Student Research Report: Compressive Sensing Framework for Embedded Signal Processing Systems

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Abstract—Compressive sensing is a new signal-processing paradigm that aims to encode sparse signals by using far lower sampling rates than those in the traditional Nyquist approach. It helps acquire, store, fuse and process large data sets efficiently and accurately. This method, which links data acquisition, compression, dimensionality reduction, and optimization, has attracted significant attention from researchers and engineers in various areas. This comprehensive reference develops a unified view on how to incorporate efficiently the idea of compressive sensing over assorted wireless network scenarios, interweaving concepts from signal processing, optimization, information theory, communications, and networking to address the issues in question from an engineering perspective. This report presents a working knowledge of compressive sensing (CS), including background on the basics of compressive sensing theory, an understanding of its benefits and limitations, and the skills needed to take advantage of compressive sensing in wireless networks.

I. INTRODUCTION

A. Motivation

Over the past few decades, signal processing have been widely deployed in many computational applications in areas such as image processing, video conferencing, smart sensors network, and other real-time processing technologies. These applications are also increasingly implemented in the form of embedded systems such as mobile and wireless devices, aerospace equipment, and biomedical instruments etc.

Many signal processing systems also involve signal sampling and conversion, where one of the most crucial limitations lies in the sampling theory (the Nyquist sampling rate or Shannon theory, which states the minimum two time sampling rate requirement.) Due to limited data processing capability found in many embedded systems, excessive data samples will negatively affects their real-time performance and power efficiency, and hence forms the bottleneck of entire digital systems. Therefore, overcoming the sampling rate limitation has become a prevailing research topic, and would be of particular significance for embedded signal processing systems.

B. Research Objectives

A novel paradigm for signal processing and acquisition, the compressive sensing or compressive sampling (CS) technique, is identified to have great potential for resolving the problems mentioned above. The CS theory has been proven to be suitable for numerous computer science and electronic engineering based applications, where it is possible to overcome the limitation of the traditional sampling theory. The main idea of the research work is to further extend the CS based techniques to embedded signal processing systems. Using this approach, a novel CS-based framework for embedded signal processing systems will be proposed that encompass various areas such as data compression, data acquisition, data storage, data transmission optimal recovery and processing, but targeted for embedded deployment.

II. CS FRAMEWORK FOR SIGNAL PROCESSING

A. Compressed Sensing Paradigm

Compressed sensing (CS) announces that sparse signals can always be reconstructed from far fewer samples than traditional method of Nyquist Theorem. Consider a task of sampling a signal of $x \in R^N$ using sub-Nyquist rate. Suppose that a group of basis Ψ provides a K-sparse representation of signal x as following, where $k << N$:

$$x = \Psi s = \sum_{l=1}^{K} \psi_l s_l \qquad (1)$$

Here x is a linear combination of K basis chosen from Ψ, and s is the corresponding coefficients of representing x in the domain constructed by the basis Ψ. In CS, x can always successfully be reconstructed from M measurements where $M << N$. The measurements y is generated by projecting x over a matrix Φ incoherent with Ψ, which can be described as $y = \Phi x = \Phi \Psi s$. If the composite matrix $\Phi \Psi$ satisfies restricted isometry property (RIP) [?], then s can be exactly reconstructed by solving the following $l1$-norm minimization problem (2):

$$\hat{s} = \arg \min \|s\|_1 \quad s.t. \quad y = \Phi \Psi s \qquad (2)$$

It is shown in [?] that if the composite matrix $\Phi \Psi$ has all random variables taken from a normal distribution, RIP has relatively high probability to be satisfied, indicating that the $l1$-norm minimization (2) can provide a stable and robust reconstruction, where only $O(K \ln(N/K))$ sampling points are needed. Therefore, CS provides a novel random sampling paradigm which requires smaller observations and releases the sampling bottleneck defined by Nyquist theory.

B. Random Measurement

Although directly designing a matrix A which satifies the RIP is difficult, random measurement or random matrix whose entries are independent and identically distributed variables, are very likely satisfies the RIP [?]. Besides, as constructing a random matrix is applicable, Gaussian distribution matrix, Bernoulli matrix, random partial Fourier matrix, random Toeplitz matrix [?] are widely used for constructing the measurement in CS applications.

C. Reconstruction Algorithms

Solving this optimization problem (2) by linear programming like basis Pursuit (BP) is computationally expensive, but recent greedy algorithms provide alternative solutions that regarded as faster and more efficient, including orthogonal matching pursuit (OMP), compressed sensing matching pursuit(CoSaMP) and iterative hard thresholding (IHT) etc. Among those sparse reconstruction algorithms, CoSaMP and IHT offer the optimal solutions as it works with a minimal number of observations and performs a better recovery robustness to noise, and requires reasonable computational complexities [**?**].

1) Orthogonal Matching Pursuit: Orthogonal matching Pursuit (OMP) [**?**] is a widely used for sparse reconstruction which develops the process of matching pursuit. As shown in algorithm 1, it progressively manages to find the support of the unknown sparse signal: Give an acquisition system $y = As$ where $s \in R^N$ is K sparse and CS measurement $A \in R^{m \times N}$ ($m << N$), each iteration of OMP selects one support λ of the vector s which contributes the most to the observation y. This selection method is based on testing the correlation values between the current columns of A and residue r. The OMP iteration would not stop before the residue r becomes relatively small.

Algorithm 1 Orthogonal Matching Pursuit(OMP)

Input: measurement $A \in R^{m \times N}$, observation y.
Output: recovery $\hat{s}$.
1: $r_0 \leftarrow y$; $\Lambda_0 \leftarrow \Theta$; $i \leftarrow 0$.
2: **while** $r_i \geq theshold$ **do**
3:      $h_i \leftarrow A^T r_i$ $(match)$
4:      $\Lambda_{i+1} \leftarrow \Lambda_i \cup \{\arg \max_{1 \leq j \leq N} |h_i(j)|\}$ $(identity)$
5:      $\hat{s}_{i+1} \leftarrow \arg \min_s \|y - A_{\Lambda_{i+1}} \hat{s}_i\|_2$ $(determine)$
6:      $r_{i+1} \leftarrow y - A\hat{s}_{i+1}$ $(update)$
7: **end while**

The computational complexity of OMP is $O(KmN)$ which is significantly smaller compared to classical convex optimization such as basis pursuit whose complexity is $O(N^3)$ (in case that sparsity of $K << N$) [**?**]. However, the robustness of OMP cannot reach the quality level of traditional convex optimization, since searching local optimal solutions instead of global solutions brings more errors to OMP.

2) Compressed Sensing Matching Pursuit: In order to improve the speed and robustness of OMP, Compressed Sensing Matching Pursuit (CoSaMP) [**?**] is developed. It aims at producing a more effective way for detecting the support of input signals shown in 2: The CoSaMP firstly find $2K$ indices for maximal correlation between columns of A and the current residue r by using the operator $H_{2K}(A^T r)$ to set all but the $2K$ largest components in $A^T r$ to zero. Then CoSaMP merges these $2K$ indices with the previous support from current recovered $\hat{s}$, in order to form a new support set λ for updating the least square solution of $\hat{s}$. In the next step, the least squares solution is pruned and consequently only the K largest components are preserved.

Although other revised matching pursuits such as ROMP [**?**], StOMP [**?**] have been developed for improving the robustness of OMP. However, compared to those revise for

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Algorithm 2 Compressed Sensing Matching Pursuit(CoSaMP)

Input: measurement $A \in R^{m \times N}$, observation y, sparsity K.
Output: recovery $\hat{s}$.
1: $r_0 \leftarrow y$; $i \leftarrow 0$.
2: **while** *stopping the criterion* **do**
3:      $h_i \leftarrow H_{2K}(A^T r_i)$ $(match)$
4:      $\Lambda \leftarrow supp(h_i) \cup supp(\hat{s}_i)$ $(identity)$
5:      $\hat{s}_{i+1} \leftarrow \arg \min_s \|y - A_\Lambda \hat{s}_i\|_2$ $(determine)$
6:      $(s_{i+1})_j \leftarrow 0 \ for \ j \notin \Lambda_i$
7:      $r_{i+1} \leftarrow y - A\hat{s}_{i+1}$ $(update)$
8: **end while**

OMP, CoSaMP offers the optimal performance as it works with a minimal number of observations and performs a better recovery robustness to noise [**?**].

III. CS BASED WIRELESS COMMUNICATION

A. Wideband Compressive Sensing ADCs

Sampling rate is crucial to signal processing systems since high clock rate enhances the power consumption and increases the burden of data transmission and storage. However, it has long been bounded by the Nyquist theory which declares the minimum sampling rate for traditional acquisition methods, and couldn't reduced easily. In order to overcome this problem, recently many signal processing devices embed CS into analog-to-digital converters(ADCs) and successfully reduces the size of measurements and thus releases the limitation given by the Nyquist theory. As a consequence, the new systems using a relatively lower clock rate significantly outperform the traditional acquisition systems.

1) Random demodulator: Random demodulator (RD) [**?**] is one of the most popular analog-to-digital converters for CS-based signal acquisition and processing. It develops the stability and robustness of CS measurement, and achieves a sub-Nyquist rate ADC for signal sampling and sequentially enhances the system power efficiency (especially the performance of front-end acquisition systems).

Figure 1 shows the construction of the random demodulator: It consists of a pseudo-random sign generator, a low-pass filter, and a sub-Nyquist ADC. The input K sparse signal is first mixed with the chipping sequence $p_c(t)$ which is a waveform and constructed by pseudo-random variables of $\{\pm 1\}$. This chipping sequence is generated by the pseudo-random sign generator which alternates at the Nyquist rate N. The mixed production $x(t) \cdot p_c(t)$ then passes through a low
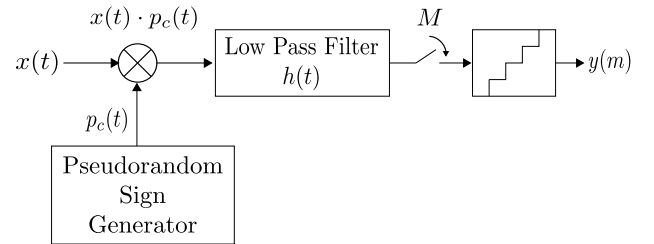

Fig. 1. Block diagram of random demodulator (RD). The components includes a pseudo-random sign generator, a low-pass filter, and a sub-Nyquist ADC
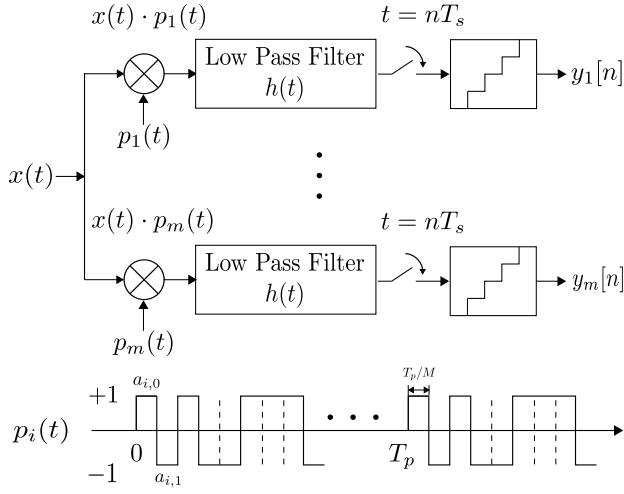
Fig. 2. Block diagram of modulated wideband convertor (MWC). The components includes parallel periodic waveforms mixers, low-pass filters, and sub-Nyquist ADCs

Fig. 3. Block diagram of typical UWB indoor communication system

pass filter $h(t)$ for anti-aliasing, and finally is sampled by a ADC uniformly at the rate of $m = O(K \log(N/K)) << N$.

2) Modulated Wideband Convertor: In the cases of sampling a multi-band signals whose carrier frequencies are unknown or time-variant (blind multi-band signal receiving), the main task is to design a receiver working independently on the carrier frequency at a low rate [**?**]. Recently, a novel architecture termed modulated wideband converter (MWC) is developed, which applies the CS theory to the traditional blind multi-band signal receivers based on non-uniform sampling [**?**]. This innovative architecture successfully provides a minimal requirement for the size of observations [**?**] thus decrease the power consumption.

The MWC consists of groups of periodic waveforms, low pass filters and sub-Nyquist rate ADCs, and can be treated as a parallel structure of the random demodulator (RD). As shown in Figure 2, each group of periodic sequence $p_i(t)$ with minimum interval of T_p/M mixes the input multi-band signal $x(t)$ for shifting the spectrum by Δf_p ($f_p = 1/T_p$ is the frequency of $p_i(t)$) and filtering it to the baseband for sampling. In the end, low rate ADCs sample the mixed productions at a rate f_s far less than the Nyquist rate f_{NYQ}.

Considering the CS measurement MWC, recovering the $\mathbf{z}(f)$ where each row $X(f - lf_p)$ presents sparse spectrum segments can be regarded as recovering a infinite set of joint sparse vectors. This problem can be solved by changing it to the problem of multiple measurement vectors (MMV) [**?**] and has been implemented.

3) CS based signal processing: Considering that many signal processing applications such as detection, classification, estimation and filtering do not require entire signal reconstruction, and only parts of certain information from measurements are needed, thus we prefer to filter out the useless information before further signal processing. Therefore, if we could estimate the useless information corresponding to some observations y_I, and then *throw* them *away* before CS based signal recovery, we can avoid a high computational complexity and thus save the power efficiency significantly. This novel idea
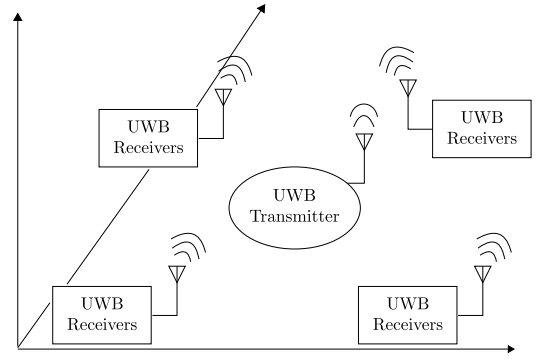
comes from the compressive signal processing (CSP) [**?**].

4) Non-Uniform Sampling: The non-uniform sampling aims at sampling signals in a local Fourier sparse representation [**?**] (LFS) (e.g. wideband signals), and provides an non-uniformly sampling pattern at pseudo-random time points with a low average rate, which establishes a sensing matrix A which can be regarded as a random partial Fourier matrix F_T that consists of randomly chosen columns of the discrete Fourier matrix(DFT) and indexed by T. According to [**?**], the matrix A satisfies the RIP in compressive sensing so the acquisition model produces a stable reconstruction of $\hat{s}$ via $l1$-minimization using $m \geq O(slog(N/s))$ samples [**?**].

B. Compressed Sensing based Ultra-Wideband Positioning

Ultra-wide band (UWB) communication is widely used in wireless communication and associated with features as extreme wide transmission bandwidth, low-power consumption, shared spectrum resources in wide ranges etc [**?**]. Among all applicable areas, UWB based accurate positioning and tracking in short range communication become popular since it provide resilient to multipath fading in hostile environment and outstanding robustness even in low signal-to-noise (SNR) situations [**?**]. In addition, the requirements of high data rate and limitations of battery supply lead the impulse-radio ultra-wide band (IR-UWB) to become a suitable communication technique in short range high data rate communication. As a result, applications based short-distance wireless sensor networks (WSNs) where large numbers of portable instrument, are widely deployed such as indoor positioning, surveillance, home automation, etc.

However, high data rate transmission puts high pressure on signal detection at ADCs in at UWB receivers, which indicates that the sampling rate becomes a main bottleneck and a critical issue for detecting signals under rich multipath channel. This paper focuses on solving this problem for IR-UWB indoor positioning system using a recent novel technique named compressed sensing. Compressed sensing (CS) is a novel paradigm which applies randomly sampling and sparse reconstruction with high successful rate. which enables a possible reconstruction of sparse signals from a relatively small group of random measurements. This indicates that the CS based IR-UWB system can possibly detect its high frequency sparse signals under a sub-Nyquist sampling rates that far below the Nyquist rate of twice the IR-UWB bandwidth. This effect is advantageous which releases the bottleneck of
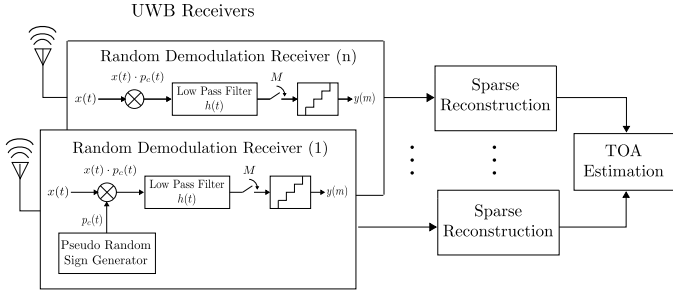
Fig. 4. Block diagram of CS Receiver implemented by random demodulator (RD). The components of RD includes a pseudo-random sign generator (PRSG), a low-pass filter (LPF), and a sub-Nyquist ADC

the large bandwidth constraints on ADC at UWB receivers, and consequently reduces storage limits and improves energy efficiency of the UWB positioning system.

1) CS Receivers for UWB Positioning: Recently some researches manage to embed the CS reconstruction algorithm, a revised orthogonal matching pursuit, at UWB receivers. This algorithm successfully improves the SNR of the received signal before it is sent to time of arrival (TOA) positioning stages, and consequently increase the performance of entire positioning accuracy [**?**]. Among these CS based systems, most of them implement a hardware structure termed the random demodulator (RD) [**?**]. As a result, new CS-UWB receivers successfully reduce the sampling rate to 30 percent of Nyquist rate or less.

In this system, each CS Receiver realizes the RD architecture that composed of a pseudo-random sign generator (PRSG), a low pass filter (LPF), and a sub-Nyquist rate analog-to-digital converter (ADC), shown in Fig.4. Then the transmitted UWB signals are collected by group of low-rate distributed ADCs only using a minimal sampling rate of $1.7K(log(N/K))$, where N stands for Nyquist rate and K is the sparsity in transmitted UWB signals [**?**]. Results in [**?**] demonstrates that the new system successfully improves positioning accuracy.

2) CS Pre-Filtering for UWB Positioning: On the other hand, some researches embed the CS technique at the UWB transmitter and regarded it as a better solution than CS receivers in terms of system hardware power efficiency. The new architecture contains CS pre-filtering at the transmitter, which is an implementation of the CS random projection before UWB signals are transmitted [**?**], shown in Fig.5. Followed by distributed sub-Nyquist rate ADCs, the downsampled signals are collected for TOA based algorithm. Simulation result shows that the new CS transmitter for UWB positioning successfully reaches positioning performance of CS receivers. Besides it meanwhile saves more energy cost since it contains less hardware mixers than CS receiver based UWB system.

In this model, the channel model is required for precise sparse reconstruction and researches embeds the framework of Bayesian by using the a priori distribution to convey the correlated information, which can further improve the precision of positioning.

IV. FUTURE WORKS ON CS BASED WIRELESS COMMUNICATION

A. Compressed channel estimation

High-rate data communication over a multipath wireless channel often requires that the channel response be known at the receiver. Training-based methods, which probe the channel in time, frequency, and space with known signals and reconstruct the channel response from the output signals, are most commonly used to accomplish this task. However, physical arguments and growing experimental evidence suggest that many wireless channels requires large signal space dimension due to large bandwidth or large number of antennas. For instance, wide band channel estimation need high frequecny sampling at receivers and provides heavy burden on data storage and power cost from extremely high sampling rate. In these cases, Compressed sensing applies multipath sparsity and present CS based estimating for multipath channels, which can potentially achieve a target reconstruction error using far less energy and, in many instances, latency and bandwidth than that dictated by the traditional least-squares-based training methods [**?**]. In our recent projection, the CS-based channel estimation which is related to an UWB positioning system is related.

B. Compressed Sensing Positioning

Various of applications of geographical positioning. In typical positioning system, In positioning, the target needs to send out signals to base stations or receive signals from base stations in order to determine the targets position, and the position of the target is calculated based on time of arrival (TOA), angle of arrival (AOA) algorithms etc. General applications of compressive sensing can be classified into 2 categorises [**?**]. One is to establish a linear mapping from the information indicating the positions to the observations and then carried out the positioning by using the CS to solve the unknowns in the mapping. Another one considers more traditional positioning techniques where the CS is used to enhance the signal SNR for positioning and also improve the power efficiency. The part 3 of this report demonstrates a CS-based UWB positioning system for indoor environment, where CS-based receivers or transmitter provide sub-Nyquist rate sampling methods for detecting high frequency UWB signals, which is proven to be effective on increasing signal SNR and position accuracy.

C. Compressed Cognitive Radio

The increasing demand for higher data rates in wireless communications in the face of limited or underutilized spectral resources has motivated the introduction of cognitive radio. Traditionally, licensed spectrum is allocated over relatively long time periods and is intended to be used only by licensees. Various measurements of spectrum utilization have shown substantial unused resources in frequency, time, and space [**?**]. The concept behind cognitive radio is to exploit these underutilized spectral resources by reusing unused spectrum in an opportunistic manner. In many cognitive radio applications, a wide band of spectrum must be sensed, which requires high sampling rates and thus high power consumption in the A/D converters. One solution to this problem is to divide the wideband channel into multiple parallel narrowband channels and to
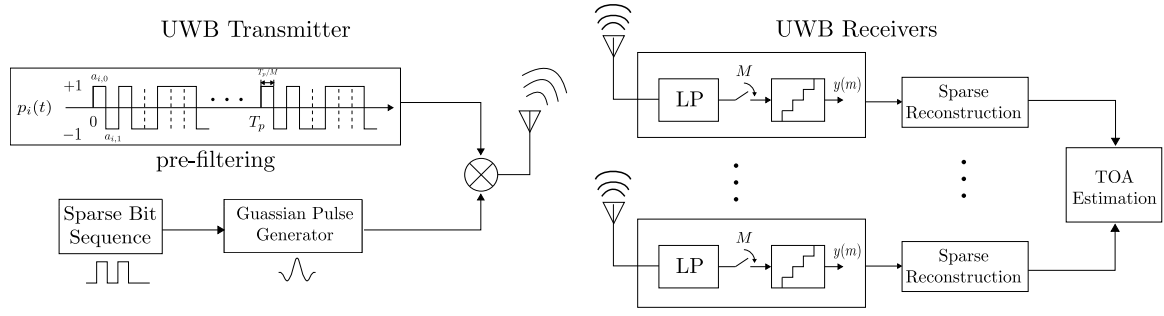
Fig. 5. Block diagram of low-rate CS pre-filtering (LRCSPF) UWB positioning system.

jointly sense transmission opportunities. opportunities on those channels. This technique is called multiband sensing. Another approach argues that the interference from the primary users can often be interpreted as being sparse in some particular domain, such as in the spectrum. In that case, compressive sensing can be used to lower the burden on the A/D converters. In cooperative versions of compressive wideband sensing have also been developed [**?**], [**?**]. Here, individual radios can make a local decision about the presence or absence of a primary user, and these results can then be fused in a centralized or decentralized manner.

D. Compressed Multiple Access

In wireless communications, an important task is the multiple access that resolves the collision of the signals sent from multiple users. Traditional studies assume that all users are active and thus the technique of multiuser detection can be applied. However, in many practical systems like wireless sensor networks, only a random and small fraction of users send signals simultaneously. In this case, we study the multiple access with sparse data traffic, in which the task is to recover the data packets and the identities of active users. Hence, based on the fact that in some special cases the useful information of active multiusers is sparse, general problem can be sovled under a CS framework efficiently. In particular, the feature of discrete unknowns will be incorporated into the reconstruction algorithm. The CS-based multiple access scheme will further be integrated with the channel coding.

REFERENCES