***Abstract*—Simultaneous Wireless Information and Power Transfer (SWIPT) is severely restricted by the low power level of the receive Radio-Frequency (RF) signal. To tackle this problem, we introduce Intelligent Reflecting Surface (IRS) that brings a high passive beamforming gain to compensate the propagation loss and boost the energy efficiency with a low power consumption. This paper investigates an IRS-aided multiuser Orthogonal Frequency Division Multiplexing (OFDM) SWIPT system based on practical nonlinear harvester model, where the single-antenna Access Point (AP) transmits information and energy simultaneously to multiple single-antenna users under the assist of IRS. We aim to maximize the weighted sum Rate-Energy (R-E) region via jointly optimizing the phase shifts at the IRS, the transmit waveform at the AP, and the power splitting ratio for all users. The problem is equivalent to a current maximization problems subject to rate constraint, and we propose an low-complexity alternating algorithm to obtain suboptimal solutions iteratively. Numerical results demonstrated that dedicated power signal is beneficial to multicarrier SWIPT, while IRS brings significant R-E region enlargement over benchmark schemes when properly configured.**

I. Introduction

A. Simultaneous Wireless Information and Power Transfer

With the great advance in communication performance (throughput, latency, outage), the main challenge of wireless network has come to energy supply. Most existing mobile devices are powered by batteries that require frequent charging or replacement, which leads to high maintenance cost and thus restricts the scale of networks. Although solar energy and inductive coupling has become popular alternatives, the former depends on the environment while the latter has a very short operation range. Simultaneous Wireless Information and Power Transfer (SWIPT) is a promising solution to connect and power mobile devices via electromagnetic (EM) waves in the Radio-Frequency (RF) band. It provides low power (in µW level) but broad coverage (up to hundreds of meters) [1] in a sustainable and controllable manner. The decreasing trend in electronic power consumption also boosts the paradigm shift from dedicated power source to Wireless Power Transfer (WPT) and SWIPT.

The concept of SWIPT were first cast in [2], where the authors investigated the Rate-Energy (R-E) tradeoff for a flat Gaussian channel and some discrete channels. Two practical receiver structures were then proposed in [3], namely Time Switching (TS) that switches between Energy Harvesting (EH) and Information Decoding (ID) modes, and Power Splitting (PS) that splits the received signal into individual components. On top of this, [4] characterized the R-E region for a Multiple-Input Multiple-Output (MIMO) broadcast system under TS and PS setup. Information and power beamforming was then considered in multiuser Multi-Input Single-Output (MISO) systems to maximize the Weighted Sum-Power (WSP) subjective to Signal-to-Interference-plus-Noise Ratio (SINR) constraints [5]. Motivated by this, [6] investigated fundamental transceiver modules, information and power scheduling, and interference management for SWIPT systems. However, [7] pointed out that the Radio Frequency-to-Direct Current (RF-to-DC) conversion efficiency depends on the harvester input power level, which also suggested a parametric harvester model based on curve fitting and proposed an iterative resource allocation algorithm. From another perspective, [8], [9] demonstrated that multisine waveform is more suitable for WPT as it outperforms single tone in both operation range and RF-to-DC efficiency. [10] derived a tractable nonlinear harvester model based on the Taylor expansion of diode I-V characteristics and proposed an adaptive waveform optimization algorithm to maximize the output DC current under rate constraints. Simulation and experiments demonstrated the benefit of modelling rectifier nonlinearity in system design [11], [12]. The work was extended to SWIPT in [13] where a superposition of modulated information waveform and multisine power waveform is optimized to enlarge the R-E region. In contrast, [14] suggested an adaptive dual-mode SWIPT, which alternates between single-tone transmission that exploits conventional modulation for high-rate applications and multisine transmission that encodes the information in the Peak-to-Average Ratio (PAPR) for power-demanding applications. By assuming On-Off-Keying (OOK) where bit 1 carries energy, [15] compared unary and Run-Length-Limited (RLL) code in terms of rate vs battery overflow/underflow probability, and adapted conventional modulation schemes to ensure WPT is only activated at the points with large offset. Also, a learning approach [16] demonstrated that the offset of the power symbol is positively correlated to the harvester energy constraint, while the information symbols are symmetrically distributed around the origin. It confirmed that the superposed waveform is feasible to enlarge R-E region when considering rectifier nonlinearity. SWIPT was also explored in the network design. [17] proposed a cooperative SWIPT Non-Orthogonal Multiple Access (NOMA) protocol with three user selection schemes such that the strong user assists the EH of the weak user. In [18], SWIPT based on Rate Splitting (RS) was formulated as a Weighted Sum-Rate (WSR) maximization problem subject to total harvested energy constraint for separated Information Receivers (IRs) and Energy Receivers (ERs).

B. Intelligent Reflecting Surface

Intelligent Reflecting Surface (IRS) adapts the wireless environment to increase spectrum and energy efficiency. In practice, an IRS consists of multiple individual reflecting elements that adjust the amplitude and phase of the incident signal through passive beamforming. Different from relay and backscatter communication, IRS assists the primary transmission without any active components, leading to low power consumption and no thermal noise added to the reflected signal. Compared with the linear increase in Amplify-and-Forward (AF) relay, the received power scales quadratically with the number of reflectors [19], since more reflectors boost the power collected by IRS and increase the array gain in its equal gain transmission.

Inspired by the advance in real-time reconfigurable metamaterials [20], [21] introduced a programmable metasurface that steers or polarizes the EM wave at specific frequency to mitigate signal attenuation. At the same time, [22] constructed an adjustable reflect array that ensures reliable millimeter-wave (mmWave) communication based on a beam-searching algorithm to reduce indoor signal blockage. Motivated by this, [23], [19] introduced an IRS-assisted MISO system and proposed a beamforming algorithm that jointly optimizes the precoder at the Access Point (AP) and the phase shifts at the IRS to maximize Signal-to-Noise Ratio (SNR). The active and passive beamforming problem was extended to the discrete phase shift case [24] and the multiuser case [25]. In [26], channel estimation for Time-Division Duplex (TDD) systems was carried through a two-stage Minimum Mean Squared Error (MMSE)-based protocol that sequentially estimates the cascaded channel of each IRS element with the others switched off. Starting from the impedance equation, [27] investigated the influence of phase shift on the reflection amplitude and proposed a parametric IRS model via curve fitting. Recent research also explored the opportunity of integrating IRS with Orthogonal Frequency-Division Multiplexing (OFDM) systems. [28] exploited spatial correlation to reduce estimation overhead and design complexity by assuming adjacent elements share a common reflection coefficient. On top of this, group-based OFDM channel estimation was investigated in [29]. By adjusting IRS over time slots, [30] introduced artificial diversity within coherence time and investigated resource allocation and IRS configuration per Resource Block (RB). Real-time high-definition video transmission was performed over a prototype constructed with Positive Intrinsic-Negative (PIN) diodes, which demonstrated the feasibility and benefit of IRS at GHz and mmWave frequency [31].

To the best of authors' knowledge, most existing papers assume a Frequency-Flat (FF) IRS where all elements reflect the signal of different frequency equally. Although Frequency-Selective Surface (FSS) has received much attention for wideband communications, it is different from IRS since active FSS requires RF-chains [32], [5] while passive FSS has fixed physical characteristics [33].

C. IRS-aided SWIPT

The effective channel enhancement and low power consumption of IRS are expected to bring more opportunities to SWIPT. Based on linear harvester model and energy interference, [34] proved that at most one energy beam is required to maximize the WSP subject to SINR constraints. The fairness issue was then considered in [35], which maximize the minimum output power on the assumption of perfect energy interference cancellation. [36] proposed a novel penalty-based algorithm, whose inner layer employs Block Coordinate Descent (BCD) method to update precoders, phase shifts and auxiliary variables while the outer layer updates the penalty coefficients. It demonstrated that Line-of-Sight (LoS) links can boost the harvested power, as the rank-deficient channels are highly correlated and a single energy stream can satisfy the energy constraints of all ERs. In [37], the WSR maximization of MIMO SWIPT was first transformed to Weighted Minimum Mean Square Error (WMMSE) problem then solved by BCD with low-complexity iterative algorithms. However, most existing IRS-SWIPT papers focus on narrowband transmission over linear harvester model.

D. Objective and Methodology

In this paper, we study an IRS-aided multiuser MISO SWIPT system where the IRS assists the information and energy transmission of all users. A multicarrier unmodulated power waveform (deterministic multisine) is superposed to a multicarrier modulated information waveform (e.g. OFDM) to boost the energy transfer efficiency without introducing additional interference. The transmit waveform, IRS phase shift and receive splitting ratio are jointly optimized to maximize the R-E tradeoff. Different from previous research, this paper investigates the fundamental impact of harvester nonlinearity on waveform and IRS design. We transform the R-E region characterization problem into multiple current maximization problems subject to different rate constraints. To reduce the design complexity, we propose an Alternating Optimization (AO) algorithm that updates the channel and transceiver iteratively based on Semi-Definite Relaxation (SDR) technique. It was demonstrated that dedicated power waveform boosts the energy transmission efficiency such that TS and PS are preferred for low-rate and high-rate applications respectively. Also, IRS brings a significant channel amplification and R-E enhancement especially when located near the transmitter, and the performance loss compared with ideal Frequency-Selective (FS) IRS decreases as the number of reflectors increases.

II. System Model

Consider an IRS-aided multiuser MISO OFDM SWIPT system where the M-antenna transmitter delivers information and power simultaneously, through a single L-reflector IRS, to K single-antenna users over N orthogonal subbands. We assume a total bandwidth B with evenly-spaced subbands around center frequency f_c. Each subband is allocated to one user per time slot, and denote the frequency of the n-th subband as f_n ($n = 1, \dots, N$). Perfect Channel State Information (CSI) with negligible training overhead is assumed to explore the analytical upper-bound of the proposed design. A quasi-static block fading channel model is considered for all links and we focus on one particular block where the channels are approximately unchanged. We characterize and compare the R-E region achieved by TS and PS receivers. Specifically, TS divides each time slot into orthogonal data and energy slots and performs a time sharing between WPT and Wireless Information Transfer (WIT). In comparison, PS splits the received signal into data and energy streams for ID and EH, and the splitting ratio ρ is coupled with waveform and IRS design. Perfect synchronization is assumed among the three parties in both scenarios, and signals reflected by IRS for two and more times are omitted.

A. Transmit Signal

Denote $\tilde{x}_{I,n}(t)$ as the information symbol transmitted over subband n, which belongs to one user at time t and follows a capacity-achieving i.i.d. Circular Symmetric Complex Gaussian (CSCG) distribution $\tilde{x}_{I,n} \sim \mathcal{CN}(0, 1)$. Let $\alpha_{k,n}$ be the allocation indicator, namely if subband n is given to user k ($k = 1, \dots, K$), we have $\alpha_{k,n} = 1$ and $\alpha_{k',n} = 0$ for

$k' \neq k$. Let $\boldsymbol{\alpha}_k = [\alpha_{k,1}, \ldots, \alpha_{k,N}]^T \in \mathbb{C}^{N \times 1}$. The superposed transmit signal at time t on antenna m ($m = 1, \ldots, M$) is

$$x_m(t) = \Re\left\{\sum_{n=1}^{N} \left(w_{I,n,m}\tilde{x}_{I,n}(t) + w_{P,n,m}\right) e^{j2\pi f_n t}\right\} \quad (1)$$

where $w_{I/P,n,m} = s_{I/P,n,m}e^{j\psi_{I/P,n,m}}$ denotes the weight on the information and power signal transmitted by antenna m at subband n. Let $\boldsymbol{w}_{I/P,n} = [w_{I/P,n,1}, \ldots, w_{I/P,n,M}]^T \in \mathbb{C}^{M \times 1}$ such that the transmit information and power signals write as

$$\boldsymbol{x}_I(t) = \Re\left\{\sum_{n=1}^{N} \boldsymbol{w}_{I,n}\tilde{x}_{I,n}(t)e^{j2\pi f_n t}\right\} \quad (2)$$

$$\boldsymbol{x}_P(t) = \Re\left\{\sum_{n=1}^{N} \boldsymbol{w}_{P,n}e^{j2\pi f_n t}\right\} \quad (3)$$

We also define $\boldsymbol{w}_{I/P} = [\boldsymbol{w}_{I/P,1}^T, \ldots, \boldsymbol{w}_{I/P,N}^T]^T \in \mathbb{C}^{MN \times 1}$ and let $\boldsymbol{s}_{I/P} = |\boldsymbol{w}_{I/P}|$, $\boldsymbol{\psi}_{I/P} = \angle \boldsymbol{w}_{I/P}$ be the magnitude and phase of the waveforms.

B. Composite Channel

At subband n, denote the AP-user k direct channel as $\boldsymbol{h}_{D,k,n}^H \in \mathbb{C}^{1 \times M}$, AP-IRS incident channel as $\boldsymbol{H}_{I,n} \in \mathbb{C}^{L \times M}$, and IRS-user k reflective channel as $\boldsymbol{h}_{R,k,n}^H \in \mathbb{C}^{1 \times L}$. At the IRS, element l ($l = 1, \ldots, L$) redistributes the incoming signal by adjusting the reflection amplitude $\gamma_l \in [0, 1]$ and phase shift $\theta_l \in [0, 2\pi)$ [1]. On top of this, the IRS matrix collects the reflection coefficients onto the main diagonal entries as $\boldsymbol{\Theta} = \mathrm{diag}(\gamma_1 e^{j\theta_1}, \ldots, \gamma_L e^{j\theta_L}) \in \mathbb{C}^{L \times L}$. The IRS-aided extra link can be modeled as a concatenation of the AP-IRS channel, IRS reflection, and IRS-user channel. Hence, the composite channel for user k over subband n is

$$\boldsymbol{h}_{k,n}^H = \boldsymbol{h}_{D,k,n}^H + \boldsymbol{h}_{R,k,n}^H \boldsymbol{\Theta} \boldsymbol{H}_{I,n} = \boldsymbol{h}_{D,k,n}^H + \boldsymbol{\phi}^H \boldsymbol{V}_{k,n} \quad (4)$$

where $\boldsymbol{\phi} = [\gamma_1 e^{j\theta_1}, \ldots, \gamma_L e^{j\theta_L}]^H \in \mathbb{C}^{L \times 1}$ and $\boldsymbol{V}_{k,n} = \mathrm{diag}(\boldsymbol{h}_{R,k,n}^H)\boldsymbol{H}_{I,n} \in \mathbb{C}^{L \times M}$. Similarly, we define $\boldsymbol{h}_k = [\boldsymbol{h}_{k,1}^T, \ldots, \boldsymbol{h}_{k,N}^T]^T \in \mathbb{C}^{MN \times 1}$, $\boldsymbol{V}_k = [\boldsymbol{V}_{k,1}^T, \ldots, \boldsymbol{V}_{k,N}^T]^T \in \mathbb{C}^{L \times MN}$ and let $\boldsymbol{a}_k = |\boldsymbol{h}_k|$, $\varsigma_k = \angle \boldsymbol{h}_k$ be the magnitude and phase of the composite channel for user k.

C. Receive Signal

The information and power signals received by user k through both direct and IRS-aided links write as

$$y_{I,k}(t) = \Re\left\{\sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \boldsymbol{w}_{I,n}\tilde{x}_{I,n}(t)e^{j2\pi f_n t}\right\} \quad (5)$$

$$y_{P,k}(t) = \Re\left\{\sum_{n=1}^{N} \boldsymbol{h}_{k,n}^H \boldsymbol{w}_{P,n}e^{j2\pi f_n t}\right\} \quad (6)$$

[1]To investigate the performance upper bound of IRS, we suppose the reflection coefficient is maximized $\gamma_l = 1\ \forall l$ while the phase shift is a continuous variable over $[0, 2\pi)$.

D. Information Decoder

A major benefit of the proposed waveform is that the power component $y_{P,k}(t)$ creates no interference to the information component $y_{I,k}(t)$. Hence, the achievable rate of user k is

$$R_k(\boldsymbol{w}_I, \boldsymbol{\phi}, \rho, \boldsymbol{\alpha}_k) = \sum_{n=1}^{N} \alpha_{k,n} \log_2\left(1 + \frac{(1-\rho)|\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{I,n}|^2}{\sigma_n^2}\right) \quad (7)$$

where σ_n^2 is the variance of the noise at RF band and during RF-to-BB conversion on tone n. Rate 7 is achievable with either waveform cancellation or translated demodulation [38].

E. Energy Harvester

Consider a nonlinear diode model based on the Taylor expansion of a small signal model [10], [38], which highlights for user k the dependency of harvester output DC current on the received waveform as

$$i_k(\boldsymbol{w}_I, \boldsymbol{w}_P, \boldsymbol{\phi}, \rho) \approx \sum_{i=0}^{\infty} k_i' \rho^{i/2} R_{\text{ant}}^{i/2} \mathcal{E}\left\{\mathcal{A}\left\{y_k(t)^i\right\}\right\} \quad (8)$$

where R_{ant} is the impedance of the receive antenna, $k_0' = i_s(e^{-i_k R_{\text{ant}}/nv_t} - 1)$, $k_i' = i_s e^{-i_k R_{\text{ant}}/nv_t}/i!(nv_t)^i$ for $i = 1, \ldots, \infty$, i_s is saturation current, n is diode ideality factor, v_t is thermal voltage. For a fixed channel and waveform, $\mathcal{A}\{.\}$ extracts the DC component of the received signal while $\mathcal{E}\{.\}$ covers the expectation over $\tilde{x}_{I,n}$.

With the assumption of evenly spaced frequencies, it holds that $\mathcal{E}\left\{y_k(t)^i\right\} = 0$ for odd i and the related terms has no contribution to DC components. For simplicity, we truncate the infinite series to the n_0-th order. As proved in [10], maximizing a truncated i_k is equivalent to maximizing a monotonic function

$$z_k(\boldsymbol{w}_I, \boldsymbol{w}_P, \boldsymbol{\phi}, \rho) = \sum_{i \text{ even}, i \geq 2}^{n_0} k_i \rho^{i/2} R_{\text{ant}}^{i/2} \mathcal{E}\left\{\mathcal{A}\left\{y_k(t)^i\right\}\right\} \quad (9)$$

where $k_i = i_s/i!(nv_t)^i$. We choose $n_0 = 4$ to investigate fundamental nonlinearity and let $\beta_2 = k_2 R_{\text{ant}}$ and $\beta_4 = k_4 R_{\text{ant}}^2$. Note that $\mathcal{E}\left\{|\tilde{x}_{I,n}|^2\right\} = 1$ and $\mathcal{E}\left\{|\tilde{x}_{I,n}|^4\right\} = 2$, which can be interpreted as a modulation gain on the nonlinear terms of the output DC current.

Similar to [39], we define $\boldsymbol{W}_{I/P} = \boldsymbol{w}_{I/P}\boldsymbol{w}_{I/P}^H$, $\boldsymbol{H}_k = \boldsymbol{h}_k \boldsymbol{h}_k^H$ and let $\boldsymbol{W}_{I/P,n}$, $\boldsymbol{H}_{k,n}$ keep their n-th ($n = -N + 1, \ldots, N-1$) block diagonal and null the remaining entries, respectively. Due to the positive definiteness of $\boldsymbol{W}_{I/P}$ and $\boldsymbol{H}_k$, we have $\boldsymbol{W}_{I/P,-n} = \boldsymbol{W}_{I/P,n}^H$ and $\boldsymbol{H}_{k,-n} = \boldsymbol{H}_{k,n}^H$. On top of this, the nonzero terms in 9 are expressed by 10 – 17 and z_k reduces to 18 – 20.

F. Weighted Sum Rate-Energy Region

Define the achievable weighted sum R-E region as

$$C_{R-Z}(P) \triangleq \left\{(R, Z) : R \leq \sum_{k=1}^{K} u_{I,k} R_k, Z \leq \sum_{k=1}^{K} u_{P,k} z_k, \frac{1}{2}(\boldsymbol{w}_I^H \boldsymbol{w}_I + \boldsymbol{w}_P^H \boldsymbol{w}_P) \leq P\right\} \quad (21)$$

where P is the transmit power budget and $u_{I,k}, u_{P,k}$ are the information and power weight of user k.

$$\mathcal{E}\left\{\mathcal{A}\left\{y_{I,k}^2(t)\right\}\right\} = \frac{1}{2}\sum_n (\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{I,n})(\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{I,n})^H \tag{10}$$

$$= \frac{1}{2}\boldsymbol{h}_k^H \boldsymbol{W}_{I,0}\boldsymbol{h}_k = \frac{1}{2}\boldsymbol{w}_I^H \boldsymbol{H}_{k,0}\boldsymbol{w}_I \tag{11}$$

$$\mathcal{E}\left\{\mathcal{A}\left\{y_{I,k}^4(t)\right\}\right\} = \frac{3}{4}\left(\sum_n (\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{I,n})(\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{I,n})^H\right)^2 \tag{12}$$

$$= \frac{3}{4}(\boldsymbol{h}_k^H \boldsymbol{W}_{I,0}\boldsymbol{h}_k)^2 = \frac{3}{4}(\boldsymbol{w}_I^H \boldsymbol{H}_{k,0}\boldsymbol{w}_I)^2 \tag{13}$$

$$\mathcal{A}\left\{y_{P,k}^2(t)\right\} = \frac{1}{2}\sum_n (\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{P,n})(\boldsymbol{h}_{k,n}^H \boldsymbol{w}_{P,n})^H \tag{14}$$

$$= \frac{1}{2}\boldsymbol{h}_k^H \boldsymbol{W}_{P,0}\boldsymbol{h}_k = \frac{1}{2}\boldsymbol{w}_P^H \boldsymbol{H}_{k,0}\boldsymbol{w}_P \tag{15}$$

$$\mathcal{A}\left\{y_{P,k}^4(t)\right\} = \frac{3}{8}\sum_{\substack{n_1,n_2,n_3,n_4\\ n_1+n_2=n_3+n_4}} (\boldsymbol{h}_{k,n_1}^H \boldsymbol{w}_{P,n_1})(\boldsymbol{h}_{k,n_2}^H \boldsymbol{w}_{P,n_2})(\boldsymbol{h}_{k,n_3}^H \boldsymbol{w}_{P,n_3})^H(\boldsymbol{h}_{k,n_4}^H \boldsymbol{w}_{P,n_4})^H \tag{16}$$

$$= \frac{3}{8}\sum_{n=-N+1}^{N-1}(\boldsymbol{h}_k^H \boldsymbol{W}_{P,n}^*\boldsymbol{h}_k)(\boldsymbol{h}_k^H \boldsymbol{W}_{P,n}^*\boldsymbol{h}_k)^H = \frac{3}{8}\sum_{n=-N+1}^{N-1}(\boldsymbol{w}_P^H \boldsymbol{H}_{k,n}^*\boldsymbol{w}_P)(\boldsymbol{w}_P^H \boldsymbol{H}_{k,n}^*\boldsymbol{w}_P)^H \tag{17}$$

$$z_k(\boldsymbol{w}_I, \boldsymbol{w}_P, \boldsymbol{\phi}, \rho) = \beta_2\rho\left(\mathcal{E}\left\{\mathcal{A}\left\{y_{I,k}^2(t)\right\}\right\} + \mathcal{A}\left\{y_{P,k}^2(t)\right\}\right) + \beta_4\rho^2\left(\mathcal{E}\left\{\mathcal{A}\left\{y_{I,k}^4(t)\right\}\right\} + \mathcal{A}\left\{y_{P,k}^4(t)\right\} + 6\mathcal{E}\left\{\mathcal{A}\left\{y_{I,k}^2(t)\right\}\right\}\mathcal{A}\left\{y_{P,k}^2(t)\right\}\right) \tag{18}$$

$$\begin{aligned} &= \frac{1}{2}\beta_2\rho(\boldsymbol{h}_k^H \boldsymbol{W}_{I,0}\boldsymbol{h}_k + \boldsymbol{h}_k^H \boldsymbol{W}_{P,0}\boldsymbol{h}_k) \\ &\quad + \frac{3}{8}\beta_4\rho^2\left(2(\boldsymbol{h}_k^H \boldsymbol{W}_{I,0}\boldsymbol{h}_k)^2 + \sum_{n=-N+1}^{N-1}(\boldsymbol{h}_k^H \boldsymbol{W}_{P,n}^*\boldsymbol{h}_k)(\boldsymbol{h}_k^H \boldsymbol{W}_{P,n}^*\boldsymbol{h}_k)^H\right) \\ &\quad + \frac{3}{2}\beta_4\rho^2(\boldsymbol{h}_k^H \boldsymbol{W}_{I,0}\boldsymbol{h}_k)(\boldsymbol{h}_k^H \boldsymbol{W}_{P,0}\boldsymbol{h}_k) \end{aligned} \tag{19}$$

$$\begin{aligned} &= \frac{1}{2}\beta_2\rho(\boldsymbol{w}_I^H \boldsymbol{H}_{k,0}\boldsymbol{w}_I + \boldsymbol{w}_P^H \boldsymbol{H}_{k,0}\boldsymbol{w}_P) \\ &\quad + \frac{3}{8}\beta_4\rho^2\left(2(\boldsymbol{w}_I^H \boldsymbol{H}_{k,0}\boldsymbol{w}_I)^2 + \sum_{n=-N+1}^{N-1}(\boldsymbol{w}_P^H \boldsymbol{H}_{k,n}^*\boldsymbol{w}_P)(\boldsymbol{w}_P^H \boldsymbol{H}_{k,n}^*\boldsymbol{w}_P)^H\right) \\ &\quad + \frac{3}{2}\beta_4\rho^2(\boldsymbol{w}_I^H \boldsymbol{H}_{k,0}\boldsymbol{w}_I)(\boldsymbol{w}_P^H \boldsymbol{H}_{k,0}\boldsymbol{w}_P) \end{aligned} \tag{20}$$

III. Single-User Optimization

Consider a single-user waveform and IRS optimization problem where $\boldsymbol{\alpha} = \boldsymbol{1}^{N\times 1}$. We characterize the rate-energy region through a current maximization problem subject to transmit power, rate, and IRS constraints

$$\max_{\boldsymbol{w}_I, \boldsymbol{w}_P, \boldsymbol{\phi}, \rho} \quad z(\boldsymbol{w}_I, \boldsymbol{w}_P, \boldsymbol{\phi}, \rho) \tag{22a}$$

$$\text{s.t.} \quad \frac{1}{2}(\boldsymbol{w}_I^H \boldsymbol{w}_I + \boldsymbol{w}_P^H \boldsymbol{w}_P) \leq P, \tag{22b}$$

$$\sum_n \log_2\left(1 + \frac{(1-\rho)|(\boldsymbol{h}_{D,n}^H + \boldsymbol{\phi}^H \boldsymbol{V}_n)\boldsymbol{w}_{I,n}|^2}{\sigma_n^2}\right) \geq \bar{R}, \tag{22c}$$

$$|\phi| = 1, \quad l = 1, \ldots, L, \tag{22d}$$

$$0 \leq \rho \leq 1 \tag{22e}$$

Problem 22 is intricate due to coupled variables and non-convex objective function 22a and constraint 22c. To reduce the design complexity, we propose an suboptimal AO algorithm that updates the IRS phase shift, transmit waveform, and receive splitting ratio iteratively.

A. IRS Phase Shift

We optimize the IRS phase shift $\boldsymbol{\phi}$ for any given waveform $\boldsymbol{w}_{I/P,n}$ and splitting ratio ρ. Since

$$\begin{aligned}\boldsymbol{h}_n\boldsymbol{h}_n^H &= (\boldsymbol{h}_{D,n}+\boldsymbol{V}_n^H\boldsymbol{\phi})(\boldsymbol{h}_{D,n}^H+\boldsymbol{\phi}^H\boldsymbol{V}_n)\\ &= \boldsymbol{h}_{D,n}\boldsymbol{h}_{D,n}^H+\boldsymbol{V}_n^H\boldsymbol{\phi}\boldsymbol{h}_{D,n}^H+\boldsymbol{h}_{D,n}\boldsymbol{\phi}^H\boldsymbol{V}_n+\boldsymbol{V}_n^H\boldsymbol{\phi}\boldsymbol{\phi}^H\boldsymbol{V}_n\\ &= \boldsymbol{M}_n^H\boldsymbol{\Phi}\boldsymbol{M}_n\end{aligned} \tag{23}$$

where t is an auxiliary variable with unit modulus and

$$\boldsymbol{M}_n=\begin{bmatrix}\boldsymbol{V}_n\\ \boldsymbol{h}_{D,n}^H\end{bmatrix}\in\mathbb{C}^{(L+1)\times M},\quad \bar{\boldsymbol{\phi}}=\begin{bmatrix}\boldsymbol{\phi}\\ t\end{bmatrix},\quad \boldsymbol{\Phi}=\bar{\boldsymbol{\phi}}\bar{\boldsymbol{\phi}}^H \tag{24}$$

Therefore, it holds that

$$\begin{aligned}|(\boldsymbol{h}_{D,n}^H+\boldsymbol{\phi}^H\boldsymbol{V}_n)\boldsymbol{w}_{I,n}|^2 &= \boldsymbol{w}_{I,n}^H\boldsymbol{M}_n^H\boldsymbol{\Phi}\boldsymbol{M}_n\boldsymbol{w}_{I,n}\\ &= \mathrm{Tr}(\boldsymbol{M}_n\boldsymbol{w}_{I,n}\boldsymbol{w}_{I,n}^H\boldsymbol{M}_n^H\boldsymbol{\Phi})\\ &= \mathrm{Tr}(\boldsymbol{C}_n\boldsymbol{\Phi})\end{aligned} \tag{25}$$

with

$$\boldsymbol{C}_n=\boldsymbol{M}_n\boldsymbol{w}_{I,n}\boldsymbol{w}_{I,n}^H\boldsymbol{M}_n^H\in\mathbb{C}^{(L+1)\times(L+1)} \tag{26}$$

Similarly, we have

$$\boldsymbol{h}\boldsymbol{h}^H=\boldsymbol{M}^H\boldsymbol{\Phi}\boldsymbol{M} \tag{27}$$

where

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{h}_D^H\end{bmatrix}\in\mathbb{C}^{(L+1)\times MN} \tag{28}$$

Next, introduce auxiliary variables

$$\begin{aligned}t_{I/P,n} &= \boldsymbol{h}^H\boldsymbol{W}_{I/P,n}^*\boldsymbol{h}\\ &= \mathrm{Tr}(\boldsymbol{h}\boldsymbol{h}^H\boldsymbol{W}_{I/P,n}^*)\\ &= \mathrm{Tr}(\boldsymbol{M}^H\boldsymbol{\Phi}\boldsymbol{M}\boldsymbol{W}_{I/P,n}^*)\\ &= \mathrm{Tr}(\boldsymbol{M}\boldsymbol{W}_{I/P,n}^*\boldsymbol{M}^H\boldsymbol{\Phi})\\ &= \mathrm{Tr}(\boldsymbol{C}_{I/P,n}\boldsymbol{\Phi})\end{aligned} \tag{29}$$

with

$$\boldsymbol{C}_{I/P,n}=\boldsymbol{M}\boldsymbol{W}_{I/P,n}^*\boldsymbol{M}^H\in\mathbb{C}^{(L+1)\times(L+1)} \tag{30}$$

Therefore, 19 rewrites as

$$\begin{aligned}z(\boldsymbol{\Phi}) &= \frac{1}{2}\beta_2\rho(t_{I,0}+t_{P,0})\\ &+\frac{3}{8}\beta_4\rho^2\left(2t_{I,0}^2+\sum_{n=-N+1}^{N-1}t_{P,n}t_{P,n}^*\right)\\ &+\frac{3}{2}\beta_4\rho^2 t_{I,0}t_{P,0}\end{aligned} \tag{31}$$

We use first-order Taylor expansion to approximate the second-order terms in 31. Based on the variables optimized at iteration $i-1$, the local approximation at iteration i suggests [40]

$$(t_{I,0}^{(i)})^2\geq 2t_{I,0}^{(i)}t_{I,0}^{(i-1)}-(t_{I,0}^{(i-1)})^2 \tag{32}$$

$$t_{P,n}^{(i)}(t_{P,n}^{(i)})^*\geq 2\Re\left\{t_{P,n}^{(i)}(t_{P,n}^{(i-1)})^*\right\}-t_{P,n}^{(i-1)}(t_{P,n}^{(i-1)})^* \tag{33}$$

$$\begin{aligned}t_{I,0}^{(i)}t_{P,0}^{(i)} &= \frac{1}{4}(t_{I,0}^{(i)}+t_{P,0}^{(i)})^2-\frac{1}{4}(t_{I,0}^{(i)}-t_{P,0}^{(i)})^2\\ &\geq \frac{1}{2}(t_{I,0}^{(i)}+t_{P,0}^{(i)})(t_{I,0}^{(i-1)}+t_{P,0}^{(i-1)})\\ &\quad -\frac{1}{4}(t_{I,0}^{(i-1)}+t_{P,0}^{(i-1)})^2-\frac{1}{4}(t_{I,0}^{(i)}-t_{P,0}^{(i)})^2\end{aligned} \tag{34}$$

32 – 34 provide lower bounds to the corresponding terms in 31, and the approximated current function at iteration i is given in 35. Hence, problem 22 is transformed to

$$\max_{\boldsymbol{\Phi}}\quad \tilde{z}(\boldsymbol{\Phi}) \tag{36a}$$

$$\text{s.t.}\quad \sum_n\log_2\left(1+\frac{(1-\rho)\mathrm{Tr}(\boldsymbol{C}_n\boldsymbol{\Phi})}{\sigma_n^2}\right)\geq\bar{R}, \tag{36b}$$

$$\boldsymbol{\Phi}_{l,l}=1,\quad l=1,\dots,L+1, \tag{36c}$$

$$\boldsymbol{\Phi}\succeq 0, \tag{36d}$$

$$\mathrm{rank}(\boldsymbol{\Phi})=1 \tag{36e}$$

We then relax the rank constraint 36e and solve the optimal IRS matrix $\boldsymbol{\Phi}^\star$ iteratively by interior-point method. If $\mathrm{rank}(\boldsymbol{\Phi}^\star)=1$, the optimal phase shift vector $\bar{\boldsymbol{\phi}}^\star$ is attained by eigenvalue decomposition (EVD). Otherwise, a solution can be extracted through Gaussian randomization method [41]. We first perform EVD on $\boldsymbol{\Phi}^\star$ as $\boldsymbol{\Phi}^\star=\boldsymbol{U}_{\boldsymbol{\Phi}^\star}\boldsymbol{\Sigma}_{\boldsymbol{\Phi}^\star}\boldsymbol{U}_{\boldsymbol{\Phi}^\star}^H$. Then, we generate Q CSCG random vectors $\boldsymbol{r}_q\sim\mathcal{CN}(\boldsymbol{0},\boldsymbol{I}_{L+1}),\ q=1,\dots,Q$ and construct the corresponding candidates by

$$\bar{\boldsymbol{\phi}}_q=e^{j\arg\left(\boldsymbol{U}_{\boldsymbol{\Phi}^\star}\boldsymbol{\Sigma}_{\boldsymbol{\Phi}^\star}^{\frac{1}{2}}\boldsymbol{r}_q\right)} \tag{37}$$

The candidate maximizing the objective function 36a is chosen as the optimal solution $\bar{\boldsymbol{\phi}}^\star$. Finally, we can retrieve the phase shift by $\theta_l=\arg(\phi_l^\star/\phi_{L+1}^\star),\ l=1,\dots,L$. The algorithm for the phase shift optimization is summarized in Algorithm 1.

B. Waveform and Splitting Ratio

1) Geometric Programming: Following [38], it can be observed from 7 and 18 – 20 that the optimal phases of information and power waveform are both match to the composite channel as

$$\boldsymbol{\psi}_I^\star=\boldsymbol{\psi}_P^\star=-\boldsymbol{\varsigma} \tag{38}$$

By such a phase selection, we have

$$|\boldsymbol{h}_n^H\boldsymbol{w}_{I,n}|^2=(\boldsymbol{a}_n^H\boldsymbol{s}_{I,n})^2=\sum_{m_1,m_2}\prod_{j=1}^{2}a_{n,m}s_{I,n,m}\triangleq E_n \tag{39}$$

Therefore, the original problem 22 is reduced to an amplitude optimization problem

$$\max_{\boldsymbol{s}_I,\boldsymbol{s}_P,\rho}\quad z(\boldsymbol{s}_I,\boldsymbol{s}_P,\rho) \tag{40a}$$

$$\text{s.t.}\quad \frac{1}{2}(\boldsymbol{s}_I^H\boldsymbol{s}_I+\boldsymbol{s}_P^H\boldsymbol{s}_P)\leq P, \tag{40b}$$

$$\sum_n\log_2\left(1+\frac{(1-\rho)E_n}{\sigma_n^2}\right)\geq\bar{R} \tag{40c}$$

Algorithm 1 IRS Phase Shift

Input $\beta_2, \beta_4, \boldsymbol{h}_D, \boldsymbol{h}_I, \boldsymbol{h}_R, Q, \bar{R}, \epsilon, \rho, \boldsymbol{w}_{I/P}, \sigma_n \ \forall n$
Initialize $i \leftarrow 0, \boldsymbol{\Phi}^{(0)}, t_{I/P,n}^{(0)} \ \forall n$ by 29
Construct $\boldsymbol{M}, \boldsymbol{M}_n, \boldsymbol{C}_n, \boldsymbol{C}_{I/P,n} \ \forall n$ by 28, 24, 26, 30
repeat;
  $i \leftarrow i+1$
  Obtain IRS matrix $\boldsymbol{\Phi}^{(i)}$ by solving problem 36
  Update auxiliary $t_{I/P,n}^{(i)} \forall n$ by 29 for SCA
until $|(z^{(i)} - z^{(i-1)})/z^{(i)}| \leq \epsilon$
Perform EVD $\boldsymbol{\Phi}^\star = \boldsymbol{U}_{\boldsymbol{\Phi}^\star} \boldsymbol{\Sigma}_{\boldsymbol{\Phi}^\star} \boldsymbol{U}_{\boldsymbol{\Phi}^\star}^H$
Generate CSCG random vectors $\boldsymbol{r}_q \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{L+1}) \ \forall q$
Construct candidate IRS vectors $\bar{\boldsymbol{\phi}}_q = e^{j \arg\left(\boldsymbol{U}_{\boldsymbol{\Phi}^\star} \boldsymbol{\Sigma}_{\boldsymbol{\Phi}^\star}^{\frac{1}{2}} \boldsymbol{r}_q\right)}$ and matrices $\boldsymbol{\Phi}_q = \bar{\boldsymbol{\phi}}_q \bar{\boldsymbol{\phi}}_q^H \ \forall q$
Select the best solution $\boldsymbol{\Phi}^\star$ and $\bar{\boldsymbol{\phi}}^\star$ for problem 36
Compute phase shift by $\theta_l^\star = \arg(\phi_l^\star / \phi_{L+1}^\star), \ l = 1, \ldots, L$
Output $\theta_l^\star \ \forall l$

with z expressed in 41. We introduce an auxiliary variable t'' and rewrite problem 40 as

$$\min_{\boldsymbol{s}_I, \boldsymbol{s}_P, \rho, \bar{\rho}, t''} \quad \frac{1}{t''} \tag{42a}$$
$$\text{s.t.} \quad \frac{1}{2}(\boldsymbol{s}_I^H \boldsymbol{s}_I + \boldsymbol{s}_P^H \boldsymbol{s}_P) \leq P, \tag{42b}$$
$$\frac{t''}{z(\boldsymbol{s}_I, \boldsymbol{s}_P, \rho)} \leq 1, \tag{42c}$$
$$\frac{2^{\bar{R}}}{\prod_n (1 + \bar{\rho} E_n / \sigma_n^2)} \leq 1, \tag{42d}$$
$$\rho + \bar{\rho} \leq 1 \tag{42e}$$

Problem 42 can be transformed into standard GP by first decomposing the information and power posynomials as sum of monomials, then deriving their upper bounds using Arithmetic Mean-Geometric Mean (AM-GM) inequality [38], [42]. Specifically, define

$$z(\boldsymbol{s}_I, \boldsymbol{s}_P, \rho) = \sum_{m_P} g_{m_P}(\boldsymbol{s}_I, \boldsymbol{s}_P, \rho) \tag{43}$$
$$1 + \frac{\bar{\rho} E_n}{\sigma_n^2} = \sum_{m_{I,n}} g_{m_{I,n}}(\boldsymbol{s}_{I,n}, \bar{\rho}) \tag{44}$$

On top of this, problem 42 is equivalent to

$$\begin{aligned}
\min_{\boldsymbol{s}_I, \boldsymbol{s}_P, \rho, \bar{\rho}, t''} \quad & \frac{1}{t''} \\
\text{s.t.} \quad & \frac{1}{2}(\boldsymbol{s}_I^H \boldsymbol{s}_I + \boldsymbol{s}_P^H \boldsymbol{s}_P) \leq P, \\
& t'' \prod_{m_P} \left(\frac{g_{m_P}(\boldsymbol{s}_I, \boldsymbol{s}_P, \rho)}{\gamma_{m_P}} \right)^{-\gamma_{m_P}} \leq 1, \\
& 2^{\bar{R}} \prod_n \prod_{m_{I,n}} \left(\frac{g_{m_{I,n}}(\boldsymbol{s}_{I,n}, \bar{\rho})}{\gamma_{m_{I,n}}} \right)^{-\gamma_{m_{I,n}}} \leq 1, \\
& \rho + \bar{\rho} \leq 1
\end{aligned} \tag{45}$$

where $\gamma_{m_P}, \gamma_{m_{I,n}} \geq 0$ and $\sum_{m_P} \gamma_{m_P} = \sum_{m_{I,n}} \gamma_{m_{I,n}} = 1$. The tightness of the AM-GM inequality depends on $\{\gamma_{m_P}, \gamma_{m_{I,n}}\}$ that require successive update [38]. At iteration i, we choose

$$\gamma_{m_{I,n}}^{(i)} = \frac{g_{m_{I,n}}(\boldsymbol{s}_{I,n}^{(i-1)}, \bar{\rho}^{(i-1)})}{1 + \bar{\rho}^{(i-1)} E_n^{(i-1)} / \sigma_n^2} \tag{46}$$
$$\gamma_{m_P}^{(i)} = \frac{g_{m_P}(\boldsymbol{s}_I^{(i-1)}, \boldsymbol{s}_P^{(i-1)}, \rho^{(i-1)})}{z(\boldsymbol{s}_I^{(i-1)}, \boldsymbol{s}_P^{(i-1)}, \rho^{(i-1)})} \tag{47}$$

$$\begin{aligned}
\tilde{z}(\boldsymbol{\Phi}^{(i)}) = {} & \frac{1}{2}\beta_2 \rho (t_{I,0}^{(i)} + t_{P,0}^{(i)}) \\
& + \frac{3}{8}\beta_4 \rho^2 \left(4(t_{I,0}^{(i)})(t_{I,0}^{(i-1)}) - 2(t_{I,0}^{(i-1)})^2 + \sum_{n=-N+1}^{N-1} 2\Re\left\{ t_{P,n}^{(i)} (t_{P,n}^{(i-1)})^* \right\} - t_{P,n}^{(i-1)} (t_{P,n}^{(i-1)})^* \right) \\
& + \frac{3}{2}\beta_4 \rho^2 \left(\frac{1}{2}(t_{I,0}^{(i)} + t_{P,0}^{(i)})(t_{I,0}^{(i-1)} + t_{P,0}^{(i-1)}) - \frac{1}{4}(t_{I,0}^{(i-1)} + t_{P,0}^{(i-1)})^2 - \frac{1}{4}(t_{I,0}^{(i)} - t_{P,0}^{(i)})^2 \right)
\end{aligned} \tag{35}$$

$$\begin{aligned}
z(\boldsymbol{s}_I, \boldsymbol{s}_P, \rho) = {} & \frac{1}{2}\beta_2 \rho \sum_{n=1}^{N} \sum_{m_1, m_2} \left(\prod_{j=1}^{2} a_{n,m_j} s_{I,n,m_j} + \prod_{j=1}^{2} a_{n,m_j} s_{P,n,m_j} \right) \\
& + \frac{3}{8}\beta_4 \rho^2 \left(\sum_{n_1, n_2} \sum_{m_1, m_2, m_3, m_4} \prod_{j=1,3} a_{n_1, m_j} s_{I, n_1, m_j} \prod_{j=2,4} a_{n_2, m_j} s_{I, n_2, m_j} + \sum_{\substack{n_1, n_2, n_3, n_4 \\ n_1 + n_2 = n_3 + n_4}} \sum_{m_1, m_2, m_3, m_4} \prod_{j=1}^{4} a_{n_j, m_j} s_{P, n_j, m_j} \right) \\
& + \frac{3}{2}\beta_4 \rho^2 \left(\sum_{n_1, n_2} \sum_{m_1, m_2, m_3, m_4} \prod_{j=1,3} a_{n,m_j} s_{I,n,m_j} \prod_{j=2,4} a_{n,m_j} s_{P,n,m_j} \right)
\end{aligned} \tag{41}$$

then solve problem 45. The GP algorithm is summarized in Algorithm 2.

Algorithm 2 GP: Waveform and Splitting Ratio

Input $\beta_2, \beta_4, \boldsymbol{h}, P, \bar{R}, \epsilon, \sigma_n \ \forall n$
Initialize $i \leftarrow 0$, $\boldsymbol{s}_{I/P}^{(0)}$, $\rho^{(0)}$
Retrieve channel magnitude and phase $\boldsymbol{a}, \varsigma$
repeat
  $i \leftarrow i+1$
  Update GM exponents $\{\gamma_{m_P}, \gamma_{m_{I,n}}\}$ by 46, 47
  Obtain waveform amplitude $\boldsymbol{s}_{I/P}^{(i)}$ and power splitting ratio $\rho^{(i)}$ by solving problem 45
  Compute output DC current $z^{(i)}$ by 41
until $|(z^{(i)} - z^{(i-1)})/z^{(i)}| \leq \epsilon$
Recover waveform $\boldsymbol{w}_{I/P}^{\star}$ by 38
Output $\boldsymbol{w}_{I/P}^{\star}, \rho^{\star}$

2) Semi-Definite Relaxation: In this case, waveform and splitting ratio are updated iteratively until convergence.

a) Transmit Waveform: Consider the waveform optimization subproblem. Once $\boldsymbol{\phi}$ and ρ are obtained, introduce auxiliary variables

$$t'_{I/P,n} = \boldsymbol{w}_{I/P}^{H} \boldsymbol{H}_n^{*} \boldsymbol{w}_{I/P} = \mathrm{Tr}(\boldsymbol{H}_n^{*} \boldsymbol{W}_{I/P}) \tag{48}$$

Therefore, 20 rewrites as

$$\begin{aligned} z(\boldsymbol{W}_I, \boldsymbol{W}_P) = {} & \frac{1}{2}\beta_2 \rho (t'_{I,0} + t'_{P,0}) \\ & + \frac{3}{8}\beta_4 \rho^2 \left(2(t'_{I,0})^2 + \sum_{n=-N+1}^{N-1} t'_{P,n} (t'_{P,n})^* \right) \\ & + \frac{3}{2}\beta_4 \rho^2 t'_{I,0} t'_{P,0} \end{aligned} \tag{49}$$

Since 49 and 31 are in the same form, we reuse 32 – 34 and bound $z(\boldsymbol{W}_I, \boldsymbol{W}_P)$ by replacing $t_{I/P,n}$ with $t'_{I/P,n}$ in 35. Also, denote $\boldsymbol{W}'_{I,n}$ as the n-th block in the main diagonal of $\boldsymbol{W}_I$ such that

$$|\boldsymbol{h}_n^H \boldsymbol{w}_{I,n}|^2 = \boldsymbol{w}_{I,n}^H \boldsymbol{h}_n \boldsymbol{h}_n^H \boldsymbol{w}_{I,n} = \mathrm{Tr}(\boldsymbol{h}_n \boldsymbol{h}_n^H \boldsymbol{W}'_{I,n}) \tag{50}$$

Hence, problem 22 is transformed to

$$\max_{\boldsymbol{W}_I, \boldsymbol{W}_P} \quad \tilde{z}(\boldsymbol{W}_I, \boldsymbol{W}_P) \tag{51a}$$

$$\text{s.t.} \quad \sum_n \log_2 \left(1 + \frac{(1-\rho)\mathrm{Tr}(\boldsymbol{h}_n \boldsymbol{h}_n^H \boldsymbol{W}'_{I,n})}{\sigma_n^2} \right) \geq \bar{R}, \tag{51b}$$

$$\frac{1}{2}\left(\mathrm{Tr}(\boldsymbol{W}_I) + \mathrm{Tr}(\boldsymbol{W}_P)\right) \leq P, \tag{51c}$$

$$\boldsymbol{W}_{I/P} \succeq 0, \tag{51d}$$

$$\mathrm{rank}(\boldsymbol{W}_{I/P}) = 1 \tag{51e}$$

We then perform SDR and solve the optimal waveform matrix $\boldsymbol{W}_{I/P}^{\star}$ iteratively by interior-point method. $\boldsymbol{w}_I^{\star}$ and $\boldsymbol{w}_P^{\star}$ can be extracted using randomized vectors $\boldsymbol{r}_q \in \mathbb{C}^{MN \times 1}$ whose entries are uniformly distributed on the unit circle. The algorithm is summarized in Algorithm 3.

Algorithm 3 SDR: Transmit Waveform

Input $\beta_2, \beta_4, \boldsymbol{h}_D, \boldsymbol{h}_I, \boldsymbol{h}_R, P, Q, \bar{R}, \epsilon, \rho, \sigma_n \ \forall n$
Initialize $i \leftarrow 0$, $\boldsymbol{W}_{I/P}^{(0)}$, $t'^{(0)}_{I/P,n} \ \forall n$ by 48
Construct $\boldsymbol{H}_n \ \forall n$
repeat
  $i \leftarrow i+1$
  Obtain waveform matrices $\boldsymbol{W}_{I/P}^{(i)}$ by solving problem 51
  Update auxiliary $t'^{(i)}_{I/P,n} \forall n$ by 48 for SCA
until $|(z^{(i)} - z^{(i-1)})/z^{(i)}| \leq \epsilon$
Perform EVD $\boldsymbol{W}_{I/P}^{\star} = \boldsymbol{U}_{\boldsymbol{W}_{I/P}^{\star}} \boldsymbol{\Sigma}_{\boldsymbol{W}_{I/P}^{\star}} \boldsymbol{U}_{\boldsymbol{W}_{I/P}^{\star}}^{H}$
Generate random vectors $\boldsymbol{r}_q \ \forall q$ with entries uniformly distributed on the unit circle
Construct candidate waveform vectors $\boldsymbol{w}_{I/P,r} = \boldsymbol{U}_{\boldsymbol{W}_{I/P}^{\star}} \boldsymbol{\Sigma}_{\boldsymbol{W}_{I/P}^{\star}}^{\frac{1}{2}} \boldsymbol{r}_q$ and matrices $\boldsymbol{W}_{I/P,q} = \boldsymbol{w}_{I/P,q} \boldsymbol{w}_{I/P,q}^{H} \ \forall q$
Select the best solution $\boldsymbol{W}_{I/P}^{\star}$ and $\boldsymbol{w}_{I/P}^{\star}$ for problem 51
Output $\boldsymbol{w}_{I/P}^{\star}$

b) Receive Splitting Ratio: We then optimize the power splitting ratio ρ for any fixed phase shift $\boldsymbol{\phi}$ and waveform $\boldsymbol{w}_{I/P}$. In this case, $z(\rho)$ is also expressed in 49 with constant $t'_{I/P,n}$ given by 48. Since $z(\rho)$ is a quadratic function that monotonically increases over $\rho \in [0,1]$, we replace the convex objective function with affine ρ and transform problem 22 to

$$\max_{\rho} \quad \rho \tag{52a}$$

$$\text{s.t.} \quad \sum_n \log_2 \left(1 + \frac{(1-\rho)|\boldsymbol{h}_n^H \boldsymbol{w}_n|^2}{\sigma_n^2} \right) \geq \bar{R}, \tag{52b}$$

$$0 \leq \rho \leq 1 \tag{52c}$$

The optimal power splitting ratio $\rho^{\star}$ can be obtained by solving problem 52.

c) Inner Loop: As summarized in Algorithm 4, the inner loop updates the waveform and splitting ratio iteratively until convergence.

Algorithm 4 Inner Loop: Waveform and Splitting Ratio

Input $\beta_2, \beta_4, \boldsymbol{h}_D, \boldsymbol{h}_I, \boldsymbol{h}_R, P, Q, \bar{R}, \epsilon, \rho, \sigma_n \ \forall n$
Initialize $\boldsymbol{w}_{I/P}^{(0)}$
$i \leftarrow 0$
repeat
  $i \leftarrow i+1$
  Update splitting ratio $\rho^{(i)}$ by solving problem 52
  Update waveform $\boldsymbol{w}_{I/P}^{(i)}$ by Algorithm 3
until $|(z^{(i)} - z^{(i-1)})/z^{(i)}| \leq \epsilon$
Output $\boldsymbol{w}_{I/P}^{\star}, \rho^{\star}$

C. R-E Region Characterization

To characterize the R-E region, we initialize the algorithm to WPT mode and increase the rate constraint gradually to obtain S boundary points. At each point, information and power waveform are initialized to WIT and WPT solutions.

1) WIT: Consider a rate maximization problem with given information waveform $\boldsymbol{w}_I$ as

$$\begin{aligned} \max_{\boldsymbol{\Phi}} \quad & \sum_n \log_2\left(1 + \frac{\mathrm{Tr}(\boldsymbol{C}_n\boldsymbol{\Phi})}{\sigma_n^2}\right) && \text{(53a)} \\ \text{s.t.} \quad & \boldsymbol{\Phi}_{l,l} = 1, \quad l = 1, \ldots, L+1, && \text{(53b)} \\ & \boldsymbol{\Phi} \succeq 0, && \text{(53c)} \\ & \mathrm{rank}(\boldsymbol{\Phi}) = 1 && \text{(53d)} \end{aligned}$$

Problem 53 can be solved after SDR, and a best solution $\bar{\boldsymbol{\phi}}^\star$ can be obtained via Gaussian randomization method. $\boldsymbol{\phi}^\star$ can be recovered by $\phi_l^\star = \phi_l^\star/\phi_{L+1}^\star,\ l = 1, \ldots, L$ and $\boldsymbol{h}$ can be constructed by 4. On top of this, the optimal information waveform can be obtained by Maximum-Ratio Transmission (MRT). The IRS phase shift and information waveform are updated iteratively until convergence.

2) WPT: The current maximization problem can be solved by setting $\rho = 1$, removing the rate constraint in problem 36 and 51 then solving them iteratively.

3) R-E Sample: We propose a two-layer AO algorithm to maximize the R-E region where the outer loop updates the phase shifts and the inner loop updates the splitting ratio and waveform until convergence. It is summarized in Algorithm 5.

Algorithm 5 Outer Loop: Phase Shift, Waveform and Splitting Ratio

Input $\beta_2, \beta_4, \boldsymbol{h}_D, \boldsymbol{h}_I, \boldsymbol{h}_R, P, Q, S, \epsilon, \sigma_n\ \forall n$
Initialize $i \leftarrow 0, \boldsymbol{\phi}^{(0)}, \boldsymbol{h}^{(0)}, \boldsymbol{w}_{I/P}^{(0)}$ by WIT/WPT solutions
Obtain splitting ratio $\rho^{(0)}$ and waveform $\boldsymbol{w}_{I/P}^{(0)}$ by GP (Algorithm 2) or inner loop (Algorithm 4)
repeat
  $i \leftarrow i + 1$
  Update IRS phase shift $\boldsymbol{\phi}^{(i)}$ by Algorithm 1
  Update composite channel $\boldsymbol{h}^{(i)}$ by 4
  Update splitting ratio $\rho^{(i)}$ and waveform $\boldsymbol{w}_{I/P}^{(i)}$ by GP (Algorithm 2) or inner loop (Algorithm 4)
until $|(z^{(i)} - z^{(i-1)})/z^{(i)}| \leq \epsilon$
Output $\boldsymbol{w}_{I/P}^\star, \rho^\star, R^\star, z^\star$

References

[1] D. W. K. Ng, T. Q. Duong, C. Zhong, and R. Schober, Eds., *Wireless Information and Power Transfer*. Chichester, UK: John Wiley & Sons, Ltd, dec 2018. [Online]. Available: http://doi.wiley.com/10.1002/9781119476863

[2] L. R. Varshney, "Transporting information and energy simultaneously," *IEEE International Symposium on Information Theory - Proceedings*, pp. 1612–1616, 2008.

[3] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: Architecture design and rate-energy tradeoff," *IEEE Transactions on Communications*, vol. 61, no. 11, pp. 4754–4767, 2013.

[4] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 1989–2001, 2013.

[5] J. Xu, L. Liu, and R. Zhang, "Multiuser miso beamforming for simultaneous wireless information and power transfer," *IEEE Transactions on Signal Processing*, vol. 62, no. 18, pp. 4798–4810, 2014.

[6] I. Krikidis, S. Timotheou, S. Nikolaou, G. Zheng, D. W. K. Ng, and R. Schober, "Simultaneous Wireless Information and Power Transfer in modern communication systems," *IEEE Communications Magazine*, vol. 52, no. 11, pp. 104–110, 2014.

[7] E. Boshkovska, D. W. K. Ng, N. Zlatanov, and R. Schober, "Practical non-linear energy harvesting model and resource allocation for SWIPT systems," *IEEE Communications Letters*, vol. 19, no. 12, pp. 2082–2085, 2015.

[8] M. S. Trotter, J. D. Griffin, and G. D. Durgin, "Power-optimized waveforms for improving the range and reliability of RFID systems," *2009 IEEE International Conference on RFID, RFID 2009*, pp. 80–87, 2009.

[9] A. S. Boaventura and N. B. Carvalho, "Maximizing DC power in energy harvesting circuits using multisine excitation," *IEEE MTT-S International Microwave Symposium Digest*, vol. 1, no. 1, pp. 1–4, 2011.

[10] B. Clerckx and E. Bayguzina, "Waveform Design for Wireless Power Transfer," *IEEE Transactions on Signal Processing*, vol. 64, no. 23, pp. 6313–6328, 2016.

[11] J. Kim, B. Clerckx, and P. D. Mitcheson, "Experimental Analysis of Harvested Energy and Throughput Trade-off in a Realistic SWIPT System," 2019. [Online]. Available: http://arxiv.org/abs/1908.08272

[12] ——, "Signal and System Design for Wireless Power Transfer : Prototype, Experiment and Validation," pp. 1–31, 2019. [Online]. Available: http://arxiv.org/abs/1901.01156

[13] B. Clerckx and J. Kim, "On the Beneficial Roles of Fading and Transmit Diversity in Wireless Power Transfer with Nonlinear Energy Harvesting," *IEEE Transactions on Wireless Communications*, vol. 17, no. 11, pp. 7731–7743, 2018.

[14] J. J. Park, J. H. Moon, K. Y. Lee, and D. I. Kim, "Dual Mode SWIPT: Waveform Design and Transceiver Architecture with Adaptive Mode Switching Policy," *IEEE Vehicular Technology Conference*, vol. 2018-June, pp. 1–5, 2018.

[15] J. Hu, Y. Zhao, and K. Yang, "Modulation and Coding Design for Simultaneous Wireless Information and Power Transfer," *IEEE Communications Magazine*, vol. 57, no. 5, pp. 124–130, 2019.

[16] M. Varasteh, E. Piovano, and B. Clerckx, "A Learning Approach to Wireless Information and Power Transfer Signal and System Design," in *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, may 2019, pp. 4534–4538. [Online]. Available: https://ieeexplore.ieee.org/document/8682485/

[17] Y. Liu, Z. Ding, M. Elkashlan, and H. V. Poor, "Cooperative Non-orthogonal Multiple Access with Simultaneous Wireless Information and Power Transfer," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 4, pp. 938–953, 2016.

[18] Y. Mao, B. Clerckx, and V. O. Li, "Rate-Splitting for Multi-User Multi-Antenna Wireless Information and Power Transfer," *IEEE Workshop on Signal Processing Advances in Wireless Communications, SPAWC*, vol. 2019-July, 2019.

[19] Q. Wu and R. Zhang, "Intelligent Reflecting Surface Enhanced Wireless Network via Joint Active and Passive Beamforming," *IEEE Transactions on Wireless Communications*, vol. 18, no. 11, pp. 5394–5409, 2019.

[20] T. J. Cui, M. Q. Qi, X. Wan, J. Zhao, and Q. Cheng, "Coding metamaterials, digital metamaterials and programmable metamaterials," *Light: Science & Applications*, vol. 3, no. 10, pp. e218–e218, 2014.

[21] C. Liaskos, S. Nie, A. Tsioliaridou, A. Pitsillides, S. Ioannidis, and I. Akyildiz, "Realizing Wireless Communication Through Software-Defined HyperSurface Environments," *19th IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks, WoWMoM 2018*, 2018.

[22] X. Tan, Z. Sun, D. Koutsonikolas, and J. M. Jornet, "Enabling Indoor Mobile Millimeter-wave Networks Based on Smart Reflect-arrays," *Proceedings - IEEE INFOCOM*, vol. 2018-April, pp. 270–278, 2018.

[23] Q. Wu and R. Zhang, "Intelligent Reflecting Surface Enhanced Wireless Network: Joint Active and Passive Beamforming Design," *IEEE Transactions on Wireless Communications*, vol. 18, no. 11, pp. 5394–5409, sep 2018. [Online]. Available: http://arxiv.org/abs/1809.01423

[24] ——, "Beamforming Optimization for Intelligent Reflecting Surface with Discrete Phase Shifts," in *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, may 2019, pp. 7830–7833. [Online]. Available: https://ieeexplore.ieee.org/document/8683145/

[25] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, "Weighted Sum-Rate Optimization for Intelligent Reflecting Surface Enhanced Wireless Networks," pp. 1–13, 2019. [Online]. Available: http://arxiv.org/abs/1905.07920

[26] Q.-U.-A. Nadeem, A. Kammoun, A. Chaaban, M. Debbah, and M.-S. Alouini, "Intelligent Reflecting Surface Assisted Wireless Communication: Modeling and Channel Estimation," pp. 1–7, 2019. [Online]. Available: http://arxiv.org/abs/1906.02360

[27] S. Abeywickrama, R. Zhang, and C. Yuen, "Intelligent Reflecting Surface: Practical Phase Shift Model and Beamforming Optimization," pp. 1–30, 2019. [Online]. Available: http://arxiv.org/abs/1907.06002

[28] Y. Yang, B. Zheng, S. Zhang, and R. Zhang, "Intelligent Reflecting Surface Meets OFDM: Protocol Design and Rate Maximization," pp. 1–32, 2019. [Online]. Available: http://arxiv.org/abs/1906.09956

[29] B. Zheng and R. Zhang, "Intelligent Reflecting Surface-Enhanced OFDM: Channel Estimation and Reflection Optimization," *IEEE Wireless Communications Letters*, pp. 1–1, 2019.

[30] Y. Yang, S. Zhang, and R. Zhang, "IRS-Enhanced OFDMA: Joint Resource Allocation and Passive Beamforming Optimization," *IEEE Wireless Communications Letters*, pp. 1–1, 2020.

[31] L. Dai, M. D. Renzo, C. B. Chae, L. Hanzo, B. Wang, M. Wang, X. Yang, J. Tan, S. Bi, S. Xu, F. Yang, and Z. Chen, "Reconfigurable Intelligent Surface-Based Wireless Communications: Antenna Design, Prototyping, and Experimental Results," *IEEE Access*, vol. 8, pp. 45 913–45 923, 2020.

[32] D. H. Kim and J. I. Choi, "Design of a multiband frequency selective surface," *ETRI Journal*, vol. 28, no. 4, pp. 506–508, 2006.

[33] R. S. Anwar, L. Mao, and H. Ning, "Frequency selective surfaces: A review," *Applied Sciences (Switzerland)*, vol. 8, no. 9, pp. 1–47, 2018.

[34] Q. Wu and R. Zhang, "Weighted Sum Power Maximization for Intelligent Reflecting Surface Aided SWIPT," *IEEE Wireless Communications Letters*, pp. 1–6, 2019.

[35] Y. Tang, G. Ma, H. Xie, J. Xu, and X. Han, "Joint Transmit and Reflective Beamforming Design for IRS-Assisted Multiuser MISO SWIPT Systems," 2019. [Online]. Available: http://arxiv.org/abs/1910.07156

[36] Q. Wu and R. Zhang, "Joint Active and Passive Beamforming Optimization for Intelligent Reflecting Surface Assisted SWIPT under QoS Constraints," pp. 1–30, 2019. [Online]. Available: http://arxiv.org/abs/1910.06220

[37] C. Pan, H. Ren, K. Wang, M. Elkashlan, A. Nallanathan, J. Wang, and L. Hanzo, "Intelligent Reflecting Surface Aided MIMO Broadcasting for Simultaneous Wireless Information and Power Transfer," pp. 1–33, 2019. [Online]. Available: http://arxiv.org/abs/1908.04863

[38] B. Clerckx, "Wireless Information and Power Transfer: Nonlinearity, Waveform Design, and Rate-Energy Tradeoff," *IEEE Transactions on Signal Processing*, vol. 66, no. 4, pp. 847–862, 2018.

[39] Y. Huang and B. Clerckx, "Large-Scale Multiantenna Multisine Wireless Power Transfer," *IEEE Transactions on Signal Processing*, vol. 65, no. 21, pp. 5812–5827, 2017.

[40] T. Adali and S. Haykin, *Adaptive Signal Processing*. Hoboken, NJ, USA: John Wiley & Sons, Inc., mar 2010. [Online]. Available: http://doi.wiley.com/10.1002/9780470575758

[41] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 664–678, 2010.

[42] M. Chiang, *Geometric programming for communication systems*, 2005, vol. 2, no. 1.