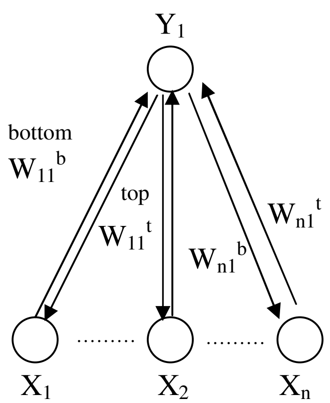
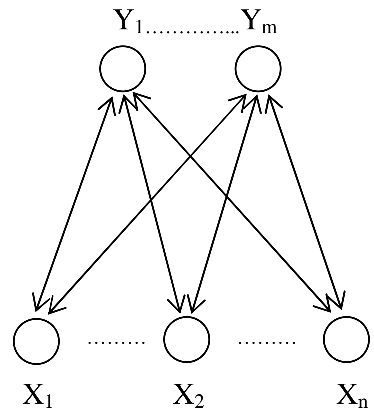
Adaptive Resonance Theory

ART1：takes only binary input
ART2：takes continuous or binary input

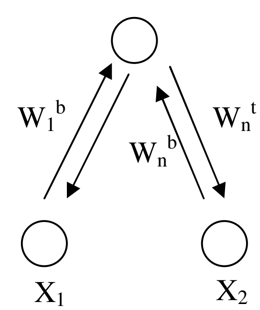
1. ART is proposed by grossberg in 1976.
2. This network features：
   ①the bottom-up competitive learning & the top-bottom ouster pattern learning.
   ②the unsupervised learning network
   ③the forward & backward process until the message resonate
   ④the dynamic generating the output node, when unfamiliar input is fed in
3. The network architecture:





Input layer： $X \in \{0,1\}$
Output layer：a cluster layer
The network starts from only one node and grows until all the input pattern are learned.
connections：every input node has one bottom-up link to output node and one top-down link to input node.

Method：



1. Setup network
2. $W_{i1}^{t} = 1 \quad w_{i1}^{b} = \dfrac{1}{1+N}$、
3. Input the pattern X
4. Calculate "matching value" for every output node j.
   $net_j = \sum_i W_{ij}^{b} \cdot X_i$ （at the beginning） j=1
5. Find the winning node j^*

6. Calculate "similar value" $V_j^* = \dfrac{\sum W_{ij^*}^t \cdot X_i}{\sum X_i}$

7. Do the vigilance test for winning node

   Case 1：if $V_j < \rho$（vigilance value）

   This means the input pattern does not similar to the connected weights and hence, does not belongs to this j* cluster.

   Find the next winning output node to see if it can pass the vigilance test, otherwise, generate a new output node.

   $$W_{ik}^t = X_i \qquad W_{ik}^b = \frac{X_i}{0.5 + \sum_i X_i}$$

   Setup output node：

   if $j=j^*$ then $Y_k=1$

   else $Y_k=0$

   Case2：if $V_j \geqq \rho$

   This means the input pattern matches to the output node j*. Therefore, the j* node is the cluster for this pattern X.

   ∴ update weights.

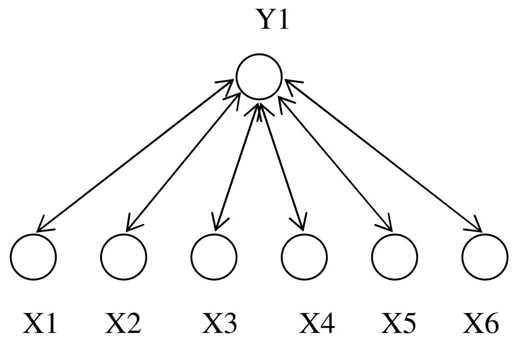
   $$W_{ij^*}^t = W_{ijx}^t \cdot X_i$$

   $$W_{ij^*}^b = \frac{W_{ij}^t \cdot X_i}{0.5 + \sum_i Wij*^t \cdot X_i}$$

   Ex：Please find the cluster for the following patterns.

XOX OXO	OXO XOX	XXX OOO	OOO XXX	XXO OXX	OXX XXO
XOO OXO	OOX OXO	OXO XOO	OXO OOO	XXX OXO	OXO XXX

X 代表 1　O 代表Φ



1. Let ρ=0.5

   $W_{11}^t = 1 \quad W_{12}^t = 1 \ldots\ldots \quad W_{16}^t = 1$

   $W_{10}^t = \{1,1,1,1,1,1\}$

   $W_1^b = \{\frac{1}{7}, \frac{1}{7}, \frac{1}{7} \ldots\ldots \frac{1}{7}\}$

2. Input 1st pattern X_1={1,0,1,0,1,0}
3. calculate matching value（匹配值）

   $net_1 = \sum_i W_{i1}^b \cdot X_2 = \frac{3}{7}$

   $\underset{j}{Max}[net_j] \Rightarrow net_1$ j*=1

4. Calculate "similar value"

   $$V_j^* = \frac{\sum W_{ij*}^t \cdot Xi}{\sum Xi} = \frac{3}{3} = 1$$

   ∵Vj*＞ρ（=0.5）(通過引用 case 2）
   update weights

   $W_{ij*}^t = W_{ij*}^t \cdot X_i \Rightarrow W_1^t\{1,0,1,0,1,0\}$.

   $$W_{ij*}^b = \frac{W_{ij*}^t \cdot X_i}{0.5+3} \Rightarrow W_1^b = \{\frac{1}{3.5}, \frac{0}{3.5}, \frac{1}{3.5}, \frac{0}{3.5}, \frac{1}{3.5}, \frac{0}{3.5}\}$$

5. Input 2nd pattern:X={0,1,0,1,0,1}
   match value $net_1 = \Phi$
   $net_j* = net_1$
   Similar value:

   $$V_{j*} = \frac{\sum W_{ij*}^t \cdot X_i}{\sum X_i} = \frac{0}{3} = 0$$

$\because V_{j*} < \rho\,(=0.5)$ $\therefore$引用 Case1

$\because$沒有另一個 output node

$\therefore$generate Y_2

$W_2^t=\{0,1,0,1,0,1\}$

$W_2^b=\{\frac{0}{3.5},\frac{1}{3.5},\frac{0}{3.5},\frac{1}{3.5},\frac{0}{3.5},\frac{1}{3.5}\}$

6. Input 3th pattern X={1,1,1,0,0,0,}

match value $net_1=\frac{2}{3.5} \Rightarrow 0.57 \rightarrow net_1$

$net_2=\frac{1}{3.5} \Rightarrow 0.285 \rightarrow net_1$

$V_{j*}=\frac{2}{3}=0.66 > \rho$ （=0.5）

Case2 update weight

$W_1^t=\{1,0,1,0,0,0\}$

$W_1^b=\{\frac{1}{3.5},\frac{0}{3.5},\frac{1}{3.5},\frac{0}{3.5},\frac{0}{3.5},\frac{0}{3.5},\}$