

# Ejercicio 1

a)

$$T(0) = 6$$

$$T(1) = \frac{5}{2} + \frac{5}{2} + 6 = 5 + 6 = 11$$

$$T(2) = \frac{5 \cdot 4}{2} + \frac{5 \cdot 2}{2} + 6 = 10 + 5 + 6 = 21$$

$$T(0) = 6$$

$$T(1) = 11$$

$$T(2) = 21$$

$$T(n) = 7T(n-2) - 6T(n-3)$$

$$T(n) + 0T(n-1) - 7T(n-2) + 6T(n-3) = 0$$

$$x^3 + 0x^2 - 7x^1 + 6x^0 = 0$$

$$x^3 - 7x + 6 = 0$$

$$\begin{array}{r|rrrr} & 1 & 0 & -7 & 6 \\ 1 & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & \underline{0} \\ 2 & & 2 & 6 & \\ \hline & 1 & 3 & \underline{0} & \\ -3 & & -3 & & \\ \hline & 1 & \underline{0} & & \end{array}$$

$$\begin{aligned} &\rightarrow x^3 - 7x + 6 = 0 \\ &(x-1)(x-2)(x+3) = 0 \end{aligned}$$

$$\begin{cases} r_1 = 1 & m_1 = 1^n \\ r_2 = 2 & m_2 = 2^n \\ r_3 = -3 & m_3 = (-3)^n \end{cases}$$

Solución parcial:  $T(n) = a + b \cdot 2^n + c \cdot (-3)^n$

$$[1] \begin{cases} T(0) = a + b + c \\ T(0) = 6 \end{cases}$$

$$[2] \begin{cases} T(1) = a + 2b - 3c \\ T(1) = 11 \end{cases}$$

$$[3] \begin{cases} T(2) = a + 4b + 9c \\ T(2) = 21 \end{cases}$$

$$\begin{cases} a + b + c = 6 & [1] \\ a + 2b - 3c = 11 & [2] \\ a + 4b + 9c = 21 & [3] \end{cases}$$

$$[1] \quad a = 6 - b - c$$

$$\begin{cases} (6-b-c) + 2b - 3c = 11 \\ (6-b-c) + 4b + 9c = 21 \end{cases} \rightarrow \begin{cases} b - 4c = 5 & [A] \\ 3b + 8c = 15 & [B] \end{cases}$$

$$[A] \quad b = 4c + 5$$

$$[A] \quad b = 4(0) + 5$$

$$[B] \quad 3(4c + 5) + 8c = 15$$

$$b = 5$$

$$12c + 15 + 8c = 15$$

$$[1] \quad a = 6 - (5) - (0)$$

$$20c = 0$$

$$a = 1$$

$$c = 0$$

$$(a, b, c) = (1, 5, 0)$$

$$T(n) = (1) + (5) \cdot 2^n + (0) \cdot 3^n$$

$$T(n) = 5 \cdot 2^n + 1$$

$$T(n) \in \Theta(2^n)$$

b)

$$T(n) \in \begin{cases} a < b^d \rightarrow O(n^{\log_b(a)}) \\ a = b^d \rightarrow O(n^d \cdot \log(n)) \\ a > b^d \rightarrow O(n^d) \end{cases}$$

$$a) \quad a=4, b=2, d=1$$

$$b) \quad a=2, b=2, d=1$$

$$c) \quad a=4, b=2, d=2$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

c)

$$f_1(n) = \log(\log(n))$$

$$f_2(n) = (\log(n))^2$$

## Ejercicio 2

$0 \in \mathbb{Z}^+$   
//pre { int[] V, V.length = n,  $n > 0$ ,  $i \in [0, n-1] : V[i] > 0$  }

//post { int x :  $\left( i \in [0, n-1] : \prod_{i=0}^{n-1} (V[i] \% 2 \neq 0) = x \right) \}$

## Ejercicio 3

Completar

## Ejercicio 4

a) //pre  $\{ \text{int}[] A, A.\text{length} = n, A[0] = k, n, k > 0, \forall i \in [0, n-2]: A[i] < A[i+1] \}$

//post  $\{ \text{int } f : (\forall i \in [0, n-2], \forall j \in [i, n-2] : (A[i+1] \neq A[i]+1) \wedge \neg(A[i+1] \neq A[i]+1, i < j) \Rightarrow A[j] = f) \}$

$A_1 = \{1\} \rightarrow 2$

$A_2 = \{1, 2, 4\} \rightarrow 3$

$A_3 = \{1, 2, 3, 4\} \rightarrow 5$

$A_4 = \{1, 2, 4, 5, 7, 8\} \rightarrow 3$

Corregir

$\vee (A.\text{length} = 1 \Rightarrow A[0]+1 = f)$

b)

```
private int menorSuperior(int[] A) {  
    if (A.length == 1) {  
        return A[0] + 1;  
    } else {  
        int i = 0;  
        while (A[i] + 1 != A[i+1] && i < A.length - 1) {  
            i++;  
        }  
        return A[i] + 1;  
    }  
}
```

c) Complejidad  $n$  porque es un bucle y en el peor de los casos, lo recorre entero.

Como está en un if y su complejidad es la mayor del if o el else, se mantiene para  $n$  en el peor caso.

$\text{menorSuperior}(\text{int}[] V) \in O(n)$

d)

```
public int menorSuperior(int[] A) {  
    if (A.length == 1) {  
        return A[0] + 1;  
    } else {  
        return buscar(A, 0, A.length);  
    }  
}
```

```
private int buscar(int[] A, int i, int s) {  
    if (i == s) {  
        if (A[i] - 1 == A[0]) {  
            min = A[i] + 1;  
        } else {  
            min = A[i - 1] + 1;  
        }  
    } else {  
        int m = (i + s) / 2;  
        if (A[m] - m == A[0]) {  
            min = buscar(A, i - 1, s);  
        } else {  
            min = buscar(A, i, s - 1);  
        }  
    }  
}
```

e)  $T(n) = 1T\left(\frac{n}{2}\right) + cte, \quad cte \in \mathbb{R}$

$$\begin{cases} a=1 \\ b=2 \\ d=0 \end{cases} \longrightarrow a = b^d \rightarrow T(n) \in \Theta(n^d \cdot \log(n))$$
$$T(n) \in \Theta(\log(n))$$

$$S(n) = ?$$