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Theory of dissipative density-gradient-driven turbulence in the tokamak edge

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A theory of resistive, density-gradient-driven turbulence is presented and compared with tokamak edge fluctuation measurements. In addition to linear driving, the theory accounts for relaxation of the density gradient through a nonlinear process associated with emission from localized density fluctuation elements. From a fluid model for isothermal electrons in toroidal geometry, equations are obtained and solved analytically, retaining both coherent and incoherent contributions. The effect of collisions on the density blobs is treated. A Reynolds number parameterizes the magnitude of the turbulent scattering relative to the collisional viscous diffusion. The analytic results indicate that the spectrum is characterized by linewidths which increase as a function of the Reynolds number and may reach $\Delta\omega/\omega \gtrsim 1$. Energy lies predominantly in the small wavenumbers ($k_\perp \rho_s \sim 0.1$). For larger wavenumbers and frequency, the spectrum decays as $k^{-17/6}$ and ω^{-2}. The fluctuation level scales as $1/k_\perp L_n$ and may reach $-$ 30% for parameters typical of the pretext edge. Particle diffusion is Bohm-like in magnitude but does not follow Bohm scaling, going instead as $n^{2/3}T_e^{1/6}$. The density fluctuations exhibit nonadiabatic character caused by the incoherent mode coupling. An expression for the departure from adiabaticity is given.

I. INTRODUCTION

Recently, the nature of turbulent fluctuations in tokamaks has received increased attention.[1] The edge region, in particular, has been a focus of interest. Measurements of particle loss in the edge region and comparison with total particle loss suggest that edge fluctuations play an important role in determining global particle confinement.[2] Further stimulus for investigation of edge turbulence has been provided by a need to understand edge phenomena in order to effectively design limiters and divertors. Discovery of the H mode,[3] with its improved confinement time, and the correlation of the transition to H mode operation with edge-related processes such as divertor operation underscore the need to understand edge turbulence and its effect on confinement.

Furthermore, since the edge region is accessible to probes as well as scattering diagnostics, a significant amount of information is being obtained on the nature of edge turbulence.[1,2,4–6] Measurements show edge turbulence to be characterized by very large fluctuations having $\delta n/n \sim 10\%$–50%, in contrast to the center where levels of 2% are typical. Both wavenumber and frequency spectra are broad. Strong nonadiabatic behavior of density fluctuations is observed ($\delta n/n \neq e\varphi/T$). Typical poloidal wavenumbers and frequencies are $k_\vartheta \rho_s \sim 1/10$ and $\omega \sim \omega_{*e}$, respectively. Both wavenumber and frequency spectra are broad, and $\Delta\omega_k$, the frequency linewidth at fixed k_ϑ, is comparable to the frequency ω.

Recent measurements of edge plasma parameters in Ohmically heated tokamaks typically indicate that $\eta_e < 1[\eta_e = L_n/L_t = (n/T)(dT/dn)]$, and that the electron mean free path $x_{\rm mfp}$ is short ($\lambda_{\rm mfp} < Rq$). Indeed, even for the hot, low collisionality plasma characteristic of the H mode discharges,[3] edge proximity to the divertor separatrix (where $q \to \infty$) implies that $\lambda_{\rm mfp} < Rq$. Hence, a fluid model of density-gradient-driven turbulence is one possible simple paradigm for edge fluctuation dynamics. However, the observed large density fluctuation level and broad frequency linewidth suggest that a nonlinear density gradient relaxation process, rather than a linear instability such as the collisional or dissipative drift mode, drives the edge turbulence.

In this paper, a nonlinear theory of resistive density-gradient-driven turbulence is presented. In this theory, the basic constituents of the steady turbulent state are *broadened collective resonances*, rather than waves or eigenmodes (collective resonances of zero width).[7] The collective resonances are broadened (driven) by emission from localized density fluctuation elements (blobs) produced by gradient relaxation and destroyed by relative $\tilde{E} \times B_0$ convection (perpendicular shear stress) and parallel collisional diffusion. The density blobs resemble eddies in a turbulent fluid rather than perturbations associated with linear waves. These localized density blobs broaden collective resonances by behaving as a source of noise emission, which must be balanced by damping (and thus broadening) of collective resonances in a steady state. Hence, the stationary turbulent state is viewed as a "soup" comprised of waves and eddy-like blobs.

In contrast to one-point theories involving only waves, here the wavenumber and frequency spectrum is treated as the basic quantity to be computed. Accordingly, closure methods are applied directly to the density two-point correlation function equation. Thus, both coherent and incoherent density fluctuations are accounted for.[7,8] In particular, incoherent density fluctuations are identified as the density blobs which act as the nonlinear noise source broadening the collective resonances. Furthermore, since the localized, eddy-like density blobs are fundamentally non-wave-like in character, it is not surprising that their mechanism for extraction of expansion free energy from the average density

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gradient differs from that of waves. This extraction can occur at a rate which exceeds that given by quasilinear relaxation estimates, thus enhancing the (effective) growth rate in comparison to those of linear theory. This enhanced growth rate is a consequence of the fact that the coupling of potential fluctuations to electrons and ions are related in two ways. First, quasineutrality requires that an electron blob convecting down the density gradient be accompanied by an ion density fluctuation. Thus, electron expansion free energy relaxation is directly related to a fluctuation-ion coupling. Second, a steady oversaturated state requires nonlinear ion dissipation to balance growth and nonlinear noise emission. Thus ion heating, certainly a consequence of fluctuation-ion coupling, is necessary for stationary turbulence. These two constraints on electron and ion coupling to potential fluctuations result in the enhanced growth.

At this point it is useful to discuss energy flow in wavenumber space occurring in this model. In particular, there are two different processes simultaneously at work. Density blob size is determined by the competition of relative $\hat{E} \times B_0$ convection and parallel diffusion with gradient relaxation. Hence, blobs are torn apart by nonlinear shear stresses, thus generating smaller scales. These small scales are then dissipated by (parallel) viscosity. In this respect, blob dynamics resemble cascades in conventional fluid turbulence. However, it should be noted that the most significant collective resonance broadening is caused by emission from the large-scale blobs. The second process is wave (collective resonance) scattering to longer, damped wavelengths. This is basically a modified weak turbulence process. However, since the effective Reynolds number is always less than unity, it provides a sufficient energy sink in the problem. Furthermore, the sink is most effective where blob emission is largest, thus allowing a steady state.

Two regimes of turbulence are treated in this theory. For frequency linewidth $\Delta\omega$ on the order of or less than ω, the Reynolds number is less than unity, where Reynolds number is the parameter indicating the relative strength of $\hat{E} \times B_0$ convection to parallel collisional diffusion. In this regime of small to moderate Reynolds numbers, weak turbulence modified to include the incoherent spectrum constituent as outlined above is applicable and quantitative analytic solutions and predictions are possible. For Reynolds numbers in excess of unity, quantitative results are very difficult to obtain, though qualitative insights are possible through strong turbulence arguments. The discussion is divided into subsections dealing with these two regimes. While the quantitative predictions are obtained in the moderate turbulence regime ($\Delta\omega \lesssim \omega_*$), the strong turbulence analysis shows that scalings for fluctuation level and transport and asymptotic behavior of the spectrum carry over into the strong ($\Delta\omega > \omega_*$) turbulence regime. The principal difference between the regimes is one of magnitude of linewidths and fluctuation level.

The principle results are now summarized:

(i) The effect of nonlinear noise on gradient relaxation rates has been quantitatively calculated. Substantial growth rate enhancement relative to the growth rate of the linear dissipative drift wave instability is shown to occur.

(ii) Analytical expressions for the frequency and wavenumber spectrum are obtained. These expressions indicate that:

(a) energy is predominantly in the small wavenumbers, $k_\perp \rho_s \lesssim 0.1$,

(b) for larger $k_\perp$ and ω, the spectrum decays as $k^{-17/6}$ and ω^{-2}. These are in excellent agreement with experiment.[4,5]

(c) Here $\Delta\omega_k$, the frequency linewidth at fixed k, is found to be broad, and $\Delta\omega_k \sim \omega$. An explicit expression for $\Delta\omega_k$ is obtained.

(iii) Qualitatively, the fluctuation level $e\varphi/T_e$ scales as $e\varphi/T_e \sim 1/k_\perp L_n$, and is typically of the order of 30% for parameters consistent with the PRETEXT edge.

(iv) The *magnitude* of the predicted particle diffusion is of the order of Bohm diffusion. However, the diffusion coefficient *scales* as $D \sim n^{2/3} T_e^{1/6}$, quite unlike Bohm scaling.

(v) The predicted incoherent mode coupling and density fluctuations offer a possible explanation of the strong nonadiabatic character of density fluctuations observed in experiments. These predictions are in qualitative[2,4,5] agreement with recent experimental results.

(vi) The theory predicts the presence of density structures (blobs) with lifetimes a few times the average correlation time. Such structures have recently been observed in the CalTech tokamak.[9]

In addition to obtaining the above features, this investigation constitutes a first attempt at the application of ideas and methods of renormalized two-point theory and nonlinear relaxation to fluid plasma systems. The basic ideas utilized were originally proposed in the context of phase space density turbulence.[8] In that case, the phase space density field f is conserved along particle trajectories. In this case, the basic model is dissipative, and the effect of collisions on small-scale density correlation (parallel viscosity) must be considered. Specifically, at very small scales where parallel collisional diffusion times are sufficiently fast relative to nonlinear density element relaxation times (eddy turnover time associated with $\hat{E} \times B_0$ convections), small-scale density correlation decays with the coherent dissipative relaxation time and incoherent fluctuations are negligible. In order for frequency broadening caused by incoherent fluctuations to occur at a particular scale, the effects of the $\hat{E} \times B_0$ nonlinearity must be stronger than those of viscous diffusion. A Reynolds number parameterizes this relative strength. Quantitatively, it is shown that substantial incoherent frequency broadening occurs for moderate Reynolds numbers, i.e., for Reynolds numbers from a few tenths to order unity, up to large Reynolds numbers, i.e., greater than unity. The density correlation function exhibits Reynolds number similarity over scales where appreciable frequency broadening occurs. Observe that this is a necessary, but not sufficient, condition for the existence of an inertial range of density fluctuations. Also, the theory quantitatively links the frequency broadening process with nonadiabatic density fluctuation behavior. Finally, the theory gives a qualitatively accurate model for density blob behavior.

Recently, edge turbulence theories based on rippling modes and dissipative drift waves[10,11] have been proposed.

In the dissipative drift wave theory of Hasegawa and Wakatani, numerical solution of two-dimensional evolution equations for density and vorticity exhibits broad frequency line width, large fluctuation levels, and Bohm-like particle diffusion[11] (magnitude and scaling). Such a model is attractive because of its simplicity and because it exhibits many features observed in studies of strong plasma turbulence, including broad frequency spectra.[11–13] However, it omits the important energy sink associated with parallel ion heating and relies on (relatively small) perpendicular ion viscosity for saturation and treats the destabilization mechanism in an *ad hoc* fashion. For these reasons, the general significance of the results for frequency width, fluctuation level, and diffusion are unclear. Furthermore, several advantages are realized from the analytic treatment of the incoherent nonlinearity. Aside from obtaining analytic formulas, the basic physics involved in the line broadening process is illuminated and its impact on other observable spectrum properties is determined. As it turns out, the line broadening is related to a nonlinear instability that can strongly affect the saturation amplitude and transport.[7,14] The analytical theory elucidates the effect of this process.

The paper is organized as follows: in Sec. II we present the basic equations and review the relevant linear theory. Section III develops the analytic theory of coherent and incoherent mode coupling. Some introductory comments preceed two major subsections, one on the physics of the driving mechanism of incoherent fluctuations and the other on the evolution of small-scale correlation, which determines the solution of the two-point density equation. In Sec. IV we derive the detailed predictions of the theory for edge turbulence in tokamaks. The linewidth, wavenumber spectrum, and particle diffusion are discussed. Section V includes a summary and conclusions.

II. BASIC EQUATIONS AND LINEAR BEHAVIOR

A. Electron and ion dynamics

At the plasma edge the electron temperature is sufficiently low so that the electron–ion collision frequency exceeds the inverse transit time $\nu_{ei} > \omega_{T_e} = v_{te}/Rq$. We consider the electrons to be isothermal and hence neglect temperature gradients and fluctuations. Since $\nu_{ei} \gg \omega$, electron inertia is also neglected. Thus, the electron fluid equations are[15]

$$\frac{\partial \hat{n}}{\partial t} + n\nabla_{\parallel} v_{\parallel e} + \nabla\cdot(n\mathbf{v}_{\perp e}) = 0, \tag{1}$$

$$\frac{dv_{\parallel e}}{dt} + \nu_{ei} v_{\parallel e} = v_{T_e}^2 \nabla_{\parallel}\left(\frac{e\hat{\varphi}}{T_e} - \hat{n}\right), \tag{2}$$

where we have assumed that $v_{\parallel e} \gg v_{\parallel}$, and

$$n = n_0 + \hat{n}, \quad \mathbf{v}_{\perp e} = \mathbf{v}_{de} + \mathbf{v}_E, \tag{3}$$

where

$$\mathbf{v}_E = -(c/B_0)\nabla\varphi \times \hat{n}_0,$$

$$\mathbf{v}_{de} = -(cT_e/eB_0)(n\times\nabla \ln B).$$

We substitute Eqs. (2) and (3) into Eq. (1). Noting that

$$\nabla\cdot(nv_{\perp e}) \cong v_{de}\cdot\nabla(\hat{n} - |e|\hat{\varphi}/T_e) + \frac{cT_e}{|e|B}\frac{d}{dx}\ln n_0\nabla_{\vartheta}\left(\frac{|e|\varphi}{T_e}\right) + \mathbf{v}_E\cdot\nabla\hat{n},$$

we write Eq. (1) in terms of the nonadiabatic electron density $H_e = \hat{n} - |e|\hat{\varphi}/T_e$. Since the $E\times B$ drift does not convect the adiabatic density ($\mathbf{v}_E\cdot\nabla e\hat{\varphi}/T_e = 0$), it follows that the electron dynamics are described by

$$\frac{\partial}{\partial t}H_e - \frac{v_{T_e}^2}{\nu_{ei}}\nabla_{\parallel}^2 H_e + \mathbf{v}_{de}\cdot\nabla H_e - \frac{c}{B_0}\nabla\varphi\times\hat{n}_0\cdot\nabla\hat{H}_e = -\frac{|e|}{T_e}\left(\frac{\partial\hat{\varphi}}{\partial t} + v_D\nabla_{\vartheta}\hat{\varphi}\right), \tag{4}$$

where

$$v_D = \frac{cT_e}{eB}\frac{d}{dx}\ln n_0.$$

We consider toroidal geometry and apply the ballooning transformation:

$$\begin{Bmatrix}\hat{\varphi}\\ H_e\end{Bmatrix} = \sum_n \exp(in\phi)\sum_m \exp(-im\vartheta) \times \int d\eta\,\exp(i[m - nq(r)]\eta)\begin{Bmatrix}\hat{\varphi}_n(\eta)\\ H_n(\eta)\end{Bmatrix},$$

where ϕ is the toroidal angle and η is the variation in the direction of the magnetic field. Using this transformation and Fourier transforming in time, Eq. (4) is

$$-i(\omega - \omega_{de})H_{\substack{n\\ \omega}} - \frac{v_{T_e}^2}{\nu_{ei}(Rq)^2}\frac{\partial^2}{\partial\eta^2}H_{\substack{n\\ \omega}} + N_{\substack{n\\ \omega}} = \frac{i|e|}{T_e}(\omega - \omega_{*e})\hat{\varphi}_{\substack{n\\ \omega}}, \tag{5}$$

where

$$\omega_{de} = \frac{cT_e}{eB}\frac{k_\vartheta}{R}(\cos\eta + \hat{s}\eta\sin\eta),$$

$$\omega_{*e} = v_D k_\vartheta, \quad k_\vartheta = nq/r, \quad \hat{s} = rq'/q,$$

and

$$N_{\substack{n\\ \omega}} = \sum_{n'}\sum_m \frac{c}{B_0}k_\vartheta k'_\vartheta(2\pi m\hat{s}) \times \exp[2\pi i n' q(r)m]\,\hat{\varphi}_{\substack{-n'\\ -\omega'}}(\eta + 2\pi m)\,H_{\substack{n+n'\\ \omega+\omega'}}(\eta). \tag{6}$$

The ballooning representation provides a compact form of the $E\times B$ nonlinearity, in which the interaction is between n and n' at r along η.[7,14] Reviewing Eq. (5), the model contains a magnetic curvature drift convection in toroidal angle ϕ represented by ω_{de}, parallel viscous diffusion, $[v_{T_e}^2/(Rq)^2 \times \nu_{ei}]\partial^2/\partial\eta^2$, the $E\times B$ nonlinear mixing and the usual drift wave density gradient source term.

As previously mentioned, we consider warm, low-frequency ion dynamics. Parallel ion motion which gives ion-sound shear damping and diamagnetic drifts which introduce toroidal coupling and permit quasibounded eigenstates for the toroidicity-induced branch[15] are retained to determine the mode structure. The ions provide a sink of fluctuation energy, and hence a potential saturation mechanism, in that the combined effects of nonlinearity and parallel ion dynamics can result in (nonlinear) ion heating.

The linear theory of dissipative drift waves is discussed in Ref. 15. It is instructive to review the scalings of growth rate with respect to collisionality and to qualitatively estimate the size of the nonlinear instability for regimes relevant to the tokamak edge. The growth rate scalings may be succinctly obtained from a variational integral representation of the electron part of the linear dielectric. This alternate derivation of the linear growth rate relies on the fact that the toroidicity-induced branch is localized in local potential wells induced by finite toroidal coupling. Outward convection of wave energy is impeded by the potential barrier and occurs only through tunnelling. Shear damping is therefore negligible. The dissipative mechanism is collisional parallel diffusion of the electron density and this determines the linear growth rate. The growth rate is proportional to the imaginary part of the electron contribution to the dielectric, which is given by

$$\Delta = \int_{-\infty}^{\infty} d\eta\, \hat{\varphi}_{\underset{\omega}{n}}(\eta) H_{\underset{-\omega}{-n}}(\eta), \tag{7}$$

where $H_{\underset{-\omega}{-n}}(\eta)$ is the solution of the linearized form of Eq. (7). We note that Δ is variational in $\hat{\varphi}$, recovering the electron part of the linear eigenmode equation upon variation with $\hat{\varphi}$. Evaluation of Eq. (7) with $\hat{\varphi}$ determined from the full eigenmode equation gives the electron part of the dielectric. The imaginary part of the electron dielectric is well approximated by retaining only the dissipative mechanism associated with collisional viscosity in the propagator for $H_{\underset{-\omega}{-n}}(\eta)$:

$$\operatorname{Im}\Delta \cong \operatorname{Im}\frac{i|e|}{T_e}\left(1-\frac{\omega_{*e}}{\omega}\right)\int_{-\infty}^{\infty} d\eta\, \hat{\varphi}_{\underset{\omega}{n}}(\eta) \times\left(-i-\frac{v_{T_e}^2}{(Rq)^2\nu_{ei}\omega}\frac{\partial^2}{\partial\eta^2}\right)^{-1}\hat{\varphi}_{\underset{-\omega}{-n}}(\eta). \tag{8}$$

Expressing $\hat{\varphi}_{\underset{\omega}{n}}(\eta)$ in terms of the Fourier transformed eigenfunction

$$\hat{\varphi}_{\underset{\omega}{n}}(\eta) = \int d\kappa\, \exp(-i\kappa\eta)\hat{\varphi}_{\underset{\omega}{n}}(\kappa),$$

Δ is expressed as

$$\Delta = \frac{i|e|}{T_e}\left(1-\frac{\omega_{*e}}{\omega}\right)\int_{-\infty}^{\infty}\frac{d\kappa\left[\hat{\varphi}_{\underset{\omega}{n}}(\kappa)\right]^2}{\left[-i+v_{T_e}^2\kappa^2/(Rq)^2\nu_{ei}\omega\right]}, \tag{9}$$

where we have used the fact that $\hat{\varphi}(\kappa)$ is even in κ. For convenience we define $\alpha = v_{T_e}^2/(Rq)^2\nu_{ei}$. With the transformation $\kappa = \sqrt{\omega/\alpha}\; y$,

$$\Delta = \frac{i|e|}{T_e}\left(1-\frac{\omega_{*e}}{\omega}\right)\left(\frac{\omega}{\alpha}\right)^{1/2}\int_{-\infty}^{\infty} dy\,\frac{\left[\hat{\varphi}_{\underset{\omega}{n}}(\sqrt{\omega/\alpha}\; y)\right]^2}{(i+y^2)}, \tag{10}$$

and two limits are apparent assuming toroidicity-induced mode structure with its extent in κ expressed by $\Delta\kappa$. For $\sqrt{\omega/\alpha}(\Delta\kappa)^{-1} < 1$, the y variation of the integrand is controlled by the denominator and the mode structure has little effect on the growth rate scaling. This limit is equivalent to $k_\parallel^2 v_{T_e}^2/(\omega_{*e}\nu_{ie}) > 1$ and corresponds to an adiabatic electron regime. Evaluating Eq. (10), we obtain

$$\operatorname{Im}\Delta \cong \frac{|e|}{T_e}\left(1-\frac{\omega_{*e}}{\omega}\right)\left(\frac{\omega}{\alpha}\right)^{1/2}|\varphi(0)|^2\frac{\pi}{\sqrt{8}}, \tag{11}$$

which implies that the growth rate scales as $\nu_{ei}^{1/2}$:

$$\frac{\gamma}{\omega} \sim \left(\frac{\omega\nu_{ei}}{v_{T_e}^2/(Rq)^2}\right)^{1/2}. \tag{12}$$

We note that replacing ν_{ei} with ω recovers the proper scaling of collisionless toroidicity-induced drift waves.

The opposite limit, $\sqrt{\omega/\alpha}\,(\Delta\kappa)^{-1} > 1$, implies that $k_\parallel^2 v_{T_e}^2/\omega_{*e}\nu_{ei} < 1$ and corresponds to the hydrodynamic electron regime. In this limit the mode structure controls the y variation of the integrand of Eq. (10) yielding

$$\operatorname{Im}\Delta \cong \frac{|e|}{T_e}\left(1-\frac{\omega_{*e}}{\omega}\right)\left(\frac{\omega}{\alpha}\right)^{1/2}\int_{-\infty}^{\infty} dy\, y^2\left[\hat{\varphi}_{\underset{\omega}{n}}(\sqrt{\omega/\alpha}\; y)\right]^2 \cong \frac{|e|}{T_e}\left(1-\frac{\omega_{*e}}{\omega}\right)\left(\frac{\alpha}{\omega}\right)(\Delta\kappa)^2\frac{\sqrt{\pi}}{2}.$$

The growth rate thus scales as ν_{ei}^{-1}:

$$\gamma/\omega \sim v_{T_e}^2/(Rq)^2\nu_{ei}\omega. \tag{13}$$

Because $\omega_{*e} > k_\parallel^2 v_{T_e}^2/\nu_{ei}$ for this regime, and because incoherent emission tends to produce an uncertainty in frequency $\Delta\omega$ on the order of ω for toroidicity-induced structure,[14] the uncertainty in phase velocity induced by the nonlinear instability exceeds the parallel flow velocity, $\Delta\omega > k_\parallel^2 v_{T_e}^2/\nu_{ei}$. This is suggestive of a strong electron mode coupling process requiring a renormalized electron response. The nonlinear destabilization is large and linear structure is probably irrelevant. In the adiabatic regime, parallel flow velocity increases relative to the uncertainty in the phase velocity. The associated parallel viscosity decreases the lifetime of incoherent fluctuations, and the electron nonlinearity is effectively weakened in comparison to the hydrodynamic regime. Weak turbulence is thus possible in the adiabatic regime provided the parallel viscosity is sufficiently large. Weak and strong turbulence limits will be distinguished in the next section and related to the Reynolds number. We anticipate that in the hydrodynamic regime, magnetohydrodynamic (MHD) instabilities such as rippling modes become relevant.

III. PHYSICS OF COHERENT AND INCOHERENT MODE COUPLING

The incoherent fluctuations, which are responsible for broad frequency spectra, are produced by the mode coupling associated with the nonlinearity. The nonlinearity, as a convolution in Fourier transform space, naturally provides a component of the perturbed distribution at wavenumber k which is proportional to the potential at some other wavenumber k'. This is the incoherent component. In nonperturbative treatments of mode coupling equations, i.e., mode simulations, the incoherent mode coupling is handled numerically. Analytic treatments of mode coupling rely on renormalization techniques. These are usually applied to one-point equations for the evolution of perturbed densities and fields. Such theories are intrinsically coherent and do not retain the incoherent component. In order to treat incoherent fluctuations, it is necessary to use a two-point equation. Dupree has shown that the renormalized two-point equation has a response exhibiting distinct behavior at opposite asymptotic limits of the relative separation variable.[8] At large scales, the correlation time is identical with the eigenmode lifetime, the eigenmode lifetime being defined as the decay time of the mode at a point, as obtained from one-point renormalized theory. The closure of the one-point equation is intrinsically coherent so the eigenmode lifetime is a coherent relaxation time. Thus the two-point density correlation at large scale is coherent. At small scales, however, the correlation time exceeds the eigenmode lifetime, increasing logarithmically to infinity as the scale diminishes to zero. This strong scale dependence arises as a consequence of the spatial correlation in the turbulent field at small scale. Since the one-point theory is incapable of treating two-point correlations it naturally retains only the coherent, scale independent response. The component of two-point correlation which outlives the eigenmodes is the incoherent response. Because this possibility only occurs for small scales, the incoherent correlation is strongly peaked at the small scales and falls to zero at the scale where the two-point correlation time asymptotes to the coherent eigenmode lifetime. The incoherent density is thus grainy and the graininess has been referred to as "clumps."[8] In the steady state, when the decay of two-point correlation is offset by a driving source, the steady-state correlation is given approximately as the product of the source term with the lifetime. The coherent part of the correlation may be extracted by subtracting the coherent time from the lifetime, leaving the incoherent response.

The two-point equation is derived from the one-point equation. For incoherent fluctuations in the electron species, we multiply the one-point equation, Eq. (4), by the nonadiabatic density at a second point, we then ensemble average and symmetrize. The resulting equation is

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_{de}(1)\cdot\nabla_1 + \mathbf{v}_{de}(2)\cdot\nabla_2 - \frac{v_{T_e}^2}{\nu_{ei}}(\nabla_{\|_1}^2 + \nabla_{\|_2}^2)\right)$$
$$\times\langle H_e(1)H_e(2)\rangle - (c/B_0)\langle\nabla_1\hat{\varphi}(1)\times\hat{n}_0\cdot\nabla_1 H_e(1)H_e(2)\rangle$$
$$-(c/B_0)\langle\nabla_2\hat{\varphi}(2)\times\hat{n}_0\cdot\nabla_2 H_e(2)H_e(1)\rangle$$
$$=\left(\frac{-|e|}{T_e}\right)\left\{\left\langle H_e(2)\left(\frac{\partial\hat{\varphi}(1)}{\partial t} + V_D\nabla_{\vartheta_1}\hat{\varphi}(1)\right)\right\rangle\right.$$
$$\left.+\left\langle H_e(1)\left(\frac{\partial\hat{\varphi}(2)}{\partial t} + V_D\nabla_{\vartheta_2}\hat{\varphi}(2)\right)\right\rangle\right\}. \quad (14)$$

Employing the ballooning representation, we rewrite Eq. (14) as

$$\left[\frac{\partial}{\partial t} + V_{de}(\eta_1)\frac{q(r_1)}{r_1}\frac{\partial}{\partial\phi_1} + v_{de}(\eta_2)\frac{q(r_2)}{r_2}\frac{\partial}{\partial\phi_2}\right.$$
$$\left.-\frac{v_{T_e}^2}{(Rq)^2\nu_{ei}}\left(\frac{\partial^2}{\partial\eta_1^2} + \frac{\partial^2}{\partial\eta_2^2}\right)\right]\langle H(\eta_1,\phi_1)H(\eta_2,\phi_2)\rangle$$
$$+\sum_{\substack{n\\ \omega}}\sum_{\substack{n'\\ \omega'}}\sum_{\substack{n''\\ \omega''}}\Big\langle\exp(in\phi_1)\exp(in'\phi_1)\exp(in''\phi_2)$$
$$\times\frac{c}{B_0}\sum_{m'}k_\vartheta k'_\vartheta\hat{s}(2\pi m')$$
$$\times\exp(-i2\pi n'qm')\hat{\varphi}_{\substack{n'\\ \omega'}}(\eta_1 + 2\pi m')H_{\substack{n''\\ \omega''}}(\eta_2)\Big\rangle$$
$$=\Bigg(\sum_{\substack{n'\\ \omega'}}\frac{i|e|}{T_e}\exp[in'(\phi_1-\phi_2)]$$
$$\times(\omega' - \omega'_{*e})\langle H(\eta_2)\hat{\varphi}(\eta_1)\rangle_{\substack{n'\\ \omega'}} + (1\leftrightarrow 2)\Bigg), \quad (15)$$

where ϕ is the toroidal angle.[7]

Equation (15) possesses the necessary properties for the formation of density granulations. The turbulent $E\times B$ mixing goes to zero as the relative spatial separation of the two points diminishes to zero. The right-hand side of Eq. (15) describes the rearrangement of the mean density by the turbulent potential, and in contrast to the $E\times B$ mixing, remains finite as the relative separation vanishes. Correlations of all scales are driven by this source. Turbulent mixing quickly destroys all but small-scale correlations on the time scale of the correlation time. These, too, decay with time, but for sufficiently small scale they persist beyond the lifetime of the turbulent fields and are turbulently scattered as macroscopic fluid elements. We note then that it is the spatial variation of the turbulent mixing process which gives rise to the strong scale dependence of the two-point correlation time. Dissipative parallel viscosity is included in this problem. Unlike the turbulent mixing, it produces a diffusion which stays finite as the relative separation goes to zero. This scale-independent decay mechanism reduces small-scale correlation.

A. Free energy relaxation

The right-hand side of Eq. (15), or source as it is commonly called, plays a fundamental role in determining the incoherent spectrum. The source term is proportional to the rate of relaxation of mean density, and hence the expansion free energy provides the driving energy for incoherent fluctuations. The growth of incoherent fluctuations under this nonlinear instability induces finite amplitudes in the eigenmodes because they shield the granulations. For eigenmodes already at finite amplitude because of linear instability, incoherent fluctuations excite the modes to higher levels through the shielding response. The damping of these overdriven modes necessary to obtain a saturated state is the width $\Delta\omega_k$ of the frequency spectrum. A spectrum balance relation which describes the role of these processes in the steady state is obtained from the solution of Eq. (15) when the source term is expressed solely in terms of the incoherent part of the two-point correlation. Rewriting the source from Eq. (15),

$$S = \sum_{n',\omega'}\frac{i|e|}{T_e}\exp[in'(\phi_1-\phi_2)](\omega' - \omega'_{*e})\langle H(\eta_2)\varphi(\eta_1)\rangle_{\substack{n'\\ \omega'}}, \quad (16)$$

we express $H(\eta_2)$ as a decomposition into its coherent component $H^c(\eta_2)$ and its incoherent component $\tilde{H}(\eta_2)$:

$$S = \sum \frac{i|e|}{T_e} \exp[in'(\phi_1 - \phi_2)](\omega' - \omega'_{*e}) \times \Big[\langle H^c(\eta_2) \hat{\varphi}(\eta_1) \rangle_{\substack{n' \\ \omega'}} + \langle \tilde{H}(\eta_2) \varphi(n_1) \rangle_{\substack{n' \\ \omega'}} \Big]. \tag{17}$$

Because the coherent density is phase coherent with the potential, it may, in general, be written in terms of response function

$$H^{(c)}_{\substack{n \\ \omega}}(\eta_2) = R_{\substack{n \\ \omega}} \frac{|e|\hat{\varphi}_n}{T_e}. \tag{18}$$

In the present calculation, the response function includes both linear and nonlinear contributions. To make contact with the present model, we write the linear part of the response which is given by the formal operator

$$R^L_{\substack{n \\ \omega}} = \frac{(\omega_{*e} - \omega)}{[\omega - (v^2_{T_e}/(Rq)^2 i\nu_{ei})(\partial^2/\partial\eta^2) - \omega_{de}]}.$$

The function $\varphi_{\substack{n \\ \omega}}$ is the potential fluctuation, determined from the quasineutrality condition

$$n^I - n^E = L^{\text{ion}}_{\substack{n \\ \omega}} \frac{e\hat{\varphi}_{\substack{n \\ \omega}}}{T_e} - \frac{e\hat{\varphi}_{\substack{n \\ \omega}}}{T_e} \frac{R_{\substack{n \\ \omega}} e\hat{\varphi}_{\substack{n \\ \omega}}}{T_e} - \tilde{H} = 0, \tag{19}$$

where $L^{\text{ion}}_{\substack{n \\ \omega}}$ is the ion response function. The coherent part of the quasineutrality condition specifies the eigenfunction operator $L_{\substack{n \\ \omega'}}$,

$$L^{\text{ion}}_{\substack{n \\ \omega}} \frac{e\hat{\varphi}_{\substack{n \\ \omega}}}{T_e} - \frac{e}{T_e}\hat{\varphi}_{\substack{n \\ \omega}} - R_{\substack{n \\ \omega}} \frac{e\hat{\varphi}_{\substack{n \\ \omega}}}{T_e} \equiv L_{\substack{n \\ \omega}} \frac{e\hat{\varphi}_{\substack{n \\ \omega}}}{T_e}, \tag{20}$$

with the full quasineutrality condition expressing the shielding of the incoherent fluctuations by the eigenmodes,

$$L_{\substack{n \\ \omega}}(\eta)(e\hat{\varphi}_{\substack{n \\ \omega}}/T_e) = \tilde{H}(\eta). \tag{21}$$

This last relationship allows us to write the shielding potential in terms of the shielded clumps, so that Eq. (17) becomes

$$S = \sum_{\substack{n' \\ \omega'}} \frac{i|e|}{T_e} \exp[in'(\phi_1 - \phi_2)](\omega' - \omega'_{*e}) \times \Big[(T_e/|e|) R_{\substack{-n' \\ -\omega'}}(\eta_2) L^{-1}_{\substack{-n' \\ -\omega'}}(\eta_2) L^{-1}_{\substack{n' \\ \omega'}}(\eta_1) \langle \tilde{H}(\eta_2)\tilde{H}(\eta_1) \rangle_{\substack{n' \\ \omega'}} + (T_e/|e|) L^{-1}_{\substack{n' \\ \omega'}}(\eta_1) \langle \tilde{H}(\eta_2)\tilde{H}(\eta_1) \rangle_{\substack{n' \\ \omega'}} \Big]. \tag{22}$$

We multiply the last term by the unit operator

$$L_{\substack{-n' \\ -\omega'}}(\eta_2) L^{-1}_{\substack{-n' \\ -\omega'}}(\eta_2) = \Big(L^{\text{ion}}_{\substack{-n' \\ -\omega'}} - 1 - R_{\substack{-n' \\ -\omega'}} \Big) L^{-1}_{\substack{-n' \\ -\omega'}}(\eta_2)$$

and note that the nonadiabatic electron response completely cancels out of the expression. Accordingly the source becomes

$$S = \sum_{\substack{n' \\ \omega'}} i(\omega' - \omega'_{*e})(L^{\text{ion}}_{\substack{-n' \\ -\omega'}} - 1) L^{-1}_{\substack{-n' \\ -\omega'}}(\eta_2) L^{-1}_{\substack{n' \\ \omega'}}(\eta_1) \times \langle \tilde{H}(\eta_2)\tilde{H}(\eta_1) \rangle_{\substack{n' \\ \omega'}}.$$

Since S is real,

$$S = \sum_{\substack{n' \\ \omega'}} (\omega' - \omega'_{*e})(L^{\text{ion}}_{\substack{n' \\ \omega'}}) \mathrm{Im}\, L^{-1}_{\substack{-n' \\ -\omega'}} L^{-1}_{\substack{n' \\ \omega'}} \langle \tilde{H}(\eta_2)\tilde{H}(\eta_1) \rangle_{\substack{n' \\ \omega'}}, \tag{23}$$

where

$$L^{\text{ion}}_{\substack{-n' \\ -\omega'}} = \mathrm{Re}(L^{\text{ion}}_{\substack{n' \\ \omega'}}) - i\, \mathrm{Im}(L^{\text{ion}}_{\substack{n' \\ \omega'}}).$$

The scaling of the source term for electron small-scale correlation with ion dissipation is a familiar feature of clump phenomena. This derivation clearly shows the scaling to be a result of quasineutrality, or equivalently, ambipolarity. In particular, since the electron density, $H(\eta) = R\, e\varphi/T_e + \tilde{H}$, is shielded by the neutralizing ion density $(L^{\text{ion}})|e|\varphi/T_e$, the relaxation of the electron density distribution function (by scattering of electron clumps down the density gradient) is constrained by the quasineutrality condition, which requires that the ions respond to neutralize the scattered electron clump. This constraint underlies the result that electron relaxation is proportional to ion dissipation. Note the eigenmode frequency is less than ω_{*e} and when the shielding distribution is energy absorbing $[\mathrm{Im}(L^{\text{ion}}) < 0]$, a source is available which permits free energy extraction from the equilibrium gradient through an electron clump channel. The energy goes from the gradient to electron small-scale correlation, to the modes via coupling, and finally to the ions which damp the modes through the collective resonance.

The quasineutral shielding process is most transparent in the fluid description where the quasineutrality condition, Eq. (19), allows for a direct replacement in Eq. (17) of $H^{(c)} + \tilde{H}$ by $L^{\text{ion}} - 1$. This holds rigorously for the general nonlinear response R_n. In the kinetic description, where the density is distributed on a velocity continuum as well as in configuration space, the analysis is more complicated. There the ballistic nature of clumps selects a particle velocity at the wave–particle resonance. The nonlinear response, however, broadens the resonance and the replacement of $H^{(c)} + \tilde{H}$ by L^{ion} at the ballistic velocity is not in general exact.

This free energy extraction mechanism is a significant channel for the relaxation of expansion free energy, particularly when the modes are linearly unstable from electron dissipation. In the steady state the ion damping is necessarily large and negative in order to balance the linear growth and incoherent excitation. There is thus an enhanced source for small-scale correlation.

We complete the source calculation by writing the inverse eigenfunction operator $L^{-1}_{\substack{n \\ \omega}}(\eta)$ in terms of the Green's function $\hat{\varphi}_{\substack{n \\ \omega}}(\eta)$ and dielectric response $\epsilon(n,\omega)$.

$$L^{-1}_{\substack{n \\ \omega}}(\eta) F(\eta) = \frac{1}{\epsilon(n,\omega)} \hat{\varphi}_{\substack{n \\ \omega}}(\eta) \int d\eta'\, \varphi_{\substack{n \\ \omega}}(\eta') F(\eta'). \tag{24}$$

The source is then given by

$$S = \sum_{\substack{n' \\ \omega'}} (\omega' - \omega'_{*e}) \frac{\epsilon^{\text{ion}}_{\text{Im}}(n',\omega')}{|\epsilon(n',\omega')|^2} |\hat{\varphi}_{\substack{-n' \\ -\omega'}}(\eta_2)\hat{\varphi}_{\substack{n' \\ \omega'}}(\eta_1)| \overline{\langle \tilde{H}^2 \rangle}_{\substack{n' \\ \omega'}}, \tag{25}$$

where

$$\langle \tilde{H}^2 \rangle_{\substack{n' \\ \omega'}} = \int d\eta_1 \int d\eta_2 \, \hat{\varphi}_{\substack{n' \\ \omega'}}(\eta_1) \hat{\varphi}_{\substack{-n' \\ -\omega'}}(\eta_2) \langle \tilde{H}(\eta_2) \tilde{H}(\eta_1) \rangle_{\substack{n' \\ \omega'}} \tag{26}$$

and $\epsilon_{\rm Im}^{\rm ion}(n',\omega')$ is the imaginary part of the ion contribution to the shielding dielectron response.

B. Evolution of small-scale correlation

In this section the solution of the two-point equation, Eq. (15), is presented. Here $\langle H(1)H(2)\rangle$ evolves according to

$$\left[\frac{\partial}{\partial t} + V_{de}(\eta_1) \frac{q(r_1)}{r_1} \frac{\partial}{\partial \phi_1} + V_{de}(\eta_2) \frac{q(r_2)}{r_2} \frac{\partial}{\partial \phi_2} - \frac{V_{T_e}^2}{(Rq)^2} \frac{1}{\nu_{ei}} \left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2} \right) \right] \langle H(1)H(2) \rangle + T_{12} = S, \tag{27}$$

where S is given by Eq. (25) and T_{12}, the triplet $E \times B$ nonlinearity is

$$T_{12} = \sum_{\substack{n \\ \omega}} \sum_{\substack{n' \\ \omega'}} \sum_{\substack{n'' \\ \omega''}} \Big\langle \exp(in\phi_1) \exp(in'\phi_1) \exp(in''\phi_2) \frac{c}{B_0} \times \sum_{m'} k_\vartheta k'_\vartheta \hat{S}(2\pi m') \exp(-i2\pi n' q m') \times \hat{\varphi}_{\substack{n' \\ \omega'}}(\eta_1 + 2\pi m') H_{\substack{n \\ \omega}}(\eta_1) H_{\substack{n'' \\ \omega''}}(\eta_2) \Big\rangle.$$

It is necessary to renormalize Eq. (27) by finding a closure for the triplet correlation in the nonlinearity. We use standard weak coupling methods associated with the direct interaction approximation. Writing the toroidal angle ϕ in terms of relative angle $\phi_- = 1/2(\phi_1 - \phi_2)$ and composite angle $\phi_+ = \frac{1}{2}(\phi_1 + \phi_2)$, the ensemble average is accomplished by integrating over ϕ_+, a Kronnecker delta $\delta_{n+n'+n'',0}$ being the result. The wave–wave coupling contribution arising from the induced field $\hat{\varphi}_{n+n'}$ is neglected, so that

$$\begin{aligned} T_{12} = \frac{c}{B_0} \sum_{\substack{n \\ \omega}} \sum_{\substack{n' \\ \omega'}} \Big\{ & \exp(in\phi_-) \sum_{m'} k_\vartheta k'_\vartheta \hat{s}(2\pi m') \exp(2\pi i n' q m') \\ & \times \Big\langle \hat{\varphi}_{\substack{-n' \\ -\omega'}}(\eta_1 + 2\pi m') H_{\substack{n+n' \\ \omega+\omega'}}(1) H_{\substack{-n \\ -\omega}}(2) \Big\rangle \\ & - \exp(in\phi_- + in'\phi_-) \\ & \times \sum_{m'} k_\vartheta k'_\vartheta \hat{s}(2\pi m') \exp(2\pi i n' q m') \\ & \times \Big\langle \hat{\varphi}_{\substack{-n \\ -\omega}}(\eta_1 + 2\pi m') H_{\substack{-n' \\ -\omega'}}(1) H_{\substack{n+n' \\ \omega+\omega'}}(2) \Big\rangle \Big\}. \end{aligned} \tag{28}$$

To obtain driven density $H_{n+n'}$, we solve Eq. (5) to second order, writing $n + n'$ for n,

$$\begin{aligned} H_{n+n'} = -L_{n+n'}^{-1} \frac{c}{B_0} \sum_{n''} \sum_{m'} k''_\vartheta (k_\vartheta + k'_\vartheta) \hat{s}(2\pi m') \\ \times \exp(2\pi i n'' q m') \hat{\varphi}_{-n''}(\eta + 2\pi m') H_{n+n'+n''}(\eta), \end{aligned} \tag{29}$$

where

$$L_{n+n'} = \omega + \omega' - \frac{v_{T_e}^2 (\partial^2/\partial \eta^2)}{i(Rq)^2 \nu_{ei}} - \widetilde{\omega}_{de}(n + n'). \tag{30}$$

From the sum over n'' in Eq. (29) the directly interacting triplet $(n'' = -n')$ is selected for substitution into the first term in Eq. (28) and $(n'' = -n)$ for substitution into the second term. We neglect the renormalization of the potential from the choice $(n'' = -n)$ for the first term and $(n'' = -n')$ for the second. Under these approximations the triplet is

$$\begin{aligned} T_{12} = \frac{-c^2}{B_0^2} \sum_n \sum_{n'} L_{\substack{n+n' \\ \omega+\omega'}}(1) \sum_m (2\pi m)^2 k_\vartheta^2 k'^2_\vartheta \\ \times \hat{s}^2 (2\pi m)^2 \Big[\exp(in\phi_-) \langle \hat{\varphi}^2(1) \rangle_{\substack{n' \\ \omega'}} \langle H(1)H(2) \rangle_{\substack{n \\ \omega}} \\ + \exp[i(n+n')\phi_-] \\ \times \exp(2\pi i m r_-/\Delta) \langle \hat{\varphi}(1)\hat{\varphi}(2) \rangle_{\substack{n' \\ \omega'}} \langle H(1)H(2) \rangle_{\substack{n \\ \omega}} \Big\}, \end{aligned} \tag{31}$$

where $\Delta = 1/k\hat{s}$. One further approximation, a Markovian approximation, will be made so that $L_{n'}$ will replace $L_{n+n'}$. The results, obtained from the solution of the two-point equation, Eq. (27), are not sensitive to the details of the renormalization. Hence, the simple closure scheme used above is adequate. This is because the renormalization affects the decay of small-scale correlation but not the driving. It has been argued that the net effect of approximations in the renormalization is to produce a change in the two-point correlation time of at most a factor of 3 (Ref. 16). Because the most important neglected processes pertain to self-binding, omitted in this closure, the actual correlation time is longer than the approximate correlation time, so that nonlinear growth rates, fluctuation levels, and transport coefficients obtained from this theory are lower bounds.

The renormalized triplet, Eq. (31), approximates the $\mathrm{E} \times \mathrm{B}$ mixing process as a diffusion in toroidal angle. The critical property, in so far as small-scale correlated triplet is concerned, is preserved in the approximated triplet, i.e., the diffusion vanishes as the relative separation goes to zero. In the coordinates of the relative separation, (r_-, ϕ_-, η_-), we may write T_{12} as

$$T_{12} = D_- \frac{\partial^2}{\partial y_-^2}, \tag{32}$$

where

$$D_- = 2D - D^{(1,2)} - D^{(2,1)}, \tag{33}$$

$$D = \frac{c^2}{B_0^2} \sum_{\substack{k' \\ \omega'}} k'^2_\vartheta \hat{s}^2 \operatorname{Re}\Big(L^{-1}_{\substack{-k' \\ -\omega'}} \Big) \sum_m (2\pi m)^2 \langle \hat{\varphi}(\eta + 2\pi m)^2 \rangle_{\substack{k' \\ \omega'}}, \tag{34}$$

and

$$\begin{aligned} D^{(1,2)} = D^{(2,1)} = \frac{c^2}{B_0^2} \sum_{\substack{k' \\ \omega'}} \exp(ik'y_-) k'^2_\vartheta \hat{s}^2 \operatorname{Re}\Big(L^{-1}_{\substack{k' \\ \omega'}} \Big) \\ \times \sum_m (2\pi m)^2 \exp(2\pi i m r_- k' \hat{s}) \\ \times \langle \hat{\varphi}(\eta_1 + 2\pi m) \varphi(\eta_2 + 2\pi m) \rangle_{\substack{k' \\ \omega'}}, \end{aligned} \tag{35}$$

and $y_- = r\phi_-/q$, and $k = nq/r$. The relative diffusion D_- is seen to consist of two parts, an independent diffusion D and correlated diffusion $D^{(1,2)}, D^{(2,1)}$. It is readily ascertainable that as $(r_-, y_-, \eta_-) \to 0$, $D^{(1,2)} \to D$, and D_- vanishes accordingly. In the opposite limit, as the separation becomes large, $D^{(1,2)} \to 0$. The relative diffusion is thus independent at large separation and accounts for the fact that turbulent correlation time asymptotes to the coherent correlation time or eigenmode lifetime $\tau_{E\times B} = (k^2 D)^{-1}$. Here $D^{(1,2)}$ and $D^{(2,1)}$ incorporate the small-scale correlation and are responsible for a correlation time τ_{cl} which exceeds τ_c on the scale for which $D^{(1,2)}$ differs from zero. The scale for $D^{(1,2)} \neq 0$ defines the clump scale. For a spectrum which is Gaussian,

$$D_- \cong 2D\left[1 - \cos(k_0' y_-)\exp(-y_-^2/2\alpha^2) \times \exp(-\eta_-^2/\Delta\eta^2)\exp(-r_-^2 \hat{s}^2/2\alpha^2)\right], \quad (36)$$

where k_0' corresponds to most probable wavenumber and α^{-1} corresponds to the wavenumber spread in the k spectrum and is a measure of the root mean square wavenumber. At small scale, D_- is quadratic in the relative variables

$$D_- \sim 2D(y_-^2 k_0^2 + \eta_-^2/\Delta\eta^2 + r_-^2 k_0^2 \hat{s}^2), \quad (37)$$

where $k_0^{-1}\left[=(k_0'^2 + 1/2\alpha^2)^{-1/2}\right]$, $\Delta\eta$ and $k_0^{-1}\hat{s}^{-1}$ are the clump scales in y_-, η_-, and r_-, respectively.

Having renormalized the triplet nonlinearity, we substitute Eq. (32) into Eq. (27) and arrive at the renormalized two-point equation,

$$\left(\frac{\partial}{\partial t} + v_{de}'(\eta_+)\eta_- \frac{\partial}{\partial y_-} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2} - D_- \frac{\partial^2}{\partial y_-^2}\right) \times \langle H(1)H(2)\rangle = S, \quad (38)$$

where $D_\parallel = v_{T_e}^2/\nu_{ei}$. The diamagnetic drift term and viscous diffusion have been expressed in terms of coordinates η_+, η_-, assuming variation in η_- about a fixed η_+. The prime denotes differentiation with respect to η_+. Equation (38) states that the electron two-point correlation evolves by the balance of drive by average density relaxation (source) with decay by relative $E\times B$ diffusion ($E\times B$ shear stress), relative magnetic drift, and collisional diffusion.

We now consider the problem of solving Eq. (38). A solution may be obtained from the Green's function satisfying the homogeneous equation

$$\left(\frac{\partial}{\partial t} + v_{de}'(\eta_+)\eta_- \frac{\partial}{\partial y_-} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2} - D_- \frac{\partial^2}{\partial y_-^2}\right) \times g(\eta_-, y_-, r_-, t\,|\,\eta_-', y_-', r_-', t') = 0, \quad (39)$$

and the condition

$$g(\eta_-, y_-, r_-, t\,|\,\eta_-', y_-', r_-', t) = \delta(\eta_- - \eta_-')\delta(y_- - y_-')\delta(r_- - r_-') \quad (40)$$

$$\langle H(1)H(2)\rangle = \int d\eta_-'\, dy_-'\, dr_-'\, dt' \times g(\eta_-, y_-, r_-, t\,|\,\eta_-', y_-', r_-', t') \times S(\eta_-', y_-', r_-'). \quad (41)$$

It is desirable to define a Reynolds number parameter to distinguish regimes where either linear behavior (parallel viscosity) or nonlinear behavior (turbulent diffusion) dominates. Therefore let

$$R_e \equiv \frac{Dk^2 (Rq)^2 \Delta\eta_c^2}{D_\parallel} = \frac{\tau_\parallel}{\tau_{E\times B}}, \quad (42)$$

where D is the independent diffusion, Eq. (34), and $\Delta\eta_c$ is the parallel correlation length. As noted, the Reynolds number is the ratio of the parallel correlation time (viscous diffusion) to the coherent nonlinear relaxation rate. In the high Reynolds number regime the full coherent (one–point) relaxation rate given by

$$\tau_c = \left(\frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2} + D\frac{\partial^2}{\partial y_-^2}\right)^{-1} \quad (43)$$

is dominated by turbulent $E\times B$ diffusion in toroidal angle so that $\tau_c \sim \tau_{E\times B} = (k_0^2 D)^{-1}$. The decay of two-point correlation is dominated by the scale dependent relative diffusion D_-. Consequently, the density granulation process is important and a large incoherent component of the density is anticipated. In this nonlinear regime, the separation of neighboring density elements is exponential, giving rise to a two-point correlation time τ_{cl} which is bigger than the coherent correlation time τ_c by a factor which goes as the log of one over the relative separation. At the smallest scales, however, τ_{cl} no longer diverges exponentially as it does in the collisionless case. The logarithmic divergence is cut off at the scale where $D_\parallel/(Rq)^2\Delta y_c^2 \sim D_- k_0^2$. In the low Reynolds number regime the coherent relaxation rate is controlled by the linear parallel viscous diffusion so that $\tau_c \sim \tau_\parallel$. Decay of two-point correlation is also dominated by the viscous diffusion. However, inside the clump scale it still holds that $D_\parallel^2/(Rq)^2\Delta\eta_c^2 + D_- k_0^2 < D_\parallel/(Rq)^2\Delta\eta_c^2 + Dk_0^2$ so that $\tau_{cl} > \tau_c$. Because the linear parallel viscosity predominates a power law governs the separation of neighboring density elements and τ_{cl} no longer exhibits a logarithmic scale dependence. In this regime we will adopt a low Reynolds number perturbation treatment in order to solve the two-point equation and ultimately $\langle \tilde{H}^2\rangle$ will be proportional to $\tau_\parallel/\tau_{E\times B}$.

In general, an exact solution of the Green's function [Eq. (39)], or equivalently the two-point correlation, given the scale dependent diffusion, is very difficult. Previous papers have instead computed moments of the Green's function.[7,8,14,17] Defined as

$$\langle y_-^2\rangle = \int dy_-'\, d\eta_-'\, dr_-'\, dt'\, y_-'^2 \times g(y_-, \eta_-, r_-, t\,|\,y_-', \eta_-', r_-', t'), \quad (44)$$

the y_-^2 moment gives the time evolution of the separation of two trajectories in toroidal angle. If the two-point correlation decays principally by toroidal diffusion, then the correlation time or clump lifetime τ_{cl} is given by the time it takes for two density elements, initially separated by $(y_-, \eta_-, r_-) < (k_0^{-1}, \Delta\eta, k_0^{-1}\hat{s}^{-1})$ to diverge to the clump scale. The clump lifetime enters the solution of the two-point equation as follows. From Eq. (38) the steady-state solution of the two-point equation is formally

$$\langle H^2\rangle = \left(v_{de}'\eta_- \frac{\partial}{\partial y_-} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2} - D_- \frac{\partial^2}{\partial y_-^2}\right)^{-1} S,$$

or approximately

$$\langle H^2\rangle \approx \tau_{cl} S. \quad (45)$$

A differential equation for $\langle y_-^2\rangle$ is obtained by taking the partial derivative with respect to time of Eq. (44). Equation (39),

$$\frac{\partial}{\partial t} g = -\left(v'_{de}\eta_- \frac{\partial}{\partial y_-} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2} - D_- \frac{\partial^2}{\partial y_-^2}\right) g,$$

is used to replace the time derivative with the spatial operator, and partial integrations are used to transfer the operator from g to y_-^2. The resulting moment equations are

$$\frac{\partial}{\partial t}\langle y_-^2 \rangle = 2k_0^2 D\left(\langle y_-^2\rangle + \frac{\langle \eta_-^2\rangle}{k_0^2 \Delta\eta_c^2}\right) + 2v'_{de}\langle y_- \eta_-\rangle, \tag{46}$$

$$\frac{\partial}{\partial t}\langle \eta_- y_-\rangle = v'_{de}\langle \eta_-^2\rangle, \tag{47}$$

$$\frac{\partial}{\partial t}\langle \eta_-^2\rangle = \frac{2D_\parallel}{(Rq)^2}. \tag{48}$$

Combining Eqs. (46) through (48) yields the equivalent equation

$$\frac{\partial^3}{\partial t^3}\langle y_-^2\rangle = 2k_0^2 D \frac{\partial^2}{\partial t^2}\langle y_-^2\rangle + \frac{4v'_{de}D_\parallel}{(Rq)^2}. \tag{49}$$

Equation (49) is integrated using the initial conditions

$$\frac{\partial}{\partial t}\langle y_-^2\rangle\Big|_{t=0} = 2k_0^2 D\left(y_-^2 + \frac{\eta_-^2}{k_0^2\Delta\eta_c^2}\right) + 2v'_{de}\eta_- y_-, \tag{50a}$$

$$\frac{\partial^3}{\partial t^3}\langle y_-^2\rangle\Big|_{t=0} = 2k_0^2 \frac{\partial^2}{\partial t^2}\langle y_-^2\rangle\Big|_{t=0} + \frac{4v'_{de}D_\parallel}{(Rq)^2}, \tag{50b}$$

$$\frac{\partial^2}{\partial t^2}\langle y_-^2\rangle\Big|_{t=0} = 2k_0^2 D\frac{\partial}{\partial t}\langle y_-^2\rangle\Big|_{t=0} + \frac{4D\,D_\parallel}{\Delta\eta_c^2(Rq)^2} + 2v'^2_{de}\eta_-^2. \tag{50c}$$

The resulting evolution equation for $\langle y_-^2\rangle$ is

$$\begin{aligned}\langle y_-^2\rangle = {} & \left(y_-^2 + \frac{\eta_-^2}{k_0^2\Delta\eta_c^2} + \frac{v'_{de}\eta_- y_-}{k_0^2 D} + \frac{D_\parallel}{k_0^4 D\Delta\eta_c^2(Rq)^2} + \frac{v'^2_{de}\eta_-^2}{2k_0^4 D^2} + \frac{v'^2_{de}D_\parallel}{2k_0^6 D^3(Rq)^2}\right)\exp(2k_0^2 Dt) \\ & - \frac{v'^2_{de}D_\parallel}{(Rq)^2 k_0^2 D}t^2 - \left(\frac{v^2_{de}D_\parallel}{k_0^4 D^2(Rq)^2} + \frac{v'^2_{de}\eta_-^2}{k_0^2 D} + \frac{2D_\parallel}{k_0^2\Delta\eta_c^2(Rq)^2}\right)t \\ & - \left(\frac{\eta_-^2}{k_0^2\Delta\eta_c^2} + \frac{v'_{de}\eta_- y_-}{k_0^2 D} + \frac{D_\parallel}{k_0^4\Delta\eta_c^2(Rq)^2} + \frac{v'^2_{de}\eta_-^2}{2k_0^4 D^2} + \frac{v'^3_{de}D_\parallel}{2k_0^6 D^3(Rq)^2}\right).\end{aligned} \tag{51}$$

Then $\langle \eta_-^2\rangle$ evolution, obtained from Eq. (48) is

$$\langle \eta_-^2\rangle = \eta_-^2 + [2D_\parallel/(Rq)^2]t. \tag{52}$$

From the inversion of Eqs. (51) and (52) we obtain decay rates corresponding to the diffusion in the toroidal and parallel directions, respectively. The decay rate τ_{cl} of the two-point correlation is determined from these two processes. In the high Reynolds number limit the diffusion is predominantly toroidal and τ_{cl} is well approximated by the toroidal decay time. In the low Reynolds number regime, however, though diffusion is predominantly parallel, it is not sufficient to approximate τ_{cl} with the parallel diffusion time. Incoherent fluctuations arise from scale dependence in the *toroidal* diffusion and are therefore a higher-order effect. The zero-order approximation $\tau_{cl} \approx \tau_\parallel$ thus includes no incoherent effects (clumps). In order for τ_{cl} to correctly represent small-scale correlation, it is necessary to include both toroidal and poloidal evolution in the dissipation range ($R_e < 1$).

With these facts in mind we examine the toroidal and parallel relaxation times by inverting Eqs. (51) and (52). Equation (51), owing to its transcendental nature, may not be inverted analytically for arbitrary Reynolds number. Approximations for the toroidal relaxation time in the high and low Reynolds number limit may be obtained as follows. For high Reynolds numbers $\tau_{E\times B} = k_0^{-2}D^{-1} < \tau_\parallel$. Within the clump scale ($y_-^2 < k_0^{-2}$, $\eta_-^2 < \Delta\eta_c^2$), $\tau_{cl} > \tau_{E\times B}$, and the exponential dependence dominates the power law dependence in Eq. (51). Inverting the exponential we obtain

$$\tau_y \approx \tau_{cl} = \tau_{E\times B}\ln\left\{\left[k_0^2\left(y_-^2 + \frac{\eta_-^2}{k_0^2\Delta\eta_c^2} + \frac{v'_{de}\eta_- y_-}{k_0^2 D} + \frac{v'^2_{de}\eta_-^2}{2k_0^4 D^2} + \frac{D_\parallel}{k_0^4 D\Delta\eta_c^2(Rq)^2} + \frac{v'^2_{de}D_\parallel}{2k_0^6 D^3(Rq)^2}\right)\right]^{-1}\right\}. \tag{53}$$

The last two terms are proportional to R_e^{-1} and are small for y_-, η_- near the clump scale. Inside the clump scale, the denominator decreases and τ_{cl} increases logarithmically until $y_- k_0$, $\eta_-/\Delta\eta_c \sim R_e^{-1}$. The $D_\parallel/D$ terms cut off the singularity which occurs in the collisionless theory and we have $\tau_{cl}\sim\ln(R_e)$ for y_-, $\eta_- \to 0$. For $R_e\to\infty$ the collisionless results are recovered. We also note the similarity of Eq. (53) with the kinetic result.[7,10,14] Here the absence of v_- terms confirms the existence of density granulations in the fluid theory.

In the low Reynolds number regime $\tau_{E\times B} \gg \tau_\parallel$, $\tau_y \gg \tau_c \sim \tau_\parallel$, implying that $\tau_y/\tau_{E\times B} < 1$. Therefore, the toroidal evolution in the low Reynolds number regime is approximated by expanding the exponential in Eq. (51) for small argument. Neglecting the diamagnetic drift terms yields a quadratic equation in t whose lowest-order solution is

$$\tau_y = (\tau_\parallel \tau_{E\times B}/2)^{1/2}(1 - k_0^2 y_-^2)^{1/2}. \tag{54}$$

The toroidal variation of the toroidal decay rate in both Reynolds number regimes is summarized in Fig. 1. It is in-

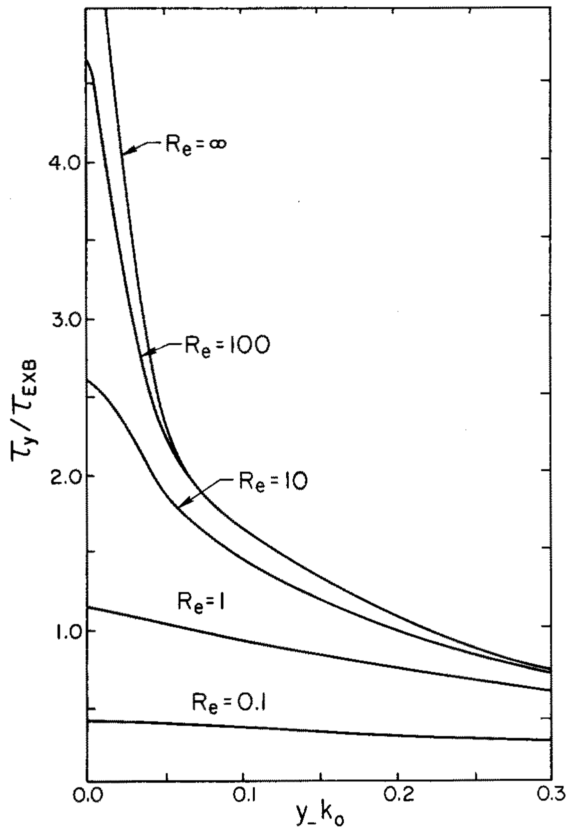

FIG. 1. Toroidal correlation time as a function of relative separation in the toroidal angle for several values of the Reynolds number.

teresting to note the Reynolds number similarity exhibited in Fig. 1 for the inertial range. The two-point correlation is equal for large Reynolds numbers with departures occuring on the scale where collisional viscosity becomes important. This scale decreases for increasing Reynolds number. These are standard features of inertial flows (compare with Stewart and Townsend[18]). In the inertial range ($R_e > 1$) where the clump lifetime τ_{cl} is given by the toroidal decay time, we may proceed as in previous theories.[7,10,14] Writing $\langle \tilde{H}^2 \rangle = (\tau_{cl} - \tau_c)S$, we shall obtain in part B of Sec. V the spectrum balance equation, from which the nonlinear growth rate, spectrum width, fluctuation level, and transport coefficients are derived using qualitative strong turbulence approximations.

In the dissipation range neither τ_y or $\tau_\parallel$ correctly approximate τ_{cl}. The appropriate combination of these rates to yield τ_{cl} is not obvious and the approximate solution $\langle \tilde{H}^2 \rangle = (\tau_{cl} - \tau_c)S$ is not of use. In this regime it is most advantageous to directly solve Eq. (38) using a perturbation theory for small Reynolds numbers. Part A of Sec. IV treats the $R_e < 1$ perturbation solution of Eq. (38), and a quantitative description of the spectrum in terms of wavenumber and frequency dependence and associated transport is presented. These results are valid for small ($R_e \ll 1$) and moderate ($R_e \lesssim 1$) Reynolds numbers and indicative of spectrum features at Reynolds numbers somewhat above unity. In part B, the large Reynolds number case is considered using Eqs. (45) and (53).

IV. THE SPECTRUM OF TOKAMAK EDGE TURBULENCE

A. Moderate Reynolds number regime

1. The spectrum balance equation

With the neglect of the small diamagnetic drift term and using Eq. (36) for D_-, we seek the steady-state solution of

$$\left\{\frac{\partial}{\partial t} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2} - D\left[1 - \cos(k_0 y_-)\exp\left(\frac{-y_-^2}{2\alpha^2}\right)\times\exp\left(\frac{-\eta_-^2}{\Delta\eta^2}\right)\right]\frac{\partial^2}{\partial y_-^2}\right\} H(y_-,\eta_-,t) = S(\eta_- y_-), \tag{55}$$

where $H(y_-,\eta_-,t) = \langle \tilde{H}(y_1,\eta_1,t)\tilde{H}(y_2,\eta_2,t)\rangle$. We undertake a peturbation solution for small Reynolds numbers with

$$H(y_-,\eta_-,t) = H^{(0)} + H^{(1)} + \cdots,$$

where $H^{(0)}$ is the solution of

$$\left(\frac{\partial}{\partial t} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2}\right) H^{(0)} = S, \tag{56}$$

and

$$\left(\frac{\partial}{\partial t} - \frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial \eta_-^2}\right) H^{(1)} = D\left[1 - \cos(k_0 y_-)\exp\left(\frac{-y_-^2}{2\alpha^2}\right)\exp\left(\frac{-\eta_-^2}{\Delta\eta^2}\right)\right] \times \frac{\partial^2}{\partial y_-^2} H^{(0)} \tag{57}$$

is the first-order equation for $H^{(1)}$. The zero-order solution is

$$H^{(0)}(y_-,\eta_-,t) = \int d\eta'_- \frac{1}{[\pi D_\parallel t/(Rq)^2]^{1/2}} \times \exp\left(\frac{-(\eta_- - \eta'_-)^2}{4D_\parallel t/(Rq)^2}\right) u(\eta'_-, y_-) - \frac{(Rq)^2}{D_\parallel}\int^{\eta_-} d\eta'_- \int^{\eta'_-} d\eta''\, S(\eta''_-, y_-), \tag{58}$$

where the first term represents a decaying transient and the second term is the steady-state part driven by $S(\eta_-, y_-)$. We are concerned with the steady state and therefore ignore the transients. The first-order solution is then

$$H^{(1)}(y_-,\eta_-,t) = \frac{(Rq)^4}{D_\parallel^2} D \int^{\eta_-} d\eta'_- \times \int^{\eta'_-} d\eta''_- \left[1 - \cos(k_0 y_-) \times \exp\left(\frac{-y_-^2}{2\alpha^2}\right)\exp\left(\frac{-\eta''^2_-}{\Delta\eta^2}\right)\right] \times \frac{\partial^2}{\partial y_-^2}\int^{\eta''_-} d\eta_3 \int^{\eta_3} d\eta_4\, S(\eta_4, y_-). \tag{59}$$

To obtain the incoherent two-point correlation $\tilde{H}(y_-, \eta_-, t) = \langle \tilde{H}(1)\tilde{H}(2)\rangle$ we subtract out the coherent correlation. The zero-order correlation is coherent; there is no incoherent correlation in that order. The first-order result is obtained by subtracting D from D_-:

$$\tilde{H}(y_-, \eta_-) = \frac{-(Rq)^4}{D_\parallel^2} D \int^{\eta_-} d\eta' \times \int^{\eta'} d\eta'' \cos(k_0 y) \exp\left(\frac{-y_-^2}{2\alpha^2}\right) \times \exp\left(\frac{-\eta''^2}{\Delta\eta^2}\right) \frac{\partial^2}{\partial y_-^2} \int^{\eta''} d\eta_3 \int^{\eta_3} d\eta_4 S(\eta_4, y_-). \tag{60}$$

We Fourier transform the source term and substitute from Eq. (25),

$$S(\eta_4, y_-) = \int d\kappa \sum_k \exp(i\kappa\eta_4) \exp(iky_-) S(\kappa, k), \tag{61}$$

$$S(\kappa, k) = \int d\omega (\omega - \omega_{*e}) \epsilon_{\mathrm{Im}}^{\mathrm{ion}}(k,\omega) \frac{|\hat{\varphi}_{\substack{k \\ \omega}}(\kappa_1) \hat{\varphi}_{\substack{-k \\ -\omega}}(\kappa_2)|}{|\epsilon(k,\omega)|^2} \overline{\langle \tilde{H}^2 \rangle}_{\substack{k \\ \omega}}. \tag{62}$$

Finally, we Fourier transform the left-hand side of Eq. (60) and obtain the two-point, Fourier-transformed incoherent spectrum:

$$\langle \tilde{H}^2(\kappa_- k_+) \rangle_k = \frac{(Rq)^4}{D_\parallel^2} D (2)^{1/2} \pi\alpha\Delta\eta \int dk' \left(\frac{\delta\eta^2}{4} + \frac{1}{\Delta_{k'}^2}\right)^{1/2} \frac{k'^2}{\kappa_-^2} \exp\left(\frac{-\kappa_-^2 \Delta\eta^2}{4}\right) \{\exp[-(k-k'-k_0)^2\alpha^2/2] + \exp[-(k-k'+k_0)^2\alpha^2/2]\} \int d\omega'(\omega' - \omega'_{*e}) \epsilon^{\mathrm{ion}}_{\mathrm{Im}\,\substack{k' \\ \omega'}}(\kappa_+) \left[|\hat{\varphi}^2(\eta_+)|_{\substack{k' \\ \omega'}} / |\epsilon(k',\omega')|^2\right] \overline{\langle \tilde{H}\tilde{H} \rangle}_{\substack{k' \\ \omega'}}, \tag{63}$$

where $\Delta_{k'}^{-1}$ is the shielding eigenmode width in the parallel direction. Several integrations have been performed to arrive at Eq. (63). Details may be found in the Appendix.

This result will be further simplified in the next section when we obtain from it the spectrum balance equation. We note at this time an important difference between the two-point correlation for fluids and the two-point phase space density correlation of kinetic theory. We observe that fluid incoherent fluctuations or density granulation are dependent on the coherent fluctuation level. This dependence enters through the decay process, not the source, and reflects the mixing of density granulations by the turbulent potential. A decay rate dependence of $D/D_\parallel^2$ for $R_e < 1$ and $1/D$ for $R_e > 1$ indicates the tendency of incoherent fluctuations to be smaller when the dominant decay mechanism is stronger. While a $1/D$ dependence also occurs in the decay rate for phase space density (kinetic) granulations, the width in velocity of the granulation is given by the trapping velocity and is also proportional to D. In the process of integrating over velocity to obtain the incoherent density correlation the amplitude dependence is thus lost. Consequently, the spectrum balance, which describes the steady-state balance between the shielding eigenmodes and driven incoherent fluctuations according to Poisson's equation, carries no explicit amplitude dependence in the kinetic case, while it does in the fluid case.

We turn now to a consideration of the spectrum balance and address the properties of the steady state turbulence in the presence of incoherent emission.

2. Linewidth

The steady-state solution of the two-point density correlation equation, Eq. (60), details the balance which occurs between driven incoherent fluctuations and the damped shielding eigenmodes when the incoherent fluctuations have steady amplitude. Having expressed the driving source in terms of the incoherent correlation it is possible to eliminate the incoherent correlation from Eq. (63) and solve to find the damping in the shielding response. This provides a formula for the frequency linewidth as well as a condition for saturation from which the wavenumber-spectrum and transport coefficients may be obtained. Proceeding, we construct from the left hand side of Eq. (63) the eigenfunction-projected correlation defined in Eq. (26). The projection is accomplished by multiplying $\langle \tilde{H}^2 \rangle$ by the shielding response structure functions, $\hat{\varphi}_{\substack{k' \\ \omega'}}(\kappa_1) \hat{\varphi}_{\substack{-k' \\ -\omega'}}(\kappa_2)$, and integrating over κ_1 and κ_2. The integrations over κ_1 and κ_2 may be transformed to κ_+ and κ_-. The projection then allows the shielding response structure functions to sample the dependence of the clump on the two parallel scales: The fast scale sampling (κ_-) accounts for the evolution or clump decay and the slow scale sampling reflects the degree to which the mode structure in the parallel direction shields the clumps. For the wave functions $\varphi(\kappa_\pm)$ we assume a Gaussian of width Δ_k: $\varphi(\kappa_\pm) = (\Delta_k \sqrt{\pi})^{-1} \exp[-\kappa_\pm^2 / \Delta_k^2]$. The integration over κ_+ is trivial; for the κ_- integration we use Eq. (A6) with $u = 0$ and obtain

$$\overline{\langle \tilde{H}\tilde{H} \rangle}_k = \frac{-(Rq)^4}{(D_\parallel)^2} D (8\pi)^{1/2} \alpha\Delta\eta \frac{1}{\Delta_k} \left(\frac{\Delta\eta^2}{4} + \frac{1}{\Delta_k^2}\right) \times \int dk' \, k'^2 \frac{\Delta k'}{(\Delta_k^2 + \Delta_{k'}^2)^{1/2}} \times (e^{-(k-k'-k_0)^2\alpha^2/2} + e^{-(k-k'+k_0)^2\alpha^2/2}) \times \int d\omega'(\omega' - \omega'_{*e}) \frac{\epsilon_{\mathrm{Im}}^{\mathrm{ion}}(k',\omega')}{|\epsilon(k',\omega')|^2} \overline{\langle \tilde{H}\tilde{H} \rangle}_{\substack{k' \\ \omega'}}. \tag{64}$$

We expand the dielectric about the wavenumber $k_r(\omega')$ corresponding to the eigenmode frequency ω', provided the system is weakly turbulent, or equivalently, that the spectrum broadening is not too large. With this expansion, the k' integration is performed by evaluating the residue at the pole corresponding to the eigenmode.

The two-time correlation $\langle \tilde{H}\tilde{H} \rangle_{\substack{k_r(\omega') \\ \omega'}}$ is obtained from the equal-time correlation $\overline{\langle \tilde{H}\tilde{H} \rangle}_{k,(\omega')}$ by multiplication with the one-particle propagator. Thus,

$$\overline{\langle\widetilde{H}\widetilde{H}\rangle_{k_r(\omega')}}_{\omega'} = 2\,\mathrm{Re}\left[\left(-i\omega' + \frac{T_e\Delta_k^2}{m\nu_{ei}(Rq)^2}\right)^{-1}\right]\overline{\langle\widetilde{H}\widetilde{H}\rangle_{k_r(\omega')}}, \qquad (65)$$

reflecting the coupling of turbulence with parallel diffusion. We substitute Eq. (65) into Eq. (64) and consider the integration over ω'. There are two decaying functions of ω' in the integrand and their relative widths determine the integration. One function is the propagator just given with width $D_\parallel\Delta_k^2/(Rq)^2$ and the other is the function $(\exp\{-[k-k_r(\omega')-k_0]^2\alpha^2/2\} + \exp\{-[k-k_r(\omega') + k_0]^2\alpha^2/2\})$. This function describes the decay and modulation of two-point correlation arising from the spatial dependence of the relative diffusion. For drift waves with poloidal correlation on the order of ρ_s the width of this function in frequency is on the order of ω_{*e} as determined by expanding $k_r(\omega')$ about the eigenfrequency. For the adiabatic regime $\omega_{*e} < D_\parallel\Delta_k^2/(Rq)^2$ we may approximate the propagator by its value at $\omega' = 0$, giving

$$\begin{aligned}\overline{\langle\widetilde{H}\widetilde{H}\rangle}_k = {} & \frac{-(Rq)^4}{D_\parallel^2}D(2\pi)^{3/2}\frac{\alpha\Delta\eta}{\Delta_k}\left(\frac{\Delta\eta}{4} + \frac{1}{\Delta_k^2}\right)\frac{(Rq)^2}{D_\parallel\Delta_k^2} \\ & \times\int d\omega'\frac{\Delta_{k_{r}(\omega')}}{(\Delta_k^2 + \Delta_{k_r(\omega')}^2)^{1/2}} \\ & \times\left(e^{-(k-k_r(\omega')-k_0)^2\alpha^2/2} + e^{-(k-k_r(\omega')+k_0)^2\alpha^2/2}\right) \\ & \times(\omega' - \omega'_{*e})\frac{\epsilon_{\mathrm{Im}}^{\mathrm{ion}}(k_r(\omega'),\omega')\overline{\langle\widetilde{H}\widetilde{H}\rangle_{k_{r(\omega')}}}}{\epsilon_{\mathrm{Im}}(k_r(\omega'),\omega')(\partial\epsilon/\partial k_r)}. \qquad (66)\end{aligned}$$

We consider two limits in evaluating Eq. (66), depending on the value of k_0 relative to α^{-1}. The most probable wavenumber k_0 arises from the oscillation of the relative diffusion occurring as the remnant of the periodicity of the scattering waves in the turbulent spectrum. This oscillation may be viewed as a consequence of the fact that a density element is correlated not only with closely neighboring density elements but density elements whose separation is at multiples of the wavelength of the scattering field. If we assume, as in previous work, that $k_0 < \alpha^{-1}$, then k_0 drops out and the sum of exponentials becomes $2\exp\{-[k-k_r(\omega')]^2\alpha^2/2\}$. Expanding $k_r(\omega')$ about the eigenfrequency and assuming that the rest of the integrand is slowly varying in ω', we obtain upon integration over ω',

$$\begin{aligned}\overline{\langle\widetilde{H}\widetilde{H}\rangle}_k = {} & \frac{-(Rq)^6}{D_\parallel^3}D(4\pi)^{3/2}\frac{\alpha\Delta\eta}{\Delta_k^3}\left(\frac{\Delta\eta^2}{4} + \frac{1}{\Delta_k^2}\right)\left(\frac{\partial k_r}{\partial\omega_k}\alpha\right)^{-1} \\ & \times k^2(\omega_k - \omega_{*e})\frac{\epsilon_{\mathrm{Im}}^{\mathrm{ion}}(k,\omega_k)}{\epsilon_{\mathrm{Im}}(k,\omega_k)(\partial\epsilon/\partial k)}\overline{\langle\widetilde{H}\widetilde{H}\rangle}_k. \qquad (67)\end{aligned}$$

The correlation now cancels out of the equation leaving an expression which relates the total dissipation in the eigenmodes to the ion contribution of the dissipation.

$$\epsilon_{\mathrm{Im}}(k,\omega_k) = C(k,\omega_k)\epsilon_{\mathrm{Im}}^{\mathrm{ion}}(k,\omega_k), \qquad (68)$$

where

$$\begin{aligned}C(k,\omega_k) = {} & \frac{(Rq)^6}{D_\parallel^3}D(4\pi)^{3/2}\frac{\alpha\Delta\eta}{\Delta_k^3}\left(\frac{\Delta\eta^2}{4} + \frac{1}{\Delta_k^2}\right) \\ & \times\left(\frac{dk_r}{d\omega_k}\alpha\right)^{-1}k^2(\omega_{*e} - \omega_k)\left(\frac{d\epsilon}{\partial k}\right)^{-1}. \qquad (69)\end{aligned}$$

From Chen *et al.*,[11]

$$\omega(k) = \frac{(1 - \pi\epsilon_n\hat{s})v_{de}k}{1 + k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)}, \qquad (70)$$

from which we obtain

$$\frac{dk_r}{d\omega_k} = \frac{k}{\omega_k}\left(\frac{1 + k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)}{1 - k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)}\right). \qquad (71)$$

The function $d\epsilon/dk$ is given approximately by

$$\frac{d\epsilon}{dk} \simeq \frac{1}{k}\left[1 + 3k^2\rho_s^2\left(1 + \frac{\hat{s}^2\pi^2}{4}\right)\right]. \qquad (72)$$

Substituting these results into Eq. (66) we have

$$\begin{aligned}C(k,\omega_k) = {} & R_e(k\rho_s)^4\left(\frac{Rq}{L_n}\right)^4\bar{\nu}^2(4\pi)^{3/2}\frac{1}{\Delta\eta\Delta_k^3}\left(\frac{\Delta\eta^2}{4} + \frac{1}{\Delta_k^2}\right) \\ & \times\frac{[1 - k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)](1 - \pi\epsilon_n\hat{s})[\pi\epsilon_n\hat{s} + k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)]}{[1 + k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)]^3[1 + 3k^2\rho_s^2(1 + \hat{s}^2\pi^2/4)]}, \qquad (73)\end{aligned}$$

where

$$\bar{\nu} = (\nu_{ei}/\omega_{*e})(m_e/m_i).$$

Equation (68) describes the steady-state shielding eigenmode response to incoherent emission. The total dissipation $\epsilon_{\mathrm{Im}}(k,\omega_k)$ is composed of electron and ion contributions, $\epsilon_{\mathrm{Im}} = \epsilon_{\mathrm{Im}}^{\mathrm{elec}} - |\epsilon_{\mathrm{Im}}^{\mathrm{ion}}|$, and is necessarily negative ($\epsilon_{\mathrm{Im}} < 0$) in order to balance the incoherent emission. Rewriting Eq. (65) to introduce the electron dissipation we obtain a saturation condition

$$\epsilon_{\mathrm{Im}}^{\mathrm{ion}}(k,\omega_k) = \epsilon_{\mathrm{Im}}^{\mathrm{elec}}(k,\omega_k)/[1 - C(k,\omega_k)]. \qquad (74)$$

This expresses the balance in the steady state between the linear electron destabilization [Eq. (11)] of the collision-driven toroidal dissipative drift wave with the enhancement by incoherent emission $[1 - C(k,\omega_k)]^{-1}$ and the ion damping, both linear and nonlinear. A related expression

$$\epsilon_{\mathrm{Im}}(k,\omega_k) = \frac{C(k,\omega_k)}{[1 - C(k,\omega_k)]}\epsilon_{\mathrm{Im}}^{\mathrm{elec}}(k,\omega_k) \qquad (75)$$

gives the total dissipative mode response to the incoherent emission process in terms of a numerical enhancement of the linear electron dissipation. The width of the frequency spectrum at fixed k is just $\epsilon_{\mathrm{Im}}/(\partial\epsilon/\partial\omega)$ and is given by

$$\Delta\omega_k = \gamma_k^{\mathrm{elec}}\{C(k,\omega_k)/[1 - C(k,\omega_k)]\}, \qquad (76)$$

where γ_k^{elec} is given approximately in Eq. (12), and $\epsilon(k,\omega_k)$ is given by Eq. (73). The physics of the shielding of incoherent fluctuations by the eigenmodes is reflected in the formula for $C(k,\omega_k)$. As previously discussed, the factor $(Rq)^4 D/D_{\parallel}^2$ enters from the decay of two-point correlation by the viscous diffusion. The factor $(\Delta\eta^2/4 + 1/\Delta_k^2)\Delta\eta/\Delta_k$ arises from the shielding of granulations of scale $\Delta\eta$ by eigenmode structures of scale Δ_k^{-1}. When the clump scale exceeds the mode width the shielding is only partial and the excitation of the modes is weaker than in cases where the mode width is comparable to or greater than the clump scale. The factor $(Rq)^2/(\Delta_k^2 D_{\parallel})$ reflects coupling of the turbulence with parallel diffusion.

To determine the magnitude of the incoherent emission in this regime we evaluate the linewidth from Eq. (76). We use the parameter values obtained in measurements of the PRETEXT edge plasma.[5] These measurements indicate that L_n lies within a range, $0.5 < L_n < 3$, which because of the L_n^{-4} dependence of $C(k,\omega_k)$, corresponds to a large variation in possible values of $C(k,\omega_k)$. At an intermediate value of $L_n = 2$ cm, and with $(k_\vartheta \rho_s)^2 = 0.1$, $\hat{s} = 1$, $\nu_{ei} = 10^6$ sec^{-1}, $\omega_{*e} = 0.4\times 10^6$ sec^{-1}, and $\Delta\eta \approx \Delta_k^{-1} = \pi/2$, we obtain $C(k,\omega_k) = 1.38\, R_e$. The linewidth is an increasing function of R_e and becomes substantial $(\Delta\omega_k/\omega_{*e} \geqslant 1.0)$ for Reynolds numbers above 0.5. For lower Reynolds numbers the parallel viscosity is increasingly dominant in the decay of small-scale correlation and the linewidth quickly decreases. For example, with $R_e = 0.2$, $\Delta\omega_k/\omega_{*e} = 0.15$. The dependence of the linewidth with Reynolds number is illustrated in Fig. 2. Because the low Reynolds number expansion and weak turbulence approximations break down in the region where $\Delta\omega_k/\omega_{*e} > 1$, the linewidth formula is not reliable in regimes of very large linewidth. It does, however, indicate the range of Reynolds numbers for which the linewidth is broad, i.e., $R_e > 0.5$, and it exhibits the dependence on mode structure, collisionality, and other parameters. Because of the L_n^{-4} dependence, broad linewidths below $R_e = 0.5$ are possible for density scale lengths less than 2 cm. Finally, note that appreciable frequency broadening is possible for $R_e < 1$.

We now consider the spectrum balance in the limit where the most probable wavenumber k_0 exceeds the correlation length scale α^{-1}. In this case there is granulation at

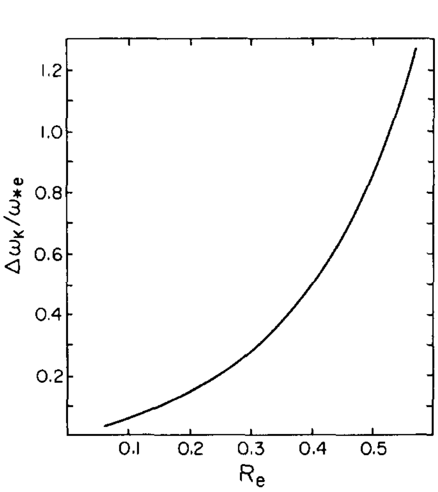

FIG. 2. Width of the frequency spectrum for a fixed wavenumber $(k_\vartheta\rho_s)^2 = 0.1$ as a function of Reynolds number.

the small scale and at multiples of k_0^{-1}. Returning to Eq. (68) we expand $k_r(\omega')$ about $\omega_{k\pm k_0}$,

$$k_r(\omega') = k_r(\omega_{k\pm k_0}) + (\omega' - \omega_{k\pm k_0})\frac{\partial k_r}{\partial \omega_{k\pm k_0}} + \cdots,$$

and observe that the argument of the exponent becomes

$$\frac{[k - k_r(\omega') \pm k_0]\alpha^2}{2} = \frac{(\omega' - \omega_{k\pm k_0})(\partial k/\partial\omega_{k\pm k_0})\alpha^2}{2}.$$

We integrate over ω' as before. Now, however, we obtain a spectrum balance equation which is nonlocal in k due to the periodicity in the scattering spectrum,

$$\begin{aligned}\overline{\langle \tilde{H}\tilde{H}\rangle}_k = {} & \frac{(Rq)^6}{D_{\parallel}^3} D (4\pi)^{3/2} \frac{\Delta\eta}{\Delta_k^3}\left(\frac{\Delta\eta^2}{4} + \frac{1}{\Delta_k^2}\right) \\ & \times\Bigg(\frac{(k-k_0)^2(\omega_{*e}(k-k_0) - \omega_{k-k_0})(\partial k/\partial\omega_{k-k_0})}{\epsilon_{\rm Im}(k-k_0,\omega_{k-k_0})[\partial\epsilon/\partial(k-k_0)]} \\ & \times \epsilon_{\rm Im}^{\rm ion}(k-k_0,\omega_{k-k_0})\overline{\langle\tilde{H}\tilde{H}\rangle}_{k-k_0} \\ & + \frac{(k+k_0)^2[\omega_{*e}(k+k_0) - \omega_{k+k_0}](\partial k/\partial\omega_{k+k_0})}{\epsilon_{\rm Im}(k+k_0,\omega_{k+k_0})[\partial\epsilon/\partial(k+k_0)]} \\ & \times \epsilon_{\rm Im}^{\rm ion}(k+k_0,\omega_{k+k_0})\overline{\langle\tilde{H}\tilde{H}\rangle}_{k+k_0}\Bigg). \qquad (77)\end{aligned}$$

The spectrum is summed [Eq. (35)] and the periodicity is represented by a single moment of the spectrum k_0. The nonlocality is thus a shift in wavenumber and the spectrum balance equation is a difference equation in k.

3. Wavenumber spectrum

By specifying the details of the ion saturation mechanism it is possible to determine the wavenumber spectrum. The saturation condition, Eq. (74), gives the appropriate relation between the damping and growth in the spectrum. For the damping mechanism we consider nonlinear ion–wave interaction or ion Compton scattering. This damping mechanism is an important and effective saturation mechanism for electron instabilities in drift waves and is appropriate for the small to moderate level of incoherent emission in the dissipation range. Recently, the nonlinear ion–wave interaction has been calculated for modes with toroidicity-induced mode structure.[19] The nonlinear dielectric operator is nonlocal and provides an important channel to the ion resonance through the mode structure. In the long wavelength part of the spectrum the ions mediate a transfer to longer wavelength modes where energy in the modes reaches a broadened ion resonance. In addition, the scattering of short wavelength fluctuations to long wavelength fluctuations is strong for ion Compton scattering. The nonlinear transfer rate exceeds the electron excitation rate above wavenumbers on the order of a few tenths ρ_s^{-1}. Such a cutoff is a feature of experimental observations. For simplicity we will neglect the linear shear damping in the spectrum balance as it is small for the toroidicity-induced mode structure.

From Similon and Diamond[19] we obtain the nonlinear damping from the ion–wave interaction

$$\frac{\gamma_{\rm nl}^{\rm ion}}{\omega_{*e}} = \frac{(\omega-\omega_{Di})^2}{\omega_{ti}^2}\frac{1}{(\omega-\omega_{*i})}\frac{1}{\Gamma_0}\int dk'\sum_m (2\pi m k_\vartheta \hat{s})^2 \times\left(\frac{\omega'_{*i}-\omega'_{Di}}{\omega'-\omega'_{Di}}-\frac{\omega_{*i}-\omega_{Di}}{\omega-\omega_{Di}}\right) \times \rho\frac{\langle |v_{k'_\vartheta}(\vartheta-2\pi m)|^2\rangle}{-i(\omega''-\omega''_{Di}+\Delta\omega'')}, \quad (78)$$

where $\langle |v_{k_\vartheta}|^2\rangle = c^2k_\vartheta^2\langle|\varphi_{k_\vartheta}|^2\rangle/B^2$ is the electrostatic field spectrum, $\omega_{Di} = 2\epsilon_n\omega_{*i}[\cos\vartheta + \hat{s}(\vartheta-\vartheta_k)\sin\vartheta]$ is the magnetic drift, $\Gamma_0 = \Gamma_0(\rho_i^2k_\perp^2(\vartheta))$, $\omega_{T_i} = V_{T_i}/Rq$, $\omega_{*i} = -(T_i/T_c)\omega_{*e}$, and $\omega'' = \omega - \omega'$. Here $\Delta\omega$ incorporates the linear and nonlinear broadenings of the ion propagator. These include ion parallel motion (ω_{T_i}); the nonlinear decorrelation from $E\times B$ turbulent motion,

$$d_k = \left(\frac{c}{B}\right)^2\sum_{k'_\vartheta,m'}(2\pi m')^2k_\vartheta^2k'^2_\vartheta\hat{s}^2\left[J_0\left(\frac{v_\perp k'_\perp}{\Omega_i}\right)\right]^2 \times\frac{1}{-i(\omega''-\omega''_{Di})+d_{k'}}\langle\varphi\varphi\rangle_{k'_\vartheta}(\vartheta-2\pi m'); \quad (79)$$

and the incoherent frequency broadening $\Delta\omega_k$. The nonlinear transfer rate, Eq. (78), comprises the nonlinear potential of the nonlocal eigenmode problem. This potential is determined from the quasineutrality condition with linear and nonlinear dynamics incorporated in an ion gyrokinetic equation, and adiabatic electrons with a nonadiabatic part for the collision driven growth. This nonlinear eigenvalue problem is used to determine nonlinear contributions to mode structure which affect energy transfer in the spectrum at finite amplitude. The mode structure it predicts is consistent with the mode structure assumed in obtaining the linear growth and incoherent emission. The linear contribution of ions to the mode structure has been discussed in Sec. II. The nonlinear ion dynamics comprising the nonlinear transfer rate are obtained from a coherent direct interaction approximation of the ion gyrokinetic equation. In this renormalization both the driven ion fluctuations $h_k^{(2)}$ and the shielding plasma response $\varphi_k^{(2)}$ are computed. The driven potential $\varphi_k^{(2)}$ produces an induced scattering associated with $h_k^{(2)}$. Similon and Diamond incorporate the induced scattering reduction into the formula for the ion Compton scattering transfer rate by including the factor ρ which they give as $\rho\sim 0.1$.

Two interactions typify the nonlinear transfer. For a linear dispersion whose frequency is an increasing function of k_ϑ at low k_ϑ and then a decreasing function at higher k_ϑ, it is possible to satisfy the condition for similar frequencies ($\omega_{*e} \gg \omega-\omega'$) with values of k_ϑ and k'_ϑ that are widely separated. The beating of a high k_ϑ and low k'_ϑ gives a beat fluctuation k'' with large perpendicular wavenumber and hence large momentum transfer to the ions. In this part of the spectrum ($k_\vartheta > k_m$, where k_m is the wavenumber corresponding to maximum frequency: $d\omega/dk|_{k_m} = 0$) the nonlinear transfer rate exceeds the growth rate. As a result the spectrum is only weakly populated for $k_\vartheta > k_m$. From Eq. (70) we calculate k_m to be $k_m\rho_s = (1+\hat{s}^2\pi^2/4)^{-1/2}$. In the long wavelength part of the spectrum, the scattering interaction occurs between modes with similar k_ϑ, k'_ϑ. Energy cascades to lower k_ϑ where it is absorbed by shear damping aided by a broadened ion resonance. The spectrum at long wavelength is given by the saturation condition, Eq. (74), with the left-hand side given by the ion Compton scattering transfer rate, Eq. (78):

$$\frac{(\omega-\omega_{Di})^2}{\omega_{T_i}^2}\frac{1}{(\omega-\omega_{*i})}\frac{1}{\Gamma_0}\sum_{k'_\vartheta,m}(2\pi k_\vartheta\hat{s}m)^2 \times\left(\frac{\omega'_{*i}-\omega'_{Di}}{\omega'-\omega'_{Di}}-\frac{\omega_{*i}-\omega_{Di}}{\omega-\omega_{Di}}\right)\rho\frac{\langle|V_{k'_\vartheta}(\vartheta-2\pi m)|^2\rangle}{-i(\omega''-\omega''_{Di})+\Delta\omega''}$$
$$= k_\vartheta^2\rho_s^2\left(1+\frac{\hat{s}^2\pi^2}{4}\right)\left(\frac{\nu_{ei}\omega_{*e}(Rq)^2}{v_{te}^2}\right)^{1/2}[1-C(k_\vartheta,\omega_{k_\vartheta})]^{-1}$$
$$\approx k_\vartheta^2\rho_s^2\left(1+\frac{\hat{s}^2\pi^2}{4}\right)\left(\frac{\nu_{ei}\omega_{*e}(Rq)^2}{v_{te}^2}\right)^{1/2}[1+C(k_\vartheta,\omega_{k_\vartheta})]. \quad (80)$$

We have assumed here that the incoherent emission level is not too strong. This holds over most of the dissipation range, and we approximate $(1-C)^{-1}\approx 1+C$. The function $C(k,\omega_k)$ is proportional to D, the $E\times B$ diffusion, Eq. (34). The two-time correlation in D is written as the single particle propagator times the equal time correlation $\langle|\varphi(\eta+2\pi m)|^2\rangle_{k'}^{\omega'} = {\rm Re}\,L_{k'}^{\omega'}\langle|\varphi(\eta+2\pi m)|^2\rangle_{k'}$ with $L_k^\omega = (-i\omega + D_\|\Delta_k^2/(Rq)^2)^{-1}$. The ω' integration is performed yielding

$$D = \frac{1}{4}\frac{(Rq)^2}{D_\|\Delta_k^2}\int dk'\,\hat{s}^2\sum_m(2\pi m)^2\langle|v_{k'_\vartheta}(\eta+2\pi m)|^2\rangle. \quad (81)$$

Substituting this result into Eq. (69) and Eq. (69) into Eq. (80), we obtain the integral equation for the saturated spectrum,

$$k_\vartheta^2\rho_s^2\left(1+\frac{\hat{s}^2\pi^2}{4}\right)\left(\frac{\nu_{ei}\omega_{*e}(Rq)^2}{v_{te}^2}\right)^{1/2} = \frac{(\omega-\omega_{Di})^2}{\omega_{ti}^2}\frac{1}{(\omega-\omega_{*i})}\frac{1}{\Gamma_0} \times\int\frac{dk'}{2\pi}\left[\left(\frac{\omega'_{*i}-\omega'_{Di}}{\omega'-\omega'_{Di}}-\frac{\omega_{*i}-\omega_{Di}}{\omega-\omega_{Di}}\right) \times\frac{\rho}{(-i(\omega''-\omega''_{Di})+\Delta\omega'')}-\frac{\omega_{ti}^2(\omega-\omega_{*i})\Gamma_0}{(\omega-\omega_{Di})^2} \times k_\vartheta^2\rho_s^2\left(\frac{1+\hat{s}^2\pi^2}{4}\right)\left(\frac{\nu_{ei}\omega_{*e}(Rq)^2}{v_{te}^2}\right)^{1/2}T_k\tau_\|\right] \times\sum_m(2\pi m k_\vartheta\hat{s})^2\langle|v_{k'_\vartheta}(\eta+2\pi m)|^2\rangle, \quad (82)$$

where

$$T_k = \frac{5}{16}\frac{(Rq)^6}{D_\|^3}(4\pi)^{3/2}\left(\frac{\pi}{2}\right)^6(\omega_* - \omega_k)\left(\frac{\partial k}{\partial\omega_k}\frac{\partial\epsilon}{\partial k}\right)^{-1}.$$

With the extended mode structure the interaction is strongest for $\eta = \pm\pi$ where the k_ϑ and k'_ϑ modes have maximum overlap, and $m = \pm 1$ contributions dominate in the sum over m. Since $k_\vartheta - k'_\vartheta$ is small, we expand the functions $\langle |v_{k'_\vartheta}(\pi)|^2\rangle$ and $[(\omega'_{*i}-\omega'_{Di})/(\omega'-\omega'_{Di}) - (\omega_{*i}-\omega_{Di})/(\omega-\omega_{Di})]$ about k_ϑ. The integration over k'_ϑ is then trivial and the spectral equation becomes an ordinary differential equation:

$$k_\vartheta^2\rho_s^2\Big(1+\frac{\hat{s}^2\pi^2}{4}\Big)\Big(\frac{\nu_{ei}\omega_{*e}(Rq)^2}{v_{te}^2}\Big)^{1/2} = \Big\{\frac{(\omega-\omega_{Di})^2}{\omega_{t_i}^2}\frac{1}{(\omega-\omega_{*i})}\frac{1}{\Gamma_0} \times\frac{(1/3)\rho\Delta k_\vartheta^3}{\Delta\omega''}\frac{d}{dk_\vartheta}\Big(\frac{\omega_{*i}-\omega_{Di}}{\omega-\omega_{Di}}\Big)\frac{d}{dk_\vartheta}\langle|v_{k_\vartheta}(\pi)|^2\rangle - \Delta k_\vartheta T_k\tau_\| k_\vartheta^2\rho_s^2\Big(1+\frac{\hat{s}^2\pi^2}{4}\Big)\Big(\frac{\nu_{ei}\omega_{*e}(Rq)^2}{v_{te}^2}\Big)^{1/2} \times\langle|v_{k_\vartheta}(\pi)|^2\rangle\Big\}[(2\pi k_\vartheta\hat{s})^2/(2\pi)]. \tag{83}$$

The function $\frac{1}{3}\Delta k_\vartheta^3$ comes from the integration of $(k'_\vartheta-k_\vartheta)^2$ and is the wavenumber range of the mode interaction. The function $\Delta\omega$ is the particle decorrelation. The particle decorrelation is determined by ion parallel motion ($\Delta\omega''=\omega_{ti}$) at the lowest turbulence levels. For higher turbulence levels the turbulent decorrelation rate is dominant and $\Delta\omega''=d_{k''}$. The wavenumber range Δk_ϑ is given by the wavenumber corresponding to the distance traveled by a wave in the correlation time, $\Delta k_\vartheta=\Delta\omega/v_*$. This is limited, however, by the spectrum width so that Δk_ϑ is the smaller of $\Delta\omega/v_*$ and k_m. For the low Reynolds number regime with moderate linear excitation but relatively weak incoherent emission, the turbulent decorrelation d_k is sufficiently strong to dominate the particle decorrelation but still give a wavenumber range within the spectrum width. Thus $\Delta\omega''=d_{k''}$ and $\Delta k_\vartheta=d_{k'}/v_*$ where

$$d_{k''}\approx(1/2\pi)k_\vartheta^2(2\pi)^2\hat{s}^2(0.2)(1/v_*)\langle|v_{k_\vartheta}(0)|^2\rangle. \tag{84}$$

To lowest order in the Reynolds number, solution of Eq. (83) yields the following spectrum,

$$\frac{\langle|v_{k_\vartheta}(0)|^2\rangle}{v_*^2}\approx\frac{5}{\hat{s}^2(2\pi)^{4/3}}\Big(\frac{1-\pi\epsilon_n\hat{s}}{1+\hat{s}^2\pi^2/4}\Big)^{1/3} \times\Big(\frac{\rho_s}{k_m^5}\Big)^{1/6}\Big(\frac{\nu_{ei}c_s}{\omega_{te}^2L_n}\Big)^{1/6}\Big(\frac{k_m^{5/2}}{k_\vartheta^{5/2}}-1\Big)^{1/3}. \tag{85}$$

The next order correction, nominally first order in the Reynolds number,

$$\frac{\langle|v_{k_\vartheta}(0)|^2\rangle^{(1)}}{v_*^2}\approx\frac{144}{3}\Big(\frac{\pi}{2}\Big)^7(1-\pi\epsilon_n\hat{s})\Delta\eta_c^2\rho_s^{9/2}k_m^{7/2}\Big(\frac{k_\vartheta}{k_m}\Big)^{5/2} \times\Big[1-\Big(\frac{k_\vartheta}{k_m}\Big)^{5/2}\Big]^{-2/3} \times\Big[1-\Big(\frac{k_\vartheta}{k_m}\Big)^{11/6}\Big]\Big(\frac{\nu_{ei}c_s}{\omega_{te}^2L_n}\Big)^4$$

is actually proportional to R_e^2 when the turbulent diffusion coefficient D is evaluated for the integrated intensity given below in Eq. (87):

$$\frac{\langle|v_{k_\vartheta}(0)|^2\rangle^{(1)}}{v_*^2}\approx R_e^2\Big(\frac{k_\vartheta}{k_m}\Big)^{5/2}(k_m\rho_s)^{7/2} \times\Big[1-\Big(\frac{k_\vartheta}{k_m}\Big)^{5/2}\Big]^{-2/3}\Big[1-\Big(\frac{k_\vartheta}{k_m}\Big)^{11/6}\Big] \times 10^4\rho_s(1-\pi\epsilon_n\hat{s})\Delta\eta_c^2\Big(\frac{\nu_{ei}c_s}{\omega_{te}^2L_n}\Big)^{1/6}. \tag{86}$$

This result indicates that in the dissipation range the wavenumber spectrum and integrated intensity are relatively insensitive to incoherent emission even when there is a broadened linewidth.

The lowest-order spectrum, Eq. (85), diverges as $k_\vartheta^{-5/6}$ near the origin and has negligible energy above $k_m\cong\rho_s^{-1}(1+\hat{s}^2\pi^2/4)^{-1/2}$. The spectrum is illustrated in Fig. 3, where theoretical and experimental results are superimposed. Inasmuch as the experimental spectrum is given in arbitrary units,[5] a comparison of magnitudes should not be implied from the figure. The experimental and theoretical spectra have been aligned vertically to allow comparison of the slopes. Asymptotically, the theoretical spectrum goes as $k_\vartheta^{-17/6}$, for large k_ϑ, in excellent agreement with experiment. Though divergent at $k_\vartheta=0$, the spectrum is integrable and the integrated intensity is

$$\frac{\langle v_E^2\rangle}{v_*^2}\approx0.06\frac{1}{\hat{s}^3}\Big(\frac{\nu_{ei}c_s}{\omega_{te}^2L_n}\Big)^{1/6}, \tag{87}$$

where

$$\langle v_E^2\rangle=\int\frac{dk'_\vartheta}{2\pi}\langle|v_{k'_\vartheta}(0)|^2\rangle.$$

In terms of $e\varphi/T_e$ the saturation level is

$$\frac{e\langle|\varphi|\rangle}{T_e}\approx\Big(\frac{0.25}{k_\perp L_n}\Big)\frac{1}{\hat{s}^{3/2}}\Big(\frac{\nu_{ei}c_s}{\omega_{te}^2L_n}\Big)^{1/12}.$$

We note that the k_m cutoff of the k spectrum arising from the

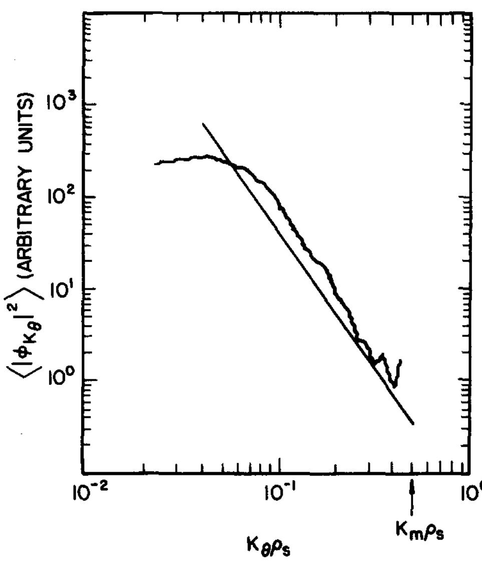

FIG. 3. Wavenumber spectrum showing agreement with experimental spectrum.[5] As the experimental spectrum is given in arbitrary units, there is no absolute comparison. The two spectra have been superimposed to show agreement of slopes. Note that the theoretical cutoff k_m coincides at the largest wavenumber measured.

strong turbulent scattering above the maximum frequency places most of the energy in modes well below the cutoff of $k_\vartheta \rho_s \cong 0.5$.

The asymptotic scaling of the fluctuation spectrum for large ω has also been measured experimentally. For purposes of comparison we recall from Eq. (21) that $|\phi_{\substack{k\\ \omega}}|^2 = (T_e^2/|e|^2)(L_{\substack{k\\ \omega}}^{-1})^2 \langle \tilde{H}^2 \rangle_{\substack{k\\ \omega}}$. We approximate $(L_{\substack{k\\ \omega}}^{-1})^2$ with $(\omega_*/\omega - 1)^2$, which goes as a constant asymptotically. The ω dependence of $\langle \tilde{H} \rangle^2_{\substack{k\\ \omega}}$ is incorporated in the propagator which takes the one-time correlation into the two-time correlation [see Eq. (65)]:

$$\langle \tilde{H}^2 \rangle_{\substack{k\\ \omega}} = 2\,\mathrm{Re}\left[-i\omega + v_{T_e}^2/\nu_{ei}(Rq)^2(\Delta\eta)^2 \right]^{-1} \langle \tilde{H}^2 \rangle_k .$$

Asymptotically, the propagator goes as $v_{T_e}^2/(\omega^2 \nu_{ei}(Rq)^2(\Delta\eta)^2)$ implying that $|\varphi_k|^2 \sim 1/\omega^2$ as $\omega \to \infty$.

An important consequence of incoherent emission is the possible deviation from an adiabatic electron response in a regime where the condition for an adiabatic response, $\omega\nu_{ei} < v_{T_e}^2 k_\parallel^2$, is satisfied. This occurs when a significant portion of the density fluctuation is incoherent. The electron density consists of the adiabatic part and the nonadiabatic contribution which in turn is comprised of coherent and incoherent contributions,

$$\hat{n}_E = \frac{|e|}{T_e}\hat{\varphi} + H_E = \frac{|e|}{T_e}\hat{\varphi} + H_E^{(c)} + \tilde{H}_E .$$

From quasineutrality, the electron density may be expressed in terms of the ion operator $L^{\mathrm{ion}}(k,\omega)$,

$$\hat{n}_E/n_0 = L^{\mathrm{ion}}(k,\omega)(|e|/T_e)\hat{\varphi}_{\substack{k\\ \omega}} .$$

The density fluctuation level $\langle \delta n^2 \rangle_{\substack{k\\ \omega}}$ is thus

$$\begin{aligned}\langle (\delta n)^2 \rangle_{\substack{k\\ \omega}} &= |L^{\mathrm{ion}}(k,\omega)|^2 (|e|^2/T_e^2) \langle \hat{\phi}^2_{\substack{k\\ \omega}} \rangle \\ &= \left[|L^{\mathrm{ion}}_{\mathrm{Re}}(k,\omega)|^2 + |L^{\mathrm{ion}}_{\mathrm{Im}}(k,\omega)|^2 \right] (|e|^2/T_e^2) \langle \hat{\varphi}^2_{\substack{k\\ \omega}} \rangle .\end{aligned} \tag{88}$$

From the dispersion relation $|L^{\mathrm{ion}}_{\mathrm{Re}}|^2 \sim 1$ for the small values of k_ϑ typical in the spectrum. From the saturation condition, Eq. (74), we replace L^{I}_{Im} with $\epsilon^{\mathrm{elec}}_{\mathrm{Im}}/(1-C)$. Thus

$$\left\langle \left(\frac{\delta n}{n} \right)^2 \right\rangle_{\substack{k\\ \omega}} \simeq \left[1 + \left(\frac{\epsilon^{\mathrm{elec}}_{\mathrm{Im}}(k,\omega_k)}{1 - C(k,\omega_k)} \right)^2 \right] \frac{|e|^2}{T_e^2} \langle \hat{\varphi}^2 \rangle_{\substack{k\\ \omega}} \tag{89}$$

and the departure from an adiabatic response is incorporated in the term $[\epsilon^{\mathrm{elec}}_{\mathrm{Im}}/(1-C)]^2$. We note that for $C = 0$, corresponding to no incoherent emission, the departure from adiabaticity is just the term $(\epsilon^{\mathrm{elec}}_{\mathrm{Im}})^2$ which arises from the coherent part of the nonadiabatic density. This term is proportional to $(\gamma/\omega_*)^2$ which is small in the adiabatic regime. With enhancement by incoherent emission, however, it is possible for $\epsilon^{\mathrm{elec}}_{\mathrm{Im}}/(1-C)$ to exceed one even with $\epsilon^{\mathrm{elec}}_{\mathrm{Im}} < 1$.

4. Particle diffusion

Having obtained the spectrum and fluctuation level, it is straightforward to determine the anomalous particle transport. The anomalous particle flux Γ_N from $E \times B$ convection of the perturbed density is given in terms of the density-potential cross correlation by

$$\begin{aligned}\Gamma_n &= \sum_{k'_\vartheta} (-i) k'_\vartheta \rho_s c_s \frac{|e|}{T_e} \langle \varphi \hat{H} \rangle_{k'_\vartheta} \\ &= \sum_{k'_\vartheta} (-i) k'_\vartheta \rho_s c_s \frac{|e|}{T_e} (\langle \hat{\varphi} \hat{H}^c \rangle_{k'_\vartheta} + \langle \hat{\varphi} \tilde{H} \rangle_{k'_\vartheta}),\end{aligned} \tag{90}$$

where we have expressed the electron density $\hat{H}_{k'_\vartheta}$ in terms of its coherent and incoherent components $H^{(c)}_{k'_\vartheta}$ and $\tilde{H}_{k'_\vartheta}$ respectively. Using Eqs. (18) and (21) we express the coherent density $H^{(c)}$ in terms of its response function and the incoherent density in terms of the shielding potential,

$$\begin{aligned}&\langle \hat{\varphi} H^{(c)} \rangle_{k'_\vartheta} + \langle \hat{\varphi} \tilde{H} \rangle_{k'_\vartheta} \\ &\quad = \left\langle \hat{\varphi} R_{\substack{k'\\ \omega'}} \frac{|e|}{T_e} \hat{\varphi} \right\rangle_{k'_\vartheta} + \left\langle \hat{\varphi} L_{\substack{k'\\ \omega'}} \frac{|e|}{T_e} \hat{\varphi} \right\rangle_{k'_\vartheta} .\end{aligned} \tag{91}$$

The electron and ion response functions determine the operator, $(L^{\mathrm{ion}}_{\substack{k\\ \omega}} - 1 - R_{\substack{k\\ \omega}}) = L_{\substack{k\\ \omega}}$, allowing us to express the incoherent contribution [second term in Eq. (89)] in terms of $R_{\substack{k\\ \omega}}$ and $L^{\mathrm{ion}}_{\substack{k\\ \omega}}$. The adiabatic density is not convected by $E \times B$ drift and as with the source term, the nonadiabatic part of the density cancels out, leaving

$$\langle \hat{\varphi} H^{(c)} \rangle_{k'_\vartheta} + \langle \hat{\varphi} \tilde{H} \rangle_{k'_\vartheta} = (e/T_e) \langle \hat{\varphi} L^{\mathrm{ion}}_{\substack{k'\\ \omega'}} \hat{\varphi} \rangle_{k'_\vartheta} .$$

The ion response is the damping which balances the clump-enhanced electron dissipation as expressed in the saturation condition, Eq. (74). Thus $L^{\mathrm{ion}}_{\substack{k'\\ \omega'}} = L^{\mathrm{elec}}_{\substack{k'\\ \omega'}}/[1 - C(k,\omega_k)]$ and Eq. (90) becomes

$$\Gamma_N = \sum_{k'_\vartheta} (-i) k'_\vartheta \rho_s c_s \frac{|e|}{T_e} \frac{1}{[1 - C(k',\omega_{k'})]} \langle \hat{\varphi}\, L^{\mathrm{elec}}_{\substack{k'\\ \omega_{k'}}} \hat{\varphi} \rangle_{k'_\vartheta} . \tag{92}$$

This formula indicates that the particle flux is essentially the quasilinear flux with an enhancement caused by incoherent emission.

Proceeding, we obtain the electron response from Eq. (5),

$$L^{\mathrm{elec}}_{\substack{k\\ \omega}} = \left(-i\omega + \alpha \frac{\partial^2}{\partial \eta^2} \right)^{-1} \frac{(-i)|e|}{T_e} \omega_{*e} \left(\frac{\omega}{\omega_{*e}} - 1 \right) \hat{\varphi}_{k_\vartheta}$$

where $\alpha = \omega_{T_e}^2/\nu_{ei}$. Substituting into Eq. (92) yields

$$\begin{aligned}\Gamma_N &= \sum_{k'_\vartheta} (k'_\vartheta \rho_s) c_s \left(1 - \frac{\omega_{*e}}{\omega} \right) \frac{|e|^2}{T_e^2} \\ &\quad \times \frac{1}{[1 - C(k',\omega_{k'})]} \left\langle \hat{\varphi} \left(-i + \frac{\alpha}{\omega} \frac{\partial^2}{\partial \eta^2} \right)^{-1} \hat{\varphi} \right\rangle_{k'_\vartheta} .\end{aligned} \tag{93}$$

The average denoted by $\langle\ \rangle$ is accomplished with an integration over η. The integral $\int d\eta\, \varphi [-1 + (\alpha/\omega)(\partial^2/\partial\eta^2)]^{-1} \varphi$ has been worked out in Sec. II [see Eqs. (9) and (10)]. Thus, in the adiabatic limit,

$$\Gamma_N = -\sum_{k'_\vartheta}|k'_\vartheta\rho_s|c_s\left(1-\frac{\omega'_{*e}}{\omega}\right)\pi\left(\frac{\omega'_k\nu_{ei}}{\omega^2_{T_e}}\right)^{1/2}$$
$$\times\frac{|e^2|}{T_e^2}\frac{1}{[1-C(k',\omega_{k'})]}|\hat\varphi_{k'_\vartheta}|^2$$
$$=\sum_{k'_\vartheta}\frac{v_*^2}{c_s}\left\langle\frac{v_{k'}^2}{v_*^2}\right\rangle\pi\left(\frac{\nu_{ei}c_s}{\omega^2_{T_e}L_n}\right)^{1/2}\frac{1}{(k_\vartheta\rho_s)^{1/2}}$$
$$\times\left[\left(1-\frac{\omega'}{\omega'_{*e}}\right)\left(\frac{\omega'_{*e}}{\omega}\right)^{1/2}\right]\frac{1}{[1-C(k',\omega_{k'})]}. \tag{94}$$

From Eq. (87) we substitute for the integrated intensity

$$\left\langle\frac{v_e^2}{v_*^2}\right\rangle\approx\sum_{k_\vartheta}\left\langle\frac{v_k^2}{v_*^2}\right\rangle=0.5\frac{1}{\hat s^3}\left(\frac{\nu_{ei}c_s}{\omega^2_{T_e}L_n}\right)^{1/6},$$

obtaining the final form for the particle flux

$$\Gamma_N=\frac{v_*^2}{c_s}\left(\frac{\nu_{ei}c_s}{\omega^2_{T_e}L_n}\right)^{2/3}\frac{\pi}{2}$$
$$\times\overline{\frac{1}{\hat s^3}\left[\left(\frac{1}{k_\vartheta\rho_s}\frac{\omega_{*e}}{\omega_k}\right)^{1/2}\left(1-\frac{\omega_k}{\omega_*}\right)\frac{1}{[1-C(k,\omega_k)]}\right]}, \tag{95}$$

where the bar over the factor in the large brackets indicates evaluation at a mean wavenumber in the spectrum. The particle diffusion is given by $D=\Gamma_N L_n$. For pretext tokamak parameters[5] with $L_n=0.5$ cm and $T_e=25$ eV the *quasilinear* part of the diffusion is 7.5×10^3 cm^2/sec. With incoherent emission sufficient to produce a linewidth of $\Delta\omega_k/\omega_{*e}=1$, the particle diffusion is 2.4×10^4 cm^2/sec, which is slightly larger than $D_{\mathrm{Bohm}}=T_e/(16eB)\approx1.8\times10^4$. For $T_e=25$ eV, $B=8$ kG. The scaling of the particle diffusion is *not* Bohm-like, but goes as

$$D\sim T_e^{1/6}n^{2/3}. \tag{96}$$

The weak temperature dependence is a natural consequence of the near offset of the intrinsic drift wave particle diffusion scaling with respect to temperature, which goes as ρ_s, and the scaling intrinsic to collisionally driven systems which tends to go as T_e to a negative power. This scaling is in rough accord with recent measurements, which find that the diffusion decreases with increasing minor radius.[5]

B. High Reynolds number regime

We now consider the turbulent spectrum in the limit opposite of that treated in the first half of this section. In the inertial range or high Reynolds number regime the $E\times B$ turbulent diffusion dominates the parallel collisional viscosity. We also assume that it dominates other linear processes, in particular, the magnetic drifts, ω_{De}. Thus, the high Reynolds number regime is intrinsically a strong turbulence regime and turbulent diffusion determines not only the two-point diffusion, but fixes time scales as well. It is not possible to describe the turbulence with the same detail of weak turbulence theory as many assumptions no longer hold. From mixing length arguments, we can, however, infer features of the spectrum, such as linewidth and wavenumber dependence, as well as estimate fluctuation levels and determine transport scalings. From this emerges a picture which is consistent with results obtained in the dissipation range.

1. Spectrum properties

As noted in the previous section, two-point diffusion is predominantly the toroidal $E\times B$ diffusion. The standard approximate solution of the two-point equation applies; $\langle\widetilde H\widetilde H\rangle=(\tau_{cl}-\tau_c)S$ with τ_{cl} obtained from the inversion of the time evolution of the toroidal separation of neighboring points. From Eq. (53),

$$\tau_{cl}=(\tau_{E\times B}/2)\ln\{1/[k_0^2y_-^2+\eta_-^2/\Delta\eta^2+\mathrm{Re}^{-1}]\}, \tag{97}$$

where the terms proportional to magnetic drift have been dropped. With the source term S given in Eq. (25), the incoherent two-point correlation is

$$\langle\widetilde H(\eta_1,y_1)\widetilde H(\eta_2,y_2)\rangle$$
$$=\tau_{E\times B}\ln\{1/[k_0^2y_-^2+\eta_-^2/\Delta\eta^2+\mathrm{Re}^{-1})\}$$
$$\times\sum_{k',\omega'}\left(\frac{(\omega'-\omega'_{*e})\epsilon^{\mathrm{ion}}_{\mathrm{Im}}(k',\omega')|\hat\varphi_{k'}(\eta)|^2_{\omega'}}{|\epsilon(k',\omega')|^2}\right.$$
$$\left.\times\frac{N_{k'\omega'}}{(\omega'^2+N^2_{k'\omega'})}\overline{\langle\widetilde H^2\rangle_{k'}}\right). \tag{98}$$

As before, the two-time, two-point correlation has been expressed as the product of the single density element propagator and the two-point, one time correlation,

$$\langle H(1)H(2)\rangle_{k\omega}=2\,\mathrm{Re}\,R(k,\omega)\langle\widetilde H(1)\widetilde H(2)\rangle_k. \tag{99}$$

The single density element propagator,

$$R(k,\omega)=\left(-i\omega-\frac{D_\parallel}{(Rq)^2}\frac{\partial^2}{\partial\eta^2}+N_{k\omega}\right)^{-1}$$

is dominated by $E\times B$ nonlinearity $N_{k'\omega}$ [Eq. (6)], so that $2\,\mathrm{Re}\,R(k,\omega)\cong2N_{k\omega}/(\omega^2+N^2_{k\omega})$. We renormalize the $E\times B$ nonlinearity and from the results of Sec. III, we find that $N_{k'\omega'}\approx Dk'^2$. We also recall that $\tau_{E\times B}=1/k_0^2D$, where $k_0^2\equiv\langle k^2\hat\varphi^2(k)\rangle/\langle\hat\varphi^2(k)\rangle$. The product $\tau_{E\times B}N_{k'\omega'}$ is thus approximated as $\tau_{E\times B}N_{k'\omega'}\approx k'^2/k_0^2$, and we note that the short wavelength is favored in the turbulent scattering process. Again, we operate on both sides of Eq. (98) in order to express $\langle\widetilde H(\eta_1y_1)\widetilde H(\eta_2y_2)\rangle$ as $\overline{\langle\widetilde H^2\rangle_k}$ and obtain a balance condition. The operations include the Fourier transform in y_- and the eigenfunction projections in η_+ and η_-. These integrals have been worked out in previous calculations,[7,14,17] and the details will not be repeated here. The η_+ and η_- integrations yield

$$\overline{\langle\widetilde H^2\rangle_k}=\int dy_-\,e^{-iky_-}[(1-l^2)^{1/2}-|l|\cos^{-1}|l|]$$
$$\times\sum_{k'}\sum_{\omega'}\frac{(\omega'-\omega'_{*e})\epsilon^{\mathrm{ion}}_{\mathrm{Im}}(k',\omega')k'^2\overline{\langle\widetilde H^2\rangle_{k'}}}{|\epsilon(k',\omega')|^2(\omega'^2+N^2_{k'\omega'})k_0^2}, \tag{100}$$

where $l^2 = k^2y_-^2 + R_e^{-2}$. While the y_- integration may be performed approximately for R_e finite, it is easier to let $R_e \to \infty$, which is the limit we are considering in any case. In this limit the y_- integration has also been given previously. Using these results we obtain the spectrum balance equation

$$\overline{\langle \tilde{H}(1)\tilde{H}(2)\rangle}_k = \frac{2\pi}{k_0}\left(\frac{k_0^2}{k^2}\right)\left[1 - J_0\left(\frac{2k}{k_0}\right)\right] \times \sum_{k',\omega'} \frac{(\omega' - \omega'_{*e})\epsilon_{\rm Im}^{\rm ion}(k_1,\omega')}{|\epsilon(k',\omega')|^2} \times \frac{\overline{\langle \tilde{H}(1)\tilde{H}(2)\rangle}_{k'}}{(\omega'^2 + N_{\substack{k'\\ \omega'}}^2)}\left(\frac{k'^2}{k_0^2}\right). \tag{101}$$

We note from Eq. (101) the absence of any linear dissipative time scale. The only relaxation process occurring is nonlinear. Consequently, the nonlinear relaxation rate and linewidth $\Delta\omega$ are related to the frequency ω which in turn is related to ω_{*e}. This is seen more readily when we eliminate the density correlation from Eq. (101). We multiply both sides of Eq. (101) by $(\omega - \omega_{*e})\epsilon_{\rm Im}^{\rm ion}(k,\omega)(k^2/k_0^2)/\left[|\epsilon(k,\omega)|^2(\omega^2 + N_{\substack{k\\ \omega}}^2)\right]$ and sum over k. The product of this factor and the correlation cancels out, leaving

$$1 = \sum_{\substack{k\\ \omega}} \frac{2\pi}{k_0}\left(\frac{k_0^2}{k^2}\right)\left[1 - J_0\left(\frac{(2k)}{k_0}\right)\right] \times \frac{(\omega - \omega_{*e})\epsilon_{\rm Im}^{\rm ion}(k,\omega)}{|\epsilon(k,\omega)|^2(\omega^2 + N_{\substack{k\\ \omega}}^2)}\frac{k^2}{k_0^2}. \tag{102}$$

Passing to the continuous limit, and replacing ω and k with dimensionless variables $\alpha = \omega/\omega_{*e}$, $d\alpha = dw/\omega_{*e}$; $\beta = k\rho_s$, $d\beta = \rho_s dk$,

$$1 = \int d\beta \int d\alpha \frac{A(\beta,\beta_0)(1-\alpha)|\epsilon_{\rm Im}^{\rm ion}(\beta,\alpha,N_{\substack{\alpha\\ \beta}}/\omega_{*e},\beta_{\substack{\alpha\\ \beta}}/\omega_{*e})|(\beta^2/\beta_0^2)}{(\alpha^2 + N_{\substack{\alpha\\ \beta}}^2/\omega_{*e}^2)\left[|\epsilon_r(\alpha,\beta)|^2 + |\epsilon_{\rm Im}(\alpha,\beta,N_{\substack{\alpha\\ \beta}}/\omega_{*e},\beta_{\substack{\alpha\\ \beta}}/\omega_{*e})|^2\right]}, \tag{103}$$

where $A(\beta,\beta_0) = (2\pi/\beta_0)(\beta_0^2/\beta^2)[1 - J_0(2\beta/\beta_0)]$. The notation indicates that the imaginary part of the dielectrics $\epsilon_{\rm Im}^{\rm ion}$ and $\epsilon_{\rm Im}$ are dependent on the variables α and β and the quantities $N_{\substack{\alpha\\ \beta}}/\omega_{*e}$ and $\beta_{\substack{\alpha\\ \beta}}/N_{*e}$, which are the normalized turbulent decorrelation and polarization, respectively. Examples of these dielectrics are given by $\epsilon_{\rm Im}^{E} = \mathrm{Re}\,R(\beta,\alpha)(\alpha - 1) = (\alpha - 1)N_{\substack{\alpha\\ \beta}}/(\alpha^2 + N_{\substack{\alpha\\ \beta}}^2/\omega_{*2}^2)$ for the electron contribution and Eq. (80) as illustrative of an ion dielectric. From dimensional analysis of Eq. (103) it follows that the high Reynolds number regime is typified by $\Delta\alpha \sim 1$, or equivalently, $\Delta\omega \sim \omega_{*e}$, $\beta \sim \beta_0 \sim 1$, $N_{\substack{k\\ \omega}} \sim \omega_{*e}$, and $\beta_{\substack{k\\ \omega}} \sim \omega_{*e}$. Accordingly, this regime is fundamentally a strongly turbulent regime with intrinsically broad linewidth.

The equivalence of the nonlinear relaxation rate with ω_{*e} effectively determines the saturation level. The nonlinear relaxation rate is

$$N_{\substack{k\\ \omega}} = \frac{T_e^2c^2}{e^2B^2}\sum_{k'_\vartheta,\omega'}\sum_m k_\vartheta^2 k_\vartheta'^2(2\pi m\hat{s})^2\frac{e^2}{T_e^2} \times \langle\varphi(\eta + 2\pi m)^2\rangle_{\substack{k'_\vartheta\\ \omega}}\frac{N_{\substack{k+k'\\ \omega+\omega'}}}{(\omega''^2 + N_{\substack{k+k'\\ \omega+\omega'}}^2)}.$$

From a mixing length approximation $(\omega'' \to 0)$,

$$N_{\substack{k\\ \omega}}^2 \sim k_\vartheta^2\rho_s^2c_s^2\hat{s}^2\bar{k}_\vartheta^2\left(\frac{2\pi\varphi(2\pi)}{\varphi(0)}\right)^2\frac{e^2\langle\varphi^2\rangle}{T_e^2}. \tag{104}$$

The condition $N_{\substack{k\\ \omega}} \sim \omega_{*e}$ then provides an approximate fluctuation level:

$$\frac{e\langle\varphi\rangle_{\rm rms}}{T_e} \sim \frac{\rho_s}{L_n}\left(\langle k_\vartheta^2\rho_s^2\rangle^{1/2}\hat{s}\frac{2\pi\varphi(2\pi)}{\varphi(0)}\right)^{-1}. \tag{105}$$

This estimate exhibits the standard drift wave scaling $e\varphi/T_e \sim \rho_s/L_n$ and with $\varphi(2\pi)/\varphi(0) = 0.2$, as previously discussed, and for typical wavenumbers $\langle k_\vartheta^2\rho_s^2\rangle^{1/2} \sim 0.3$. The saturation level is somewhat above that of the low Reynolds number regime and consistent with experimental measurements in the 30% range. Mixing length estimates are based on an assumption of maximal turbulent energy transfer and ignore details of the energy sink. If the sink removes energy more slowly than the turbulent transfer, energy will accumulate and the fluctuation level will be higher than that predicted from a mixing length approximation. Thus, the mixing length estimate represents a lower bound.

In the mixing length limit, no wavenumber dependence of the amplitude is obtained. The spectrum balance equation provides a further relation from which the asymptotic wavenumber dependences of the saturated spectrum may be obtained. From the spectrum balance equation, Eq. (98), we obtain

$$\overline{\langle \tilde{H}(1)\tilde{H}(2)\rangle}_k \cong A(k)\frac{\rho_s^2}{L_n^2}\left(\langle k_\vartheta^2\rho_s^2\rangle^{1/2}\hat{s}\frac{2\pi\varphi(2\pi)}{\varphi(0)}\right)^{-2}M,$$

where we have assumed $(\delta n/n)_{\rm rms} \sim (\rho_s/L_n)[\langle k_\vartheta^2\rho_s^2\rangle^{1/2}\hat{s}2\pi\varphi(2\pi)/\varphi(0)]^{-1}$ from Eq. (102) and M is a number related to the dimensionless integral, Eq. (100). Referring back to the eigenfunction representation of the operators and the relationship between incoherent density and the potential, i.e., Eqs. (21), (24), and (26), $\langle \tilde{H}(1)\tilde{H}(2)\rangle_k = \langle\varphi(1)\varphi(2)\rangle_k\overline{\langle \tilde{H}(1)\tilde{H}(2)\rangle}_k$. With this relationship the spectrum balance equation is written

$$\langle \tilde{H}(1)\tilde{H}(2)\rangle_k = \bar{A}(k_\vartheta)\langle\hat{\varphi}(1)\hat{\varphi}(2)\rangle_{k_\vartheta}, \tag{106}$$

where

$$\bar{A}(k_\vartheta) = \frac{\rho_s^2}{L_n^2}\left(\langle k_\vartheta^2\rho_s^2\rangle^{1/2}\hat{s}\frac{2\pi\varphi(2\pi)}{\varphi(0)}\right)^{-2}MA(k_\vartheta).$$

We seek a relationship between the electron density δn and the incoherent nonadiabatic density $\widetilde{H}$. By definition, $\delta n = (1 + R_{k\omega})(e\varphi/T_e) + \widetilde{H}_{k\omega}$. Replacing φ with $\widetilde{H}$ through the quasineutral shielding condition, Eq. (19), yields

$$\delta n = (1 + L_{k\omega}^{-1} + R_{k\omega} L_{k\omega}^{-1})\widetilde{H}_{k\omega}. \tag{107}$$

The electron response $R_{k\omega}$, together with the ion operator and the adiabatic piece comprise the eigenoperator:

$$L_{k\omega}^{\text{ion}} - (R_{k\omega} + 1) = L_{k\omega}$$

or

$$R_{k\omega} = -L_{k\omega} + L_{k\omega}^{\text{ion}} - 1.$$

Substituting for $R_{k\omega}$ in Eq. (107), taking the complex conjugate square of both sides, and substituting for $|\widetilde{H}_{k\omega}|^2$ from Eqs. (99) and (106) gives

$$\langle \delta n^2 \rangle_{k\omega} = |L_{k\omega}^{\text{ion}} L_{k\omega}^{-1}|^2 \Big[N_{k\omega}/(\omega^2 + N_{k\omega}^2) \Big] |\hat{\varphi}_k(\eta)|^2 \overline{A}(k_\vartheta)$$

$$= \frac{N_{k\omega}}{(\omega^2 + N_{k\omega}^2)} \overline{A}(k_\vartheta) |\varphi(\eta)|^2 \left(1 + \frac{|\epsilon^E(k_\vartheta \omega)|^2}{|\epsilon^{\text{ion}}(k_\vartheta, \omega)|^2} \right)^{-1}. \tag{108}$$

Since $|\omega + i\Delta\omega| > v_{T_e}^2/(\nu_{ei}\Delta\eta_c^2)$, both species are hydrodynamic, $\epsilon \sim \omega_* /(\omega + iN_{k\omega})$, and the term in the large parentheses asymptotes to a constant in both limits $(k \to 0, \infty)$. The nonlinear decorrelation rate, $N_{k\omega}$, goes like $k_\vartheta^2 D$ and $\overline{A}(k_\vartheta)$ goes like k_ϑ^2. Thus

$$\langle \delta n^2 \rangle_{k_\vartheta \omega} \sim \begin{cases} 1/\omega^2 & (\omega \to \infty) \\ 1/k_\vartheta^4 & (k \to \infty) \end{cases} \tag{109}$$

integrating over ω, we obtain

$$\langle \delta n^2 \rangle_{k_\vartheta} \approx \overline{A}(k_\vartheta) |\varphi_{k_\vartheta}(\eta)|^2 \sim \begin{cases} 1/k_\vartheta^2 & (k \to \infty) \\ k_\vartheta & (k \to 0). \end{cases} \tag{110}$$

With these estimates, we may derive particle transport scaling relationships in the same way we did for the low Reynolds number regime. Summarizing these results we have

$$\Gamma_N \sim \rho_s^2 c_s / L_n^2 \tag{111}$$

and

$$D \sim \rho_s^2 c_s / L_n. \tag{112}$$

V. CONCLUSIONS

The thrust of this paper has been twofold. In working with this fluid model a theoretical framework for the analytical treatment of mode coupling equations such as the Hasegawa–Mima equation has been developed. In this theoretical framework the physics of the broad linewidth of the frequency spectrum has been retained in such a way as to illuminate the underlying causes and its relationship to other observable properties of the turbulence. Based on the equation for two-point density correlation, this theory is the extension of Dupree's theory of two-point phase space density correlation to fluid systems. Here, incoherent fluctuations in the density arising from the mode coupling contributions to the density at wavenumber k proportional to the potential at some other wavenumber k' are related to density granulations. The density granulations result from the fact that the relative turbulent diffusion vanishes as the spatial separation of two points decrease to zero. Driving granulations in the electron correlation is a source which is proportional to the ion dissipation. This proportionality is exact in the fluid theory and arises as a consequence of the fact that the total electron density (coherent and incoherent) is related to the ion dynamics by quasineutrality.

In addition, the effect of particle collisions on the density granulations has been included in this dissipative fluid model. Particle collisions are incorporated in the model as a parallel collisional viscosity, which results in a relative diffusion inversely proportional to the collision frequency. The magnitude of the parallel collisional viscosity relative to the $E \times B$ mixing introduces two limits and a parameter to differentiate them. These two limits are identified by a high Reynolds number regime (inertial range) in which the turbulent $E \times B$ diffusion dominates the linear parallel diffusion of the collisional viscosity and a low Reynolds number regime (dissipation range) in which the linear parallel collisional viscosity is dominant.

The Reynolds number is defined as $R_e = k^2 D (Rq)^2 \Delta\eta_c^2 \nu_{ei}/v_{T_e}^2 = \tau_\parallel/\tau_{E \times B}$ and is order unity for experiment. In the high Reynolds number regime the separation of trajectories of neighboring fluid elements is exponential. The lifetime of the small-scale correlation is logarithmic but not singular since the linear collisional diffusion eventually separates fluid elements. The incoherent correlation at scales below the clump scale is large, approaching a value of $\tau_c \ln(R_e) S$, as the separation vanishes, where τ_c is the one-point relaxation time, R_e is the Reynolds number, and S is the source. The linewidth in the high Reynolds number regime is intrinsically broad.

In the low Reynolds number regime, trajectories of neighboring fluid elements separate according to a power law. The incoherent correlation is nonzero below the clump scale but small. In this regime it is not possible to estimate the incoherent correlation from the evolution under the dominant diffusion process as the correlation involves a complicated combination of both dominant diffusion (parallel collisional viscosity) and the smaller $E \times B$ diffusion. In a Reynolds number expansion of the incoherent correlation, the lowest order term is first order in R_e, indicating that the incoherent correlation vanishes entirely as $R_e \to 0$.

Based on the theory of two-point density correlation, we present a study of tokamak edge turbulence, providing formulas for wavenumber and frequency spectra, fluctuation level, and particle transport for steady-state turbulence. For calculating spectrum properties in the low Reynolds number limit we adopt ion Compton scattering as the saturation mechanism. We find a linewidth which is small for most $R_e \lesssim 0.3$. As R_e approaches 1 it is possible for the linewidth to be as large as ω_{*e}. The wavenumber spectrum is computed by balancing the ion Compton scattering-damping rate with the linear growth rate enhanced by the incoherent emission, which is weak for $R_e < 1$. The spectrum is strongly peaked at the longest wavelengths and has a cutoff at $k_m = \rho_s^{-1}(1 + \hat{s}^2\pi^2/4)^{-1/2}$. Modes with wavenumbers above this cutoff are only weakly excited because of efficient scattering to long wavelengths which overcomes the linear growth. The steady-state fluctuation level scales as $(\rho_s/L_n)[\nu_{ei}c_s/(\omega_{T_e}^2 L_n)]^{1/12}$. The ρ_s/L_n scaling has been observed on many experiments. The scaling with other parameters is extremely weak, going as the 1/12 power. The amplitude is given approximately as $e\varphi/T_e \sim 2\rho_s/L_n$. The particle diffusion is outward and scales as $T_e^{1/6}n^{2/3}$.

For the large Reynolds number regime, strong turbulence estimates apply because of the strong excitation relative to linear relaxation. Properties of the spectrum are calculated from general mode coupling saturation in the mixing length approximation. In this regime the linewidth is of the order of or exceeds the diamagnetic drift frequency ω_{*e}. The fluctuation level is of the order of $e\varphi/T_e \sim 7\rho_s/L_n$. The particle diffusion scales identically as it does for $\mathrm{Re} < 1$.

In general terms, we find that linewidths on the order of ω_{*e} and fluctuation levels in the neighborhood of those observed experimentally for the tokamak edge are possible for dissipative drift waves. The scaling of the fluctuation level is consistent with experiment both in its dependence on ρ_s/L_n and its lack of dependence on other parameters, most notably ν_{ei}. The particle diffusion has very weak temperature dependence and moderate density dependence. This scaling is decidedly non-Bohm-like and seems to be suggestive of the type of diffusion scaling which is measured experimentally.

Recently, new experimental results[20,21] indicate that a large, rapidly varying radial electric field is present at the plasma edge. Inclusion of the effects associated with this electric field can *modify* the density-gradient-driven turbulence model proposed in this paper or introduce a *new* source of instability associated with $E_0 \times B_0$ velocity shear. Furthermore, the scale size associated with velocity shear may replace the radial mode width as the radial correlation length. These considerations will be discussed in a future publication.

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APPENDIX: SOLUTION OF THE SPECTRUM BALANCE EQUATION

Here we provide the details of the derivation of Eq. (63), starting with Eqs. (60)–(62). Substituting Eqs. (61) and (62) into Eq. (60) we perform the η_3 and η_4 integrations and take the derivatives in y_- to yield a factor of $(k^2/\kappa^2)e^{i\kappa\eta''}$. Completing the square in η'' permits the integrations in η'' and η'. The integration constants are chosen so that $\langle \widetilde{H} \rangle^2$ remains finite. This means that η' and η'' are integrated from $|\eta_+|$ to ∞ and $|\eta'|$ to ∞, respectively. The integrations yield

$$\widetilde{H}(y_-,\eta_-) = -\frac{(Rq)^4}{D_\parallel^2} D \frac{\sqrt{\pi}}{2} \int d\kappa \int dk\, e^{iky_-} e^{-y_-^2/2\alpha^2} \cos(k_0 y_-) \Delta\eta^2 \mathrm{i\,erfc}(|\eta_-|/\Delta\eta - i\kappa\Delta\eta/2) e^{-\kappa^2\Delta\eta^2/4}$$
$$\times \int dw (\omega - \omega_{*e}) \epsilon_{\mathrm{Im}}^{\mathrm{ion}}(\kappa\omega) \frac{|\hat{\varphi}_{\substack{k\\ \omega}}(\kappa_1)\hat{\varphi}_{\substack{-k\\ -\omega}}(\kappa_2)|}{|\epsilon(k,\omega)|^2} \overline{\langle \widetilde{H}^2 \rangle_{\substack{k\\ \omega}}}, \tag{A1}$$

where

$$\mathrm{i\,erfc}(Z) = \int_Z^\infty \mathrm{erfc}(t)\,dt = -Z\,\mathrm{erfc}(Z) + \frac{1}{\sqrt{\pi}} e^{-Z^2}, \tag{A2}$$

and $\mathrm{erfc}(Z)$ is the complimentary error function.[22] We take the Fourier transform in η_-, η_+, and y_- of both sides of Eq. (A1). On the left-hand side we have

$$\int d\eta_+\, e^{-i\kappa_+\eta_+} \int d\eta_-\, e^{-i\kappa\eta_-} \int dy_-\, e^{-iky_-} \widetilde{H}(y_-,\eta_+) = \langle \widetilde{H}^2(\kappa) \rangle_k.$$

On the right-hand side we write $\cos(k_0 y_-) = [\exp(ik_0 y_-) + \exp(-ik_0 y_-)]/2$ and complete the square in y_-. Integrating in y_- we obtain

$$\langle \widetilde{H}^2(\kappa) \rangle_k = \frac{-(Rq)^4}{D_\parallel^2} D \frac{\sqrt{\pi}}{2} \int d\eta_-\, e^{-i\kappa\eta} \int d\kappa' \int dk'\, \Delta\eta^2 \frac{\alpha}{\sqrt{2}} \left(e^{-(k-k'-k_0)^2\alpha^2/2} + e^{-(k-k'+k_0)^2\alpha^2/2}\right) \mathrm{i\,erfc}\left(\frac{|\eta_-|}{\Delta\eta} - \frac{i\kappa'\Delta\eta}{2}\right)$$
$$\times \left(\frac{k'^2}{\kappa'^2}\right) e^{-k'^2\Delta\eta^2/4} \int dw' (\omega' - \omega'_{*e}) \frac{\epsilon_{\mathrm{Im}}^{\mathrm{ion}} |\hat{\varphi}^2|_{\substack{k'\\ \omega'}} \langle \widetilde{H}^2 \rangle_{\substack{k'\\ \omega'}}}{|\epsilon|^2}. \tag{A3}$$

The integration over η_- may be carried out next using the identity

$$\int d\eta' \, e^{-i\eta'\kappa} \mathrm{i\,erfc}\left(\frac{\Delta}{2}|\eta'| - \frac{iu}{\Delta}\right) = -\frac{\Delta}{2} e^{u^2/\Delta^2} \frac{1}{\kappa^2}\left[\mathrm{erfc}\left(-\frac{i}{\Delta}(u+\kappa)\right) e^{-(u-\kappa)^2/\Delta^2}\right]$$

$$(\kappa \neq 0). \tag{A4}$$

Upon taking the real part we have

$$\langle \tilde{H}^2 \rangle_k = \frac{(Rq)^4}{D_\parallel^2} D\alpha\Delta\eta \frac{\sqrt{\pi}}{2} \int d\kappa' \int dk' \, e^{-(\kappa-\kappa')^2\Delta\eta^2/4} \times \frac{k'^2}{\kappa^2\kappa'^2}\left(e^{-(k-k'-k_0)^2\alpha^2/2} + e^{-(k-k'+k_0)^2\alpha^2/2}\right) \times \int d\omega'(\omega' - \omega'_{*e}) \frac{\epsilon_{\mathrm{Im}}^{\mathrm{ion}} |\hat{\varphi}|^2_{\substack{k' \\ \omega'}}}{|\epsilon|^2} \overline{\langle \tilde{H}^2 \rangle}_{\substack{k' \\ \omega'}}. \tag{A5}$$

The variable κ' is the "wavenumber" corresponding to the relative parallel variation η_-. The squared wavefunction $|\hat{\varphi}|^2_{\substack{k' \\ \omega'}}$ depends on both κ' and κ_+, where κ_+ corresponds to η_+. We assume that the wavefunction is Gaussian in η with width Δ_k^{-1}. Then

$$|\varphi_k(\eta_1)\varphi_{-k}(\eta_2)| \cong e^{-\eta_1^2\Delta_k^2} e^{-\eta_2^2\Delta_k^2} = e^{-2\eta_+^2\Delta_k^2} e^{-2\eta_-^2\Delta_k^2}$$

and

$$|\varphi^2(\kappa + \kappa')| = e^{-\kappa_+^2/8\Delta_k^2} e^{-\kappa'^2/8\Delta_k^2}.$$

With this assumption we may perform the integration over κ' by completing the square and using the identity

$$\int d\kappa \frac{1}{\kappa^2} e^{-(\kappa-u)^2/\Delta^2} = -\left(\frac{2}{\Delta}\right) e^{-u^2/\Delta^2} \mathrm{i\,erfc}\left(-\frac{iu}{\Delta}\right). \tag{A6}$$

Upon taking the real part we arrive at Eq. (63) of the text.

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