

《数字图像处理》测试题

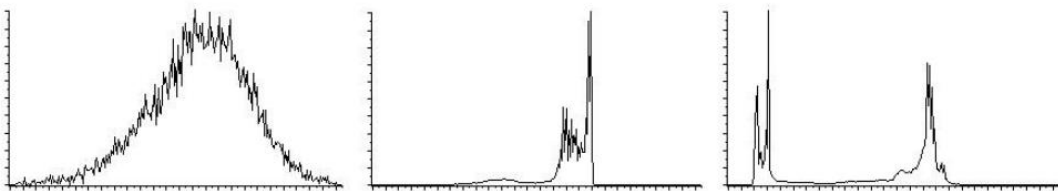
(测试时间：共 1 小时)

方向：_____ 姓名：_____ 学号：_____

1. (20 pts) In an image compression system, 192000 bits are used to represent a 800x600 image with 256 gray levels. What is the compression ratio for this system?

$$\frac{192000}{800 \times 600 \times 8} \times 100\% = 5\%$$

2. (20 pts) Among the 3 proposed histograms, which is the one corresponding to the shown image? Why?



A

B

C

Histogram A does not show a peak for the background, so it can't be the one. The designated pixel is the darkest in the image, but there are quite a lot of pixels in Histogram C that are darker. So the right histogram is B. Other arguments: in the upper left corner is the transition between the tool and the background very smooth, which means that gray levels are close to each other in that region. It justifies that the 2 peaks corresponding to the 2 objects are close to each other.

3. (30 pts)

$$\begin{pmatrix} 1 & 2 & 0 \\ 4 & 1 & 2 \\ 0 & 4 & 7 \end{pmatrix}$$

- a) Filtering the given 3×3 gray level image with 3×3 median filter (Zero Padding)

0	1	0
1	2	1
0	1	0

- b) Design a spatial filter to sharpen the given 3×3 gray level image. Then compute the value of the center pixel in the filtered image.

0	-1	0
-1	5	-1
0	-1	0

the center pixel = -7
or

-1	-1	-1
-1	9	-1
-1	-1	-1

the center pixel = -11

4. (30 pts)

Consider a 3×3 spatial mask that averages the 4 neighbors of a point (x, y) including itself:

$$\begin{pmatrix} 0 & 1/8 & 0 \\ 1/8 & 1/2 & 1/8 \\ 0 & 1/8 & 0 \end{pmatrix}$$

- a) Find the equivalent filter $H(u, v)$ in the frequency domain.

Tips: $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$

The filtered function is given by

$$g(x, y) = \frac{1}{8}f(x + 1, y) + \frac{1}{8}f(x - 1, y) + \frac{1}{8}f(x, y + 1) + \frac{1}{8}f(x, y - 1) + \frac{1}{2}f(x, y).$$

From property 3 in Table 4.3,

$$\begin{aligned}
 G(u, v) &= \frac{1}{8} F(u, v) e^{\frac{j2\pi u}{M}} + \frac{1}{8} F(u, v) e^{-\frac{j2\pi u}{M}} + \frac{1}{8} F(u, v) e^{\frac{j2\pi v}{N}} + \frac{1}{8} F(u, v) e^{-\frac{j2\pi v}{N}} \\
 &\quad + \frac{1}{2} F(u, v) \\
 &= \frac{1}{8} \left[e^{\frac{j2\pi u}{M}} + e^{-\frac{j2\pi u}{M}} + e^{\frac{j2\pi v}{N}} + e^{-\frac{j2\pi v}{N}} + 4 \right] F(u, v) \\
 &= H(u, v) F(u, v)
 \end{aligned}$$

where $H(u, v)$ is the filter function:

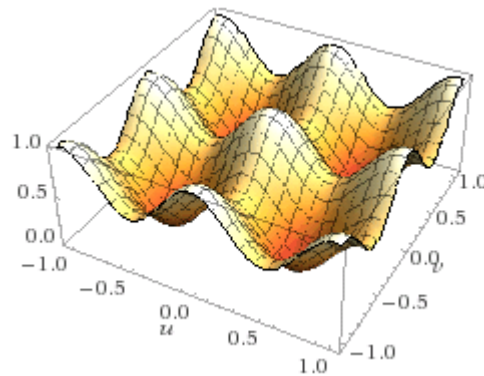
$$\begin{aligned}
 H(u, v) &= \frac{1}{8} \left[e^{\frac{j2\pi u}{M}} + e^{-\frac{j2\pi u}{M}} + e^{\frac{j2\pi v}{N}} + e^{-\frac{j2\pi v}{N}} + 4 \right] \\
 &= \frac{1}{4} \left[\cos\left(\frac{2\pi u}{M}\right) + \cos\left(\frac{2\pi v}{N}\right) + 2 \right]
 \end{aligned}$$

- b) What kind of filter it is? High-pass or low-pass?

Shifting the filter to the center of the frequency rectangle gives

$$H(u, v) = \frac{1}{4} \left[\cos \frac{2\pi \left[u - \frac{M}{2} \right]}{M} + \cos \frac{2\pi \left[v - \frac{N}{2} \right]}{N} + 2 \right]$$

When $(u, v) = (M/2, N/2)$ (the center of the shifted filter), $H(u, v) = 1$. For values away from the center, $H(u, v)$ decreases because of the order in which derivatives are taken. The important point is the dc term is preserved and other low frequencies are passed, which is the characteristic of a low-pass filter.



5. (optional +30 pts)

- a) Compute the 2-D discrete Fourier transform of the following image

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Tips:

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

$$\begin{aligned} F(0, 0) &= \frac{1}{16} \sum_{x=0}^3 \sum_{y=0}^3 f(x, y) e^{-j2\pi(0x/4 + 0y/4)} \\ &= \frac{1}{16} (1 + 0 + \dots + 2 + \dots + 1 + \dots) = 1/4 \end{aligned}$$

$$\begin{aligned} F(0, 1) &= \frac{1}{16} \sum_{x=0}^3 \sum_{y=0}^3 f(x, y) e^{-j2\pi(0x/4 + y/4)} \\ &= -1/8 \end{aligned}$$

.....

$$F(u, v) = \frac{1}{8} \begin{pmatrix} 2 & -1 & 2 & -1 \\ -j & 1+j & -j & 1+j \\ 0 & 1 & 0 & 1 \\ j & 1-j & j & 1-j \end{pmatrix}$$

- b) Filter the same image of previous question with the following frequency domain filter H

$$H = \frac{1}{4} \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$G(u, v) = F(u, v)H(u, v)$$

$$\begin{aligned} &= \frac{1}{8} \begin{pmatrix} 2 & -1 & 2 & -1 \\ -j & 1+j & -j & 1+j \\ 0 & 1 & 0 & 1 \\ j & 1-j & j & 1-j \end{pmatrix} \cdot \frac{1}{4} \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{32} \begin{pmatrix} 4 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
 g(x, y) &= \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} G(u, v) e^{j2\pi(ux/M + vy/N)} \\
 &= \frac{1}{16} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix}
 \end{aligned}$$