Probability Certificate of Correctness (PCC) Driver

User Tutorial for the Probability Certificate of Correctness (PCC) Driver

May 2, 2014

1.0 Purpose

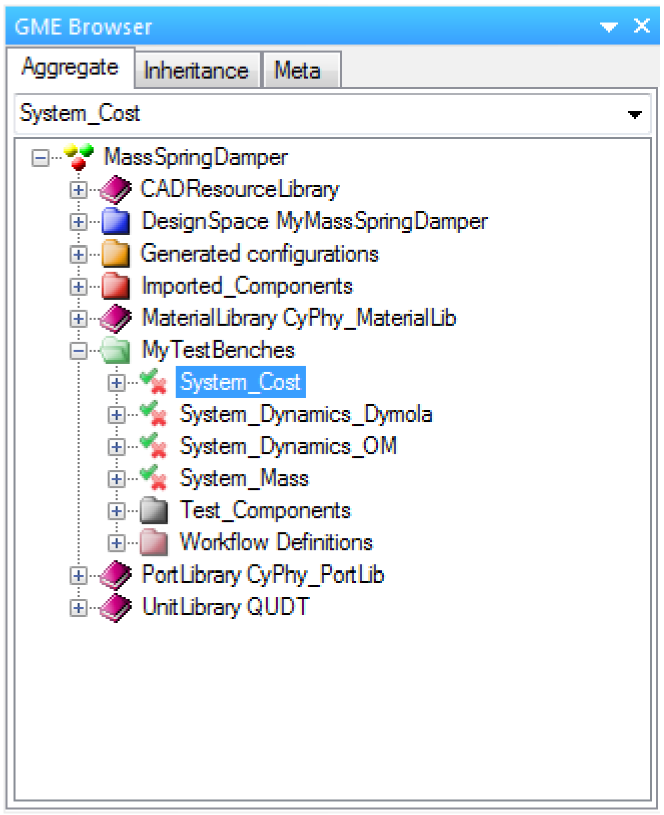
To estimate how well a model meets a set of requirements, given parameters with specified probability distributions.

2.0 Prerequisites

This tutorial assumes you will be using a mass-spring driver model, available for download here, or easy to create using the directions in the MSD Tutorial.

3.0 Getting Started

Examining the GME Browser, you should see a structure like this:

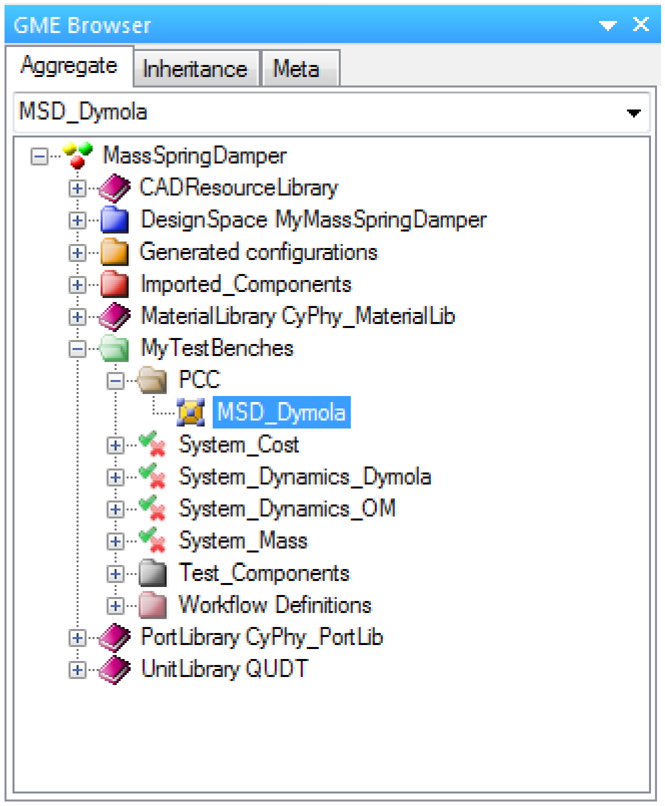


```
MassSpringDamper
  CADResourceLibrary
  DesignSpace MyMassSpringDamper
  Generated configurations
  Imported_Components
  MaterialLibrary CyPhy_MaterialLib
  MyTestBenches
    System_Cost
    System_Dynamics_Dymola
    System_Dynamics_OM
    System_Mass
    Test_Components
    Workflow Definitions
  PortLibrary CyPhy_PortLib
  UnitLibrary QUDT
```

Figure 1

We will need to create an empty Parametric Exploration Model in order to proceed. To do this, right-click on "MyTestBenches" and select "Insert Folder" -> "Parametric Exploration". Name this new folder "PCC". Then right-click the PCC folder and select

"Insert Model" -> "Parametric Exploration". Name this model "MSD_Dymola" (or MSD_OM if working with OpenModelica). The GME Browser will now look like this:



```
MassSpringDamper
    CADResourceLibrary
    DesignSpace MyMassSpringDamper
    Generated configurations
    Imported_Components
    MaterialLibrary CyPhy_MaterialLib
    MyTestBenches
        PCC
            MSD_Dymola
        System_Cost
        System_Dynamics_Dymola
        System_Dynamics_OM
        System_Mass
        Test_Components
        Workflow Definitions
    PortLibrary CyPhy_PortLib
    UnitLibrary QUDT
```

Figure 2

Open up the newly created parametric exploration model by double clicking on "MSD_Dymola" in the GME browser. This will open a new editing window.
If you are using the model you created using the MSD Tutorial, you may need to add parameters for "Mass" and "spring_constant" to the System_Dynamics_Dymola (or System_Dynamics_OM if using OpenModelica) editing window. This may be accomplished by copy/pasting the "Mass" and "spring_constant" parameters from the "DesignSpace MyMassSpringDamper" -> "MyMassSpringDamper" folder in the GME Browser. If this is done, the window will appear as follows:

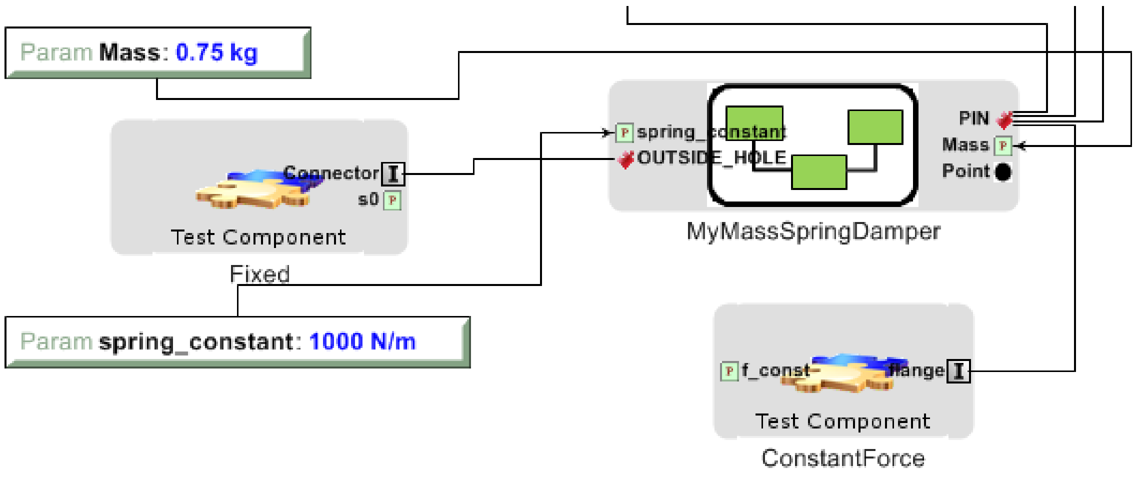


Figure 3

Step 1: Inserting the Test Bench Model

Copy the test bench model under study from the GME Browser to the MSD_Dymola editing window. You can do this by right-clicking “System_Dynamics_Dymola” (or “System_Dynamics_OM” if using OpenModelica) in the GME Browser and selecting “Copy”. Then right-click in the editing window and use “Paste Special” -> “As Reference” to paste in a reference to the model. The editing window will now contain a reference, which appears as follows:

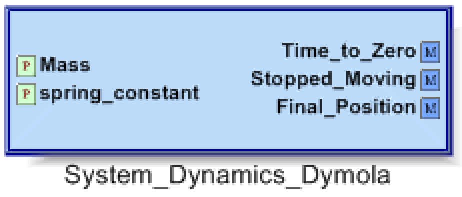


Figure 4

Step 2: Inserting a PCCDriver

From the Part Browser, drag a PCCDriver object into the editing window:

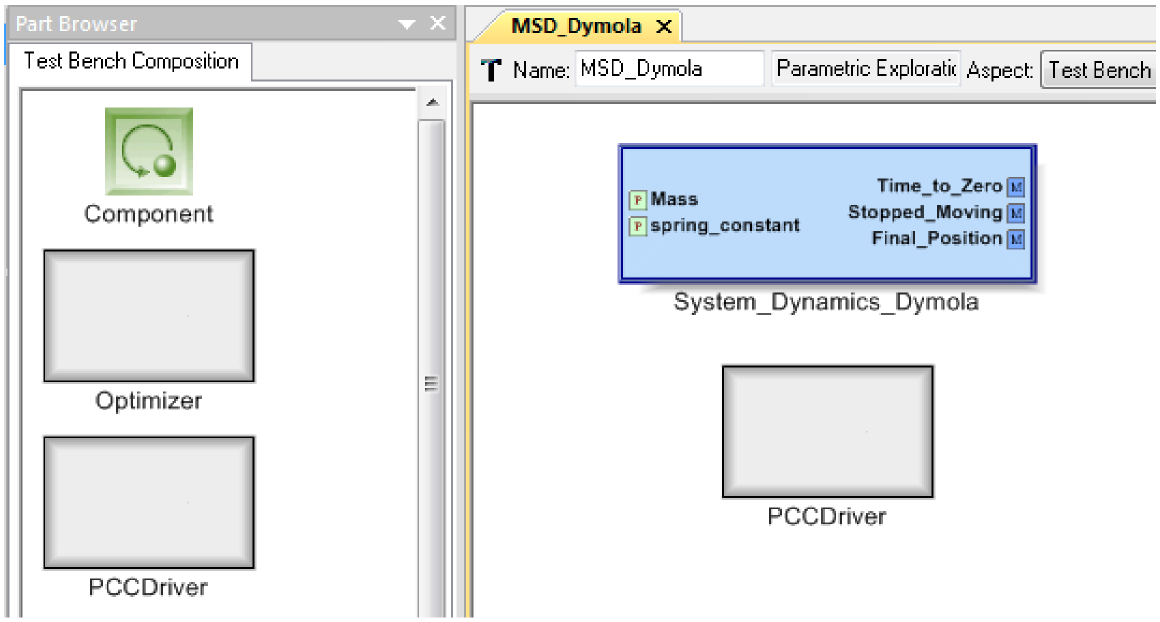


Figure 5

Step 3: Adding parameter objects

Open the new PCCDriver object by double-clicking on it. The PCCDriver now needs parameters added to match each parameter in the test bench model under study – in this case, the parameters “Mass” and “spring_constant”. Each parameter also needs a distribution. For this tutorial, we assume that the parameters are normally distributed. Drag two icons for the normal distribution from the Part Browser to the editing window, and rename them “Mass” and “spring_constant”, as follows:

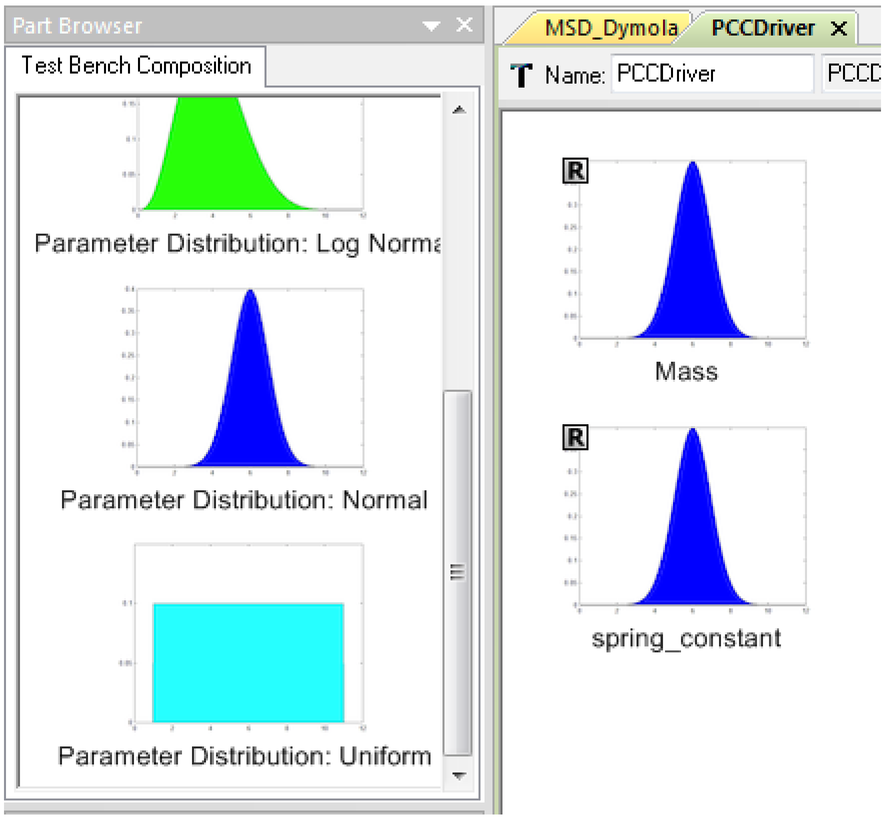

Part Browser
Test Bench Composition
Parameter Distribution: Log Norma
Parameter Distribution: Normal
Parameter Distribution: Uniform
MSD_Dymola
PCCDriver
Name: PCCDriver
PCCD
Mass
spring_constant

Figure 6

Any object can be renamed easily using the Object Inspector. The four available parameter types each represent a different random variable distribution, each defined by distribution-specific parameters:

The Beta Distribution

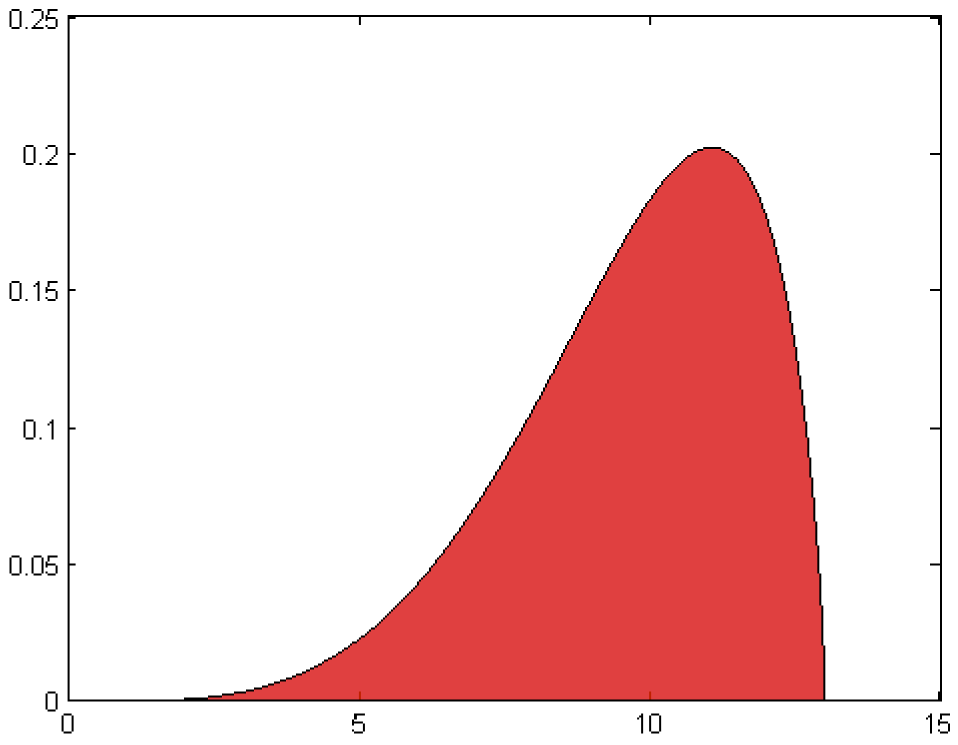


Figure 7

Unlike the normal distribution, the Beta distribution has finite upper and lower bounds (defined by upper and lower limits) and can be symmetric or non-symmetric (defined by two shape parameters).

The Log-Normal Distribution

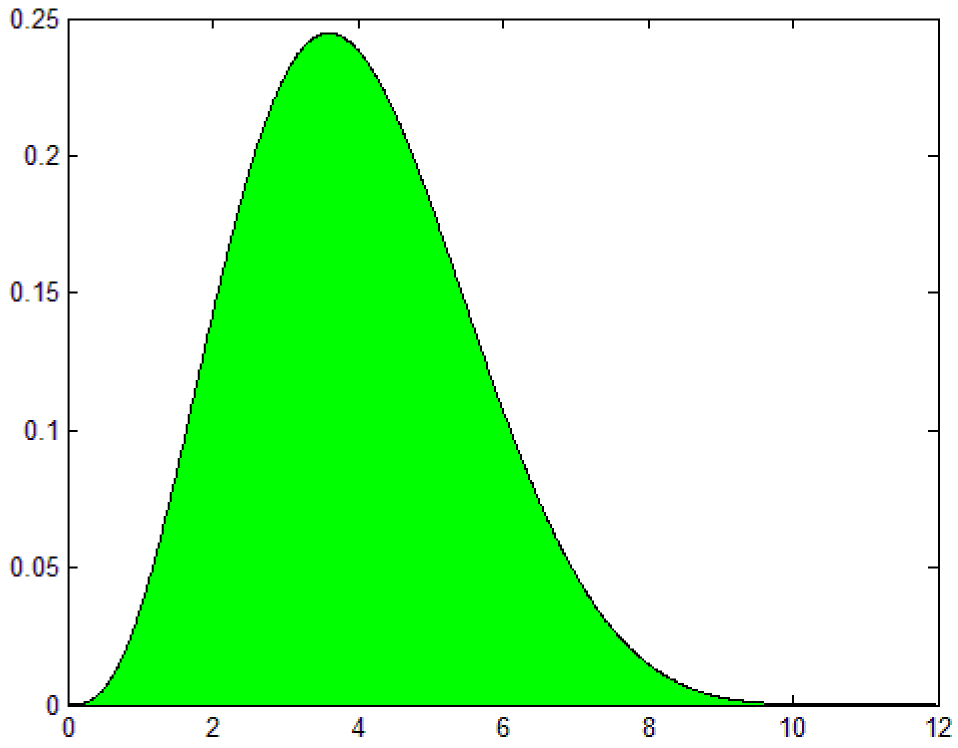


Figure 8

Defined by a shape and a log scale parameter. The log-normal distribution is similar to the normal distribution but has a lower limit of 0 instead of –infinity.

The Normal Distribution

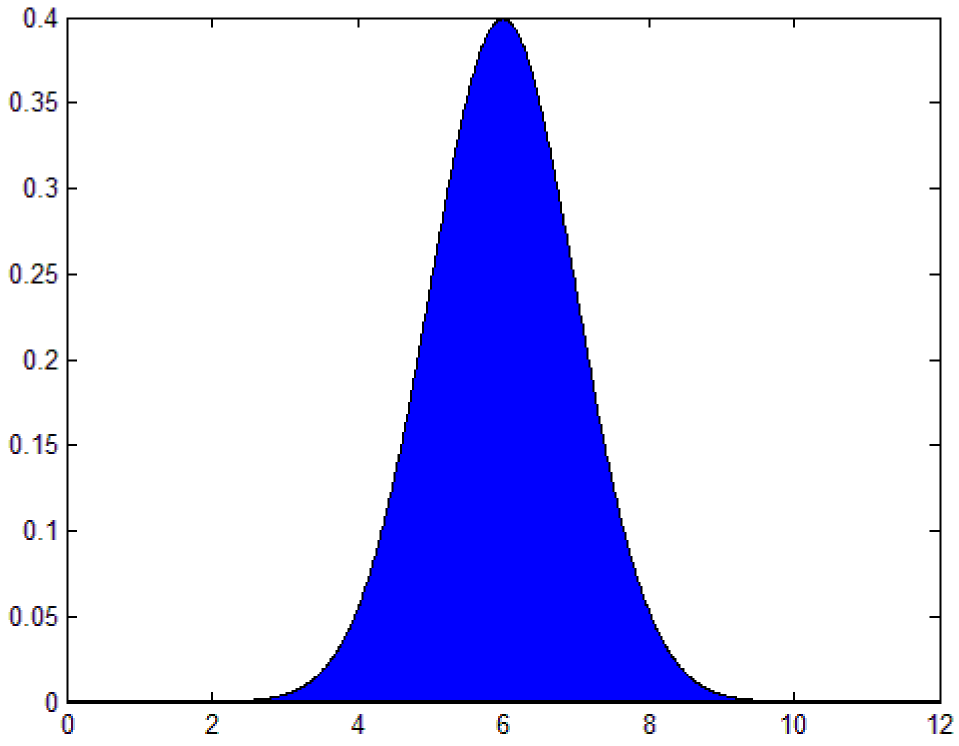


Figure 9

Defined by its mean and standard deviation. The normal distribution is always symmetric and is defined over the range [–infinity, infinity].

The Uniform Distribution

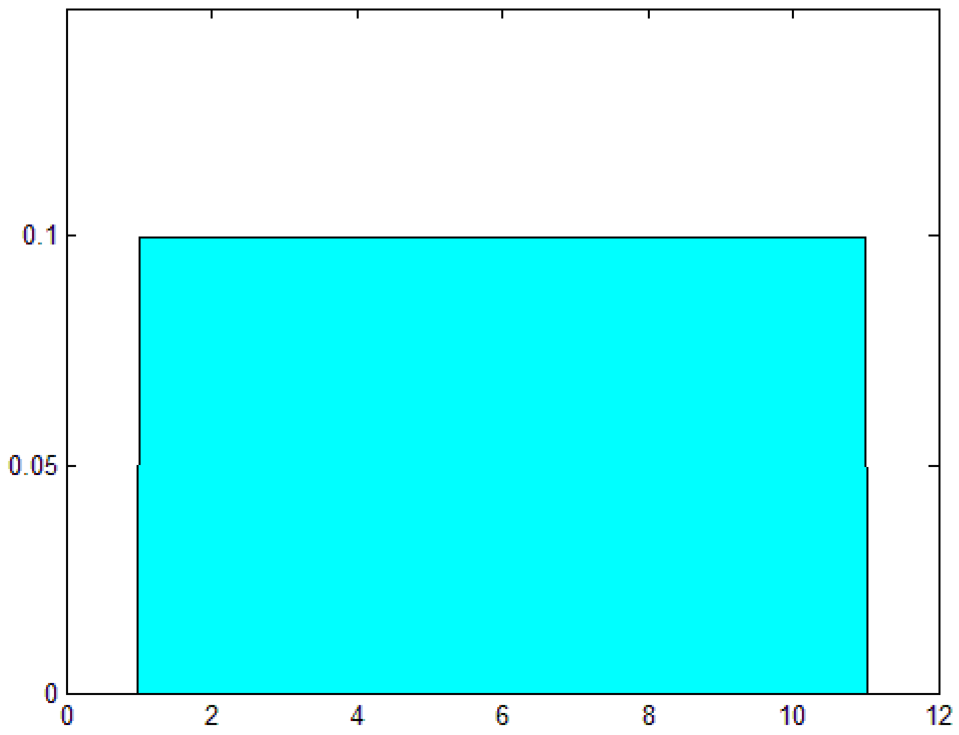
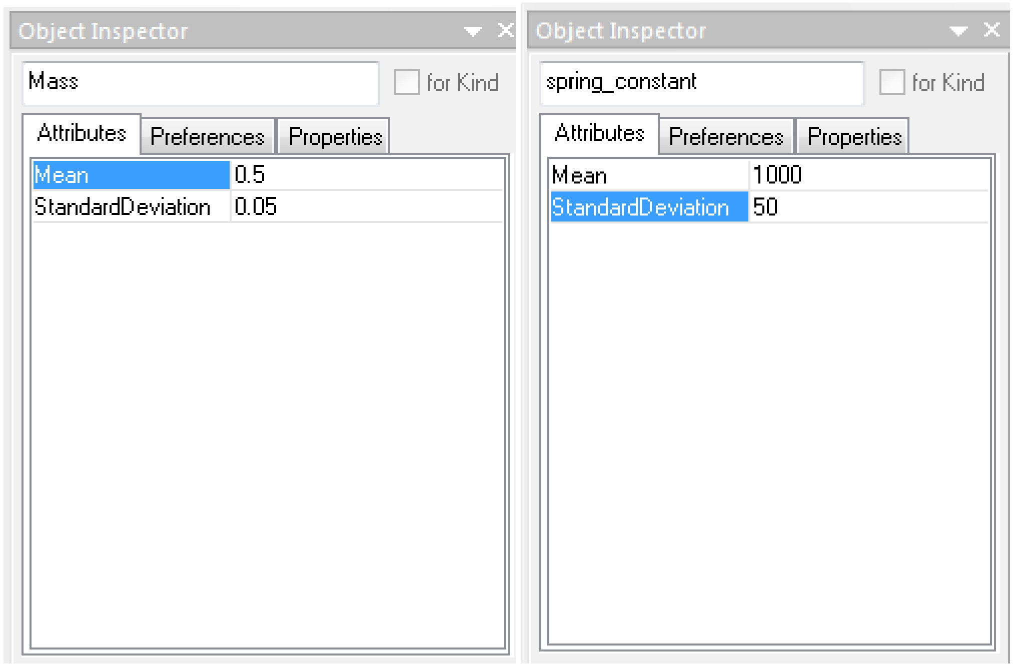


Figure 10

Defined by its upper and lower limits. Every outcome between the limits is equally probable.

Step 4: Setting the distribution-specific parameters for each parameter

To do this, select a parameter in the editing window, then modify the parameter fields in the Attributes tab of the Object Inspector. We will set the mean of the Mass parameter to ".5" and the standard deviation to ".05". Likewise, we'll set the mean of the spring_constant parameter to "1000" and the standard deviation to "50", as shown below:



Mass

Mean	0.5
StandardDeviation	0.05

spring_constant

Mean	1000
StandardDeviation	50

Figure 11

Step 5: Adding PCCOutput objects

Drag a PCCOutput object from the Part Browser to the editing window for each output of the test bench model it is desired to measure, and rename them appropriately. In this case, we are interested in the two continuous outputs, “Time_to_Zero” and “Final_Position”:

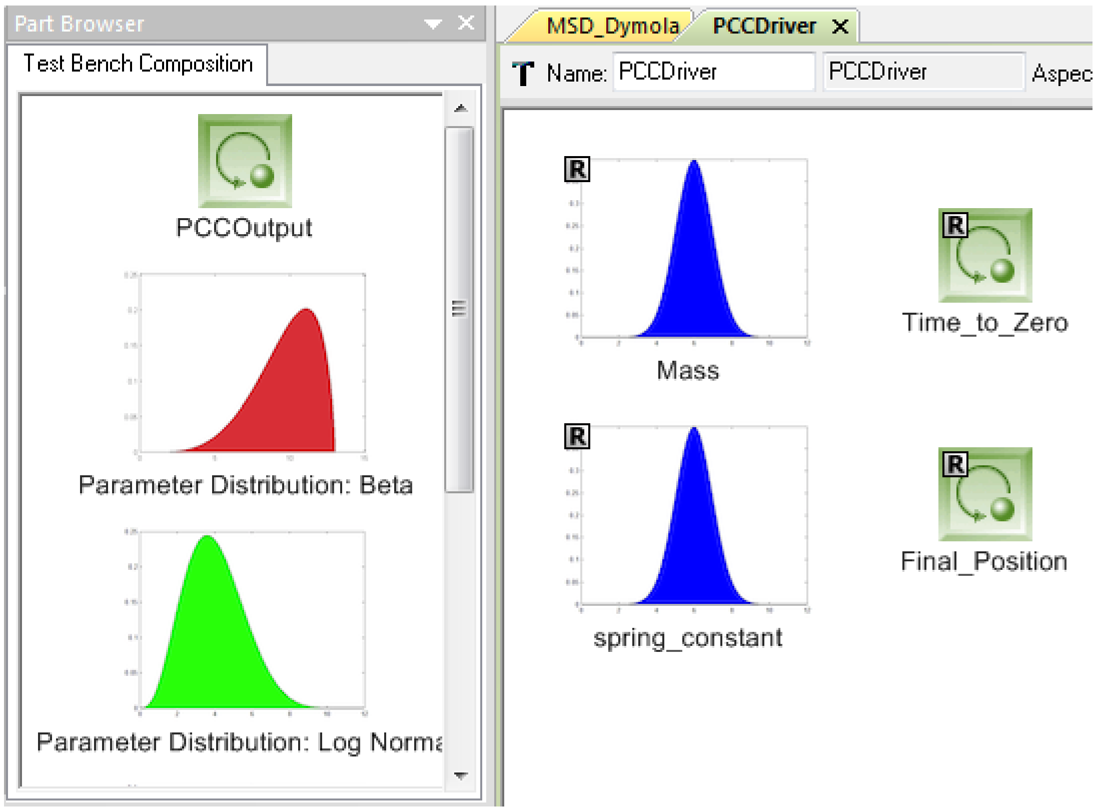
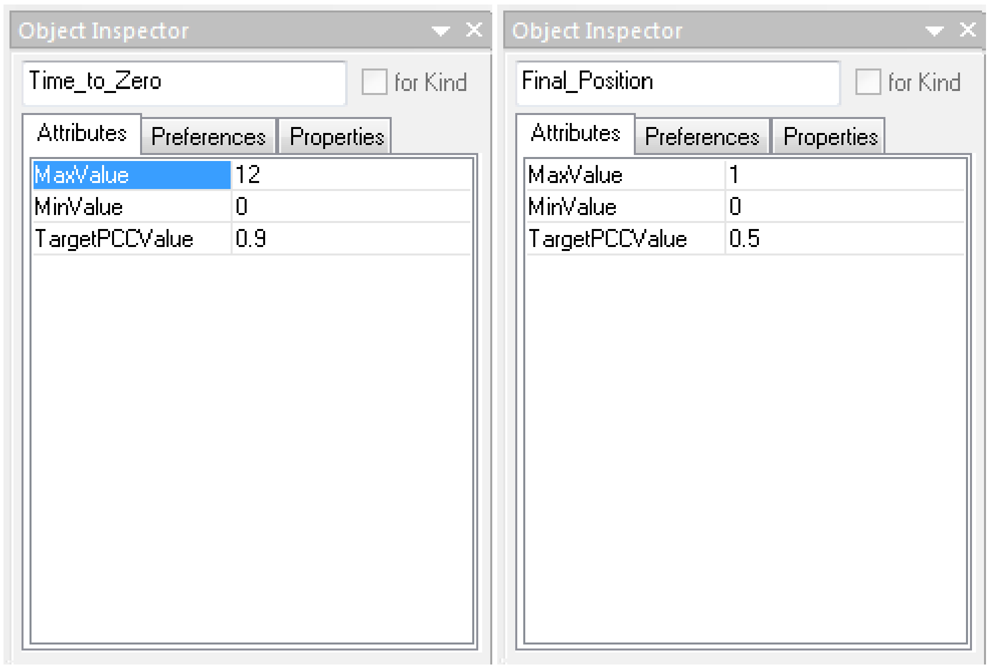


Figure 12

Step 6: Configuring the PCCOutput objects

Per design specifications, each model output variable has an acceptable range, i.e. a range of values an output may take and be considered a successful simulation. After selecting a PCCOutput object, its acceptable range must be set by modifying the MaxValue and MinValue fields in the Attributes tab of the Object Inspector. The simulator will later calculate a PCC value for this range of outputs, which captures the probability that the output will remain in the specified ranges given the input distributions. The TargetPCCValue specifies the minimum probability that this calculated PCC value will fall between its MaxValue and MinValue values and still be considered a successful configuration.
For this tutorial, set the Min and Max values of Time_to_Zero to 0 and 12 respectively, with a TargetPCCValue of 0.9. This specifies that we desire a 90% probability that the mass in our MSD model will reach 0 velocity within 12 seconds. Likewise, set the Min and Max values of Final_Position to 0 and 1 respectively, with a 0.5 TargetPCCValue. This specifies that we desire at least a 50% probability that the final position of the mass will lie between 0 and 1:



Time_to_Zero

MaxValue	12
MinValue	0
TargetPCCValue	0.9

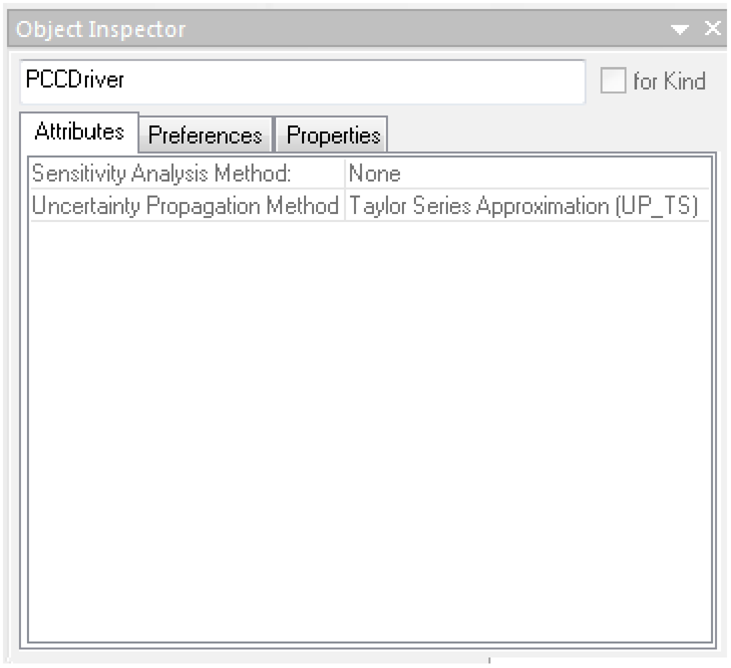
Final_Position

MaxValue	1
MinValue	0
TargetPCCValue	0.5

Figure 13

Step 7: Selecting the Uncertainty Propagation (UP) and Sensitivity Analysis (SA) Methods

After selecting the PCCDriver object by clicking on it in the MSD_Dymola editor window, we can select the UP_Method and/or SA_Method we wish to use via drop-down menus in the Attributes tab of the Object Inspector. We will be using the default option of a Taylor Series approximation for this tutorial, as it is very fast to calculate and requires only 5 simulations given the two inputs we are using:



Sensitivity Analysis Method:	None
Uncertainty Propagation Method	Taylor Series Approximation (UP_TS)

Figure 14

Each of the available methods differs in the way it samples from the space defined by the parameter distributions:

UP Methods

- **Monte Carlo Simulation (MCS):** Random samples eventually cover the entire space. This is the most robust method, but requires thousands, or even millions, of samples to provide an accurate representation of the output distribution. Provides an approximation of the entire output distribution space.
- **Taylor Series (TS) Approximation:** Also known as the Mean Value First Order Second Moment (MVFOSM), this method performs a first-order Taylor expansion very near the mean values of all parameters. The fastest method.

Accurate only for linear or nearly linear problems. Provides a Normally distributed approximation of the entire output distribution space.

- **Most Probable Point (MPP) Method:** Also known as the Hasofer and Lind (HL) First Order Reliability Method (FORM), this method searches the parametric space iteratively, looking for the most probable point of failure. The method then performs a first-order Taylor expansion around that point. Provides only the probabilities of being between the limits.
- **Full Factorial Numerical Integration (FFNI):** Applies the Gaussian quadrature numerical integration technique. Sampling is done only at the quadrature nodes. Provides an approximation of the entire output distribution space.
- **Univariate Dimension Reduction (UDR) Method:** Like the FFNI method, but ignoring interaction effects between the parameters. Almost as fast as TS, yet much more accurate. Provides an approximation of the entire output distribution space.
- **Polynomial Chaos Expansion (PCE):** A polynomial expression in terms of standard normal variables is fitted as a surrogate model. An MCS is then applied to the surrogate, which is very fast to evaluate. Provides an approximation of the entire output distribution space.

For more information on the various uncertainty propogation methods, please consult the paper "System-level Design Reliability Enabled by Copulas", by Christopher Hoyle and Irem Tumer.

SA Methods

The Sobol method, also known as the (Quasi) Monte Carlo method, uses the first order sensitivity index by decomposing the model function into summands of increasing dimensionality. The approach has been expanded by subsequent researchers to include computation of the total sensitivity index. The integrals utilized in the analysis can be computed with Monte Carlo methods.
The main idea underlying the Fourier amplitude sensitivity testing (FAST) method is to convert the k-dimensional integral into a one dimensional integral. Each uncertain input factor is related to a frequency ω and transformed by X(s) = Gi(sin(ωs)), where Gi is a suitably defined parametric equation which allows each factor to be varied in its range, as the parameter s is varied. The set {ω1,..., ωk} are linearly independent integer frequencies. FAST only provides the first-order indices. In 1999, researchers proposed an improvement of the FAST method. They called it the Extended Fourier Amplitude Sensitivity Test (EFAST). With this method they could estimate the total effect indices, as in the Sobol method, by estimating the variance in the complementary set. This is done by assigning a frequency ω for the factor X (usually high) and almost identical frequencies to the rest ωi (usually low).

Step 8: Connecting the Test Bench Model to the PCCDriver

By this point, the parametric exploration model should have two components, a reference to the Test Bench Model, and the PCCDriver. Using the Connect Mode, connect each of the parameters on the PCCDriver to the corresponding parameter on the reference to the test bench model. Then connect each of the outputs from the reference to the Test Bench Model to the corresponding outputs on the PCCDriver, as follows:

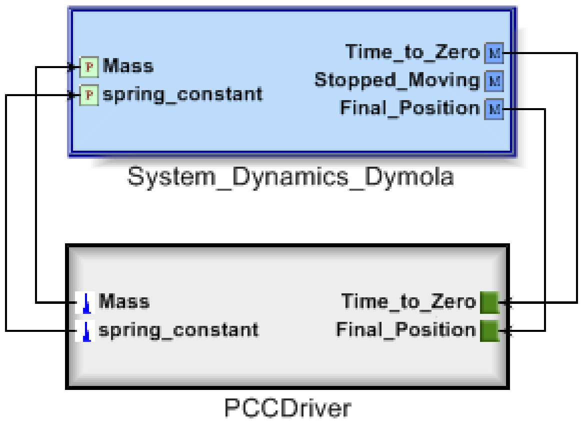


Figure 15

The "PCCDriver" object can be widened by setting "Preferences" -> "PortLabel Length" to "0" in the Object Inspector.

Step 9: Running the Simulations

Click the CyPhy Master Interpreter button ( ) in the toolbar. If a visual representation of the output is desired, select "Open dashboard". Select the configurations to run, then click OK.

Step 10: Interpreting the results

For a visual interpretation of the results, the dashboard may be used. A tutorial on using the dashboard with PCC is given in the MSD tutorial. This section will cover how to drill deeper into the textual output of PCC, and interpret the results. There are two ways the PCC text output can be accessed once the simulation is finished. One way is to right-click the job in the JobManager when it is finished and

select “Show in explorer”. The second is to click on the link provided in the GME Console after the words “Generated files are here:”.
Once you have entered the PCC directory, you will see several files that were used to either run the simulation, or were output from the simulation. There are three files of interest to us: “**parameters.csv**”, “**stdout.txt**” (or “**consoleText.txt**” if you ran PCC remotely), and “**testbench_manifest.json**”.
The parameters.csv is a comma-separated file containing several columns which record the inputs sent to and outputs received from the model. For example, you might see results similar to this for the mass-spring damper model:

Mass	spring_constant	TestBench.Final_Position	TestBench.Time_to_Zero
0.689253812	997.2664058	1.002741098	9
0.694253812	997.2664058	1.002741098	8.399999619
0.684253812	997.2664058	1.002741098	10
0.689253812	1002.266406	0.997738719	9
0.689253812	992.2664058	1.007793903	8.600000381

Table 1

The first column(s) (Mass and spring_constant here) list the inputs given to the simulation. The values in these columns are the points sampled from the distribution specified by the designer and are specific to the PCC method used.
The remaining columns (in this case TestBench.Final_Position and TestBench.Time_to_Zero) are the outputs corresponding to the sampled input values. To fit a distribution to these output values, the value of the output must vary with one or more inputs.

If we open stdout.txt/consoleText.txt and scroll to the bottom, we will see an output similar to this:

```
Correlation:
[[ 1.  1.]
 [ 1.  1.]]
Moments:
  Mean = [ 1.          6.80000019]
  Variance = [ 0.00250014  0.99999809]
  Skewness = [[ 0.]
[ 0.]]
  Kurtosis = [[ 3.]
[ 3.]]
PCC: [0.5, 0.99999990035306252, 0.5]
Complexity estimates: [0.2038907047155587, 4.1327274127362008]
Done!
Elapsed time:  25.0759999752 seconds
```

These are some of the statistical data generated from the PCC run. We can see a correlation matrix, moments of the output distributions, and the PCC calculated for each of the outputs. The PCC values displayed here show that there is a 50% chance that the limits of the first output (in this case, "Final_Position") will be met, and nearly a 100% chance that the limits of Time_to_Zero will be met. Together, these produce a probability of 50% that ALL limits will be met, which is the final value shown in the PCC vector. It is these PCC values that are tested against the "TargetPCCValues" of the various PCCOutputs discussed above. A complexity estimate for the output distributions is also provided.
The last file of use is **testbench_manifest.json**. This file contains a summary of all the inputs and outputs of the simulation. Covariance and correlation matrices are available here, as well as the moments, PCC values, and complexities of the various output distributions.

Step 11: Troubleshooting

There are several common problems people have run into when using PCC. Here are a few you might see:

No output variation warnings:

You might see warnings like “One or more outputs does not vary over given parameter variation”, and/or “The standard deviation of output X is zero. No distribution or correlation can be calculated for it.”
This warning usually means one of your outputs is not varying given the input distributions you have specified. You can see “parameters.csv” for more details to confirm that a model output is not varying with the varying input. To fix this, you can try specifying more inputs, a wider input distribution, or alter the model.
If you are using Taylor Series approximation, you may observe this message if the distribution on a particular output forms a parabola. This causes the variance to be calculated as zero. Here is an example of what such a parameters.csv file might look like:

MASS	SPRING_CONSTANT	TestBench.Final_Position	TestBench.Time_to_Zero
0.5	1000	1	2
0.505	1000	1	2
0.495	1000	1	2
0.5	1005	0.995024876	2.2
0.5	995	1.005025126	2.2

Table 2

As we can see, the values highlighted in red form a parabola in the output “Testbench.Time_to_Zero” given the input “SPRING_CONSTANT”. This causes the variance for that output to be calculated as zero. In this case, it is recommended that you switch to using a different method, such as UP_UDR or UP_MPP.

Limits too far away from distribution:

It is possible to specify limits that are very far away from the actual distribution of output values. This might cause PCC values of 1 or cause the UP_MPP method to fail to converge. If you experience a problem from limits that are too large, try shrinking your limits or try a different method such as UP_UDR.

Negative standard deviation:

Always specify a positive standard deviation on your inputs (if the distribution supports it). Negative standard deviations will cause an the error, “Sigma must be greater than zero for input ‘XXX’”.

If your simulator fails unexpectedly:

If your simulator (Dymola or OpenModelica, typically) crashes or fails to complete for some reason, PCC will also fail. Depending on the kind of failure experienced, it may give an error message such as “Simulator returned only 3 results of a requested 7. See parameters.csv for details.” In this case, it is possible to check “parameters.csv” to see which inputs caused the simulator to fail, and diagnose the problem. If parameters.csv does not exist, the simulator failed to return any results. In this case, the simulation should be corrected and tested with fixed inputs before attempting to compute the PCC.