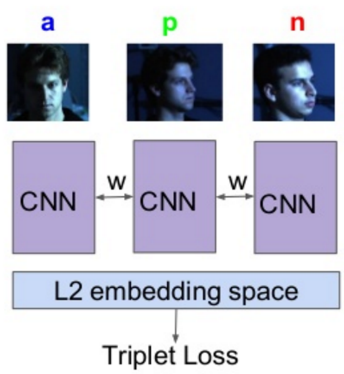
Triplet Loss

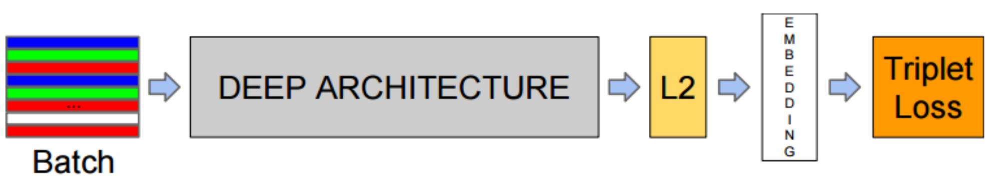
Introduction

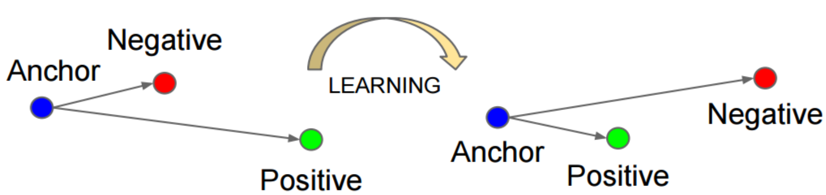
The triplet loss is ofen uses on perseon re-indentification or face recognition problems, the idea of triplet loss is to receive 3 scores, with 1 pair related to the score of similar classes and an extra score related to a "negative class". The training objective will minimize this loss by making related scores similar while making the difference between the related scores and the negative score larger.

$$Loss_{triplet} = \sum_{i=1}^{N} \left[L_{2_{distance}} \left(f_{anchor}[i] - f_{positive}[i] \right)^2 - L_{2_{distance}} \left(f_{anchor}[i] - f_{negative}[i] \right)^2 + \alpha \right]$$

The triplet loss has 3 inputs, which are the output of a neural network for 3 classes, a class called "anchor class", another class that is positive, and a negative class. This concept for person reindentification or face recognition is important because for example the anchor class could be a person, and positive class could be the same person under other circunstance, and a negative class.







Here the inputs of the triplet loss comes from 3 different networks what share the same weights, but this also can be achieved by using a single network and building the batch with the anchor, positive and negative class. This also implies that if the triplet loss has 3 inputs during backpropagation we should return 3 gradients.

Euclidian distance (Or L2 distance)

Like other distances (ie L1 distance) the L2 distance return a scalar that represent how 2 vectors(p,q) are similar to each other.

$$p := [1, 2, 3] = [1, 2, 3]$$

$$q := [1.1, 2, 3.3] = [1.1, 2, 3.3]$$

$$L_2 := (a, b, N) \rightarrow \sqrt{\sum_{i=1}^{N} (a[i] - b[i])^2} = (a, b, N) \rightarrow \sqrt{\sum_{i=1}^{N} (a_i - b_i)^2}$$

$$L_2(p, q, 3) = 0.3162277660$$

$$L_2(p_v, q_v, 3) = \sqrt{(p_{v_1} - q_{v_1})^2 + (p_{v_2} - q_{v_2})^2 + (p_{v_3} - q_{v_3})^2}$$

On the context of loss functions we use the square l2 distance to actually cancel the squar-root term.

$$\| p_i - q_i \|_2^2 = \left(\sqrt{\sum_{i=1}^{N} (a[i] - b[i])^2} \right)^2$$

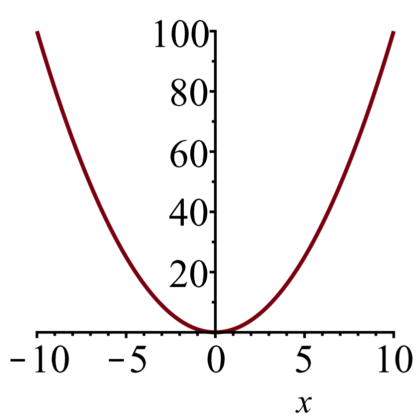
This sometimes causes confusion because now it will be called L2-norm loss

$$L_{2_{loss}} := (a, b, N) \rightarrow \sum_{i=1}^{N} (a[i] - b[i])^2 = (a, b, N) \rightarrow \sum_{i=1}^{N} (a_i - b_i)^2$$

$$L_{2_{loss}}(p, q, 3) = 0.10$$

$$L_{2_{loss}}(p_v, q_v, 3) = (p_{v_1} - q_{v_1})^2 + (p_{v_2} - q_{v_2})^2 + (p_{v_3} - q_{v_3})^2$$

Also for ilustration purposes this is how the plot of the L2-norm loss looks like $L_{2_{loss_{plot}}} := x^2 = x^2 \rightarrow$



There is a variant of the L2-norm loss called MSE (Mean squared error) loss which divide the L2-norm loss by the batch-size, the idea is to decouple the batch size from the loss value.

$$MSE_{loss} := (a, b, N) \rightarrow \frac{1}{N} \cdot \left(\sum_{i=1}^{N} (a[i] - b[i])^2 \right) = (a, b, N) \rightarrow \frac{\sum_{i=1}^{N} (a_i - b_i)^2}{N}$$

$$MSE_{loss}(p_v, q_v, 3) = \frac{1}{3} \left(p_{v_1} - q_{v_1}\right)^2 + \frac{1}{3} \left(p_{v_2} - q_{v_2}\right)^2 + \frac{1}{3} \left(p_{v_3} - q_{v_3}\right)^2$$

Get the derivative of L2-norm loss w.r.t to input

Before we delve into the gradient of the triplet-loss let's investigate how to find the derivative of the L2-norm function, by definition the idea of the norm function is to give a positive scalar number that represent the size of a vector, so we can consider the following simplification.

$\sum_{i=1}^{N} (a[i] - b[i])^2 \Rightarrow \sum_{i=1}^{N} u^2$, where u will be a function that calculate the difference $(a[i] - b[i])$ vector

Let's just lay out some derivatives rules, consider u as a function

Scalar multiple rule: $\frac{d}{dx} (\alpha \cdot u) = \alpha \frac{d}{dx} u$

Sum Rule: $\frac{d}{dx} \sum u = \sum \frac{d}{dx} u$, this is nice because I don't know yet how to calculate derivatives with sum properly on maple

$$sum \cdot \frac{\partial}{\partial a} (a - b)^2 = sum\ (2\, a - 2\, b)$$

$$sum \cdot \frac{\partial}{\partial b} (a - b)^2 = sum\ (-2\, a + 2\, b)$$

Putting all together

Now let's put together the L2-norm inside the triplet loss, and also push outside all the $\sum$ operators

$$f := (anchor, positive, negative, \alpha) \rightarrow \left[(anchor - positive)^2 - (anchor - negative)^2 + \alpha \right] =$$

$$(anchor, positive, negative, \alpha) \rightarrow \left[(anchor - positive)^2 - (anchor - negative)^2 + \alpha \right]$$

$$\frac{d}{d\, anchor} f(anchor, positive, negative, \alpha) = [-2\, positive + 2\, negative]$$

$$\frac{d}{d\, positive} f(anchor, positive, negative, \alpha) = [-2\, anchor + 2\, positive]$$

$$\frac{d}{d\, negative} f(anchor, positive, negative, \alpha) = [2\, anchor - 2\, negative]$$

Now I just isolate the factors from the expression:

$$factors(-2\, positive + 2\, negative) = [2, [[negative - positive, 1]]]$$

$$factors(-2\, anchor + 2\, positive) = [-2, [[anchor - positive, 1]]]$$

$$factors(2\, anchor - 2\, negative) = [2, [[anchor - negative, 1]]]$$

Gradients of Triplet loss

Due to the fact that the triplet loss wants to make the distance between $[f_{anchor}, f_{positive}]$ small while making the distance $[f_{anchor}, f_{negative}]$ big by an α factor the gradients will be zero if the expression:

$$\left[L_{2_{distance}}(f_{anchor} - f_{positive})^2 - L_{2_{distance}}(f_{anchor} - f_{negative})^2 + \alpha \right]$$

gives a negative number (<=0), because it would imply that $\left[L_{2_{distance}}(f_{anchor} - f_{positive})^2 \right]$ is already bigger than $\left[L_{2_{distance}}(f_{anchor} - f_{negative})^2 + \alpha \right]$

Gradient w.r.t anchor input

$$\frac{\partial}{\partial f_{anchor}} Loss_{triplet} = \sum_{i=1}^{N} \left[2(f_{negative} - f_{positive}) \right]$$

Gradient w.r.t positive input

$$\frac{\partial}{\partial f_{positive}} Loss_{triplet} = \sum_{i=1}^{N} \left[-2(f_{anchor} - f_{positive}) \right]$$

Gradient w.r.t negative input

$$\frac{\partial}{\partial f_{negative}} Loss_{triplet} = \sum_{i=1}^{N} \left[2(f_{anchor} - f_{negative}) \right]$$

References

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