{(M-1)}{Mlog_2M} Q\left(1 - \sqrt{\frac{3\gamma log_2M/(M^2-1)}{\frac{3\gamma log_2M}{M^2-1}+1}}\right)$..............(2)

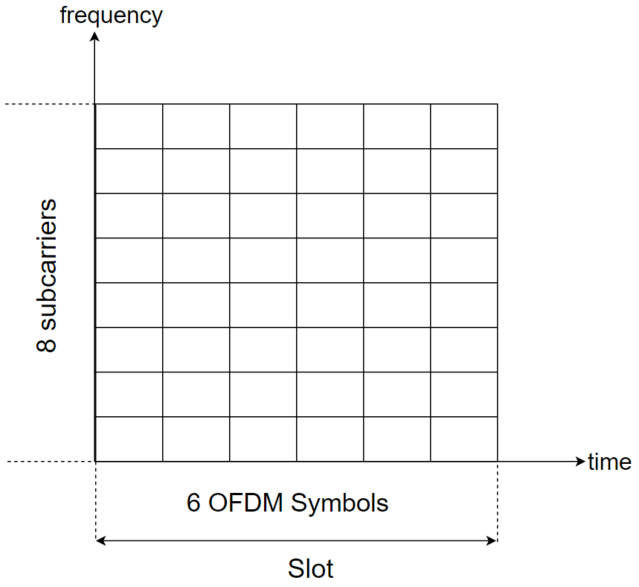
- $SNR_t = SNR_f + 10log\left(\frac{N_{used}}{N}\right)$ [dB]................................(3)

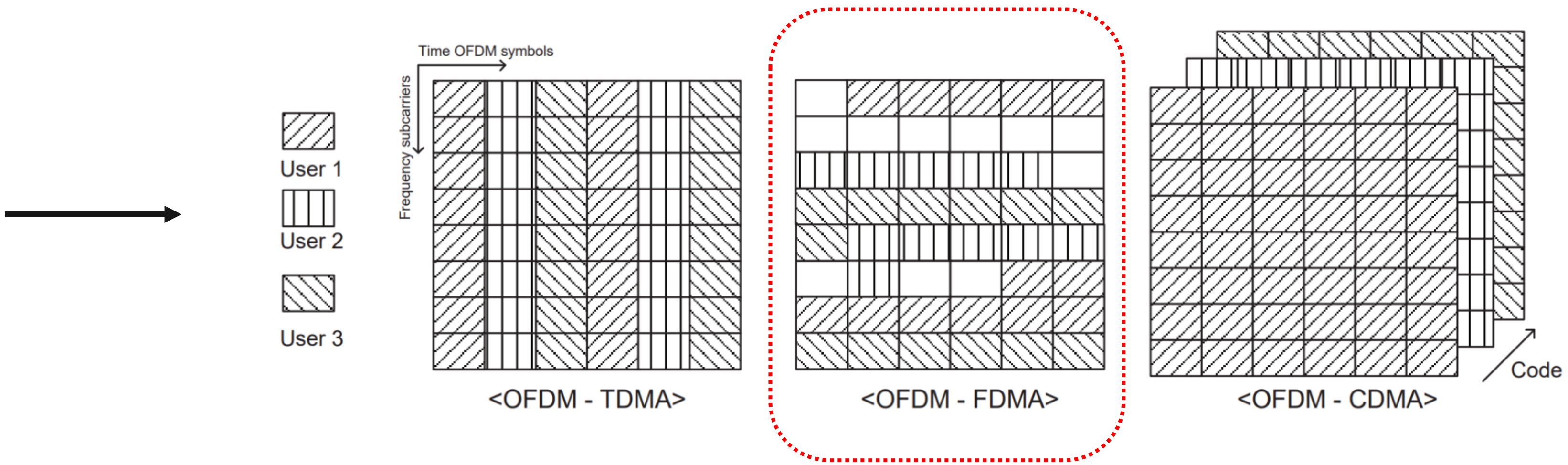
MATLAB Simulations



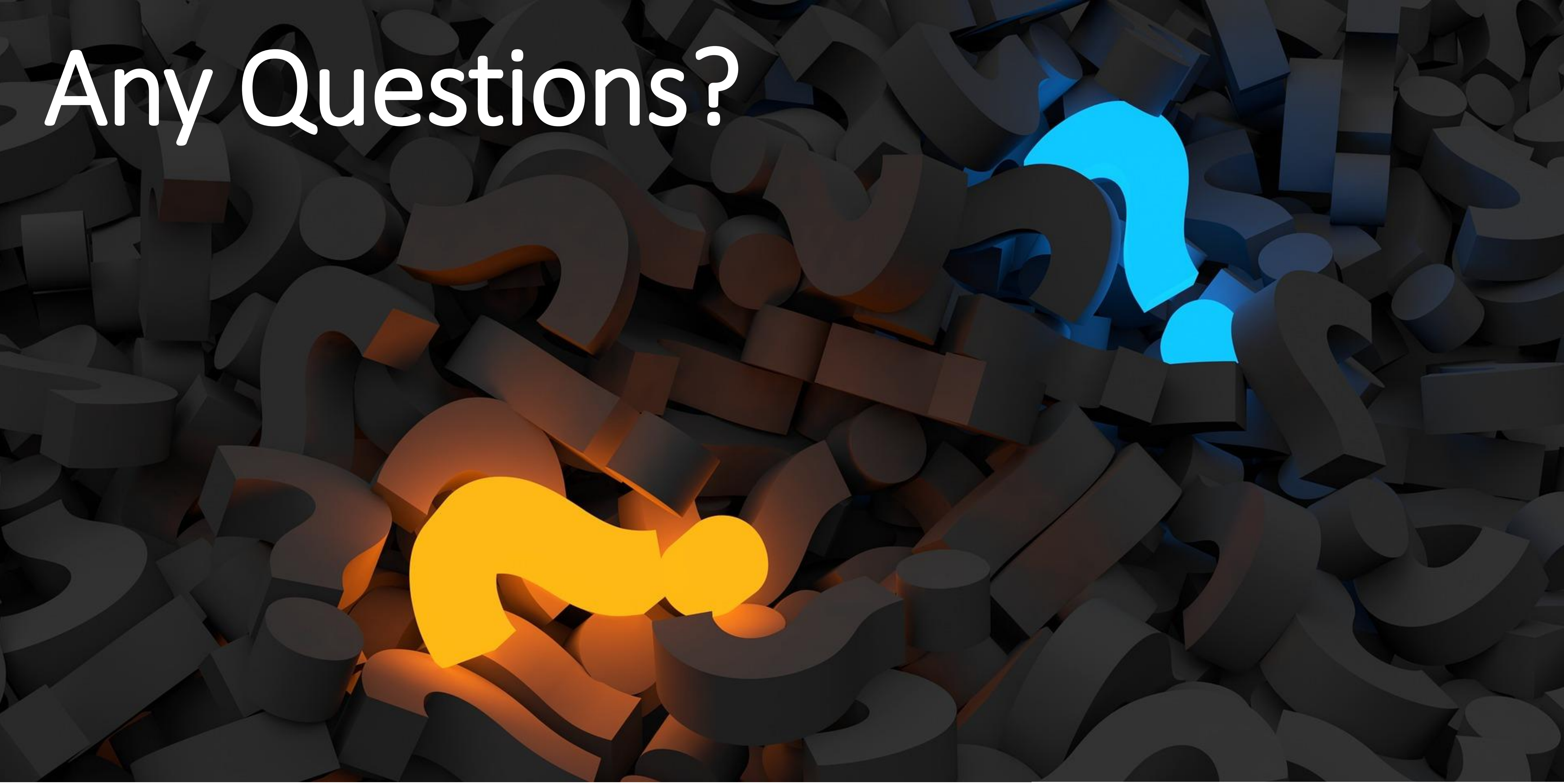
OFDMA: Multiple Access Extension of OFDM

- In OFDM, ***all*** subcarriers are used for transmitting the symbols of ***a single user***.
- OFDM can be combined with TDMA, FDMA, and CDMA for a multi-user system.





Any Questions?



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