Least-Square Learning Rule：

$X_j =（X_0, X_1, 、、、X_n）^t$ （i.e. $X = \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_n \end{bmatrix}$） $1 \le j \le p$

$W_0 =（W_0, W_1, \cdots W_n）^t$ （i.e. $W = \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_n \end{bmatrix}$）

$Net_j = \sum_{i=0}^{n} W^t X_j$ （i.e., $Net_j = W_o X_o + W_1 X_1 + \cdots W_n X_n$）

By applying the least-square learning rule the weights is ：

$$\Rightarrow \begin{matrix} W^* = R^{-1}P \\ RW^* = P \end{matrix} \quad \text{where} \quad \begin{cases} R：\text{correlation matrix} \\ R = \dfrac{R'}{P} \quad R' = R_1' + R_2' + \cdots + R_P' = \sum_{j=1}^{P} X_j X_j^t \\ P = \dfrac{\sum_{j=1}^{-p} T_j X_j^t}{P} \end{cases}$$

EX： $X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $X_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T_1=1$ $T_2=1$ $T_3=-1$

	X1	X2	X3	Tj
X1	1	1	0	1
X2	1	0	1	1
X3	1	1	1	-1

Sol: 先算 R

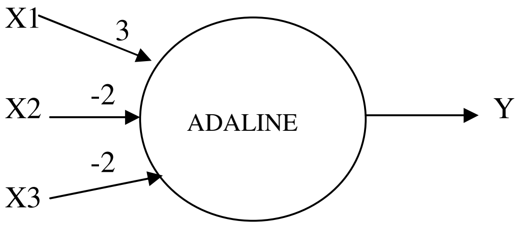
$$\left.\begin{aligned} R_1' &= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}(110) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ R_2' &= \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}(101) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \\ R_3' &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(111) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \end{aligned}\right\} \quad R = \frac{1}{3}\begin{pmatrix} 3 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

$$\Downarrow$$

$$= \begin{pmatrix} 1 & 2/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 2/3 & 1/3 & 2/3 \end{pmatrix}$$

$$\left.\begin{aligned} P_1^{\,t} &= 1\cdot(1,1,0) = (110) \\ P_2^{\,t} &= 1\cdot(1,0,1) = (101) \\ P_3^{\,t} &= -1\cdot(1,1,1) = (-1-1-1) \end{aligned}\right\} \quad P^t = \frac{1}{3}(100) = \left(\frac{1}{3},0,0\right)$$

$$R\cdot W^* = P \Rightarrow \begin{pmatrix} 1 & 2/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 2/3 & 1/3 & 2/3 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left\{\begin{aligned} 3W_1 + 2W_2 + 2W_3 &= 1 \\ 2W_1 + 2W_2 + W_3 &= 0 \\ 2W_1 + W_2 + 2W_3 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} W_1&=3 \\ W_2&=-2 \\ W_3&=-2 \end{aligned}$$



Verify the net:

代入（1,1,0）

net=$3X_1$-$2X_2$-$2X_3$=1　　Y=1　ok

代入（1,0,1）

net=$3X_1$-$2X_3$=1　　Y=1　ok

代入（1,1,1）

net=$3X_1$-$2X_2$-$2X_2$= -1　　Y=-1　ok

$< \varepsilon_k^2 > \Longrightarrow \Phi$ minimum best solution

練習:計算反矩陣的方法