E.S.S. invader

x is an ESS: $y \neq x$:

1 $u(x,x) > u(y,x)$

2 $\boxed{u(x,x) = u(y,x)}$ & $u(x,y) > u(y,y)$ $\Downarrow$ $\Vert$

conversely if (1) or (2) holds then x is ESS.

$\boxed{\text{Proof:}}$

x is ESS:

$$u(x, x_\varepsilon) > u(y, x_\varepsilon)$$

$$(1-\varepsilon)\,\cancel{u(x,x)} + \varepsilon\, u(x,y) > \cancel{(1-\varepsilon)\, u(y,x)} + \varepsilon\, u(y,y) \qquad \Vert$$

if (1) holds: for ε small enough condition holds. If (2) holds then condition holds.

- if $u(x,x) < u(y,x)$ (not (1)) then for small enough ε condition won't hold

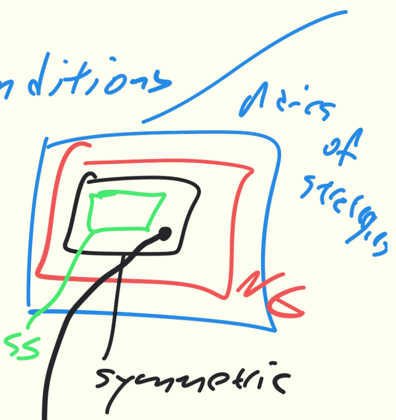
- if $u(x,x) = u(y,x)$ & $u(x,y) \leq u(y,y)$ (not (2)) then condition doesn't hold.

To find ESS

1. write $(A, A^T) \in \mathbb{R}^{n \times n^2}$

2. Identify all symmetric N.E.

3. test the conditions



$$A = \begin{pmatrix} 1 & 3 \\ 4 & 1 \end{pmatrix}$$

.4 .6

~~$\{(1,0), (0,1)\}$~~

~~$\{(0,1), (1,0)\}$~~

$\{(.4, .6), (.4, .6)\}$

$u(x,x) > u(y,x)$

$\boxed{u(x,x) = u(y,x)}$

$u(x,y) > u(y,y)$

?

let $y = (y_1, y_2)$

$y_2 = 1 - y_1$

$$u(x,y) = y_1 + 1.8$$

$$u(y,y) = 5y_1 - 5y_1^2 + 1$$

$$u(x,y) - u(y,y) = -4y_1 + 5y_1^2 + .8$$

$$= 5(y_1 - .4)^2 > 0 \quad y \neq x,\ y_1 \neq .4$$

$\boxed{\text{thus } x = (.4, .6) \text{ is an ESS}}$