


HW5

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2019.9.25



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[5] 球面上均匀分布点, 以球面上 (θ, φ) 为参数的概率密度函数为: 因为 $0 \sim 2\pi$ 的 $\varphi - \varphi + 2\pi$ 区间内概率正比于该区间对应球上面积

$\sin \theta d\theta d\varphi$,

$\therefore p(\theta, \varphi) \propto \sin \theta$, 归一化后为 $p(\theta, \varphi) = \frac{\sin \theta}{4\pi}$.

变换到 xy 平面上, 为 $\begin{cases} x = \sin \theta \cos \varphi \\ y = \sin \theta \sin \varphi \end{cases}$.

则按新的概率密度为 $p(x, y) = \left| \frac{\partial(\theta, \varphi)}{\partial(x, y)} \right| p(\theta, \varphi)$

$$= \frac{\sin \theta}{4\pi} \cdot \frac{1}{\left| \frac{\partial(x, y)}{\partial(\theta, \varphi)} \right|}$$

$$= \frac{\sin \theta}{4\pi} \cdot \frac{1}{r \sin \theta \cos \theta}$$

$$= \frac{1}{4\pi r \cos \theta}$$

$$= \frac{1}{4\pi z}$$

$$= \frac{1}{4\pi \sqrt{1-x^2-y^2}}$$

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Marsaglia 采样法. (u, v) 服从在 $1/\pi$ 的 xy 概率密度为

$$p(x, y) = \left| \frac{\partial(u, v)}{\partial(x, y)} \right| p(u, v) = \frac{1/\pi}{\left| \frac{\partial(x, y)}{\partial(u, v)} \right|} = \frac{\sqrt{1-u^2-v^2}}{4(1-u^2)}$$

$$= \frac{1}{\pi} \cdot \frac{1}{\left| \begin{matrix} \frac{\partial x}{\partial u} - \frac{u}{\sqrt{1-u^2-v^2}} \frac{\partial v}{\partial u} & \frac{\partial x}{\partial v} - \frac{v}{\sqrt{1-u^2-v^2}} \frac{\partial v}{\partial u} \\ -\frac{u}{\sqrt{1-u^2-v^2}} \frac{\partial u}{\partial v} & \frac{\partial y}{\partial v} - \frac{v}{\sqrt{1-u^2-v^2}} \frac{\partial u}{\partial v} \end{matrix} \right|}}$$

$$= \frac{1}{\pi} \cdot \frac{1}{4-8(u^2+v^2)} = \frac{1}{4\pi(1-2r^2)} = \frac{1}{4\pi z}$$

$$= \frac{1}{4\pi \sqrt{1-x^2-y^2}}, \text{ 与两维的 } p(x, y) \text{ 相同.}$$

模拟结果见图.

Simulation results of the Marsaglia Sampling.

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