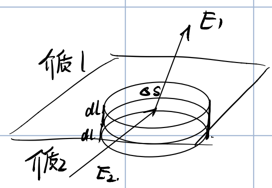
介质分界面 电磁场不连续.

└→ 利用 Maxwell 方程积分形式.

$$\boxed{\begin{aligned}\oint \vec{E}\cdot d\vec{l} &= -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}\\ \oint \vec{H}\cdot d\vec{l} &= I_f+\frac{d}{dt}\int \vec{D}\cdot d\vec{S}\end{aligned}}\ \Big\}\ \text{电磁场 切向关系.}$$

$$\boxed{\begin{aligned}\oint \vec{D}\, d\vec{S} &= Q_f\\ \oint B\, dS &= 0\end{aligned}}\ \Big\}\ \text{法向关系.}$$

法向分量场的突变关系.



$$\varepsilon_0 \oint E\, dS = Q_f + Q_p$$

$$\varepsilon_0 (E_{2n} - E_{1n})\Delta S = (\sigma_f + \sigma_p)\Delta S$$

$$\varepsilon_0 (E_{2n} - E_{1n}) = \sigma_f + \sigma_p$$

$$\sigma_p = -\vec{n}(\vec{P}_2 - \vec{P}_1) = -(P_{2n} - P_{1n})$$

$$\varepsilon_0 (E_{2n} - E_{1n}) + (P_{2n} - P_{1n}) = \sigma_f$$

$D = \varepsilon_0 E + P.$

$D_{2n} - D_{1n} = \sigma_f$ 　面电荷分布的高斯定理微分形式.

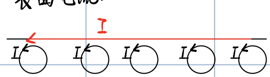
由于没有发现磁荷, 可直接推出 $B_{2n} = B_{1n}$

$$\begin{cases} D_{2n} - D_{1n} = \sigma_f \\ B_{1n} = B_{2n} \end{cases}$$

电场法向分量不连续.
磁场法向分量连续.

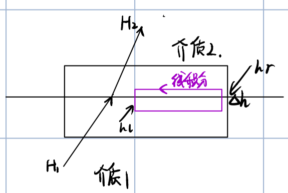
切向分量突变关系.

表面电流.



把薄层电流当作面电流

引入线密度 α 　通过 Δl 段的电流, $I = \alpha \Delta l$



$$\oint \vec{H}\cdot d\vec{l} = I_f + \frac{d}{dt}\int \vec{D}\cdot d\vec{S}$$

$$\oint \vec{H}\cdot d\vec{l} = (H_{2t} - H_{1t})\Delta l + (H_{1n} + H_{2n})\frac{\Delta h_l}{2} - (H_{1n} + H_{2n})\frac{\Delta h_r}{2},\quad \text{取 } h_l = h_r.$$

$$= (H_{2t} - H_{1t})\Delta l$$

$$\int \vec{D}\cdot d\vec{S} \longrightarrow 0 \quad (ds \to 0)$$

$$I_f = \alpha_f \Delta l$$

$\Rightarrow H_{2t} - H_{1t} = \alpha_f$

$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}$ （写成矢量）

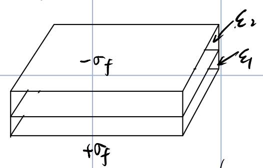
同理. $\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$

$\begin{cases} \vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0 & \text{电场 切向分量连续.} \\ \vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha} & \text{磁场 切向分量不连续.} \end{cases}$

（边界处安培环路定理）

$$\Rightarrow \begin{cases} \vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0 \\ \vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha} \\ \vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma \\ \vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \end{cases}$$

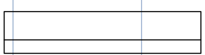
eg.



求 E 与 束缚电荷分布

$\begin{cases} \vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0 \\ \vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma \end{cases}$ E 只有垂直分量

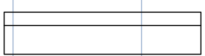
极板内部 $E = 0$



$\vec{n} \cdot (\vec{D}_1 - \vec{D}_{导}) = \sigma_f$

$D_{导} = 0$

$\vec{D}_1 = \sigma_f$



$D_2 = \sigma_f$

$\begin{cases} E_1 = \dfrac{\sigma_f}{\varepsilon_1} \\ E_2 = \dfrac{\sigma_f}{\varepsilon_2} \end{cases}$

介质表面, $\sigma_f = 0$　束缚

$\varepsilon_0 (E_{2n} - E_{1n}) = \sigma_f + \sigma_p$

$\varepsilon_0 \left(\dfrac{\sigma_f}{\varepsilon_1} - \dfrac{\sigma_f}{\varepsilon_2} \right) = \sigma_p$

导体的 σ_f.

求导体、介质 接触面 σ

σ_p^b (下极板) $= \varepsilon_0 E_1 - \sigma_{f} = -(1 - \varepsilon_0/\varepsilon_1)\sigma_f$

σ_p^u (上极板) $= \sigma_f - \varepsilon_0 E_2 = (1 - \varepsilon_0/\varepsilon_2)\sigma_f$.

$\sigma_p^u + \sigma_p^l + \sigma_p = 0$, 电荷守恒.