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Ratings quality over the business cycle[☆]

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ABSTRACT

Credit rating agencies (CRAs) have long held that reputational concerns discipline their behavior. The value of reputation, however, depends on economic fundamentals that vary over the business cycle. In a model of ratings incorporating endogenous reputation and a market environment that varies, we find that ratings quality is countercyclical. Specifically, a CRA is more likely to issue less-accurate ratings when fee-income is high, competition in the labor market for analysts is tough, and securities' default probabilities are low. Persistence in economic conditions can diminish our results, while mean reversion exacerbates them. The presence of naive investors reduces overall quality, but quality remains countercyclical. Finally, we demonstrate that competition among CRAs yields similar results.

1. Introduction

The current financial crisis has prompted an examination of the role of credit-rating agencies (CRAs). With the rise of structured finance products, the agencies rapidly expanded their ratings business and earned dramatically higher profits (Moody's, for example, tripled its profits between 2002 and 2006). Yet ratings quality seems to have suffered, as the three main agencies increasingly gave top ratings to structured finance products shortly before the financial markets collapsed. This type of behavior has been brought to the public's (and regulators') attention many times, such as during the East Asian Financial Crisis (1997) and the failures of Enron (2001) and Worldcom (2002). Beyond the issue of why the CRAs were off-target, these repeated instances raise the question of *when* CRAs are more likely to be off-target.

In this paper, we examine theoretically how CRAs' incentives to provide quality ratings change in different economic environments, specifically in the booms and recessions of business cycles. Our analysis highlights that both the effective costs of providing high-quality ratings and the benefits to the CRA of doing so vary through the business cycle. Specifically, we show that reputational incentives lead naturally to countercyclical ratings quality.

Several economic fundamentals suggest that ratings quality is lower in booms and improves in recessions. In a boom, it is more expensive to hire skilled analysts.[1]

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[1] For example: "At the height of the mortgage boom, companies like Goldman offered million-dollar pay packages to workers like Mr. Yukawa who had been working at much lower pay at the rating

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The CRA can earn more in a boom, and so may be tempted to capitalize on the opportunity to earn the higher revenues available. The CRA could do so both directly—through higher volume of issues and, perhaps, through higher fees—and indirectly—through advisory and other ancillary services. In addition, if issues are relatively unlikely to default in boom periods, monitoring a CRA's activities is less effective, and the CRA's returns from investing in ratings quality are likely to be diminished. Default probabilities may actually be higher towards the end of a boom if lower-quality issuers seek to be rated.

We examine the impact of each of these economic fundamentals in a simple infinite-horizon model of ratings reputation in which CRAs invest in ratings quality each period and earn fees as long as no rated product defaults. Our analysis demonstrates that the simple intuitions in the above paragraph have force. In particular, when future shocks to economic fundamentals are independent and identically distributed (i.i.d.) draws from a probability distribution, these forces lead to countercyclical ratings quality. However, in the more realistic case in which states are not independent across time, new effects arise. Our findings may be diminished when there is substantial persistence in shocks (positive correlation), but may actually be exacerbated when there is mean-reversion. Introducing naive customers reduces accuracy in both booms and recessions, but the comparison between the states does not change. We also extend the model to allow for competition between CRAs, and we demonstrate that similar results hold.

The idea that ratings quality may be countercyclical is consistent with recent empirical work on the market for structured finance products. As a relatively new market for hard-to-evaluate investments, the structured finance market opened up the possibility for accuracy and reputation management by CRAs. Ashcraft, Goldsmith-Pinkham, and Vickery (2010) show that the mortgage-backed security-issuance boom from 2005 to mid-2007 led to ratings-quality declines. Griffin and Tang (2012) demonstrate that CRAs made mostly positive adjustments to their models' predictions of credit quality and that the amount adjusted increased substantially from 2003 to 2007. These adjustments were positively related to future downgrades.

Our results are relevant to the current policy debate regarding the role of CRAs. We show that if reputation losses are higher, there are greater incentives to provide accurate ratings. Recent Securities and Exchange Commission (SEC) rules promoting full disclosure of ratings history can make it easier for investors to know when a CRA is performing poorly and to punish it. The Dodd-Frank financial reform bill makes CRAs more exposed to liability claims for poor performance.[2] This may give the investors a stick to make punishment credible.

White (2010) highlights the role that regulation has played in enhancing the importance and market power of the three major rating agencies (by granting them a special status and having capital and investment requirements tied to ratings). Given the "protected" position of these agencies,[3] the reputational concerns that constrain CRAs' behavior should be understood somewhat more broadly than the reduced-form approach taken in our model.[4] The model views these reputational concerns as arising from investors' withdrawal of their business from CRA-rated products. Although this might appear stark, it may apply well to innovative financial instruments, which have been the focus of public and policy concerns. Indeed, the structured finance market (and the need for ratings) dried up as the crisis hit, and stock market valuations for Moody's fell significantly. In addition, concerns regarding a regulatory environment that is relatively more or less sympathetic to the CRAs may also determine the CRAs' reputational incentives. Lastly, although something similar has not occurred in the recent crisis, the downfall of Arthur Andersen represents a severe punishment to a certification intermediary in a similar line of business (auditing).

Our paper may also be cast more broadly in terms of incentives for certification intermediaries who are paid by those seeking certification. In this light, evidence on equity analysts appears consistent with our model. Michaely and Womack (1999) and Lin and McNichols (1998) find substantial evidence of biased recommendations when analysts' employers had underwriting relationships with the firms being analyzed. Jackson (2005) shows that both optimistic recommendations and better reputation (in the form of a higher ranking in an investor survey) generate higher trading volumes with the analyst's employer. Hong and Kubik (2003) show that accuracy becomes less important (and optimism somewhat more important) for equity analysts moving up the job hierarchy in boom times.[5]

In the following subsection, we review related theoretical work. In Section 2, we set up the model. In Section 3, we analyze the case of a monopoly CRA. In Section 4, we examine the robustness of the model to correlation and naive investors. In Section 5, we study the duopoly case. In Section 6, we formulate the predictions of the model as hypotheses and examine support from recent empirical work on ratings. Section 7 concludes. Unless indicated that the proof is in the Online Appendix, all proofs are in the Appendix.

(*footnote continued*)
agencies, according to several former workers at the agencies. Around the same time that Mr. Yukawa left Fitch, three other analysts in his unit also joined financial companies like Deutsche Bank." This excerpt is from "Prosecutors ask if 8 banks duped rating agencies," by L. Story, *New York Times*, May 12, 2010.

[2] The higher standard of liability for CRAs required by the Dodd-Frank bill has not been enforced, as an initial attempt to do so caused CRAs to pull their ratings from asset-backed securities, freezing the market. The SEC decided, in response, to delay implementation for further study, and there is discussion about eliminating the requirement (see Jessica Holzer, "House panel votes to free raters from ABS liability," *Wall Street Journal* Online, July 20, 2011).

[3] The Dodd-Frank bill and current rulemaking by the SEC will most likely diminish regulatory barriers to entry.

[4] And in related models of endogenous reputation, such as Mathis, McAndrews, and Rochet (2009).

[5] Specifically, they compare accuracy in the 1996–2000 boom period with accuracy in the period 1986–1995.

1.1. Related theoretical literature

Mathis, McAndrews, and Rochet (2009) is the closest paper to this one in examining how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truthtelling to build up/exploit its reputation. The authors focus on whether an equilibrium in which the CRA tells the truth in every period exists, and they demonstrate that truthtelling incentives are weaker when the CRA has more business from rating complex products.[6] Strausz (2005) is similar in structure to Mathis, McAndrews, and Rochet (2009), but examines information intermediaries in general. Our model considers a richer environment in which CRA incentives are linked to a broad set of economic fundamentals that fluctuate and may persist through time. Our paper also examines competition and develops a connection with labor-market conditions.

Our model also builds on and further develops the understanding of firm behavior in business cycles. Several papers analyze how firms maintain collusive behavior through the business cycle, while we analyze incentives to build up or milk reputation. Rotemberg and Saloner (1986) and Dal Bó (2007) consider future states to be i.i.d. draws from a known distribution. Haltiwanger and Harrington (1991) consider a deterministic business cycle. Bagwell and Staiger (1997) and Kandori (1991) add correlation between periods, as we do in our model.

In addition to Mathis, McAndrews, and Rochet (2009), there are several other recent theoretical papers on CRAs. Faure-Grimaud, Peyrache, and Quesada (2009) look at corporate governance ratings in a market with truthful CRAs and rational investors. They show that issuers may prefer to suppress their ratings if they are too noisy. They also find that competition between rating agencies can result in less information disclosure. Mariano (2012) considers how reputation disciplines a CRA's use of private information when public information is also available. Fulghieri, Strobl, and Xia (2011) focus on the effect of unsolicited ratings on CRA and issuer incentives. Bolton, Freixas, and Shapiro (2012) demonstrate that competition among CRAs may reduce welfare due to shopping by issuers. Conflicts of interest for CRAs may be higher when exogenous reputation costs are lower and there are more naive investors. Skreta and Veldkamp (2009) and Sangiorgi, Sokobin, and Spatt (2009) assume that CRAs relay their information truthfully, and they demonstrate how noisier information creates more opportunity for issuers to take advantage of a naive clientele through shopping. Bouvard and Levy (2010) examine the two-sided nature of CRA reputation. In Pagano and Volpin (2012), CRAs also have no conflicts of interest, but can choose ratings to be more or less opaque depending on what the issuer asks for. They show that opacity can enhance liquidity in the primary market but may cause a market freeze in the secondary market. Winton and Yerramilli (2011) study a related market, that of originate-to-distribute lending, in a dynamic model in which reputation may discipline monitoring incentives.

Our model explores the interaction between the business cycle and incentives. Bond and Glode (2011) analyze a problem where individuals may become bankers or regulators (of bankers). They find that in booms, banks grab the most-talented regulators, making the regulation of the system more fragile. In this paper, this type of cream-skimming by banks from CRAs forms the basis of our observations on the analyst labor market. Relatedly, Bar-Isaac and Shapiro (2011) examine incentives for analysts at CRAs and find that CRA accuracy is non-monotonic in the probability that analysts have outside offers from banks; it increases at first because of more effort from the analysts, but then may decrease due to lower CRA training incentives. Povel, Singh, and Winton (2007) study firms' incentives to commit fraud when investors may engage in costly monitoring, and they find that fraud is more likely to occur in good times.

2. The model

We begin by presenting a model with a single CRA and many issuers and investors who can interact over an infinite number of discrete periods.[7] Economic fundamentals change from period to period. We suppose that there are two states $s \in \{R,B\}$, R corresponding to a recession and B to a boom. We specify the difference between the two states after defining the model.

Each period, an issuer has a new investment, which can be good (G) or bad (B). A good investment never defaults and pays out 1, while a bad investment defaults with probability p_s. If it defaults, its payout is zero; otherwise, its payout is 1. The probability that an investment is good is λ_s. The issuer has no private information about the investment. This implies that the CRA can have a welfare-increasing role of information production by identifying the quality of the investment. Both the issuer and the investors observe the ratings and performance of the investment.

The issuer approaches the CRA at the beginning of the period to evaluate its investment. If the CRA gives a good rating, the issuer pays the CRA an amount π_s. The CRA is not paid for bad ratings. This is a version of the shopping effect described in Bolton, Freixas, and Shapiro (2012) and Skreta and Veldkamp (2009). Mathis, McAndrews, and Rochet (2009) assume that no issue takes place if the rating is bad and that the CRA is not paid in this case, which is equivalent to our approach.

Our focus is on the CRA's ratings policy—i.e., how they invest in increasing the likelihood that their analyses are correct. We model this as a direct cost to the CRA for improving its accuracy. There is no direct conflict of interest, as in Bolton, Freixas, and Shapiro (2012),

[6] Mathis, McAndrews, and Rochet (2009) provide examples of reputation cycles where the CRA's reputational incentives fluctuate, depending on the current level of reputation. These are not related to economic fundamentals of the business cycle, as they are in our model.

[7] Issuers and investors may be long- or short-lived in the model, whereas the CRA is long-lived.

and we remain agnostic about whether CRAs intentionally produce worse quality ratings. In our model, increasing rating quality is costly, and the CRA maximizes profits given the reality of the business environment.

The cost that the CRA pays for accurate ratings could represent improving analytical models and computing power; performing due diligence on the underlying assets; the staffing resources allocated to ratings; or hiring and retaining better analysts. For the sake of concreteness, we will focus on the employment channel: Hiring better analysts is more costly to the CRA.[8] Investors cannot directly observe the CRA's policy, but must infer it from their equilibrium expectations and from their previous observations of defaults on rated investments.

We model the analyst labor market in a reduced-form manner. In a given period, a CRA pays a wage $w_s \in [0,\overline{w}]$ to get an analyst of ability $z(w_s,\gamma_s) \in [0,1]$, where γ_s is a parameter that captures labor-market conditions.[9] When there is no confusion, we suppress the arguments and write ability as z_s. We suppose that it is harder to attract and retain higher-ability analysts and that it becomes even harder at the top end of the wage distribution, meaning that $\partial z/\partial w_s > 0$ and $\partial^2 z/\partial w_s^2 < 0$. We also assume that $\partial z/\partial w_s \to \infty$ as $w_s \to 0$, $z(0,\gamma_s) = 0$, and $\partial z/\partial w_s|_{\overline{w}} = 0$. With respect to the labor-market conditions, we suppose that when γ_s is larger, the labor market is tighter, and it is more difficult to get high-quality workers, so that $\partial z/\partial \gamma_s < 0$ and $\partial^2 z/\partial \gamma_s \partial w_s < 0$. This implies that a higher wage must be paid in order to maintain quality if the labor market is tighter.

Ability is important for gathering information and determining whether the investment is good or bad. All analysts can identify a good investment perfectly ($p(G|G) = 1$). They may, however, make an error about a bad investment with positive probability $1-z$, where $p(B|B) = z$. Therefore, the CRA, through its wage, is choosing its tolerance for mistakes based on both the costs of hiring *and* the incentives for accuracy that are embedded in the dynamics of the model.

These incentives for accuracy arise since we assume that if investors suspect that the CRA is not investing sufficiently in ratings quality (say, $z < \overline{z}$), then they would not purchase the investment product; however, if investors believe that the CRA has invested sufficiently ($z \geq \overline{z}$), then the rating is of sufficient quality to lead investors to purchase. The cutoff $\overline{z}$ is exogenous here, but it represents the investor's decision to allocate money to this investment as opposed to other opportunities; that is, it could be derived from a participation-constraint or portfolio-allocation problem for the investor.[10] We suppose throughout that while the CRA maintains its reputation, the constraint $z \geq \overline{z}$ does not bind. Trivially, if the constraint is ever violated, then investors would not purchase, and so issuers would not seek ratings; in such circumstances, the CRA would not be active.

As in any infinitely repeated game, there are many equilibria. We focus on the equilibrium in which the CRA is most likely to report honestly or, equivalently, minimize mistakes: i.e., the equilibrium supported by grim-trigger strategies (see Abreu, 1986). Issuers and investors observe only three states: a good report where the investment returns 1; a bad report; and a good report where the investment defaults. A grim-trigger strategy here is that investors never purchase an investment rated by a CRA that had previously produced a good report for an investment that subsequently defaulted. This grim outcome is an equilibrium in the continuation game since, if investors do not purchase, then it is optimal for the CRA to set $w=0$ so that $z < \overline{z}$. Moreover, this equilibrium of the infinitely repeated game has a natural interpretation corresponding to reputation. As in the seminal work of Klein and Leffler (1981), and developed in a wide-ranging literature discussed in Section 4 of Bar-Isaac and Tadelis (2008), the CRA sustains its reputation as long as it is not found to give a good rating to a bad investment, but loses its reputation if it is ever found to do so.

We distinguish between booms and recessions by assuming throughout the paper that booms involve higher fees ($\pi_B > \pi_R$), tighter labor-market competition ($\gamma_B > \gamma_R$), a greater proportion of good projects ($\lambda_B > \lambda_R$), and lower probabilities of default ($p_B < p_R$).[11]

These economic fundamentals change through time, but the state of the economy in each period depends on the state of the economy in the previous period. Define τ_s as the probability that there is a transition from the current state s to the other state. Note that both τ_B and $1-\tau_R$ represent the probabilities of moving to a recessionary state in the next period (when starting from the boom and recessionary states, respectively). When $\tau_B = 1-\tau_R$, each period's state is an independent and identically distributed draw from the same distribution. When $\tau_B < 1-\tau_R$, there is persistence or positive correlation among states: A boom state is more likely to follow a boom state than a recessionary state, and a recessionary state is more likely to follow a recessionary state. When $\tau_B > 1-\tau_R$, there is reversion to the mean among states. These transition probabilities are related to the duration of a boom or recession: A higher value of τ_s implies a

[8] While there is no empirical work on CRA staffing, internal emails uncovered by the Senate Permanent Subcomittee on Investigations (2010) shed light on the CRAs' staffing situation right before the recent crisis. For example, a Standard & Poor's (S&P) employee wrote on 10/31/2006: "While I realize that our revenues and client service numbers don't indicate any ill [e]ffects from our severe understaffing situation, I am more concerned than ever that we are on a downward spiral of morale, analytical leadership/quality and client service."

[9] Note that here, the wage w is the wage per issue rated so lower quality might reflect either a less-able analyst or an analyst (of equal quality) who spends less time on a rating. This latter interpretation is also present in Khanna, Noe, and Sonti (2008), who demonstrate in a static setting that initial public offering (IPO) screening may suffer when the demand for financing increases due to a fixed supply of labor available to underwriters.

[10] In order to endogenize $\overline{z}$, we would have to consider the investor's problem explicitly and introduce additional parameters to capture his preferences and opportunities. Note that although these may vary according to the business cycle, it is sufficient to consider $\overline{z}$ the highest threshold in any state. Similarly, the threshold $\overline{z}$ might vary across different versions of the model (e.g., the duopoly model in Section 5).

[11] In reality, λ_s may actually be decreasing, and p_s may be increasing at some point if booms attract lower-quality issuers or investments to get ratings. These cases can be analyzed easily in our model, given our comparative statics results.

shorter duration for the state s and a rapid move towards the other state.

3. Countercyclical ratings quality

In this section, we analyze the model for the situation in which economic shocks are independent and identically distributed draws. We use this to prove our main result: that ratings quality is countercyclical. In subsequent sections, we examine the robustness of these conclusions by extending the model in several directions.

The CRA maximizes by choosing the current wage, and takes the continuation values as given. We assume that investors anticipate that the ratings-quality is high enough in each state such that they would purchase after observing a good rating; that is, $z(w_s,\gamma_s)>\overline{z}$ for $s=R$ or B. These conditions can be verified after characterizing the equilibrium wages w_R^* and w_B^*.

It is convenient to introduce notation for the probability that an investment gets a good rating in state $s\in\{B,R\}$:

$$\alpha_s := \lambda_s+(1-\lambda_s)(1-z_s). \tag{1}$$

The CRA gives a good rating when the project is good (with probability λ_s) or when the project is bad and is misreported (that is, with probability $(1-\lambda_s)(1-z_s)$).

We also introduce notation for the probability that the CRA "survives" into the future—i.e., the probability that it does not give a good rating to an investment that subsequently defaults. This probability is given by

$$\sigma_s := 1-(1-\lambda_s)(1-z_s)p_s, \tag{2}$$

where $s\in\{B,R\}$.

Here, the only possibility for an investment to be rated as good and then default is when it is bad, which occurs with probability $(1-\lambda_s)$; rated as good, which occurs with probability $(1-z_s)$; and defaults, which happens with probability p_s. Note that the probabilities α_s and σ_s are endogenous since they depend on z_s, which, in turn, depends on the CRA's strategic choices of investment in ratings quality w_s.

The value functions from the beginning of a period are denoted as V_s when the state is s and the value from the end of a period in state s (i.e., before it is known whether the economy will be in a boom or a recession in the next period) are denoted as EV_s. Given that τ_s captures the probability of moving from s to the other state, it is immediate that

$$EV_B := (1-\tau_B)V_B+\tau_B V_R \tag{3}$$

and

$$EV_R := (1-\tau_R)V_R+\tau_R V_B. \tag{4}$$

This notation allows us to simply write down a value function for each state:

$$V_B = \max_{w_B} \pi_B\alpha_B - w_B+\delta\sigma_B EV_B,$$
$$V_R = \max_{w_R} \pi_R\alpha_R - w_R+\delta\sigma_R EV_R. \tag{5}$$

Knowing the current state, the CRA pays the wage w_s and earns the fee whenever it reports a good project, which occurs with probability α_s. The probability that the project is bad, the agency misreports, and the project defaults is $1-\sigma_s$; then, in the continuation, no issuer returns to the CRA (anticipating that the CRA would set $w_s=0$), and the CRA's continuation value is zero. Otherwise, the CRA earns the expected continuation value EV_s with probability σ_s.

We denote equilibrium values with an asterisk (*). We begin by proving the existence and uniqueness of a solution.

Lemma 1. There exists a unique solution (V_B^*,V_R^*) with associated w_B^* and w_R^* to the system of Eq. (5).

We are interested in the difference between accuracy during booms and during recessions. We begin by writing the first-order conditions for the decision variables w_B and w_R, respectively:

$$\frac{\partial z}{\partial w}(w_B^*,\gamma_B)=\frac{1}{1-\lambda_B}\frac{1}{\delta p_B EV_B^*-\pi_B}, \tag{6}$$

$$\frac{\partial z}{\partial w}(w_R^*,\gamma_R)=\frac{1}{1-\lambda_R}\frac{1}{\delta p_R EV_R^*-\pi_R}. \tag{7}$$

Given the first-order conditions (6) and (7), it follows that $w_B^*\le w_R^*$, and there is more accuracy in recessions than in booms when:

$$(1-\lambda_B)(\delta p_B EV_B^*-\pi_B)\le(1-\lambda_R)(\delta p_R EV_R^*-\pi_R). \tag{8}$$

As we stated earlier, when $\tau_B=1-\tau_R$, each period's state is an i.i.d. draw from the same distribution. That is, the likelihood of transitioning to a boom (or a recession) is the same, irrespective of whether there is a recession or boom today. This implies that the continuation values from a boom and from a recession are identical, $EV_B^*=EV_R^*$. In this case, we get a very strong result: Ratings quality is lower in boom states than in recessionary states.

Proposition 2. If states are independent across time ($\tau_B=1-\tau_R$), then there is more investment in ratings quality in a recession than in a boom.

This is a key result. It says that ratings are countercyclical—there is higher ratings quality in recessions than in booms. This result is intuitive. The assumption that economic shocks are i.i.d. draws makes the CRA essentially treat the future as fixed in all states of the world, with its current investment affecting only the current payoff and the probability of surviving to the future. Therefore, the higher fees that arise in a boom mean that the CRA wants to be less accurate to collect more fees. There is less reason to invest in ratings quality in a boom if, in any case, more of the investments are good investments. Lower default probabilities in a boom imply a lower likelihood of getting caught for reduced accuracy. Finally, a tighter labor market in a boom means that hiring good analysts is more costly. All of these point to lower accuracy in boom states.

4. Robustness

In this section, we push the monopoly CRA model in two directions to study how general the countercyclical ratings-quality results are. First, we look at correlation between economic shocks. Second, we allow for some of the investors to be naive. Then, in Section 5, we consider the case of competition between CRAs.

4.1. Correlation between shocks

If booms and recessions do not arise independently of history, then Proposition 2 cannot be applied directly. Condition (8) is not necessarily easy to verify since the continuation values EV_B^* and EV_R^* are endogenously determined.

One can see from the first-order conditions that the value to the CRA of being in a boom or recession plays a key role in the CRA's investment decision. The higher fees, larger proportion of good projects, and lower default probabilities that characterize a boom suggest that the value to a CRA of being in a boom is larger than the value of being in a recession. However, the fact that labor markets are tighter in booms provides a countervailing effect, as the extra wages a CRA must pay in a boom might eliminate some of the advantages.

These intuitions are formalized in Proposition 3 below:

Proposition 3. The difference between the value of being in a boom rather than in a recession ($V_B^*-V_R^*$):

(i) *decreases in the probability of default in a boom* (p_B) *and the competitiveness of labor-market conditions* (γ_B) *and increases in the proportion of good projects* (λ_B) *and the fee* (π_B);
(ii) *increases in the probability of default in a recession* (p_R) *and the competitiveness of labor-market conditions* (γ_R) *and decreases in the proportion of good projects* (λ_R) *and the fee* (π_R);
(iii) *decreases in the probability of transitioning from a boom to a recession* (τ_B) *and increases in the probability of transitioning from a boom to a recession* (τ_R) *if and only if it is more valuable to be in the boom state* ($V_B^* > V_R^*$).

In general, the comparison between V_B^* and V_R^* is ambiguous. However, as $V_B^* > V_R^*$ seems to be the interesting (and intuitive) case, we assume this to be true for presentation purposes throughout the remainder of the paper.[12] Although this is an assumption on endogenous values, it trivially holds where γ_B and γ_R are close enough.

Assumption A1. The value to a CRA of being in a boom is larger than the value of being in a recession ($V_B^* > V_R^*$)

Given Assumption A1, we can state the following:

Proposition 4. If there is mean reversion between states, then there is more investment in ratings quality in a recession than in a boom.

This is a direct result of Condition (8). Mean reversion implies that the future expected value is larger in a recession than in a boom because of the increased likelihood of transitioning to the boom. In the recession, the CRA builds up its reputation so as to reap the benefits of the approaching boom. In the boom, the incentive is to milk reputation since the recession is likely to come soon. This, then, implies that there are more-accurate ratings in a recession than in a boom.

In the case of persistence, or, equivalently, positive correlation, we find that ratings may also be countercyclical. Formally, Condition (8) is slack when states are independent across time, suggesting that at least for "small" levels of positive correlation, the condition would not be violated and that ratings are countercyclical. Numerical simulations suggest that the range in which countercyclical ratings arise can be significant, as illustrated in Fig. 1. In the figure, the area below the dashed diagonal line is where there is positive correlation between shocks. The shaded subset of that area is where ratings quality is procyclical. In this area, positive correlation is at its maximum. The rest of the figure, which is in white, has countercyclical ratings quality.

To better understand the implications of correlation, we now study continuous changes in the correlation between states (increasing/decreasing the amount of correlation). As described above, changing the extent of correlation between states has an empirical counterpart in the length of a boom or recession; for example, decreasing the probability of transitioning from boom to recession (reducing τ_B) is equivalent to supposing that booms, on average, last longer. We explore how the expected length of booms and recessions affects ratings quality in Proposition 5:

Proposition 5. (i) *Longer booms (i.e., a reduction in* τ_B*) increase investment in ratings quality in both states.*
(ii) *Longer recessions (a reduction in* τ_R*) decrease investment in ratings quality in both states.*

This useful result highlights that changing expectations of the likely severity of recessions or of the extent of moderation can impact ratings quality. Longer booms increase ratings quality in both states. In the boom, there is less likelihood that the good times will end soon, meaning that there is less desire to milk reputation. In the recession, the payoff of a transition to a boom increases, meaning that it is a good time to build up reputation. For analogous reasons, longer recessions have the reverse effect.

In the U.S. from 1854–2009, the average duration of a boom was 42 months and that of a recession was 16 months.[13] Taking this average as representative of expectations, in terms of the parameters of our model, $\tau_R \simeq (42/16)\tau_B$.[14] In Fig. 1, this would be represented by a line between the point $\tau_B = \tau_R = 0$ and the point where $\tau_R = 1$ and $\tau_B = 16/42$. Thus, this may be consistent with either procyclical or countercyclical accuracy. However, the empirical work discussed in the Introduction and in

[12] Results on the opposite case ($V_B^* < V_R^*$) can be summarized easily, given the proofs in the Appendix.

[13] Taken from the National Bureau of Economic Research (NBER) Business Cycle Dating Committee (http://www.nber.org/cycles.html).

[14] As we do not specify the length of a period in our model, we can only calibrate so that the relative frequency of booms and recessions is correct.

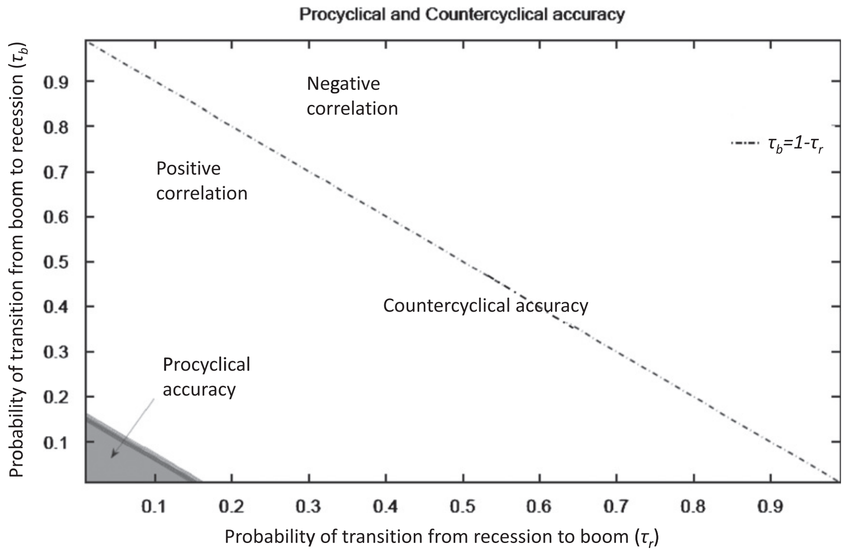

Fig. 1. Countercyclical or procyclical accuracy as a function of correlation of shocks over time (parameters used: $\delta = 0.95$, $p_B = p_R = 0.5$, $\lambda_B = \lambda_R = 0.5$, $\gamma_B = \gamma_R = 0.5$, $\pi_B = 0.5$, $\pi_R = 0.25$; $z(w,\gamma) = \sqrt{w}/\gamma$).

Section 6 suggests that ratings accuracy has been countercyclical. Proposition 5 is interesting in highlighting that concerns about a prolonged recession may have the additional consequence of reduced investment in ratings quality.

The results on the countercyclicality of ratings provide insight into the recent crisis and past events. While CRAs often claim that reputational incentives are a sufficient disciplinary device to maintain quality ratings, we show that ratings quality fluctuates with economic fundamentals. Are these fluctuations in ratings quality necessarily bad? We do not undertake a full welfare analysis in this paper, as doing so would require fully modeling investor utility. However, in the online Appendix, we present a reduced-form analysis of how a social planner would choose ratings quality. We demonstrate that when there are fewer good issues, higher default probabilities, and a looser labor market (as in a recession), the social planner would also want to invest more in ratings quality. There are two important caveats. First, if outside options are larger for investors in booms than in recessions (which is likely), this would give the social planner an incentive to invest more in ratings quality in booms. Second, we are unable to compare the levels of investment from the social planner's solution to the CRA's choices, due to the fact that the social planner's problem incorporates investors' outside options explicitly, and the CRA's problem does not.

4.2. *Naive investors*

Some of the potential investors in rated issues are sophisticated: They understand a CRA's incentives and rationally withdraw their business from rated investments when evidence of poor rating quality persists. However, it is likely that a fraction of investors are naive, in that they are willing to buy investments with good ratings regardless of the quality of the ratings. This may be due to poor incentives to do due diligence; an example may be pension fund managers whose compensation depends only marginally on the ex post return of the assets they manage. Moreover, the complexity of some investments makes it more costly to evaluate their worth. Regulation that forces managers to purchase only investments with good ratings could also provide incentives to be trusting.[15] In this section, we study the effect of incorporating naive investors into our dynamic model.

We model naive investors as being willing to invest in products with good ratings irrespective of any evidence—such as defaults—on poor accuracy. This will generally impede the reputation mechanism, as a fraction of well-rated products have a guaranteed market. The proportion of fees that CRAs generate from good ratings given to issuers who sell to naive investors will be denoted by ω. For simplicity, we will assume that this proportion is constant across states.[16] We define $\overline{V}_s$ as the continuation value for the CRA in state s when sophisticated investors will no longer purchase rated products but naive investors continue to do so. In this subgame, the CRA pays a wage $w_s^* = 0$ for $s \in \{B,R\}$, as it retains the trust of naive investors regardless

[15] In a study of the CRA credit-watch mechanism, Boot, Milbourn, and Schmeits (2006) model investors who take ratings at face value, calling them institutional investors. Similarly, Hirshleifer and Teoh (2003) model investors with "limited attention and processing power." Skreta and Veldkamp (2009) model naive investors as not realizing the selection bias that shopping for rating agencies induces. In Bolton, Freixas, and Shapiro (2012), naive investors also take ratings at face value, but punish rating agencies when evidence (default) proves malfeasance.

[16] Although this proportion is fixed across states, this leads to different levels of fees across states.

of performance. We can write $\overline{V}_s$ for $s \in \{B,R\}$ as

$$\overline{V}_s = (1-\tau_s)(\omega\pi_s + \delta\overline{V}_s) + \tau_s(\omega\pi_{-s} + \delta\overline{V}_{-s}).$$

Solving these equations for $\overline{V}_s$ gives us:

$$\overline{V}_s = \frac{1-\tau_s-\delta(1-\tau_s-\tau_{-s})}{(1-\delta)(1-\delta(1-\tau_s-\tau_{-s}))}\omega\pi_s + \frac{\tau_s}{(1-\delta)(1-\delta(1-\tau_s-\tau_{-s}))}\omega\pi_{-s}. \tag{9}$$

The value function for state s (where $s \in \{B,R\}$) when sophisticated investors are still willing to purchase rated investments is

$$V_s = \max_{w_s} \pi_s\alpha_s - w_s + \delta\sigma_s((1-\tau_s)V_s + \tau_s V_{-s}) + \delta(1-\sigma_s)\overline{V}_s. \tag{10}$$

Existence and uniqueness of equilibrium can be shown using the approach of Lemma 1.

As one would expect, the effort that a CRA invests in accuracy while sophisticated investors are still purchasing rated investments decreases as the fraction of naive investors grows larger. The reduction of market discipline from investors reduces this investment in accuracy.

Proposition 6. Investment in ratings quality in both states $s \in \{B,R\}$ decreases as the proportion of fees generated from naive investors grows larger.

The proof is in the Online Appendix.

Now we examine the effect of naive investors on the countercyclicality of ratings accuracy.

We begin by writing the first-order conditions for the decision variables w_s, $s \in \{B,R\}$:

$$\frac{\partial z}{\partial w}(w_s^*, \gamma_s) = \frac{1}{1-\lambda_s}\frac{1}{\delta p_s((1-\tau_s)V_s^* + \tau_s V_{-s}^*) - \delta p_s \overline{V}_s - \pi_s}. \tag{11}$$

It follows that $w_B^* \leq w_R^*$, and there is more accuracy in recessions than in booms when:

$$(1-\lambda_B)(\delta p_B(EV_B^* - \overline{V}_B) - \pi_B) \leq (1-\lambda_R)(\delta p_R(EV_R^* - \overline{V}_R) - \pi_R). \tag{12}$$

By definition, $EV_s^* > \overline{V}_s$. We also know that $\pi_B > \pi_R$ implies that $\overline{V}_B > \overline{V}_R$. Given Assumption A1, this implies that $EV_B^* - \overline{V}_B < EV_R^* - \overline{V}_R$ when states are independent across time or when there is mean reversion between states. Therefore, our results of countercyclical ratings accuracy are robust to the presence of naive investors.

5. Duopoly

In the main model, we considered a monopoly CRA. Nevertheless, it is important to learn whether the main insights of that model hold when competition is taken into account. While S&P, Moody's, and Fitch certainly exercise some market power, they also compete for market share. In this section, we model competition between two rating agencies. In order to deal with the tractability of an infinite-period reputation model of competition, we model competition in a very simple fashion, by supposing that the fee that the CRA charges (and/or the volume of issues) depends not only on the state, but also on the extent of competition among CRAs. Specifically, we write $\pi_{D,s}$ to denote the fee charged by a duopolist in state s and $\pi_{M,s}$ to denote the fee charged by a monopolist in state s, where $\pi_{M,s} > \pi_{D,s}$ and $s \in \{B,R\}$.

We allow for correlation between the products that the agencies are rating. In practice, the fraction of issues that CRAs rate in common varies by type of product.[17] We maintain the assumption from the previous section that in each period, a CRA rates one product, and we will incorporate correlation by defining ρ as the probability that CRA i and CRA j are rating the same product. We suppose that, although investors know ρ, they cannot determine whether an issue with only one good rating means that only one CRA rated that issue or that both CRAs rated the issue but only one gave a good rating; that is, we view the absence of a rating and a bad rating as equivalent.[18]

In analyzing the reputational equilibria and CRA incentives, we consider two possibilities for the way that investors react when they detect ratings inflation. (Recall that investors react when observing investments given a good rating that subsequently default.) Specifically, we first consider a grim-trigger-strategy equilibrium in which investors who observe the default of an issue with a good rating from CRA j stop buying investments rated by CRA j. This is similar to our punishment strategy in the monopoly case, and we will call it the "independent" punishment strategy, as punishments for the CRAs are not connected.

The second possibility that we address links the punishment of the CRAs: If both CRAs give the same issue that defaults a good rating, they are not punished. Any other default of an issue with a good rating from CRA j has investors withdrawing their business from CRA j.[19] We will call this the "linked" punishment strategy. This might be thought of as reflecting that investors are unsure whether the joint error is a problem with the CRAs' investment in accuracy or a one-time shock that was difficult to predict. Rating agencies certainly made this

[17] While in the corporate bond market, almost all rated issues are rated by S&P and Moody's, with Fitch's share varying a large amount; see Becker and Milbourn (2011), in the structured finance market, there is substantially more variance in terms of which firms are rating. Benmelech and Dlugosz's (2010) sample of asset-backed securities shows that approximately 75% were rated by two or fewer CRAs (Table 10). Among those that were rated by two CRAs (about 60%), about 88% were rated by S&P and Moody's. White (2010) displays figures from an SEC filing in 2009 that shows that S&P, Moody's, and Fitch rated 198, 200, 109, 281, and 77, 480 asset-backed securities, respectively. Some of the non-overlap in structured products is the result of ratings shopping by issuers. Modeling shopping is beyond the scope of this paper, but insight on shopping can be found in Bolton, Freixas, and Shapiro (2012) and Skreta and Veldkamp (2009).

[18] The alternate assumption—that investors can observe one good and one bad rating on the same issue, thus ascertain that the CRA giving the good rating is making a mistake, and punish this CRA appropriately—would lead to slightly different expressions but qualitatively similar results.

[19] Stolper (2009) examines a similar type of joint reputation in a game in which a regulator is actively monitoring and punishing CRAs. There are additional punishment mechanisms that are of interest. For example, consumers may stop trusting all CRAs if one was found to have incorrectly rated an investment. In this case, the analysis would look similar to that of the monopoly case but with lower per-period payoffs. In addition, CRAs may collude; however, this may require the somewhat unreasonable assumption that they observe each other's wage policies.

claim regarding their mistakes on mortgage-backed securities. As we discussed in the Introduction, there did seem to be some punishment, but it was not as severe as being forced out of business.

5.1. The independent punishment strategy

When one CRA loses the confidence of investors, the market becomes a monopoly. When a CRA acts as a monopolist, the analysis of Sections 3 and 4 applies. It is straightforward in this case to characterize optimal wages in each state, $w^*_{M,s}$, the continuation value associated with each state, $V^*_{M,s}$, and the expected continuation value, $EV^*_{M,s}=(1-\tau_s)V^*_{M,s}+\tau_s V^*_{M,-s}$ (where $-s$ represents the other state). These have properties identical to those characterized in Sections 3 and 4.

Using this characterization of the monopoly case, we can, in effect, work backwards to consider duopoly behavior. In particular, we can write down the value for CRA i of being in a duopoly in state s and paying a wage $w_{i,s}$, given that its rival, CRA j, is expected to be paying a wage $w_{j,s}$:

$$V_{i,s}=\pi_{D,s}\alpha_{i,s}-w_{i,s}+\delta[\rho\widehat{\sigma}^{IP}_{ij,s}+(1-\rho)\sigma_{i,s}\sigma_{j,s}]EV^*_{D,s} + \delta(1-\sigma_{j,s})[\rho z_{i,s}+(1-\rho)\sigma_{i,s}]EV^*_{M,s}, \tag{13}$$

where $EV^*_{D,s}=(1-\tau_s)V^*_{i,s}+\tau_s V^*_{i,-s}$ and $s\in\{R,B\}$, and $\alpha_{i,s}$ and $\sigma_{i,s}$ are the expressions for CRA i of α_s and σ_s defined in expressions (1) and (2), which reflect the probability of issuing a good rating and of surviving to the next period, respectively. We introduce new notation:

$$\widehat{\sigma}^{IP}_{ij,s} := 1-(1-\lambda_s)(1-z_{i,s}z_{j,s})p_s \tag{14}$$

to denote the probability that both CRAs i and j survive when they rate the same issue. Note that both of them would not survive when rating the same issue only when the issue is a bad issue (which occurs with probability $(1-\lambda_s)$); at least one rates it as good (with probability $(1-z_{i,s}z_{j,s})$); and it defaults (with probability p_s).

The value function defined in (13) is the duopoly analogue of Eq. (5). Again, the CRA earns fees (here, of $\pi_{D,s}$) if it gives a good rating (with probability $\alpha_{i,s}$) and incurs the costs of paying the analyst ($w_{i,s}$). Here, however, the future value for CRA i, if it succeeds in sustaining its reputation, incorporates both the possibility that its rival sustains its reputation, so that the CRA continues as a duopolist in the future, and the possibility that the rival firm is found to have assigned a good rating to a bad investment that defaulted, in which case the CRA becomes a monopolist. The CRA remains a duopolist if both CRAs survive when rating the same investment (with probability $\rho\widehat{\sigma}_{ij,s}$) or when they rate different investments (with probability $(1-\rho)\sigma_{i,s}\sigma_{j,s}$). It becomes a monopolist if it identifies a bad investment that the other does not (with probability $\rho(1-\sigma_{j,s})z_{i,s}$) or if they rate different investments and it is the only agency to survive (which occurs with probability $(1-\rho)(1-\sigma_{j,s})\sigma_{i,s}$).

In the following lemma, we demonstrate an important property of CRA choices.

Lemma 7. The CRAs' investments in ratings quality are strategic substitutes.

This lemma demonstrates that if CRA i raises its investment, CRA j would lower its investment in response, and vice versa.[20] By raising its investment, CRA i increases the likelihood that it would be around in the subsequent periods. This, then, reduces the future payoffs for CRA j (and maintains its current payoffs), creating an incentive for CRA j to reduce its quality.

The lemma also ensures that there is a unique symmetric equilibrium.[21] We use this to derive the following result on investment in ratings quality when states are i.i.d. draws.

Proposition 8. When states are independent across time ($\tau_B=1-\tau_R$), there is lower investment in ratings quality in a boom than in a recession if labor-market conditions do not vary over time. The effect of varying labor-market conditions is ambiguous.

There are now two effects on the incentives of the CRA: the direct effect, as in the monopoly case, and a strategic effect. The direct effect clearly has a similar impact on incentives to provide quality ratings as in our monopoly model (described in Proposition 2). A strategic effect arises since a change in the parameters affects the action of a rival CRA, which may then affect the probability of becoming a monopolist rather than a duopolist in the future, altering the CRA's tradeoff between current and future payoffs. Our analysis demonstrates that the direct effect outweighs the strategic effect for three of the four parameters: fees, default probability, and the proportion of good investments. This is not true for labor-market tightness. Tighter labor-market conditions (an increase in γ_s), holding all else constant, increase the rival's cost, which leads to a reduction in the quality of the rival's ratings. Hence, the rival's reduction in ratings quality gives a CRA an incentive to raise quality, which is in opposition to the direct effect of a larger cost for accuracy. This leads to the ambiguous result on labor-market tightness.

For the duopoly model with correlation, as in the case of monopoly, we assume that it is more valuable for a CRA to be in a boom state than in a recessionary state. Thus, we use Assumption A1 here, which, with the new notation, amounts to $V^*_{M,B}>V^*_{M,R}$. We also add an analogous assumption for the duopoly case:

Assumption A2. The value to a CRA in a duopoly of being in a boom is larger than that of being in a recession ($V^*_{D,B}>V^*_{D,R}$).

Proposition 9. If there is mean reversion between states ($\tau_B>1-\tau_R$), and A1 *and* A2 *hold, there is lower investment in ratings quality in a boom than in a recession if labor-market conditions do not vary over time. The effect of varying labor-market conditions is ambiguous.*

[20] Perotti and Suarez (2002) find that banks' decisions to take on more risk in a dynamic framework are strategic substitutes for a reason similar to the accuracy decision in our model: If one bank is more risky, it may be more likely to stop operating, giving market power to the remaining bank and making that bank less likely to take on risk.

[21] This does not rule out the existence of asymmetric equilibria.

Therefore, countercyclical ratings quality may also be a feature of a competitive ratings market. While competition here changes the value of maintaining a CRA's reputation relative to a market dominated by a monopolist, the economic fundamentals shift incentives in a way very similar to that of the monopolist. The exception, of course, is that tighter labor markets in booms can bring about either procyclical or countercyclical accuracy in ratings.

5.2. The linked punishment strategy

We now consider a different punishment strategy: If both CRAs give the same issue that defaults a positive rating, they are not punished. Any other default of an issue with a positive rating has investors withdrawing their business from the culpable CRA. The value function for a duopolist in state s (where $s \in \{R,B\}$) is, therefore:

$$V_{i,s} = \pi_{D,s}\alpha_{i,s} - w_{i,s} + \delta[\rho\hat{\sigma}^{LP}_{ij,s} + (1-\rho)\sigma_{i,s}\sigma_{j,s}]EV^{*}_{D,s} + \delta(1-\sigma_{j,s})[\rho z_{i,s} + (1-\rho)\sigma_{i,s}]EV^{*}_{M,s}, \tag{15}$$

where

$$\hat{\sigma}^{LP}_{ij,s} := 1-(1-\lambda_s)(z_{i,s}+z_{j,s}-2z_{i,s}z_{j,s})p_s. \tag{16}$$

The only difference between Eqs. (15) and (13) is that in (15), $\hat{\sigma}^{LP}_{ij,s}$ replaces $\hat{\sigma}^{IP}_{ij,s}$. Recall that $\hat{\sigma}^{IP}_{ij,s}$ reflected the probability under independent punishments that both CRAs rate the same issue and survive. The expression $\hat{\sigma}^{LP}_{ij,s}$ captures the same situation but $\hat{\sigma}^{LP}_{ij,s} > \hat{\sigma}^{IP}_{ij,s}$, reflecting that under linked punishments, both CRAs survive when both rate the same issue that subsequently defaults—a circumstance that would not lead to their survival in the case of independent punishments.

In the following lemma, we demonstrate an important property of CRA strategies.

Lemma 10. (1) *If* $[\rho+(1-\rho)(1-\lambda_s)p_s][EV^{*}_{D,s} - EV^{*}_{M,s}]+\rho EV^{*}_{D,s} < 0$, *the CRAs' investments in ratings quality are strategic substitutes.*

(2) *If* $[\rho+(1-\rho)(1-\lambda_s)p_s][EV^{*}_{D,s} - EV^{*}_{M,s}]+\rho EV^{*}_{D,s} > 0$, *the CRAs' investments in ratings quality are strategic complements.*

This lemma demonstrates that the strategic nature of investments has changed. Both the likelihood of being in a duopoly and the benefit of being incorrect are larger (due to no punishment if both CRAs get the rating wrong).

If CRA i lowers its rating quality, then CRA j faces a higher probability of becoming a monopolist, which should increase CRA j's investment in quality. However, there is an additional effect present now. If CRA i lowers its rating quality, the state where CRA j escapes punishment for calling a bad investment good becomes more likely. This latter effect implies that CRA j should decrease its investment in quality. Either effect can dominate. When the former dominates, then CRA i's lower investment in quality leads CRA j to increase its own investment—the case of strategic substitutes. If the second effect dominates, then lower investment from CRA i would lead to lower investment from CRA j—the strategic complements case. As we highlight below, the two cases require somewhat different analyses.

5.2.1. Strategic substitutes

When the CRAs' strategies are strategic substitutes, as characterized in part 1 of Lemma 10, there is a unique symmetric equilibrium.[22] We analyze this symmetric equilibrium.

Given that the nature of the strategic interaction is similar to the case of the independent punishment strategy, which we analyzed in Section 5.1, we find results analogous to Propositions 8 and 9, as one might expect.

Proposition 11. When punishments are linked and investments in ratings quality are strategic substitutes, if states are independent across time ($\tau_B = 1-\tau_R$), *there is lower investment in ratings quality in a boom than in a recession if labor-market conditions do not vary over time. The effect of varying labor-market conditions is ambiguous.*

The proof is in the Online Appendix.

Proposition 12. When punishments are linked and investments in ratings quality are strategic substitutes, if there is mean reversion between states ($\tau_B > 1-\tau_R$), *and* A1 *and* A2 *hold, there is lower investment in ratings quality in a boom than in a recession if labor-market conditions do not vary over time. The effect of varying labor-market conditions is ambiguous.*

The proof is in the Online Appendix.

5.2.2. Strategic complements

In the case of strategic complements, characterized in part 2 of Lemma 10, it is possible to have multiple symmetric equilibria and/or a corner solution. We start by writing out conditions that guarantee existence and uniqueness of a symmetric equilibrium, all of which are consistent with the model. We then analyze the difference between CRA investments in accuracy in booms and in recessions, and we find results similar to the cases of strategic substitutes.

The following conditions are sufficient to guarantee existence and uniqueness of an equilibrium. This equilibrium will be symmetric—i.e., both CRAs will make the same investments in ratings quality in both states.

$$EV^{*}_{M,s} > EV^{*}_{D,s} \quad \text{for } s \in \{B,R\},$$

$$\pi_{D,s} \quad \text{is small for } s \in \{B,R\},$$

$$\frac{\partial^3 z}{\partial w^3} \le 0.$$

Condition 1 states that the expected equilibrium value of a CRA is larger if the CRA is a monopolist than if it is a duopolist. Condition 2 is consistent with Condition 1. Both of these could represent a situation of Bertrand competition, in which case $\pi_{D,s} = 0$ for $s \in \{B,R\}$, and $\pi_{M,s} > 0$ for $s \in \{B,R\}$. In the proof of existence and uniqueness, we will be more specific about how small $\pi_{D,s}$ should be. The last condition is on the third derivative of $z(w)$. This is satisfied by a range of functions.

[22] This does not rule out the existence of asymmetric equilibria.

Proposition 13. When punishments are linked, investments in ratings quality are strategic complements, and Conditions 1–3 *hold, there exists a unique equilibrium.*

We now analyze how the investments in ratings quality differ between booms and recessions given the unique equilibrium.

Proposition 14. When punishments are linked, investments in ratings quality are strategic complements, and Conditions 1–3 *hold, if states are independent across time* ($\tau_B = 1-\tau_R$), *there is lower investment in ratings quality in a boom than in a recession.*

The proof is in the Online Appendix.

The results from the strategic substitute case for fees, probability of default, and the fraction of good issues remain the same. Moreover, we now get an unambiguous result with respect to labor-market tightness. When the labor market is tighter, investment in ratings quality strictly decreases, as it did in the monopoly case. The obvious reason is that the strategic effect has switched; an increase in labor-market tightness makes the competitor CRA lower its investment, which induces the CRA to lower its own investment (which it wants to do in any case because its costs have also gone up).

The result on mean reversion holds as well.

Proposition 15. When punishments are linked, investments in ratings quality are strategic complements, and Conditions 1–3, A1, *and* A2 *hold, if there is mean reversion between states* ($\tau_B > 1-\tau_R$), *there is lower investment in ratings quality in a boom than in a recession.*

The proof is in the Online Appendix.

Once again, there is countercyclical ratings quality for all variables. Labor-market tightness is no longer ambiguous due to the strategic effect going in the same direction as the direct effect.

6. Empirical implications

In this section, we examine evidence related to testable implications of the model. To examine our hypotheses, we use a set of very recent empirical papers focused on CRAs and ratings quality.

The model shows that ratings quality may be countercyclical. This effect is likely to be exacerbated when economic shocks are negatively correlated and diminished when economic shocks are positively correlated. While we are unable to find direct evidence relating the nature of business cycles to ratings quality, some recent papers document a decrease in ratings quality in the recent boom. Ashcraft, Goldsmith-Pinkham, and Vickery (2010) find that as the volume of mortgage-backed security (MBS) issuance increased dramatically from 2005 to mid-2007, the quality of ratings declined. Specifically, when conditioning on the overall risk of the deal, subordination levels[23] for subprime and Alt-A MBS deals decreased over this period. Furthermore, subsequent ratings downgrades for the 2005 to mid-2007 cohorts were dramatically greater than for previous cohorts. Griffin and Tang (2012) find that adjustments by CRAs to their models' predictions of credit quality in the collateralized debt obligation (CDO) market were positively related to future downgrades. These adjustments were overwhelmingly positive, and the amount adjusted (the width of the AAA tranche) increased sharply from 2003 to 2007 (from 6% to 18.2%). The adjustments are not well explained by natural covariates (such as past deals by collateral manager, credit enhancements, and other modeling techniques). Furthermore, 98.6% of the AAA tranches of CDOs in their sample failed to meet the CRAs' reported AAA standard (for their sample from 1997 to 2007). They also find that adjustments increased CDO value by, on average, $12.58 million per CDO.

Larger current revenues should lead to lower ratings quality. He, Qian, and Strahan (forthcoming) find that MBS tranches sold by larger issuers performed significantly worse (market prices decreased) than those sold by small issuers during the boom period of 2004–2006. They define larger by market share in terms of deals. As a robustness check, they also look at market share in terms of dollars and find similar results. Faltin-Traeger (2009) shows that when one CRA rates more deals for an issuer in a half-year period than does another CRA, the first CRA is less likely to be the first to downgrade that issuer's securities in the next half-year.

More-complex investments imply lower ratings quality. Increasing the complexity of investments has two implications for ratings quality. First, it implies more noise regarding the performance of the investment, making it harder to detect whether a CRA can be faulted for poor ratings quality. Second, it implies that CRAs may require more expensive/specialized workers to maintain a given level of quality. Both of these channels decrease the return to investing in ratings quality. Structured finance products are certainly more complex (and the methodology for evaluating them less standardized) than corporate bonds, which is suggestive for the recent performance of structured finance ratings. Within the structured finance arena, Ashcraft, Goldsmith-Pinkham, and Vickery (2010) find that the MBS deals that were most likely to underperform were ones with more interest-only loans (because of limited performance history) and lower documentation—i.e., loans that were more opaque or difficult to evaluate.

We also offer two outcomes of the model that are testable but not yet examined, to the best of our knowledge.

(1) Ratings-quality decisions between CRAs may be strategic substitutes or strategic complements. When one CRA chooses to produce better ratings, do other CRAs have incentives to worsen or improve their ratings? With the independent punishment strategy, we show that choices are likely to be strategic substitutes.

[23] The subordination level that they use is the fraction of the deal that is junior to the AAA tranche. A smaller fraction means that the AAA tranche is less "protected" from defaults and, therefore, less costly from the issuer's point of view.

However, if the linked punishment strategy is more likely, then it is possible that CRA investments will be strategic complements. Kliger and Sarig (2000) use a natural experiment that seems tailored for testing this question: Moody's switch to a finer ratings scale. While their focus is on the informativeness of ratings, it would be interesting to study the strategic aspect of how this affected the quality of Standard and Poor's ratings.

(2) When forecasts of growth/economic conditions are better, ratings quality should be higher. This is because reputation-building is needed for milking in good times, and forecasts should be directly related to CRAs' future payoffs. This is also a prediction of the models of Mathis, McAndrews, and Rochet (2009) and Bolton, Freixas, and Shapiro (2012).

7. Conclusion

In this paper, we analyze how CRAs' incentives to provide high-quality ratings vary over the business cycle. We define booms as having tighter labor markets, larger revenue for CRAs, and lower average default probabilities than recessions. When economic shocks are i.i.d., booms have strictly lower quality ratings than do recessions, due to the incentive to milk reputation. These incentives are exacerbated when shocks are negatively correlated (mean reversion) and diminished when shocks are positively correlated. Adding naive investors does not change the qualitative results. We also put forth a model of competition that accounts for CRAs rating similar investments and different market reactions to evidence of CRA inaccuracy. This model demonstrates that countercyclical ratings quality also holds in a competitive environment. Lastly, we find some empirical support for the model and make suggestions for future empirical work.

In order to make our model tractable, we have simplified both investor and issuer behavior. Providing more structure on their decision-making might provide additional insight and allow us to endogenize some of the parameters that we have taken to be exogenous. It could also prove useful to model the business cycle in a more realistic manner. Finally, we have focused on initial ratings, but CRAs follow investments over time and develop reputations from the upgrading/downgrading process.

Appendix A

Proof of Lemma 1. First, consider existence. Note that both $\pi_B\alpha_B - w_B$ and $\pi_R\alpha_R - w_R$ are bounded from above. Say that both are strictly less than A; then, trivially, $V_B < A/(1-\delta)$ and $V_R < A/(1-\delta)$. Define two functions from Eq. (5), $V_B(V_R)$ and $V_R(V_B)$. Note that both are increasing and continuous functions, and that both $V_B(0) > 0$ and $V_R(0) > 0$ are positive. Since $V_B(A/(1-\delta)) < A/(1-\delta)$ and $V_R(A/(1-\delta)) < A/(1-\delta)$, it follows that there must be an odd number of solutions. This is easy to see graphically in Fig. 2. However, we argue below that $V_B(\cdot)$ and $V_R(\cdot)$ are convex and, thereby, show that there cannot be more than two solutions. This, then, proves that the solution is unique.

It remains to demonstrate that $V_B(\cdot)$ and $V_R(\cdot)$ are convex. Note, first, that we can consider:

$$w_B^* = \arg\max_w \pi_B\alpha_B - w + \delta\sigma_B((1-\tau_B)V_B + \tau_B V_R). \tag{17}$$

First, we claim that $dw_B^*/dV_R > 0$. We use the implicit function theorem to do so. Consider the first-order

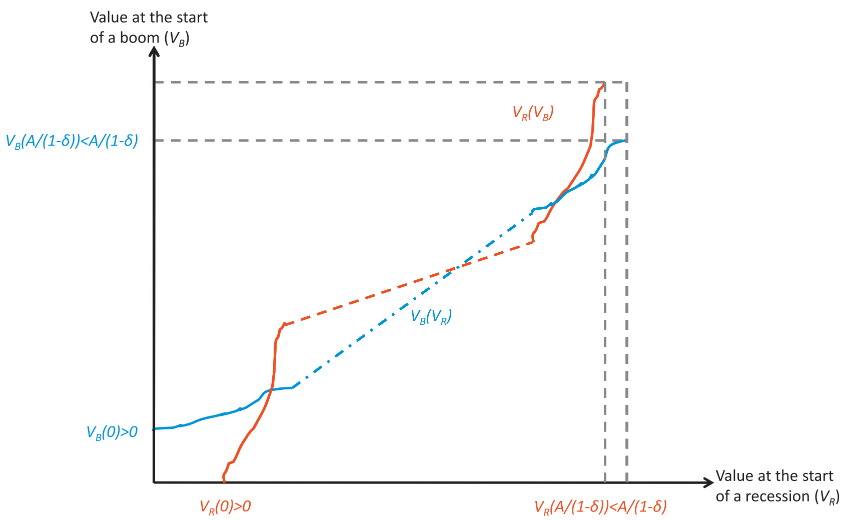

Fig. 2. Odd number of solutions.

condition (FOC) of the CRA's maximization problem:[24]

$$-\pi_B \frac{\partial z_B}{\partial w}\bigg|_{w_B^*} - \frac{1}{1-\lambda_B} + \delta p_B((1-\tau_B)V_B + \tau_B V_R)\frac{\partial z_B}{\partial w}\bigg|_{w_B^*} = 0. \quad (18)$$

Taking the derivative of the FOC with respect to V_R and rearranging yields

$$\frac{dw_B^*}{dV_R} = \frac{\tau_B \delta p_B}{\pi_B - \delta p_B((1-\tau_B)V_B + \tau_B V_R)} \frac{\frac{\partial z_B}{\partial w}}{\frac{\partial^2 z_B}{\partial w^2}}. \quad (19)$$

Note that the assumption that the CRA's second-order condition is negative implies that the denominator of the first fraction is negative, and so, since $\partial^2 z_B/\partial w^2 < 0$ and $\partial z_B/\partial w > 0$, it follows that $dw_B^*/dV_R > 0$.

Now,

$$\frac{dV_B}{dV_R} = \frac{\partial V_B}{\partial w_B^*}\frac{dw_B^*}{dV_R} + \frac{\partial V_B}{\partial V_R} = \frac{\partial V_B}{\partial V_R} \quad (20)$$

since w_B^* is chosen to maximize V_B (the envelope condition), and so we can write

$$\frac{dV_B}{dV_R} = \left(\tau_B + (1-\tau_B)\frac{dV_B}{dV_R}\right)\delta\sigma_B = \frac{\tau_B \delta\sigma_B}{1-(1-\tau_B)\delta\sigma_B} > 0. \quad (21)$$

Next, to prove convexity, note that

$$\frac{d^2 V_B}{dV_R^2} = \frac{d}{dz_B}\left(\frac{\tau_B\delta\sigma_B}{1-(1-\tau_B)\delta\sigma_B}\right)\frac{\partial z_B}{\partial w}\frac{dw_B^*}{dV_R}$$
$$= \frac{(1-\lambda_B)p_B\tau_B\delta}{(1-(1-\tau_B)\delta\sigma_B)^2}\frac{\partial z_B}{\partial w}\frac{dw_B^*}{dV_R} > 0. \quad (22)$$

Analogously, $d^2V_R/dV_B^2 > 0$. □

Proof of Proposition 3. We start by introducing some additional notation:

$$G_s(p_R, p_B, \gamma_R, \gamma_B, \pi_R, \pi_B, \lambda_R, \lambda_B, \delta)$$
$$:= -V_s + \pi_s\alpha_s - w_s + \delta\sigma_s((1-\tau_s)V_s + \tau_s V_{-s}). \quad (23)$$

We suppress the arguments for G_B and G_R and can then rewrite Eq. (5) as $G_B = G_R = 0$.

We apply the implicit function theorem, which, here, implies that

$$\frac{dV_R^*}{da} = -\frac{\det\begin{bmatrix}\frac{\partial G_B}{\partial a} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial a} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}} \quad \text{and} \quad \frac{dV_B^*}{da} = -\frac{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial a}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial a}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}}, \quad (24)$$

where a is an arbitrary parameter. We begin by analyzing the (common) denominator of both expressions.

As we show in the lemma below, this determinant is negative. □

Lemma 16. $(\partial G_B/\partial V_R^*)(\partial G_R/\partial V_B^*) - (\partial G_B/\partial V_B^*)(\partial G_R/\partial V_R^*)$ *is negative.*

Proof. First, note:

$$\frac{\partial G_B}{\partial V_B^*} = \delta\sigma_B(1-\tau_B) - 1 < 0, \quad (25)$$

$$\frac{\partial G_B}{\partial V_R^*} = \delta\sigma_B\tau_B > 0, \quad (26)$$

$$\frac{\partial G_R}{\partial V_B^*} = \delta\sigma_R\tau_R > 0, \quad (27)$$

$$\frac{\partial G_R}{\partial V_R^*} = \delta\sigma_R(1-\tau_R) - 1 < 0, \quad (28)$$

where we have used the envelope theorem to simplify expressions. This, then, allows us to rewrite

$$\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}$$
$$= \delta^2\tau_B\tau_R\sigma_R\sigma_B - (1-\delta\sigma_B(1-\tau_B))(1-\delta\sigma_R(1-\tau_R)). \quad (29)$$

Next, note that

$$\frac{\partial}{\partial \tau_B}\left(\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}\right) = \delta\sigma_B(\delta\sigma_R - 1) < 0, \quad (30)$$

where the inequality follows since $1 > \sigma_s > 0$.

Finally, note that at $\tau_B = 0$,

$$\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*} = -(1-\delta\sigma_B)(1-\delta\sigma_R(1-\tau_R)) < 0. \quad \square \quad (31)$$

Resumption of Proof of Proposition 3. Given Lemma 16, we can apply the implicit function theorem and note that $(d/da)(V_B^* - V_R^*)$ has the same sign as

$$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial a}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial a}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial a} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial a} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}.$$

We consider several parameters of interest; proofs for other parameters are similar, and so are omitted.

The effect of a change in the probability of default in a boom (p_B): Consider, first, the comparative static with respect to p_B:

$$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial p_B}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial p_B}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial p_B} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial p_B} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = -\frac{\partial G_B}{\partial p_B}\left(\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*}\right),$$

since $\partial G_R/\partial p_B = 0$. Now $\partial G_B/\partial p_B = -\delta(1-\lambda_B)(1-z_B)\,((1-\tau_B)V_B^* + \tau_B V_R^*) < 0$ and $\partial G_R/\partial V_R^* + \partial G_R/\partial V_B^* = -1 + \delta\sigma_R < 0$.

Consequently, $d(V_B^* - V_R^*)/dp_B < 0$.

The effect of a change in labor-market conditions in a recession (γ_R):

$$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \gamma_R}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \gamma_R}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \gamma_R} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \gamma_R} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = \frac{\partial G_R}{\partial \gamma_R}\left(\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*}\right)$$

since $\frac{\partial G_B}{\partial \gamma_R} = 0$.

Note that $\partial G_B/\partial V_R^* + \partial G_B/\partial V_B^* = \delta\sigma_B - 1 < 0$ and since $\partial G_R/\partial\gamma_R = (\delta(1-\lambda_R)p_R((1-\tau_R)V_R^* + \tau_R V_B^*) - \pi_R(1-\lambda_R))\;\; \partial z/\partial\gamma_R < 0$ by the second-order condition and since $\partial z/\partial\gamma_R < 0$, it follows that $d(V_B - V_R)/d\gamma_R > 0$.

The effect of a change in the transition probabilities: (i) First, we examine the change with respect to a change in τ_B

$$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_B}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_B}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \tau_B} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \tau_B} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = -\frac{\partial G_B}{\partial \tau_B}\left(\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*}\right)$$

since $\frac{\partial G_R}{\partial \tau_B} = 0$.

[24] It can be shown that the second-order condition is satisfied when λ_B, λ_R, and δ are close enough to 1.

As above, $\partial G_R/\partial V_R^* + \partial G_R/\partial V_B^* < 0$ and $\partial G_B/\partial \tau_B = -\delta\sigma_B (V_B^* - V_R^*)$. It follows that $\text{sign}(\partial G_B/\partial \tau_B) = -\text{sign}(V_B^* - V_R^*)$. Therefore, sign $d(V_B^* - V_R^*)/d\tau_B = -\text{sign}(V_B^* - V_R^*)$.

(ii) Second, we examine the change with respect to a change in τ_R

$$\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_R} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_R} \end{bmatrix} - \det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_R} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_R} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix} = \left(\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*}\right)\frac{\partial G_R}{\partial \tau_R}.$$

As above, $\partial G_B/\partial V_B^* + \partial G_B/\partial V_R^* < 0$. Also, $\partial G_R/\partial \tau_R = \delta\sigma_R (V_B^* - V_R^*)$. It follows that $\text{sign}(\partial G_R/\partial \tau_R) = \text{sign}(V_B^* - V_R^*)$. Therefore, sign $d(V_B^* - V_R^*)d\tau_R = -\text{sign}(V_B^* - V_R^*)$. □

Proof of Proposition 5. First, consider the first-order condition that characterizes w_B^*:

$$-\pi_B(1-\lambda_B)\frac{\partial z_B}{\partial w} - 1 + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)V_B^* + \tau_B V_R^*) = 0. \tag{32}$$

Taking the total derivative with respect to τ_B, we obtain

$$\begin{aligned} 0 = {} & \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}(V_R^* - V_B^*) + (1-\lambda_B)\frac{\partial^2 z_B}{\partial w^2}\frac{dw_B^*}{d\tau_B} \\ & \times(\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B) \\ & + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}\left((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_B} + \tau_B\frac{\partial V_R^*}{\partial \tau_B}\right). \end{aligned} \tag{33}$$

First, note that (using the results from Lemma 16 and the definition of σ_s $(s = B,R)$:

$$\begin{aligned} \text{sign}\left(\frac{\partial V_B^*}{\partial \tau_B}\right) &= \text{sign}\left(\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_B} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_B} \end{bmatrix}\right) = -\text{sign}\frac{\partial G_B}{\partial \tau_B}\frac{\partial G_R}{\partial V_R^*} \\ &= -\text{sign}(\delta\sigma_B(1-\delta\sigma_R(1-\tau_R)))(V_B^* - V_R^*) \\ &= -\text{sign}(V_B^* - V_R^*) \end{aligned}$$

and

$$\begin{aligned} \text{sign}\left(\frac{\partial V_R^*}{\partial \tau_B}\right) &= \text{sign}\left(\det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_B} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_B} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}\right) = \text{sign}\left(\frac{\partial G_B}{\partial \tau_B}\frac{\partial G_R}{\partial V_B^*}\right) \\ &= \text{sign}(\delta^2\sigma_B\sigma_R\tau_R(V_R^* - V_B^*)) = -\text{sign}(V_B^* - V_R^*). \end{aligned}$$

Now, consider (33): Since $\delta p_B(1-\lambda_B)\partial z_B/\partial w > 0$ and $\partial^2 z_B/\partial w^2 < 0$ and $1-\lambda_B > 0$, it follows that $dw_B^*/d\tau_B$ has the same sign as $\text{sign}(V_B^* - V_R^*) \times \text{sign}(\pi_B - \delta p_B\ ((1-\tau)V_B^* + \tau V_R^*))$. Rearranging the FOC as $-\pi_B + \delta p_B((1-\tau)V_B^* + \tau V_R^*) = (1/(1-\lambda_B))\partial z_B/\partial w$ and noting that the right-hand side is positive gives the result for w_B^*; that is, $\text{sign}(dw_B^*/d\tau_B) = -\text{sign}(V_B^* - V_R^*)$.

Analogously,
$\text{sign}(dw_R^*/d\tau_R) = -\text{sign}(V_R^* - V_B^*) = \text{sign}(V_B^* - V_R^*)$.

Next, we consider $dw_B^*/d\tau_R$.

Taking the derivative of (32) with respect to τ_R, we obtain

$$\begin{aligned} 0 = {} & (1-\lambda_B)\frac{\partial^2 z_B}{\partial w^2}\frac{dw_B^*}{d\tau_R}(\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B) \\ & + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}\left((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R} + \tau_B\frac{\partial V_R^*}{\partial \tau_R}\right). \end{aligned}$$

As above, $(\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B) > 0$ and $\partial^2 z_B/\partial w^2 < 0$ so that $\text{sign}(dw_B^*/d\tau_R) = \text{sign}(\delta p_B(1-\lambda_B)(\partial z_B/\partial w)((1-\tau_B)\partial V_B^*/\partial\tau_R + \tau_B\partial V_R^*/\partial\tau_R)) = \text{sign}((1-\tau_B)\partial V_B^*/\partial\tau_R + \tau_B\partial V_R^*/\partial\tau_R))$

where the second inequality follows since $\delta p_B(1-\lambda_B)\,\partial z_B/\partial w > 0$.

Consider

$$\begin{aligned} \text{sign}\left(\frac{\partial V_B^*}{\partial \tau_R}\right) &= \text{sign}\left(\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_R} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_R} \end{bmatrix}\right) = \text{sign}\left(\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial \tau_R}\right) \\ &= \text{sign}(\delta^2\sigma_B\tau_B\sigma_R(V_B^* - V_R^*)) = \text{sign}(V_B^* - V_R^*) \end{aligned} \tag{34}$$

and

$$\begin{aligned} \text{sign}\left(\frac{\partial V_R^*}{\partial \tau_R}\right) &= \text{sign}\left(\det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_R} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_R} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}\right) = -\text{sign}\left(\frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial \tau_R}\right) \\ &= \text{sign}((1-\delta\sigma_B(1-\tau_B))\delta\sigma_R(V_B^* - V_R^*)) \\ &= \text{sign}(V_B^* - V_R^*). \end{aligned}$$

This implies $\text{sign}(dw_B^*/d\tau_R) = \text{sign}(V_B^* - V_R^*)$. Analogously, $\text{sign}(dw_R^*/d\tau_B) = -\text{sign}(V_B^* - V_R^*)$. □

Proof of Lemma 7. In equilibrium, $w_{i,s}^*$ is optimally chosen and satisfies the following first-order condition:

$$\begin{aligned} 0 = -1 + \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{ & -\pi_{D,s} + \delta p_s[\rho z_{j,s} + (1-\rho)\sigma_{j,s})]EV_{D,s}^* \\ & + \delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\sigma_{j,s})]EV_{M,s}^*\}. \end{aligned}$$

For notational convenience, define

$$\begin{aligned} Y_s := (1-\lambda_s)\{ & -\pi_{D,s} + \delta p_s[\rho z_{j,s} + (1-\rho)\sigma_{j,s}]EV_{D,s}^* \\ & + \delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\sigma_{j,s})]EV_{M,s}^*\}. \end{aligned}$$

Assuming that the first-order condition is satisfied, we know that $Y_s > 0$.

We begin by taking the derivative of CRA i's first-order condition with respect to $w_{j,s}$:

$$\begin{aligned} & \frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}}Y_s + \frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta[\rho(1-\lambda_s)p_s + (1-\rho)(1-\lambda_s)^2 p_s^2] \\ & [EV_{D,s}^* - EV_{M,s}^*] = 0. \end{aligned}$$

Rearranging gives us:

$$\frac{dw_{i,s}}{dw_{j,s}} = -\frac{\dfrac{\partial z_{i,s}}{\partial w}\dfrac{\partial z_{j,s}}{\partial w}\delta[\rho(1-\lambda_s)p_s + (1-\rho)(1-\lambda_s)^2 p_s^2][EV_{D,s}^* - EV_{M,s}^*]}{\dfrac{\partial^2 z_{i,s}}{\partial w^2}\,Y_s}. \tag{35}$$

Since

$$\frac{\dfrac{\partial z_{i,s}}{\partial w}\dfrac{\partial z_{j,s}}{\partial w}}{\dfrac{\partial^2 z_{i,s}}{\partial w^2}} < 0, \quad Y_s > 0,$$

and, as shown in the lemma below, $EV_{D,s}^* - EV_{M,s}^* < 0$, it follows that $dw_{i,s}/dw_{j,s} < 0$. □

Lemma 17. $EV_{M,s}^* > EV_{D,s}^*$.

Proof. We will prove by contradiction. Suppose that $EV_{M,B}^* < EV_{D,B}^*$. Then, it must be the case that either $V_{M,B}^* < V_{D,B}^*$ or $V_{M,R}^* < V_{D,R}^*$, or both; that is, there are three cases to consider: (i) $V_{M,B}^* < V_{D,B}^*$ and $V_{M,R}^* > V_{D,R}^*$; (ii) $V_{M,B}^* > V_{D,B}^*$ and $V_{M,R}^* < V_{D,R}^*$; and, (iii) $V_{M,B}^* < V_{D,B}^*$ and $V_{M,R}^* < V_{D,R}^*$. Note that in the text and in other proofs, we represent CRA i's payoffs in a duopoly in state s using $V_{i,s}^*$, $z_{i,s}$, and $w_{i,s}$. Here, we denote the same expressions as $V_{D,s}^*$, $z_{D,s}$, and $w_{D,s}$ to clarify the difference between monopoly and duopoly. All expressions will be from the point of view of CRA i.

We consider each case in turn:

Case (i): $EV^*_{M,B} < EV^*_{D,B}$, $V^*_{M,B} < V^*_{D,B}$ and $V^*_{M,R} > V^*_{D,R}$.

Substituting $EV^*_{D,B}$ for $EV^*_{M,B}$ in Eq. (13) allows us to write

$$V^*_{D,B} < \pi_{D,B}\alpha^*_{D,B} - w^*_{D,B} + \delta\sigma^*_{D,B} EV^*_{D,B}$$
$$= \pi_{D,B}\alpha^*_{D,B} - w^*_{D,B} + \delta\sigma^*_{D,B}((1-\tau_B)V^*_{D,B} + \tau_B V^*_{D,R}),$$

where $w^*_{D,B}$ is the optimal duopoly wage in the boom state and $z^*_{D,B}$ the associated accuracy, and defining $\alpha^*_{D,B}$ and $\sigma^*_{D,B}$ in the obvious way.

Since $\pi_{M,B} > \pi_{D,B}$ and $V^*_{M,R} > V^*_{D,R}$, it follows that

$$V^*_{D,B} < \pi_{M,B}\alpha^*_{D,B} - w^*_{D,B} + \delta\sigma^*_{D,B}((1-\tau_B)V^*_{D,B} + \tau_B V^*_{M,R}). \tag{36}$$

Now, let

$$V_1 := \max_w \pi_{M,B}\alpha_B(w) - w + \delta\sigma_B(w)((1-\tau_B)V^*_{D,B} + \tau_B V^*_{M,R}),$$

where $\alpha_B(w)$ and $\sigma_B(w)$ highlight the dependence of these endogenous probabilities on w.

Then, V_1 is greater than or equal to the right-hand side of Eq. (36), and so $V_1 > V^*_{D,B}$.

Next, define

$$V_n := \max_w \pi_{M,B}\alpha_B(w) - w + \delta\sigma_B(w)((1-\tau_B)V_{n-1} + \tau_B V^*_{M,R}).$$

Then, following similar reasoning, $V_n \geq V_{n-1}$. It is clear that every element of $\{V_n\}$ is bounded above by $\pi_{M,B}/(1-\delta)$, and since the sequence is non-decreasing and bounded, it must converge and the limit $V_n \to V^*_{M,B}$ since $V^*_{M,B}$ solves

$$V^*_{M,B} := \pi_{M,B}\alpha^*_B - w^*_{M,B} + \delta\sigma^*_B((1-\tau_B)V^*_{M,B} + \tau_B V^*_{M,R}). \tag{37}$$

It follows that $V^*_{M,B} > V^*_{D,B}$, which is a contradiction.

Case (ii): $EV^*_{M,B} < EV^*_{D,B}$, $V^*_{M,B} > V^*_{D,B}$, and $V^*_{M,R} < V^*_{D,R}$.

Given that $EV^*_{M,B} < EV^*_{D,B}$, we can conclude

$$V^*_{D,R} := \pi_{D,R}\alpha^*_{D,R} - w^*_{D,R} + \delta[\rho\hat{\sigma}^{IP*}_{D,R} + (1-\rho)(\sigma^*_{D,R})^2]$$
$$\times((1-\tau_R)V^*_{D,R} + \tau_R V^*_{D,B}) + \delta(1-\sigma^*_{D,R})[\rho z^*_{D,R} + (1-\rho)\sigma^*_{D,R}]$$
$$\times((1-\tau_R)V^*_{M,R} + \tau_R V^*_{M,B}) < \pi_{D,R}\alpha^*_{D,R} - w^*_{D,R}$$
$$+\delta[\rho\hat{\sigma}^{IP*}_{D,R} + (1-\rho)(\sigma^*_{D,R})^2]((1-\tau_R)V^*_{D,R} + \tau_R V^*_{M,B})$$
$$+\delta(1-\sigma^*_{D,R})[\rho z^*_{D,R} + (1-\rho)\sigma^*_{D,R}]((1-\tau_R)V^*_{D,R} + \tau_R V^*_{M,B})$$
$$= \pi_{D,R}\alpha^*_{D,R} - w^*_{D,R} + \delta\sigma^*_{D,R}((1-\tau_R)V^*_{D,R} + \tau_R V^*_{M,B}),$$

where the second expression follows from the first by noting that $V^*_{M,B} > V^*_{D,B}$ and $V^*_{M,R} < V^*_{D,R}$, and the third line follows by rearranging terms.

Working from the last expression and following the same reasoning as in Case (i) ensures that $V^*_{M,R} > V^*_{D,R}$ and provides a contradiction.

Case (iii): $EV^*_{M,B} < EV^*_{D,B}$, $V^*_{M,B} < V^*_{D,B}$, and $V^*_{M,R} < V^*_{D,R}$.

For this case, it must also be true that $EV^*_{M,R} < EV^*_{D,R}$. Since $EV^*_{M,s} < EV^*_{D,s}$ for $s \in \{B,R\}$, and substituting $EV^*_{D,s} = (1-\tau_s)V_{D,s} + \tau_s V_{D,-s}$, we can write:

$$V_{D,s} < \pi_{D,s}\alpha^*_{D,R} - w^*_{D,s} + \delta\sigma^*_{D,s}((1-\tau_s)V_{D,s} + \tau_s V_{D,-s}), \tag{38}$$

for $s \in \{B,R\}$.

We can then construct a $V^n_{D,s}$ and V^n_{D-s} as follows. First, define

$$V^1_{D,s} := \max_{w_s} \pi_{D,s}\alpha_s(w_s) - w_s + \delta\sigma_s(w_s)((1-\tau_s)V^1_{D,s} + \tau_s V_{D,-s}).$$

Then, $V^1_{D,s}$ is larger than the right-hand side of (38), and so $V^1_{D,s} > V_{D,s}$. We can recursively define

$$V^n_{D,s} := \max_{w_s} \pi_{D,s}\alpha_s(w_s) - w_s + \delta\sigma_s(w_s)((1-\tau_s)V^n_{D,s} + \tau_s V^{n-1}_{D,-s}).$$

$V^n_{D,s}$ and $V^n_{D,-s}$ are non-decreasing in n, and since for any n, both are bounded above $\pi_{D,B}/(1-\delta)$, the limits as $n \to \infty$ must be well defined; let $\tilde{V}_{D,s} := \lim_{n\to\infty} V^n_{D,s}$; then, it is immediate that $(1-\tau_s)\tilde{V}_{D,s} + \tau_s\tilde{V}_{D,-s} \geq EV^*_{D,s}$.

Now, compare $\tilde{V}_{D,s}$ with $V^*_{M,s}$:

$$\tilde{V}_{D,s} = \max_w \pi_{D,s}\alpha_s(w) - w + \delta\sigma_s(w)((1-\tau_s)\tilde{V}_{D,s} + \tau_s\tilde{V}_{D,-s}). \tag{39}$$

$$V^*_{M,s} = \max_w \pi_{M,s}\alpha_s(w) - w + \delta\sigma_s(w)((1-\tau_s)V^*_{M,s} + \tau_s V^*_{M,-s}). \tag{40}$$

Note that the only difference between $\tilde{V}_{i,s}$ and $V^*_{M,s}$ are the fees $\pi_{M,s}$ and $\pi_{D,s}$, which are exogenous. Define Eq. (40) as two equations (one for each state) where the $V_{M,s}$ is brought over to the right-hand side and the left-hand side equals zero. Label these two equations G_B and G_R. We can now apply the implicit function theorem as in Proposition 3. We define σ_s as in Lemma 16. This implies that

$$\frac{dV^*_{M,R}}{d\pi_{M,R}} = -\det\begin{bmatrix} \frac{\partial G_B}{\partial \pi_{M,R}} & \frac{\partial G_B}{\partial V^*_{M,B}} \\ \frac{\partial G_R}{\partial \pi_{M,R}} & \frac{\partial G_R}{\partial V^*_{M,B}} \end{bmatrix} \Big/ \det\begin{bmatrix} \frac{\partial G_B}{\partial V^*_{M,R}} & \frac{\partial G_B}{\partial V^*_{M,B}} \\ \frac{\partial G_R}{\partial V^*_{M,R}} & \frac{\partial G_R}{\partial V^*_{M,B}} \end{bmatrix}$$
$$= -\frac{(\lambda_R + (1-\lambda_R)(1-z_{M,R}))(1-\delta\sigma_B(1-\tau_B))}{\delta^2\tau_B\tau_R\sigma_R\sigma_B - (1-\delta\sigma_B(1-\tau_B))(1-\delta\sigma_R(1-\tau_R))}$$

and

$$\frac{dV^*_{M,R}}{d\pi_{M,B}} = -\det\begin{bmatrix} \frac{\partial G_B}{\partial \pi_{M,B}} & \frac{\partial G_B}{\partial V^*_{M,B}} \\ \frac{\partial G_R}{\partial \pi_{M,B}} & \frac{\partial G_R}{\partial V^*_{M,B}} \end{bmatrix} \Big/ \det\begin{bmatrix} \frac{\partial G_B}{\partial V^*_{M,R}} & \frac{\partial G_B}{\partial V^*_{M,B}} \\ \frac{\partial G_R}{\partial V^*_{M,R}} & \frac{\partial G_R}{\partial V^*_{M,B}} \end{bmatrix}$$
$$= -\frac{(\lambda_B + (1-\lambda_B)(1-z_{M,B}))\delta\sigma_R\tau_R}{\delta^2\tau_B\tau_R\sigma_R\sigma_B - (1-\delta\sigma_B(1-\tau_B))(1-\delta\sigma_R(1-\tau_R))}.$$

In Lemma 16, we demonstrated that the denominator of both expressions is negative. The overall expressions are, therefore, positive. It is easy to see that both $dV^*_{M,B}/d\pi_{M,B}$ and $dV^*_{M,B}/d\pi_{M,R}$ are also positive. Therefore, since $\pi_{M,s}$ is larger than $\pi_{D,s}$ for $s \in \{B,R\}$, $V^*_{M,s}$ is larger than $\tilde{V}_{D,s}$ for $s \in \{B,R\}$. This, then, implies that $EV^*_{M,s} > EV^*_{D,s}$ for $s \in \{B,R\}$, which is a contradiction.

Proof of Proposition 8. In equilibrium, $w^*_{i,s}$ is optimally chosen and satisfies the following first-order condition:

$$0 = -1 + \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_{j,s} + (1-\rho)\sigma_{j,s}]EV^*_{D,s}$$
$$+\delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\sigma_{j,s})]EV^*_{M,s}\}. \tag{41}$$

Imposing symmetry, we write the equilibrium wage for this duopoly case as $w^*_{D,s}$, drop the i and j subscripts on the $z_{i,s}$ and $z_{j,s}$ functions, and rewrite the CRA's first-order condition, Eq. (41), as

$$0 = -1 + \frac{\partial z_s}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_s + (1-\rho)\sigma^*_{D,s})]EV^*_{D,s}$$
$$+\delta p_s[\rho(1-z_s) + (1-\rho)(1-\sigma^*_{D,s})]EV^*_{M,s}\}. \tag{42}$$

Define A_s as the right-hand side of Eq. (42) and

$$\hat{Y}_s = (1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_s + (1-\rho)\sigma^*_{D,s})]EV^*_{D,s} + \delta p_s[\rho(1-z_s) + (1-\rho)(1-\sigma^*_{D,s})]EV^*_{M,s}\}.$$

Then,

$$\frac{dA_s}{dw} = \frac{\partial^2 z_s}{\partial w^2}\hat{Y}_s - \delta\left(\frac{\partial z_s}{\partial w}\right)^2 (1-\lambda_s)p_s[\rho + (1-\rho)(1-\lambda_s)p_s] (EV^*_{M,s} - EV^*_{D,s}) < 0,$$

where the inequality follows since $\hat{Y}_s > 0$ (assuming that there is an interior solution), $\partial^2 z_s/\partial w^2 < 0$, $\partial z_s/\partial w > 0$ and $EV^*_{D,s} < EV^*_{M,s}$ by Lemma 17 above.

Note that we can represent the first-order condition as

$$\frac{1}{\frac{\partial z_s}{\partial w}} = \hat{Y}_s. \tag{43}$$

The left-hand side is increasing in w. It also shifts up if γ is larger.

The right-hand side is decreasing in w. We now examine what happens to the right-hand side when the parameters change. In particular, how can we compare $\hat{Y}_B$ and $\hat{Y}_R$? This is a similar exercise to the one we performed for the monopoly case.

When states are independent across time, $\tau_B = 1-\tau_R$, which implies that $EV^*_{D,B} = EV^*_{D,R}$ and $EV^*_{M,B} = EV^*_{M,R}$.

We consider each of our parameters in turn.

First, consider $\pi_{D,s}$:

$$\frac{d\hat{Y}_s}{d\pi_{D,s}} = -(1-\lambda_s) < 0. \tag{44}$$

Next,

$$\frac{d\hat{Y}_s}{dp_s} = (1-\lambda_s)\{\delta(2(1-\rho)p_s + \rho)EV^*_{D,s} + \delta(1-z_s) \times[\rho + 2(1-\rho)(1-\lambda_s)p_s](EV^*_{M,s} - EV^*_{D,s})\} > 0. \tag{45}$$

Turning next to λ_s,

$$\frac{d\hat{Y}_s}{d\lambda_s} = -\frac{\hat{Y}_s}{1-\lambda_s} + (1-\lambda_s)\delta(1-\rho)(1-z_s)p_s^2(EV^*_{D,s} - EV^*_{M,s}) < 0. \tag{46}$$

Finally, we consider γ_s,

$$\frac{d\hat{Y}_s}{d\gamma_s} = -\frac{\partial z_s}{\partial \gamma_s}\delta p_s(1-\lambda_s)[\rho + (1-\rho)(1-\lambda_s)p_s](EV^*_{M,s} - EV^*_{D,s}) > 0. \tag{47}$$

Using Eq. (43) and the subsequent results, it is clear that when $\pi_{D,B} > \pi_{D,R}$, $p_B < p_R$, and $\lambda_B > \lambda_R$, it is true that $w^*_B < w^*_R$. This effect on wages is ambiguous for $\gamma_B > \gamma_R$, as higher γ_s pushes both the left-hand side and right-hand side of Eq. (43) up. □

Proof of Proposition 9. Given Assumption A1, $EV^*_{M,B} < EV^*_{M,R}$. Assumption A2 implies that $EV^*_{D,B} < EV^*_{D,R}$. Using the proof of the previous proposition, this implies that $\hat{Y}_B < \hat{Y}_R$ when $\pi_{D,B} > \pi_{D,R}$, $p_B < p_R$, and $\lambda_B > \lambda_R$ hold (and setting $\gamma_B = \gamma_R$), which means that $w^*_{D,B} < w^*_{D,R}$. As in the previous proposition, setting $\gamma_B > \gamma_R$ has an ambiguous effect. □

Proof of Lemma 10. In equilibrium, the wage $w^*_{i,s}$ for $s \in \{R,B\}$ is optimally chosen and so satisfies the first-order condition:

$$0 = -1 + \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho(-1+2z_{j,s}) + (1-\rho)\sigma^*_{D,s})]EV^*_{D,s} + \delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\sigma^*_{D,s})]EV^*_{M,s}\}. \tag{48}$$

For notational convenience, define

$$\tilde{Y}_s = (1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho(-1+2z_{j,s}) + (1-\rho)\sigma^*_{D,s}]EV^*_{D,s} + \delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\sigma^*_{D,s})]EV^*_{M,s}\}. \tag{49}$$

Assuming that the first-order condition is satisfied, we know that $\tilde{Y}_s > 0$.

We begin by taking the derivative of CRA i's first-order condition with respect to $w_{j,s}$:

$$0 = \frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}}\tilde{Y}_s + \frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s[\rho + (1-\rho)(1-\lambda_s)p_s] \times[EV^*_{D,s} - EV^*_{M,s}] + \frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s\rho EV^*_{D,s}, \tag{50}$$

and, so

$$\frac{dw_{i,s}}{dw_{j,s}} = -\frac{\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s\{[\rho + (1-\rho)(1-\lambda_s)p_s][EV^*_{D,s} - EV^*_{M,s}] + \rho EV^*_{D,s}\}}{\frac{\partial^2 z_{i,s}}{\partial w^2}\tilde{Y}_s}. \tag{51}$$

Since

$$\frac{\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}} < 0 \quad \text{and} \quad \tilde{Y}_s > 0,$$

the result follows. □

Proof of Proposition 13. Define the reaction functions for state s as $w_{i,s}(w_{j,s})$ and $w_{j,s}(w_{i,s})$. From the definition of strategic complements, we know that $w'_{i,s}(w_{j,s})$ and $w'_{j,s}(w_{i,s})$ are greater than zero. Next, in order to ensure that the curves cross, we need two conditions. First, we need $w_{i,s}(0)$ and $w_{j,s}(0)$ to be greater than zero. The reaction functions are implicitly defined by the first-order conditions given by Eq. (48). We will look at the case of CRA i. Taking $w_{j,s} = 0$, we know that $z_{j,s}(0) = 0$. The first-order condition simplifies to

$$1 = \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho + (1-\rho)(1-\lambda_s)p_s] \times(EV^*_{M,s} - EV^*_{D,s}) + \delta p_s(1-\rho)EV^*_{D,s}\}.$$

In order for $w_{i,s}(0) > 0$ to be true, we need the expression in curly brackets to be positive. That would imply that $\partial z_{i,s}/\partial w > 0$. Conditions 1 and 2 guarantee this.

We also need the reaction functions to cross the 45-° line. Consider $w_{j,s} = \overline{w}$, the maximum wage. Denote $z(\overline{w}) = \overline{z} \le 1$. Then, the first-order condition in Eq. (48) is

$$1 = \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s(1-\overline{z})[\rho + (1-\rho)(1-\lambda_s)p_s] \times(EV^*_{M,s} - EV^*_{D,s}) + \delta p_s(1-\rho(1-\overline{z}))EV^*_{D,s}\}.$$

Once again, from Conditions 1 and 2, we know that the expression in curly brackets is positive. This implies that

$\partial z_{i,s}/\partial w$ is positive and finite. Since $\partial z^2/\partial w^2 < 0$ and $\partial z/\partial w|_{\overline{w}} = 0$, it must be that $w_{i,s}(\overline{w}) < \overline{w}$.

Lastly, for uniqueness, we will show that Condition 3 implies that the reaction functions are concave. To reduce notation slightly, label the strategic complements condition $[\rho+(1-\rho)(1-\lambda_s)p_s][EV^*_{D,s}-EV^*_{M,s}]+\rho EV^*_{D,s}$ as Ψ. The slope of the reaction function $w_{i,s}(w_{j,s})$ is given by Eq. (51), which we can rewrite as

$$\frac{dw_{i,s}}{dw_{j,s}} = -\frac{\dfrac{\partial z_{i,s}}{\partial w}\dfrac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s\Psi}{\dfrac{\partial^2 z_{i,s}}{\partial w^2}\tilde{Y}_s}.$$

The second-order condition for the reaction function is given by

$$\frac{d^2 w_{i,s}}{dw_{j,s}^2} = -\delta(1-\lambda_s)p_s\Psi\frac{\left(\begin{array}{c}\left(\dfrac{\partial^2 z_{i,s}}{\partial w^2}\dfrac{dw_{i,s}}{dw_{j,s}}\dfrac{\partial z_{j,s}}{\partial w}+\dfrac{\partial z_{i,s}}{\partial w}\dfrac{\partial^2 z_{j,s}}{\partial w^2}\right)\dfrac{\partial^2 z_{i,s}}{\partial w^2}\tilde{Y}_s \\ -\left(\dfrac{\partial^3 z_{i,s}}{\partial w^3}\dfrac{dw_{i,s}}{dw_{j,s}}\tilde{Y}_s+\dfrac{\partial^2 z_{i,s}}{\partial w^2}\dfrac{\partial \tilde{Y}_s}{\partial w_{j,s}}\right)\dfrac{\partial z_{i,s}}{\partial w}\dfrac{\partial z_{j,s}}{\partial w}\end{array}\right)}{\left(\dfrac{\partial^2 z_{i,s}}{\partial w^2}\tilde{Y}_s\right)^2}.$$

Note that from the property of strategic complements, $\Psi > 0$ and $dw_{i,s}/dw_{j,s} > 0$. As above, Conditions 1 and 2 give us that $\tilde{Y}_s > 0$. Condition 3 gives us that $\partial^3 z_{i,s}/\partial w^3$ is positive. The only remaining term left to sign is $\partial\tilde{Y}_s/\partial w_{j,s}$. Taking the derivative $\tilde{Y}_s$, as in Eq. (49), gives us $\delta(1-\lambda_s)p_s(\partial z_{j,s}/\partial w)\Psi$, which is positive. This proves that $d^2 w_{i,s}/dw_{j,s}^2$ is negative. □

Appendix B. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jfineco.2012.11.004.

References

Abreu, D., 1986. Extremal equilibria of oligopolistic supergames. Journal of Economic Theory 39, 191–225.

Ashcraft, A., Goldsmith-Pinkham, P., Vickery, J., 2010. MBS ratings and the mortgage credit boom. Unpublished working paper. Federal Reserve Bank of New York.

Bagwell, K., Staiger, R., 1997. Collusion over the business cycle. Rand Journal of Economics 28, 82–106.

Bar-Isaac, H., Shapiro, J., 2011. Credit ratings accuracy and analyst incentives. American Economic Review Papers and Proceedings 101, 120–124.

Bar-Isaac, H., Tadelis, S., 2008. Seller reputation. Foundations and Trends in Microeconomics 4, 273–351.

Becker, B., Milbourn, T., 2011. How did increased competition affect credit ratings? Journal of Financial Economics 101, 493–514.

Benmelech, E., Dlugosz, J., 2010. The credit ratings crisis. NBER Macroeconomics Annual 2009, 161–207.

Bolton, P., Freixas, X., Shapiro, J., 2012. The credit ratings game. Journal of Finance 67, 85–112.

Bond, P., Glode, V., 2011. Bankers and regulators. Unpublished working paper. University of Minnesota and University of Pennsylvania.

Boot, A.W.A., Milbourn, T., Schmeits, A., 2006. Credit ratings as coordination mechanisms. Review of Financial Studies 19, 81–118.

Bouvard, M., Levy, R., 2010. Humoring both parties: a model of two-sided reputation. Unpublished working paper. McGill University and University of Mannheim.

Dal Bó, P., 2007. Tacit collusion under interest rate fluctuations. RAND Journal of Economics 38, 533–540.

Faltin-Traeger, O., 2009. Picking the right rating agency: issuer choice in the ABS market. Unpublished working paper. Columbia Business School.

Faure-Grimaud, A., Peyrache, E., Quesada, L., 2009. The ownership of ratings. RAND Journal of Economics 40, 234–257.

Fulghieri, P., Strobl, G., Xia, H., 2011. The economics of solicited and unsolicited credit ratings. Unpublished working paper. University of North Carolina at Chapel Hill and University of Texas at Dallas.

Griffin, J.M., Tang, D.Y., 2012. Did subjectivity play a role in CDO credit ratings? Journal of Finance 67, 1293–1328.

Haltiwanger, J., Harrington, J., 1991. The impact of cyclical demand movements on collusive behavior. Rand Journal of Economics 22, 89–106.

He, J., Qian, J., Strahan, P. Are all ratings created equal? The impact of issuer size on the pricing of mortgage-backed securities. Journal of Finance, forthcoming.

Hirshleifer, D., Teoh, S.H., 2003. Limited attention, information disclosure and financial reporting. Journal of Accounting and Economics 36, 337–386.

Hong, H., Kubik, J., 2003. Analyzing the analysts: career concerns and biased earnings forecasts. Journal of Finance 58, 313–351.

Jackson, A., 2005. Trade generation, reputation, and sell-side analysts. Journal of Finance 60, 673–717.

Kandori, M., 1991. Correlated demand shocks and price wars during booms. Review of Economic Studies 58, 171–180.

Khanna, N., Noe, T.H., Sonti, R., 2008. Good IPOs draw in bad: inelastic banking capacity and hot markets. Review of Financial Studies 21, 1873–1906.

Klein, B., Leffler, K.B., 1981. The role of market forces in assuring contractual performance. Journal of Political Economy 89, 615–641.

Kliger, D., Sarig, O., 2000. The information value of bond ratings. Journal of Finance 55, 2879–2902.

Lin, H., McNichols, M., 1998. Underwriting relationships, analysts' earnings forecasts and investment recommendations. Journal of Accounting and Economics 25, 101–127.

Mariano, B., 2012. Market power and reputational concerns in the ratings industry. Journal of Banking & Finance 36, 1616–1626.

Mathis, J., McAndrews, J., Rochet, J.C., 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies?. Journal of Monetary Economics 56, 657–674.

Michaely, R., Womack, K., 1999. Conflict of interest and the credibility of underwriter analyst recommendations. Review of Financial Studies 12, 653–686.

Pagano, M., Volpin, P., 2012. Securitization, transparency, and liquidity. Review of Financial Studies 25, 2417–2453.

Perotti, E., Suarez, J., 2002. Last bank standing: What do I gain if you fail?. European Economic Review 46, 1599–1622.

Povel, P., Singh, R., Winton, A., 2007. Booms, busts, and fraud. Review of Financial Studies 20, 1219–1254.

Rotemberg, J., Saloner, G., 1986. A supergame-theoretic model of price wars during booms. American Economic Review 76, 390–407.

Sangiorgi, F., Sokobin, J., Spatt, C., 2009. Credit-rating shopping, selection and equilibrium structure of ratings. Unpublished working paper. Stockholm School of Economics, Securities and Exchange Commission, and Carnegie Mellon University.

Senate Permanent Subcomittee on Investigations, 2010. Hearing on Wall Street and the financial crisis: the role of credit rating agencies (Exhibits).

Skreta, V., Veldkamp, L., 2009. Ratings shopping and asset complexity: a theory of ratings inflation. Journal of Monetary Economics 56, 678–695.

Stolper, A., 2009. Regulation of credit rating agencies. Journal of Banking & Finance 33, 1266–1273.

Strausz, R., 2005. Honest certification and the threat of capture. International Journal of Industrial Organization 23, 45–62.

White, L.J., 2010. Markets: the credit rating agencies. Journal of Economic Perspectives 24 (2), 211–226.

Winton, A., Yerramilli, V., 2011. Lender moral hazard and reputation in originate-to-distribute markets. Unpublished working paper. University of Minnesota and University of Houston.