算符的基本运算

$$\left[\frac{\partial}{\partial x}, f(x)\right]\psi = \frac{\partial}{\partial x}\left(f(x)\psi(x)\right) - f(x)\left(\frac{\partial}{\partial x}\psi(x)\right)$$

$$= \frac{\partial f}{\partial x}\psi + f\cdot\frac{\partial \psi}{\partial x} - f\frac{\partial \psi}{\partial x}$$

$$= \frac{\partial f}{\partial x}\psi$$

$$\Rightarrow \left[\frac{\partial}{\partial x}, f(x)\right] = \frac{\partial f}{\partial x}$$

特例 $\left[\frac{\partial}{\partial x}, x\right] = 1$

$$\left[\frac{\partial}{\partial x}, f(x)\right] = \underbrace{\frac{\partial}{\partial x}(f(x))}_{\text{算符积}} - f(x)\frac{\partial}{\partial x}$$

$$= \underbrace{\frac{\partial f}{\partial x}}_{\text{偏导数}} + f\frac{\partial}{\partial x} - f\frac{\partial}{\partial x} = \frac{\partial f}{\partial x}$$

*. 运算规则

$$[A, B+C] = [A,B] + [A,C]$$

$$[A, BC] = B[A,C] + [A,B]C$$

$$[AB, C] = A[B,C] + [A,C]B$$

*. 算符的逆

$$A\psi = \varphi \quad ①$$

$$\Rightarrow A^{-1}\varphi = \psi \quad ②$$

$$A^{-1}A = I \leftarrow \text{单位算符}$$

$$AA^{-1} = I$$

* 算符转置

$$\int \psi^*(\tilde{o}\varphi)\,d\vec{x} = \int \varphi\, o\, \psi^*\, d\vec{x}$$

$$(\psi, \tilde{o}\varphi) = (\varphi^*, o\psi^*) = (\tilde{o}^*\psi, \varphi)$$

e.g. $$\int \varphi \frac{\partial}{\partial x}\psi^* dx = \int \varphi\, d\psi^* = \varphi\psi^*\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty}\psi^*\frac{\partial}{\partial x}\varphi\, dx$$

束缚态，为0.

$$= -\int_{-\infty}^{+\infty}\psi^*\frac{\partial}{\partial x}\varphi\, dx$$

故. $$\widetilde{\frac{\partial}{\partial x}} = -\frac{\partial}{\partial x}$$

定义. 厄密共轭. $O^{+} = (\tilde{O})^{*}$ （ $\tilde{O}^{*} = \widetilde{O^{*}}$ ）

$$\left(\psi, O^{+}\varphi\right) = (\psi, \widetilde{O^{*}}\varphi) = (O\psi, \varphi)$$

O是Hermit算符. $(\psi, O\varphi) = (O\psi, \varphi) \Rightarrow O^{+} = O$

证明. $P_x^{+} = P_x$, 即 P_x是厄密算符

$P_x \Rightarrow -i\hbar\frac{\partial}{\partial x}$ $\qquad \tilde{P}_x = i\hbar\frac{\partial}{\partial x}$

$$(\tilde{P}_x)^{*} = -i\hbar\frac{\partial}{\partial x} = P_x^{+} = P_x.$$