Signal and channel

MISO, one L-reflector IRS, single-user, N-subband

- transmit signal at antenna m:

$$x_m(t) = \Re\left\{\sum_{n=1}^{N}\left(w_{I,n,m}\tilde{x}_{I,n}(t) + w_{P,n,m}\right)e^{j2\pi f_n t}\right\} \tag{1}$$

- composite channel at subband n:
  - AP-user direct channel $\boldsymbol{h}_{D,n}^H \in \mathbb{C}^{1\times M}$
  - IRS-aided extra channel: concatenate the following terms
    - AP-IRS incident channel $\boldsymbol{H}_{I,n} \in \mathbb{C}^{L\times M}$
    - IRS reflection diagonal matrix $\boldsymbol{\Theta} = \mathrm{diag}(\gamma_1 e^{j\theta_1}, \ldots, \gamma_L e^{j\theta_L}) \in \mathbb{C}^{L\times L}$
    - IRS-user reflective channel $\boldsymbol{h}_{R,n}^H \in \mathbb{C}^{1\times L}$

$$\boldsymbol{h}_n^H = \boldsymbol{h}_{D,n}^H + \boldsymbol{h}_{R,n}^H\boldsymbol{\Theta}\boldsymbol{H}_{I,n} = \boldsymbol{h}_{D,n}^H + \boldsymbol{\phi}^H\boldsymbol{V}_n \tag{2}$$

where $\boldsymbol{\phi} = [\gamma_1 e^{j\theta_1}, \ldots, \gamma_L e^{j\theta_L}]^H \in \mathbb{C}^{L\times 1}$ and $\boldsymbol{V}_n = \mathrm{diag}(\boldsymbol{h}_{R,n}^H)\boldsymbol{H}_{I,n} \in \mathbb{C}^{L\times M}$.

- received signal

$$y(t) = \Re\left\{\sum_{n=1}^{N}\boldsymbol{h}_n^H\left(\boldsymbol{w}_{I,n}\tilde{x}_{I,n}(t) + \boldsymbol{w}_{P,n}\right)e^{j2\pi f_n t}\right\} \tag{3}$$

Problem formulation

$$
\begin{aligned}
\max_{\boldsymbol{w}_I, \boldsymbol{w}_P, \phi, \rho} \quad & z(\boldsymbol{w}_I, \boldsymbol{w}_P, \phi, \rho) && \text{(4a)} \\
\text{s.t.} \quad & \frac{1}{2}(\boldsymbol{w}_I^H \boldsymbol{w}_I + \boldsymbol{w}_P^H \boldsymbol{w}_P) \leq P, && \text{(4b)} \\
& \sum_n \log_2 \left(1 + \frac{(1-\rho)|(\boldsymbol{h}_{D,n}^H + \phi^H \boldsymbol{V}_n)\boldsymbol{w}_{I,n}|^2}{\sigma_n^2}\right) \geq \bar{R}, && \text{(4c)} \\
& |\phi_l| = 1, \quad l = 1, \ldots, L, && \text{(4d)} \\
& 0 \leq \rho \leq 1 && \text{(4e)}
\end{aligned}
$$

where $\boldsymbol{w}_{I/P} = [\boldsymbol{w}_{I/P,1}^T, \ldots, \boldsymbol{w}_{I/P,N}^T]^T \in \mathbb{C}^{MN \times 1}$.

Challenge: the fourth-order term in current expression

$$\mathcal{A}\left\{y_P^4(t)\right\} = \frac{3}{8} \sum_{\substack{n_1,n_2,n_3,n_4 \\ n_1+n_2=n_3+n_4}} (\boldsymbol{h}_{n_1}^H \boldsymbol{w}_{P,n_1})(\boldsymbol{h}_{n_2}^H \boldsymbol{w}_{P,n_2})(\boldsymbol{h}_{n_3}^H \boldsymbol{w}_{P,n_3})^H(\boldsymbol{h}_{n_4}^H \boldsymbol{w}_{P,n_4})^H \tag{5}$$

Two known approaches so far:

GP

$$\mathcal{A}\left\{y_P^4(t)\right\} = \frac{3}{8} \sum_{\substack{n_1,n_2,n_3,n_4 \\ n_1+n_2=n_3+n_4}} s_{P,n_1} s_{P,n_2} s_{P,n_3} s_{P,n_4} \|\boldsymbol{h}_{n_1}\|\|\boldsymbol{h}_{n_2}\|\|\boldsymbol{h}_{n_3}\|\|\boldsymbol{h}_{n_4}\| \tag{6}$$

where $s_{P,n}$ is real scalar.

SDP

$$\mathcal{A}\left\{y_P^4(t)\right\} = \frac{3}{8} \sum_{n=-N+1}^{N-1} (\boldsymbol{h}^H \boldsymbol{W}_{P,n}^* \boldsymbol{h})(\boldsymbol{h}^H \boldsymbol{W}_{P,n}^* \boldsymbol{h})^H = \frac{3}{8} \sum_{n=-N+1}^{N-1} (\boldsymbol{w}_P^H \boldsymbol{H}_{k,n}^* \boldsymbol{w}_P)(\boldsymbol{w}_P^H \boldsymbol{H}_{k,n}^* \boldsymbol{w}_P)^H \tag{7}$$

where $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_N^T]^T \in \mathbb{C}^{MN\times 1}$, $\boldsymbol{W}_{I/P,n}$, $\boldsymbol{H}_{k,n}$ keep the n-th $(n = -N+1, \ldots, N-1)$ block diagonal and null the remaining entries.

GP-based approach

Joint design of ϕ, $s_{I/P,n}$, ρ

GP is not applicable for ϕ since its real and imaginary parts could take negative value.

Alternating optimization of $(s_{I/P,n}, \rho)$ and ϕ

Cannot formulate IRS design as convex problem:

- need to approximate $\|\boldsymbol{h}_{n_1}\|\|\boldsymbol{h}_{n_2}\|\|\boldsymbol{h}_{n_3}\|\|\boldsymbol{h}_{n_4}\|$
- even if we provide first-order approximation to the fourth-order term above, $\|\cdot\|$ (reduce to $|\cdot|$ for SISO) produces convex objective function
  - can solve dual problem but no idea to prove zero duality gap

SDP-based approach: alternating optimization of $(\boldsymbol{w}_{I/P,n}, \rho)$ and ϕ

IRS design problem:

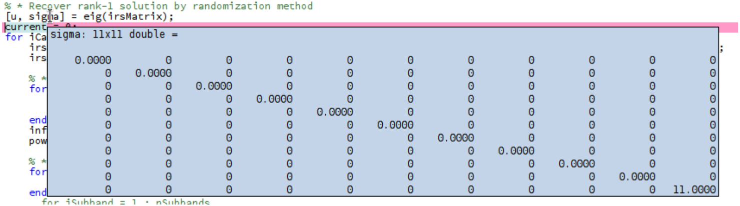
$$
\begin{aligned}
\max_{\boldsymbol{\Phi}} \quad & \tilde{z}(\boldsymbol{\Phi}) = \mathrm{Tr}(\boldsymbol{A}\boldsymbol{\Phi}) && \text{(8a)} \\
\text{s.t.} \quad & \sum_n \log_2\left(1 + \frac{(1-\rho)\mathrm{Tr}(\boldsymbol{C}_n\boldsymbol{\Phi})}{\sigma_n^2}\right) \geq \bar{R}, && \text{(8b)} \\
& \boldsymbol{\Phi}_{l,l} = 1, \quad l = 1, \dots, L+1, && \text{(8c)} \\
& \boldsymbol{\Phi} \succeq 0, && \text{(8d)} \\
& \mathrm{rank}(\boldsymbol{\Phi}) = 1 && \text{(8e)}
\end{aligned}
$$

Despite being convex, problem 8 is not a standard SDP due to rate constraint 8b, and there is no proof that the loss from SDR would be negligible. In other words, we only obtain $\boldsymbol{\Phi}^\star$.

$$
\begin{aligned}
\min_{\boldsymbol{X} \in \mathbb{S}^n} \quad & \mathrm{Tr}(\boldsymbol{C}\boldsymbol{X}) \\
\text{s.t.} \quad & \mathrm{Tr}(\boldsymbol{A}_i\boldsymbol{X}) \trianglerighteq_i b_i, \quad i = 1, \dots, m, \\
& \boldsymbol{X} \succeq \boldsymbol{0}, \quad \mathrm{rank}(\boldsymbol{X}) = 1.
\end{aligned}
$$

Simulation results

For SISO only, simulation show that $\mathbf{\Phi}^{\star}$ is (always almost) rank-1, and $\phi^{\star}$ extracted by Gaussian randomization method achieves the same performance as $\mathbf{\Phi}^{\star}$.



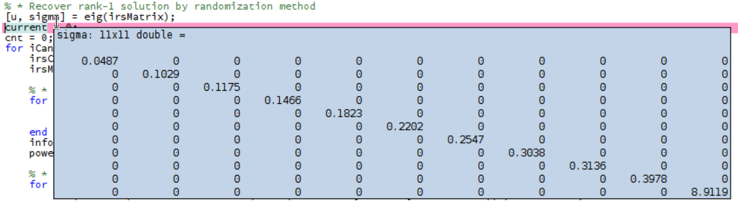
```
% * Recover rank-1 solution by randomization method
[u, sigma] = eig(irsMatrix);
```

sigma: 11x11 double =

0.0000	0	0	0	0	0	0	0	0	0	0
0	0.0000	0	0	0	0	0	0	0	0	0
0	0	0.0000	0	0	0	0	0	0	0	0
0	0	0	0.0000	0	0	0	0	0	0	0
0	0	0	0	0.0000	0	0	0	0	0	0
0	0	0	0	0	0.0000	0	0	0	0	0
0	0	0	0	0	0	0.0000	0	0	0	0
0	0	0	0	0	0	0	0.0000	0	0	0
0	0	0	0	0	0	0	0	0.0000	0	0
0	0	0	0	0	0	0	0	0	0.0000	0
0	0	0	0	0	0	0	0	0	0	11.0000

Figure: Eigenvalue of $\mathbf{\Phi}^{\star}$ for Tx $= 1$

For MISO this is not the case.



```
% * Recover rank-1 solution by randomization method
[u, sigma] = eig(irsMatrix);
```

sigma: 11x11 double =

0.0487	0	0	0	0	0	0	0	0	0	0
0	0.1029	0	0	0	0	0	0	0	0	0
0	0	0.1175	0	0	0	0	0	0	0	0
0	0	0	0.1466	0	0	0	0	0	0	0
0	0	0	0	0.1823	0	0	0	0	0	0
0	0	0	0	0	0.2202	0	0	0	0	0
0	0	0	0	0	0	0.2547	0	0	0	0
0	0	0	0	0	0	0	0.3038	0	0	0
0	0	0	0	0	0	0	0	0.3136	0	0
0	0	0	0	0	0	0	0	0	0.3978	0
0	0	0	0	0	0	0	0	0	0	8.9119

Figure: Eigenvalue of $\mathbf{\Phi}^{\star}$ for Tx $= 2$

R-E plots

We optimize $(\boldsymbol{w}_{I/P,n}, \rho)$ and ϕ alternatively. IRS is always optimized by SDR, and the plots compare waveform optimized by GP and SDR.

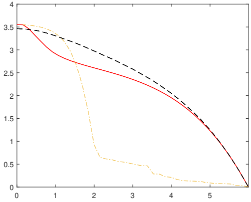


Figure: GP (black) vs SDR

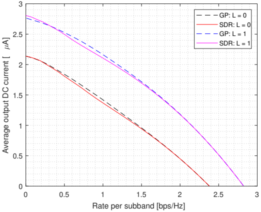


Figure: No IRS vs single reflector

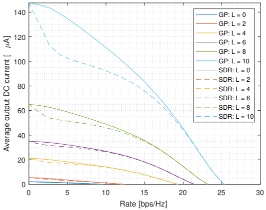


Figure: Number of reflectors

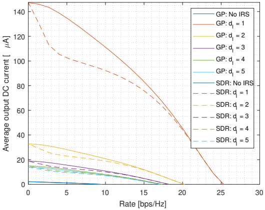


Figure: AP-IRS distance

Problem

- Although $\mathbf{\Phi}^\star$ turns out rank-1 for SISO, we have no idea on its demonstration
- Even if we rewrite sum-rate constraint 8b as quadratic form $\mathrm{Tr}(\boldsymbol{C}\mathbf{\Phi})$, the approximation accuracy γ requires further check (due to this additional constraint)

Complex constant-modulus QP	For $\boldsymbol{C} \succeq \mathbf{0}$,
$$\begin{array}{ll} \max\limits_{\boldsymbol{x} \in \mathbb{C}^n} & \boldsymbol{x}^H \boldsymbol{C} \boldsymbol{x} \\ \text{s.t.} & \lvert x_i \rvert^2 = 1,\ i = 1, \ldots, n \end{array}$$	$$\gamma = \pi/4 = 0.7854.$$ Remark: coincide with complex k-ary QP as $k \to \infty$.

Figure: Approximation accuracy for SDP with magnitude constraint only

Thank you

Thank you.