

Signal and channel

MISO, one L -reflector IRS, single-user, N -subband

- transmit signal at antenna m :

$$x_m(t) = \Re \left\{ \sum_{n=1}^N (w_{I,n,m} \tilde{x}_{I,n}(t) + w_{P,n,m}) e^{j2\pi f_n t} \right\} \quad (1)$$

- composite channel at subband n :

- ▶ AP-user direct channel $\mathbf{h}_{D,n}^H \in \mathbb{C}^{1 \times M}$
- ▶ IRS-aided extra channel: concatenate the following terms
 - ★ AP-IRS incident channel $\mathbf{H}_{I,n} \in \mathbb{C}^{L \times M}$
 - ★ IRS reflection diagonal matrix $\mathbf{\Theta} = \text{diag}(\gamma_1 e^{j\theta_1}, \dots, \gamma_L e^{j\theta_L}) \in \mathbb{C}^{L \times L}$
 - ★ IRS-user reflective channel $\mathbf{h}_{R,n}^H \in \mathbb{C}^{1 \times L}$

$$\mathbf{h}_n^H = \mathbf{h}_{D,n}^H + \mathbf{h}_{R,n}^H \mathbf{\Theta} \mathbf{H}_{I,n} = \mathbf{h}_{D,n}^H + \phi^H \mathbf{V}_n \quad (2)$$

where $\phi = [\gamma_1 e^{j\theta_1}, \dots, \gamma_L e^{j\theta_L}]^H \in \mathbb{C}^{L \times 1}$ and $\mathbf{V}_n = \text{diag}(\mathbf{h}_{R,n}^H) \mathbf{H}_{I,n} \in \mathbb{C}^{L \times M}$.

- received signal

$$y(t) = \Re \left\{ \sum_{n=1}^N \mathbf{h}_n^H (\mathbf{w}_{I,n} \tilde{x}_{I,n}(t) + \mathbf{w}_{P,n}) e^{j2\pi f_n t} \right\} \quad (3)$$

Problem formulation

$$\max_{\mathbf{w}_I, \mathbf{w}_P, \phi, \rho} z(\mathbf{w}_I, \mathbf{w}_P, \phi, \rho) \quad (4a)$$

$$\text{s.t.} \quad \frac{1}{2}(\mathbf{w}_I^H \mathbf{w}_I + \mathbf{w}_P^H \mathbf{w}_P) \leq P, \quad (4b)$$

$$\sum_n \log_2 \left(1 + \frac{(1-\rho)|(\mathbf{h}_{D,n}^H + \phi^H \mathbf{V}_n) \mathbf{w}_{I,n}|^2}{\sigma_n^2} \right) \geq \bar{R}, \quad (4c)$$

$$|\phi_l| = 1, \quad l = 1, \dots, L, \quad (4d)$$

$$0 \leq \rho \leq 1 \quad (4e)$$

where $\mathbf{w}_{I/P} = [\mathbf{w}_{I/P,1}^T, \dots, \mathbf{w}_{I/P,N}^T]^T \in \mathbb{C}^{MN \times 1}$.

Challenge: the fourth-order term in current expression

$$\mathcal{A}\{y_P^4(t)\} = \frac{3}{8} \sum_{\substack{n_1, n_2, n_3, n_4 \\ n_1 + n_2 = n_3 + n_4}} (\mathbf{h}_{n_1}^H \mathbf{w}_{P, n_1}) (\mathbf{h}_{n_2}^H \mathbf{w}_{P, n_2}) (\mathbf{h}_{n_3}^H \mathbf{w}_{P, n_3})^H (\mathbf{h}_{n_4}^H \mathbf{w}_{P, n_4})^H \quad (5)$$

Two known approaches so far:

GP

$$\mathcal{A}\{y_P^4(t)\} = \frac{3}{8} \sum_{\substack{n_1, n_2, n_3, n_4 \\ n_1 + n_2 = n_3 + n_4}} s_{P, n_1} s_{P, n_2} s_{P, n_3} s_{P, n_4} \|\mathbf{h}_{n_1}\| \|\mathbf{h}_{n_2}\| \|\mathbf{h}_{n_3}\| \|\mathbf{h}_{n_4}\| \quad (6)$$

where $s_{P, n}$ is real scalar.

SDP

$$\mathcal{A}\{y_P^4(t)\} = \frac{3}{8} \sum_{n=-N+1}^{N-1} (\mathbf{h}^H \mathbf{W}_{P, n}^* \mathbf{h}) (\mathbf{h}^H \mathbf{W}_{P, n}^* \mathbf{h})^H = \frac{3}{8} \sum_{n=-N+1}^{N-1} (\mathbf{w}_P^H \mathbf{H}_{k, n}^* \mathbf{w}_P) (\mathbf{w}_P^H \mathbf{H}_{k, n}^* \mathbf{w}_P)^H \quad (7)$$

where $\mathbf{h} = [\mathbf{h}_1^T, \dots, \mathbf{h}_N^T]^T \in \mathbb{C}^{MN \times 1}$, $\mathbf{W}_{I/P, n}$, $\mathbf{H}_{k, n}$ keep the n -th ($n = -N+1, \dots, N-1$) block diagonal and null the remaining entries.

GP-based approach

Joint design of ϕ , $s_{I/P,n}$, ρ

GP is not applicable for ϕ since its real and imaginary parts could take negative value.

Alternating optimization of $(s_{I/P,n}, \rho)$ and ϕ

Cannot formulate IRS design as convex problem:

- need to approximate $\|h_{n_1}\| \|h_{n_2}\| \|h_{n_3}\| \|h_{n_4}\|$
- even if we provide first-order approximation to the fourth-order term above, $\|\cdot\|$ (reduce to $|\cdot|$ for SISO) produces convex objective function
 - ▶ can solve dual problem but no idea to prove zero duality gap

SDP-based approach: alternating optimization of $(\mathbf{w}_{I/P,n}, \rho)$ and ϕ

IRS design problem:

$$\max_{\Phi} \quad \tilde{z}(\Phi) = \text{Tr}(\mathbf{A}\Phi) \quad (8a)$$

$$\text{s.t.} \quad \sum_n \log_2 \left(1 + \frac{(1-\rho)\text{Tr}(\mathbf{C}_n\Phi)}{\sigma_n^2} \right) \geq \bar{R}, \quad (8b)$$

$$\Phi_{l,l} = 1, \quad l = 1, \dots, L+1, \quad (8c)$$

$$\Phi \succeq 0, \quad (8d)$$

$$\text{rank}(\Phi) = 1 \quad (8e)$$

Despite being convex, problem 8 is not a standard SDP due to rate constraint 8b, and there is no proof that the loss from SDR would be negligible. In other words, we only obtain Φ^* .

$$\begin{aligned} \min_{\mathbf{X} \in \mathbb{S}^n} \quad & \text{Tr}(\mathbf{C}\mathbf{X}) \\ \text{s.t.} \quad & \text{Tr}(\mathbf{A}_i\mathbf{X}) \succeq_i b_i, \quad i = 1, \dots, m, \\ & \mathbf{X} \succeq \mathbf{0}, \quad \text{rank}(\mathbf{X}) = 1. \end{aligned}$$

Simulation results

For SISO only, simulation show that Φ^* is (always almost) rank-1, and ϕ^* extracted by Gaussian randomization method achieves the same performance as Φ^* .

```
% * Recover rank-1 solution by randomization method
[u, sigma] = eig(firsMatrix);
current = 0;
for iCar = 1:nCar
    for iCs = 1:nCs
        sigma = 11x11 double =
            0.0000    0    0    0    0    0    0    0    0    0    0;
            0    0.0000    0    0    0    0    0    0    0    0    0;
            0    0    0.0000    0    0    0    0    0    0    0    0;
            0    0    0    0.0000    0    0    0    0    0    0    0;
            0    0    0    0    0.0000    0    0    0    0    0    0;
            0    0    0    0    0    0.0000    0    0    0    0    0;
            0    0    0    0    0    0    0.0000    0    0    0    0;
            0    0    0    0    0    0    0    0.0000    0    0    0;
            0    0    0    0    0    0    0    0    0.0000    0    0;
            0    0    0    0    0    0    0    0    0    0.0000    0;
            0    0    0    0    0    0    0    0    0    0    11.0000;
    end
    info = 0;
    power = 0;
    % *
    for i = 1:nSuhband
        for iSuhband = 1 + nSuhband
```

Figure: Eigenvalue of Φ^* for Tx = 1

For MISO this is not the case.

```
% * Recover rank-1 solution by randomization method
[u, sigma] = eig(firsMatrix);
current = 0;
for iCar = 1:nCar
    for iCs = 1:nCs
        sigma = 11x11 double =
            0.0487    0    0    0    0    0    0    0    0    0    0;
            0    0.1029    0    0    0    0    0    0    0    0    0;
            0    0    0.1175    0    0    0    0    0    0    0    0;
            0    0    0    0.1466    0    0    0    0    0    0    0;
            0    0    0    0    0.1823    0    0    0    0    0    0;
            0    0    0    0    0    0.2202    0    0    0    0    0;
            0    0    0    0    0    0    0.2547    0    0    0    0;
            0    0    0    0    0    0    0    0.3638    0    0    0;
            0    0    0    0    0    0    0    0    0.3136    0    0;
            0    0    0    0    0    0    0    0    0    0.3978    0;
            0    0    0    0    0    0    0    0    0    0    8.9119;
    end
    info = 0;
    power = 0;
    % *
    for i = 1:nSuhband
        for iSuhband = 1 + nSuhband
```

Figure: Eigenvalue of Φ^* for Tx = 2

R-E plots

We optimize $(\mathbf{w}_{I/P,n}, \rho)$ and ϕ alternatively. IRS is always optimized by SDR, and the plots compare waveform optimized by GP and SDR.

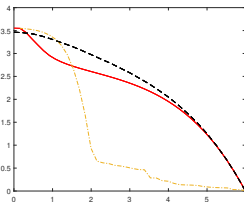


Figure: GP (black) vs SDR

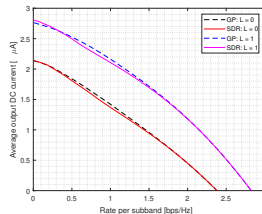


Figure: No IRS vs single reflector

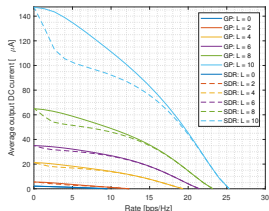


Figure: Number of reflectors

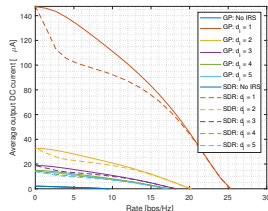


Figure: AP-IRS distance

Problem

- Although Φ^* turns out rank-1 for SISO, we have no idea on its demonstration
- Even if we rewrite sum-rate constraint 8b as quadratic form $\text{Tr}(\mathbf{C}\Phi)$, the approximation accuracy γ requires further check (due to this additional constraint)