§ 4-2 谐振动的合成

一、两同方向、同频率谐振动的合成

$$\text{分振动}\left.\begin{aligned} x_1 &= A_1\cos(\omega t+\varphi_1) \\ x_2 &= A_2\cos(\omega t+\varphi_2) \end{aligned}\right\} \longrightarrow \text{合振动 } x = x_1 + x_2\,?$$

a)解析法

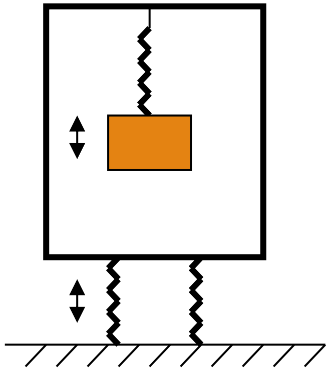
$$\cos(\omega t+\varphi_1) = \cos\varphi_1\cos\omega t - \sin\varphi_1\sin\omega t$$

$$\cos(\omega t+\varphi_2) = \cos\varphi_2\cos\omega t - \sin\varphi_2\sin\omega t$$

$$x = \underbrace{(A_1\cos\varphi_1 + A_2\cos\varphi_2)}_{A\cos\varphi}\cos\omega t - \underbrace{(A_1\sin\varphi_1 + A_2\sin\varphi_2)}_{A\sin\varphi}\sin\omega t$$

$$x = A\cos\varphi\cos\omega t - A\sin\varphi\sin\omega t$$

$$= A\cos(\omega t+\varphi)$$



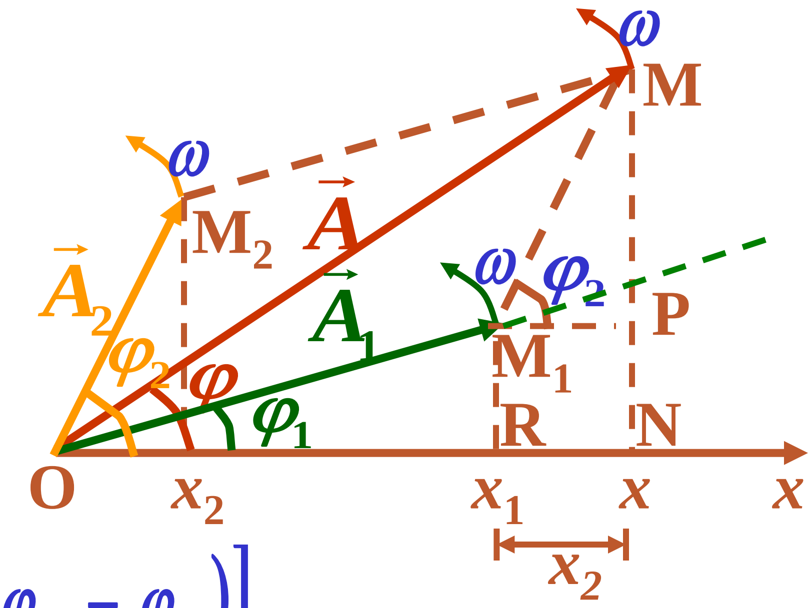
合振动是简谐振动，其频率仍为ω

b)旋转矢量法

$$x = x_1 + x_2$$

$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$x = A\cos(\omega t + \varphi)$$



$$A^2 = A_1^2 + A_2^2 - 2A_1A_2\cos\left[180^0 - (\varphi_2 - \varphi_1)\right]$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)}$$

$$\mathrm{tg}\,\varphi = \frac{\overline{MN}}{\overline{ON}} = \frac{\overline{MP} + \overline{PN}}{\overline{OR} + \overline{RN}}$$

$$\mathrm{tg}\,\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$$

讨论 $A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$

相位差对合振动起重要作用！

1) 两分振动同相：$\Delta\varphi=\varphi_2-\varphi_1=2k\pi, k=0,\pm1,\pm2,\cdots$

$$\cos(\varphi_2-\varphi_1)=1$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2}=A_1+A_2=A_{max}$$ 振动加强

若 $A_1=A_2$ 则 $A=2A_1$

2) 两分振动反相：$\Delta\varphi=\varphi_2-\varphi_1=(2k+1)\pi, k=0,\pm1,\pm2,\cdots$

$$\cos(\varphi_2-\varphi_1)=-1$$

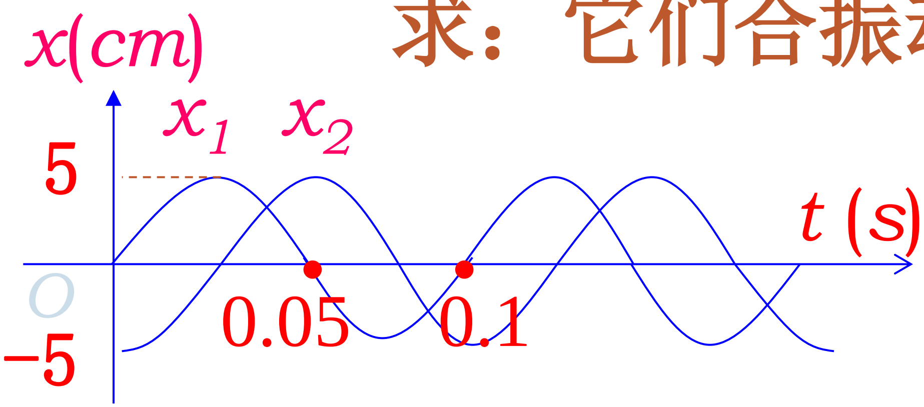
$$A=\sqrt{A_1^2+A_2^2-2A_1A_2}=|A_1-A_2|=A_{min}$$ 振动减弱

若 $A_1=A_2$ 则 $A=0$ 质点静止

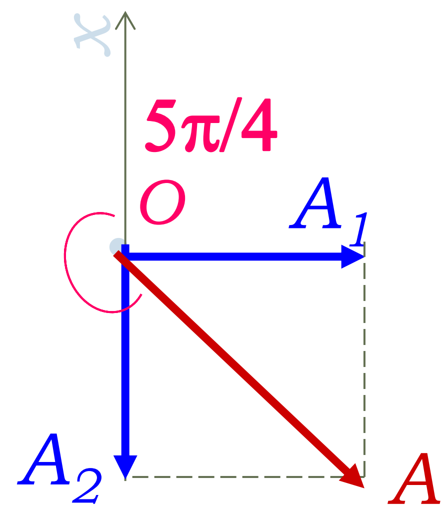
3) $\Delta\varphi=\varphi_2-\varphi_1\neq k\pi, k=0,\pm1,\pm2,\cdots$

$$A_{min}<A<A_{max} \qquad |A_1-A_2|<A<A_1+A_2$$

[例题4-6]两同频率谐振动曲线如图所示，求：它们合振动方程



利用矢量图求谐振合成



解： 由谐振曲线图知：

A=5cm, T=0.1s

$$\varphi_1 = -\frac{\pi}{2} \qquad \varphi_2 = \pi$$

$$A = 5\sqrt{2}cm \qquad \varphi = -\frac{3}{4}\pi$$

$$x_1 = 5\cos(20\pi t - \pi / 2)$$

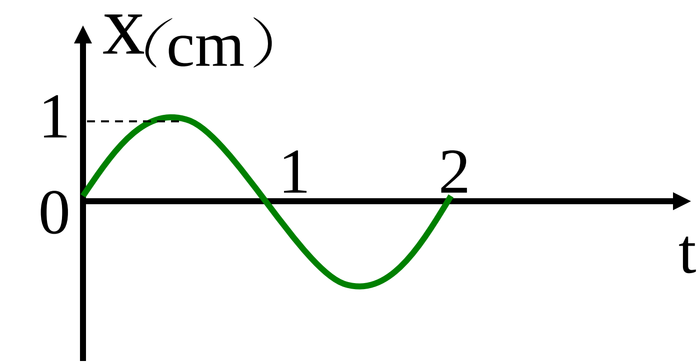
$$x_2 = 5\cos(20\pi t + \pi)$$

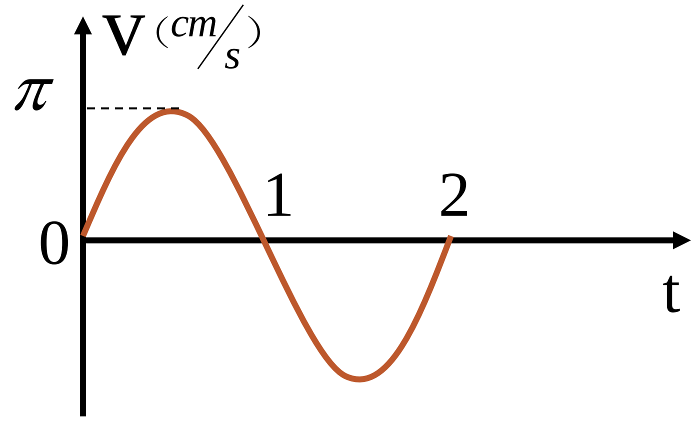
$$x = x_1 + x_2$$

$$= 5\sqrt{2}\cos(20\pi t - 3\pi / 4)cm$$

[例]:两同方向,同频率的简谐振动,振动1的$x \sim t$曲线及振动2的$v \sim t$曲线如图所示.

求:(1)$\varphi_2 - \varphi_1$　　(2)$A_{合}$





解(1).

$$\because t = 0 \begin{cases} x_{10} = 0 \\ v_{10} > 0 \end{cases} \quad \therefore \varphi_1 = -\frac{\pi}{2}$$

$$\because t = 0$$

$v_{20} = 0$且将增大(向正方向)

$$\therefore \varphi_2 = \pi$$

$$则: \varphi_2 - \varphi_1 = \pi - (-\frac{\pi}{2}) = \frac{3}{2}\pi(或 -\frac{\pi}{2})$$

(2) $A_{合} = ?$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

$$A_1=1$$

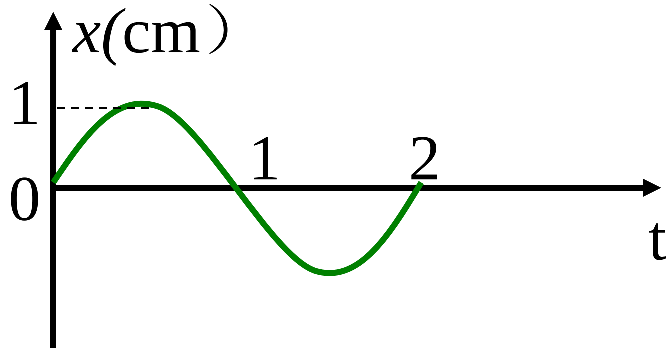
$$\varphi_2-\varphi_1=-\frac{\pi}{2}$$

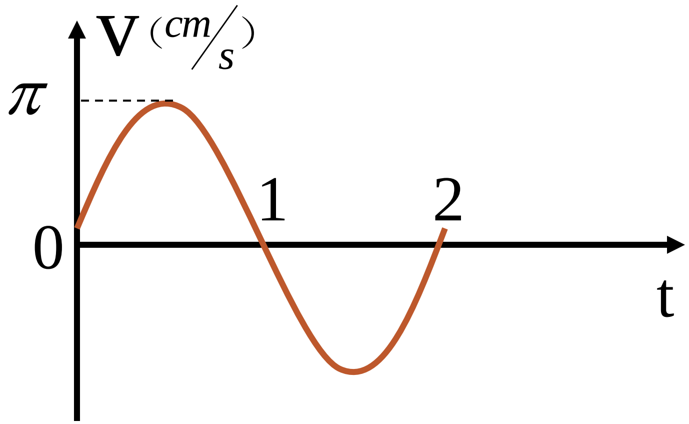
$$\because v_{2\max}=A_2\omega=\pi$$

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{2}=\pi$$

$$A_2=1$$

$$A=\sqrt{2}cm$$





二、 同方向不同频率谐振动的合成　　拍

1、一般情况

合成后的振动不再是谐振动，而是比较复杂的周期运动。

2、特殊情况($\omega_1 \approx \omega_2$)——拍现象

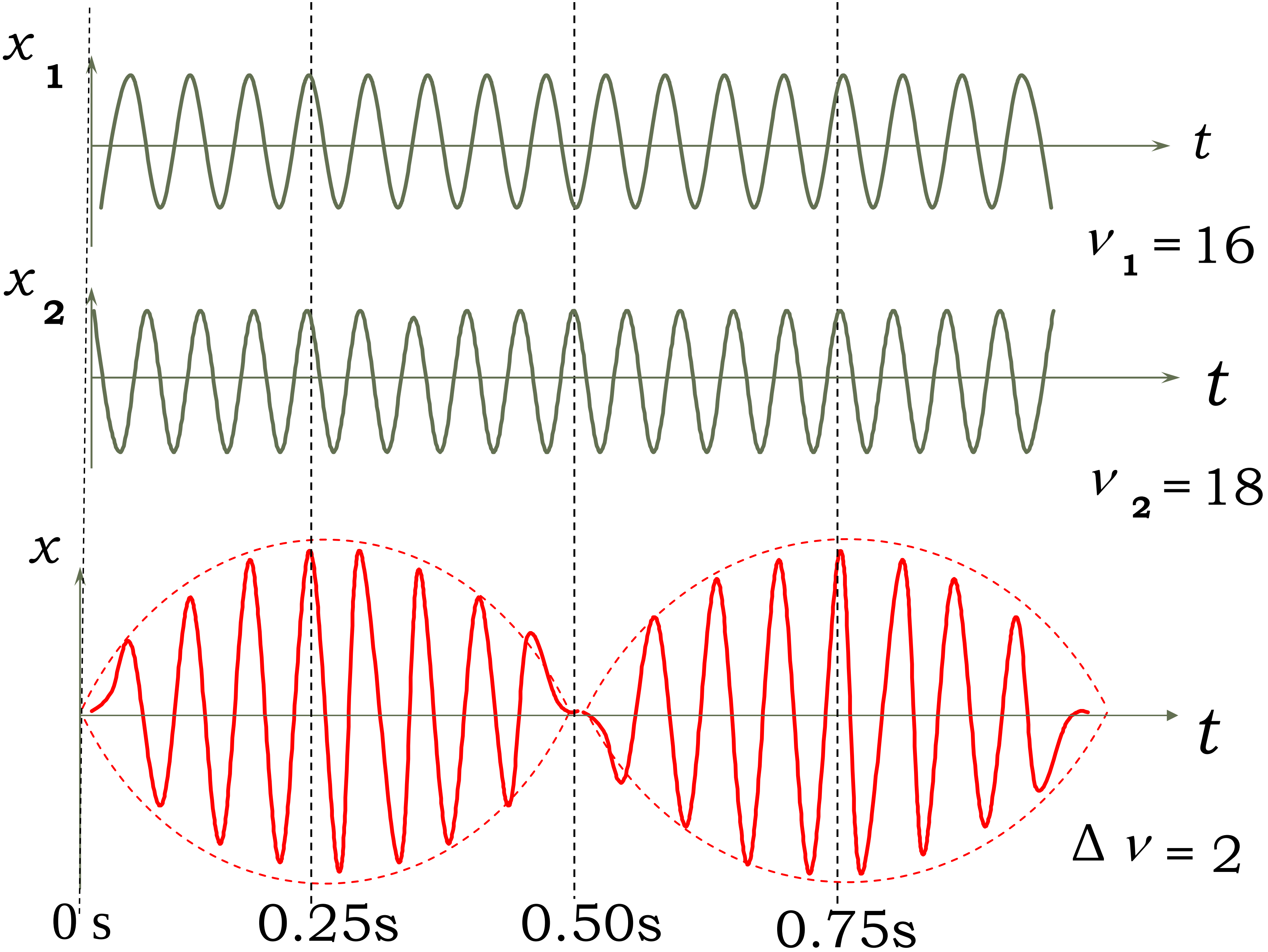
$$\left.\begin{aligned} x_1 &= A\cos(\omega_1 t+\varphi) \\ x_2 &= A\cos(\omega_2 t+\varphi) \\ x &= x_1 + x_2 \end{aligned}\right\}$$

$$x = \left(2A\cos\frac{\omega_2-\omega_1}{2}t\right)\cos\left(\frac{\omega_2+\omega_1}{2}t+\varphi\right)$$

$$= A'\cos\left(\frac{\omega_2+\omega_1}{2}t+\varphi\right)$$

（1）合振动的振幅随时间发生周期性变化。

（2）单位时间内振动加强或减弱的次数（拍频）$\nu = \nu_2 - \nu_1$


x_1
t
$\nu_1 = 16$
x_2
t
$\nu_2 = 18$
x
t
$\Delta\nu = 2$
0 s
0.25s
0.50s
0.75s

三、两互相垂直谐振动的合成

1、同频率谐振动的合成

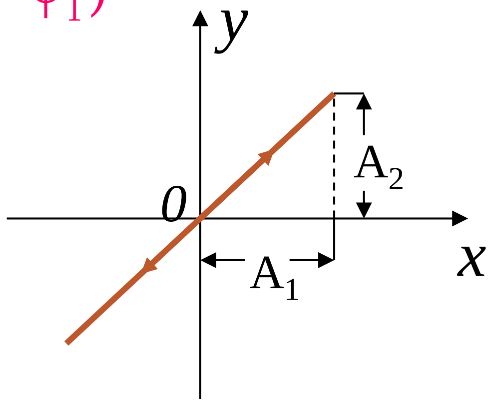
$$\begin{cases} x = A_1\cos(\omega t+\varphi_1) \\ y = A_2\cos(\omega t+\varphi_2) \end{cases}$$

合振动的轨迹方程

$$\left(\frac{x}{A_1}\right)^2+\left(\frac{y}{A_2}\right)^2-2\left(\frac{xy}{A_1A_2}\right)\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

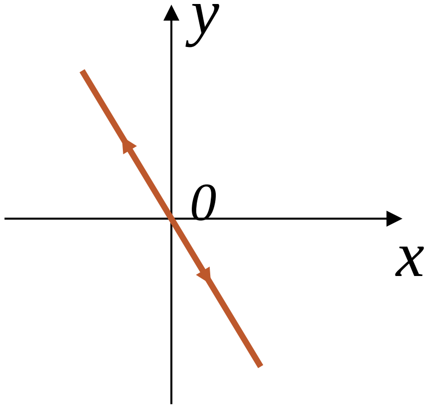
讨论： (1) $\varphi_2-\varphi_1=0$

$$\left(\frac{x}{A_1}\right)^2+\left(\frac{y}{A_2}\right)^2-2\left(\frac{xy}{A_1A_2}\right)=0 \Rightarrow y=\frac{A_2}{A_1}x$$



(2) $\varphi_2-\varphi_1=\pi$

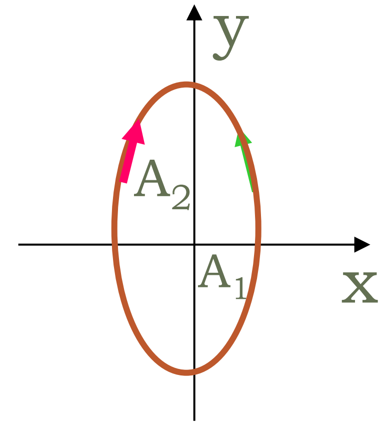
$$\left(\frac{x}{A_1}\right)^2+\left(\frac{y}{A_2}\right)^2+2\left(\frac{xy}{A_1A_2}\right)=0 \Rightarrow y=-\frac{A_2}{A_1}x$$



（3） $\varphi_2 - \varphi_1 = \dfrac{\pi}{2}$ （y分振动超前$\dfrac{\pi}{2}$）

$$\left(\frac{x}{A_1}\right)^2 + \left(\frac{y}{A_2}\right)^2 = 1$$

运行方向：**顺时针**

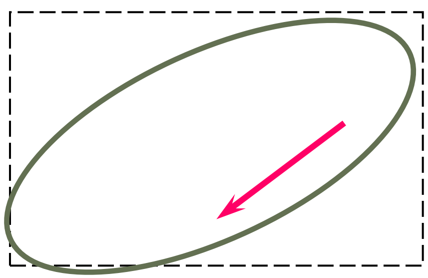


（4） $\varphi_2 - \varphi_1 = -\dfrac{\pi}{2}$ （y分振动滞后$\dfrac{\pi}{2}$）

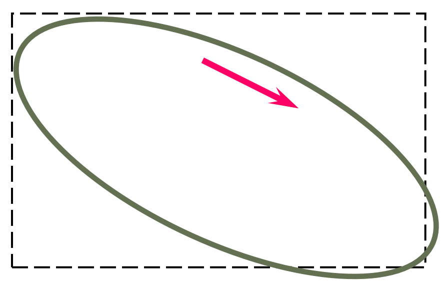
运行方向：**逆时针**

$$\left(\frac{x}{A_1}\right)^2 + \left(\frac{y}{A_2}\right)^2 - 2\left(\frac{xy}{A_1 A_2}\right)\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$$

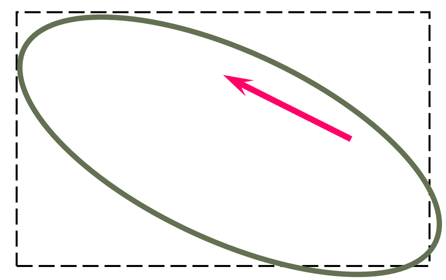
（5） $\varphi_2 - \varphi_1 =$ 其它值



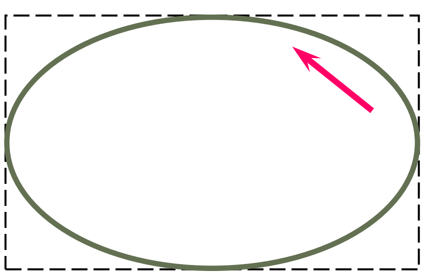
$\dfrac{\pi}{4}$



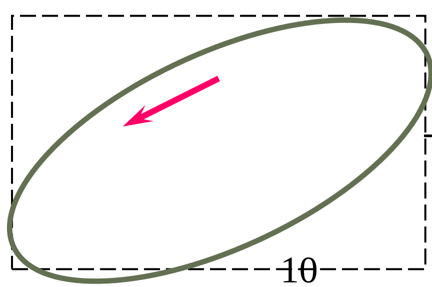
$\dfrac{3\pi}{4}$



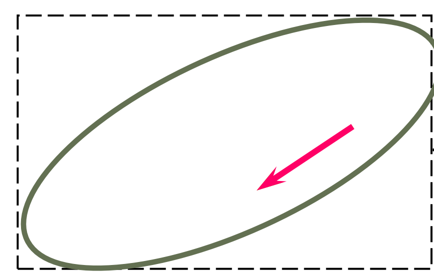
$\dfrac{5\pi}{4}$



$\dfrac{3\pi}{2}$



$\dfrac{7\pi}{4}$



$\dfrac{9\pi}{4}$

2、不同频率谐振动的合成

李萨如图：由成（简单）整数比的两个垂直方向的谐振合成而形成封闭、稳定的曲线