CAP5638 Project 2

Classification Using Linear Discriminant Functions and Boosting Algorithms

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Todo list

- Add plots for training error . . . 2
- isn't this one wrong? Use the one from adaboost.WeakClassifier . . . 2
- Multiclass: OvO . . . 5
- Multiclass: Results . . . 5
- get a plot here . . . 6
- Multiclass: UCI Wine results for One-Against-the-Other . . . 6
- USPS Handwritten One-Against-the-Rest Results . . . 6
- Multiclass: USPS Digits results for One-Against-the-Other . . . 7
- better writeup here . . . 8
- AdaBoost Results . . . 8
- anything to add here? (e.g. why multiclass is garbage) . . . 8
- should probably explain why multiclass is garbage here . . . 8
- Support vector machines . . . 10
- Kernel method for linear discriminant functions . . . 10
- Multiple-class linear machines and multiple-class boosting . . . 10

The algorithms were implemented in Python 3.5, with a dependence on the *scipy* [1] library.

1 Basic Two-Class Classification Using Perceptron Algorithms

Abstractly, the problem is as follows: Given n labeled training samples, $D = \{(x_1, L_1), (x_2, L_2), ..., (x_n, L_n)\}$, where $L_i = \pm 1$, implement Algorithm 4 (Fixed-Increment Single-Sample Perceptron Algorithm) and Algorithm 8 (Batch Relaxation with Margin) of Chapter 5 in the textbook [2].

The algorithms are:

Algorithm 5.4 (Fixed-Increment Single-Sample Perceptron)

1: **initialize** $a, k = 0$
2: **do** $k \leftarrow (k+1) \mod n$
3: **if** $\mathbf{y}_k$ is misclassified by $\mathbf{a}$ **then** $\mathbf{a} \leftarrow \mathbf{a} + \mathbf{y}_k$
4: **until** all patterns properly classified
5: **return** a

Algorithm 5.8 (Batch Relaxation with Margin)

1: **initialize** $a, \eta(\cdot), b, k \leftarrow 0$
2: **do** $k \leftarrow (k+1) \mod n$
3: $\mathcal{Y}_k = \{\}$
4: $j = 0$
5: **do** $j \leftarrow j + 1$
6: **if** $\mathbf{a}^t\mathbf{y}^j \leq b$ **then** Append $\mathbf{y}^j$ to $\mathcal{Y}_k$
7: **until** $j = n$
8: $\mathbf{a} \leftarrow \mathbf{a} + \eta(k) \sum_{y \epsilon \mathcal{Y}} \frac{b - \mathbf{a}^t\mathbf{y}}{\|\mathbf{y}\|^2}\mathbf{y}$
9: **until** $\mathcal{Y}_k = \{\}$
10: **return** a

Results

Add plots for training error

Long training times proved problematic, due to the fact that it took greater than 100000 iterations to reach convergence using the fixed relaxation rule for the UCI wine data set. As such, the most significant result here is that the higher dimensional USPS handwritten digits data set converges much more rapidly than the UCI wine data set – by several orders of magnitude. Futhermore, for the fixed-increment rule, accuracy is much greater when testing on the handwritten digits data set. The batch relaxation rule performed well for both data sets, though higher on average for the UCI wine data set.

This most likely has to do with the fact that the handwritten digits data set has a great deal more training samples than the wine data set. With more training samples to update the hyperplane, the classifier can converge to a solution much more rapidly.

isn't this one wrong? Use the one from adaboost.WeakClassifier

As for the long training times for the wine data set, the updated weights $\mathbf{a}(k)$ would oscillate between values for a long time before settling on a steady solution. To prevent this behaviour, several early termination heuristics were used. The heuristics were:

1. Always terminate after 100,000 iterations, and using the value of $\mathbf{a}$ that minimizes $J_p(\mathbf{a})$ for Algorithm 5.4 and $J_r(\mathbf{a})$ for Algorithm 5.8.

2. Terminate if, after 50,000 iterations, $\|\mathbf{a}(k+1) - \mathbf{a}(k)\| \approx 0$, and $J_p(\mathbf{a}) \approx 0$ for Algorithm 5.4 and $J_r(\mathbf{a}) \approx 0$ for Algorithm 5.8.

Most likely, the necessity for early termination is due to the fact that the mean and standard deviation of the training samples was not normalized (however, the perceptron was trained using normalized augmented features, e.g. for training label ω_1, $y_i^T = [1, \mathbf{x_i}]$ if x_i belongs to ω_1, and $y_i^T = [-1, -\mathbf{x_i}]$ if x_i is not a sample for ω_1.

It's very likely that these early-termination heuristics had a detrimental effect on the classification accuracy, since in these cases, the there was no guarantee that $\mathbf{a}^T\mathbf{y} \geq 0$ for all augmented $\mathbf{y}$.

UCI Wine Data Set

Algorithm 5.4 (Fixed-Increment Single-Sample Perceptron)

Training Statistics			
Class	**Correct (of 89) (%)**	**Iterations**	**Runtime(s)**
ω_1	60 (67.42%)	100001	63.151
ω_2	56 (62.92%)	100001	63.050
ω_3	85 (95.51%)	84710	53.850
	Total	284710	180.051

The corresponding weights after training were:

$$\mathbf{a}_{\omega_1} = \begin{bmatrix} -2. \\ -23.54 \\ -1.28 \\ -3.54 \\ -36.8 \\ -136. \\ -2.7 \\ 0.36 \\ -1.34 \\ 2.3 \\ -2.86 \\ -2.88 \\ -0.42 \\ -490. \end{bmatrix} \quad \mathbf{a}_{\omega_2} = \begin{bmatrix} 9484.0 \\ 56298.47 \\ -26970.62 \\ 9950.26 \\ -10456.6 \\ -3765. \\ 55712.08 \\ -1502.68 \\ 36611.34 \\ -40821.68 \\ -109006.33 \\ 65677.49 \\ 44053.19 \\ -430.0 \end{bmatrix} \quad \mathbf{a}_{\omega_3} = \begin{bmatrix} 451.0 \\ -1399.63 \\ -10081.3 \\ -16667.88 \\ 28250.3 \\ 2665.0 \\ -71066.41 \\ -107364.33 \\ -12267.75 \\ -24570.91 \\ 94303.98 \\ -52048.4 \\ -132265.0 \\ -958.0 \end{bmatrix}$$

Algorithm 5.8 (Batch Relaxation with Margin)

Training Statistics			
Class	**Correct (of 89) (%)**	**Iterations**	**Training Time (s)**
ω_1	87 (97.75%)	22864	27.498
ω_2	85 (95.51%)	52918	62.130
ω_3	83 (93.26%)	35600	127.227
	Total	111,382	216.855

The corresponding weights after training were:

$$\mathbf{a}_{\omega_1} = \begin{bmatrix} 0.796 \\ -0.31 \\ 0.768 \\ 0.183 \\ -0.362 \\ -0.063 \\ 0.74 \\ 0.153 \\ 0.867 \\ 0.601 \\ 0.128 \\ 0.216 \\ 0.583 \\ 0.01 \end{bmatrix} \quad \mathbf{a}_{\omega_2} = \begin{bmatrix} 261.354 \\ 869.172 \\ -1800.878 \\ -101.749 \\ 1335.277 \\ 6.557 \\ 1335.119 \\ 1928.754 \\ 99.223 \\ 921.129 \\ -6428.793 \\ 895.385 \\ 2459.442 \\ -33.943 \end{bmatrix} \quad \mathbf{a}_{\omega_1} = \begin{bmatrix} 0.855 \\ -0.161 \\ 0.587 \\ 0.188 \\ -0.108 \\ -0.017 \\ 0.295 \\ -0.334 \\ 0.758 \\ -0.18 \\ 0.882 \\ -0.084 \\ 0.086 \\ -0.003 \end{bmatrix}$$

USPS Handwritten Digits Data Set

Algorithm 5.4 (Fixed-Increment Single-Sample Perceptron)

Training Statistics			
Class	**Correct (of 2007) (%)**	**Iterations**	**Training Time (s)**
ω_0	1928 (96.06%)	17	0.031s
ω_1	1982 (98.75%)	9	0.018s
ω_2	1873 (93.32%)	14	0.026s
ω_3	1896 (94.47%)	14	0.033s
ω_4	1897 (94.52%)	23	0.040s
ω_5	1905 (94.92%)	22	0.042s
ω_6	1960 (97.66%)	25	0.047s
ω_7	1926 (95.96%	25	0.045s
ω_8	1881 (93.72%)	20	0.037s
ω_9	1919 (95.62%)	41	0.073s
	Total	210	0.392

The weights are too large to display in the report, however they can be displayed by invoking the following command using the included run.py script:

python3 run.py fixed bin/digits_train.txt bin/digits_test.txt

Algorithm 5.8 (Batch Relaxation with Margin)

Training Statistics			
Class	**Correct (of 2007) (%)**	**Iterations**	**Training Time (s)**
ω_0	1837 (91.53%)	717	1.297s
ω_1	1917 (95.52%)	742	1.277s
ω_2	1850 (92.18%)	606	1.084s
ω_3	1894 (94.37%)	434	0.744s
ω_4	1843 (91.83%)	597	1.130s
ω_5	1900 (94.67%)	825	1.571s
ω_6	1891 (94.22%)	831	1.612s
ω_7	1904 (94.87%)	1358	2.581s
ω_8	1863 (92.83%)	934	1.907s
ω_9	1898 (94.57%)	1649	3.331s
	Total	18797	16.535

The weights are too large to display in the report, however they can be displayed by invoking the following command using the included run.py script:

python3 run.py relax bin/digits_train.txt bin/digits_test.txt

2 Multi-Class Classification

For this classification method, both the fixed-increment and batch relaxation training rules from the previous section were used for testing.

For one-against-the-rest classification, a sample was classified as ω_i if $\mathbf{a}_i^T\mathbf{x} \geq \mathbf{a}_j^T\mathbf{x}$ for all $i \neq j$.

For one-against-other classification...

Elaborate on one-against-other classification

Results

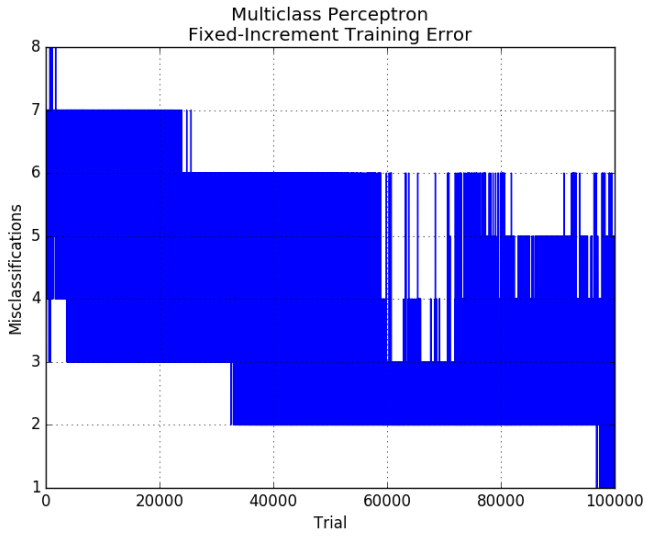
For each dataset, now train a classifier to classify all the classes using the one-against-the-rest and the one-against-the-other methods based on the two two-class algorithms, resulting in four different classifiers on each dataset and then classify the test set. Document classification accuracy, iterations in training, and classification time for test, and compare the one-against-the- rest and the one-against-the-other methods.

(Note: The runtime statistics for training are tabulated in the previous section)

UCI Wine Data Set

One-Against-the-Rest

Using the fixed increment rule (Algorithm 5.4 above), the one-against-the-rest classifier correctly classified 68 testing samples out of 89 (76.40% accuracy). There were 111382 iterations during training (taking $\approx$ 216.855 seconds).



Using the batch relaxation rule (Algorithm 5.8), the one-against-the-rest classifier correctly classified 85 testing samples out of 89 (95.51% accuracy). There were 300000 iterations during training (taking ≈ 398.827 seconds).

get a plot here

One-Against-the-Other

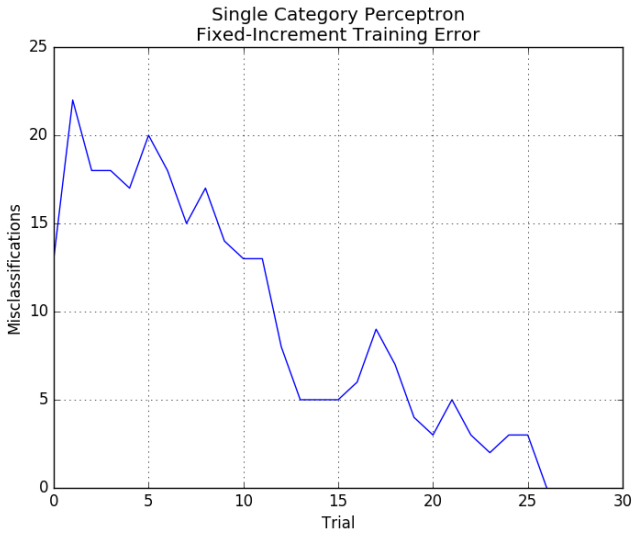
One-Against-the-Other Classification needs to be implemented still.

USPS Handwritten Digits Data Set

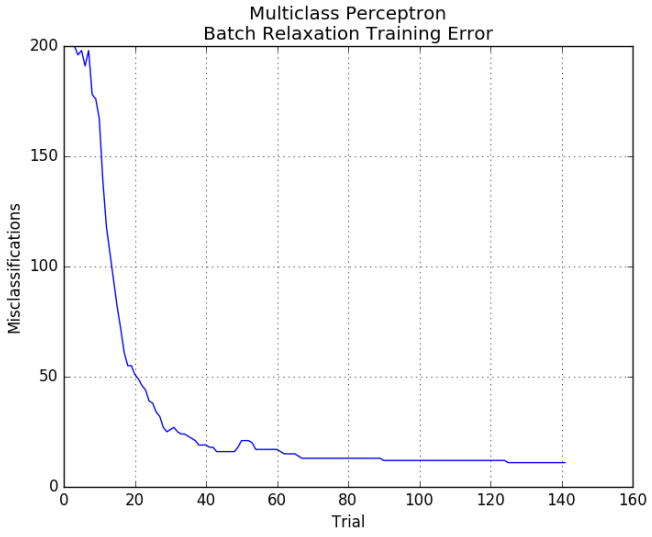
One-Against-the-Rest

Completed training after 27 trials. Total training time: 2.923 and 1617 correct out of 2007 (80.57 accuracy)

Using the fixed increment rule (Algorithm 5.4 above), the one-against-the-rest classifier correctly classified 1608 samples out of 2007 (80.12% accuracy). Training occured over 177 iterations (≈ 0.571 seconds).



Using the batch relaxation rule (Algorithm 5.8), the one-against-the-rest classifier correctly classified 1626 testing samples out of 2007 (81.02% accuracy). Training occured over 142 training trials ($\approx$ 23.032 seconds).



One-Against-the-Other

One-Against-the-Other Classification needs to be implemented still.

3 Adaboost to Create Strong Classifers

Implement Algorithm 1 (AdaBoost) in Chapter 9 of the textbook to create a strong classifier using the above linear discriminant functions.

better writeup here

Algorithm 9.1 (AdaBoost)

1: **initialize** $\mathcal{D} = \left\{\mathbf{x}^1, y_1, ..., \mathbf{x}^n, y_n\right\}, k_{max}, W_1(i) = 1/n, i = 1...n$
2: $k = 0$
3: **do** $k \leftarrow k + 1$
4: train weak learner C_k using $\mathcal{D}$ sampled according to $W_k(i)$
5: $E_k \leftarrow$ training error of C_k measured on $\mathcal{D}$ using $W_k(i)$
6: $\alpha_k \leftarrow 0.5 \ln\left[(1 - E_k)/E_k\right]$
7: $W_{k+1}(i) = \frac{W_k(i)}{Z_k} \times \begin{cases} e^{-\alpha_k} & \text{if } h_k(\mathbf{x}^i) = y_i \text{ (correct classification)} \\ e^{\alpha_k} & \text{if } h_k(\mathbf{x}^i) \neq y_i \text{ (incorrect classification)} \end{cases}$
8: **until** $k = k_{max}$
9: **return** C_k and α_k for $k = 1$ to k_{max} (ensemble of classifiers with weights)

Results

Boost Algorithm 8 to create a strong classifier for class 1 vs. class 2, class 1 vs. class 3, and class 2 vs. class 3 on the two datasets. Then classify the corresponding test samples from the relevant classes in test sets (in other words, for example, for the class 1 vs. class 2 classifier, you only need to classify test samples from classes 1 and 2); then document classification accuracy and show and analyze the improvement.

UCI Wine Data Set

anything to add here? (e.g. why multiclass is garbage)

Classes	Correct / Total (Accuracy)	Training Time (s)
ω_1 vs ω_2	34 / 34 (100%)	6.61s
ω_1 vs ω_3	34 / 34 (100%)	6.35s
ω_2 vs ω_3	34 / 34 (100%)	5.75s
Multiclass	17 / 51 (33.33%)	18.71s

USPS Handwritten Digits Data Set

should probably explain why multiclass is garbage here

Classes	**Correct / Total (Accuracy)**	**Training Time (s)**
ω_0 vs ω_1	264 / 623 (42.38%)	4.88s
ω_0 vs ω_2	198 / 557 (35.55%)	4.53s
ω_0 vs ω_3	166 / 525 (31.62%)	4.30s
ω_0 vs ω_4	200 / 559 (35.78%)	4.40s
ω_0 vs ω_5	160 / 519 (30.83%)	4.38s
ω_0 vs ω_6	170 / 529 (32.14%)	4.49s
ω_0 vs ω_7	147 / 506 (29.05%)	4.37s
ω_0 vs ω_8	166 / 525 (31.62%)	4.30s
ω_0 vs ω_9	177 / 536 (33.02%)	4.36s
ω_1 vs ω_2	462 / 462 (100.00%)	3.67s
ω_1 vs ω_3	430 / 430 (100.00%)	3.70s
ω_1 vs ω_4	464 / 464 (100.00%)	3.66s
ω_1 vs ω_5	423 / 424 (99.76%)	3.67s
ω_1 vs ω_6	434 / 434 (100.00%)	3.77s
ω_1 vs ω_7	411 / 411 (100.00%)	3.80s
ω_1 vs ω_8	430 / 430 (100.00%)	3.88s
ω_1 vs ω_9	441 / 441 (100.00%)	4.20s
ω_2 vs ω_3	364 / 364 (100.00%)	4.59s
ω_2 vs ω_4	398 / 398 (100.00%)	4.64s
ω_2 vs ω_5	358 / 358 (100.00%)	4.58s
ω_2 vs ω_6	368 / 368 (100.00%)	4.82s
ω_2 vs ω_7	345 / 345 (100.00%)	4.58s
ω_2 vs ω_8	364 / 364 (100.00%)	4.62s
ω_2 vs ω_9	375 / 375 (100.00%)	4.62s
ω_3 vs ω_4	365 / 366 (99.73%)	4.43s
ω_3 vs ω_5	326 / 326 (100.00%)	4.32s
ω_3 vs ω_6	336 / 336 (100.00%)	4.36s
ω_3 vs ω_7	313 / 313 (100.00%)	4.38s
ω_3 vs ω_8	332 / 332 (100.00%)	4.37s
ω_3 vs ω_9	343 / 343 (100.00%)	4.33s
ω_4 vs ω_5	360 / 360 (100.00%)	4.44s
ω_4 vs ω_6	370 / 370 (100.00%)	4.45s
ω_4 vs ω_7	347 / 347 (100.00%)	4.44s
ω_4 vs ω_8	366 / 366 (100.00%)	4.46s
ω_4 vs ω_9	377 / 377 (100.00%)	4.39s
ω_5 vs ω_6	330 / 330 (100.00%)	4.74s
ω_5 vs ω_7	307 / 307 (100.00%)	4.73s
ω_5 vs ω_8	326 / 326 (100.00%)	4.76s
ω_5 vs ω_9	337 / 337 (100.00%)	4.78s
ω_6 vs ω_7	317 / 317 (100.00%)	4.01s
ω_6 vs ω_8	336 / 336 (100.00%)	4.00s
ω_6 vs ω_9	347 / 347 (100.00%)	4.02s
ω_7 vs ω_8	313 / 313 (100.00%)	4.10s
ω_7 vs ω_9	324 / 324 (100.00%)	4.04s
ω_8 vs ω_9	343 / 343 (100.00%)	4.11s
Multiclass	359 / 2007 (17.89%)	209.49s

4 Extra Credit

4.1 Support vector machines

By using an available quadratic programming optimizer or an SVM library, implement a training and classification algorithm for support vector machines. Then use your algorithm on the USPS dataset. Document the classification accuracy and compare the results with that from the two basic algorithms.

4.2 Kernel method for linear discriminant functions

Given a kernel function, derive the kernel-version of Algorithm 4 and implement the algorithm, and then apply it on the given wine and USPS datasets. Document the classification accuracy and compare the results with that from the two basic algorithms without kernels. Use the polynomial function of degree three as the kernel function; optionally, you can use other commonly used kernel functions.

4.3 Multiple-class linear machines and multiple-class boosting

Use the Keslers construction to train a linear machine for multi-class classification and then use the SAMME algorithm to boost its performance on the training set. Apply the algorithm on both datasets and classify the corresponding test samples in the test sets. Document the classification accuracy and compare the results with that from the one-against-the-rest and one-against-the- other algorithms.

References

[1] Jones E, Oliphant E, Peterson P, *et al.* **SciPy: Open Source Scientific Tools for Python**, 2001-, `http://www.scipy.org/` [Online; accessed 2015-10-24].

[2] Richard O. Duda, Peter E. Hart, and David G. Stork **Pattern Classification** 2nd edition