NANYANG TECHNOLOGICAL UNIVERSITY

PROGRESS REPORT FOR QUALIFYING EXAMINATION

PH.D

COMPRESSED SENSING FOR WIDEBAND SIGNAL PROCESSING

CHEN HAO
G1302488C

SCHOOL OF COMPUTER ENGINEERING

MARCH 2015

Abstract

Signal processing have been widely used in many computational based applications such as in the areas of image and video processing, smart sensors network, and other real-time processing embedded system. Many signal processing systems also involve signal sampling and conversion, where one of the most crucial limitations lies in the sampling theory (the Nyquist sampling rate or Shannon theory.)

Due to limited data processing and storing capability found in many embedded systems, excessive data samples will negatively affect their performance and power efficiency, and hence forms the bottleneck of the entire digital systems.

Compressive sensing (or compressed sensing) (CS) technique, is a relatively novel paradigm for signal acquisition that has great potential to address the problems mentioned above. The CS theory has been shown to be suitable for numerous computer science and electronic engineering based applications, where it is possible to overcome the restriction of the conventional sampling theory.

The main focus of the research work is to further extend the CS based signal acquisition techniques for CS based data processing, particular in the wireless based applications. Using this approach, the proposed directions of the research will encompass various areas such as data compression, data acquisition, data storage, data transmission, optimal recovery and processing. Specifically, the focus will be for wideband signal processing based application since the requirement of sampling wideband spectrum are much more demanding than narrow band signal.

This report presents the research motivation that are targeted for areas involving extremely high frequency acquisition and processing scenarios, such as the ultra wideband (UWB) positioning applications and cognitive spectrum sensing (CSS) systems.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

Signal processing community has successfully developed various algorithms to capture and extract useful information from real world signals. As a result, the digital signal processing (DSP) have been widely deployed in many computational applications in areas such as image processing, video conferencing, smart sensors network, and other real-time processing applications that involve data acquisition.

However, with the amount of data collected by such systems significantly increased in recent years, mainly due to the extremely high frequency and wider bandwidth communication systems, the challenges in acquiring signals at ever higher sampling rates becomes a serious and crucial problem in the front-end sampling circuits.

The Shannon/Nyquist sampling theorem [1] states that an original signal can recover without lost of information if the sampling can be done at least two times faster than the signals bandwidth. As a result, in many DSP related applications, e.g. digital image processing and video streaming, the Nyquist rate is so high that the amount of samples generated are enormous , making compression a necessary prerequisite to storage or transmission. In other words, the Shannon/Nyquist sampling requirments not only presents the most crucial limitations in fast sampling systems, it also produces large amount of data which leads to problems in efficient storing and transmission, that further lead to power management related issues.

1.1.1 Compressed Sensing

In recent years, a novel paradigm for signal processing and acquisition, the compressive sensing or compressed sensing (CS) technique, has been shown to have great potential to address the aforementioned sampling problems, and has also been demonstrated to be suitable for numerous computer and electronic engineering based applications. The CS utilises the notion that when the signals of interest contain only a relatively small number of significant components (we call it sparsity), known as sparse signal, sampling at the Nyquist rate is not necessary as well as inefficient.

[CH:Mar08]The following words are newly added for a more clear statement for CS

If we assume the original signal as $1 \times N$ vector, the observation behaviour can be modelled as a sensing matrix A with a size of $m \times N$, and $m < N$. Then the sampling system can be described as

$$y = Ax \tag{1.1}$$

Normally we cannot uniquely reconstruct the original signal x because the system defined by (1.1) is underdetermined. But using the sparse information, the CS theorem [2] states that it is possible to uniquely reconstruct the signal using $O(klog(N/k))$ samples, where k is the sparsity. This result so attractive in signal acquisition because the magnitude of required samples can be significantly decreased, so that the aforementioned sampling problem in Shannon/Nyquist sampling theorem can be solved by CS under the assumption of signal's sparsity.

Naturally, there are many signals directly have sparse information that suitable for the CS based sampling and reconstruction, such as images with low-rank matrix representation, and wireless channels with sparse impulse response coefficients. For those signals who has not apparent sparse structures, it is also possible to derive their approximate sparsity by using orthogonal basis representation or dictionary learning algorithms. As a result, most of physical signals can approximately presents their sparsity in the basis spanned domain and suitable for the CS applications [2–5].

1.2 Motivation and Objectives

[CH:Mar08]I put the topic CSP in the latter section of cognitive radio. Now in this section, I only talk about general motivations for CS based signal processing, especially for wideband wireless signals. The sentences are all revised based on your last feedback

The main focus of the research work is to further extend the CS based signal acquisition techniques for CS based data processing, particular in the wireless based applications. Using this approach, the proposed directions of the research will encompass various areas such as data compression, data acquisition, data storage, data transmission, optimal recovery and processing. Specifically, the focus will be for wideband signal processing based application since the requirement of sampling wideband spectrum are much more demanding than narrow band signals. The following subsections present the related objectives for the focus.

1.2.1 Wideband Analog-to-Digital Converters

Modern digital signal processing applications deal with a large amount of data, resulting in enormous numbers of observations and requiring high speed sampling in the front-end analog-to-digital converters (ADCs) of systems according to the Shannon/Nyquist sampling theorem. However, non-ideality error, design complexity, and energy cost are proportional to the high sampling rate and seriously affect systems' performance.

Since the CS has the potential to reduce the required number of observations, then it is possible to reduce the sampling rate of the ADCs, then the problem caused by high sampling rate can be overcome along with the serious effects in system performance.

1.2.2 Ultra Wideband Communication Systems

[CH:Mar08]As your comment, I omit some details for UWB-transmitters and our energy aware design.

Ultra wideband (UWB) based wireless communication is associated with features such as extreme wide transmission bandwidth, low-power consumption and shared spectrum resources [6]. However, sampling of such high frequency (in ranges of GHz) or wideband signals is a challenging problem.

The UWB signal displays sparsity when viewed in time domain (Figure 1.1). Therefore, it

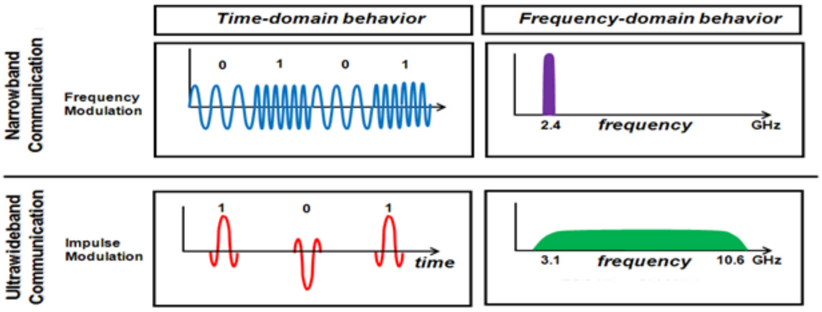


Figure 1.1: Comparison of UWB signals vs narrow band signals in time and frequency domain.

is possible to embed the CS into UWB systems to reduce the required sampling rate: the CS sensing matrix can be a combination of the multipath channel with pre-mixing waveform done at receivers; the original signal is the UWB signal with time domain sparsity.

To implement the CS on UWB, there is no need to add any complex hardware for down-conversion such as matched filters or oscillators. All needed is a random mixing procedure before ADC's input, and then the low rate sampling can be achieved based on the CS theory. As a result, compressed sensing would be appropriate to reduce the sampling rate while keeping flexibility for UWB systems.

1.2.3 Cognitive Radio

Cognitive Radio (CR) is used to address the limited spectrum resource that seriously restricts the rapidly increasing demand for more accessible bandwidth. However, studies [7] indicate that many spectrum resources are actually unused in terms of time and space domain. This is termed as the spectrum holes, which are displayed the idle channel in Figure 1.2. The idea of exploiting such spectrum holes dynamically has led to the concept of cognitive radio [8], where licensed spectrum dedicated to a primary network would be reused by secondary (cognitive) devices dynamically when the primary user is idle.

The cognitive devices cannot generate any harmful interference to the primary systems, so they needs to be kept at a distance from the primary system, or transmit at relatively low energy levels. However, the CR would need to be of sufficient wideband sensitivity to monitor the spectrum utilization situation.

As illustrated in in Figure 1.2 each secondary users needs to be fully aware of when and

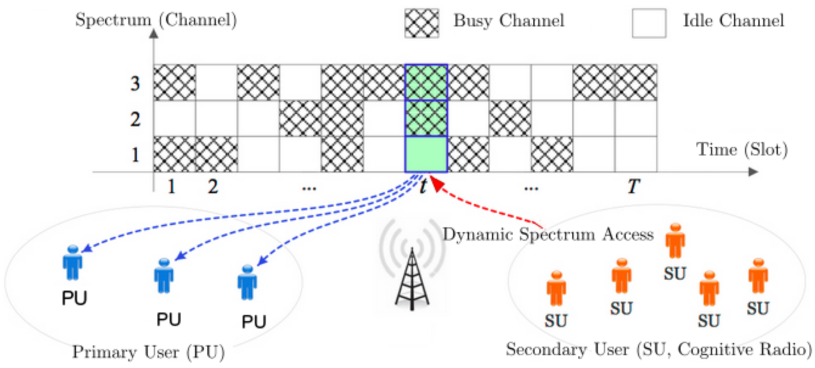


Figure 1.2: Example of spectrum reuse and dynamic spectrum access for cognitive radios.

which sub-bands are idle, such that dynamical spectrum access or hand-off can be performed efficiently. As a result, the cognitive radios have to detect the presence of weak and wideband primary signals at a low energy level, and this problem is proposed as an fundamental issue in spectrum sensing [9].

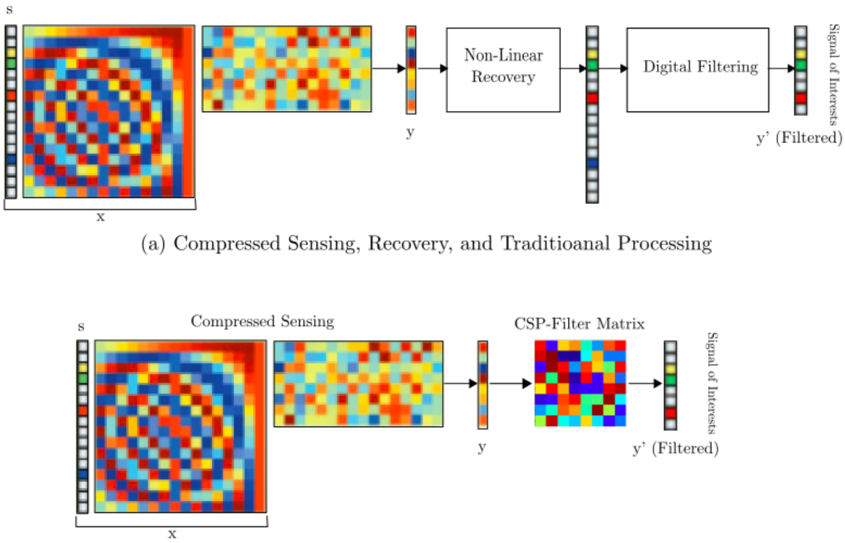
Due to the low spectrum utilisation, the wideband spectrum is inherently sparse, which meets the requirements for the use of the compressive sensing techniques. The CS spectrum sensing is hence a promising candidate to detect such wideband spectrum utilisation while using sub-Nyquist sampling rates [10].

1.2.3.1 Compressive Signal Processing

[CH:Mar08]In this part, I begin to talk about the CSP, and briefly introduce our recommend CSP-CR devices model

Most of tasks in cognitive spectrum sensing are related to detection, estimation, filtering, or classification rather than general CS task such as 'sample-then-recovery' procedures. As a result, a new research branch for the CS, the compressive signal processing (CSP) [11], can be proposed to simplify the complexity of general CS work through directly extracting useful information from the compressive samples without performing fully recovery, which is shown as Figure 1.3.

Thus the CSP becomes a suitable candidate for CR that avoid the latency and inefficiency encountered in CS recovery process, while at the same time enable the usage of flexible and programmable digital system.



(a) Compressed Sensing, Recovery, and Traditioanal Processing

(b) Compressive Signal Processing Framework

Figure 1.3: Filtering based scenario for comparison between (a) traditional digital signal processing via compressive measurements and (b) compressive signal processing (CSP) framework.

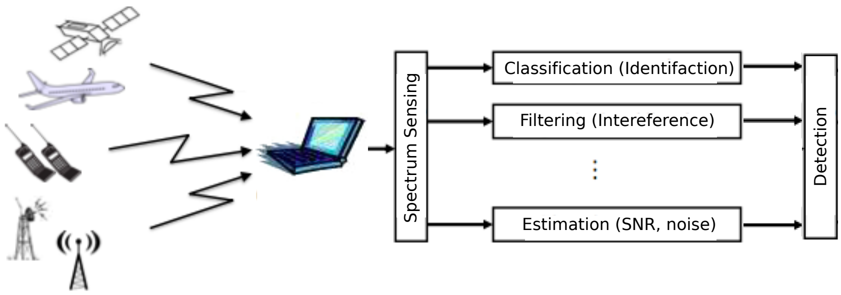


Figure 1.4: The CSP based cognitive spectrum sensing for hybrid primary signals

For instance, the Figure 1.4 presents the CSP based cognitive spectrum sensing (CSS), which is able to detect and process various co-existed primary signals by using multiple hybrid processing methods such as classification or filtering. In this way, the CSP based CSS can provide better sensing adaptivity and flexibility for modern heterogeneous CR networks.

1.3 Organization of the Report

The report is organized as follows:

1. Chapter 1 introduces our main topic for CS based signal processing, including the back-

ground, motivation and objectives in this report.

2. Chapter 2 shows presents the background of compressed sensing theory.

3. Chapter 3 shows a survey done for recent CS-ADC architectures.

4. Chapter 4 propose our related work in CS-UWB positioning systems.

5. Chapter 5 demonstrates the CS spectrum sensing in cognitive radios.

6. Chapter 6 draws a conclusion for the report and introduce our future research plans and areas that focus on compressive signal processing based cognitive spectrum sensing and access.

Chapter 2

Literature Review

2.1 Introduction

Compressed sensing is a novel techniques introduced by Donoho [12] and by Candes, Romberg, and Tao [2] in 2006, and since then has become a key concept in various areas of applied mathematics, computer science, and electrical engineering. It demonstrates that high-dimensional signals that allow a sparse representation through a suitable basis or, more generally, a frame, can be recovered by using effective algorithms from what was previously considered highly incomplete linear measurements. This chapter presents an introduction and a survey of compressed sensing.

2.2 Sampling System

In many real world applications, signals are usually sampled and stored in digital devices for further analysis, such as medical image imaging and CT tomography. From the discrete sampling and storage, the system are expected to be able to exactly reconstruct the original signals using observations as less as possible.

Assuming that the original signal is $1 \times N$ vector, the observation behaviour can be modelled as a sensing matrix A with a size of $m \times N$. Then the sampled data y, an $m \times 1$ vector, can be described as

$$y = Ax, \tag{2.1}$$

Then an important question arises:

- In order to reconstruct signal x, what is the minimum samples are needed in y?

Conventional linear algebra theory states that unique solution of (2.1) exists only if $m \geq N$. That is to say, at least N samples are needed in y. However, consider a case when the signal x is sparse, which means most of the values in x are zero or relatively small, can x be reconstructed using smaller numbers of observations y? (i.e. with $m < N$)

Conventionally, if $m < N$, the the equation (2.1) is underdetermined, such that multiple possible solutions exist. This means it is not possible to uniquely recover the original signal x. However, if the x is sparse, then the answer maybe Yes, based on the technique of Compressed Sensing (CS). That is, the CS allows a possible sparse informations reconstruction from a very few numbers of samples y ($m << N$).

The sparse characteristics can naturally exist (such as x) or generated by orthogonal basis decomposition (such as $x = As$ where s is sparse, A is the basis e.g. wavelets decomposition). As such, the CS approach provides a feasible method to theoretically reduce the number of observations required. The essence of the CS is hence to :

- Find the minimum number of samples for the sensing matrix.
- Design a reconstruction algorithm to uniquely recover original signals via those samples.

2.3 Sparsity

Consider a signal $x \in R^{N \times 1}$ which is sparse i.e., it has very few non-zero coefficients in the sense that

$$\|x\|_0 := \{i : x_i \neq 0\} \tag{2.2}$$

is small, or that there exists an orthonormal basis Ψ such that $x = \Psi s$, where x is a linear combination of few k basis chosen from Ψ, s is the corresponding coefficients of representing x in the domain spanned by the basis Ψ. k is then termed as the sparsity.

Naturally, there are many signals directly have sparse information that suitable for the CS based sampling and reconstruction, such as images with low-rank matrix representation, and wireless channels with sparse impulse response coefficients. For those signals who has not apparent sparse structures, it is also possible to derive their approximate sparsity by using orthogonal basis representation or dictionary learning algorithms. As a result, most of physical signals can approximately presents their sparsity in the basis spanned domain and suitable for the CS applications [2–5]. In this chapter, it is hence assumed throughout that x is already a sparse vector or can be sparse represented.

2.4 Sparse Reconstruction

When the x is sparse, an intuitive solution of the underdetermined equation (2.1) is equals to the solution of the problem (P0), which aims to find the minimum numbers of supports in x as follows:

$$P0: \quad min\|x\|_0 \quad s.t. \quad y = Ax \tag{2.3}$$

Then the following Theorem presents the sufficient condition to gain the unique solution:

Theorem 1 *Assume that $A \in R^{m\times N}$ is an average of the independent $2k$ column matrix. Then for any k-sparse vector x, the x can be exactly recovered by solving the problem P0 (2.3).*

Hence, according to Theorem 1, then there is a coding and decoding of (A, Δ) such that $x = \Delta(Ax)$, which means $2k$ times the number of observations is sufficient for reconstruction. However, solving the problem P0 is very difficult. In fact, P0 is an NP-complete problem [13].

Is there a more efficient decoding algorithms available? The answer maybe Yes if the condition of null space property (NSP) is satisfied ([14], Appendix). In this case, using the solution of the following problem P1, which is termed as Basis Pursuit (BP) or l1-norm minimisation, x can be uniquely reconstructed..

$$P1: \quad min\|x\|_1 \quad s.t. \quad y = Ax \tag{2.4}$$

To solve the problem P1, various types of recovery algorithms exist, such as convex optimisation and greedy algorithms, where each is having its own advantages and disadvantages. In this section, the widely used linear programming (LP) is used as an example to solve P1:

$$\begin{aligned} LP: \quad min\|t_1 + t_2 + \cdots + t_N\|_1 \quad t \in R^N \quad s.t. \quad y = Ax,\\ -t_j \le x \le t_j, \quad t_j \ge 0, \quad j = 1\cdots N \end{aligned} \tag{2.5}$$

[CH:Mar08]Here I move the details of NSP to the Appendix, because the RIP is the alternative for NSP, and the following discuss of CS recovery are all around the RIP

2.4.1 Restricted Isometry Property

The Section 2.4 and Appendix states that satisfying the NSP condition can enable a more efficient unique reconstruction through solving P1 rather than P0. However, how to verify the sensing matrix A satisfying the NSP condition is computational complex, which render the NSP approach not practically feasible. A more intuitive concept termed as Uniform Uncertainty Principle, or Restricted Isometry Property (RIP), is introduced in [2].

Definition 1 *Let A be an $m \times n$ sensing matrix. Then A has the Restricted Isometry Property (RIP) of order k, if there exists a $\delta_k \in (0, 1)$ such that*

$$(1 - \delta_k)\|x\|_2 \leq \|Ax\|_2 \leq (1 + \delta_k)\|x\|_2 \quad for\ all\ k\ sparse\ x \tag{2.6}$$

Theorem 2 *[2] A sufficient condition for the null space property (NSP) to hold and for all k-sparse signal is that RIP holds for 2k-sparse signals with*

$$\rho = \frac{\sqrt{2}\delta_{2k}}{1 - \delta_{2k}} \leq 1 \tag{2.7}$$

It follows that if $\delta_{2k} \leq \sqrt{2} - 1$ then NSP is satisfied, so that the unique reconstruction can be achieved by l1-norm minimisation (2.4).

According to the Theorem 2, an equivalent condition for the existence of a unique sparse solution of (P1) can now be stated in terms of RIP. If the sensing matrix satisfies the RIP with the $\delta_{2k} \leq \sqrt{2} - 1$, then for any k-sparse vector x, the Euclidian distance of projected x (i.e. Ax) and x still keep very close since the $\delta_{2k} \in (0, 1)$ is relatively small.

Bla bla..

2.5 Conclusion

In conclusion, everything has been bla bla all over.

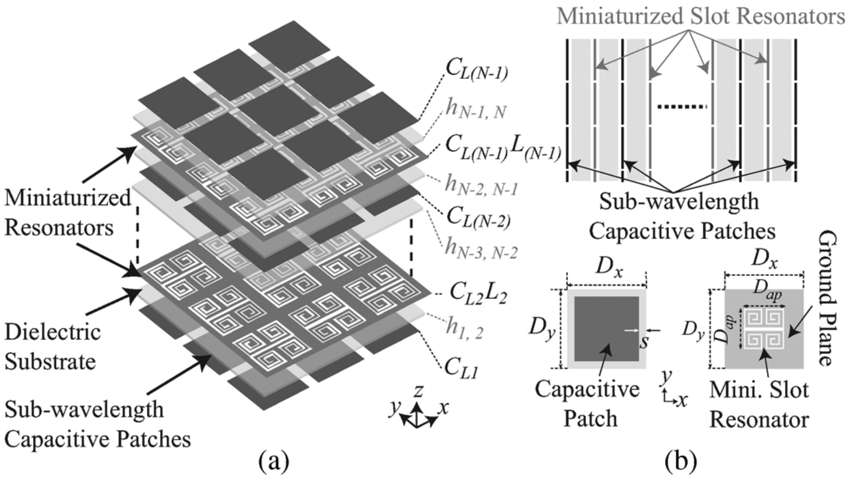


Figure 2.1: FigureA's caption.

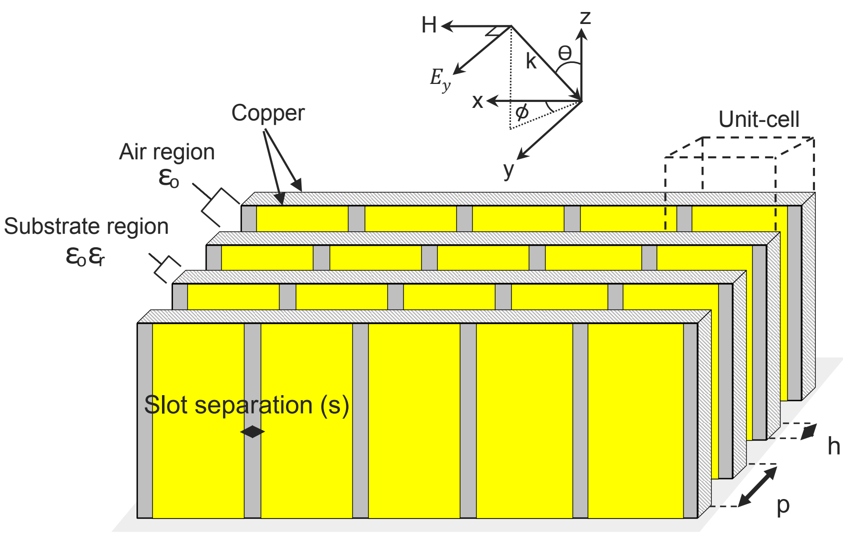


Figure 2.2: FigureB's caption.

Chapter 3

First Proposed Solution

3.1 Introduction

Introduction..

3.2 Our Proposed Solution

Our proposed solution is..

3.2.1 Design and Analysis

The design process starts by..

3.2.2 Design Example

The design example we proposed is..

3.2.2.1 Theory and Procedures

The theory states that..

3.2.2.2 Simulated and Measured Results

The results shown in..

3.3 Our Second Proposed Solution

Bla bla..

3.3.1 Design and Analysis

Bla bla..

3.3.2 Design Example

Bla bla..

3.3.2.1 Theory and Procedures

Bla bla..

3.3.2.2 Simulated and Measured Results

Bla bla..

3.4 Conclusion

In conclusion, and as before, most of it has been bla bla..

Chapter 4

Second Proposed Solution

4.1 Introduction

Introduction..

4.2 Description of the Solution

Bla bla..

4.3 Design Example

Bla bla..

4.3.1 Theory and Procedures

Bla bla..

4.3.2 Simulated and Measured Results

Bla bla..

4.4 Conclusion

And the conclusion is, same old same!

Chapter 5

Conclusion and Future Work

5.1 Introduction

Introduction..

5.2 Description of the Solution

Bla bla..

5.3 Design Example

Bla bla..

5.3.1 Theory and Procedures

Bla bla..

5.3.2 Simulated and Measured Results

Bla bla..

5.4 Conclusion

And the conclusion is, same old same! ———————————————————————————-

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In conclusion, as Brad Pitt and his chio wife said in [15], NTU researchers have no life. So, go get a life!

6.2 Future Work

A few suggestions are provided for related future works:

1. Bla..
2. Bla..
3. Bla..

List of Publications

[1] NTUnerd and NTUBigNerd, "An Approach to Reserve Tickets to Phuket While Your Supervisor is On Leave," *Whateverth Singaporean Conf. on Nerdy Yet Sexy Researchers (SgNYSR), 20whateverteen, pp. xxxx-xxxx.*

References

[1] A. D. Wyner and S. Shamai, "Introduction to communication in the presence of noiseby ce shannon," *Proc. IEEE*, vol. 86, no. 2, pp. 442–446, 1998.

[2] E. J. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *Information Theory, IEEE Transactions on*, vol. 52, no. 2, pp. 489–509, 2006.

[3] E. J. Candes and T. Tao, "Near-optimal signal recovery from random projections: Universal encoding strategies?" *Information Theory, IEEE Transactions on*, vol. 52, no. 12, pp. 5406–5425, 2006.

[4] M. Rudelson and R. Vershynin, "On sparse reconstruction from fourier and gaussian measurements," *Communications on Pure and Applied Mathematics*, vol. 61, no. 8, pp. 1025–1045, 2008.

[5] H. Rauhut, J. Romberg, and J. A. Tropp, "Restricted isometries for partial random circulant matrices," *Applied and Computational Harmonic Analysis*, vol. 32, no. 2, pp. 242–254, 2012.

[6] J. L. Paredes, G. R. Arce, and Z. Wang, "Ultra-wideband compressed sensing: channel estimation," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 1, no. 3, pp. 383–395, 2007.

[7] E. G. Larsson and M. Skoglund, "Cognitive radio in a frequency-planned environment: some basic limits," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 12, pp. 4800–4806, 2008.

[8] E. Axell, G. Leus, E. G. Larsson, and H. V. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," *Signal Processing Magazine, IEEE*, vol. 29, no. 3, pp. 101–116, 2012.

[9] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Allerton Conference on Communication, Control, and Computing.* Monticello, Illinois, 2004, pp. 1662–1671.

[10] H. Sun, A. Nallanathan, C.-X. Wang, and Y. Chen, "Wideband spectrum sensing for cognitive radio networks: a survey," *Wireless Communications, IEEE*, vol. 20, no. 2, pp. 74–81, 2013.

[11] M. A. Davenport, P. T. Boufounos, M. B. Wakin, and R. G. Baraniuk, “Signal processing with compressive measurements,” *Selected Topics in Signal Processing, IEEE Journal of*, vol. 4, no. 2, pp. 445–460, 2010.

[12] D. L. Donoho, Y. Tsaig, I. Drori, and J.-L. Starck, “Sparse solution of underdetermined systems of linear equations by stagewise orthogonal matching pursuit,” *Information Theory, IEEE Transactions on*, vol. 58, no. 2, pp. 1094–1121, 2006.

[13] G. Davis, S. Mallat, and M. Avellaneda, “Adaptive greedy approximations,” *Constructive approximation*, vol. 13, no. 1, pp. 57–98, 1997.

[14] R. A. DeVore, “Deterministic constructions of compressed sensing matrices,” *Journal of Complexity*, vol. 23, no. 4, pp. 918–925, 2007.

[15] B. Pitt and A. Jolie, “A guide to find your bride,” in *Proceedings of the Desperate Single Research Students in NTU, 20whateverteen*, 20whateverteen, pp. xxx–xxx.

.1 The Null Space Property

Our proposed solution is..