Fundamentals of Wireless Communication

Multiuser Capacity and Opportunistic Comm.

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9th February, 2022

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Introduction

What techniques achieve channel capacity in multiuser communication systems?

Successive Interference Cancellation(SIC) and Superposition Coding

Multiuser opportunistic communication and multiuser diversity.

- Applying these techniques in *uplink* and *downlink* directions **achieves capacity**

General K-User AWGN Uplink Capacity

The K-user capacity region is described by $2^k - 1$ constraints, one for each possible non-empty subset $\mathcal{S}$ of users:

$$\sum_{k \in \mathcal{S}} R_k < log\left(1 + \frac{\sum_{k \in \mathcal{S}} P_k}{N_0}\right) \ldots \ldots \ldots .(1) \;\; for\ all\ \mathcal{S} \subset \{1, \ldots, k\}$$

The sum capacity of the channel is:

$$C_{sum} = log\left(1 + \frac{\sum_{k=1}^{K} P_k}{N_0}\right) \text{ bps/Hz} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

For successive cancellation order among the users and equal power case symmetric capacity:

$$C_{sym} = \frac{1}{K}\left(1 + \frac{KP}{N_0}\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3) \quad \text{bps}$$

- The baseband downlink AWGN channel with two users is:

$$y_k[m] = h_k x[m] + w_k[m] \quad \text{for } k = 1,2$$

They **simultaneously** communicate reliably at (R_1, R_2)

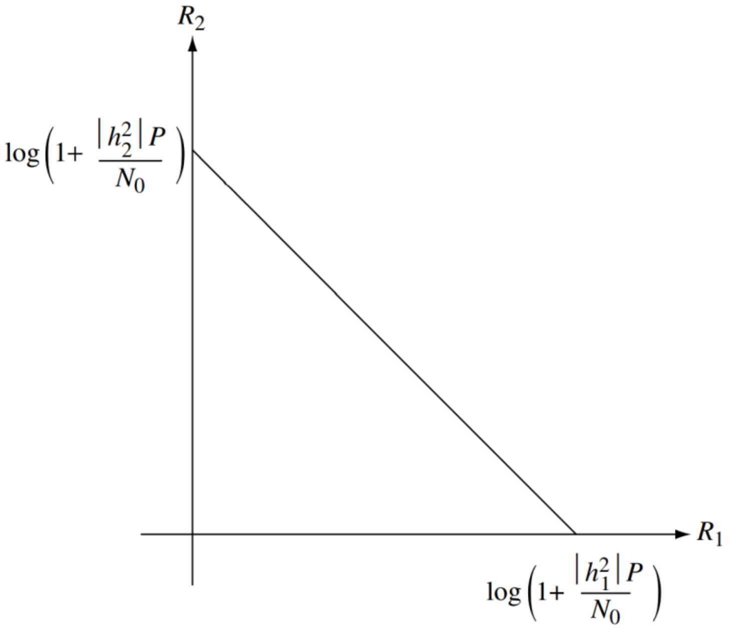
But single user bound is: $R_k < log\left(1 + \frac{P|h_k|^2}{N_0}\right)$ for $k = 1,2$

Symmetric case: $|h_1| = |h_2|$

Superposition: $x[m] = x_1[m] + x_2[m]$

$$\boldsymbol{R_1} = log\left(1 + \frac{(P_1 + P_2)|h_1|^2}{N_0}\right) - log\left(1 + \frac{P_2|h_1|^2}{N_0}\right) \ldots (4)$$

$$\boldsymbol{R_2} = log\left(1 + \frac{P_2|h_2|^2}{N_0}\right) \ldots\ldots\ldots\ldots\ldots..(5)$$



Let's consider:

No symmetric case: $|h_1| < |h_2|$

Superposition coding scheme: $x[m] = x_1[m] + x_2[m]$

$$\boldsymbol{R_1} = log\left(1 + \frac{P_1|h_1|^2}{P_2|h_1|^2 + N_0}\right) \text{ bps/Hz} \ldots\ldots\ldots\ldots\ldots..(6)$$

$$\boldsymbol{R_2} = log\left(1 + \frac{P_2|h_2|^2}{N_0}\right) \text{bps/Hz} \ldots\ldots\ldots\ldots\ldots..(7)$$

- Generally, in a multiuser system with ordering: $|h_1| \leq |h_2| \ldots \leq |h_k|$

The boundary of the capacity region of the downlink AWGN channel is given by the parameterized rate tuple

$$R_k = log\left(1 + \frac{P_k|h_k|^2}{N_0 + (\sum_{j=k+1}^{K} P_j)|h_k|^2}\right) \ldots\ldots.(8) \quad \text{for } k = 1,..,K$$

Uplink Fading Channel

Consider the complex baseband uplink flat fading channel with K users:

$$y[m] = \sum_{k=1}^{K} h_k[m] x_k[m] + w[m],\ where\ \ \{h_k[m]\}_m = fading\ process\ of\ k$$

For slow fading: $h_k[m] = h_k \ \forall\ m$, $\mathrm{SNR} = \frac{P|h_k|^2}{N_0}$

Suppose the users are transmitting at the same rate $\boldsymbol{R}$ bps/Hz, if $C_{sym} < R$:

$$P_{out}^{ul} := \mathbb{P}\left\{\log\left(1 + \mathrm{SNR}\sum_{k\in\mathcal{S}}|h_k|^2\right) < |\mathcal{S}|R, for\ some\ \mathcal{S} \subset \{1, \dots, k\}\right\} \dots\dots\dots (9)$$

$$C_\epsilon^{sym} = largest\ R\ such\ that\ (9) \leq \epsilon$$

At low SNR, $C_\epsilon^{sym} \approx \frac{C_{\epsilon/k}(\mathrm{KSNR})}{K}$

For fast fading: $\{h_k[m]\}_m$ modelled as time-varying ergodic process.

- **With only receiver CSI**:

$$C_{sum} = \mathbb{E}\left[log\left(1 + \frac{\sum_{k=1}^{K} |h_k|^2 P}{N_0}\right)\right] \ldots \ldots \ldots . (10)$$

With SIC at the receiver:

$$y[m] = h_k[m]x_k[m] + \sum_{i=k+1}^{K} h_i[m]x_i[m] + w[m]$$

$$\text{User } k \text{ rate, } R_k = \mathbb{E}\left[log\left(1 + \frac{|h_k|^2 P}{\sum_{i=k+1}^{K} |h_i|^2 P + N_0}\right)\right] \ldots \ldots \ldots (11)$$

With orthogonal multiple access scheme:

$$C_{sum} = \sum_{k=1}^{K} \frac{1}{K} \mathbb{E}\left[log\left(1 + \frac{K|h_k|^2 P}{N_0}\right)\right] = \mathbb{E}\left[log\left(1 + \frac{K|h_k|^2 P}{N_0}\right)\right] \ldots \ldots . (12)$$

- **With full CSI**:

A simple block fading model with:

- $h_k[m] = h_{k,l}$ constant over the l^{th} coherence period.
- $h_k[m] = h_{k,l}$ is $i.i.d.$ across different coherence periods.
- Channel → L parallel sub-channels, L coherence periods, independent fading

$$C_{sum} = \max_{P_{k,l}: k=1,\dots K, l=1,\dots L} \frac{1}{L} \sum_{l=1}^{L} log\left(1 + \frac{\sum_{k=1}^{K} P_{k,l} |h_{k,l}|^2}{N_0}\right)$$

Power constraint on each user: $\frac{1}{L} \sum_{l=1}^{L} P_{k,l} = \mathrm{P} \quad k = 1, \dots, K$

As $L \to \infty \Rightarrow$ power allocation policy adhere to, by users (appropriate: waterfilling)

$$C_{sum} = \mathbb{E}\left[log\left(1 + \frac{P_{k^*}(\boldsymbol{h})|h_{k^*}|^2}{N_0}\right)\right] \dots\dots\dots (13)$$

Downlink Fading Channel

Downlink fading channel with K users:

$$y_k[m] = h_k x[m] + w_k[m] \quad \text{for } k = 1, \dots, K$$

With only receiver CSI:

Assuming fading statistics symmetry and ergodicity

$$\sum_{k=1}^{K} R_k < \mathbb{E}\left[log\left(1 + \frac{P|h|^2}{N_0}\right)\right]$$

With full CSI:

Appropriate power is allocated to the best user each time in a varying channel

$$\max_{k=1,\dots,K} |h_k|^2$$

- Optimal power allocation: $P^*(\boldsymbol{h}) = \left(\frac{1}{\lambda} - \frac{N_0}{\max_{k=1,\dots,K} |h_k|^2}\right)^+$

$$C_{sum} = \mathbb{E}\left[log\left(1 + \frac{P^*(\boldsymbol{h})\left(\max_{k=1,\dots,K} |h_k|^2\right)}{N_0}\right)\right] \dots\dots\dots.(14)$$

Frequency-selective fading channel

- Flat fading analysis in the uplink and the downlink is easily extended here.

OFDM is applied to the multiuser channels.

The n^{th}sub-carrier has uplink channel:

$$\tilde{y}_n[i] = \sum_{k=1}^{K} \tilde{h}_n^{(k)}[i]\tilde{d}_n^{(k)}[i] + \widetilde{w}_n[i]$$

- Uplink: parallel multiuser sub-channels, one for each sub-carrier and each coherence time interval

The optimal strategy is to allow the best user to transmit on each of these sub-channels.

The power allocated to the best user is waterfilling over time and frequency.

Unlike flat fading case, multiple users can transmit at the same time, but over different sub-carriers.

Multiuser Diversity

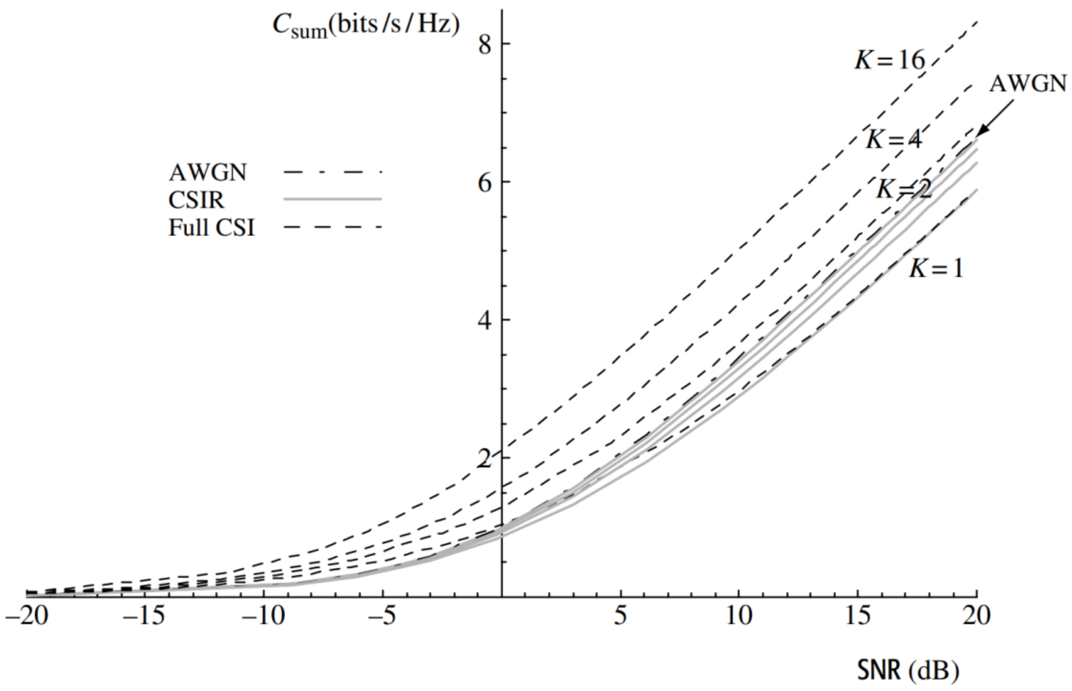
Multiuser diversity gain comes from two effects:

- increase in total transmit power in the case of the uplink ($\boldsymbol{KP}$)
- effective channel gain at time m: $|\boldsymbol{h_1}[\boldsymbol{m}]|^2 \rightarrow \max\limits_{\boldsymbol{k}=1,..,\boldsymbol{K}} |\boldsymbol{h_k}[\boldsymbol{m}]|^2$

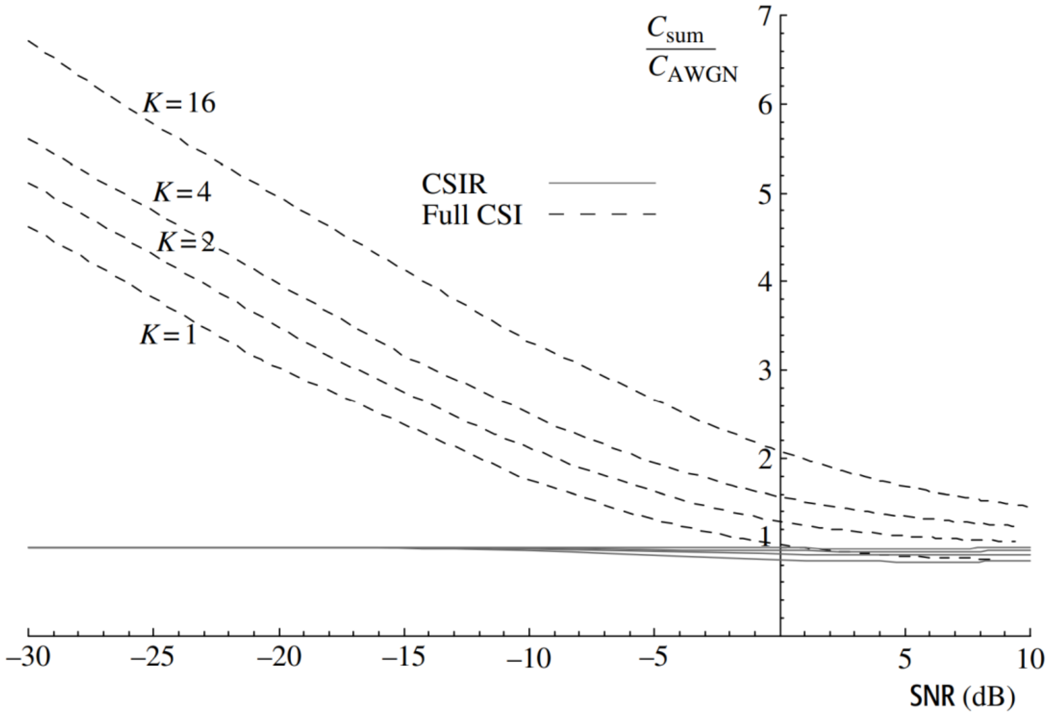
$\uparrow = KP$

$\downarrow = P$

$\max\limits_{\boldsymbol{k}} |\boldsymbol{h_k}[\boldsymbol{m}]|$



High SNR Regime



Low SNR Regime

Proportional Fair Scheduling- Hitting the peaks

The cellular system requirements to extract the multiuser diversity benefits are:

- the base-station has access to channel quality measurements.
- it can schedule transmission among users and adapt data rate as a function of instantaneous channel quality.

Two key issues arise in practice: **Fairness** and **Delay**

Solution: **decide which user to transmit information to at each time slot, based on the requested rates the base-station has previously received from the mobiles.**

The user with the highest ratio below is scheduled:

$$\frac{R_k[m]}{T_k[m]} \dots\dots\dots\dots\dots\dots. (15)$$

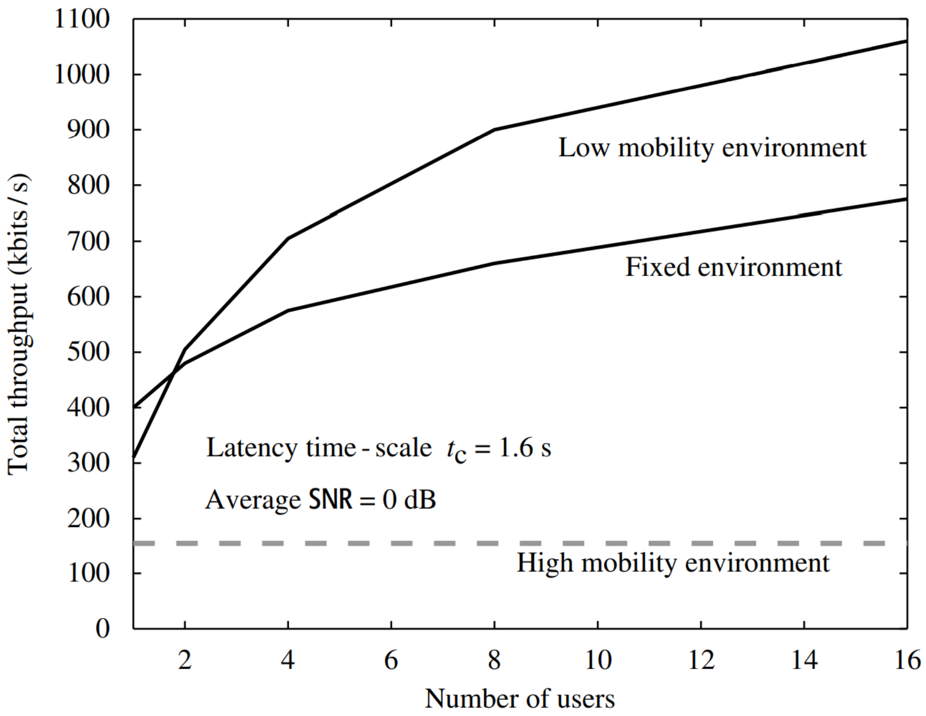
$$R_k[m] = \text{current requested rate of user } k$$
$$T_k[m] = \text{average throughput of user } k \text{ in the past } \mathrm{T_c} \text{ time slots}$$

What is the physical significance of (15)?

Special case: By symmetry, T_k is the same across all users and only $\boldsymbol{R_k}$ matters

What then happens with asymmetric user channel statistics?

Below is the performance of proportional fair scheduling



- **Fixed**: While users are fixed, there are moving objects around them (2Hz, Rician)
- **Low mobility**: users moving at walking speeds (2 Km/hr, Rayleigh)
- **High mobility**: users moving at (30 Km/hr, Rayleigh)

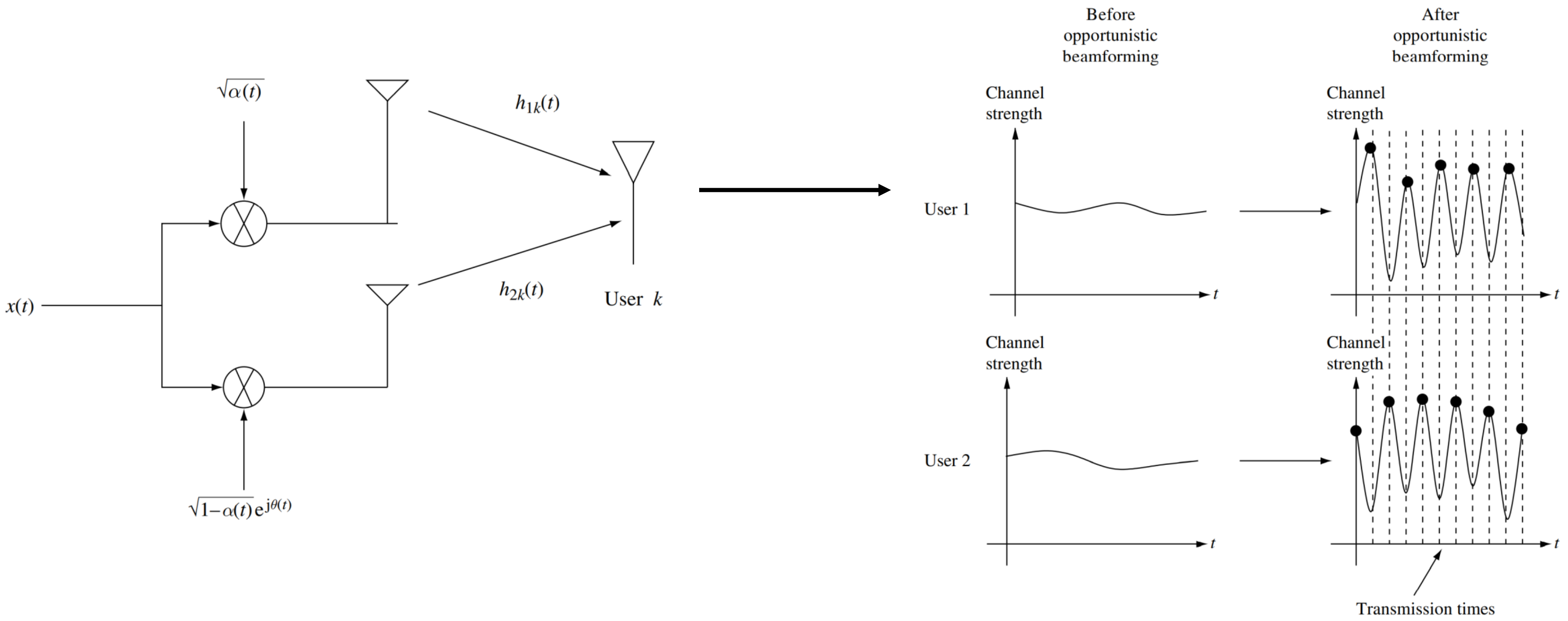
Opportunistic beamforming using dumb antennas

The amount of multiuser diversity depends on:

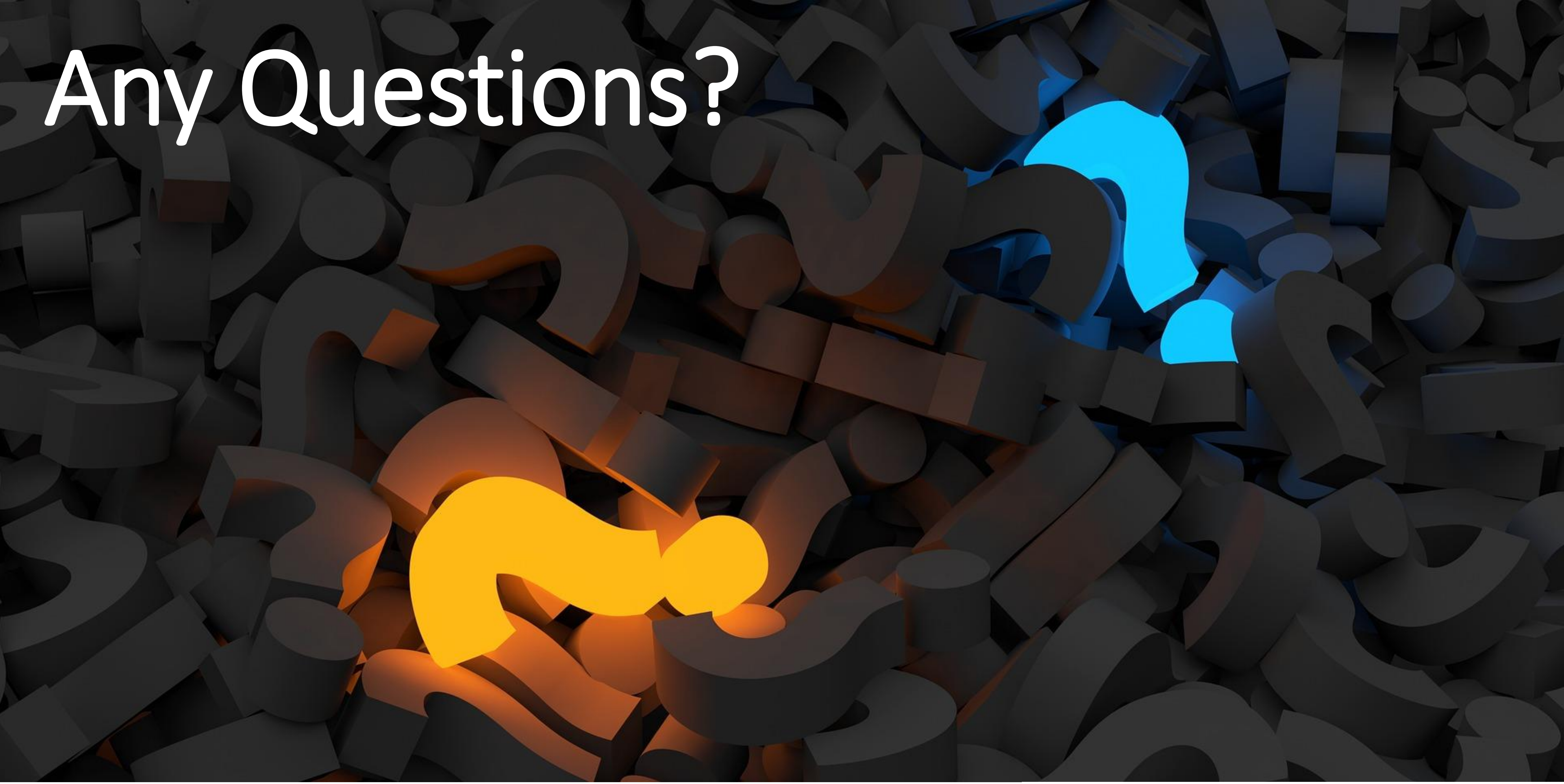
- the rate of channel fluctuations
- dynamic range of channel fluctuations

Scheduling algorithms exploits the channel fluctuations by **hitting the peaks.**

What do we do if there are not enough fluctuations?


$\sqrt{\alpha(t)}$
$h_{1k}(t)$
$h_{2k}(t)$
$x(t)$
User k
$\sqrt{1-\alpha(t)}e^{j\theta(t)}$
Before opportunistic beamforming
After opportunistic beamforming
Channel strength
User 1
User 2
t
Transmission times

Any Questions?



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