Selection

$x(t)$ $\qquad\qquad\qquad$ $y(t)$

$$\frac{dx}{dt} = ax \qquad\qquad \frac{dy}{dt} = by$$

$$\rho(t) = \frac{x(t)}{y(t)}$$

$$\frac{d\rho}{dt} = \frac{\frac{dx}{dt}y - \frac{dy}{dt}x}{y^2} = \frac{xy(a-b)}{y^2} = \frac{x}{y}(a-b)$$

$$= \rho(a-b)$$

$$\boxed{\begin{array}{l} \rho(t) = \rho_0 \, e^{(a-b)t} \\ \rho_0 = \rho(0) \end{array}}$$