# README

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## 1 GC equation

It is shown in appendix A of [2], the canonical equation of $\boldsymbol{X}$ in flux coordinate is consistent with Morozov-Solovev-Boozer formula,

$$\frac{d\boldsymbol{X}}{dt} = \frac{\rho_\parallel \nabla \times (\boldsymbol{A} + \rho_\parallel \boldsymbol{B})}{1 + \rho_\parallel \boldsymbol{B} \cdot (\nabla \times \boldsymbol{B})/B^2} \tag{1}$$

Besides that, the equation of $\rho_\parallel$ is also consistent with the one used in XGC[1].

The equations used in this module is listed as below:

$$\frac{d\boldsymbol{r}}{dt} = \frac{1}{D}\left(\frac{q\rho_\parallel}{m}\boldsymbol{B} + \frac{q\rho_\parallel^2}{m}\nabla \times \boldsymbol{B} + \boldsymbol{B} \times \nabla H/B^2\right) \tag{2}$$

$$\frac{d\rho_\parallel}{dt} = -\frac{1}{B^2 D}\left(\nabla H \cdot \boldsymbol{B} + \rho_\parallel \nabla H \cdot \nabla \times \boldsymbol{B}\right) \tag{3}$$

in which $\rho_\parallel = \frac{mv_\parallel}{qB}, H = \frac{q}{2m}\rho_\parallel^2 B^2 + \frac{\mu B}{q} + \Phi, \nabla H = \frac{q}{m}\rho_\parallel^2 B \nabla B + \mu \nabla B/q + \nabla\Phi, D = 1 + \rho_\parallel \boldsymbol{B} \cdot \nabla \times \boldsymbol{B}/B^2 = 1 + \rho_\parallel \boldsymbol{b} \cdot \nabla \times \boldsymbol{b}$.

## 2 normalization

$v_{\rm th} = \sqrt{2eT_i/m_p}$,$\bar{L}_\rho = \frac{m_p v_{\rm th}}{e\bar{B}}$,$\epsilon_L \equiv \frac{\bar{L}_\rho}{\bar{L}_B}$,$\bar{T}_\omega = \frac{m_p}{e\bar{B}}$,$\bar{T} = \bar{L}_B/v_{\rm th} = \bar{T}_\omega/\epsilon_L$,$\bar{H} = \frac{m_p v_{\rm th}^2}{2e} = T_i$,$\bar{\mu} = \frac{m_p v_{\rm th}^2}{2\bar{B}} = \frac{eT_i}{\bar{B}}$,$\bar{\phi} = T_i$,$\frac{d}{dt} = \frac{1}{\bar{T}}\frac{d}{d\hat{t}}$,$\nabla = \frac{1}{\bar{L}_B}\hat{\nabla}$

$m = m_p\hat{m}$,$q = e\hat{q}$,$B = \bar{B}\hat{B}$,$\boldsymbol{B} = \bar{B}\hat{\boldsymbol{B}}$,$v_\parallel = v_{\rm th}\hat{v}_\parallel$,$v_\perp = v_{\rm th}\hat{v}_\perp$,$\rho_\parallel = \bar{L}_\rho\hat{\rho}_\parallel$,$\boldsymbol{r} = \bar{L}_B\hat{\boldsymbol{r}}$,$t = \bar{T}\hat{t}$,$H = \bar{H}\hat{H}$,$\mu = \bar{\mu}\hat{\mu}$,$\phi = \bar{\phi}\hat{\phi}$

$R = \hat{R}\bar{L}_B, Z = \hat{Z}\bar{L}_B, h_\theta = \hat{h}_\theta\bar{L}_B, B_\theta = \bar{B}\hat{B}_\theta, B_\zeta = \bar{B}\hat{B}_\zeta$

$$\nabla H = \frac{\bar{H}}{\bar{L}_B}\hat{\nabla}\hat{H} = \frac{\bar{H}}{\bar{L}_B}\left(\frac{2\hat{q}\hat{\rho}_\parallel^2}{\hat{m}}\hat{B}\hat{\nabla}\hat{B} + \frac{\hat{\mu}}{\hat{q}}\hat{\nabla}\hat{B} + \hat{\nabla}\hat{\Phi}\right) \tag{4}$$

$$D = 1 + \epsilon_L \hat{\rho}_\| \boldsymbol{b} \cdot \hat{\nabla} \times \boldsymbol{b} \tag{5}$$

$$\frac{d\boldsymbol{r}}{dt} = \frac{\bar{L}_B d\hat{\boldsymbol{r}}}{\bar{T} d\hat{t}} = v_{\mathrm{th}} \frac{d\hat{\boldsymbol{r}}}{d\hat{t}}$$

$$\begin{aligned}
\frac{1}{D}\left(\frac{q\rho_\|}{m}\boldsymbol{B} + \frac{q\rho_\|^2}{m}\nabla\times\boldsymbol{B} + \boldsymbol{B}\times\nabla H/B^2\right) &= \frac{1}{D}\left(\frac{e\bar{L}_\rho\bar{B}}{m_p}\frac{\hat{q}\hat{\rho}_\|}{\hat{m}}\hat{\boldsymbol{B}} + \frac{e\bar{L}_\rho^2\bar{B}}{m_p\bar{L}_B}\hat{\nabla}\times\hat{\boldsymbol{B}} + \frac{\bar{H}}{\bar{B}\bar{L}_B}\hat{\boldsymbol{B}}\times\hat{\nabla}\hat{H}/\hat{B}^2\right) \\
&= \frac{v_{\mathrm{th}}}{D}\left(\frac{\hat{q}\hat{\rho}_\|}{\hat{m}}\hat{\boldsymbol{B}} + \epsilon_L\hat{\nabla}\times\hat{\boldsymbol{B}} + \frac{\epsilon_L}{2}\hat{\boldsymbol{B}}\times\hat{\nabla}\hat{H}/\hat{B}^2\right)
\end{aligned}$$

$$\frac{d\hat{\boldsymbol{r}}}{d\hat{t}} = \frac{1}{D}\left(\frac{\hat{q}\hat{\rho}_\|}{\hat{m}}\hat{\boldsymbol{B}} + \epsilon_L\hat{\nabla}\times\hat{\boldsymbol{B}} + \frac{\epsilon_L}{2}\hat{\boldsymbol{B}}\times\hat{\nabla}\hat{H}/\hat{B}^2\right) \tag{6}$$

$$\frac{d\rho_\|}{dt} = \frac{\bar{L}_\rho}{\bar{T}}\frac{d\hat{\rho}_\|}{d\hat{t}} = \epsilon_L v_{\mathrm{th}}\frac{d\hat{\rho}_\|}{d\hat{t}}$$

$$\begin{aligned}
-\frac{1}{B^2 D}\left(\nabla H\cdot\boldsymbol{B} + \rho_\|\nabla H\cdot\nabla\times\boldsymbol{B}\right) &= -\frac{\bar{H}}{\bar{B}\bar{L}_B}\frac{1}{\hat{B}^2 D}\left\{\hat{\nabla}\hat{H}\cdot\left[\hat{\boldsymbol{B}} + \epsilon_L\hat{\rho}_\|\hat{\nabla}\times\hat{\boldsymbol{B}}\right]\right\} \\
&= -\frac{v_{\mathrm{th}}}{2}\frac{1}{\hat{B}^2 D}\hat{\nabla}\hat{H}\cdot\left(\hat{\boldsymbol{B}} + \epsilon_L\hat{\rho}_\|\hat{\nabla}\times\hat{\boldsymbol{B}}\right)
\end{aligned}$$

$$\frac{d\hat{\rho}_\|}{d\hat{t}} = -\frac{1}{2\hat{B}^2 D}\hat{\nabla}\hat{H}\cdot\left(\hat{\boldsymbol{B}} + \epsilon_L\hat{\rho}_\|\hat{\nabla}\times\hat{\boldsymbol{B}}\right) \tag{7}$$

Since BOUT++ uses field aligned coordinates $(x, y, z)$, the components of (6) is expressed like this

$$\frac{d\hat{\boldsymbol{r}}}{d\hat{t}} = \frac{d\hat{\psi}}{d\hat{t}}\hat{\boldsymbol{e}}_x + \frac{d\theta}{d\hat{t}}\hat{\boldsymbol{e}}_y + \frac{dz}{d\hat{t}}\hat{\boldsymbol{e}}_z$$

And $\hat{\psi}$ is normalized to $\hat{\psi}_{new} = \frac{\hat{\psi}-\hat{\psi}_a}{\hat{\psi}_b-\hat{\psi}_a}$ further in the code, so

$$\frac{d\hat{\psi}_{new}}{d\hat{t}} = \frac{1}{\hat{\psi}_b - \hat{\psi}_a}\frac{d\hat{\psi}}{d\hat{t}}$$

# 3 output and plot of the results

This module works only in circular geometry and runs on single processor now. The results is manipulated manually. For example, the output is stored in file bout_hopper_debug.pbs.e6956740, then the 1001 lines results between line 170 and 1170 is fetched to file e6956740 and plotted using IDL.

```
examples/gc> vi bout_hopper_debug.pbs.e6956740
examples/gc>head -n 1170 bout_hopper_debug.pbs.e6956740 | tail -n 1001 >
e6956740
examples/gc> idl
IDL> plotgc,'e6956740',1001,0,1001
```

# References

[1] C S Chang, Seunghoe Ku, and H Weitzner. Numerical study of neoclassical plasma pedestal in a tokamak geometry. *Physics of Plasmas*, 11(5):2649, 2004.

[2] Shaojie Wang. Canonical Hamiltonian theory of the guiding-center motion in an axisymmetric torus, with the different time scales well separated. *Physics of Plasmas*, 13(5):052506, 2006.