Recursion, pt. 1

The Foundations

What is Recursion?

- *Recursion* is the idea of solving a problem in terms of itself.
  - For some problems, it may not be possible to find a direct solution.
  - Instead, the problem is typically broken down, progressively, into simpler and simpler versions of itself for evaluation.

What is Recursion?

- One famous problem which is solved in a *recursive* manner: the factorial.
  - n! = 1 for n = 0, n =1…
  - n! = n * (n-1)!, n > 1.
- Note that aside from the n=0, n=1 cases, the factorial’s solution is stated in terms of a *reduced* form of itself.

What is Recursion?

- As long as n is a non-negative integer, n! will eventually reach a *reduced* form for which there is an exact solution.
  - 5! = 5 * 4! = 5 * 4 * 3! = …
    = 5 * 4 * 3 * 2 * **1**

What is Recursion?

- From this point, the solution for the reduced problem will be used to determine the exact solution.

5 * 4 * 3 * 2 * **1** = 5 * 4 * 3 * **2**
= 5 * 4 * **6**
= 5 * **24**
= **120**.

Recursion

- Thus, the main idea of recursion is to reduce a complex problem to a combination of operations upon its simplest form.
  - This “simplest form” has a well-established, exact solution.

What is Recursion?

- As a result of how recursion works, it ends up being subject to a number of jokes:
  - "In order to understand recursion, you must understand recursion."
  - Or, "recursion (n): See recursion."

A Mathematical Look

- Recursion is actually quite similar to a certain fairly well-known mathematical proof technique: *induction*.

A Mathematical Look

- Proof by induction involves three main parts:
  - A *base case* with a known solution.
    - Typically, for the most basic version of the problem. Say, for $i = 0$ in a series.
  - A proposed, closed-form solution for any value k, which the base case matches.

A Mathematical Look

- Proof by induction involves three main parts:
  - A proof that shows that if the proposed solution works for time step k, it works for time step $k+1$.
    - Typically, it works by showing that the closed form solution for time step $k+1$ is equal to that given by a known, correct alternative.

A Mathematical Look

- The main idea behind how induction works is the same as that for recursion.
  - The process is merely inverted: the way that induction proves something is how recursion *will actually produce its solution*.

The Basic Process

- There are two main elements to a recursive solution:
  - The *base case*: the form (or forms) of the problem for which an exact solution is provided.
  - The recursive step: the reduction of one version the problem to a simpler form.

The Basic Process

- Note that if we progressively reduce the problem, one step at a time, we'll eventually hit the base case.
  - From there, we take that solution and modify it as necessary on the way back up to yield the true solution.

The Basic Process

- There are thus these two main elements to a recursive solution of the factorial method:
  - The *base case*: 0! and 1!
  - The recursive step: n! = n * (n-1)!
  - Note that the “n *” will be applied after the base case is reached.
  - (n-1)! is the reduced form of the problem.

Coding Recursion

- As we’ve already seen, programming languages incorporate the idea of function calls.
  - This allows us to reuse code in multiple locations within a program.
  - Is there any reason that a function shouldn’t be able to reuse *itself*?

Coding Recursion

```
int factorial(int n)
{
  if(n == 0 || n == 1)
    return 1;
  else return n * factorial(n-1);
}
```

Coding Recursion

- Potential problem: how can the program keep track of its state?
  - There will multiple versions of “n” over the different calls of the factorial function.
  - The answer: stacks!
  - The stack is a data structure we haven’t yet seen, but may examine in brief later in the course.

Coding Recursion

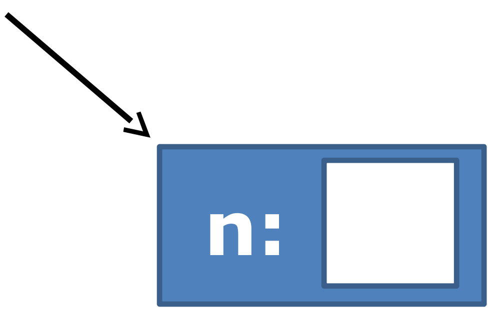
```
int factorial(int n)
{
   if(n == 0 || n == 1)
   return 1;
   else return n * factorial(n-1);
}
```

- Each individual method call within a recursive process can be called a *frame*.

Coding Recursion

```
int factorial(int n)
{
   if(n == 0 || n == 1)
      return 1;
   else return n * factorial(n-1);
}
```

- We’ll use this to denote each frame of this method’s execution.
  - Let’s try n = 5.



Coding Recursion

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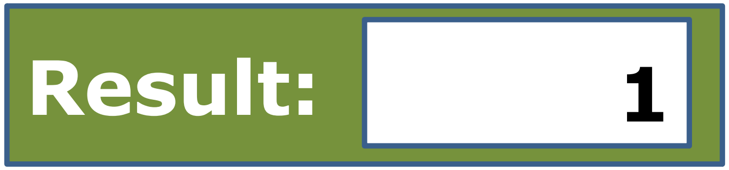

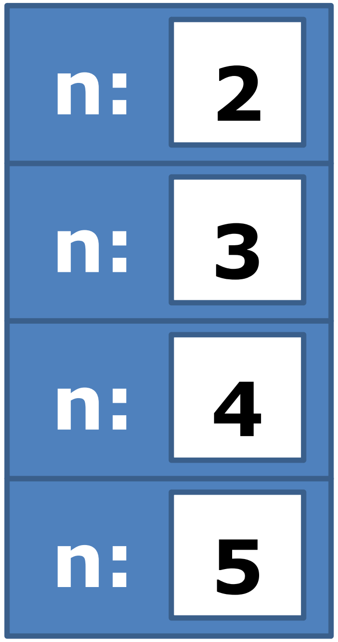

Coding Recursion

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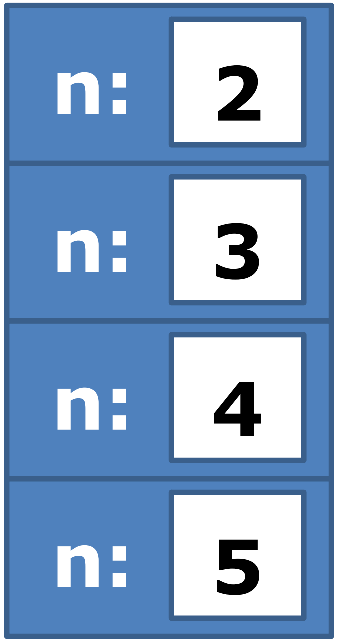

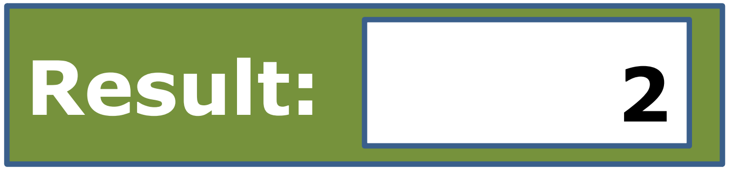

Coding Recursion

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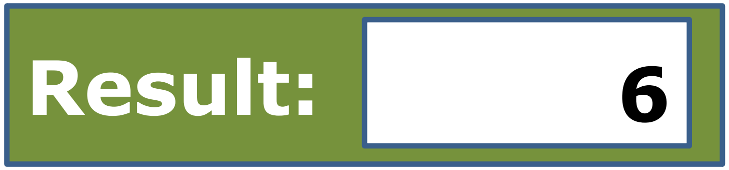

Coding Recursion

```
int factorial(int n)
{
   if(n == 0 || n == 1)
      return 1;
   else return n * factorial(n-1);
}
```

n:	4
n:	5

Result: 24

Coding Recursion

```
int factorial(int n)
{
   if(n == 0 || n == 1)
      return 1;
   else return n * factorial(n-1);
}
```

Result: 120

n: 5