META Tools – FEA Verification Case
Flat, Finite Plate with Central Hole in Tension

Dr. Richard E. Gregory, Brian Neaves, Robert Cook
Vanderbilt University, Institute for Software Integrated Systems

May 21, 2013

Abstract

This paper documents the solution of a flat, finite plate with a central hole loaded in axial tension for use in verifying the META tools. The mechanics of materials solution is presented to determine the stress concentration factor and the maximum normal stress. A set of dimensions is then chosen to present a numerical example. Results are then compared to a manual finite element solution to show the feasibility of a numerical solution. Finally the problem is solved with the META tools and compared to the mechanics of materials solution and the manual FEA solution to verify the solution process of the META tools.

Nomenclature

δ	axial deflection (m)
σ	normal stress (MPa)
σ_{ave}	average normal stress (MPa)
σ_{max}	maximum normal stress (MPa)
A	cross sectional area (m^2)
A_{net}	net cross sectional area (m^2)
D	plate width (m)
E	modulus of elasticity (GPa)
K_t	stress concentration factor (unitless)
L	plate length (m)
P	applied axial load (N)
r	hole radius (m)
t	plate thickness (m)

1 Introduction

This document contains the theoretical solution for the maximum stress in a flat, finite plate with a central hole loaded in axial tension shown in Figure 1. A complete explanation of axial stress, axial deflection, and stress concentration factors can be found in Beer [Beer et al., 2009],

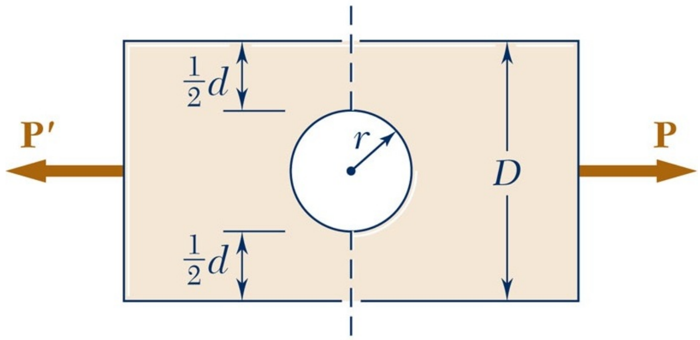

Figure 1: Flat, finite plate with central hole loaded in axial tension [Beer et al., 2009].

Boresi [Boresi and Schmidt, 2003], Norton [Norton, 2006], Pilkey [Pilkey, 2005], Pilkey [Pilkey and Pilkey, 2008], and Young [Young et al., 2012].

2 Mechanics of Materials Solution

The stresses in this case are described by the classical equation for axial stress,

$$\sigma = \frac{P}{A} \tag{1}$$

where P is the applied load, and A is the cross sectional area defined in terms of the plate thickness t and the plate width D as

$$A = tD \tag{2}$$

The maximum stress at the edge of the hole, σ_{max}, is given by

$$\sigma_{max} = K_t \sigma_{ave} \tag{3}$$

where K_t is the theoretical stress concentration factor, σ_{ave} is the average section stress, given by

$$\sigma_{ave} = \frac{P}{A_{net}} \tag{4}$$

and A_{net} is the net cross sectional area, given by

$$A_{net} = t(D - 2r) \tag{5}$$

The net cross sectional area here is used instead of the gross cross sectional area defined in Equation 2. The maximum stress defined in Equation 3 occurs only at the inside edges of the hole. The net section stress drops off quickly to the average stress as defined in Equation 4 and as shown in Figure 2.

A set of dimensions was chosen to represent an example case for use in validating various FEA codes as well as the META Tool Suite. These dimensions are listed in Table 1.

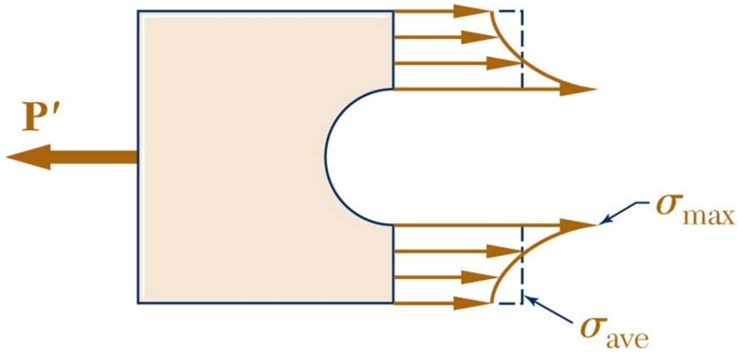

Figure 2: Stress distribution around stress concentration [?].

Table 1: Dimensions for example case.

Parameter Name	Variable	Value
Applied Load	P	1000 N
Plate Width	D	50 mm
Hole Radius	r	5 mm
Plate Thickness	t	5 mm
Plate Length	L	200 mm

Using these values, a stress concentration factor, K_t, can be determined from Figure 3. First, the parameter 2r/D must be computed and is found to be

$$\frac{2r}{D} = \frac{2(5\text{ mm})}{50\text{ mm}} = 0.20 \tag{6}$$

which yields a value of $K_t = 2.5$ for the theoretical stress concentration factor. Other references give similar stress concentrations as shown in Table 2 which show agreement to within 1%. The average stress is then computed using the net cross sectional area which is

$$A_{net} = t(D - 2r) = 5\text{ mm}(50\text{ mm} - 2(5\text{ mm})) = 200\text{ mm}^2 \tag{7}$$

and

$$\sigma_{ave} = \frac{P}{A_{net}} = \frac{1000\,N}{200\text{ mm}^2} = 5\text{ MPa} \tag{8}$$

Table 2: Stress Concentration Factors from Various References.

Reference	K_t
Beer [Beer et al., 2009]	2.5
Norton [Norton, 2006]	2.502
Pilkey [Pilkey, 2005]	2.506
Peterson [Pilkey and Pilkey, 2008]	2.519
Roark [Young et al., 2012]	2.508

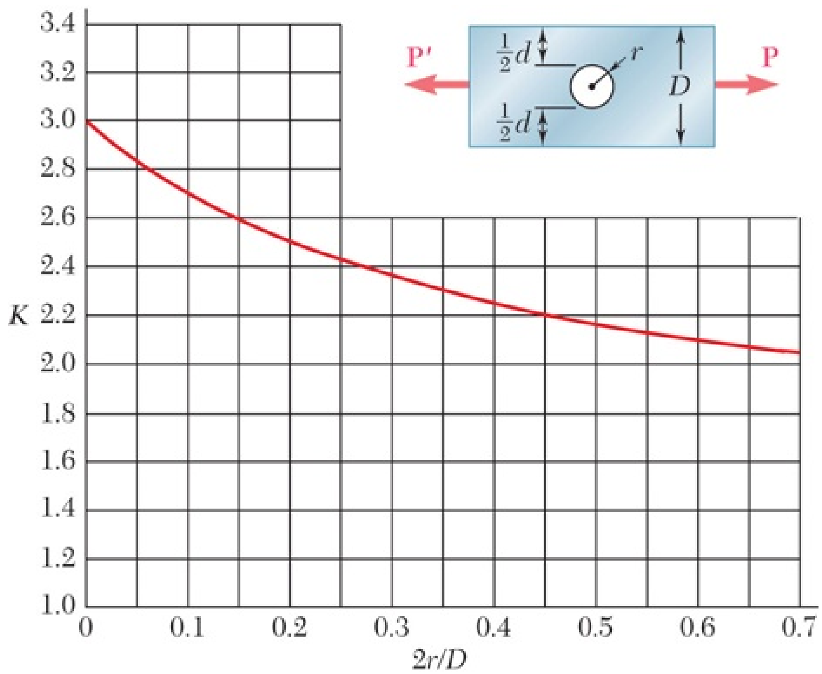

Figure 3: Theoretical stress concentration factor K_t as a function of plate geometry [Beer et al., 2009].

The maximum stress is then found to be

$$\sigma_{max} = K_t \sigma_{ave} = 2.5(5 \text{ MPa}) = 12.5 \text{ MPa} \tag{9}$$

For an additional quality check, the deflection of the plate can be calculated by

$$\delta = \frac{PL}{AE} \tag{10}$$

where the total area will be used instead of the net cross sectional area and E is the Modulus of Elasticity and will be 206.8 GPa for steel. The axial deflection is then calculated as

$$\delta = \frac{PL}{AE} = \frac{(1000 \text{ N})(200 \text{ mm})}{(5 \text{ mm})(50 \text{ mm})(206 \text{ GPa})} = 3.8835 \times 10^{-6} \text{ m} \tag{11}$$

This will underestimate the actual deflection because the hole was ignored in this computation. The presence of the hole will make the plate more compliant, thereby increasing the deflection.

3 FEA Solution

The manual FEA solution was performed in Creo 2.0 Simulate. Information from Table 1 was used to construct the model. The material values used for steel were $E = 206$ GPa, and $\nu = 0.28$. The solid model of the geometry is shown in Figure 4.

1101 elements

Figure 4: Creo 2.0 model of plate with central hole.

Figure 5: FEA model showing boundary conditions.

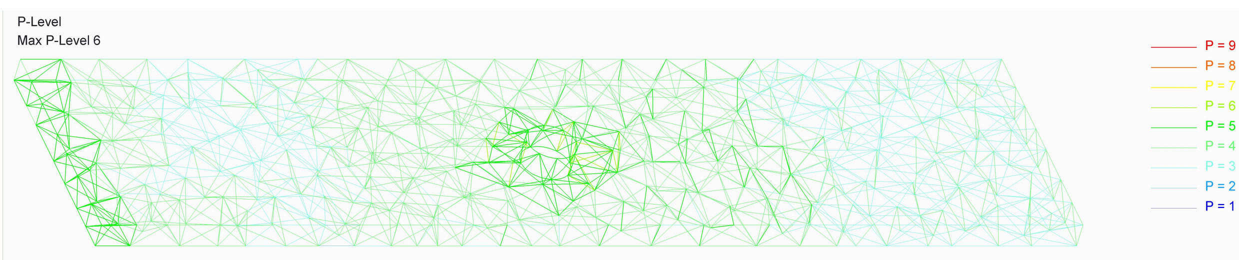

Figure 6: FEA model mesh and associated p level refinement.

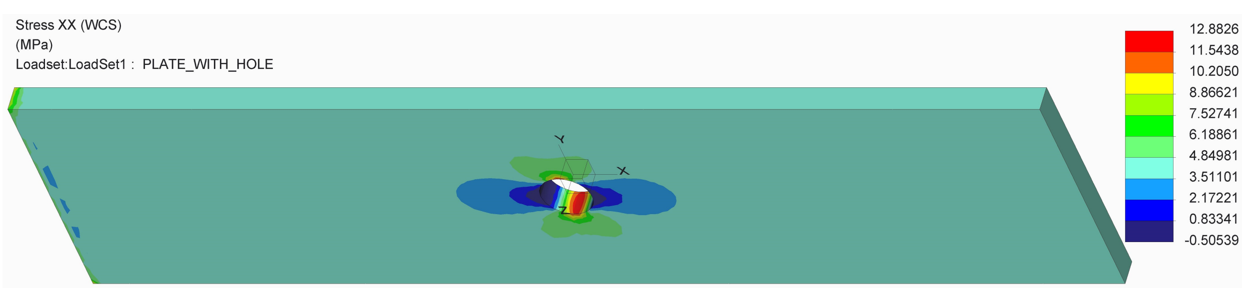

Figure 7: FEA analysis showing the σ_{xx} stress distribution.

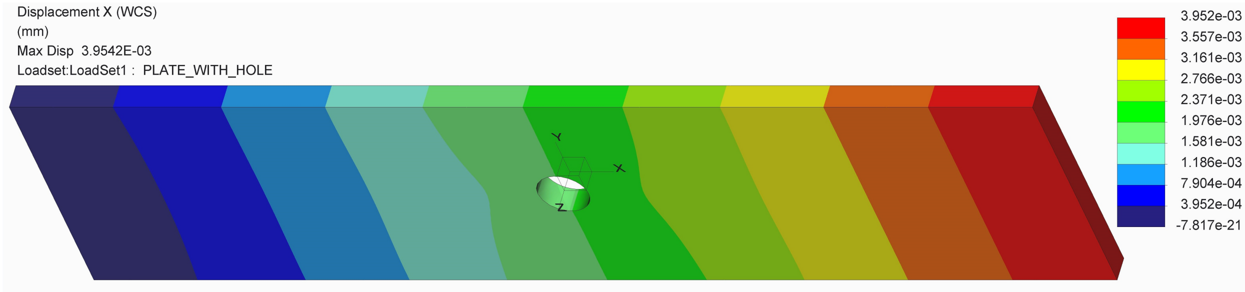

Figure 8: FEA analysis showing displacement in the *x* direction.

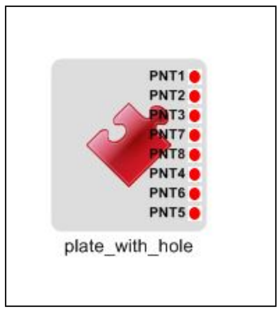

Figure 9: Curated Plate with Hole

4 META Tools Solution

Based on the FEA section, a CyPhy tools solution can be created. Figure 9 shows the assembly of the curated plate with hole. Since the stress test in question involves only a single object, the assembly of the Plate with Hole is simply one component: the plate itself. The interior of this part object can be seen in Figure 10. Note that there are a total of eight points.

Figure 11 shows the test bench. Notice that there are two polygon objects in the workspace, each receiving four points as a set of inputs. These sets correspond with the four corner points located on a given side of the plate. Note that the script labeled ”ordinal position” for each of the connections has a separate number between 1 and 4. This ordering is representative of the plotting order taken by CyPhy in producing the plane onto which the constraint or force will be applied.

Figure 12 displays the corresponding points in Creo used to define the plane onto which the displacement constraint will be placed. Note that the pattern traces the outline of the bottom going from PNT0 to PNT3. The resulting polygon is then closed, and all nodes incident on that plane will become subject to the displacement constraint. The exact same logic applies to the other end onto which the force is being loaded.

The interior of the displacement block contains two sets of elements as seen in Figure 13, Rotation and Translation. The properties of all factors within either block should be set to FIXED.

For the ForceLoad block, there is a single Force element inside. Its value should be set to 1000 newtons in the positive X direction. This concludes the setup for the META Tools Solution.

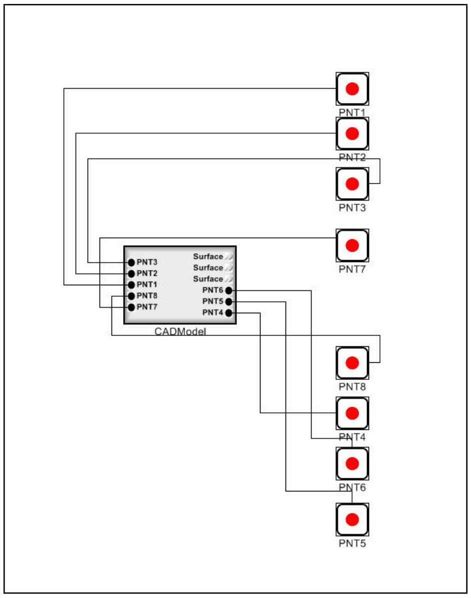

Figure 10: Curated Plate with Hole

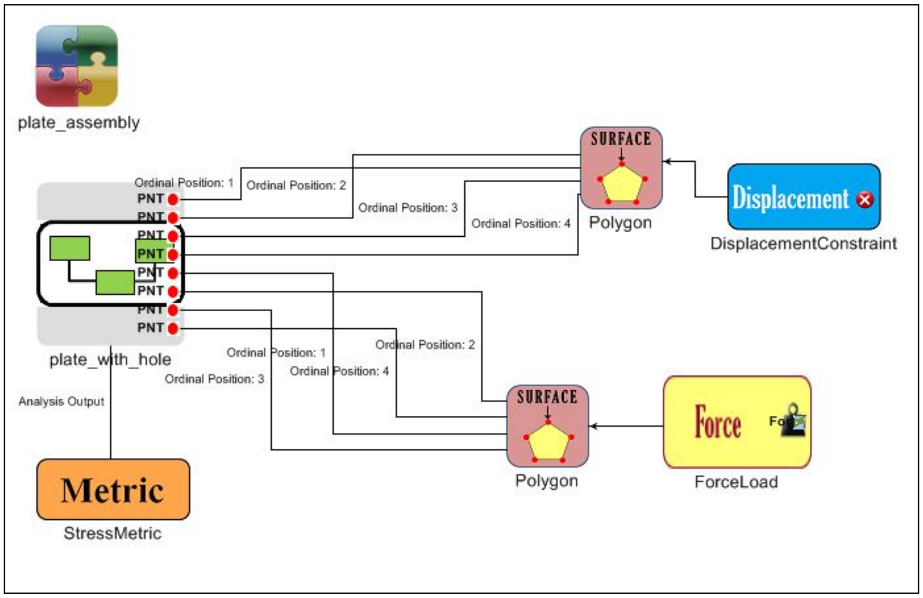

Figure 11: Curated Plate with Hole

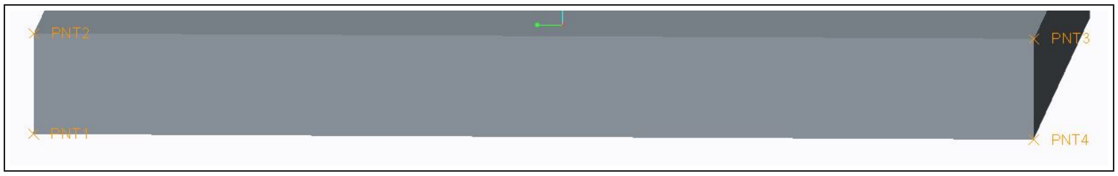

Figure 12: Plane with Constraint Points Displayed

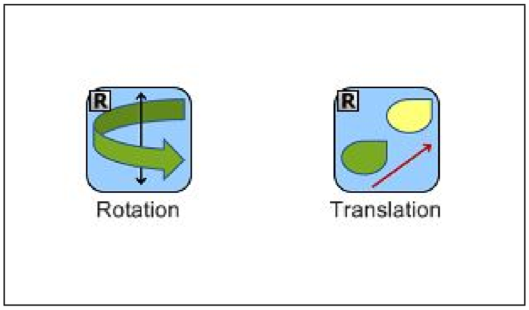

Figure 13: Interior of the Displacement Block

5 Summary

More to come.

References

[Beer et al., 2009] Beer, F. P., Johnston, Jr., E. R., DeWolf, J. T., and Mazurek, D. F. (2009). *Mechanics of Materials*. McGraw Hill, 5 edition.

[Boresi and Schmidt, 2003] Boresi, A. R. and Schmidt, R. J. (2003). *Advanced Mechanics of Materials*. John Wiley & Sons, Inc., 6 edition.

[Norton, 2006] Norton, R. L. (2006). *Machine Design: An Integraged Apporach*. Pearson Prentice Hall, 3 edition.

[Pilkey, 2005] Pilkey, W. D. (2005). *Formulas for Stress, Strain, and Structural Matrices*. John Wiley & Sons, Inc., 2 edition.

[Pilkey and Pilkey, 2008] Pilkey, W. D. and Pilkey, D. F. (2008). *Peterson's Stress Concentration Factors*. John Wiley & Sons, Inc., 3 edition.

[Young et al., 2012] Young, W. C., Budynas, R. G., and Sadegh, A. M. (2012). *Roark's Formula for Stress and Strain*. McGraw Hill, 8 edition.