Compressed Sensing for Wideband Signal Processing

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Abstract

Signal processing have been widely deployed in many computational applications in areas such as image processing, video conferencing, smart sensors network, and other real-time processing technologies. Many signal processing systems also involve signal sampling and conversion, where one of the most crucial limitations lies in the sampling theory (the Nyquist sampling rate or Shannon theory, which states the minimum two time sampling rate requirement.) Due to limited data processing and storing capability found in many wireless communication systems, excessive data samples will negatively affect their performance and power efficiency, and hence forms the bottleneck of entire digital systems.

As a novel paradigm for signal processing and acquisition, the compressive sensing or compressed sensing (CS) technique, is identified to have great potential for resolving the problems mentioned above. The CS theory has been proven to be suitable for numerous computer science and electronic engineering based applications, where it is possible to overcome the limitation of the traditional sampling theory.

The main idea of the research work is to further extend the CS based techniques to wireless systems. Using this approach, the novel systems will be proposed that encompass various areas such as data compression, data acquisition, data storage, data transmission, optimal recovery and processing. Specifically, we stress the focus on the wideband signal processing since the requirement of sampling wideband spectrum generates comparatively higher pressure than narrow band signal processing, hence dealing with wideband acquisition and processing is full of importance and necessary.

Following this idea, in this report, we focus our research aims to the areas involving extremely high frequency acquisition and processing scenarios – the ultra wideband (UWB) positioning and cognitive spectrum sensing (CSS). For many ultra wideband (UWB) communication systems, how to receive the signal over GHz at lower cost is a crucial problem.

For cognitive radios (CR), how to efficiently sense the occupation of active primary users (PU) while not creating interference to PUs is still an important and open issue.

In order to solve these problems, this report studies novel approaches which respectively embed the CS framework into those systems. In our proposed CS-UWB system, random projection is located at transmitters to support CS framework while sub-Nyquist low-rate ADCs are implemented at receivers. As a result, not only the accuracy is improved but also the sampling cost is significantly reduced. On the other hand, we study the recent CS-CSS systems' performance and analyize the cost of embedding CS into such systems. The analysis result indicates us that rather than fully reconstructing the CS measurements, the partial recovery or extraction useful information directly from unrecovered signals is more attractive and suitable in many cognitive radio spectrum sensing cases since it improves the real-time capability while keeps low sampling rate in CS. This idea intrigues our future research directions and motivations aiming at compressive signal processing (CSP, including compressive measurements analysis and partial support recovery etc) for wideband signal processing in cognitive radios and ultra wideband systems.

The organization this report is presented as: Chapter 2 firstly presents the overview of compressed sensing theory and its applications, and then focuses on wideband signal acquisition hardware design which relates to our survey for recent CS-ADC architectures. Then the following chapter 3 presents our related work in compressive UWB positioning systems. What's more, the chapter 4 introduces the compressive spectrum sensing for cognitive radios, but points out the limitation due to CS fully reconstruction. Finally, the chapter 5 proposes our recommenced future research on compressive signal processing based cognitive spectrum sensing.

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Chapter 1

Introduction

This chapter firstly introduces the challenges in signal processing with respect of signal acquiring limitation due to Nyquist sampling theorem, and then turn to focus on the compressed sensing (CS) for solving the problem. Next, potential CS based wireless applications, especially those for wideband signal processing, are proposed. Besides, it is still important to point out that there still exist several gaps between CS theory and its hardware implementations involving constraints like energy-cost, non-ideal model mismatch, real-time capacity. These gaps motivate our investigations and future research. At last, the organisation of the entire report is presented.

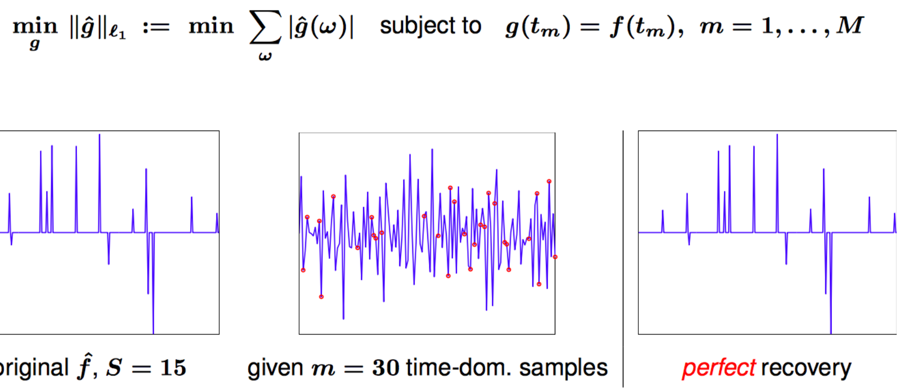
1.1 Challenges of Signal Processing

Signal processing is to processes or transfers information that contained in real world signals. The principle of signal processing is firstly founded by Alan V. Oppenheim and Ronald W. Schafer, and so far widely developed and related to modelling, analysis, extraction, learning, security etc [1].

Once Oppenheim and Schafer stated that the ”digitisation” can be applied in signal processing [2], signal processing is tightly linked with digitisation, for it can provide additional benefits such as compression and error detection [3]. Consequently, the digital signal processing (DSP) algorithms and devices have been widely deployed in image processing, video conferencing, smart sensors network, and other real-time processing tasks. Its applications are also increasingly implemented in the form of embedded systems such as mobile and wireless devices [4], aerospace equipment, and biomedical instruments [5].

However, during the digitisation, signal acquiring and quantification are always highly restricted by the sampling rate, because the famous Shannon theory states that the minimum sampling rate should be at least twice as the most highest frequency components of original signals (we also call it the Nyquist rate). As a result, DSP devices have to match the high sampling rate requirements in their input front-end. What's worse, because the input front-end of DSP is the first stage of the entire system, the limitation of sampling rate correspondingly influences the further stages for data processing and storage.

Therefore, the limitation in signal sampling not only increases the design complexity to analog-to-digital converters (ADCs), which is the input front-end of DSP, but increases the data size of storage, as well as the scale for further data processing. Consequently, the power consumption, computational cost, and commercial design cost become raising. What's worse, as the trend of developing DSPs requires faster sampling, larger dynamic range, higher dimensional data, low-energy consumption and high sensing mobility [6], if we cannot solve the sampling limitation, then not only sampling itself, which relates to faster sampling, larger range and sensing mobility, but also burden storage for high dimensional data and energy saving for low-energy consumption, will be significantly affected. In one word, all requirement refers to the key problem of sampling. From this aspect, solving the limitation in signal sampling is by no means tolerable for modern signal processing devices.



1.2 Compressed Sensing Overview

Fortunately, the compressed sensing (CS) presents us an useful approach to extract sparse interests of signal BELOW the Nyquist rate. The CS framework provides high possibility to reconstruct original signals through few randomly sampled data than traditional Nyquist sampling method. Using the l1-norm minimisation, figure 1.2 demonstrates the recovery results between CS reconstruction and conventional recovery. From the comparison, we can conclude that the CS successfully recover the sparse spectrum information from the original signals. The following paragraphs presents related topics for CS.

Sparsity The goal of signal processing is to reconstruct the original signal from a stream of samples. In the procedure of reconstruction, fewer samples of information is needed if the prior knowledge of signals frequencies is clear. By appropriate representation, e.g. Fourier basis representation, natural signals are always represented by few significant coefficients, and termed as sparse information. Mathematically, the sparse information can be presented if assuming the signal of $x \in R^N$ can be decomposed by a group of basis Ψ, where $k << N$:

$$x = \Psi s = \sum_{l=1}^{K} \psi_l s_l \tag{1.1}$$

Here x is a linear combination of K basis chosen from Ψ, and s is the corresponding coefficients of representing x in the domain constructed by the basis Ψ.

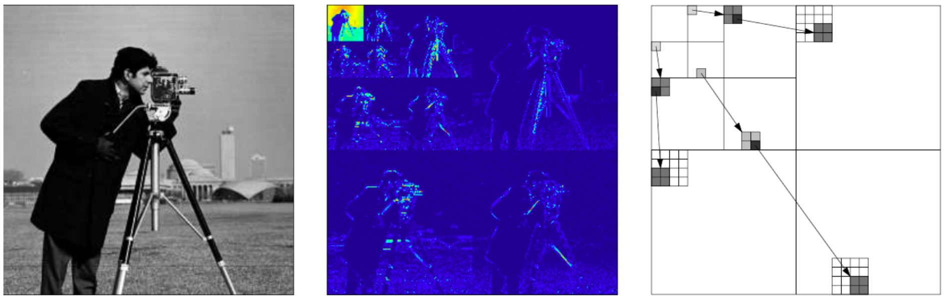


Figure 1.1: An example of Haar-wavelet decomposition. Only light coefficients are significant. Dark ones are nearly zero and can be neglected.

Incoherence The incoherence presents the idea that signals x containing a sparse representation in basis Ψ must be spread out in the basis domain in which they are acquired [7]. For instance, the spike is spread out in the frequency domain. In other words, the incoherence indicates the sampling/sensing waveforms have an extremely dense representation in the basis.

Relevance Similarly, the wavelets compression [8] e.g JPEG2000 also develops the sparse information in natural images. It represents images into approximately sparse and ignores large amount of small (nearly zero) coefficients. This is the reason why the wavelets only remain small size data while keeping the image with high information. For instance, using wavelets, natural images can be compressed into a relative small size (¡ 5 percent of original size) while still keeping high quality. An example is demonstrated in figure 1.2.

According to the principles of wavelets, the steps of this approach can be described as sample then compress. However, in the step of sampling, the Nyquist rate is still required. This is the main difficult between CS and wavelets, since the CS can overcome the limitation provided by the Nyquist rate.

Develop of CS Around 2005, Terence Tao, David Donoho, Emmanuel Cands mathematically proved that using random selection, the low-rate sampled measurements can reconstruct original data with high possibility, using $l1$-norm minimisation [9, 10] shown as equation 1.2.

$$\hat{s} = \arg\min \|s\|_1 \quad s.t. \quad y = As \tag{1.2}$$

, where it has been proven that only $O(K \ln(N/K))$ samples are needed if matrix A are incoherent with s. One of the best choice for the independence is to use i.i.d variables (independent and identically distributed variables) constructed matrix such as the Gaussian matrix. Further, more constructed matrix like Bernoulli matrix, partial Fourier matrix are proven suitable for CS. Therefore, CS provides a novel random sampling paradigm which is applicable for modern signal sampling and data processing with low rate.

1.3 Typical Compressed Sensing Applications

Compressive Radar Systems Compressed sensing has showed outstanding feature in in reducing the sampling rate in many wireless applications, e.g. radar systems, since many wireless devices communicate at high frequency which lead to heavy burden in sampling task so that the CS can be used to reduce the sampling rate at receivers [11]. In addition, in radar systems, it's necessary to have capacity to acquire wideband signals with different signal templates, such as TV bands, satellite signals, cell-phone informations etc. Traditional method always require filter-banks to down-sample these signals, while the CS can help original system directly sense the various of wide-band signal, or with less filters. The figure 1.3 presents a wide-band signal monitoring and processing radar system that receives signals from a variety of sources.

Compressive Single Pixel Camera Different from the wavelets, compressed sensing requires little samples which reduce the energy during image acquisition. For instance, the CS framework is implemented for single-pixel cameras by Rice University [12]. This new CS-camera directly acquires random projected samples from a image without first gain the

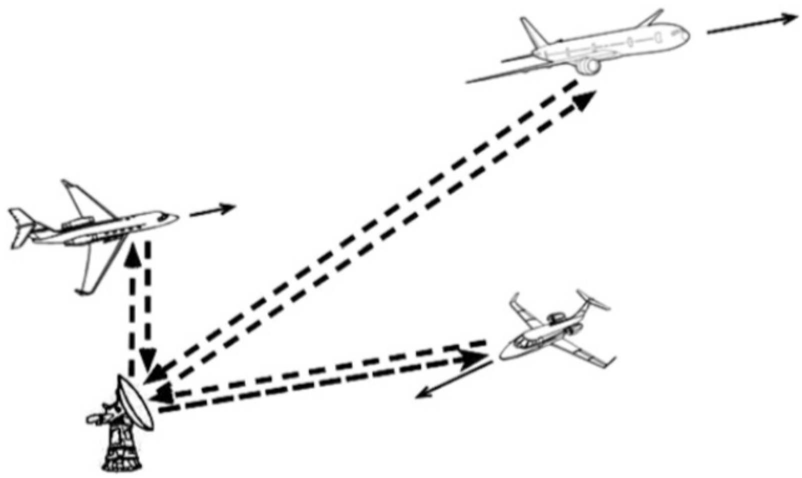


Fig. 1.7 Schematic illustration of a radar device measuring distances and velocities of objects

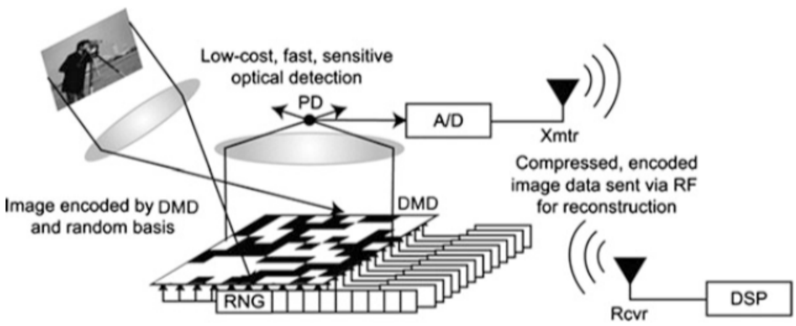


Fig. 1.5 Schematic representation of a single-pixel camera (Image courtesy of Rice University)

pixels, and the random projected samples are collected by digital micro-mirror array, which optically reflects a linear projection of the image onto pseudo-random binary patterns.

Magnetic Resonance Imaging Magnetic resonance imaging (MRI) is a widely used technology in medical imaging, applications such as brain scanning, angiography, and dynamic heart imaging are necessary for medical diagnosis [13]. However, this imaging tool burdened by an inherently slow data acquisition process since traditional approaches based on the Shannon sampling theorem requires long-time scanning if the high-resolution images are required [14]. In the cases that the patients cannot be expected to hold statically for

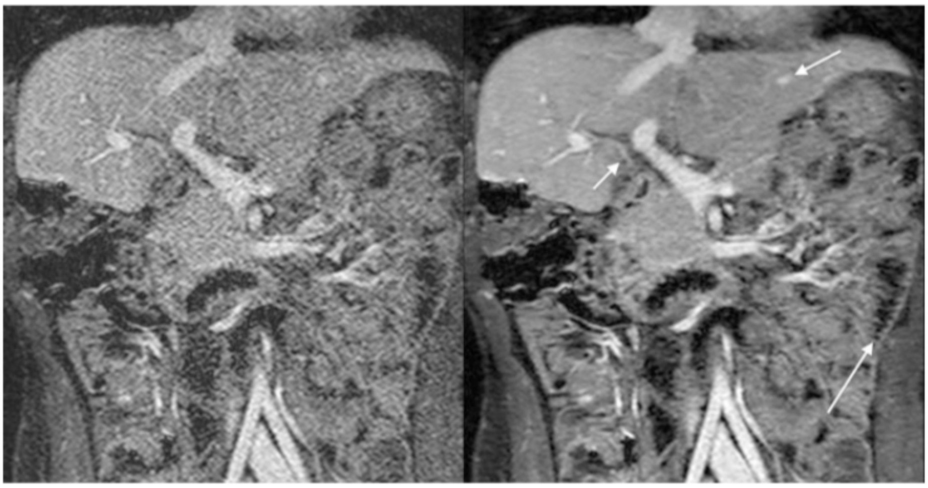


Fig. 1.6 Comparison of a traditional MRI reconstruction (*left*) and a compressive sensing reconstruction (*right*). The pictures show a coronal slice through an abdomen of a 3-year-old pediatric patient following an injection of a contrast agent. The image size was set to $320 \times 256 \times 160$ voxels. The data were acquired using a 32-channel pediatric coil. The acquisition was accelerated by a factor of 7.2 by random subsampling of the frequency domain. The *left image* is a traditional linear reconstruction showing severe artifacts. The *right image*, a (wavelet-based) compressive sensing reconstruction, exhibits diagnostic quality and significantly reduced artifacts. The subtle features indicated with *arrows* show well on the compressive sensing reconstruction, while almost disappearing in the traditional one (Image courtesy of Michael Lustig, Stanford University, and Shreyas Vasanawala, Lucile Packard Children's Hospital, Stanford University)

Figure 1.2: The comparison of a traditional MRI reconstruction (left) and a compressive sensing reconstruction (right).

long-period (e.g. children are too impatient to sit for more than minutes). The embedding CS to MRI has the potential to significantly reduce the scan time.

1.4 Compressive Wireless Communication

Over the past few decades, the demand for high frequency communication and wideband signal detection keeps increasing, which worsen the acquiring the situation. For instance, modern digital systems often require bigger data size, or higher rate communication (e.g. ultra-wideband), or more flexible networks (which emerges hybrid wireless devices, e.g. cognitive radio), and all the requirements highly rely on accurate and sensitive sampling approaches. Thus, enhancing the performance of sampling devices brooks no delay.

Therefore, our study motivation derives from the problem of signal acquisition devices, especially for those who aims at collecting wideband / high frequency signals. An suitable research object is the wireless application, for many applications are communicating at high frequency, and becoming more and more depend on high performance receivers and detectors, as well as the novel sampling approaches. Fortunately, the compressed sensing (CS) presents us an useful approach to extract sparse interests of signal BELOW Nyquist rate. This novel approach motivates a large range of application in signal processing. The next section will introduce this method.

In fact, although the CS framework solve the problem in Nyquist sampling rate theoretically, the gap between hardware applications and ideal cases still remain. For instance, as the CS theory often requires a true randomness in sampling approaches, while hardware implementations only provide pseudo-randomness instead. The constraints doesn't only stay in CS theory implementation, but design limitations or awareness such as energy consumption, real-time capability, design complexity etc. All the constraints bring about tradeoffs design schemes, so CS applications for wireless, signal processing is still an open issue.

1.4.1 Research Aims

As an important application area for CS, wireless communication is always puzzled by high frequency transmission detection or wideband signal processing, e.g. radar systems. Those applications, to our best knowledge, are mainly focus on high frequency required scenarios, including multiple-input and multiple-output (MIMO), multiple access (MA), ultra-wideband (UWB), cognitive radio (CR).

In this report, we are more focusing on CS based wireless communication, especially aims at the physical layer and MAC layer. For research applications, we select two typical and popular applications, which involves extremely high-frequency and wide bandwidth signal processing respectively – the ultra-wideband system and cognitive radio.

Compressive Ultra-Wideband Positioning The Ultra-Wideband (UWB) communication is widely used in wireless communication and associated with features as extreme wide transmission bandwidth, low-power consumption, shared spectrum resources in wide ranges etc. However, for many ultra wideband communication systems, how to energy-efficiently collect signals higher than GHz is a crucial problem. In order to solve these problems, we study the novel approach that embeds CS into the UWB positioning. In our proposed compressive UWB positioning system, random projection is located at transmitters to support CS framework while sub-Nyquist low-rate ADCs are implemented at receivers. As a result, not only the accuracy is improved but also the sampling cost is significantly reduced. The detailed design is introduced in chapter 3.

Compressive Cognitive Radio Spectrum Sensing Cognitive Radio (CR) has been attracting many attention due to its potential better utilisation for limited spectrum resources. However, for many cognitive radios, how to accurately and quickly sense the occupation of active primary users (PUs) without additional interference is a crucial problem. In order to solve this problems, we study the novel approach in chapter 4 embedding CS into recent CR spectrum sensing techniques. Then we analyse the performance and cost for applying the CS. The analysis and discussion finally motivates us for future research on compressive signal processing (CSP) for cognitive spectrum sensing in chapter 5.

1.5 Problem Statement

In this chapter, we narrow the concerned problems to CS wireless hardware designs, mainly from two aspects: (1) Although the CS is outstanding for its significance in reducing the sampling rate, however, as a sacrifice, the reconstruction procedure is non-linear and full of complexity and difficulty. (2) Besides, CS sensing model hardware implementation is not easy and simple, for building absolute incoherent and independent sensing matrix is challenging (e.g. true randomness in Gaussian matrix can hardly be implemented in computer engineering field). The following problems open up some interesting issues for our research, and some of them are focused in this report and some will be studied in our future works.

Real-Time Capability The reconstruction algorithms for CS is non-linear, which indicates that the recovered data are not directly suitable for conventional digital signal processing where a simple linear recovery using cardinal sine interpolation is needed. Besides, some of the high accuracy reconstruction algorithms based on convex optimisation are time consuming, e.g. the basis pursuit's complexity is $O(N^3)$, which generates large time delay for further data processing, and do harm to the feed-back required devices. On the other hand, even though greedy method (e.g. the orthogonal matching pursuit's complexity is $O(kMN)$) has speed up the sunning time for CS reconstruction, the recovered data is located in different domain, e.g. original data is sampled in time domain while the recovered data in frequency or spatial domain. The domain mismatch sometimes require addition transforming task which generates more cost in time and energy. In one word, CS reconstruction make the real-time capability worse, and lead many CS techniques not suitable for feed-back required processing systems.

Processing Flexibility Higher flexibility is one of the important trend and requirement for modern signal processing devices. In contrast to the task-specific hardware used in many classical acquisition systems, many hardware designed to use a compressive measurement protocol can be extremely flexible. A stylised future CS radar system demonstrate the potential signal processing pattern for wide-band signals: In figure 1.3, the CS radar receives various types of signals from televisions, cellphones, aeroplanes and satellites etc. However, the signals' templates are unknown to the system, such as signal-to-noise ratio and sparsity order which seriously affect the performance of detectors. Then what information should be collected ? How to accomplish matched filters ? Is adaptive selection needed ? These solutions sometimes computational expensive but not flexible enough. What's worse, the CS reconstruction is non-linear which does not match the traditional processing approaches and thus increase the analysis difficulty.

Energy Efficiency Compressed sensing community has provided many hardware architectures for analog-to-digital information to build sub-Nyquist rate analog-to-digital converters (ADCs), such as random demodulator (RD) [15] and modulated wideband converter (MWC) [16] mentioned in chapter 2. In facts, however, most CS based ADC architectures do NOT eliminate Nyquist rate in entire systems – although the sampling rate reduced, the mixing rate for random projection remains Nyquist rate. Besides, some CS framework involves random filtering, however, the randomised coefficients for filter's impulse response is hard to implement. In addition, aforementioned CS non-linear reconstruction cost still suffers the entire system in energy consumption. Therefore, concerning the design complexity and energy cost and correspondingly making acceptable tradeoffs is a necessary topic.

Sensing Matrix Construction Different from the Nyquist sampling approaches which samples at uniform time grid, the compressed sensing framework acquires data relying on the inner product [17]. Consequently, the different sampling approach generates different implementations in ADC design. However, the CS sensing model hardware implementation is not easy and simple, since building absolute incoherent and independent sensing matrix is challenging. For instance, true random variable comprised sensing matrix can hardly be implemented in computer engineering field. Consequently, approximations for CS theory to CS implementation are often required, such as using pseudo-randomness, or using less independent sensing matrix etc. As a result, the non-ideality of the sensing sampling in hardware design generates in-negligible errors.

1.6 Major Contributions

This report studies novel energy-aware and real-time ability improved approaches for embedding the CS framework into cognitive radios (CR) and ultra-wideband (UWB) systems. The brief introduction about the contributions are shown as follows:

Implementation for energy-aware design in compressive UWB positioning The compressed sensing is proven effective to improve the accuracy of IR-UWB positioning. However, most of papers do not concern the design complexity and energy cost in their implementations, for instance, the random mixing waveform is of extremely high frequency (in GHz). Our proposed compressive UWB positioning present a novel view of energy tradeoff design, which is implemented by a low-rate random-projection at transmitters and low rate ADCs at receivers. It is clear that this design significantly reduce the peak frequency in the system with only acceptable rate increase in receivers' ADC sampling rate.

Exploration for direct signal processing in compressive detector in CR The compressed sensing does not directly match the traditional processing algorithms (section 1.5). However, in cognitive radio, most of tasks for spectrum sensing are related to detection, estimation, filtering, classification. This contradiction leads the large additional loss in energy usage and time utilisation when we applies the CS to reduce the sampling limitation in wideband sensing for CR. Then noticing the fully reconstruction is always not needed, we discuss the future direction in directly processing compressively sampled data in CR, which aims at extracting effective information without fully recovery so that the entire real-time capability and energy efficiency will be significantly increased.

1.7 Organisation

The main idea of the research work is to further extend the CS based techniques to wideband signal processing systems for wireless communication, in order the solve the limitation of the sampling rate. Using this approach, a novel CS-based framework for signal processing will be proposed that encompass various areas such as data compression, data acquisition, data storage, data transmission optimal recovery and processing, and also targeted for embedded deployment. In addition, since embedded signal processing system are popular and suitable in many wireless network applications such as battery supplied mobile devices for wireless communication. The rest of this report is organised as follows:

Chapter 2 firstly presents the overview of compressed sensing theory and its applications, and then focuses on wideband signal acquisition hardware design which relates to our survey for recent CS-ADC architectures. Then the following chapter 3 presents our related work in compressive UWB positioning systems. Whats more, the chapter 4 introduces the compressive spectrum sensing for cognitive radios, but points out the limitation due to CS fully reconstruction. Finally, the chapter 5 proposes our recommenced future research on compressive signal processing based cognitive spectrum sensing.

Chapter 2

Compressed Sensing and Low Rate ADC

In this chapter, we first introduce basic compressed sensing (CS) theory including sparse representation, incoherence, and reconstruction. Then we focus on how to build CS based ADC architectures, and presents our survey work for recent CS-ADCs. This work establish a good preparation for future CS applications for wireless devices. Later on, some general hardware architectures for CS reconstruction and processing are introduced. At last, a brief introduction of future CS application for wireless communication is presented.

2.1 Introduction

In today's big data era, signal processing devices are always perplexed by its heavy power and storage cost due to the size and sampling requirements. The sampling requirements are mainly due to the Shannon sampling theory which states that the minimum sampling rate should be at least twice as the most highest frequency components in original signal (we call it the Nyquist rate). As a fact, the limitation in signal sampling not only increases the design complexity to analog-to-digital converters (ADCs), but increases the data size of storage, as well as the scale for further data processing. Consequently, the power consumption, computational cost, and commercial design cost become raising. What's worse, if we cannot solve the sampling limitation, then not only sampling itself, which relates to faster sampling, larger range and sensing mobility, but also burden storage for high dimensional data and energy saving for low-energy consumption, will be significantly affected. In conclusion, all requirement refers to the key problem of sampling. From this aspect, solving the limitation in signal sampling is by no means tolerable for modern signal processing devices.

Since acquisition approach is crucial, many recent researches aim at how to select useful interest of signals more efficiently. Examples such as wavelet [8] have significantly reduced the compressed size, and extracted very sparse information from original data. However, before we can extract such interests, the Nyquist rate of sampling is STILL required, thus the heavy burden in ADCs could NOT be neglected. In other words, the limitation from Nyquist rate does not eliminated for hardware devices.

Fortunately, the compressed sensing (CS) presents us an useful alternative to extract sparse interests of signal BELOW Nyquist rate [5]. The CS framework provide high possibility to reconstruct original signals through randomly sampled measurements, and the sampling rate is far below the Nyquist rate.As a result, the CS framework is becoming popular in many high frequency or wideband signal acquisition required systems and devices.

2.2 Compressed Sensing Theory

This section mainly introduces the theory in compressed sensing.

2.2.1 Compressed Sensing Paradigm

Compressed sensing (CS) announces that sparse informations can always be reconstructed from far fewer samples than traditional method of Nyquist Theorem. Assume the sensing system is $y = \Phi x$, where $y \in R^m$ is the observation, $\Phi_{m\times N}$ is the sensing matrix, and $x \in R^N$ is the original signal to be reconstructed. Here $m << N$ which implies the sampling rate is relatively low. The following paragraphs in this section briefly introduce the CS framework.

Sparse representation Consider a task of sampling a signal of x using sub-Nyquist rate. Suppose that a group of basis Ψ provides a K-sparse representation of signal x as following, where $k << N$.

$$x = \Psi s = \sum_{l=1}^{K} \psi_l s_l \tag{2.1}$$

Here x is a linear combination of K basis chosen from Ψ, and s is the corresponding coefficients of representing x in the domain constructed by the basis Ψ.

Reconstruction Then Candes, Romberg and Tao [9] showed that one could almost always recover the signal x exactly by solving the following $l1$-norm minimisation problem (convex program, equation 2.2):

$$\hat{s} = \arg\min \|s\|_1 \quad s.t. \quad y = \Phi\Psi s \tag{2.2}$$

Theorem 1. *This formulation is known as basis pursuit (BP). Assume that x is K-sparse and we are given m Fourier coefficients with frequencies selected uniformly at random. Suppose that the number of observations obeys*

$$m \leq C \cdot K \cdot logN \tag{2.3}$$

Then minimizing $l1$ reconstructs x exactly with overwhelming probability which exceeds $1O(N^{\delta})$.

Further, reconstruction from noisy measurements can be stated in a relaxed form as a basis pursuit de-noising (BPDN) [18] problem, which allows some measurement mismatch of $\epsilon > 0$:

$$\hat{s} = \arg\min \|s\|_1 \quad s.t. \quad \|y - \Phi\Psi s\|_2 \leq \epsilon \tag{2.4}$$

Restricted Isometry Property We also need means to quantify the robustness of CS under noisy and only approximately sparse conditions. The most prominent criterion for is the restricted isometry property (RIP) in [19]. This property essentially requires that every set of columns of the sensing matrix behaves like an orthonormal system. Namely, the RIP measures how well distances are preserved in a linear transformation. A matrix A fulfills the RIP with restricted isometry constant δ_K if

$$(1-\delta_k)\|s\|_2 \leq As \leq (1+\delta_k)\|s\|_2 \tag{2.5}$$

Candes et al. [20] introduces an error bound for the solution of BPDN equation 2.4 under the condition that $\delta_{3K} + 3\delta_{4K} < 2$ and that the error $\|e\|_2 < \epsilon$:

$$\|x-\hat{x}\|_2 \leq C_1\epsilon + C_2\delta_K(x)_1/\sqrt{K} \tag{2.6}$$

where the C1 and C2 are small constant, and $\delta_K(x)_1$ is the minimal $l1$-error introduced by the best possible K-sparse approximation.

Incoherence A criterion related to the RIP is the incoherence of the projection Φ and the sparsifying basis Ψ. The mutual coherence is defined as:

$$\mu(\Phi,\Psi) = \max_{i\in[1,m];j\in[1,N]} |<\phi_i,\psi_j>| \tag{2.7}$$

The less coherent the two matrix Φ and Ψ are, the better the reconstruction works. The greater the incoherence of the measurement/sparsity pair (Φ,Ψ), the smaller the number of measurements needed.

Sensing Matrix In order to successfully recover the s, we have to compose and construct sensing matrix A by Φ and Psi to satisfy the RIP. However, calculating the restricted isometry constant of a given matrix A is itself a problem of combinatorial complexity. Hence, artificially designing and testing the RIP of a sensing matrix is difficult. Fortunately, random variable composed matrix, whose entries are independent and identically distributed (i.i.d) variables, are very likely satisfies the requirements [21]. Besides, if the projection matrix Φ's variables are i.i.d and basis matrix Ψ is Fourier, or wavelets alike matrix, then the overall sensing matrix $A = \Phi\Psi$ still keeps high probability to achieve a successful recovery (similar to the conclusion in theorem 1). What's more, as long as the incoherence of Φ and Ψ great enough, exactly reconstruction from equation 2.4 is possible. As researches in projection matrix Φ develops, sub-Gaussian distribution measurement, Bernoulli measurement, random partial Fourier measurement, random Toeplitz measurement [22] are all proven suitable for CS sampling and reconstruction, and widely used for constructing the measurement in CS applications.

2.2.2 Reconstruction Algorithms

The $l1$-minimization plays an important role in successfully designing computationally stable CS reconstruction. This approach can be treated as convex optimization such as basis pursuit [18] and Bregman [23]. However, although they recover signal with lower average error, but they are computationally burdensome. On the other hand, some other sort of algorithms recover sparse signal termed combinatorial algorithms including chaining pursuit [24] and HHS pursuit [25] can achieve extremely rapid computational speed for sparse signal reconstruction but seems to be sensitive to noise as they strongly rely on the group testing of highly structured samples.

Compared to convex optimization and combinatorial algorithms, greedy pursuit algorithms particularly are intermediate of the fast running time and sampling efficiency, and become a better trade-off between speed, stability and robustness. Popular greedy pursuits include OMP [26], CoSaMP [27], HTP [28] etc. In recent CS based signal acquisition and reconstruction systems where both time saving and recovery precision are crucial, hence greedy pursuits are widely utilized. The following subsections introduces prevailing greedy pursuit algorithms.

Orthogonal Matching Pursuit Orthogonal matching Pursuit (OMP) [29] is a widely used for sparse reconstruction which develops the process of matching pursuit. As shown in algorithm 1, it progressively manages to find the support of the unknown sparse signal: Give an acquisition system $y = As$ where $s \in R^N$ is K sparse and CS measurement $A \in R^{m\times N}$ $(m << N)$, each iteration of OMP selects one support λ of the vector s which contributes the most to the observation y. This selection method is based on testing the correlation values between the current columns of A and residue r. The OMP iteration would not stop before the residue r becomes relatively small.

Algorithm 1: Orthogonal Matching Pursuit(OMP)

input : measurement $A \in R^{m\times N}$, observation y.

output: recovery $\hat{s}$.

$r_0 \leftarrow y$; $\Lambda_0 \leftarrow \Theta$; $i \leftarrow 0$; ;

while $r_i \geq theshold$ **do**

  $h_i \leftarrow A^T r_i \quad //match$;

  $\Lambda_{i+1} \leftarrow \Lambda_i \cup \{\arg\max_{1\leq j\leq N} |h_i(j)|\} \quad //identity$;

  $\hat{s}_{i+1} \leftarrow \arg\min_s \|y - A_{\Lambda_{i+1}}\hat{s}_i\|_2 \quad //determine$;

  $r_{i+1} \leftarrow y - A\hat{s}_{i+1} \quad //update$;

end

The computational complexity of OMP is $O(KmN)$ which is significantly smaller compared to classical convex optimization such as basis pursuit whose complexity is $O(N^3)$ (in case that sparsity of $K << N$) [30]. However, the robustness of OMP cannot reach the quality level of traditional convex optimization, since searching local optimal solutions instead of global solutions brings more errors to OMP.

Compressed Sensing Matching Pursuit In order to improve the speed and robustness of OMP, Compressed Sensing Matching Pursuit (CoSaMP) [27] is developed. It aims at producing a more effective way for detecting the support of input signals shown in 2: The CoSaMP firstly find $2K$ indices for maximal correlation between columns of A and the current residue r by using the operator $H_{2K}(A^T r)$ to set all but the $2K$ largest components in $A^T r$ to zero. Then CoSaMP merges these $2K$ indices with the previous support from current recovered $\hat{s}$, in order to form a new support set λ for updating the least square solution of $\hat{s}$. In the next step, the least squares solution is pruned and consequently only the K largest components are preserved.

Algorithm 2: Compressed Sensing Matching Pursuit(CoSaMP)

input : measurement $A \in R^{m \times N}$, observation y, sparsity K.

output: recovery $\hat{s}$.

```
r_0 ← y; i ← 0. ;
while stopping the criterion do
    h_i ← H_2K(A^T r_i)    //match ;
    Λ ← supp(h_i) ∪ supp(ŝ_i)    //identity ;
    ŝ_{i+1} ← arg min_s ||y − A_Λ ŝ_i||_2    //determine ;
    (s_{i+1})_j ← 0 for j ∉ Λ_i ;
    r_{i+1} ← y − A ŝ_{i+1}    //update ;
end
```

Although other revised matching pursuits such as ROMP [31], StOMP [32] have been developed for improving the robustness of OMP. However, compared to those revise for OMP, CoSaMP offers the optimal performance as it works with a minimal number of observations and performs a better recovery robustness to noise [27].

Hard Thresholding Pursuit(HTP) Similar to CoSaMP, HTP enhances the stability and robustness by improving the efficiency for detecting the support set Λ from obeservation y, through considering larger range of correlations between residues r and measurement

A [33]. In this algorithm, each iteration of HTP performs a gradient descent aiming at the final solution in order to estimate k supports that contributes most to the observation y. Next, by solving a pruned least squares problem, HTP updates its solution $\hat{s}$ and residue r for the next iteration. The pesudocode of the algorithm is given in 3:

Algorithm 3: Hard Thresholding Pursuit(HTP)

input : measurement $A \in R^{m\times N}$, observation y, step size v, sparsity K.

output: recovery $\hat{s}$.

$r_0 \leftarrow y; i \leftarrow 0.$;

while *stopping the criterion* **do**

  $h_i \leftarrow \hat{s}_i + vA^T r_i \quad //gradientdecent$;

  $\Lambda \leftarrow supp(H_k(h_i)) \quad //identity$;

  $\hat{s}_{i+1} \leftarrow \arg\min_s \|y - A_\Lambda \hat{s}_i\|_2 \quad //determine$;

  $(s_{i+1})_j \leftarrow 0 \; for \; j \notin \Lambda_i$;

  $r_{i+1} \leftarrow y - A\hat{s}_{i+1} \quad //update$;

end

2.2.3 Performance of Greedy Pursuits

Among the prevailing greedy pursuit algorithms of OMP, ROMP, StOMP, CoSaMP, HTP, the CoSaMP and HTP outperform other greedy pursuits in terms of stability [33]. In addition, according to [34], by comparing the output Signal-to-Noise(SNR) in recovered sparse signals, it suggests that average SNR provided by CoSaMP and HTP are approximately very close to the traditional BP, which indicates that revised greedy pursuits such as CoSaMP finally reach the high quality level of robustness provided by the classical convex optimizations. Besides, most of the greedy pursuit algorithms remain lower computational complexity than convex optimizations. For instance, the complexity of ROMP, StOMP, CoSaMP and HTP are $O(KmN)$, $O(N\ln N)$, $O(mN)$ and $O(MN)$respectively, and all surpass that in convex algorithms such as BP ($O(N^3)$) [35]. In conclusion, the revised greedy pursuits preserves the fast speed of its own, and successfully develops higher stability and robustness. Consequently, they become suitable and widely implemented in CS applications involved fast sparse reconstructions.

2.3 Compressive ADC Architectures

Analog-to-Digital converter (ADC) is utilized in many consumer electronics products that interact with real-world signals which are predominantly analog in nature. The conversion of the analog signals involves the sampling process has long been bounded by the Nyquist theory which declares the minimum sampling rate for traditional acquisition methods, and couldn't reduced easily. In order to overcome this problem, recently many signal processing devices embed CS into analog-to-digital converters(ADCs) and successfully reduces the size of measurements and thus releases the limitation given by the Nyquist theory. As a consequence, the new systems using a relatively lower clock rate significantly outperform the traditional acquisition systems.

2.3.1 Random Demodulator

Random demodulator (RD) [15] is one of the most popular analog-to-digital converters for CS-based signal acquisition and processing. It develops the stability and robustness of CS measurement, and achieves a sub-Nyquist rate ADC for signal sampling and sequentially enhances the system power efficiency (especially the performance of front-end acquisition systems).

Figure 2.1 shows the construction of the random demodulator: The K-sparse input signal x is first mixed with the chipping sequence $p_c(t)$ which is a waveform constructed by pseudo-random variables of $\{\pm 1\}$. The mixed product component $x(t) \cdot pc(t)$ is then passed through an anti-aliasing low pass filter h(t), before being sampled at a uniform in-

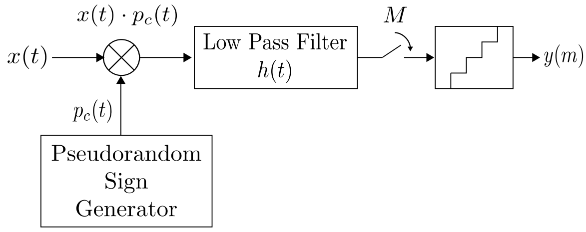


Figure 2.1: Block diagram of random demodulator (RD). The components includes a pseudo-random sign generator, a low-pass filter, and a sub-Nyquist ADC

terval but at a lower sampling rate of m which is of the order $(Klog(N/K))$ which is much lower than the normal required rate corresponds to N (since $K << N$). The most widely used reconstruction algorithms for RD are those based on OMP and CoSaMP greedy pursuit. Experimental results [15] demonstrates that the minimal sampling rate needed is only $1.7K(log(N/K))$ and in addition, it provides better SNR performance when compared with conventional Nyquist rate based ADCs.

2.3.2 Modulated Wideband Convertor

In the cases of sampling a multi-band signals whose carrier frequencies are unknown or time-variant (blind multi-band signal receiving), the main task is to design a receiver working independently on the carrier frequency at a low rate [36]. Recently, a novel architecture termed modulated wideband converter (MWC) is developed, which applies the CS theory to the traditional blind multi-band signal receivers based on non-uniform sampling [37]. This innovative architecture successfully provides a minimal requirement for the size of observations [38] thus decrease the power consumption. Besides, compared to the RD, MWC provides robustness against the noise and model mismatches.

The MWC consists of groups of periodic waveforms, low pass filters and sub-Nyquist rate ADCs, and can be treated as a parallel structure of the random demodulator (RD). As shown in Figure 2.2, each group of periodic sequence $p_i(t)$ with minimum interval of

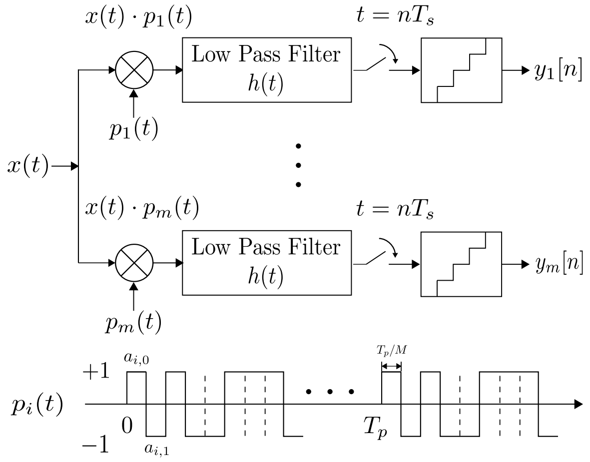


Figure 2.2: Block diagram of modulated wideband convertor (MWC). The components includes parallel periodic waveforms mixers, low-pass filters, and sub-Nyquist ADCs

T_p/M mixes the input multi-band signal $x(t)$ for shifting the spectrum by Δf_p ($f_p = 1/T_p$ is the frequency of $p_i(t)$) and filtering it to the baseband for sampling. In the end, low rate ADCs sample the mixed productions at a rate f_s far less than the Nyquist rate f_{NYQ}.

Reconstruction is done by subspace detection via continuous to finite block (CTF) [36]. The CTF block is comprised of frame construction, and the multiple measurement vectors (MMV) that can be solved by greedy pursuits based algorithms. The signal reconstruction can be achieved by a direct pseudo-inverse operation based on results of the CTF block.

2.3.3 Non-Uniform Sampling

Apart from the modulated based CS architectures such as random demodulators and modulated wideband converters which uniformly samples the mixed input analog signals at a slow rate, another novel CS architecture – the non-uniform sampling which is based on the theory of information recovery from random samples [39] develops recently. This architecture aims at sampling signals in a local Fourier sparse representation [21] (LFS) (e.g. wideband signals), and provides an non-uniformly sampling pattern at pseudo-random

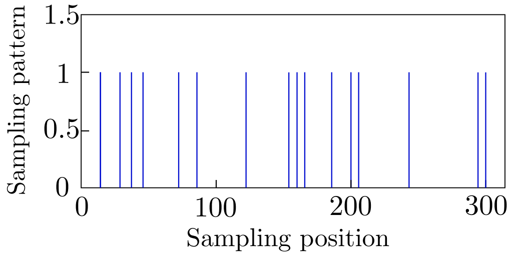


Figure 2.3: A pseudo-randomly sample pattern for single pure tone signal of length 300

time points with a low average rate. An example of this sampling pattern is shown in Figure 2.4.

The matrix multiplication between F and s stands for the input signal x which contains sparse spectrum s, and F is the full discrete time Fourier transform (DFT) matrix. The diagonal matrix D represents the behavior of non-uniform sampling, where the value ϵ of diagonal items are chosen pseudo-randomly from $\{0, 1\}$.

The NUS based CS acquisition model establishes a sensing matrix A which equal DF and can be regarded as a random partial Fourier matrix F_T which consists of randomly chosen columns of the discrete Fourier matrix (DFT) and indexed by T. According to [39], the matrix A satisfies the RIP in compressive sensing so the acquisition model produces a stable reconstruction of $\hat{s}$ via $l1$-minimisation using $m \geq O(slog(N/s))$ samples [39]. In addition, many greedy algorithms such as OMP and CoSaMP are also implemented as fast reconstruction for this architecture [40].

2.3.4 Compressive Multiplexer

Common implementations of the CS based non-uniform sampling architectures varies from [39–42]. Among these implementations, the random sampling compressive sensing ADC (RS-ADC) is likely to be the prevailing design shown in Figure 2.4. This architecture is comprised of input multiplexers (MUX), analog queues (AQs), demultiplexer (DEMUX) and a low-rate successive ADC.

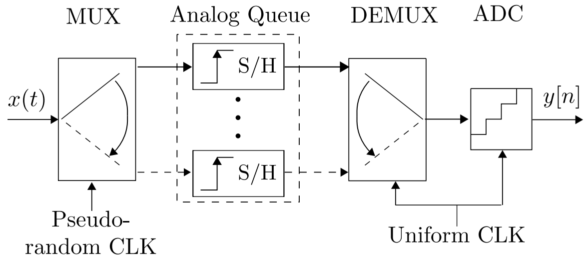


Figure 2.4: Block diagram of the random sampling CS ADC (RS-ADC). The components includes a pseudo-random sign generator, a low-pass filter, and a sub-Nyquist ADC

This implementation uses an input multiplexer driven by a non-uniform clock to switch the signal among several parallel S/H based analog queues. A low rate ADC (e.g. Successive Approximation ADC) is used to convert the stored samples, performing at uniform intervals but operating at sub-Nyquist average rate. Greedy pursuit algorithms such as OMP and CoSaMP are then used to perform fast reconstruction for this architecture. However, the main problem of applying RS- ADC lies in sampling high frequency signals: since the ADC and input MUX perform inherent bandwidth limitations modeled as a lowpass filter preceding the uniform sampling, acquisitions for high frequency signals results in a loss of the spectrum components. Besides, the high switching speed of the MUX increases noise and reduces the power efficiency.

2.4 Conclusion

The Nyquist sampling theorem states that signal should be sampled at least two times faster than the signal bandwidth so that the aliasing would not occur. However, modern applications produce a large amount of data, resulting in large numbers of samples which requires high speed sampling. To overcome this problem, the compressive ADC architectures are studied in this chapter. First we present the theory of CS framework, then a comprehensive survey of the novel compressive ADC (CS-ADC) designs are studied.

Major types of CS-ADCs, such as random demodulators, modulated wideband converters, non-uniform samplers, and compressive multiplexer are explained in detail.

Chapter 3

Compressive UWB Positioning

In this chapter, we first present an overview of UWB communication, and then focus on on our related work in Impulse-radio ultra-wideband (UWB) positioning system. The proposed work shows the potential advantages if we implement CS projection in the transmitter of the UWB positioning system according to the experiment result. Besides, Our proposed compressive UWB positioning presents a novel view of energy trade-off design, which is implemented by a low-rate random-projection at transmitters and low rate ADCs at receivers. It is clear that this design significantly reduce the peak frequency in the system with acceptable rate increase in receivers' ADC sampling rate.

3.1 Introduction

Ultra-wide band (UWB) communication is widely used in wireless communication and associated with features as extreme wide transmission bandwidth, low-power consumption, shared spectrum resources in wide ranges etc [43]. Among all applicable areas, UWB based accurate positioning and tracking in short range communication become popular since it provides resilient to multipath fading in hostile environment and outstanding robustness even in low signal-to-noise (SNR) situations [44]. In addition, the requirements of high data rate and limitations of battery supply lead the impulse-radio ultra-wide band (IR-UWB) to become a suitable communication technique in short range high data rate communication. As a result, applications based short-distance wireless sensor networks (WSNs) where large numbers of portable instrument, are widely deployed such as indoor positioning, surveillance, home automation, etc.

However, high data rate transmission puts huge pressure on signal detection at ADCs at receivers, which indicates the sampling rate becomes a main bottleneck to the IR-UWB system. This paper focuses on the IR-UWB indoor positioning, aims at solving the bottleneck of sampling rate by using a recent novel technique termed compressed sensing (CS), aforementioned in Chapter 2. Compressed sensing is a novel paradigm which applies randomly sampling and sparse reconstruction, which enables a possible reconstruction strategy for sparse signals from a relatively small group of random measurements. It indicates that CS based IR-UWB systems can possibly detect high frequency sparse signals under a sub-Nyquist sampling rates that far below the Nyquist rate (twice the IR-UWB bandwidth). This result is advantageous which releases the bottleneck of the large bandwidth constraints on ADC at UWB receivers, and consequently reduces storage limits and improves energy efficiency in IR-UWB positioning systems.

Recently some researches manage to embed CS reconstruction algorithms, e.g. revised orthogonal matching pursuit, at UWB receivers. These algorithms successfully improve the SNR of the received signal before they are sent to the stages of time of arrival (TOA) based positioning algorithm. Consequently, this method increases the performance of entire positioning accuracy [45]. For hardware implementation of the new CS based UWB receivers, most of them apply the hardware structure termed the random demodulator (RD) [46], and consequently this new CS-UWB receiver successfully reduces the sampling rate significantly compared to the Nyquist rate [47].

On the other hand, some researches embed the CS technique mainly at UWB transmitters. They develop a waveform-based precoding transmitter, in order to fulfil a random projection of the UWB generated pulses [48]. Followed by sub-Nyquist sampling ADCs at receivers, the sampled signals are sent to TOA based algorithm for calculating the location of the UWB transmitter. Simulation results show that the new CS-UWB transmitter manage to significantly decrease the sampling rate of receivers successfully improves accuracy of traditional UWB positioning system.

However, both CS-UWB receivers and CS-UWB transmitters suffer from high-data rate random mixing operation, where PN sequence at mixers is required to reach the extremely high Nyquist rate (e.g. beyond 10 GHz). This requirement generates heavy burden on bandwidth of hardware mixers and additionally increases a high frequency noise. To solve this problem, this paper propose an advanced low-rate CS-UWB positioning system: For the CS-UWB transmitter, it implements a relatively low-rate random projection matrix to slow down the mixing rate. For CS-UWB receivers, they sacrifices a small degree of compression ratio to keep equivalent performance as those positioning system using traditional CS-UWB transmitters. As a result, the trade-off between the random mixing rate and the sub-Nyquist sampling rate makes our system to become a more energy balanced, and consequently gains a better performance in entire power consumption and energy efficiency.

3.2 Model of Traditional UWB Positioning

Extremely wide transmission bandwidths of the IR-UWB offers outstanding multipath resolutions for accurate positioning in indoor environment. Consider a typical UWB indoor communication model where distributed UWB receivers (base stations) are placed in an area to detect the location of a moving UWB transmitter (tag). The transmitter periodically broadcasts Gaussian shaped pulse $p(t)$ through an indoor multipath channel, and receivers detect signals for time of arrival (TOA) based positioning calculation. The received signals can be described as (3.1):

$$r(t) = p(t) * h(t) + n(t) = \sum_{l=1}^{L} a_l p(t - \tau_l) + n(t) \tag{3.1}$$

where $p(t)$ is the transmitted Gaussian pulse, $n(t)$ stands for zero-mean additive white Gaussian noise (AWGN), and $h(t)$ refers to the standard UWB channel model denoted by IEEE 802.15.4a (3.2):

$$h(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l) \tag{3.2}$$

Here a_l and τ_l are the gain and delay corresponding to the i-th path in the channel model. The L defines the total number of propagation paths, and $\delta(t)$ is the Dirac delta function. Based on the fact that geometrical difference yields different time of arrivals, the received signals at the different receivers are collected for TOA based algorithm. At last accurate position of the transmitter is calculated based on TOA [49]. In addition, since both transmitted pulses and components of multipath channel can be regarded as approximately sparse, the received IR-UWB signals becomes sparse, and consequently the CS framework is applicable for UWB positioning [47].

3.3 Compressive UWB Positioning

This section first discusses popular CS based architectures for UWB positioning. Next, a new structure of CS-UWB positioning will be introduced as the main contribution of this paper.

3.3.1 Compressive Receivers

Typical recent researches embed the CS reconstruction algorithm at UWB receivers to improve the SNR of the received signal. As a result, it not only increases the performance of the positioning accuracy [45], but also reduces the sampling rate to a relatively low level

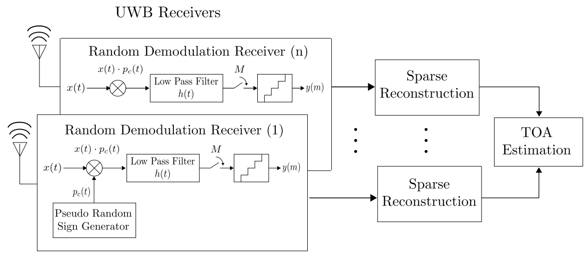


Figure 3.1: Block diagram of compressive receiver implemented by random demodulator (RD). The components of RD includes a pseudo-random sign generator (PRSG), a low-pass filter (LPF), and a sub-Nyquist ADC

compared to the Nyquist rate. Among these compressive receivers, most of them develop the random demodulator (RD) [46] as the main structure of CS based UWB receivers [47].

In this system, each compressive receiver realises the RD architecture that composed of a pseudo-random sign generator (PRSG), a low pass filter (LPF), and a sub-Nyquist rate analog-to-digital converter (ADC), shown in Fig.3.1.

Then the transmitted UWB signals are collected by group of low-rate distributed ADCs only using a minimal sampling rate of $1.7K(log(N/K))$ [46], where N stands for Nyquist rate and K is the sparsity in transmitted UWB signals. Results in [50] demonstrates that the new system successfully improves positioning accuracy.

3.3.2 Compressive Transmitters

On the other hand, some researches embed the CS technique at the UWB transmitter and regarded it as a better solution than compressive receivers in terms of system hardware power efficiency. The new architecture contains a random tap FIR at the transmitter, which accomplishes the CS random projection before UWB signals are transmitted [48]. Followed by distributed sub-Nyquist rate ADCs, the down-sampled signals are collected for TOA based algorithm. Simulation result [48] shows that the new compressive transmitter is suitable for detecting the indoor channel model with higher accuracy than traditional

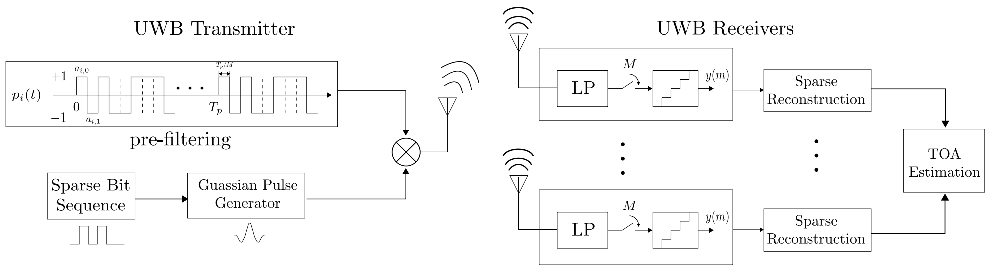


Figure 3.2: Block diagram of low-rate compressive random mixing UWB positioning system.

TOA based method. This architecture is also suitable for indoor positioning because the transmitted signal pulse and channel model keep the same. Besides, it meanwhile saves more energy cost since it contains less hardware mixers than compressive receiver based UWB system. However, the complexity of implementation a high data rate random tap FIR filters brings additional cost and becomes a main difficulty in applications.

3.4 Energy Aware Random Mixing Transmitters

As mentioned, both compressive receivers and compressive transmitters suffer from high-data rate random projection operation, where the alternative rate of pseudo-random sequence is required (often equal or above 5 GHz in such cases). This requirement generates a heavy burden on bandwidth of hardware mixers and high frequency noise.

In order to solve this problem but meanwhile preserve the energy efficient feature in compressive transmitters, this paper propose an low-rate compressive random mixing (CRM) UWB positioning system shown in Fig.3.2: At the transmitter side, the new system provides a relatively low-rate pseudo-random sequence to mix the generated Gaussian pulses. At the receivers side, it sacrifices the compression ratio (or sampling rate) to catch up with the equivalent performance as that in the compressive receivers based UWB system. As a result, the trade-off between the random projection rate and the sub-Nyquist sampling rate offers a more energetic balanced system, where no hardware devices suffers from the limitation brought from the high data rate sequence. In addition, the positioning accuracy is successfully improved compared to the original UWB system.

System Model Based on the IR-UWB indoor positioning system model in (3.1), the new structure of our system can be regarded as shown in Fig.3.2. At the transmitter, the a pseudo-random sequence (PRS) whose variables are chosen from $-1, +1$ are generated using a relatively low sub-Nyquist alternative rate. When a UWB pulse generated, first it will be randomly mixed by the pseudo-random sequence, and then broadcasted to indoor environment from the transmitter. Afterwards, receivers directly down-sample the signals at a relatively low rate. Finally, after processing the sparse reconstruction (i.e OMP or BP), the original TOA based algorithm can work for indoor positioning. In addition, these steps at receivers can be executed pipelined as [47]. Hence, based on the workflow of the new architecture, the math model of this system can be represented as following matrix form (3.3):

$$y = D * H * P * x \tag{3.3}$$

where y are discrete sampled observations from low rate ADCs at receivers, and x is the generated UWB Gaussian pulses at traditional transmitters. Here H is the correspondingly Toeplitz matrix which presents the signal convolution using IEEE 802.15.4a model in (3.2), and F stands for the random projection step, which is a diagnose matrix whose variables are randomly chosen from $\{-1, +1\}$ but alternatives at a sub-Nyquist low rate. The D represents the downsampling behaviour. It is a $m \times N$ matrix ($m << N$) with 0-1 entries , and each of its rows contains a block of $\frac{m}{l}N$ contiguous ones, which has been used for simulation in [15]. Then $\hat{x}$ can be reconstructed from various sparse reconstruction algorithm such as BP, OMP and CoSaMP. At last, the recovered signal $\hat{x}$ is applied for TOA based positioning estimation.

3.5 Simulation Results

This experiment uses Matlab to evaluate the performance of the proposed system. In the simulation, the UWB waveform is a periodic simple pulse which is shaped by the second derivative Gaussian wave with a pulse duration of 1ns. The bandwidth of this signal is 8GHz. The UWB channel model is based on the IEEE 802.15.4a CM1 model for line of sight (LOS) indoor environment, and zero-mean additive white Gaussian noise (AWGN) is added to generate an average SNR of 10dB. The simulation results cover random points in an area of 10m $\times$ 10m $\times$ 10m space.

CS Basis pursuit denoising (BPDN) algorithm is used at the receiver to perform the reconstruction process. Figure 4 shows the results of the reconstruction successful rate against the receivers sampling rates. Existing RD based system achieves 100% successful reconstruction starting at 200MHz sampling rate, but requires 10GHz PN sequence at each receiver. Our proposed system use much lower rate PN sequence (1GHz or 500MHz) at the transmitter and achieve 100% successful reconstruction at 350MHz and 500MHz sampling rate. Hence, accuracy of the lower rate PN sequence system can be improved by using higher sampling rate at the receiver. Therefore, the simulation result shows that the proposed system can apply trade-off among random projection rate, sub-Nyquist sampling rate and reconstruction rate. This design significantly reduce the peak frequency (Nyquist rate in mixing waveform or receiving end) in the system with acceptable rate increase in receivers' ADC sampling rate.

Table I compares the accuracy of the two CS based systems against conventional UWB based system which uses hypothetical 10GHz Nyquist sampling rate. Both CS based systems produces much lower errors then the conventional UWB based system, while the proposed RP system has similar performance as the RD based system.

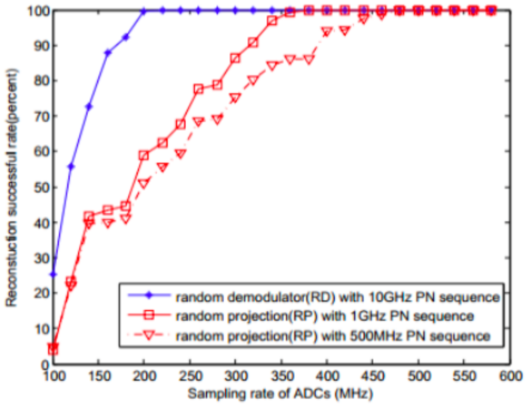


Fig. 4. Simulated performance comparison.

Table I: UWB Positioning Systems' Errors Comparison

	Sampling rate (ADC)	PN rate	Average error	Max error	Min error
Conventional	10GHz	none	27mm	46mm	8mm
RD	250MHz	10GHz	9mm	17mm	1mm
RP	500MHz	500MHz	9mm	19mm	1mm

3.6 Conclusion

The compressed sensing is proven effective to improve the accuracy of IR-UWB positioning. However, most of papers do not concern the design complexity and energy cost in their implementations, for instance, the random mixing waveform is of extremely high frequency (in GHz). Our proposed compressive UWB positioning present a novel view of energy trade-off design, which is implemented by a low-rate random-projection at transmitters and low rate ADCs at receivers. This design significantly reduce the peak frequency (Nyquist rate in mixing waveform or receiving end) in the system with acceptable rate increase in receivers' ADC sampling rate.

In this paper, following the architecture of waveform-based pre-coding at the transmitter [48], we proposed a new compressive random mixing transmitters for UWB positioning. The main purpose of this system is to slow down the high rate mixing operation when building CS framework at transmitters. Simulation results demonstrate that our proposed

CS-based UWB positioning scheme successfully decreases the high rate mixing operation by sacrificing a slight compression ratio ($< 10\%$) or small increase on average error ($<$ 1mm). The new CS-UWB TOA positioning system can achieve a much higher positioning accuracy than the traditional system in mm level. Future research will focus on how to reduce the reconstruction running time so that the real-time performance can be enhanced. This aim relates to the compressive signal processing that will be mentioned in the next chapter.

Chapter 4

Wideband Cognitive Spectrum Sensing

Cognitive Radio (CR) has been attracting many attention in recent researches with respect to the potential better utilisation performance of limited spectrum resources. In this chapter, we first propose an overview of cognitive radio networks. Then we focus on the bottleneck in its front-end sampling devices and investigate the typical CS framework for spectrum sensing in CR.

4.1 Introduction

As the wireless techniques keep fast developing, the limited spectrum resource seriously restricts the fast increasing demand for more accessible bandwidth. As a result, the dynamic spectrum access (DSA) becomes necessary and popular, which enables unused spectrum accessed opportunistically, shown in figure 4.1.(a).

Following this idea, the cognitive radio (CR) develops aiming at optimizing utilisation of idle bands for communications, without doing harm to the primary users (licensed spectrum) [51]. Correspondingly, the CR devices have to sense the environment (including spectrum usage, noise level etc) quickly and accurately, and reconfigure themselves to adapt the varying circumstance.

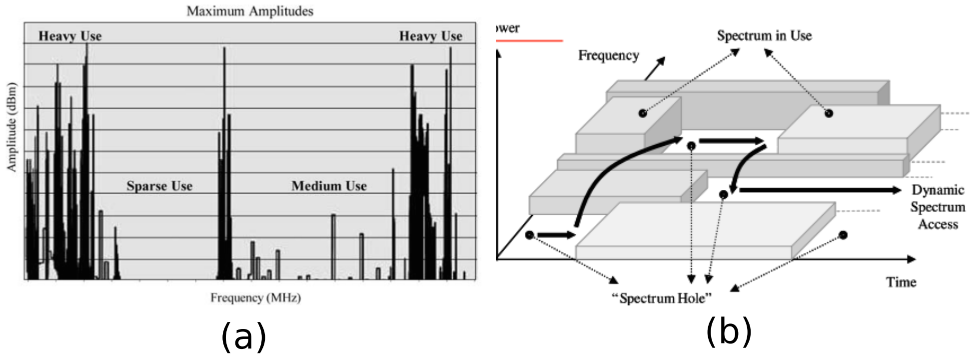


Figure 4.1: (a) The existence of unused spectrum resources; (b) The concept of spectrum holes

In cognitive radios, the first step is to sense the unused bandwidth, termed as the spectrum holes and shown in figure 4.1.(b). If the band is detected as unused, the CR networks will use it for further communication. Otherwise, the CR moves to find other spectrum holes, or stays in the same band but avoid interference by changing its transmission power or modulation model.

However, trends of communicating requires higher frequency and wider bandwidth. As a result, signal acquisition significant is crucial according to the Nyquist sampling theory. What's worse, since CR should not generates additional interference to the licensed users, CR must limits its working power to a relatively low level (if CR do not change its modulation model). The demand for sensing with low power contradicts with the requirement for sensing in high sensitivity. Thus this contradiction makes the signal acquisition much more difficult.

In conclusion, spectrum sensing becomes one of the most crucial problem for cognitive radios, and it is still an open issue. Later, compressed sensing will be introduced to be embedded into traditional spectrum sensing algorithms, in order to solve the problem and enhance the overall performance.

4.2 Hypothesis Test Model

The aim of spectrum sensing is to decide whether a particular sub-band of the spectrum is available or not. In other words, the procedure is to discriminate based on two hypotheses in equation 4.1:

$$\begin{aligned} H0 &: y[n] = w[n] \\ H1 &: y[n] = w[n] + x[n] \end{aligned} \tag{4.1}$$

, where $x[n]$ is the primary user's signal, $y[n]$ is the vectorial observation, $w[n]$ is the noise, and n refers to time slots. The hypothesis 1 suggests that the primary user's signal exists, while hypothesis 0 suggests no. Typically, the decision is made by comparing a predetermined threshold with test statistic $\Lambda(y)$ in equation 4.2:

$$\Lambda(y) \underset{H_1}{\overset{H_0}{\lesseqgtr}} \alpha \tag{4.2}$$

Then the performance of a detector is quantified by the receiver operating characteristics (ROC) curve, which presents the probability of detection $P_D = Prob(\Lambda(y) > \alpha, H1)$ and false alarm probability $P_fa = Prob(\Lambda(y) > \alpha, H0)$.

4.3 Narrowband Detection

In this section, typical spectrum sensing approaches are introduced. Narrowband sensing algorithms can be suitably applied when the channel frequency response is flat. The following figure 4.2 demonstrates most typical architectures in narrowband spectrum sensing.

Matched filter In the figure 4.2.(a), the matched filtering (MF) detector [52] is presented. when the signal to be detected is perfectly known (i.e. mean and variance), the optimal test statistic is produced by matched filter by correlating the received signal to a template. However, the signal cannot always be known in practise, so sometimes it's not applicable. Besides, the carrier synchronisation is also a remained difficult problem.

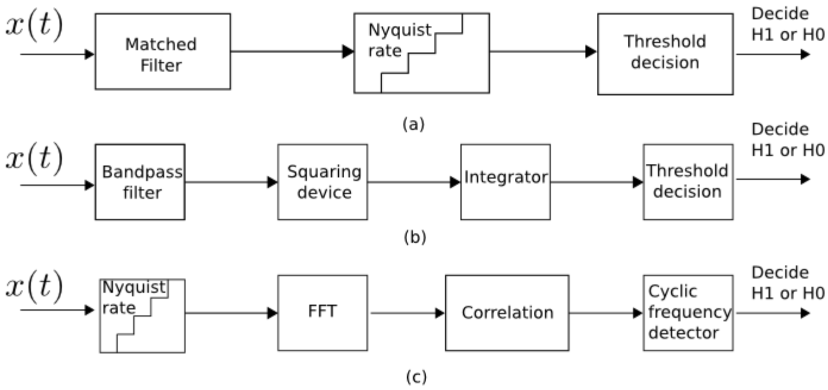


Figure 4.2: Block diagrams for traditional narrowband detection architectures: $a)$ matched filtering detector; $b)$ energy detector; $c)$ feature detector

Energy detector In the figure 4.2.(b), the energy detector (ED) [53] is presented. In the case where the signal to be detected does not present structure template, the ED can produce the optimal test statistic by directly analysis the power and variance of the received signal. The implementation of ED is simple, but it suffers from poor detection results in low SNR environment. Besides, the ED cannot distinguish different primary signals at the same time.

Feature detector In the figure 4.2.(c), cycle-stationary feature detection (FD) [54] is presented. If discrimination for primary signals and higher detection performance are required, the FD exploits the cyclic non-stationary features from primary signals. The cyclic features can be found in many typical modulated signals, for instance, in the orthogonal frequency-division multiplexing (OFDM) contains cyclic features in correlation structure due to the cyclic prefix (CP) between transmitted data. However, the computational cost is relatively high and long running time delay is always existing.

4.4 Nyquist Wideband Detection

In the scenarios where the bandwidth is sufficiently larger than coherence bandwidth of channel, wideband sensing is more suitable than narrowband sensing. For instance, it can be used for sensing the ultra-high-frequency (UHF) TV band, ranging from 300 MHz to 3 GHz, while the narrowband sensing providing single binary decision over whole spectrum is always not suitable for identifying individual spectrum access opportunities. The following figure 4.3 demonstrates the typical architectures for wideband spectrum sensing and detection.

Multiband joint detector In figure 4.3.(a), the multiband joint detector (MJD) [55] is presented. The MJD first uses serial-to-parallel conversion (S/P) to divide samples into parallel data streams, then it process the FFT to divide spectrum X(f) into groups of narrowband spectrum. Then each binary hypothesis detection is performed and joint optimised at last. The high sampling rate and lower speed of joint optimisation is the main bottleneck.

Wavelet detector In figure 4.3.(b), the wavelet detector [56] is introduced. The wavelet analysis of power spectral density (PSD) can provide significant border symbols of two neighbour sub-bands, the aim of detection becomes a spectral edge detection problem. However, the high sampling rate is also the bottleneck.

Sweep-tune detector In figure 4.3.(c), the sweep-tune detector [55,57] is displayed. This detector uses a special frequency mixing technique that 'sweep' across the frequency range of interest, to down-converts signals to a lower frequency. The adaptive local oscillator (LO) is used for 'sweep' procedure. However, the procedure of 'sweep' mixing generates too much time to wait.

Filter-bank detector Also using the idea of down-conversion, the figure 4.3.(d) shows the structure of filter-bank detector [57]. Not only following the technique which 'sweep'

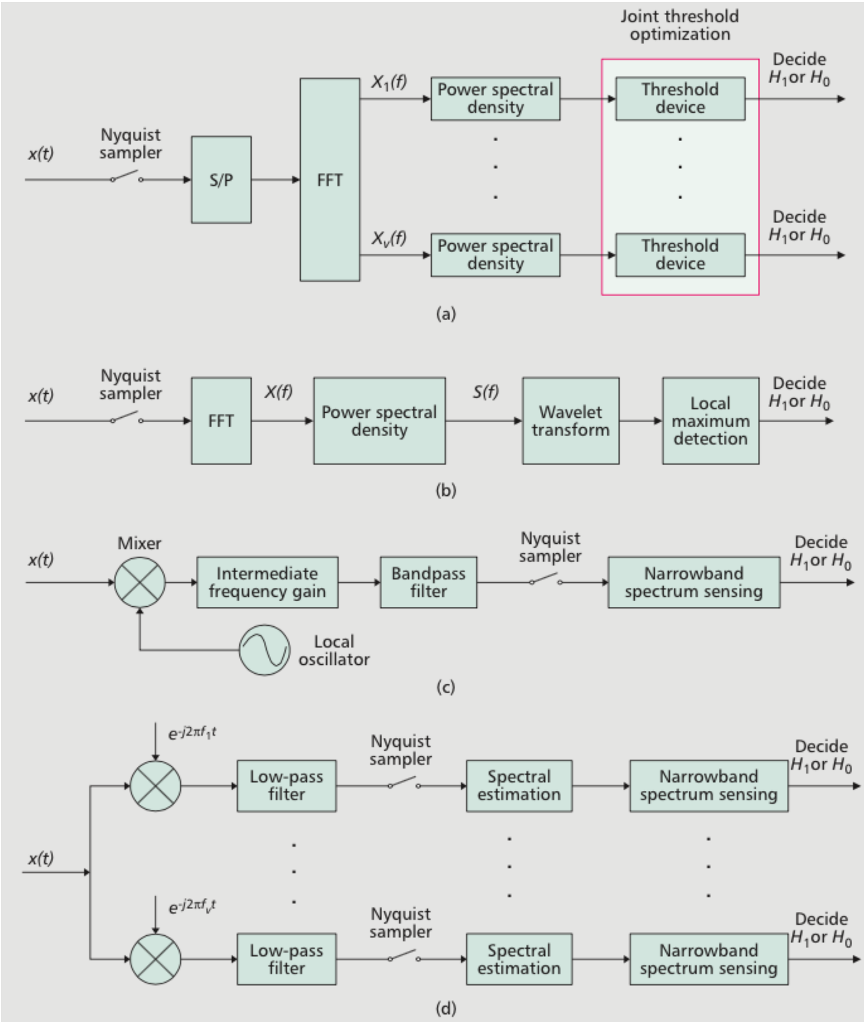


Figure 4.3: Block diagrams for Nyquist wideband detectors architectures: $a)$ multiband joint detector; $b)$ wavelet detector; $c)$ sweep-tune detector; $d)$ filter-bank detector

mixing the interest of signals, it also applies parallel structure to speed up the processing time by using filter-bank. As a result, the time cost of mixing reduces but the implementation cost largely increase.

4.5 Compressive Wideband Detection

Different from Nyquist wideband detection, the sub-Nyquist spectrum sensing applies the multi-coset (MC) sampling, multi-rate (MR) sampling, or compressed sensing (CS) to reduce the required sampling rate. Before talking about CS based sampling, first we take brief look at MC and MR sampling.

The multi-coset (MC) sampling [58] applies blocks of parallel consecutive samples with special time offsets to sample, so that each channel has a task of low-rate sampling. Then joint spectrum recovery and further detection can be performed. The main difficulty is how to perform sampling channel synchronisation with highly accurate time offsets. The quality of the specific offsets is crucial for robustness in its spectral reconstruction.

The multi-rate (MR) sampling [59] uses various sampling rates to wrap different sparse spectrum onto individual channels, and then use joint sparse spectrum recovery for further energy detection. Time synchronisation is no longer needed compared to the MC. But instead, the sacrifice is the hardware cost for parallel structure, as well as the increased sampling rate compared to original CS although the MR's sampling rate is still less than Nyquist rate.

4.5.1 Compressive Spectrum Sensing

Cognitive spectrum sensing is another typical application suitable for compressed sensing. The applicability mainly lies in two aspects:

Sampling Rate The trend of higher frequency transmission is also suitable for cognitive radio, which leads to higher rate sampling rate at receivers. Thus it's reasonable to develop the CS based spectrum sensing techniques to reduce the sampling rate.

Flexibility and Energy The cognitive radio requires flexibility for sensing various types of signals (TV signal, cell phone, satellites etc) in a relatively wide bandwidth. However, normal wideband spectrum detection uses filtering or mixing for down-conversion (then low-rate sampling). This approach require difficult analog implementations such as adaptive local oscillator(LO) for filter-banks(FB). Inversely, if the CS is used, then the system can get wider sensing (frequency) ranges without the hardware of LO or FB.

4.5.2 Compressive Detectors Overview

Therefore, the CS become popular in cognitive spectrum sensing. The figure 4.4 demonstrates the typical architectures in wideband spectrum sensing, and briefly analyse CS detectors. The similar CS based architecture can also be viewed in chapter 2.

Random Demodulation based Detector Figure 4.4.(a) presents the random demodulation (RD) based detector [15], which is an analog-to-information converter (AIC) for finite-length and discrete-time signals, and consists of pseudorandom wave generator, a mixer, and a low-rate ADC. The detailed architecture of RD is introduced in chapter 2. The reconstruction of RD sampled data involves $l1$-norm minimization (e.g. based BP, LASSO) or greedy method (e.g. OMP). This design is simple, but easily affected by model mismatches and design imperfections.

Modulated Wideband Converter based Detector Figure 4.4.(b) displays the modulated wideband converter (MWC) based detector [16] for the case where multichannel signals are designed to be detected. The MWC can be considered as a parallel structure of RD, and its

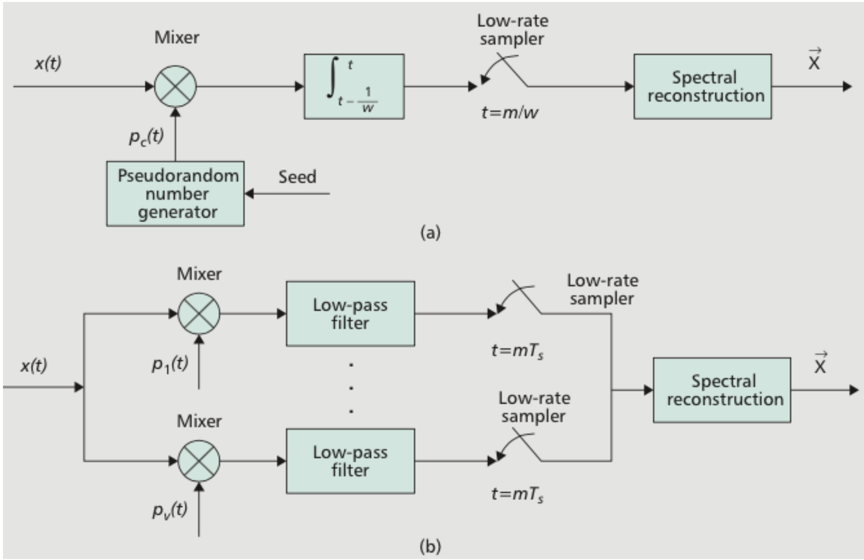


Figure 4.4: Block diagrams for compressive wideband sensing architectures: a) random demodulation based detector; b) modulated wideband converter-based detector

architecture is introduced in chapter 2. The reconstruction of MWC involves the multiple measurement vector (MMV) sparse recovery which exploits the fact that the columns of original spectrum coefficients share the same sparsity pattern. Compared to the RD, MWC provides robustness against the noise and model mismatches.

Cooperative Compressive Spectrum Sensing Cooperative versions of spectrum sensing is an open issue, which aims at solving the hidden terminal problem [51] and improve sensing accuracy. The cooperative version of compressive wideband sensing have been developed [60, 61]. Here, individual radios can make a local decision about the presence or absence of a primary user, and these results can then be fused in a centralised or decentralised manner. However, a greater cooperation gain can be achieved by fusing all the compressed measurements, again in a centralised or decentralised manner. In general, such measurement fusion requires that each cognitive radio knows the channel state information (CSI) from all primary users to itself [60], which is cumbersome. But recent extensions

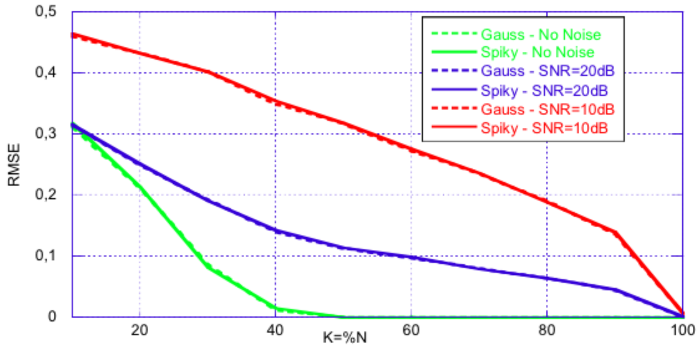


Figure 4 – RMSE for channel occupancy reconstruction as a function of K (percentage of N) for different SNR values.

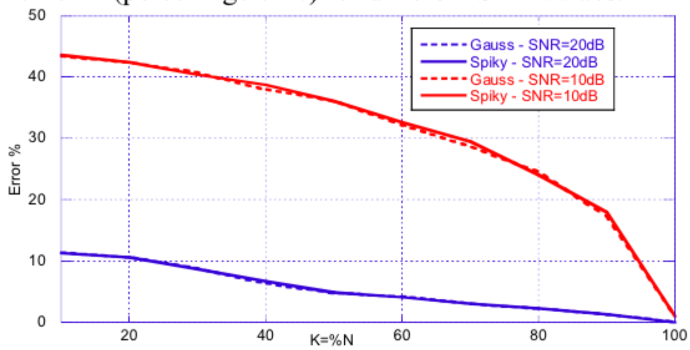


Figure 5 – Percentage error on the decision of channel occupancy as a function of K (percentage of N) for different SNR values.

Figure 4.5: Block diagrams for mean square error and error detection performance of the proposed compressive cognitive radar systems

show that measurement fusion can also be carried out without CSI knowledge [62].

Compressive Radar System In [63] the CS is embedded to enhance the performance of cognitive radar that use wide operating frequency bandwidths for spectrum sensing and sharing. The compressive cognitive radar utilises the typical CS techniques, including the random demodulation (RD) for signal acquiring, basis pursuit de-noising (BPDN) algorithm and discrete cosine basis (DCT) for sparse reconstruction, and classical energy detector (ED) for hypothesis analysis.

The experimental results figure 4.5 shows the CS based radar has appealing performance in low sampling rate (using only $< 30\%$ of the total samples of the original signal) and low detection error in high SNR cases. But in low SNR environment, the proposed system still struggles and suffers.

4.6 Challenges and Discussion

The compressed sensing based wideband spectrum sensing for cognitive radio provide its outstanding feature in reducing the sampling rate. However, the corresponding drawbacks emerge in real-time ability and energy consumption, mainly due to its computational expensive non-linear reconstruction and energy consuming characters:

Long time feedback delay Accuarcy is cruical for primary detection. Hence, if we applies CS for sampling, the convex optimisation (e.g.basis pursuit) is always needed for data recovery since it provides better accuracy and robustness comparing to greedy methods (chapter 2). However, convex optimisation is time consuming, which cause too long time to fast feed-back. Since feed back is very important for CR, which is responsible to avoid interference and quick reconfiguration, agility and reconfigurablity may reduce.

Mismatch for signal processing The reconstruction algorithms for CS is non-linear, which indicates that the recovered data are not directly suitable for conventional digital signal processing where traditional recovering only requires cardinal sinc interpolation (linear process). This brings difficulty in directly reuse the traditional method for sensed data.

high energy cost The heavy reconstruction for CS not only brings large time-delay, but also additional energy cost. Compared to linear recovery in traditional approaches, the CS based signal detection additionally required the block for spectrum recovery before further hypothesis detection.

4.7 Conclusion

Cognitive radio has been widely used and attracting many research attentions in its spectrum sensing techniques. In this chapter, traditional sensing approaches such as energy detection and feature detection are introduced. In order to solve the detection task for wideband and high frequency signals, compressed sensing based spectrum sensing (CS-CSS) is introduced and demonstrated. However, the compressively sampled data does not directly match the traditional processing algorithms (section 1.5). Then here comes the question: what if we directly perform hypothesis detection without CS reconstruction? If the idea is achievable, the additional energy cost will be eliminated so that the entire energy reduces. Besides, not only detection, if we can expand this idea to filtering, estimation, then more intelligent-based scheme for cognitive radio can be supported in physical layer. The answer refers to our future research aims that shown in the next chapter.

Chapter 5

Compressive CRN Processing

In the last chapter 4, we have discussed the main drawback in compressed sensing based spectrum sensing (CS-CSS) which derives from the computational non-linear CS reconstruction. As a fact, the CS framework sacrifices the time and energy performance in reconstruction produce (in return, the sampling rate is reduced). In order to solve this problem, this chapter focuses on exploring the potential approach of directly analysing compressive measurements without fully CS reconstruction. The new approach is termed as compressive signal processing (CSP), and it will be used for compressive spectrum sensing (CSS) in cognitive radio network (CRN), so that the drawback will be overcome.

5.1 Introduction

The compressed sensing based wideband spectrum sensing for cognitive radio provides its outstanding feature in reducing the sampling rate. However, the corresponding drawbacks emerge in real-time ability and energy cost, mainly due to its computational complex non-linear reconstruction and energy cost in entire cognitive radio systems.

However, many signal processing applications such as detection, classification, estimation and filtering do NOT require entire signal reconstruction [64]. For instance, cognitive radios are aiming at processing the hypothesis detection rather than fully recovering the primary signals. In these cases, the aim of CS fully recovery is no longer necessary, so processing schemes like 'directly analysis without recovery' or 'partial recovery then analysis' become possible and applicable for cognitive spectrum sensing.

This novel idea derives from the compressive signal processing (CSP) [64], which aims at extracting information directly from compressive samples without fully recovery. Further, this idea is developed in both theory and applications. In [65], the shift retrieval problem for compressive samples is researched. Rather than recovering CS data then analysing the shifted distance, the author develops efficient algorithms and proofs for directly recognising the shift distance from CS data. Valsesia et al [66] develops the circulant sensing matrix based processing for compressive measurements, which displays an potential CSP application for convolution based models and is suitable for filtering or channel impulse response involved cases. Especially, for cognitive radio spectrum detection, a CSP based energy detector is designed in [67]. Further, Guo et al develops CSP for feature detector in [68] and pattern clustering is achieved.

Since the paper [68] in year 2013 GLOBECOM conference says, "So far the CSP concept is only defined in the theory level and has not been applied to CR field", we believe there still exists many worthy research area and cases for CSP based CR spectrum sensing. In conclusion, CSP based cognitive spectrum sensing is still a relative new approach which applies the CS but throw away some CS drawbacks for CR specrum sensing. The following sections are organised as different function introduction for CSP based cognitive radio spectrum sensing, including filtering, detection, estimation. Some related potential application for CS-UWB and CS demodulation are also introduced.

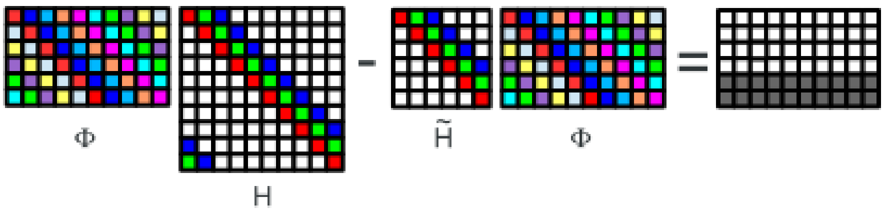


Figure 5.1: Block diagram of exchanging order of circulant matrix. H is corresponding impulse response of filter, Φ is the compressed sensing matrix. The exchanging sacrifice is the loss of rows in the results (matrix in the right side of equal sign).

5.2 CSP Filtering for Cognitive Spectrum Sensing

In scenarios where cognitive spectrum sensing requires filters, the design cost for analog filters are expensive. Then we can use CSP based filtering to transform the analog filter to digital filter, which omits the implementation of hardware design for filters.

Filter domain transform Assume that the filter's impulse response is h (with length of N_h) and its corresponding matrix form is H, then H is a circulant sensing matrix. Then, according to a CSP related theory in [66], it possible to exchange the H and Φ (compressed sensing matrix), transform the analog filter into digital domain for filters within CS frameworks. In other words, the compressive measurements y can be directly processed by filtering H without recovery in some cases 5.1 as follows:

$$\hat{y} = \Phi H x = H \Phi x = H y \quad , where \quad i \in [1, m - N_h + 1]. \tag{5.1}$$

, where the Φ is compressed sensing matrix with size of $m \times N$. Consequently, by moving the filtering from analog to digital part, large amount of hardware complexity is omitted.

Interference filtering In cases where useless information is redundancy for further recovery in CS framework, or specifically where some sub-bands are priorly known in the compressive cognitive spectrum sensing, system using CSP can regard those useless information as interference and apply CSP-filtering to sample-then-filter the compressive data rather than sample-recover-then-filter it.

Assume a sparse signal $x \in R^N$ which consists of two components:

$$x = x_s + x_I, \quad x_S \in S_S \; and \; x_I \in S_I \tag{5.2}$$

, where the x_S contains the spectrum of interest and the x_I stands for the useless information. After the CS acquisition, we gain $y = \Phi x = \Phi(x_S + x_I)$. Then our aim is to wipe out the contribution of Φx_I from the observation y before recovering x. The following theorem provide the applicability of this idea:

Theorem 2. *[64] Suppose that Φ satisfies the δ-stable RIP for all $x_S \in \Sigma_S$ and $x_I \in \Sigma_I$, where I is a K_I dimensional subspace of R^N. Assume that Ψ_I is an matrix whose columns constructs an orthogonal basis for the K_I dimensional subspace. Define the matrix $P_\Omega = \Phi\Psi_I(\Phi\Psi_I)^\dagger$ and $P_{\Omega^\perp} = I - P_\Omega$. For any $x \in \Sigma_S \cup \Sigma_I$, we regard $x = x_S + x_I$, where $x_S \in s_S$ and $x_I \in s_I$, then*

$$P_{\Omega^\perp}\Phi x = P_{\Omega^\perp}\Phi\hat{x}, \quad \frac{\delta}{1-\delta} \leq \|P_{\Omega^\perp}\Phi\hat{x}\|_2 \leq 1 + \delta \tag{5.3}$$

The theorem 2 proofs that there exists a matrix $P_{\Omega^\perp}$ which is nearly orthogonal to the useless information x_I such that $P_{\Omega^\perp}x_I \approx 0$. Hence the matrix $P_{\Omega^\perp}$ is the filter for selecting the desired signal x_S. we can apply this theory to construct a $filteringmatrix$ in CS acquisition model if the support of the x_I is known.

5.3 CSP Detection for Cognitive Spectrum Sensing

CSP based energy detector Davenport et al [64] develops the theorem to directly build up hypothesis detection through compressed samples. Assume the detection is based

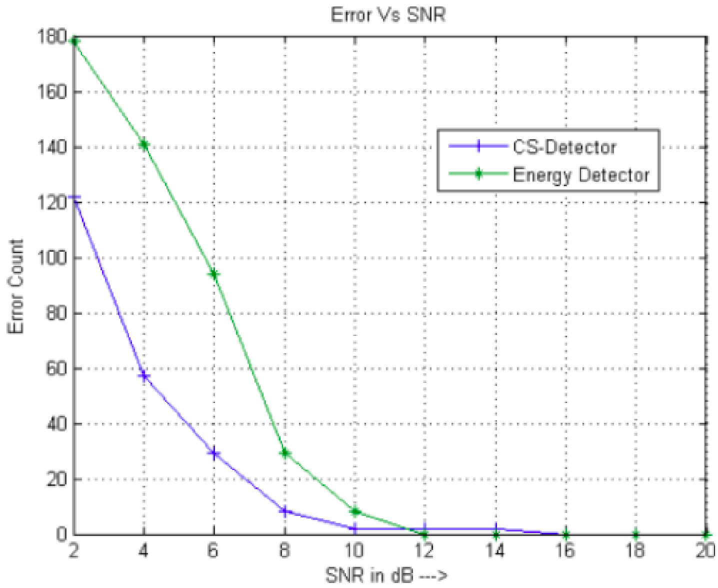


Figure 5.2: Block diagram of performance comparison between the CSP detector (compressive signal processing based detector) and ED (energy detector) from aspect of signal to noise ratio and detection error.

on two hypotheses in

$$\begin{aligned} H0 &: y = \Phi w \\ H1 &: y = \Phi(w + s) \end{aligned} \tag{5.4}$$

, where s is the primary user's signal, y is the vectorial observation, Φ is compressed sensing matrix (e.g. the random demodulation), and w stands for the noise. Then the following equation 5.5 presents the structure of CSP based detector.

$$\Lambda(y) = (\Phi\Phi^T)^{-1}\Phi s \quad \underset{H_1}{\overset{H_0}{\lessgtr}} \quad \alpha \tag{5.5}$$

In [67], the experimental results of comparing CSP-detector and traditional energy detector (ED), which is shown in figure 5.2. One of the defect is that the theorem does not drive other blind sensing techniques for cases where the information of primary signal s is unknown to us. Thus further developing for CSP based blind sensing is a potential direction.

CSP based cyclic feature detector Guo et al develops CSP for feature detector in [68] and achieves pattern clustering. The work generates the compressive CR spectrum measurement by utilizing both the cyclic-stationary feature and sparsity prior knowledge at the spectrum sensing front end. Then the paper applies the compressive CSP without the need of signal or feature reconstruction. However, as the author says in [68], the CSP feature detection model is still simple. How to analyze the spectrum pattern recognition performance in a more complicated CRN environment is open issue. For instance, with large-scale SUs and non-Poisson PU traffic models. Also, More efficient machine learning schemes (such as information geometry) can be used to recognize the PUs signal patterns after obtaining the CS samples via CSP scheme.

5.4 CSP Estimation for Cognitive Spectrum Sensing

Since the signal-to-noise-ratio, sparsity order are crucial factors which seriously affects the performance of compressive detectors, CSP based estimation becomes popular for cognitive radio. This technique make CR predict the level of sparsity or noise, so that CR can vary the sampling rate or changing detection algorithms:

Sparsity order estimation For instance, the sparsity order of the spectrum occupancy is time-varying in CR networks. If CR intends fully exploit the CS framework, the sampling rate of CS receiver should keeping adjusting to sparsity orders. Then [69] develops an approach which analysis sparsity order through asymptotic eigenvalue probability distribution function (aepdf) from the covariance matrix of the compressively sampled primary signal. In details, the aepdf is related to the sparsity order via a lookup table in figure 5.3.

Noise level estimation For CR networks, the signal-to-noise ratio (SNR) affects the performance of spectrum detectors. For example, the energy detector (ED) is simple and fast, but with poor performance in low SNR scenarios; Feature detectors (FD) are complex and

Scenarios	**I**	**I**	**II**	**II**	**III**	**III**
	SNR=2 dB	SNR=2 dB	DR=6.02 dB	DR=6.02 dB	SCN=4 SNR=0 dB	SCN=4 SNR=0 dB
Sparsity Level	**Compressive**	**Full**	**Compressive**	**Full**	**Compressive**	**Full**
1	9.63	7.44	12.80	10.19	6.93	5.86
0.9	9.03	7.14	11.80	9.58	6.53	5.65
0.8	8.41	6.83	11.03	9.08	6.17	5.44
0.7	7.85	6.53	10.22	8.60	5.80	5.24
0.6	7.26	6.21	9.36	8.08	5.43	5.02
0.5	6.67	5.88	8.42	7.44	5.08	4.81
0.4	6.05	5.52	7.51	6.84	4.74	4.60
0.3	5.44	5.15	6.63	6.24	4.43	4.40
0.2	4.85	4.76	5.66	5.57	4.17	4.20
0.1	4.25	4.31	4.69	4.80	3.96	4.00
0	3.79	3.79	3.79	3.79	3.79	3.79

Figure 5.3: Block diagram of lookup table for sparsity order estimation through signal-to-noise ratio (SNR) and asymptotic eigenvalue probability distribution function (aepdf). For example, if the value of aepdf = 7.26 for the compressive case in SNR = 2db, it can be estimated that the sparsity order of spectrum occupancy is 0.6

slow, but have strong robustness to noise. In [70], the author similarily build up a lookup table relating the noise estimation with the eigenvalue probability distribution function in covariance matrix of the compressively sampled primary signal.

Hybrid spectrum sensing If we have known the SNR, we can design a more intelligent two stage detector. In the coarse stage, a quick search is done over a wide bandwidth, and in the fine stage, the sensing is done over the individual candidate sub-bands in that bandwidth, one at a time. the coarse stage is based on energy detection due to its fast processing. If the test statistics is larger than a predefined threshold, then the band is considered occupied. Otherwise, a fine stage is performed where a cycle-stationary detector is implemented due to its robustness at the low SNR regime. (For sparsity level, we can judge whether the CS based detector suitable if there exists dense channel occupancy by primary users)

Adaptive spectrum Sensing In case when a CR successfully know the primary signals' SNR, the CR can intelligently choose optimal detection algorithms based on SNR related performance of detectors. For instance, if the SNR is detected to be very low, then cycle-stationary detection algorithm can be used due to its robustness at the low SNR regime.

Otherwise, the energy detector can be used since it's fast and accurate enough in high SNR regime. Extending this idea of selecting various of detector based on CSP estimation, we can select the optimal detector adaptively. This will provide future cognitive radio more flexibility to the varying environment.

5.5 Other Related Wideband Processing with CSP

5.5.1 CSP TOA Algorithm for Ultra-Wideband Positioning

Maximizing the cross-correlation between the two signals is one of the key steps in time-of-arrival (TOA) algorithms which have been applied in compressive UWB positioning in chapter 3. In that system, the procedures at receiving end can be abstracted as (1) low-rate sampling, then (2) CS fully reconstruction, finally (3) TOA algorithm based locationing. However, the (2) CS fully reconstruction may sometimes Not necessary if we borrow the theorem in [65], where the shift retrieval problem for compressive samples is investigated. Rather than recovering CS data then analysing the shifted distance, the author develops efficient algorithms and proofs for directly recognising the shift distance from CS data. The paper illustrates the results by running a Monte Carlo simulation shown in figure 5.4. According to this result, the CSP-TOA positioning becomes possible.

5.5.2 CSP Demodulation

A compressive sensing phase-locked loop (CS-PLL) [71], is designed for directly extracting the phase and frequency from compressively sampled modulated signal $without$ sparse recovery. Since the restricted isometry property (RIP) of CS ensures that the standard inner product between $x[n]$ and $u[n]$ is approximately the same as that in the compressively sensed version produced by $y[m]$ and $u[n]$ [64], hence, the inner products in the standard PLL and that in the CS-PLL are nearly the same, ensuring the consistence be-

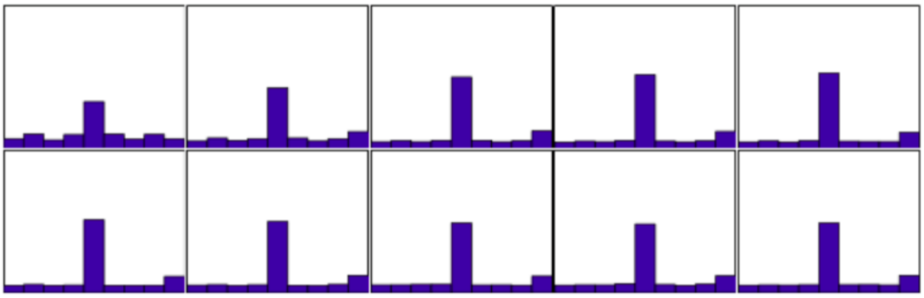


Figure 5.4: Histogram for the estimated shift in SNR = 2. From left to right, top to bottom, compression ratio = 0.1 ... 1. The true shift was set to 5 in all trials

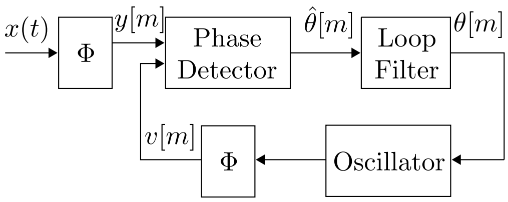


Figure 5.5: Block diagram of the compressive sensing phase-locked loop (CS-PLL). The components includes the pre-CS sampler, phase detector, loop filter, oscillator and the feedback CS multiplier

tween the standard PLL $\theta[n]$ and the CS-PLL's output $\theta[m]$. This idea is similar to the CSP estimation related theorem in section 5.4. The presentation of the $\theta[m]$ can be presented as $\theta[m] = \sum_k y[k]v[k]h[m-k]$, where the index m indicates the lower sampling rate compared with the Nyquist rate index n. In addition, the compressive sensing operations Φ which apply the RD's architecture consist of a input pseudo-random sequence, a mixer, a integrator, and a low-rate sampling ADC, which is the same as the RD's architecture shown in Figure 2.1.

5.6 Conclusion

In the this chapter, we have followed the discussion of the main drawback in compressed sensing based spectrum sensing (CS-CSS) which derives from the computational non-linear CS reconstruction, and propose the idea of signal processing directly on compressively sampled data (CSP). Then we introduced our future potential works for cognitive spectrum sensing with CSP, including CSP filtering, CSP detection, CSP estimation. Also, potential CSP TOA Algorithm for Ultra-Wideband Positioning is introduced. We hope that by utilise the CSP in spectrum sensing, energy efficient hardware with high flexibility can be achieved in the future.

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