思考练习：

1、定解问题$\begin{cases} u_t - a^2 u_{xx} - b u_x - cu = 0, & x \in R, t > 0 \\ u(x,0) = \phi(x), & x \in R \end{cases}$

其中a, b, c是常数，关于x施行Fourier变换，原定解问题可转化为$\begin{cases} \hat{u}_t(\lambda, t) + (a^2\lambda^2 - ib\lambda - c)\hat{u}(\lambda, t) = 0 \\ \hat{u}(\lambda, 0) = \hat{\phi}(\lambda) \end{cases}$

2、对于积分$\frac{1}{2a\sqrt{\pi t}}\int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4a^2t}} y dy$,可令$\frac{x-y}{2a\sqrt{t}} = \eta$,积分可化为

$$\frac{1}{2a\sqrt{\pi t}}\int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4a^2t}} y dy = \frac{1}{2a\sqrt{\pi t}}\int_{+\infty}^{-\infty} e^{-\eta^2}(x - 2a\sqrt{t}\eta)(-2a\sqrt{t}) d\eta$$

$$= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{-\eta^2}(x - 2a\sqrt{t}\eta) d\eta = \frac{1}{\sqrt{\pi}}\left[\int_{-\infty}^{+\infty} e^{-\eta^2} x d\eta + \int_{-\infty}^{+\infty} e^{-\eta^2}(-2a\sqrt{t}\eta) d\eta\right]$$

$$= x + 0 = x$$