Compressed Sensing for Wideband Signal Processing

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A Process report submitted to the School of Computer Engineering

of Nanyang Technological University

in partial fulfillment of the requirements for the

degree of Doctor of Philosophy

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January 2015

Abstract

Acknowledgements

I would like to express my sincere and great gratitude to my supervisor, Assoc Prof. Vun Chan Hua for his kind guidance and encouragement. It is a great honor to work with him. I own my sincere appreciate to all my friends in CeMNet. Without their help, my first year study and research in NTU can not process well.

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Compressive Ultra Wideband Positioning

In this chapter, we focus on one of the popular ultra wideband (UWB) wireless application – impulse-radio ultra-wideband (IR-UWB) positioning system. The proposed UWB positioning system shows the potential advantages if we implement CS projection in the transmitter of the UWB positioning system according to the experiment result. Besides, Our proposed compressive UWB positioning presents a novel view of energy trade-off design, which is implemented by a low-rate random-projection at transmitters and low rate ADCs at receivers. It is clear that this design significantly reduce the peak frequency in the system with acceptable rate increase in receivers' ADC sampling rate.

4.1 Introduction

Ultra-wide band (UWB) communication is widely used in wireless communication and associated with features as extreme wide transmission bandwidth, low-power consumption, shared spectrum resources in wide ranges etc [1]. Among all applicable areas, UWB based accurate positioning and tracking in short range communication become popular since it provides resilient to multipath fading in hostile environment and outstanding robustness even in low signal-to-noise (SNR) situations [2]. In addition, the requirements of high data rate and limitations of battery supply lead the impulse-radio ultra-wide band (IR-UWB) to become a suitable communication technique in short range high data rate communication. As a result, applications based short-distance wireless sensor networks (WSNs) where large numbers of portable instrument, are widely deployed such as indoor positioning, surveillance, home automation, etc.

However, high data rate transmission puts huge pressure on signal detection at ADCs at receivers, which indicates the sampling rate becomes a main bottleneck to the IR-UWB system. This paper focuses on the IR-UWB indoor positioning, aims at solving the bottleneck of sampling rate by using a recent novel technique termed compressed sensing (CS), aforementioned in Chapter 2. It indicates that CS based IR-UWB systems can possibly detect high frequency sparse signals under a sub-Nyquist sampling rates that far below the Nyquist rate (twice the IR-UWB bandwidth). This result is advantageous which releases the bottleneck of the large bandwidth constraints on ADC at UWB receivers, and consequently reduces storage limits and improves energy efficiency in IR-UWB positioning systems.

4.1.1 Literature Review

Recently some researches manage to embed CS reconstruction algorithms, e.g. revised orthogonal matching pursuit, at UWB receivers. These algorithms successfully improve the SNR of the received signal before they are sent to the stages of time of arrival (TOA) based positioning algorithm. Consequently, this method increases the performance of entire positioning accuracy [3]. For hardware implementation of the new CS based UWB receivers, most of them apply the hardware structure termed the random demodulator (RD) [4], and consequently this new CS-UWB receiver successfully reduces the sampling rate significantly compared to the Nyquist rate [5].

On the other hand, some researches embed the CS technique mainly at UWB transmitters. They develop a waveform-based precoding transmitter, in order to fulfil a random projection of the UWB generated pulses [6]. Followed by sub-Nyquist sampling ADCs at receivers, the sampled signals are sent to TOA based algorithm for calculating the location of the UWB transmitter. Simulation results show that the new CS-UWB transmitter manage to significantly decrease the sampling rate of receivers successfully improves accuracy of traditional UWB positioning system.

However, both CS-UWB receivers and CS-UWB transmitters suffer from high-data rate random mixing operation, where PN sequence at mixers is required to reach the extremely high Nyquist rate (e.g. beyond 10 GHz). This requirement generates heavy burden on bandwidth of hardware mixers and additionally increases a high frequency noise.

4.1.2 Contribution

To solve this problem, our proposed work proposes an advanced low-rate CS-UWB positioning system: For the CS-UWB transmitter, it implements a relatively low-rate random projection matrix to slow down the mixing rate. For CS-UWB receivers, they sacrifices a small degree of compression ratio to keep equivalent performance as those positioning system using traditional CS-UWB transmitters. As a result, the trade-off between the random mixing rate and the sub-Nyquist sampling rate makes our system to become a more energy balanced, and consequently gains a better performance in entire power consumption and energy efficiency.

4.2 Model of Traditional UWB Positioning

Extremely wide transmission bandwidths of the IR-UWB offers outstanding multipath resolutions for accurate positioning in indoor environment. Consider a typical UWB indoor communication model where distributed UWB receivers (base stations) are placed in an area to detect the location of a moving UWB transmitter (tag). The transmitter periodically broadcasts Gaussian shaped pulse $p(t)$ through an indoor multipath channel, and receivers detect signals for time of arrival (TOA) based positioning calculation. The received signals can be described as (4.1):

$$r(t) = p(t) * h(t) + n(t) = \sum_{l=1}^{L} a_l p(t - \tau_l) + n(t) \tag{4.1}$$

where $p(t)$ is the transmitted Gaussian pulse, $n(t)$ stands for zero-mean additive white Gaussian noise (AWGN), and $h(t)$ refers to the standard UWB channel model denoted by IEEE 802.15.4a (4.2):

$$h(t) = \sum_{l=1}^{L} a_l \delta(t - \tau_l) \tag{4.2}$$

Here a_l and τ_l are the gain and delay corresponding to the i-th path in the channel model. The L defines the total number of propagation paths, and $\delta(t)$ is the Dirac delta function. Based on the fact that geometrical difference yields different time of arrivals, the received signals at the different receivers are collected for TOA based algorithm. At last accurate position of the transmitter is calculated based on TOA [7]. In addition, since both transmitted pulses and components of multipath channel can be regarded as approximately sparse, the received IR-UWB signals becomes sparse, and consequently the CS framework is applicable for UWB positioning [5].

4.2.1 Compressive Receivers

Typical recent researches embed the CS reconstruction algorithm at UWB receivers to improve the SNR of the received signal. As a result, it not only increases the performance of the positioning accuracy [3], but also reduces the sampling rate to a relatively low level compared to the Nyquist rate. Among these compressive receivers, most of them develop the random demodulator (RD) [4] as the main structure of CS based UWB receivers [5].

In this system, each compressive receiver realises the RD architecture that composed

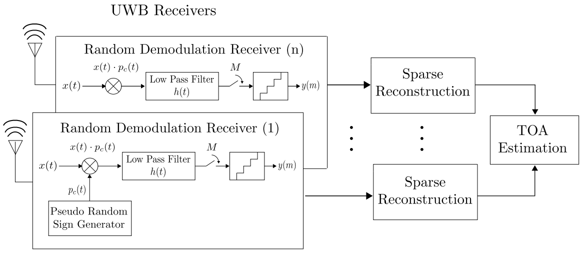

Figure 4.1: Block diagram of compressive receiver implemented by random demodulator (RD). The components of RD includes a pseudo-random sign generator (PRSG), a low-pass filter (LPF), and a sub-Nyquist ADC

of a pseudo-random sign generator (PRSG), a low pass filter (LPF), and a sub-Nyquist rate analog-to-digital converter (ADC), shown in Fig.4.1.

Then the transmitted UWB signals are collected by group of low-rate distributed ADCs only using a minimal sampling rate of $1.7K(log(N/K))$ [4], where N stands for Nyquist rate and K is the sparsity in transmitted UWB signals. Results in [8] demonstrates that the new system successfully improves positioning accuracy.

4.2.2 Compressive Transmitters

On the other hand, some researches embed the CS technique at the UWB transmitter and regarded it as a better solution than compressive receivers in terms of system hardware power efficiency. The new architecture contains a random tap FIR at the transmitter, which accomplishes the CS random projection before UWB signals are transmitted [6]. Followed by distributed sub-Nyquist rate ADCs, the down-sampled signals are collected for TOA based algorithm. Simulation result [6] shows that the new compressive transmitter is suitable for detecting the indoor channel model with higher accuracy than traditional TOA based method. This architecture is also suitable for indoor positioning because the transmitted signal pulse and channel model keep the same. Besides, it meanwhile saves more energy cost since it contains less hardware mixers than compressive receiver based

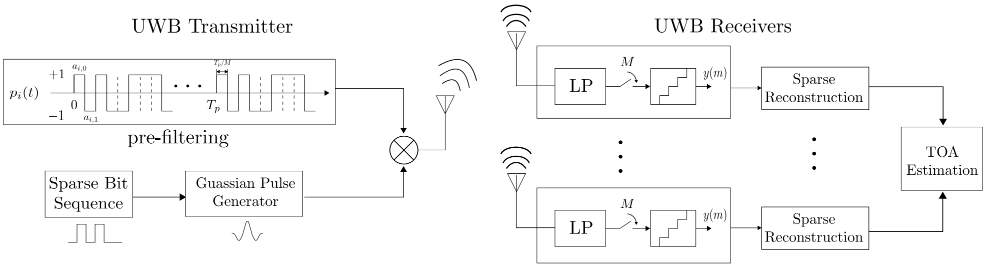

Figure 4.2: Block diagram of low-rate compressive random mixing UWB positioning system.

UWB system. However, the complexity of implementation a high data rate random tap FIR filters brings additional cost and becomes a main difficulty in applications.

4.3 Proposed Work: Energy Aware Random Mixing Transmitters

In this section, we follows the architecture of waveform-based pre-coding at the transmitter [6], we proposed a new compressive random mixing transmitters for UWB positioning. The main purpose of this system is to slow down the high rate mixing operation when building CS framework at transmitters. Simulation results demonstrate that our proposed CS-based UWB positioning scheme successfully decreases the high rate mixing operation by sacrificing a slight compression ratio ($< 10\%$) or small increase on average error ($<$ 1mm). The new CS-UWB TOA positioning system can achieve a much higher positioning accuracy than the traditional system in mm level. Future research will focus on how to reduce the reconstruction running time so that the real-time performance can be enhanced. This aim relates to the compressive signal processing that will be mentioned in the next chapter.

4.3.1 Introduction

As mentioned, both compressive receivers and compressive transmitters suffer from high-data rate random projection operation, where the alternative rate of pseudo-random sequence is required (often equal or above 5 GHz in such cases). This requirement generates a heavy burden on bandwidth of hardware mixers and high frequency noise.

In order to solve this problem but meanwhile preserve the energy efficient feature in compressive transmitters, this paper propose an low-rate compressive random mixing (CRM) UWB positioning system shown in Fig.4.2: At the transmitter side, the new system provides a relatively low-rate pseudo-random sequence to mix the generated Gaussian pulses. At the receivers side, it sacrifices the compression ratio (or sampling rate) to catch up with the equivalent performance as that in the compressive receivers based UWB system. As a result, the trade-off between the random projection rate and the sub-Nyquist sampling rate offers a more energetic balanced system, where no hardware devices suffers from the limitation brought from the high data rate sequence. In addition, the positioning accuracy is successfully improved compared to the original UWB system.

4.3.2 System Model

Based on the IR-UWB indoor positioning system model in (4.1), the new structure of our system can be regarded as shown in Fig.4.2. At the transmitter, the a pseudo-random sequence (PRS) whose variables are chosen from $-1, +1$ are generated using a relatively low sub-Nyquist alternative rate. When a UWB pulse generated, first it will be randomly mixed by the pseudo-random sequence, and then broadcasted to indoor environment from the transmitter. Afterwards, receivers directly down-sample the signals at a relatively low rate. Finally, after processing the sparse reconstruction (i.e OMP or BP), the original TOA based algorithm can work for indoor positioning. In addition, these steps at receivers can be executed pipelined as [5]. Hence, based on the workflow of the new architecture, the math model of this system can be represented as following matrix form (4.3):

$$y = D * H * P * x \tag{4.3}$$

where y are discrete sampled observations from low rate ADCs at receivers, and x is the generated UWB Gaussian pulses at traditional transmitters. Here H is the correspondingly Toeplitz matrix which presents the signal convolution using IEEE 802.15.4a model in (4.2), and F stands for the random projection step, which is a diagnose matrix whose variables are randomly chosen from $\{-1, +1\}$ but alternatives at a sub-Nyquist low rate. The D represents the downsampling behaviour. It is a $m \times N$ matrix ($m << N$) with 0-1 entries , and each of its rows contains a block of $\frac{m}{l}N$ contiguous ones, which has been used for simulation in [9]. Then $\hat{x}$ can be reconstructed from various sparse reconstruction algorithm such as BP, OMP and CoSaMP. At last, the recovered signal $\hat{x}$ is applied for TOA based positioning estimation.

4.3.3 Simulation Results

This experiment uses Matlab to evaluate the performance of the proposed system. In the simulation, the UWB waveform is a periodic simple pulse which is shaped by the second derivative Gaussian wave with a pulse duration of 1ns. The bandwidth of this signal is 8GHz. The UWB channel model is based on the IEEE 802.15.4a CM1 model for line of sight (LOS) indoor environment, and zero-mean additive white Gaussian noise (AWGN) is added to generate an average SNR of 10dB. The simulation results cover random points in an area of 10m $\times$ 10m $\times$ 10m space.

CS Basis pursuit denoising (BPDN) algorithm is used at the receiver to perform the reconstruction process. Figure 4 shows the results of the reconstruction successful rate against the receivers sampling rates. Existing RD based system achieves 100% successful reconstruction starting at 200MHz sampling rate, but requires 10GHz PN sequence at each receiver. Our proposed system use much lower rate PN sequence (1GHz or 500MHz)

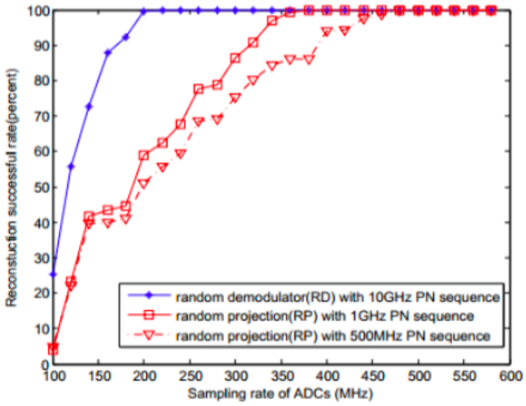

Fig. 4. Simulated performance comparison.

Table I: UWB Positioning Systems' Errors Comparison

	Sampling rate (ADC)	PN rate	Average error	Max error	Min error
Conventional	10GHz	none	27mm	46mm	8mm
RD	250MHz	10GHz	9mm	17mm	1mm
RP	500MHz	500MHz	9mm	19mm	1mm

at the transmitter and achieve 100% successful reconstruction at 350MHz and 500MHz sampling rate. Hence, accuracy of the lower rate PN sequence system can be improved by using higher sampling rate at the receiver. Therefore, the simulation result shows that the proposed system can apply trade-off among random projection rate, sub-Nyquist sampling rate and reconstruction rate. This design significantly reduce the peak frequency (Nyquist rate in mixing waveform or receiving end) in the system with acceptable rate increase in receivers' ADC sampling rate.

Table I compares the accuracy of the two CS based systems against conventional UWB based system which uses hypothetical 10GHz Nyquist sampling rate. Both CS based systems produces much lower errors then the conventional UWB based system, while the proposed RP system has similar performance as the RD based system.

4.4 Conclusion

The compressed sensing is proven effective to improve the accuracy of IR-UWB positioning. However, most of papers do not concern the design complexity and energy cost in their implementations, for instance, the random mixing waveform is of extremely high frequency (in GHz). Our proposed compressive UWB positioning present a novel view of energy trade-off design, which is implemented by a low-rate random-projection at transmitters and low rate ADCs at receivers. This design significantly reduce the peak frequency (Nyquist rate in mixing waveform or receiving end) in the system with acceptable rate increase in receivers' ADC sampling rate.

Chapter 5

Wideband Cognitive Spectrum Sensing

Cognitive Radio (CR) has been attracting many attention in recent researches with respect to the potential better utilisation performance of limited spectrum resources. In this chapter, we first propose an overview of cognitive radio networks. Then we focus on the bottleneck in its front-end sampling devices and investigate the typical CS framework for spectrum sensing in CR.

5.1 Introduction

As the wireless techniques keep fast developing, the limited spectrum resource seriously restricts the fast increasing demand for more accessible bandwidth. As a result, the dynamic spectrum access (DSA) becomes necessary and popular, which enables unused spectrum accessed opportunistically, shown in figure 5.1.(a).

Following this idea, the cognitive radio (CR) develops aiming at optimizing utilisation of idle bands for communications, without doing harm to the primary users (licensed spectrum) [10]. Correspondingly, the CR devices have to sense the environment (including spectrum usage, noise level etc) quickly and accurately, and reconfigure themselves to adapt the varying circumstance.

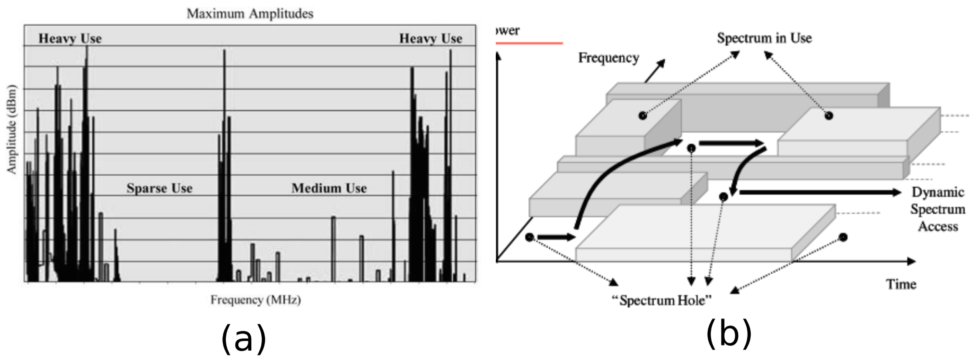

Figure 5.1: (a) The existence of unused spectrum resources; (b) The concept of spectrum holes

Therefore, in cognitive radios, the first and important procedure is to sense the unused bandwidth, termed as the spectrum holes and shown in figure 5.1.(b). If the band is detected as unused, the CR networks will use it for further communication. Otherwise, the CR moves to find other spectrum holes, or stays in the same band but avoid interference by changing its transmission power or modulation model.

However, trends of communicating requires higher frequency and wider bandwidth. As a result, signal acquisition significant is crucial according to the Nyquist sampling theory. What's worse, since CR should not generates additional interference to the licensed users, CR must limits its working power to a relatively low level (if CR do not change its modulation model). The demand for sensing with low power contradicts with the requirement for sensing in high sensitivity. Thus this contradiction makes the signal acquisition much more difficult.

In conclusion, spectrum sensing becomes one of the most crucial problem for cognitive radios, and it is still an open issue. Then, compressed sensing will be introduced to be embedded into traditional spectrum sensing algorithms, in order to solve the problem and enhance the overall performance.

5.2 Hypothesis Test Model for Spectrum Sensing

The aim of spectrum sensing is to decide whether a particular sub-band of the spectrum is available or not. In other words, the procedure is to discriminate based on two hypotheses in equation 5.1:

$$\begin{aligned} H0 : y[n] &= w[n] \\ H1 : y[n] &= w[n] + x[n] \end{aligned} \tag{5.1}$$

, where $x[n]$ is the primary user's signal, $y[n]$ is the vectorial observation, $w[n]$ is the noise, and n refers to time slots. The hypothesis 1 suggests that the primary user's signal exists, while hypothesis 0 suggests no. Typically, the decision is made by comparing a predetermined threshold with test statistic $\Lambda(y)$ in equation 5.2:

$$\Lambda(y) \underset{H_1}{\overset{H_0}{\lesseqgtr}} \alpha \tag{5.2}$$

Then the performance of a detector is quantified by the receiver operating characteristics (ROC) curve, which presents the probability of detection $P_D = Prob(\Lambda(y) > \alpha, H1)$ and false alarm probability $P_f a = Prob(\Lambda(y) > \alpha, H0)$.

5.3 Traditional Detection Method

5.3.1 Narrowband Detection

In this section, typical spectrum sensing approaches are introduced. Narrowband sensing algorithms can be suitably applied when the channel frequency response is flat. The following figure 5.2 demonstrates most typical architectures in narrowband spectrum sensing.

5.3.1.1 Matched filter

In the figure 5.2.(a), the matched filtering (MF) detector [11] is presented. when the signal to be detected is perfectly known (i.e. mean and variance), the optimal test statistic is

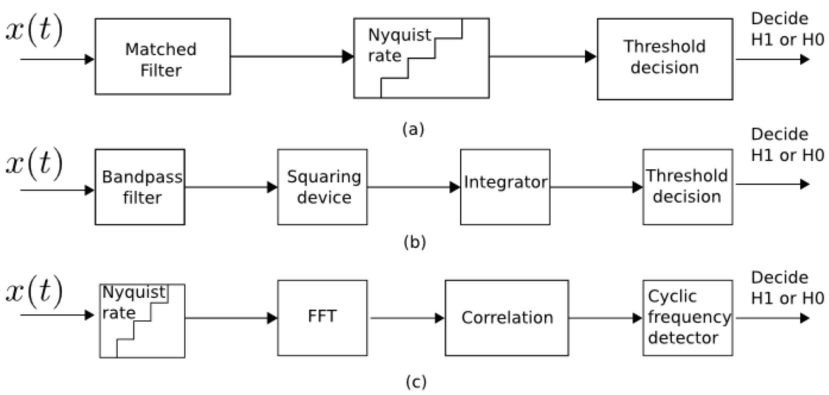

Figure 5.2: Block diagrams for traditional narrowband detection architectures: $a)$ matched filtering detector; $b)$ energy detector; $c)$ feature detector

produced by matched filter by correlating the received signal to a template. However, the signal cannot always be known in practise, so sometimes it's not applicable. Besides, the carrier synchronisation is also a remained difficult problem.

5.3.1.2 Energy detector

In the figure 5.2.(b), the energy detector (ED) [12] is presented. In the case where the signal to be detected does not present structure template, the ED can produce the optimal test statistic by directly analysis the power and variance of the received signal. The implementation of ED is simple, but it suffers from poor detection results in low SNR environment. Besides, the ED cannot distinguish different primary signals at the same time.

5.3.1.3 Feature detector

In the figure 5.2.(c), cycle-stationary feature detection (FD) [13] is presented. If discrimination for primary signals and higher detection performance are required, the FD exploits the cyclic non-stationary features from primary signals. The cyclic features can be found in many typical modulated signals, for instance, in the orthogonal frequency-division multiplexing (OFDM) contains cyclic features in correlation structure due to the cyclic prefix (CP) between transmitted data. However, the computational cost is relatively high and long running time delay is always existing.

5.3.2 Nyquist Wideband Detection

In the scenarios where the bandwidth is sufficiently larger than coherence bandwidth of channel, wideband sensing is more suitable than narrowband sensing. For instance, it can be used for sensing the ultra-high-frequency (UHF) TV band, ranging from 300 MHz to 3 GHz, while the narrowband sensing providing single binary decision over whole spectrum is always not suitable for identifying individual spectrum access opportunities. The following figure 5.3 demonstrates the typical architectures for wideband spectrum sensing and detection.

5.3.2.1 Multiband joint detector

In figure 5.3.(a), the multiband joint detector (MJD) [14] is presented. The MJD first uses serial-to-parallel conversion (S/P) to divide samples into parallel data streams, then it process the FFT to divide spectrum X(f) into groups of narrowband spectrum. Then each binary hypothesis detection is performed and joint optimised at last. The high sampling rate and lower speed of joint optimisation is the main bottleneck.

5.3.2.2 Wavelet detector

In figure 5.3.(b), the wavelet detector [15] is introduced. The wavelet analysis of power spectral density (PSD) can provide significant border symbols of two neighbour sub-bands, the aim of detection becomes a spectral edge detection problem. However, the high sampling rate is also the bottleneck.

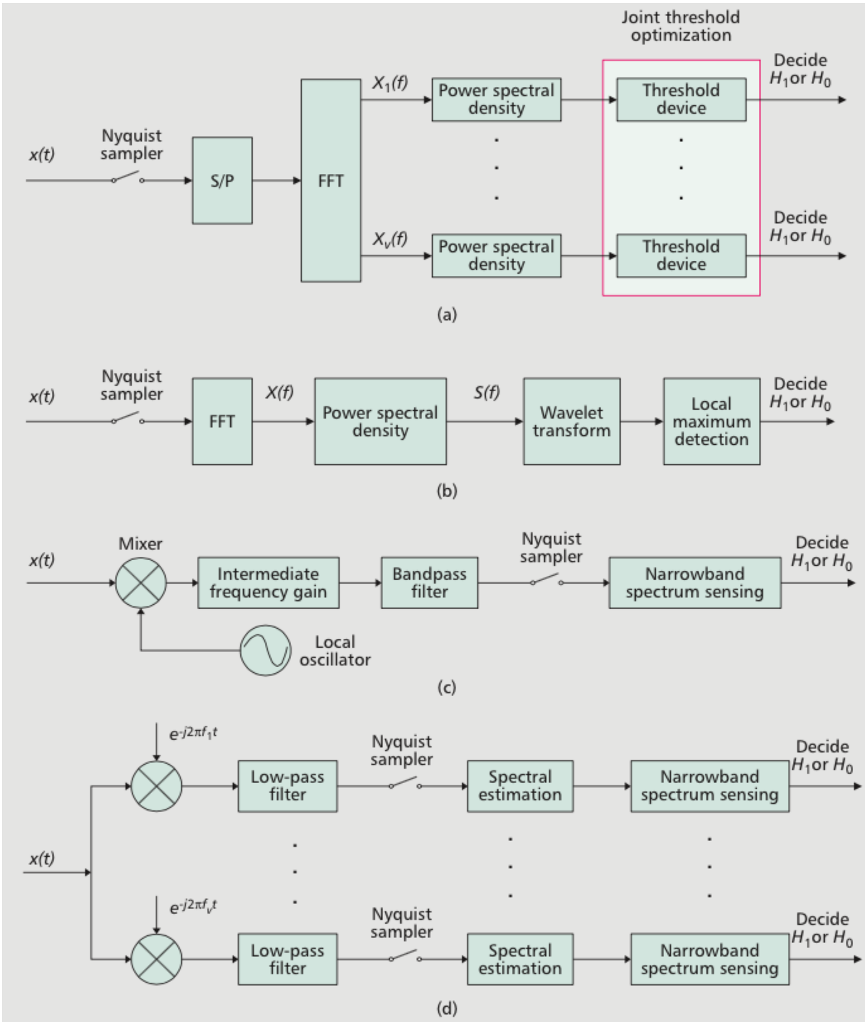

Figure 5.3: Block diagrams for Nyquist wideband detectors architectures: $a)$ multiband joint detector; $b)$ wavelet detector; $c)$ sweep-tune detector; $d)$ filter-bank detector

5.3.2.3 Sweep-tune detector

In figure 5.3.(c), the sweep-tune detector [14, 16] is displayed. This detector uses a special frequency mixing technique that 'sweep' across the frequency range of interest, to down-converts signals to a lower frequency. The adaptive local oscillator (LO) is used for 'sweep' procedure. However, the procedure of 'sweep' mixing generates too much time to wait.

5.3.2.4 Filter-bank detector

Also using the idea of down-conversion, the figure 5.3.(d) shows the structure of filter-bank detector [16]. Not only following the technique which 'sweep' mixing the interest of signals, it also applies parallel structure to speed up the processing time by using filter-bank. As a result, the time cost of mixing reduces but the implementation cost largely increase.

5.4 Sub-Nyquist Wideband Detection

Different from Nyquist wideband detection, the sub-Nyquist spectrum sensing applies the multi-coset (MC) sampling, multi-rate (MR) sampling, or compressed sensing (CS) to reduce the required sampling rate. Before talking about CS based sampling, first we take brief look at MC and MR sampling.

5.4.0.5 Multi-Coset Sampling

The multi-coset (MC) sampling [17] applies blocks of parallel consecutive samples with special time offsets to sample, so that each channel has a task of low-rate sampling. Then joint spectrum recovery and further detection can be performed. The main difficulty is how to perform sampling channel synchronisation with highly accurate time offsets. The quality of the specific offsets is crucial for robustness in its spectral reconstruction.

5.4.0.6 Multi-Rate Sampling

The multi-rate (MR) sampling [18] uses various sampling rates to wrap different sparse spectrum onto individual channels, and then use joint sparse spectrum recovery for further energy detection. Time synchronisation is no longer needed compared to the MC. But instead, the sacrifice is the hardware cost for parallel structure, as well as the increased sampling rate compared to original CS although the MR's sampling rate is still less than Nyquist rate.

5.5 CS based Detection Method

5.5.1 Introduction to Compressive Spectrum Sensing

Cognitive spectrum sensing is another typical application suitable for compressed sensing. The applicability of Compressive spectrum sensing (CSS)mainly lies in two aspects:

Sampling Rate The trend of higher frequency transmission is also suitable for cognitive radio, which leads to higher rate sampling rate at receivers. Thus it's reasonable to develop the CS based spectrum sensing techniques to reduce the sampling rate.

Flexibility and Energy The cognitive radio requires flexibility for sensing various types of signals (TV signal, cell phone, satellites etc) in a relatively wide bandwidth. However, normal wideband spectrum detection uses filtering or mixing for down-conversion (then low-rate sampling). This approach require difficult analog implementations such as adaptive local oscillator(LO) for filter-banks(FB). Inversely, if the CS is used, then the system can get wider sensing (frequency) ranges without the hardware of LO or FB.

Therefore, the CS become popular in cognitive spectrum sensing. The figure 5.4 demonstrates the typical architectures in wideband spectrum sensing, and briefly analyse CS detectors. The similar CS based architecture can also be viewed in chapter 2.

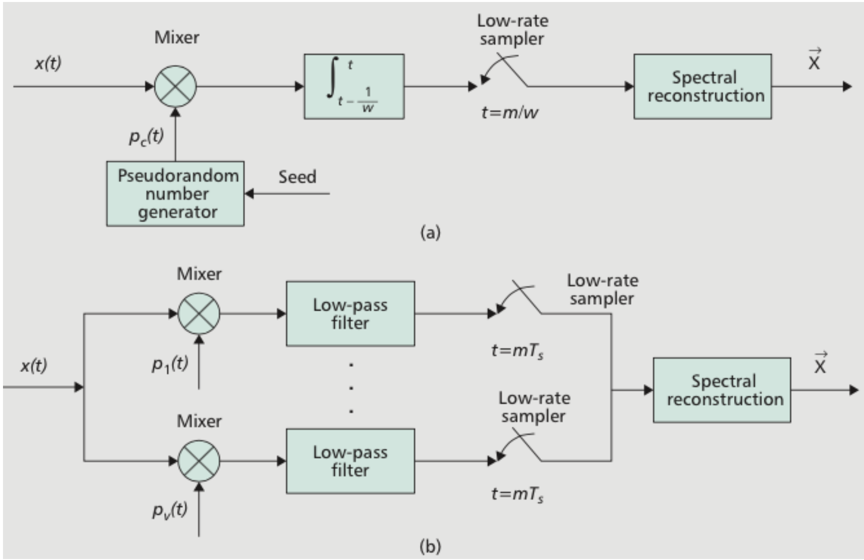

Figure 5.4: Block diagrams for compressive wideband sensing architectures: a) random demodulation based detector; b) modulated wideband converter-based detector

5.5.2 Compressive Detectors Overview

5.5.2.1 Demodulation based Detector

Figure 5.4.(a) presents the random demodulation (RD) based detector [9], which is an analog-to-information converter (AIC) for finite-length and discrete-time signals, and consists of pseudorandom wave generator, a mixer, and a low-rate ADC. The detailed architecture of RD is introduced in chapter 2. The reconstruction of RD sampled data involves $l1$-norm minimization (e.g. based BP, LASSO) or greedy method (e.g. OMP). This design is simple, but easily affected by model mismatches and design imperfections.

5.5.2.2 Modulated Wideband Converter based Detector

Figure 5.4.(b) displays the modulated wideband converter (MWC) based detector [19] for the case where multichannel signals are designed to be detected. The MWC can be considered as a parallel structure of RD, and its architecture is introduced in chapter 2. The

reconstruction of MWC involves the multiple measurement vector (MMV) sparse recovery which exploits the fact that the columns of original spectrum coefficients share the same sparsity pattern. Compared to the RD, MWC provides robustness against the noise and model mismatches.

5.5.2.3 Cooperative Compressive Spectrum Sensing

Cooperative versions of spectrum sensing is an open issue, which aims at solving the hidden terminal problem [10] and improve sensing accuracy. The cooperative version of compressive wideband sensing have been developed [20, 21]. Here, individual radios can make a local decision about the presence or absence of a primary user, and these results can then be fused in a centralised or decentralised manner. However, a greater cooperation gain can be achieved by fusing all the compressed measurements, again in a centralised or decentralised manner. In general, such measurement fusion requires that each cognitive radio knows the channel state information (CSI) from all primary users to itself [20], which is cumbersome. But recent extensions show that measurement fusion can also be carried out without CSI knowledge [22].

5.5.3 An Instance: CS based Cognitive Radar System

In [23] the CS is embedded to enhance the performance of cognitive radar that use wide operating frequency bandwidths for spectrum sensing and sharing. The compressive cognitive radar utilises the typical CS techniques, including the random demodulation (RD) for signal acquiring, basis pursuit de-noising (BPDN) algorithm and discrete cosine basis (DCT) for sparse reconstruction, and classical energy detector (ED) for hypothesis analysis.

The experimental results figure 5.5 shows the CS based radar has appealing performance in low sampling rate (using only $< 30\%$ of the total samples of the original signal) and low detection error in high SNR cases. But in low SNR environment, the proposed system still struggles and suffers.

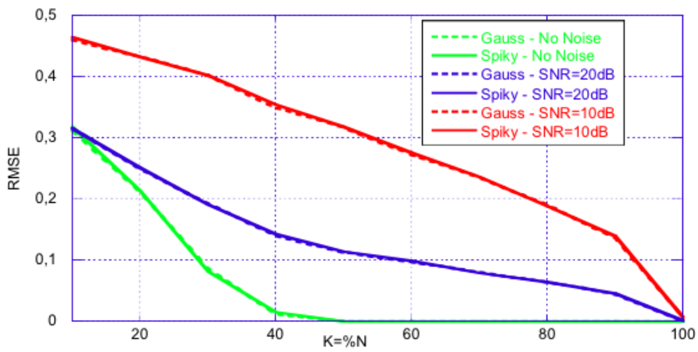

Figure 4 – RMSE for channel occupancy reconstruction as a function of *K* (percentage of *N*) for different SNR values.

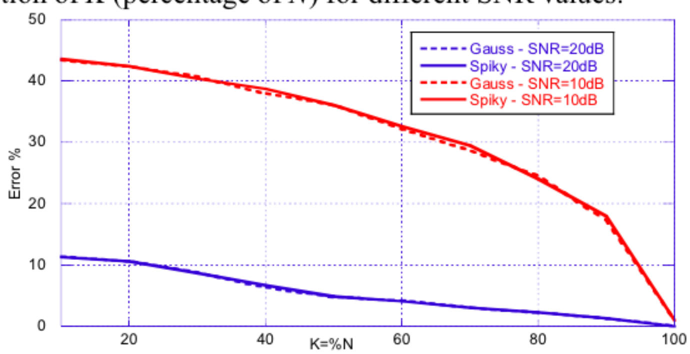

Figure 5 – Percentage error on the decision of channel occupancy as a function of *K* (percentage of *N*) for different SNR values.

Figure 5.5: Block diagrams for mean square error and error detection performance of the proposed compressive cognitive radar systems

5.6 Challenges and Discussion

The compressed sensing based wideband spectrum sensing for cognitive radio provide its outstanding feature in reducing the sampling rate. However, the corresponding drawbacks emerge in real-time ability and energy consumption, mainly due to its computational expensive non-linear reconstruction and energy consuming characters:

5.6.1 Long-Time Feedback Delay

Accuarcy is cruical for primary detection. Hence, if we applies CS for sampling, the convex optimisation (e.g.basis pursuit) is always needed for data recovery since it provides better accuracy and robustness comparing to greedy methods (chapter 2). However, convex optimisation is time consuming, which cause too long time to fast feed-back. Since feed back is very important for CR, which is responsible to avoid interference and quick reconfiguration, agility and reconfigurablity may reduce.

5.6.2 Mismatch for Traditional Signal Processing

The reconstruction algorithms for CS is non-linear, which indicates that the recovered data are not directly suitable for conventional digital signal processing where traditional recovering only requires cardinal sinc interpolation (linear process). This brings difficulty in directly reuse the traditional method for sensed data.

5.6.3 High Energy Cost

The heavy reconstruction for CS not only brings large time-delay, but also additional energy cost. Compared to linear recovery in traditional approaches, the CS based signal detection additionally required the block for spectrum recovery before further hypothesis detection.

5.7 Conclusion

Cognitive radio has been widely used and attracting many research attentions in its spectrum sensing techniques. In this chapter, traditional sensing approaches such as energy detection and feature detection are introduced. In order to solve the detection task for wideband and high frequency signals, compressed sensing based spectrum sensing (CS-CSS) is introduced and demonstrated. However, the compressively sampled data does not directly match the traditional processing algorithms.

Then here comes the question: what if we directly perform hypothesis detection without CS reconstruction? If the idea is achievable, the additional energy cost will be eliminated so that the entire energy reduces. Besides, not only detection, if we can expand this idea to filtering, estimation, then more intelligent-based scheme for cognitive radio can be supported in physical layer. The answer refers to our future research aims that shown in the next chapter.

Chapter 6

Future Work: Novel Signal Processing for CS Spectrum Sensing

In the last chapter 5, we have discussed the main drawback in compressed sensing based spectrum sensing (CS-CSS) which derives from the computational non-linear CS reconstruction. As a fact, the CS framework sacrifices the time and energy performance in reconstruction produce (in return, the sampling rate is reduced). In order to solve this problem, this chapter focuses on exploring the potential approach of directly analysing compressive measurements without fully CS reconstruction. The new approach is termed as compressive signal processing (CSP), and it will be used for compressive spectrum sensing (CSS) in cognitive radio network (CRN), so that the drawback may be overcome in our proposed CSP based CSS systems.

6.1 Introduction to CSP

The compressed sensing based wideband spectrum sensing for cognitive radio provides its outstanding feature in reducing the sampling rate. However, the corresponding drawbacks emerge in real-time ability and energy cost, mainly due to its computational complex non-linear reconstruction and energy cost in entire cognitive radio systems.

However, many signal processing applications such as detection, classification, estimation and filtering do NOT require entire signal reconstruction [24]. For instance, cognitive radios are aiming at processing the hypothesis detection rather than fully recovering the primary signals. In these cases, the aim of CS fully recovery is no longer necessary, so processing schemes like 'directly analysis without recovery' or 'partial recovery then analysis' become possible and applicable for cognitive spectrum sensing.

This novel idea derives from the compressive signal processing (CSP) [24], which aims at extracting information directly from compressive samples without fully recovery.

6.1.1 Literature Review

In [25], the shift retrieval problem for compressive samples is researched. Rather than recovering CS data then analysing the shifted distance, the author develops efficient algorithms and proofs for directly recognising the shift distance from CS data. Valsesia et al [26] develops the circulant sensing matrix based processing for compressive measurements, which displays an potential CSP application for convolution based models and is suitable for filtering or channel impulse response involved cases. Especially, for cognitive radio spectrum detection, a CSP based energy detector is designed in [27]. Further, Guo et al develops CSP for feature detector in [28] and pattern clustering is achieved.

6.1.2 Proposed Research

Since so far the CSP concept is always only defined in the theory level and has not been widely applied to CSS field, we believe there still exists many worthy research area and cases for CSP based spectrum sensing. In other words, CSP based cognitive spectrum sensing is still a relative new approach which applies the CS but throw away some CS drawbacks for CR spectrum sensing.

6.2 Potential CSP Applications In Spectrum Sensing

The following sections are organised as different function introduction for CSP based cognitive radio spectrum sensing, including filtering, detection, estimation. Some related potential application for CS-UWB and CS demodulation are also introduced.

6.2.1 CSP based Filtering

In scenarios where cognitive spectrum sensing requires filters, the design cost for analog filters are expensive. Here we may use CSP based filtering to transform the analog filter to digital filter, which omits the implementation of hardware design for filters.

6.2.1.1 Filter Domain Transform

Assume that the filter's impulse response is h (with length of N_h) and its corresponding matrix form is H, then H is a circulant sensing matrix.

Then, according to a CSP related theory in [26], it possible to exchange the H and Φ (compressed sensing matrix), sometimes in CS frameworks, the effect of analog filter equals the effect processed by digital filters. The compressive measurements y can be directly processed by filtering H without recovery in some cases 6.1 as follows:

$$\hat{y} = \Phi H x = H \Phi x = H y \quad , where \quad i \in [1, m - N_h + 1]. \tag{6.1}$$

, where the Φ is compressed sensing matrix with size of $m \times N$. Consequently, by moving the filtering from analog to digital part, large amount of hardware complexity could be omitted.

6.2.1.2 Interference filtering

In cases where useless information is redundancy for further recovery in CS framework, or specifically where some sub-bands are priorly known in the compressive cognitive spectrum sensing, system using CSP can regard those useless information as interference, which

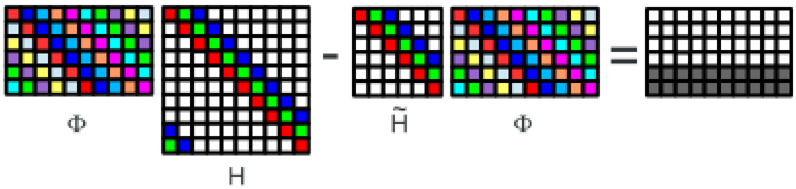

Figure 6.1: Block diagram of exchanging order of circulant matrix. H is corresponding impulse response of filter, Φ is the compressed sensing matrix. The exchanging sacrifice is the loss of rows in the results (matrix in the right side of equal sign).

applies CSP to achieve the procedure of 'sample-then-filter' to compressive data rather than sample-recover-then-filter it.

Assume a sparse signal $x \in R^N$ which consists of two components:

$$x = x_s + x_I, \quad x_S \in S_S \; and \; x_I \in S_I \tag{6.2}$$

, where the x_S contains the spectrum of interest and the x_I stands for the useless information. After the CS acquisition, we gain $y = \Phi x = \Phi(x_S + x_I)$. Then our aim is to wipe out the contribution of Φx_I from the observation y before recovering x. The following theorem provide the applicability of this idea:

Theorem 1. *[24] Suppose that Φ satisfies the δ-stable RIP for all $x_S \in \Sigma_S$ and $x_I \in \Sigma_I$, where I is a K_I dimensional subspace of R^N. Assume that Ψ_I is an matrix whose columns constructs an orthogonal basis for the K_I dimensional subspace. Define the matrix $P_\Omega = \Phi\Psi_I(\Phi\Psi_I)^\dagger$ and $P_{\Omega^\perp} = I - P_\Omega$. For any $x \in \Sigma_S \cup \Sigma_I$, we regard $x = x_S + x_I$, where $x_S \in s_S$ and $x_I \in s_I$, then*

$$P_{\Omega^\perp}\Phi x = P_{\Omega^\perp}\Phi\hat{x}, \quad \frac{\delta}{1-\delta} \leq \|P_{\Omega^\perp}\Phi\hat{x}\|_2 \leq 1 + \delta \tag{6.3}$$

The theorem 1 proofs that there exists a matrix $P_{\Omega^\perp}$ which is nearly orthogonal to the useless information x_I such that $P_{\Omega^\perp}x_I \approx 0$. Hence the matrix $P_{\Omega^\perp}$ is the filter for selecting the desired signal x_S. we can apply this theory to construct a filtering matrix in CS acquisition model if the support of the x_I is known.

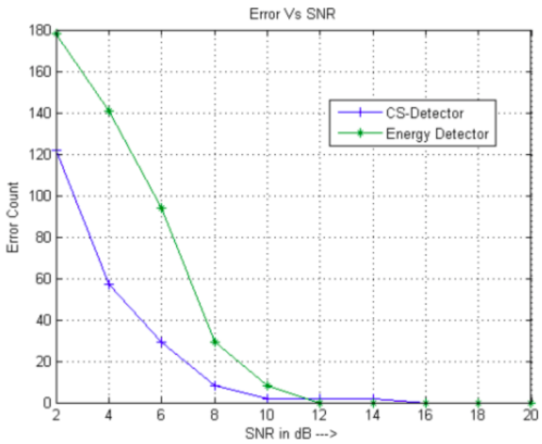

Figure 6.2: Block diagram of performance comparison between the CSP detector (compressive signal processing based detector) and ED (energy detector) from aspect of signal to noise ratio and detection error.

6.2.2 CSP based Detection

6.2.2.1 CSP based Energy Detector

Davenport et al [24] develops the theorem to directly build up hypothesis detection directly through compressed samples. Assume the detection is based on two hypotheses in

$$\begin{aligned} H0 &: y = \Phi w \\ H1 &: y = \Phi(w + s) \end{aligned} \tag{6.4}$$

, where s is the primary user's signal, y is the vectorial observation, Φ is compressed sensing matrix (e.g. the random demodulation), and w stands for the noise. Then the following equation 6.5 presents the structure of CSP based detector.

$$\Lambda(y) = (\Phi\Phi^T)^{-1}\Phi s \quad \underset{H_1}{\overset{H_0}{\lesseqgtr}} \quad \alpha \tag{6.5}$$

In [27], the experimental results of comparing CSP-detector and traditional energy detector (ED), is shown in figure 6.2. It has been shown that CS based Detector can provide better Error vs SNR performance.

One defect of this simulation result is that, the paper does not discuss blind sensing techniques for cases where the information of primary signal s is unknown to us. Thus further developing for CSP based blind sensing is a potential direction.

6.2.2.2 CSP based Cyclic Feature Detector

Guo et al [28] develops CSP for feature detector in and achieves pattern clustering. The work generates the compressive spectrum measurement by utilizing both the cyclic-stationary feature and sparsity prior knowledge at the spectrum sensing front end. Then the paper applies the compressive CSP without the need of signal or feature reconstruction.

However, as the author states that [28], the CSP feature detection model is still simple. So how to analyze the spectrum pattern recognition performance in a more complicated CRN environment is open issue. For instance, with large-scale SUs and non-Poisson PU traffic models. Also, More efficient machine learning schemes (such as information geometry) can be used to recognize the PUs signal patterns after obtaining the CS samples via CSP scheme.

6.2.3 CSP based Estimation

Since the signal-to-noise-ratio, sparsity order are crucial factors which seriously affects the performance of compressive detectors, CSP based estimation becomes popular for cognitive radio.

This technique make CR be able to directly predict the level of sparsity or noise, so that CR can vary the sampling rate or change the detection algorithms based on those predicted information.

6.2.3.1 Sparsity Order Estimation

For instance, the sparsity order of the spectrum occupancy is time-varying in CR networks. If CR intends to fully exploit the CS framework, the sampling rate of CS receiver should

Scenarios Sparsity Level	I SNR=2 dB Compressive	I SNR=2 dB Full	II DR=6.02 dB Compressive	II DR=6.02 dB Full	III SCN=4 SNR=0 dB Compressive	III SCN=4 SNR=0 dB Full
1	9.63	7.44	12.80	10.19	6.93	5.86
0.9	9.03	7.14	11.80	9.58	6.53	5.65
0.8	8.41	6.83	11.03	9.08	6.17	5.44
0.7	7.85	6.53	10.22	8.60	5.80	5.24
0.6	7.26	6.21	9.36	8.08	5.43	5.02
0.5	6.67	5.88	8.42	7.44	5.08	4.81
0.4	6.05	5.52	7.51	6.84	4.74	4.60
0.3	5.44	5.15	6.63	6.24	4.43	4.40
0.2	4.85	4.76	5.66	5.57	4.17	4.20
0.1	4.25	4.31	4.69	4.80	3.96	4.00
0	3.79	3.79	3.79	3.79	3.79	3.79

Figure 6.3: Block diagram of lookup table for sparsity order estimation through signal-to-noise ratio (SNR) and asymptotic eigenvalue probability distribution function (aepdf). For example, if the value of aepdf = 7.26 for the compressive case in SNR = 2db, it can be estimated that the sparsity order of spectrum occupancy is 0.6

keeping adjusting to sparsity orders.

Therefore, estimating the sparsity orders in very important, and the [29] develops an approach which analysis sparsity order through asymptotic eigenvalue probability distribution function (aepdf) from the covariance matrix of the compressively sampled primary signal. In details, the aepdf is related to the sparsity order via a lookup table in figure 6.3.

However, this paper does not apply the estimated information into further process in spectrum sensing to pursuit higher efficiency in adaptive sampling rate. So next we can apply the idea to further improvement of sensing efficiency.

6.2.3.2 Noise Level Estimation

For CR networks, the signal-to-noise ratio (SNR) affects the performance of spectrum detectors. For example, the energy detector (ED) is simple and fast, but with poor performance in low SNR scenarios; Feature detectors (FD) are complex and slow, but have strong robustness to noise. In [30], the author similarily build up a lookup table relating the noise estimation with the eigenvalue probability distribution function in covariance matrix of the compressively sampled primary signal.

Then if we have the access to estimate the SNR in CRN, we can achieve the following spectrum sensing approach in the future:

Hybrid Spectrum Sensing If we have known the SNR, we can design a more intelligent two stage detector. In the coarse stage, a quick search is done over a wide bandwidth, and in the fine stage, the sensing is done over the individual candidate sub-bands in that bandwidth, one at a time. the coarse stage is based on energy detection due to its fast processing. If the test statistics is larger than a predefined threshold, then the band is considered occupied. Otherwise, a fine stage is performed where a cycle-stationary detector is implemented due to its robustness at the low SNR regime. (For sparsity level, we can judge whether the CS based detector suitable if there exists dense channel occupancy by primary users)

Adaptive spectrum Sensing In case when a CR successfully know the primary signals' SNR, the CR can intelligently choose optimal detection algorithms based on SNR related performance of detectors. For instance, if the SNR is detected to be very low, then cycle-stationary detection algorithm can be used due to its robustness at the low SNR regime. Otherwise, the energy detector can be used since it's fast and accurate enough in high SNR regime. Extending this idea of selecting various of detector based on CSP estimation, we can select the optimal detector adaptively. This will provide future cognitive radio more flexibility to the varying environment.

6.2.4 Other Related Wideband Processing with CSP

6.2.4.1 CSP based TOA for UWB Positioning

Maximizing the cross-correlation between the two signals is one of the key steps in time-of-arrival (TOA) algorithms which have been applied in compressive UWB positioning in chapter 4. In that system, the procedures at receiving end can be abstracted as (1) low-rate sampling, then (2) CS fully reconstruction, finally (3) TOA algorithm based locationing.

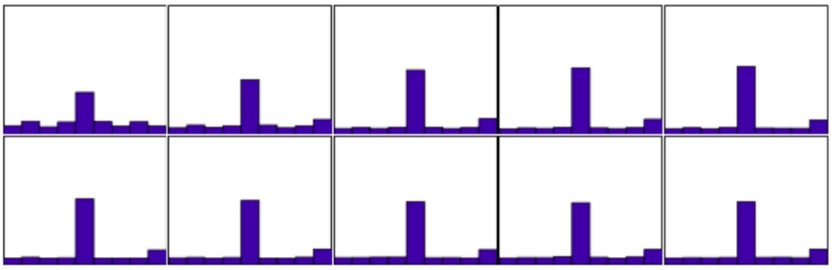

Figure 6.4: Histogram for the estimated shift in SNR = 2. From left to right, top to bottom, compression ratio = 0.1 ... 1. The true shift was set to 5 in all trials

However, the (2) CS fully reconstruction may sometimes Not necessary if we borrow the theorem.

According to [25], where the shift retrieval problem for compressive samples is investigated, we may directly recognising the shift distance from CS data without recovery. Now the thoerem has not been embedded into UWB positioning, so the CSP-TOA positioning is a possible research application in the future.

6.2.4.2 CSP based Demodulation

A compressive sensing phase-locked loop (CS-PLL) [31], is designed for directly extracting the phase and frequency from compressively sampled modulated signal $without$ sparse recovery. Since the restricted isometry property (RIP) of CS ensures that the standard inner product between $x[n]$ and $u[n]$ is approximately the same as that in the compressively sensed version produced by $y[m]$ and $u[n]$ [24], hence, the inner products in the standard PLL and that in the CS-PLL are nearly the same, ensuring the consistence between the standard PLL $\theta[n]$ and the CS-PLL's output $\theta[m]$. The presentation of the $\theta[m]$ can be presented as $\theta[m] = \sum_k y[k]v[k]h[m-k]$, where the index m indicates the lower sampling rate compared with the Nyquist rate index n. In addition, the compressive sensing operations Φ which apply the RD's architecture consist of a input pseudo-random sequence, a mixer, a integrator, and a low-rate sampling ADC, which is the same as the random demodulator's

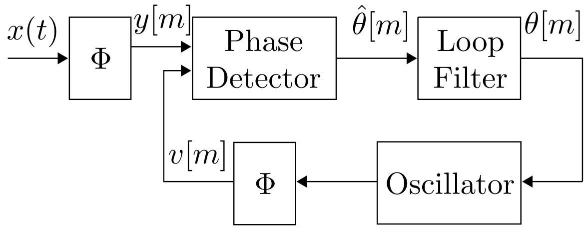

Figure 6.5: Block diagram of the compressive sensing phase-locked loop (CS-PLL). The components includes the pre-CS sampler, phase detector, loop filter, oscillator and the feedback CS multiplier

architecture in chapter 2.

This CS-PLL has many application fields if the FM demodulation is necessary in compressive sensed data. When we successfully apply this technique, we don not need recovery, but directly demodulate original information from the received modulated signals.

6.3 Conclusion

In the this chapter, we have followed the discussion of the main drawback in compressed sensing based spectrum sensing (CS-CSS) which derives from the computational non-linear CS reconstruction, and then propose the idea of signal processing directly on compressively sampled data (CSP) without fully recovery. Then we introduce our future potential work which will embed CSP into cognitive spectrum sensing, including CSP based filtering, CSP based detection, and CSP based estimation. Some other possible CSP applications in UWB positioning and signal demodulation are also introduced. We hope that by utilise the CSP in spectrum sensing, a high real-time performance, high flexible ,energy efficient hardware could be achieved in the future.

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