1. (a) Give the definition of Normal Form Game. [2]

(b) For the rest of this question, consider the Normal Form Game with the following matrix representation:

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$$

Give the utilities to both players using the following strategy pairs:

(i) $\sigma_r = (1, 0) \qquad \sigma_c = (0, 1)$ [1]

(ii) $\sigma_r = (1/2, 1/2) \qquad \sigma_c = (1/3, 2/3)$ [1]

(iii) $\sigma_r = (1/4, 3/4) \qquad \sigma_c = (0, 1)$ [1]

(c) Give the definition for the Lemke-Howson algorithm. [5]

(d) Show that the vertices for the best response polytopes are given by:

OBTAIN P AND Q

[4]

(e) Draw the best response polytopes. [2]

(f) Use the plots to carry out the Lemke-Howson algorithm with all possible initial dropped labels. [4]

(g) Use the pair of vertices found in the previous question and find an initial dropped label that gives a different Nash equilibrium than the one obtained in the previous question. [1]

(h) Give a sketch of a proof, including potential assumptions that in a non degenerate game the number of Nash equilibria is odd. [4]

2. (a) Give the general definition of the Prisoner's Dilemma. [2]

(b) What values of S, T give valid Prisoner's Dilemma games:

(i) $A = \begin{pmatrix} 3 & S \\ 5 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & T \\ -1 & 1 \end{pmatrix}$ [4]

(ii) $A = \begin{pmatrix} 2 & S \\ -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -2 \\ S & 1 \end{pmatrix}$ [4]

(c) Consider the following reactive players:

$$p = (3/5, 3/4) \qquad q = (1/2, 1/4)$$

(i) Draw a diagram showing the Markov chain corresponding to a match between these two players. [2]

(ii) Obtain the Markov chain representation of a match between these two players. [2]

(d) State a theoretic result giving the utility of two general reactive players in a Prisoner's dilemma match. [5]

(e) Consider a reactive player $p = (x, x/2)$ and an opponent $q = (1/2, 1/4)$. Show that the utility to the player p is given by:

OBTAIN UTILITY

[5]

(f) Using the above find the optimal value of x. [4]

3. (a) For a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, obtain the following equation describing the corresponding evolutionary game:

$$\frac{dx}{dt} = x(f - \phi)$$

where $f = Ax$ and $\phi = fx$. [2]

(b) Define a mutated population. [2]

(c) Define an evolutionary stable strategy. [2]

(d) State and prove a theorem giving a general condition for an Evolutionary stable strategy. [4]

(e) Consider the following game $A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}$, obtain all evolutionary stable strategies. [5]

(f) Consider the accompanying 2008 paper entitled "Studying the emergence of invasiveness in tumours using game theory" by Basanta et al.

(i) Give a general summary of the paper. [3]

(ii) There is a minor error in this paper, describe and suggest the fix. [2]

(iii) How does the theorem in part 4 of this question relate to the findings of the paper? [2]

(iv) Suggest an alternative area of game theory that could also be used. [3]

4. (a) Give the definition of a Moran process on a game. [4]

(b) State and prove a theorem giving the fixation probabilities for a general birth death process. [6]

(c) Consider the Markov process on the Prisoners Dilemma: $A = \begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix}$ Use the above theorem to obtain the fixation probabilities for each strategy for $N = 10$. [3]

(d) Consider the following two strategies for the Prisoners Dilemma:

- Tit For Tat: start by cooperating and then repeat the opponents previous message.
- Alternator: start by cooperating and then alternate between defecting and cooperating.

Assuming a match lasting 10 turns show that the utility matrix between these two strategies corresponds to:

FIND MATRIX

[5]

(e) Obtain the fixation probabilities x_1 for each strategy. [7]