An Iterative Reconstruction Algorithm for Positron Emission Tomography

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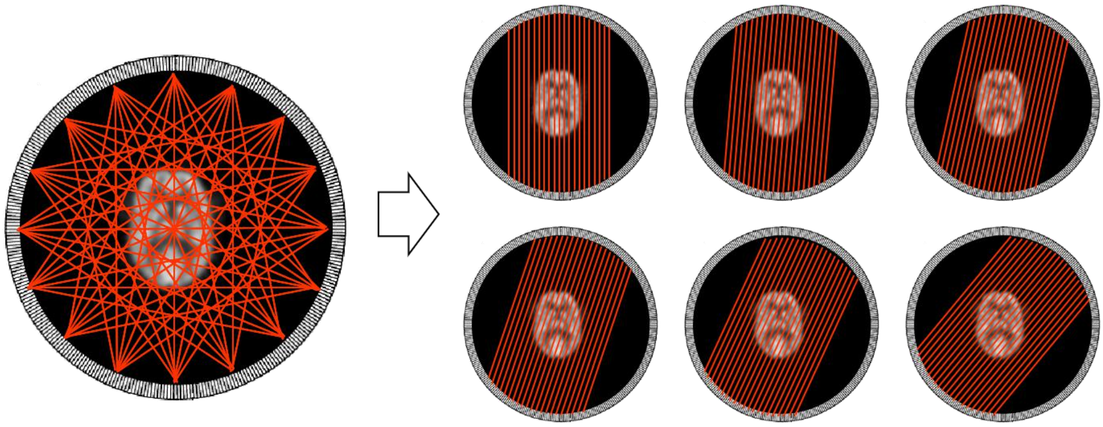
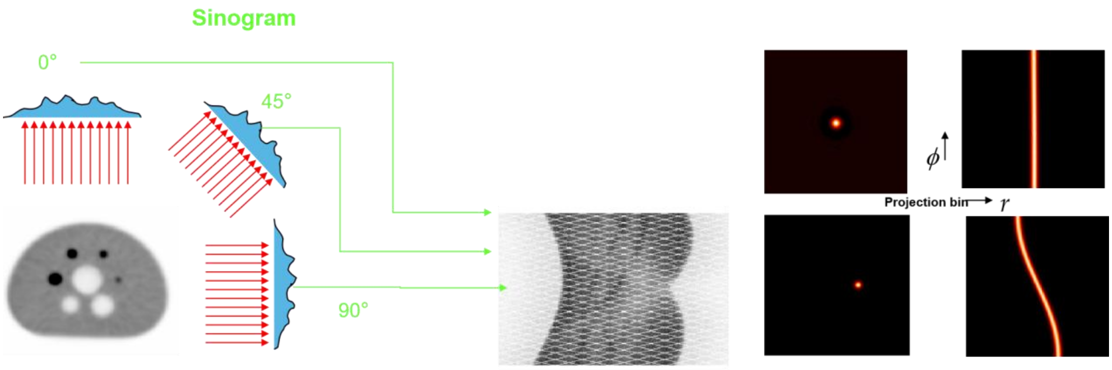
Mechanism

In this project, we used the provided sparse matrix A and scan data Y to establish an iterative reconstruction algorithm.As shown in the figure below, during the scanning process, the parallel projections of all angles are stored respectively so that one can obtain the orthographic projection. The picture required called Sinogram, as the next processing object, we can acquire Positron Emission Tomography utilized back projection.



Store parallel projections of all angles



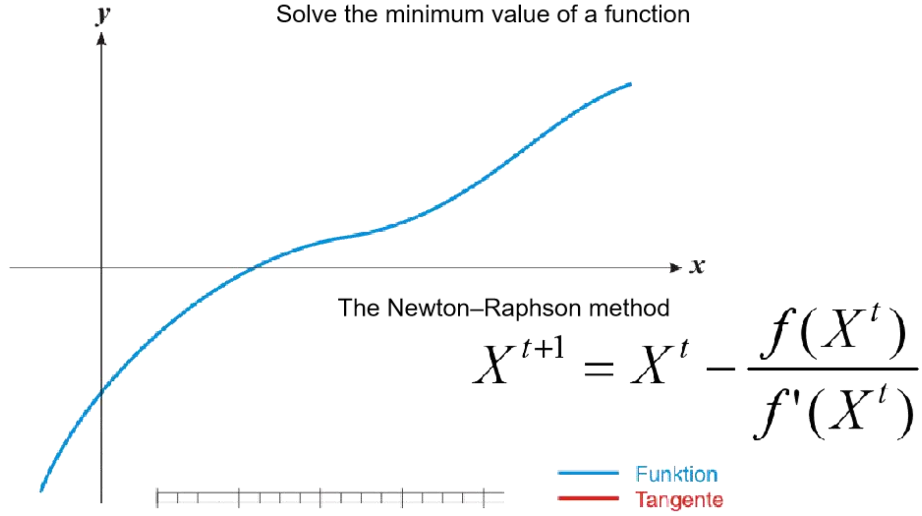
Mathematics

Iterative algorithm is a mathematically approach that can solve optimization problem by starting from a certain initial value X0, according to an iterative formula,and successively calculating X_1,

X_2,..., so that the sequence $\{X_t\}$ converges to the exact solution for the problem. As figure below shown,the Newton–Raphson method can solve the minimum value of a one-dimensional function which also called Newton iteration method.

A sparse matrix is a matrix that is comprised of mostly zero values. In this project, through the machine shown in the figure, we can measure and calculate the system sparse matrix A.

$$X^* = \arg_x \min f(x)$$



$$A_{6\times 7} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 & 4 \end{bmatrix}$$

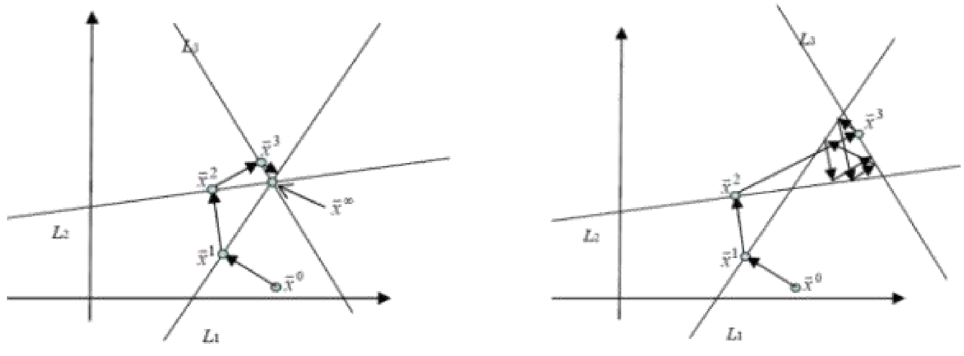
sparse matrix

	row	column	value
0	6	7	7
1	0	2	1
2	1	1	2
3	2	0	3
4	3	3	5
5	4	4	6
6	5	5	7
7	5	6	4

Algorithm

1. ART iterative Algorithms

The algebraic reconstruction technique (ART) algorithm is suitable for image reconstruction of incomplete projection data, and has strong anti-noise interference ability. Its biggest disadvantage is the large amount of calculation and the slow reconstruction speed.



(a) (b)

As image shown,L1, L2 and L3 are three straight lines, representing three equations. Their intersection point is the solution of the system of equations. When the data is consistent, each step of the calculation will project the current calculated point onto the next straight line, so that it satisfies the next equation. Eventually, this algorithm will converge to the solution of the system of equations (a). If the data is incompatible, this algorithm will cause the solution jump back and forth, leading to the equation solution to not converge (b). The ART algorithm is carried out one ray after another. When each ray is considered, the image is updated once, which is called completion of an iteration. If the image does not meet the convergence requirements, repeat the above process until the convergence conditions are met. This algorithm can be written as the formula shown.

$$\mathrm{f}_j^{(k+1)} = f_j^{(k)} + \lambda \frac{p_i - \sum_{n=1}^{N} \omega_{in} \mathrm{f}_j^{(k)}}{\sum_{n=1}^{N} \omega_{in}^{2}} \omega_{ij}$$

"Plus" ART algorithm

Where λ is the relaxation factor,λ∈(0,2). $f_j^{(k)}$ is the result of the k^{th} iteration

①$\vec{\mathrm{f}}_j = \vec{\mathrm{f}}_j^{(0)}, \; j = 1,2,3 \ldots N$

②$p_i{*} = \sum_{j=1}^{N} \omega_{ij} f_j^{(0)}$

③$\Delta_i = p_i - p_i{*}$

④$C_{ij} = \Delta_i \frac{\omega_{ij}}{\sum_j (\omega_{ij})^2}$

⑤$f_j = f_j^{(0)} + C_{ij}$

A series of iterative results $f^{(0)}$, $f^{(1)}$...$f^{(K)}$, if they complu convergence condition, there is a positive integer K for a small positive number ξ given in advance, so that when k<K, there are: $| \mathrm{f}_j^{(k)} - f_j^{(k-1)} | < \xi$

$$\mathrm{f}_j^{(k+1)} = f_j^{(k)} \left(\frac{p_i}{\sum_{n=1}^{N} \omega_{in} \mathrm{f}_n^{(k)}} \right)^{\lambda \omega_{ij}}$$

"Multiplication" ART algorithm

Specific iteration steps:

① First assign initial values to the unknown image vector

②Calculate the theoretical projection value of the i-th ray after passing through the object:

③Calculate the error between the theoretical projection value and the actual projection value:

④Calculate the correction value of the i-th equation:

⑤ Use the correction value in ④ to correct the j-th pixel value:

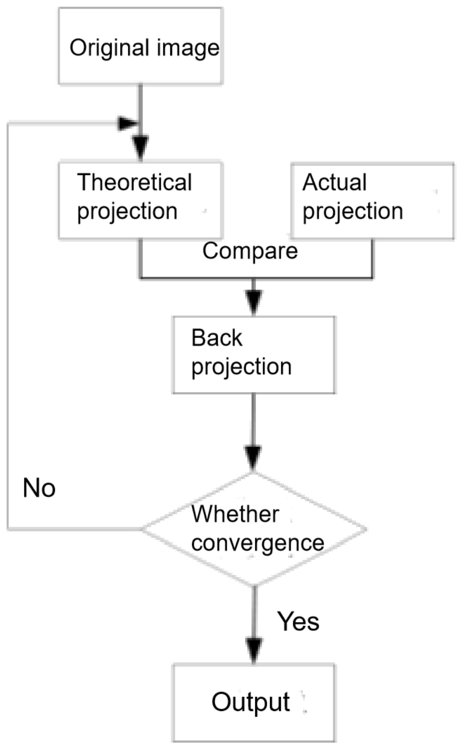
⑥ Repeat ② to ⑤ for all rays in this direction to complete the pixel value correction in this direction.

⑦ Repeat steps ① to ⑥ for the remaining projection directions until all the directions are corrected, that is, one iteration is completed.

⑧ Take the result of the iteration as the initial value of the next iteration, repeat the above steps, and get the K-th result

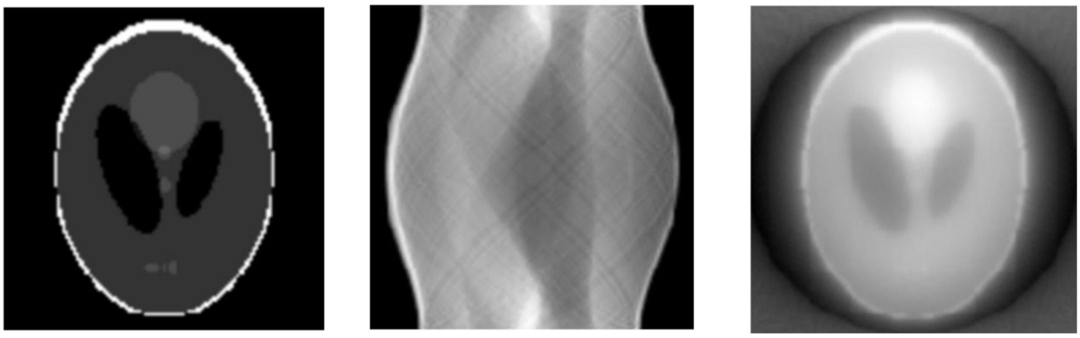
⑨If the convergence condition of the above formula is satisfied, that is, there is a positive integer K for a small positive number given in advance ξ

The ART reconstruction algorithm is completed by adding a correction value in the process of correcting the pixel value. This form of ART reconstruction algorithm is called "additive" ART algorithm. When the pixel value is corrected by multiplying by a correction value, we call it MART.



2. MLEM reconstruction algorithm

As the mechanism mentioned below,one can obtain projection data Y.Therefore, the question is: Given the projection data Y and system matrix A (also called probability matrix), find the image X.



Original image -> Orthographic projection -> Back projection

One can establish a system of equations Y=AX to obtain image X.Here A(i,j) represents the probability that the photon pair emitted by the j-th pixel being received by the i-th detector.

MLEM（maximum likelihood-expectation maximization）

$$Y = AX \Leftrightarrow \min \| Y - AX \|^2$$

$$X = (A^T A)^{-1} AY$$

$$X = \max(0, X)$$

$$X_j^{n+1} = \frac{X_j^n}{\sum_k A_{kj}} \sum_i A_{ij} \frac{Y_i}{(AX^n)_i} \Leftrightarrow X^{n+1} = X^n \cdot * \{ A^T [Y./(AX^n)] \}./S$$

S - the vector obtained by summing the matrix A by column

.* - the components of the two vectors are multiplied one by one

./ - the components of the two vectors are divided one by one

① $\sim (AX^n)(\widetilde{Y} = AX^n)$

② $\sim Y./(AX^n)(M = Y./\widetilde{Y})$

③ $\sim A^T(Y./(AX^n))$

④ $\sim \mathrm{S} = \sum_k A_{kj}$

Here, we proposed maximum likelihood-expectation maximization(MLEM) demonstrating in the formula below.① is the orthographic projection process to get the vector $\widetilde{Y}$.

② The two vectors Y and $\widetilde{Y}$ are divided item by item to get the vector M

③The transpose of A is multiplied by M

④S is equal to the sum of matrix A by column.

Therefore, the iteration term X(n+1)=X(n).*A^TM./S

Reconstruction

Amplification

In order to suppress statistical noise, a low-pass filter is added in the process of reconstructing the tomographic image by the filtered back projection method. Considering that the details of the image are preserved as much as possible, the cut-off frequency of the filter is generally set near the highest response frequency of the detector (1/FWHM).
We increased the spatial resolution (FWHM) of the detector to 0.4cm, the various frequency components of the image in the passband were added. So the signal/noise ratio was improved.

Calibration

1. Decay Calibration

Radioactive decay causes the strength of the drug to gradually decrease the lifespan of the positron nuclides according to an exponential law. The change of the radioactive intensity of the drug during the measurement can be ignored in static collection, but it must be considered for dynamic collection of whole body scanning gated collection and quantitative research. According to the exponential decay law, when the drug with the radioactivity intensity A_0 is injected, the radioactivity intensity drops to $A(t)=A_0e^{(-\lambda t)}$ when a certain frame is collected after time t. Where λ is the decay constant of the nuclide, therefore, one can obtain the intensity A_0 at the time of injection from the radioactive intensity A(t) of the drug at time t according to which $A_0=A(t)/e^{(\lambda t)}$. If the acquisition time of each frame is shorter than the half-life of the drug, you can ignore the change in radioactive intensity during the acquisition of each frame. Multiplying $e^{(\lambda t)}$ as a scale factor by the count value of each pixel in the frame can normalize the image to the time t as the injection time that is approximated by the midpoint time from the injection to the acquisition of this frame.

2. Coincidence Calibration

Scattered coincidence and accidental coincidence events are relatively uniform in spatial distribution that may respond to the contrast of image.
Scattering coincides with the distribution of radioactivity in the patient's body. Due to its uniform distribution, we can estimate that the scattering coincides with the part where is no radiopharmaceutical at the edge of the field. Subsequently, we can deduct the calculation result from the original data. This correction must be performed before attenuation calibration and image reconstruction.
If the accidental coincidence circuit is added to the original circuit and the detection unit is

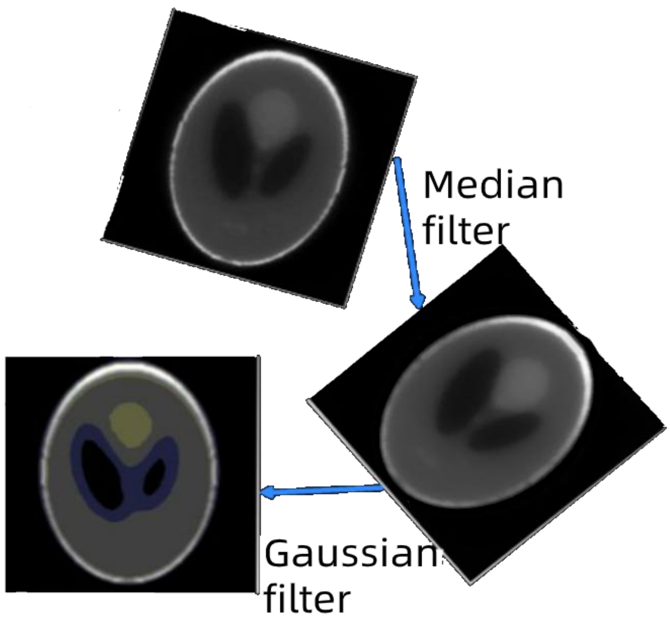
connected, we can use software to complete the accidental coincidence correction. Knowing the count rate R output by detection unit, and the width τ of the coincidence window, the accidental coincidence can be estimated by the formula $2\tau R^2$.

3. Attenuation Calibration

To obtain quantitative diagnostic conclusions from PET images, the attenuation of the human body must be corrected. The intensity of γ-ray Io after passing through the human body is I_b. We define the amount of attenuation δ, $I_b = I_o\delta$. Irrelevant in the same line with line attenuation and the position of the source point, so long as the propagation along the same path, regardless of the annihilation point O where the intensity I as measured in line are equal. We can use a line source placed outside and parallel to the axis of the human body to measure the attenuation of the conforming path through the source point. When the line source rotates around the body once to complete the transmission scan, the attenuation results along all the coincident lines can be obtained. Compared with the blank scan Io when there is no patient, the attenuation is obtained. Use it as a divisor to perform attenuation correction on the corresponding projection value.

Denoising

Here, we used Median filter and Gaussian filter. Median filter is a nonlinear smoothing filter. Its basic principle is to replace the value of a point in a digital image with the median value of each point in a field of that point. It is commonly used in pixels with a large difference in degree value.Median filter can change the pixel value closer to the surrounding pixel values, so that isolated noise points can be eliminated.

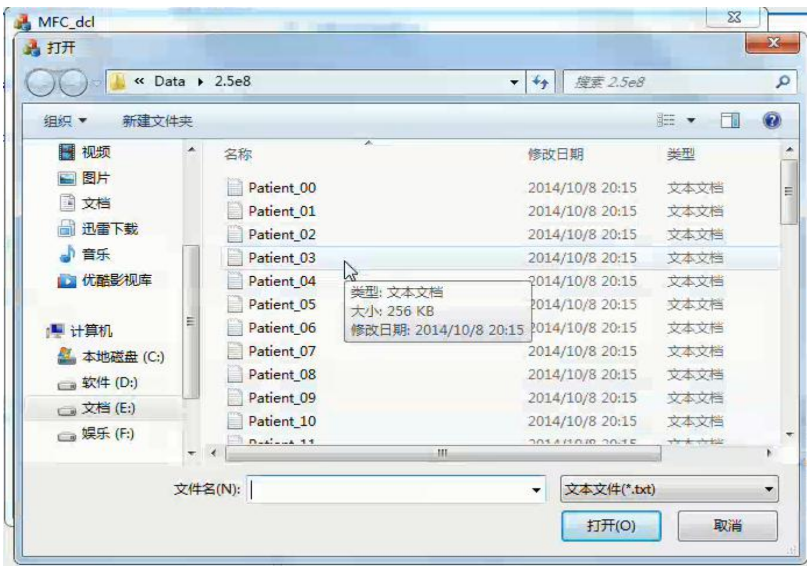


Gaussian filter is a linear smoothing filter, which is suitable for eliminating Gaussian noise. It uses a convolution to scan each pixel in the image, and uses the weighted average gray value of the pixels in the neighborhood determined by the template to replace the value of the center pixel of the template.

```
    void Iterative::SmoothGauss(Vector &pImageData, int nWidth, int nHeight, int nWidthS
tep)
{
double *sz = new double[nWidth*nHeight];
for (int k = 0; k<nWidth*nHeight;  k++)
{
sz[k] = pImageData.pdData[k];
}
int i = 0;
int j = 0;
float fValue = 0.0;
double *pLine[3] = { NULL, NULL, NULL };
int nTemplate[9] =
{
1, 2, 1,
2, 4, 2,
1, 2, 1
};
for (j = 1; j < nHeight - 1; j++)
{
```

Implement

After finishing C++ code of the iterative reconstruction part, we made a Windows Forms program (MFC) to adjust and find out the optimal values of iterations.

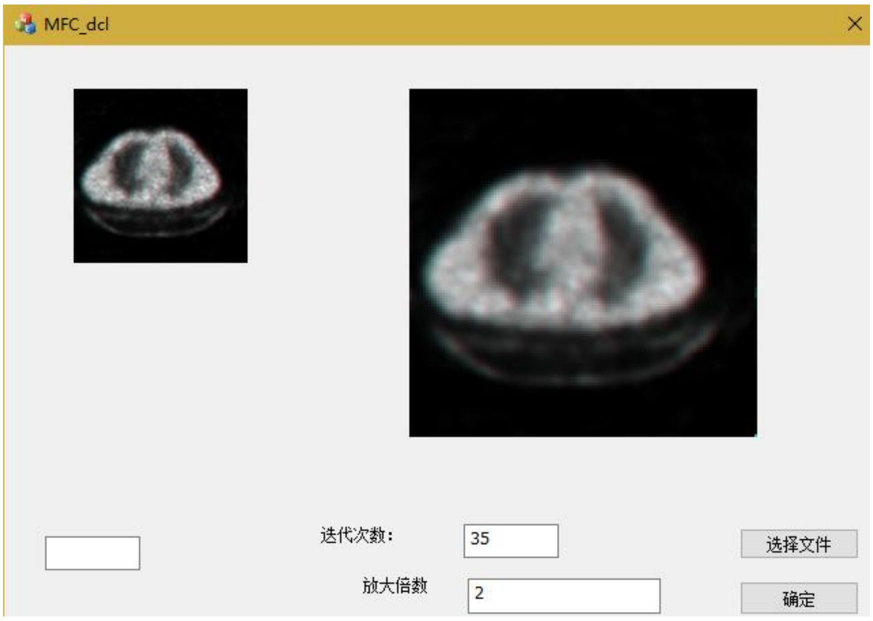


We can upload patient data, set iteration values, and then obtain reconstructed images.


MFC_dcl
迭代次数:
15
选择文件
放大倍数
2
确定


MFC_dcl
迭代次数:
25
选择文件
放大倍数
2
确定


MFC_dcl
迭代次数:
30
选择文件
放大倍数
2
确定


MFC_dcl
迭代次数：
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选择文件
放大倍数
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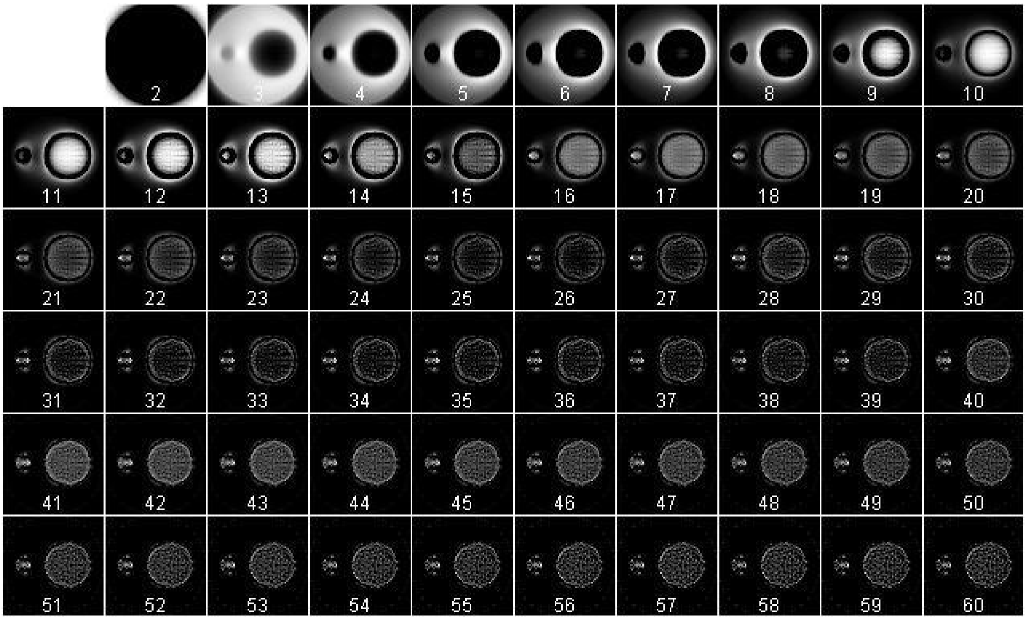
确定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