HW5

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[5] 球面上均匀随机分布坐标点，以球面上(θ,φ)的参数表示的几率密度函数为：因为 $\theta\sim\theta+d\theta$，$\varphi\sim\varphi+d\varphi$ 区间内概率正比于该区间对应球上面积 $\sin\theta\, d\theta\, d\varphi$，

$\therefore p(\theta,\varphi) \propto \sin\theta$，归一化后为 $p(\theta,\varphi) = \frac{\sin\theta}{4\pi}$.

变换到(x,y)平面上，为 $\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \end{cases}$.

则投影的几率分布为

$$p(x,y) = \left|\frac{\partial(\theta,\varphi)}{\partial(x,y)}\right| p(\theta,\varphi) = \frac{\sin\theta}{4\pi}\cdot\frac{1}{\left|\frac{\partial(x,y)}{\partial(\theta,\varphi)}\right|} = \frac{\sin\theta}{4\pi}\cdot\frac{1}{r\sin\theta\cos\theta} = \frac{1}{4\pi\cos\theta} = \frac{1}{4\pi z} = \frac{1}{4\pi\sqrt{1-x^2-y^2}}.$$

Marsaglia采样法. (u,v) 概率密度为 $\frac{1}{\pi}$. 则 x,y 概率密度为

$$p(x,y) = \left|\frac{\partial(u,v)}{\partial(x,y)}\right| p(u,v) = \frac{\frac{1}{\pi}}{\left|\frac{\partial(x,y)}{\partial(u,v)}\right|}$$

$$= \frac{1}{\pi}\cdot\frac{1}{\left|\begin{matrix} 2\sqrt{1-r^2}-2u^2/\sqrt{1-r^2} & -2uv/\sqrt{1-r^2} \\ -2uv/\sqrt{1-r^2} & 2\sqrt{1-r^2}-2v^2/\sqrt{1-r^2} \end{matrix}\right|}$$

$$= \frac{1}{\pi}\cdot\frac{1}{4-8(u^2+v^2)} = \frac{1}{4\pi(1-2r^2)} = \frac{1}{4\pi z}$$

$$= \frac{1}{4\pi\sqrt{1-x^2-y^2}}.$$ 与前推导的 $p(x,y)$ 相同.

模拟结果见下图.

(1.jpg is the full size picture)

Simulation results of the Marsaglia Sampling.