

My Final College Paper

A Thesis
Presented to
The Division of History and Social Sciences
Reed College

In Partial Fulfillment
of the Requirements for the Degree
Bachelor of Arts

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Acknowledgements

The cat, Grindle.

Preface

This is an example of a thesis setup to use the reed thesis document class.

List of Abbreviations

You can always change the way your abbreviations are formatted. Play around with it yourself, use tables, or come to CUS if you'd like to change the way it looks. You can also completely remove this chapter if you have no need for a list of abbreviations. Here is an example of what this could look like:

AI	Artificial Intelligence
MAS	Multi-Agent System
MDP	Markov Decision Processes
ML	Machine Learning

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Abstract

The preface pretty much says it all.

Dedication

You can have a dedication here if you wish.

Introduction

Welcome to the L^AT_EX thesis template. If you've never used T_EX or L^AT_EX before, you'll have an initial learning period to go through, but the results of a nicely formatted thesis are worth it for more than the aesthetic benefit: markup like L^AT_EX is more consistent than the output of a word processor, much less prone to corruption or crashing and the resulting file is smaller than a Word file. While you may have never had problems using Word in the past, your thesis is going to be about twice as large and complex as anything you've written before, taxing Word's capabilities. If you're still on the fence about using L^AT_EX, read the Introduction to LaTe_X on the CUS site as well as skim the following template and give it a few weeks. Pretty soon all the markup gibberish will become second nature.

0.1 Why use it?

L^AT_EX does a great job of formatting tables and paragraphs. Its line-breaking algorithm was the subject of a PhD. thesis. It does a fine job of automatically inserting ligatures, and to top it all off it is the only way to typeset good-looking mathematics.

0.2 Who should use it?

Anyone who needs to use math, tables, a lot of figures, complex cross-references, IPA or who just cares about the final appearance of their document should use L^AT_EX. At Reed, math majors are required to use it, most physics majors will want to use it, and many other science majors may want it also.

Chapter 1

Literature Review

This "chapter" is my attempt to get words on a page in the process of writing the literature review. It is also an attempt to structure my thought process on what I have read and what questions I want to answer. In general, the proceeding sections would like to answer the following questions:

- Why Multi-Agent Simulations?
- How do the agents model people, what are the assumptions?
- How does learning affect these simulations?
- Does the non-stationarity of multi-agent systems make learning techniques invalid as models of human behavior?

Because this is a cursory document it will move back and forth between detailed notes on a single paper and a more structured literature review where I feel stronger on the material.

1.1 Why use Agent Based Modeling?

Tries to answer :

- What questions do Agent Based Models (ABM) try to answer?
- What are their strengths?
- What are their shortcomings?
- Examples and results?

1.1.1 The Logic of Agent Based Simulations

From "Agent-based modeling: Methods and techniques for simulating human systems" Bonabeau (2002):

Agent-based modeling (ABM) is a technique that models a system as a computer simulation of autonomous decision making entities that we call agents. The agents behavior and decision making process should in some way represent the behavior of the system they are supposed to model. Models are able to produce complex behavior from even relatively simple assumptions about agents that may be insightful for understanding the system they are trying to simulate. "Sophisticated ABM sometimes incorporates neural networks, evolutionary algorithms, or other learning techniques to allow realistic learning and adaptation." The main idea of ABM's is "describing a system from the prospective of its constituent units."

"The benefits of ABM over other modeling techniques can be captured in three statements: (i) ABM captures emergent phenomena; (ii) ABM provides a natural description of a system; and (iii) ABM is flexible." The main benefit from this list being that ABM captures emergent phenomena that result from the interaction of individual agents. Example given is Traffic Jam that is result of car agents, but the traffic jam moves in the opposite direction of the cars and the traffic jam is itself an emergent behavior from the Car agents. One reason to use and ABM model is if there may be emergent phenomena; " when (i) Individual behavior is nonlinear and can be characterized by thresholds, if-then rules, or nonlinear coupling. (ii) Individual behavior exhibits memory, path dependence, and hysteresis, non-markovian behavior, or temporal correlations including learning and adaptation. (iii) Agent interactions are heterogeneous and can generate network effects. (iv) Averages will not work. Aggergate DEQ's tent to smooth out fluctuations, not ABM, which is important because under certain conditions, fluctuations can be amplified."

ABM is natural for explaining a system of behavioral entities such as traffic jams, stock markets, voters, or the inter-workings of an organization. Modeling Markets: Markets, and stock markets specifically have their resulting dynamics come from the interaction of many agents. This can be understood using ABM. NASDAQ used a an ABM to make decision about mechanism changes where there are many agents who learn via reinforcement learning and neural networks with the goal not being to emulate human behavior, but all possible behavior in regards to the change in mechanism. Found that decreasing tick-size actually increased ask-bid spread and made price discovery harder for agents. Another use is modeling auctions. EBAY lets people build agents to automate the bidding process. **important to thesis below:** "ultimately, transactions among economic software agents will constitute an essential and perhaps even dominant portion of the world economy." People at IBM have been exploring impact of shopbots on market dynamics with simulated shopbot economies of buyers and sellers. Economists have studied phenomena such as price dispersion by using ABM models where the goal is to design economic software agents. "In particular, they have been examining agent economies in which (i)search costs are nonlinear; (ii) some portion of the buyer population makes no use of search

mechanisms; and (iii) shopbots are economically motivated strategically pricing their services so as to maximize their own profits.” These strategies of ABM techniques can also be applied to studying many agents playing an economic game. Author calls this “Game Theory without the theory.” and says “Game theory is a great framework, but game theorists suffer from self-imposed constraints: being able to prove their theorems puts severe limitations to what is possible” in particular he argues it is impossible to model realistic games within their framework.

The author describes the following issues with ABM. Like all models (in social sciences especially) the model must serve a purpose and cannot be a general-purpose model. **important:** “Another issue has to do with the very nature of the systems one is modeling with ABM in social sciences: they most often involve human agents, with potentially irrational behavior, subjective choices, and complex psychology—in other words, soft factors, difficult to quantify, calibrate, and sometimes justify.” Should only use ABM to justify qualitative results (not quantitative) due to the varying degrees of accuracy input into the simulation. Bonabeau (2002)

How this applies to Thesis: This goes in depth on the “why” of Agent based modeling, specifically justifying it for bottom up modeling in the cases where that may be the best way to model something in the social sciences. The really interesting parts to me was about markets which highlights more or less exactly what I want to do, looking at markets composed of heterogeneous agents that learn via methods in AI. Some of these models are not even trying to emulate human behavior but just study the dynamics of competitive computer agents in a market setting (or how to build the best agents). The real issue I seem to be focusing in on is, what results can we get from such AI agent models when we know that different learning algorithms behave differently within the context of a non-stationary learning environment. As noted in my notes from a different section, when multiple agents are interacting and learning together they inherently create a non-stationary environment (constantly changing). This constant changing means that only under very specific circumstances and with certain algorithms can it be proved the agents will converge to an (optimal) equilibrium. Can a neural net be said to approximate a human’s learning? Probably not, but many simulations use it anyway. Does changing the composition of agents of each learning algorithm change the resulting dynamic in a meaningful way? If so, like I suspect, what can one even learn from such a simulation that generalizes at all to markets of agents artificial or otherwise?

1.1.2 Example: Schelling’s Segregation Model

Schelling’s Segregation model is an example of a ABM designed in 1969 by Thomas C. Schelling to try and explain racial segregation. Example of how agent preferences can lead to interesting aggregate structure. This model is presented in the lecture notes of Thomas Sargent and I recreated the model in python. The model is as follows:

There are two kinds of agents, orange and green of which there are 250 of each type living in on a unit square. The agents each have a location, a point on the

square, (x, y) , where $0 < x, y < 1$. The agents are *happy* if half of its neighbors, the ten closest agents in euclidean distance, are of the same type. If the agent is not *happy* then the agent is said to be *unhappy*. Initially, the agents are randomly assigned a point on the unit square i.e. each is initially assigned a point drawn from a bi-variate uniform distribution $S = (0, 1)^2$. Cycling though each agent, each agent is given a choice to stay or to move. If the agent is happy, then they stay otherwise they move according to the following procedure:

1. Draw a random location in S
2. If happy at new location, move there
3. Else, go to step 1

Continue this cycle, moving through the agents until everyone is happy and does not wish to move.

The results are shown in the following figures:

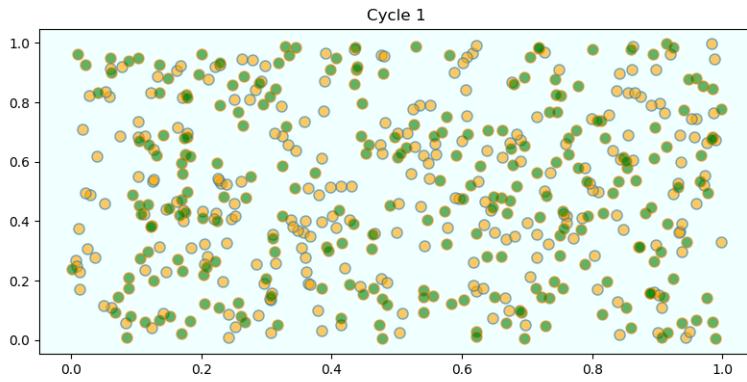


Figure 1.1: Schelling’s Segregation Model: Cycle 1

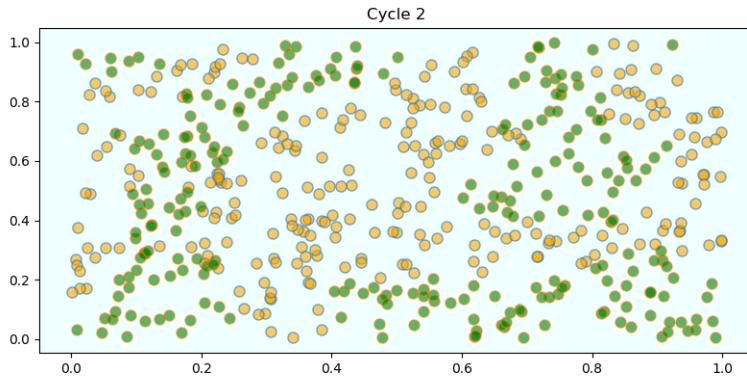


Figure 1.2: Schelling’s Segregation Model: Cycle 2

As can be seen, in cycle 1 (Figure 1.1), the agents are well distributed among each other, but as they move in cycles 2-5, they become progressively more segregated.

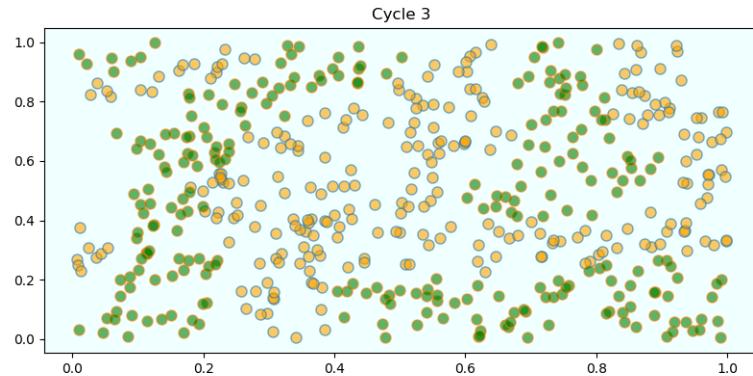


Figure 1.3: Schelling's Segregation Model: Cycle 3

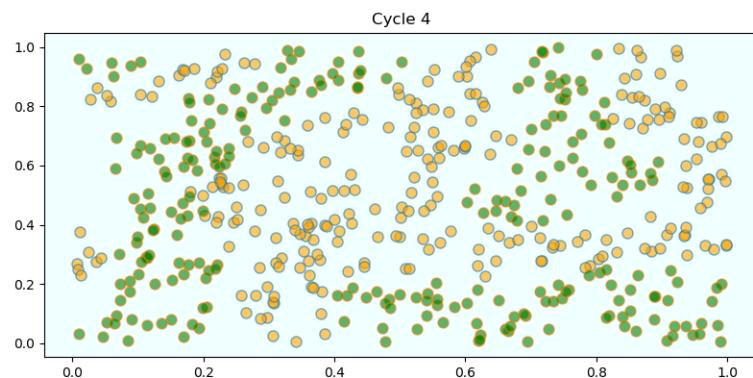


Figure 1.4: Schelling's Segregation Model: Cycle 4

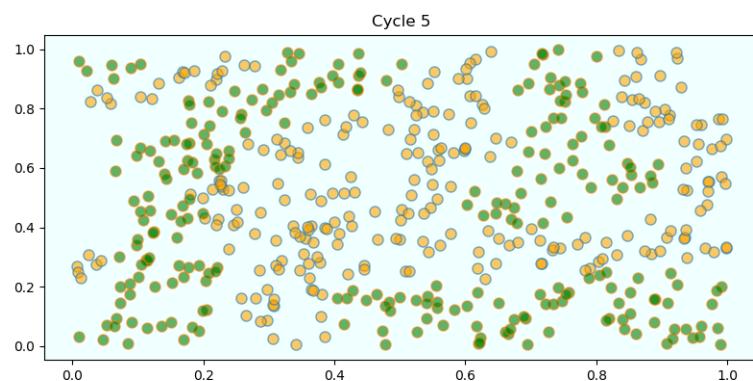


Figure 1.5: Schelling's Segregation Model: Cycle 5

After 5 cycles all agents are happy and the simulation terminates. This example highlights the benefits of using an ABM. With very few assumptions about the agents preferences (wanting half or more of their neighbors to be the same type as them), we

can see the resulting emergent behavior of segregated neighborhoods in the system as a whole.

Chapter 2

Agent Systems

This chapter lays out the background material of multi-agent systems—what they are, how they work, and why economists might be interested in them—that drives the rest of the thesis.

2.1 Agents

An **agent** is something that acts. This comes from the Latin word *agere*, to do. In general, anything might be considered an agent so long as it can perceive the environment and act upon it. A **rational agents** is something that acts to achieve the best outcome or with uncertainty, the best expected outcome in a given situation. For example we are agents, we have the ability to take an action and effect the world. We even might be considered rational since we are generally trying to navigate the world around us so as to try and lead to our preferred outcome. People are not the only agents, animals and machines are also agents capable of acting on the world around them.

2.1.1 In Economics

In economics many of the models are constructed with utility maximizing agents in line with Decision Theory and Rational Choice Theory. Because economics seeks to understand the outcomes of peoples interactions, and perhaps how these outcomes could be improved, if people can be modeled as utility maximizers, it is then possible to analytically solve for this solution and see what the results are.

The primary purpose of constructing agents in economics is to see how their interactions play out in an economic setting that can not be solved analytically.

2.1.2 In Artificial Intelligence

In artificial intelligence, the primary purpose of understanding agents is to try and create the most effective artificial agents possible. This discipline cares deeply about how good the agents they are able to make are able to solve problems, either general

or specific. To do this, they use much of the theory of agents with a deep emphasis on how a constructed agent can be taught or learn from its environment to achieve its goals.

2.2 Formal Model of Agents

This section lays out a more formal definition of agents, specifically geared towards constructed or artificial agents such as we will be constructing. An **agent** is anything that can be viewed as perceiving its environment through sensors and acting upon its environment through actuators. A **precept** is a perceptual input to an agent, and a **precept sequence** is the complete history of everything the agent has ever perceived. The agent then has an **agent function** that maps these precepts to an action. In the case of an artificial agent this is done with an **agent program** that is the physical implementation of the agent function. In the case of an agent simulated on a computer, this will be the computer code that decides what action to take. The agents actions will then cause the environment to go through a series of states and the agent will measure the outcome with some **performance measure**.

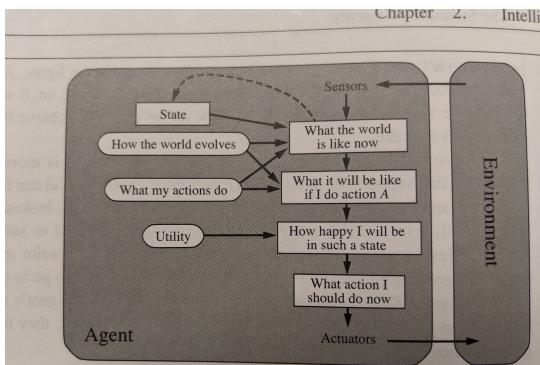


Figure 2.14 A model-based, utility-based agent. It uses a model of the world, utility function that measures its preferences among states of the world. Then it chooses an action based on the utility of the resulting states.

Figure 2.1: Utility Based Agent

For our purposes we focus on what are called **utility based agents**, where the agent has a utility function as an internalized performance measure and is able to numerically assign values of "goodness" to the outcome from the agents perspective. A rational agent will always try to maximize average expected utility over its lifetime as explained by decision theory.

2.2.1 Task Environment

The environment that an agent exists in and their relationship to it is critical for understanding the system as a whole. For example the environment can fully observable with the agent's sensors able to see the entire environment, or only partially observable to the agent. There can also be multiple agents within the system, each trying to maximize their own performance measure. This is called a **multi-agent system**

and it will be a key aspect of all of the simulations we construct. It is important to note that the agents in the multi-agent system share no internal state and must communicate (if they so wish to do so) through the environment. The environment can also be deterministic or stochastic, either depending entirely upon its current state and the agents actions or contains randomness.

2.2.2 Agents Who Learn

Learning is a critical part of being a rational agent. We expect that as agents interact with their environment, they should be able to adapt and change in order to continue to maximize their utility. The general model of an artificial agent who learns is as follows.

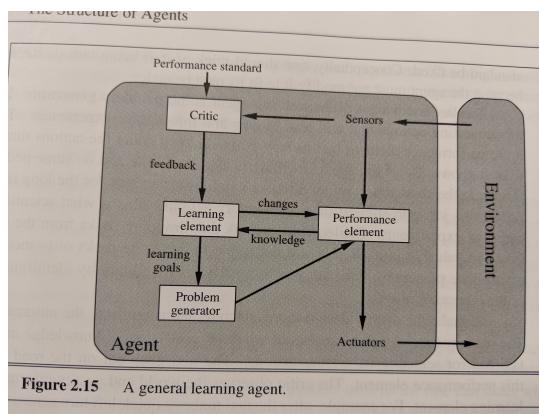


Figure 2.2: Model of Agent that Learns

First, the agent reprieves its environment and sends that information to the performance function that determines what it will do. There is now a critic that gives feedback for how the agent is doing based on aprori information and the observed state that sends feedback to a learning element that can change the behavior of the performance element. Based on what the learning element is trying improve, there is also a problem generator that suggests actions to the performance element so that different actions can be tested and evaluated. Russel & Norvig (2010)

Chapter 3

Reinforcement Learning

Reinforcement learning has become a popular way of teaching agents how to play games and interact with each other in Machine Learning (ML). Because the dynamics and behavior of a final simulation are determined by the learning algorithm used for its composing agents, we spend some time developing the ideas of reinforcement learning in both a single agent and multi-agent setting.

3.1 Single Agent Learning

Reinforcement learning is the area of machine learning where agents learn through repeated interactions with their dynamic environment. This section gives a brief overview of learning algorithms used by single agents with a focus on deep reinforcement learning. In general, reinforcement learning formalizes the interaction of agents and environment using a Markov decision process (MDP).

- Definition:** Markov Decision Process
- A Markov decision process is 5-tuple $\langle \mathcal{S}, \mathcal{A}, R, T, \gamma \rangle$ where,
1. \mathcal{S} represents a finite set of states
 2. \mathcal{A} represents a finite set of actions
 3. T is a transition function $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$ that determines the probability of transitioning from any state $s \in \mathcal{S}$ to any state $s' \in \mathcal{S}$ given any possible action $a \in A$
 4. R is the reward function $R : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow \mathbb{R}$
 5. $\gamma \in [0, 1]$ is the discount factor that balances immediate and future rewards

3.1.1 Q-Learning

Q-Learning is an algorithm that has been developed for single-agent, fully observable environments with discrete actions. Q learning involves creating a table of expected payoffs for each action a available at a state s denoted $\hat{Q}(s, a)$. Each time the agent transitions from a state s to a state s' taking an action a and receiving

payoff r , the Q table is updated according to the following assignment:

$$\hat{Q}(s, a) \leftarrow \hat{Q}(s, a) + \alpha[(r + \gamma \max_{a^1} \hat{Q}(s', a')) - \hat{Q}(s, a)]$$

where we pick $\alpha \in [0, 1]$ which we call the learning rate.

3.1.2 Deep Reinforcement Learning

Using deep learning in the form of Neural Networks can be helpful for exploring large policy spaces efficiently. It also eliminates the need to hand design features to be optimized for the reinforcement learning process. Shoham & Leyton-Brown (2008)

Chapter 4

Tables and Graphics

4.1 Tables

The following section contains examples of tables, most of which have been commented out for brevity. (They will show up in the .tex document in red, but not at all in the .pdf). For more help in constructing a table (or anything else in this document), please see the LaTeX pages on the CUS site.

Table 4.1: Correlation of Inheritance Factors between Parents and Child

Factors	Correlation between Parents & Child	Inherited
Education	-0.49	Yes
Socio-Economic Status	0.28	Slight
Income	0.08	No
Family Size	0.19	Slight
Occupational Prestige	0.21	Slight

If you want to make a table that is longer than a page, you will want to use the `longtable` environment. Uncomment the table below to see an example, or see our online documentation.

Table 4.2: Chromium Hexacarbonyl Data Collected in 1998–1999

Chromium Hexacarbonyl			
State	Laser wavelength	Buffer gas	Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$
$z^7P_4^o$	266 nm	Argon	1.5
$z^7P_2^o$	355 nm	Argon	0.57
$y^7P_3^o$	266 nm	Argon	1
$y^7P_3^o$	355 nm	Argon	0.14
$y^7P_2^o$	355 nm	Argon	0.14
$z^5P_3^o$	266 nm	Argon	1.2
$z^5P_3^o$	355 nm	Argon	0.04
$z^5P_3^o$	355 nm	Helium	0.02
$z^5P_2^o$	355 nm	Argon	0.07
$z^5P_1^o$	355 nm	Argon	0.05
$y^5P_3^o$	355 nm	Argon	0.05, 0.4
$y^5P_3^o$	355 nm	Helium	0.25
$z^5F_4^o$	266 nm	Argon	1.4
$z^5F_4^o$	355 nm	Argon	0.29
$z^5F_4^o$	355 nm	Helium	1.02
$z^5D_4^o$	355 nm	Argon	0.3
$z^5D_4^o$	355 nm	Helium	0.65
$y^5H_7^o$	266 nm	Argon	0.17
$y^5H_7^o$	355 nm	Argon	0.13
$y^5H_7^o$	355 nm	Helium	0.11
a^5D_3	266 nm	Argon	0.71
a^5D_2	266 nm	Argon	0.77
a^5D_2	355 nm	Argon	0.63
a^3D_3	355 nm	Argon	0.05
a^5S_2	266 nm	Argon	2
a^5S_2	355 nm	Argon	1.5
a^5G_6	355 nm	Argon	0.91
a^3G_4	355 nm	Argon	0.08
e^7D_5	355 nm	Helium	3.5
e^7D_3	355 nm	Helium	3
f^7D_5	355 nm	Helium	0.25
f^7D_5	355 nm	Argon	0.25
f^7D_4	355 nm	Argon	0.2
f^7D_4	355 nm	Helium	0.3
Propyl-ACT			

State	Laser wavelength	Buffer gas	Ratio of $\frac{\text{Intensity at vapor pressure}}{\text{Intensity at 240 Torr}}$
$z^7P_4^o$	355 nm	Argon	1.5
$z^7P_3^o$	355 nm	Argon	1.5
$z^7P_2^o$	355 nm	Argon	1.25
$z^7F_5^o$	355 nm	Argon	2.85
$y^7P_4^o$	355 nm	Argon	0.07
$y^7P_3^o$	355 nm	Argon	0.06
$z^5P_3^o$	355 nm	Argon	0.12
$z^5P_2^o$	355 nm	Argon	0.13
$z^5P_1^o$	355 nm	Argon	0.14
Methyl-ACT			
$z^7P_4^o$	355 nm	Argon	1.6, 2.5
$z^7P_4^o$	355 nm	Helium	3
$z^7P_4^o$	266 nm	Argon	1.33
$z^7P_3^o$	355 nm	Argon	1.5
$z^7P_2^o$	355 nm	Argon	1.25, 1.3
$z^7F_5^o$	355 nm	Argon	3
$y^7P_4^o$	355 nm	Argon	0.07, 0.08
$y^7P_4^o$	355 nm	Helium	0.2
$y^7P_3^o$	266 nm	Argon	1.22
$y^7P_3^o$	355 nm	Argon	0.08
$y^7P_2^o$	355 nm	Argon	0.1
$z^5P_3^o$	266 nm	Argon	0.67
$z^5P_3^o$	355 nm	Argon	0.08, 0.17
$z^5P_3^o$	355 nm	Helium	0.12
$z^5P_2^o$	355 nm	Argon	0.13
$z^5P_1^o$	355 nm	Argon	0.09
$y^5H_7^o$	355 nm	Argon	0.06, 0.05
a^5D_3	266 nm	Argon	2.5
a^5D_2	266 nm	Argon	1.9
a^5D_2	355 nm	Argon	1.17
a^5S_2	266 nm	Argon	2.3
a^5S_2	355 nm	Argon	1.11
a^5G_6	355 nm	Argon	1.6
e^7D_5	355 nm	Argon	1

4.2 Figures

If your thesis has a lot of figures, L^AT_EX might behave better for you than that other word processor. One thing that may be annoying is the way it handles “floats” like tables and figures. L^AT_EX will try to find the best place to put your object based on the text around it and until you’re really, truly done writing you should just leave it where it lies. There are some optional arguments to the figure and table environments

to specify where you want it to appear; see the comments in the first figure.

If you need a graphic or tabular material to be part of the text, you can just put it inline. If you need it to appear in the list of figures or tables, it should be placed in the floating environment.

To get a figure from StatView, JMP, SPSS or other statistics program into a figure, you can print to pdf or save the image as a jpg or png. Precisely how you will do this depends on the program: you may need to copy-paste figures into Photoshop or other graphic program, then save in the appropriate format.

Below we have put a few examples of figures. For more help using graphics and the float environment, see our online documentation.

And this is how you add a figure with a graphic:

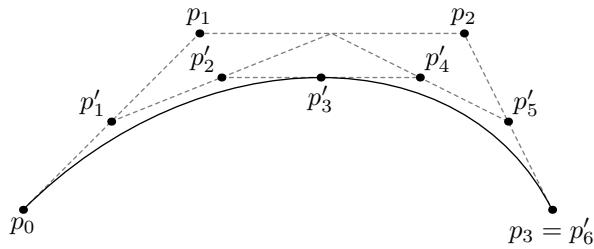


Figure 4.1: A Figure

4.3 More Figure Stuff

You can also scale and rotate figures.

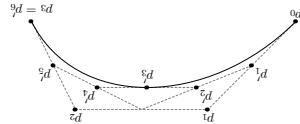


Figure 4.2: A Smaller Figure, Flipped Upside Down

4.4 Even More Figure Stuff

With some clever work you can crop a figure, which is handy if (for instance) your EPS or PDF is a little graphic on a whole sheet of paper. The viewport arguments are the lower-left and upper-right coordinates for the area you want to crop.

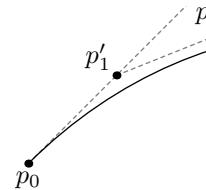


Figure 4.3: A Cropped Figure

4.4.1 Common Modifications

The following figure features the more popular changes thesis students want to their figures. This information is also on the web at web.reed.edu/cis/help/latex/graphics.html.

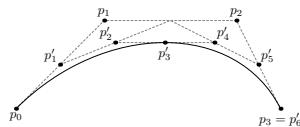


Figure 4.4: Subdivision of arc segments. You can see that $p_3 = p'_6$.

Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in L^AT_EX does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things L^AT_EX would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't* be indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

Appendix A

The First Appendix

Appendix B

The Second Appendix, for Fun

References

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