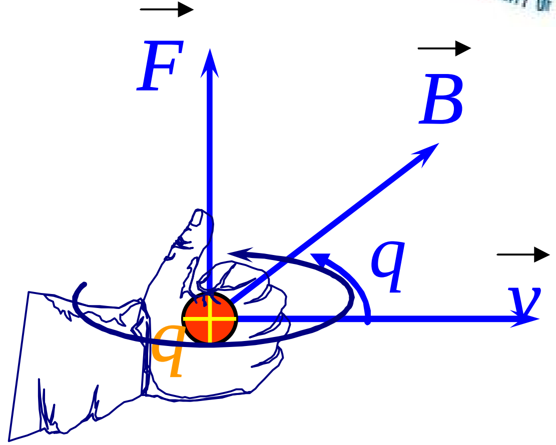
# 10.5磁场对运动电荷的作用

## 10.5.1 洛仑兹力—运动电荷在磁场中受力

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

$$\begin{cases} q > 0, \vec{F}\text{方向与}\vec{v} \times \vec{B}\text{一致} \\ q < 0, \vec{F}\text{方向与}\vec{v} \times \vec{B}\text{相反} \end{cases}$$

$\vec{F}$恒$\perp\vec{v}$表明$\vec{F}$只改$\vec{v}$方向




## 10.5.2 带电粒子在匀B中的运动

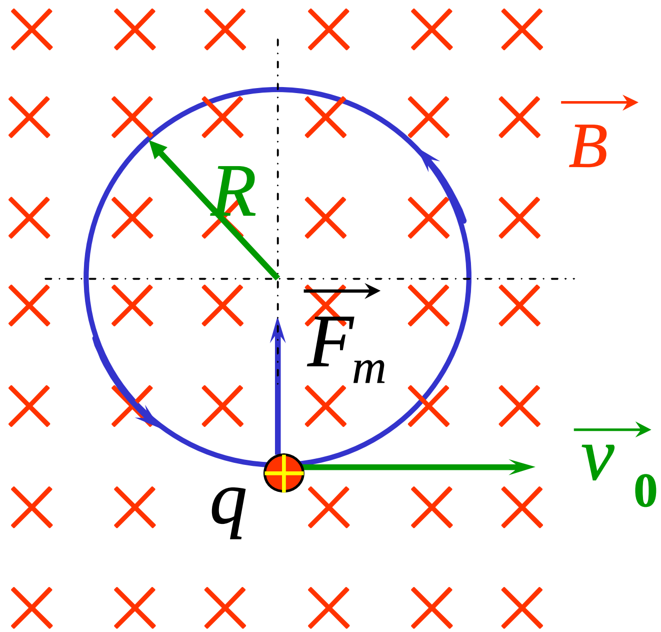
$F = ma$, 方程: $q\vec{v} \times \vec{B} = m\vec{a}$

(1)$\vec{v}_0 \perp \vec{B}$ ∵受力$f = qv_0B(= mv_0^2/R)$

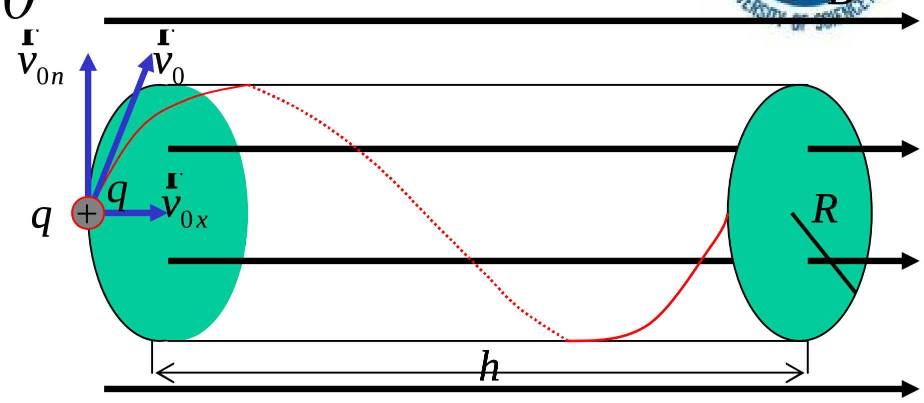
，轨迹 圆$R = mv_0/qB$

∴周期 $T = 2\pi R/v_0$

$= 2\pi m/qB$



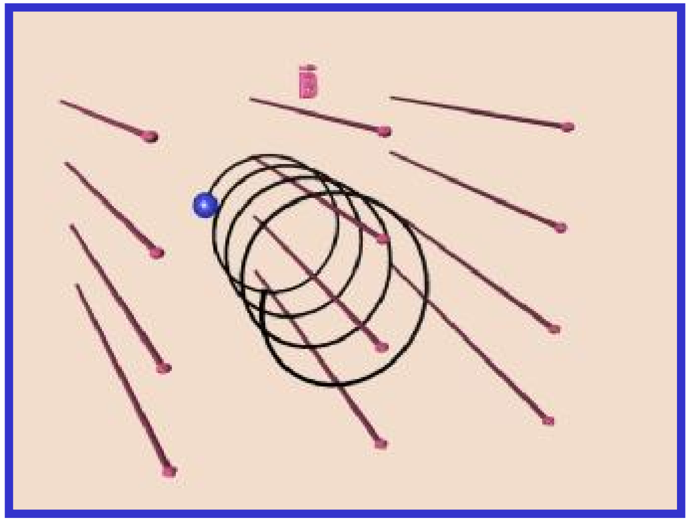
$$(2)\ \vec{v}_0 \text{与} \vec{B} \text{成} \theta \text{角} \begin{cases} v_{//} = v_0\cos\theta \\ v_{\perp} = v_0\sin\theta \end{cases}$$



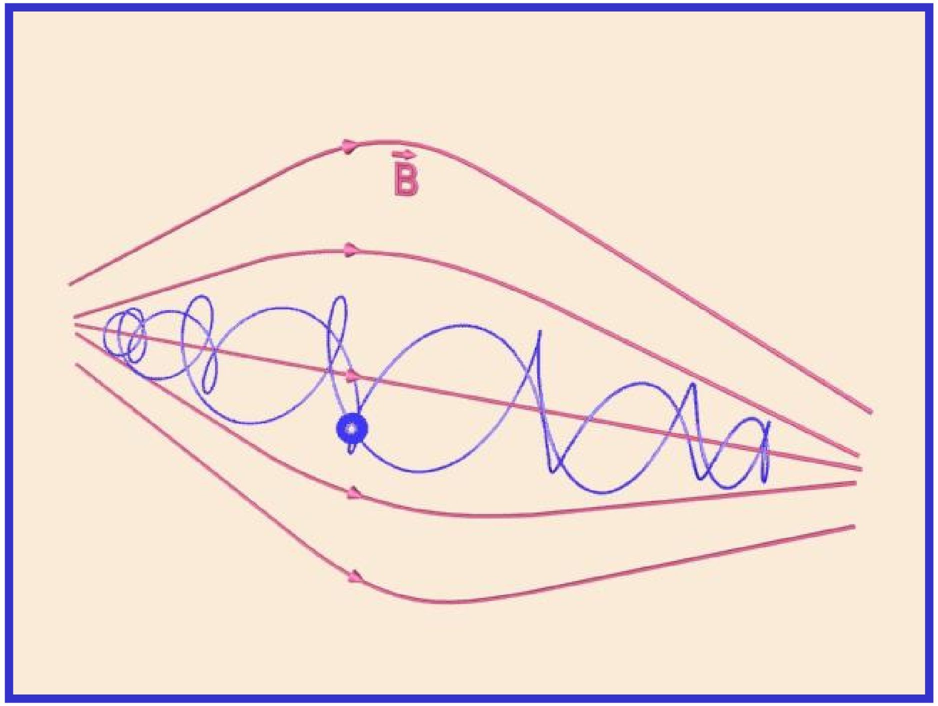

$$R = \frac{mv_{\perp}}{qB} = \frac{mv_0 \sin\theta}{qB}$$

$$T = \frac{2\pi R}{v_{\perp}} = \frac{2\pi m}{qB}$$

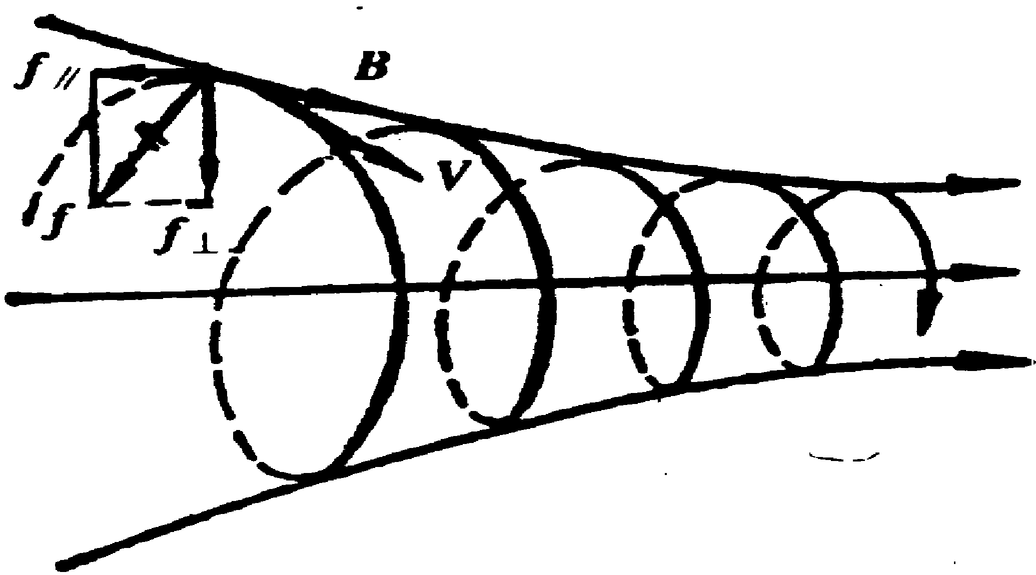
$$\text{螺距} h = v_{//} T = \frac{2\pi m v_0 \cos\theta}{qB}$$



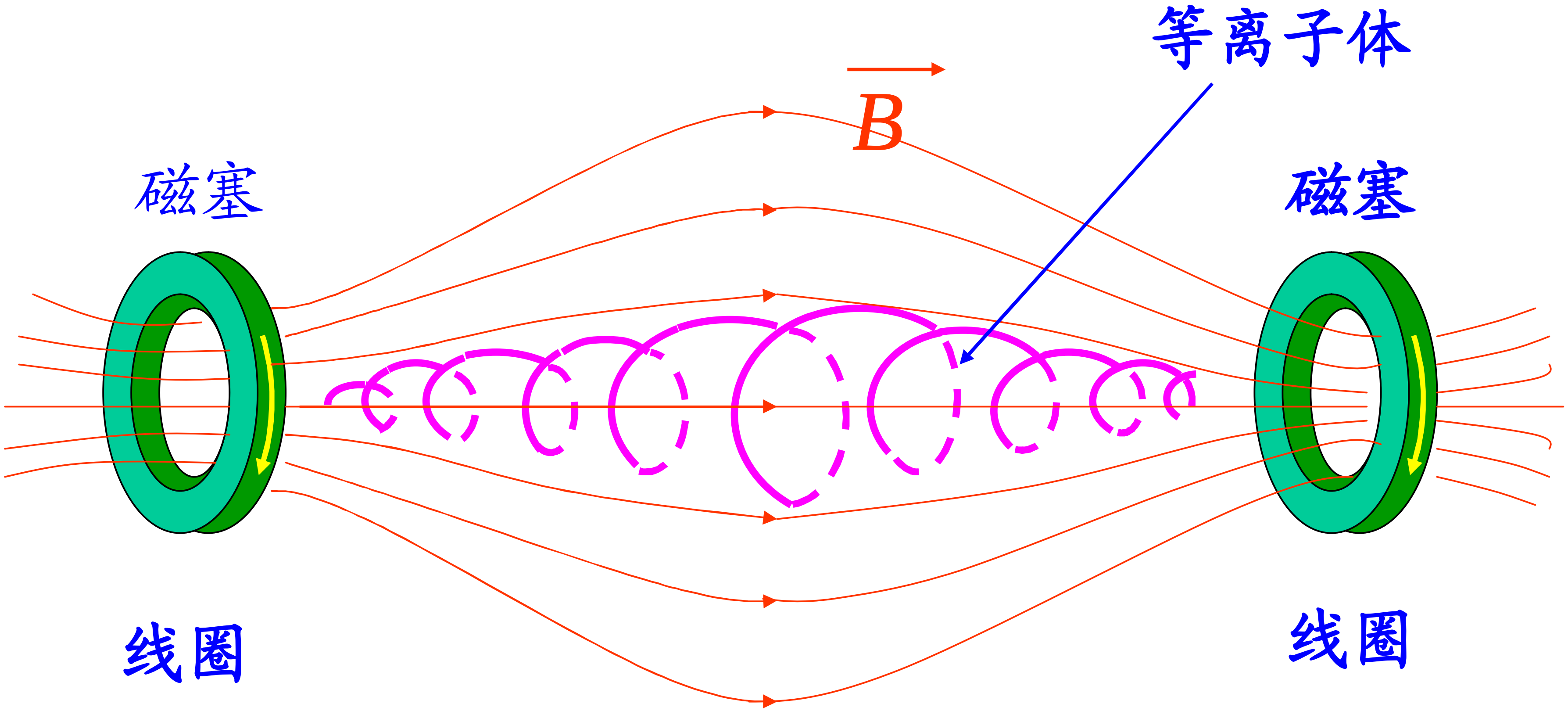
## 10.5.3 带电粒子在非均匀磁场中运动



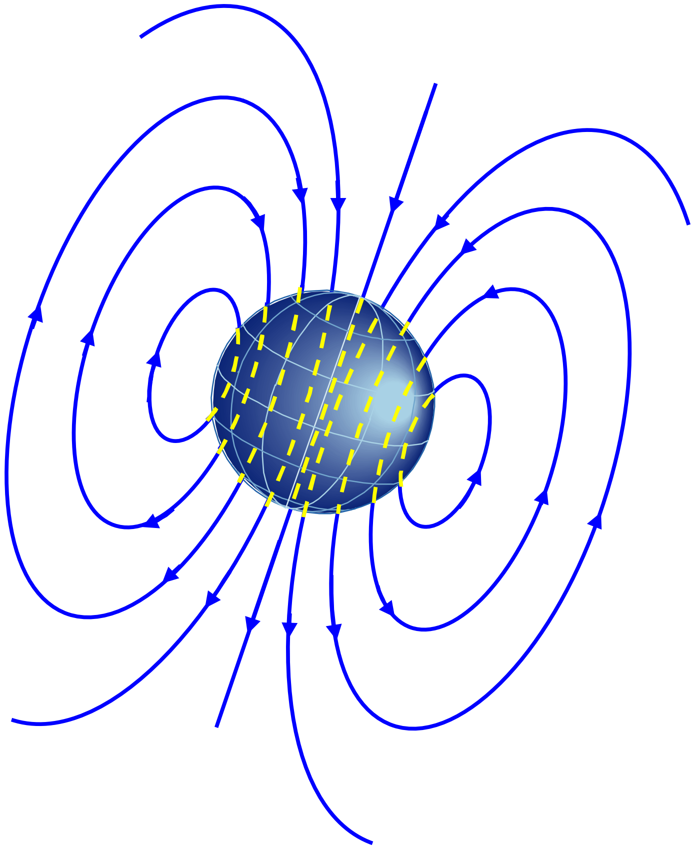

粒子受到一个与运动方向相反的力$F_x$，此力阻止粒子向磁场增强方向运动.

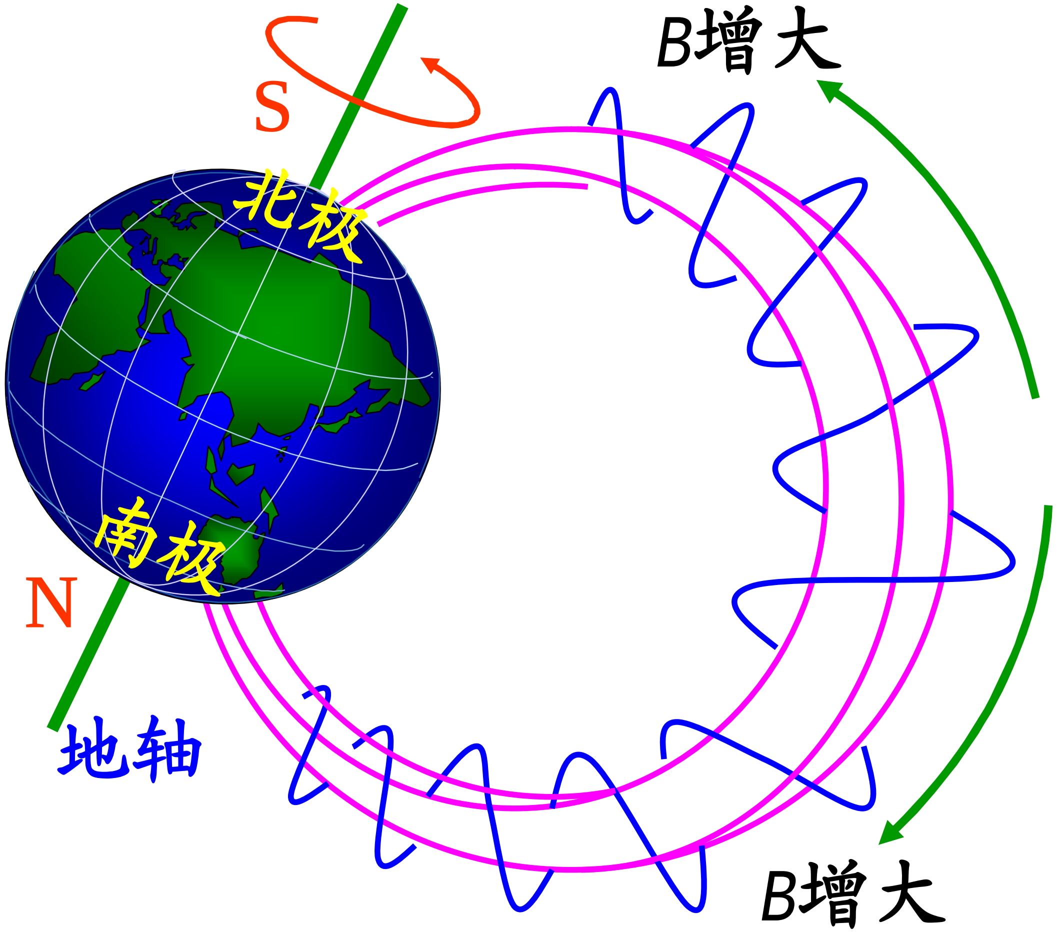


# (1)磁约束装置



# (2)范艾仑(J.A.Van Allen)辐射带



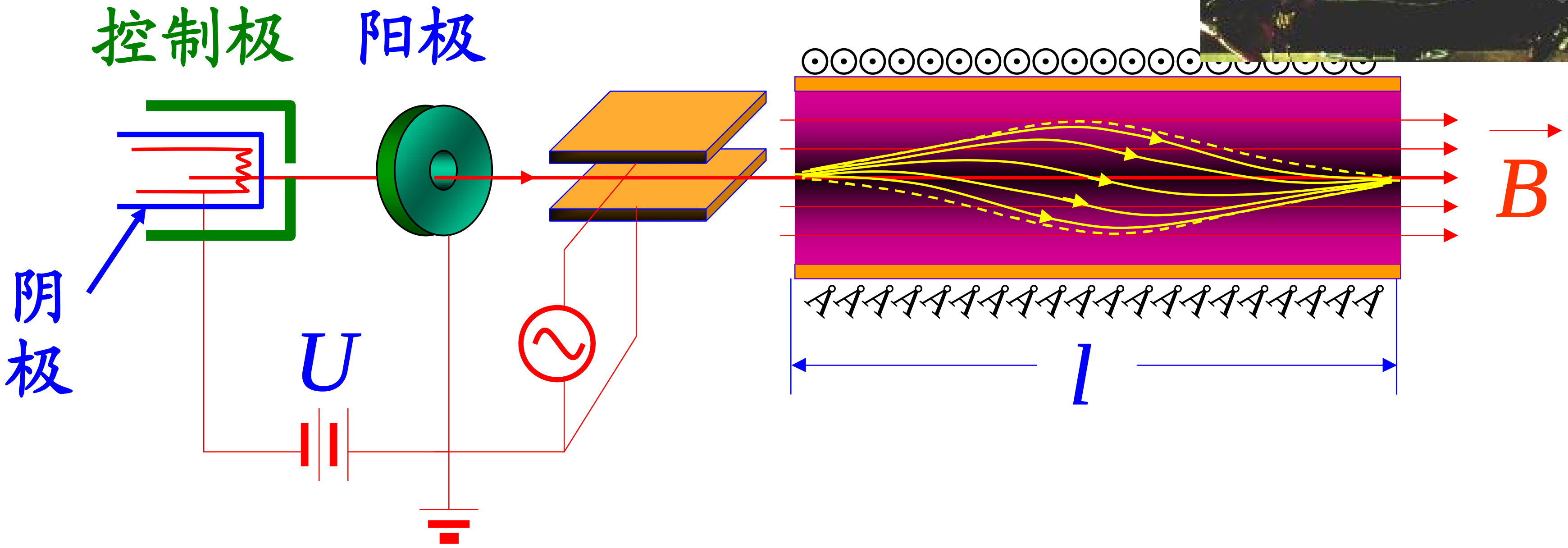
## (3)极光

在地磁两极附近 由于磁感线与地面垂直 外层空间入射的带电粒子可直接射入高空大气层内 它们和空气分子的碰撞产生的辐射就形成了极光

# 10.5.4 带电粒子在电场和磁场中运动的应用

动力学方程：

$$m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}=q\vec{E}+q\vec{v}\times\vec{B}$$



## [思考题]电子质量、电量分别为m、e，填写下表

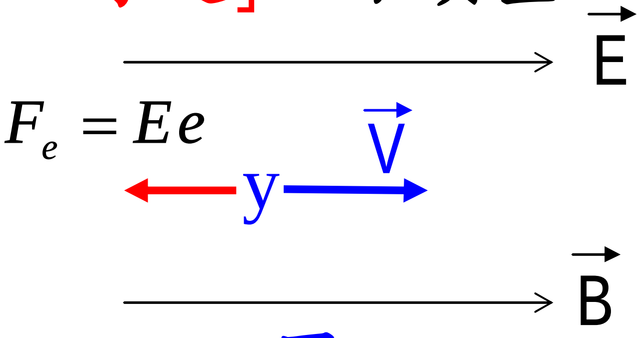



图1

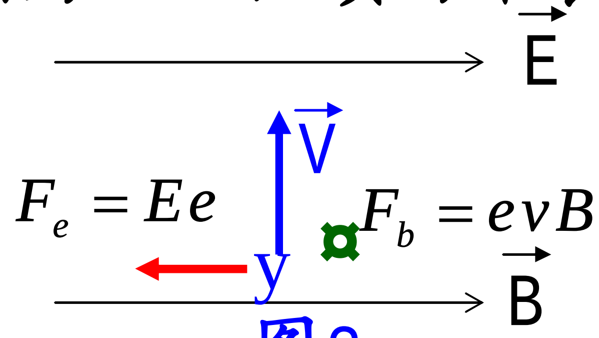



图2

图1：$a_n = 0$ ; $a_t = Ee/m$

$F_n = 0 \qquad F_t = F_e = Ee$

图2：$a_n = \dfrac{e\sqrt{E^2 + B^2 v^2}}{m}$ ; $a_t = 0$

$F_n = \sqrt{F_e^2 + F_b^2} \qquad F_t = 0$

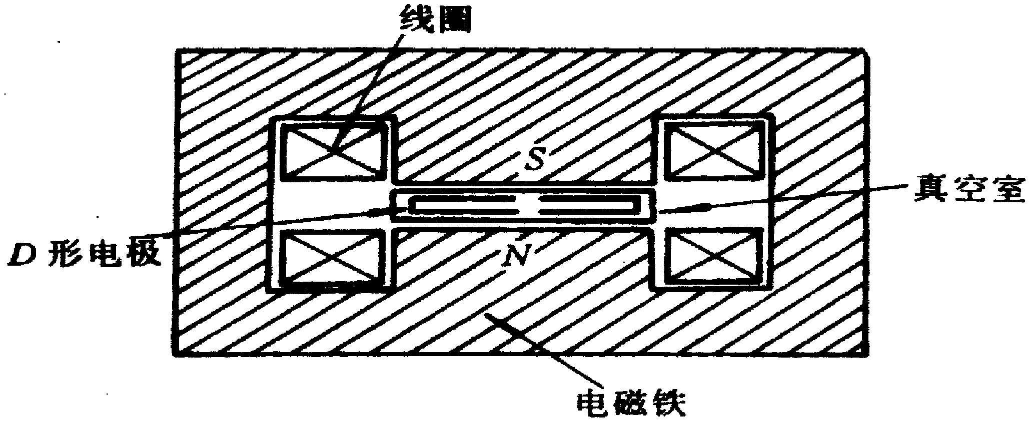
# 10.5.5技术应用 A 回旋加速器

回旋加速器是用来获得高能带电粒子的设备。

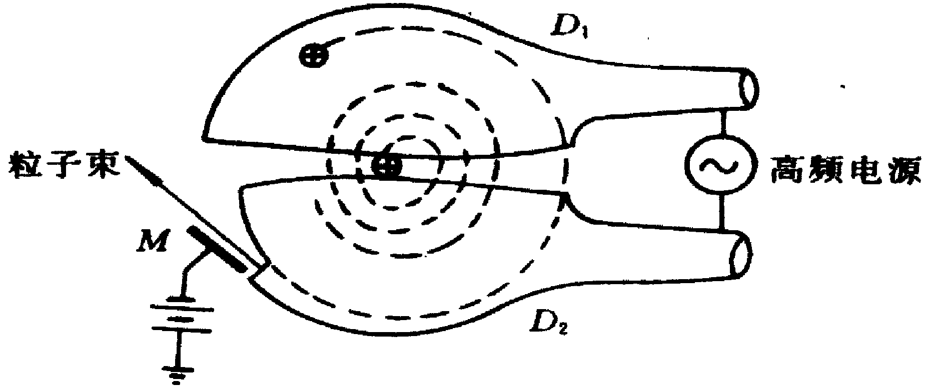
基本性能:

1. 使带电粒子在磁场的作用下作回旋运动。

2. 使带电粒子在电场的作用下得到加速。




(a)




(b)

# 回旋加速器

轨道半径 $$R=\frac{v}{(q/m)B}$$

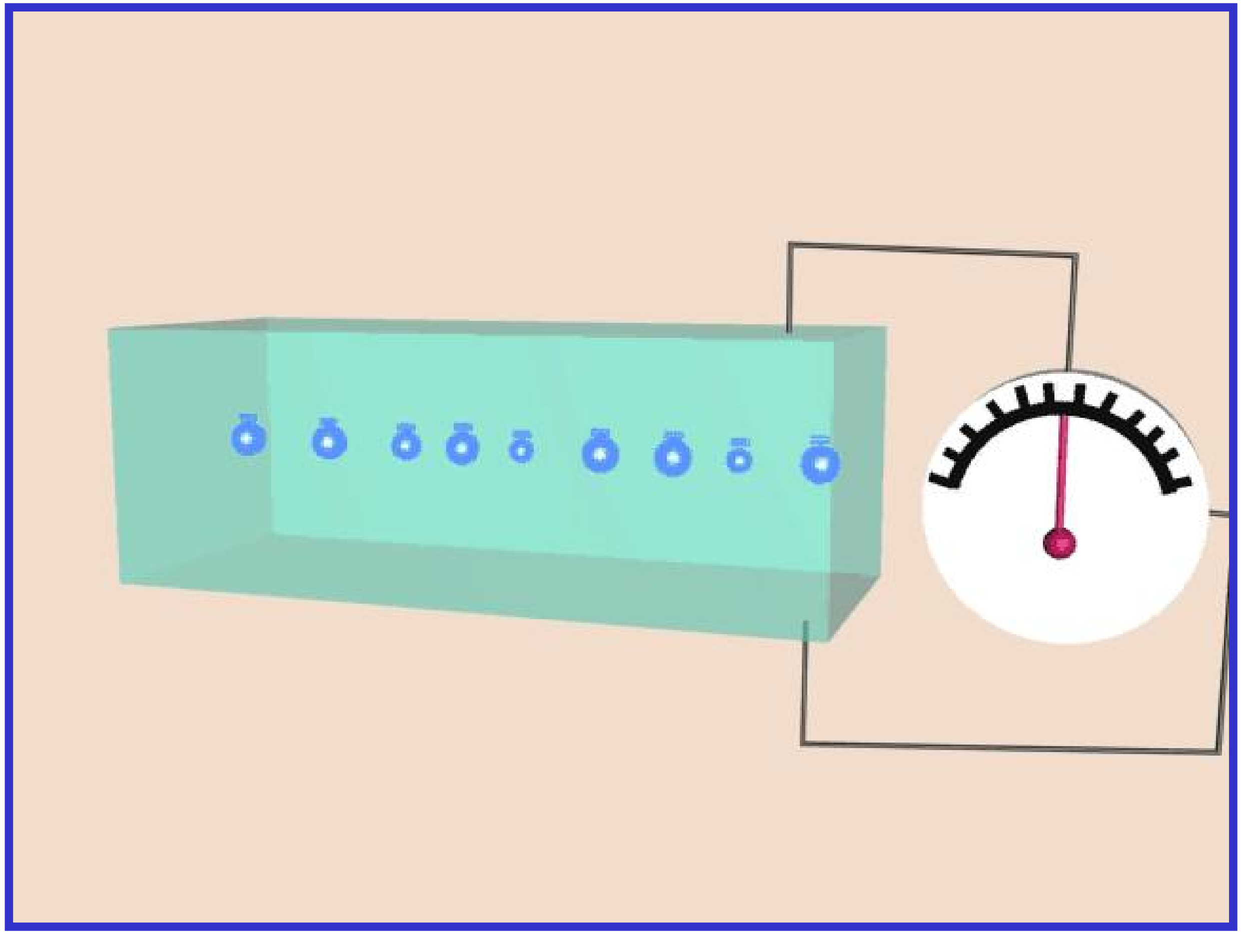
加速器

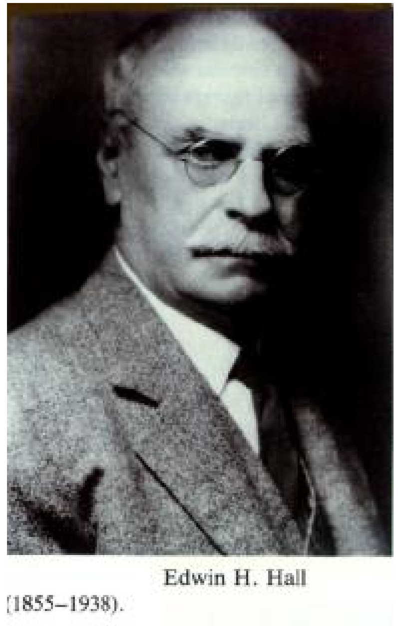
粒子引出速度 $$v=\frac{q}{m}BR$$

粒子的动能

$$E_k=\frac{1}{2}mv^2=\frac{q^2}{2m}B^2R^2$$

# 10.5.5技术应用B （E.C.Hall）霍耳效应

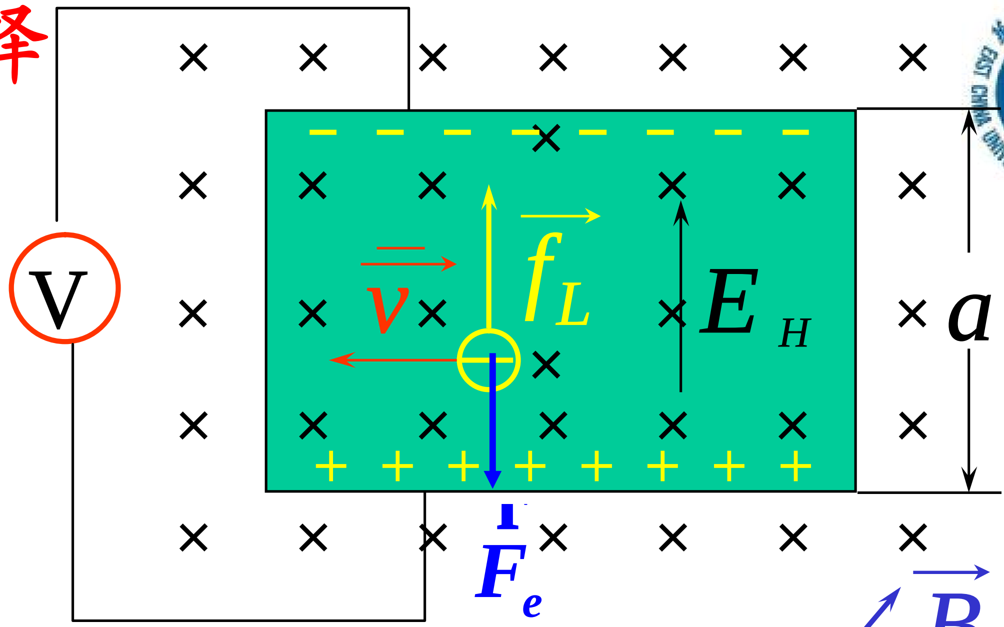





霍耳

1879年霍耳(A.H.Hall)发现:在匀强磁场中通电金属导体板的上下表面出现横向电势差.该现象称为霍耳效应

# 霍耳效应的经典解释

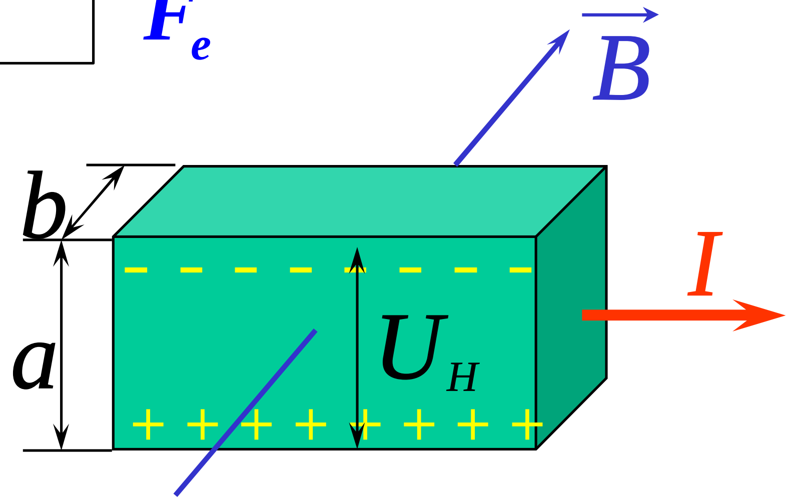



$$F_L = evB$$

$$F_e = eE_H = \frac{eU_H}{a}$$

平衡时: $F_L = F_e$

即: $evB = \frac{eU_H}{a}$



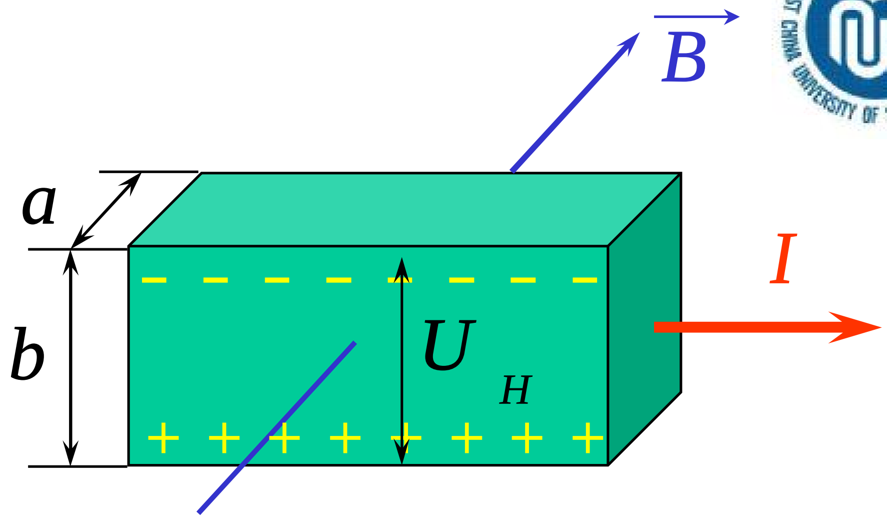

得: $U_H = vBa$

又 $\because I = nevab \Rightarrow v = \frac{I}{neab}$

$$U_H = \frac{1}{ne}\frac{IB}{b} \Rightarrow R_H = \frac{1}{ne}$$

实验指出：$U_H \propto \dfrac{IB}{a}$

$$\Rightarrow U_H = R_H \frac{IB}{a}$$




$R_H$——霍耳系数，它是和材料的性质有关的常数

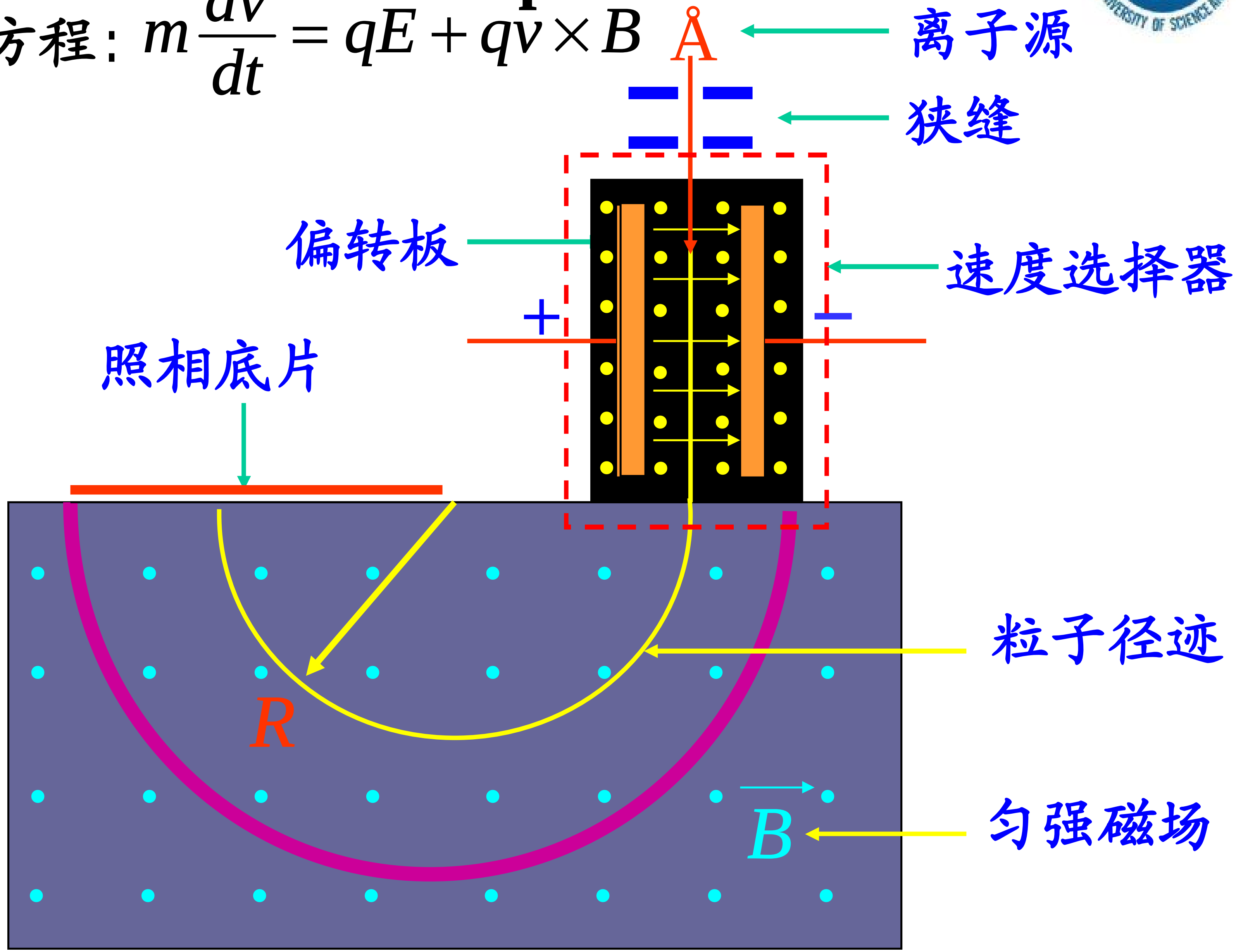
经典电子论对霍耳效应的解释得金属导体：$R_H = -\dfrac{1}{ne}$

电子数密度

对于n型半导体载流子为电子，而P型半导体体载流子为 带正电的空穴。根据$R_H$的符号可确定半导体类型，根据$R_H$大小的测定，可确定载流子浓度。

## 技术应用　C 质谱仪

动力学方程：$m\dfrac{d\vec{v}}{dt} = q\vec{E} + q\vec{v}\times\vec{B}$

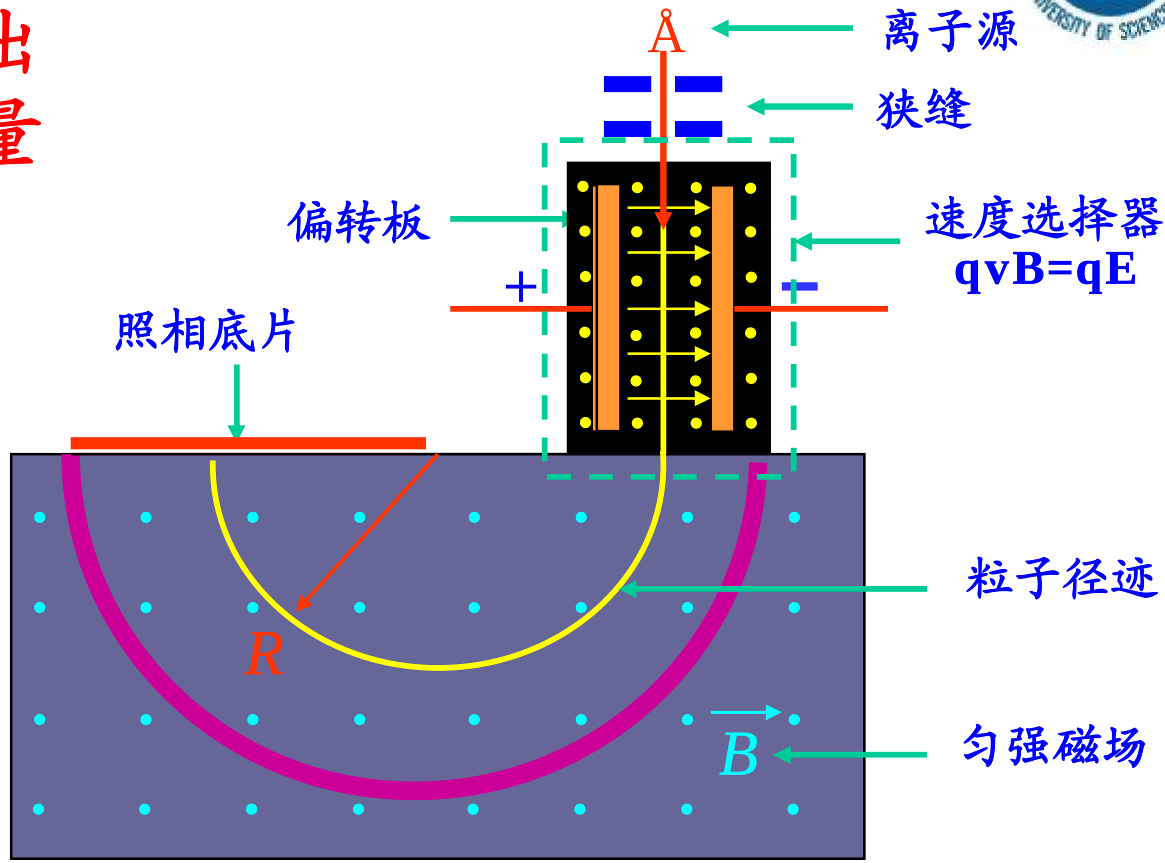


利用质谱仪可以测出元素中同位素的含量

通过速度选择器后粒子的速度

$$v=\frac{E}{B'} \qquad (1)$$

$$\frac{q}{m}=\frac{v}{RB} \qquad (2)$$

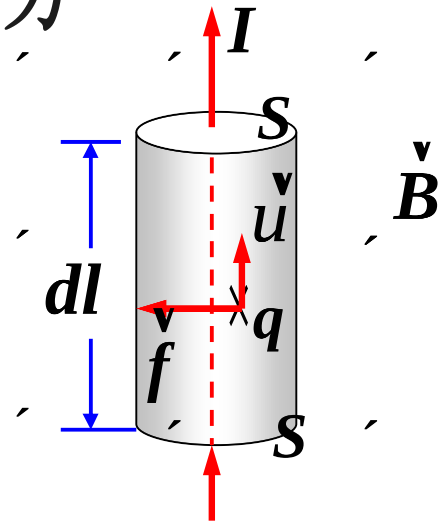



$$R\propto\frac{mE}{qB'B} \qquad \Rightarrow \quad R\propto m$$

# 10-6 磁场对电流的作用

运动电荷受磁场的作用力-- 洛仑兹力

电流元、载流导线受力?




## 10.6.1 安培定律

$$\vec{f} = q\vec{v} \times \vec{B}$$

$$dN = nSdl$$

$$d\vec{F} = dN \cdot \vec{f} = nSdl \cdot q\vec{v} \times \vec{B} = \underbrace{nSqv}_{I}\, d\vec{l} \times \vec{B}$$

$$\therefore \quad d\vec{F} = Id\vec{l} \times \vec{B}$$

大小 $dF = IdlB\sin\theta$

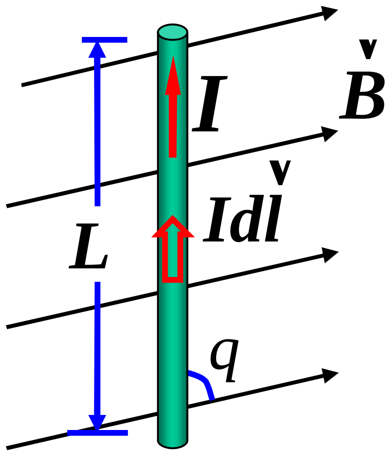
方向 $Id\vec{l} \times \vec{B}$

讨论：$d\vec{F} = Id\vec{l} \times \vec{B}_{外}$　　$dF = dq \cdot \vec{E}_{外}$

$\vec{f} = q\vec{u} \times \vec{B}_{外}$

## 有限长载流直导线在外 磁场中受力

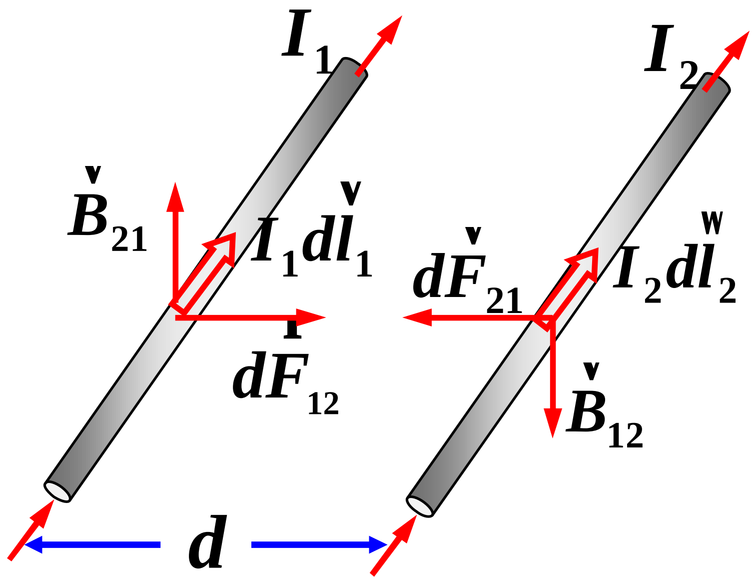
$$d\vec{F} = Id\vec{l} \times \vec{B}$$




$$\boxed{\vec{F} = \int Id\vec{l} \times \vec{B}}$$

$$F_x = \int dF_x,\ F_y = \int dF_y \Rightarrow \vec{F} = F_x\vec{i} + F_y\vec{i}$$

# 例6.无限长平行载流直导线间的相互作用力




$$d\vec{F}_{21} = I_2 d\vec{l}_2 \times \vec{B}_{12}$$

$$dF_{21} = I_2 dl_2 B_{12} \sin\theta$$

$$= I_2 dl_2 \frac{\mu_0 I_1}{2\pi d}$$

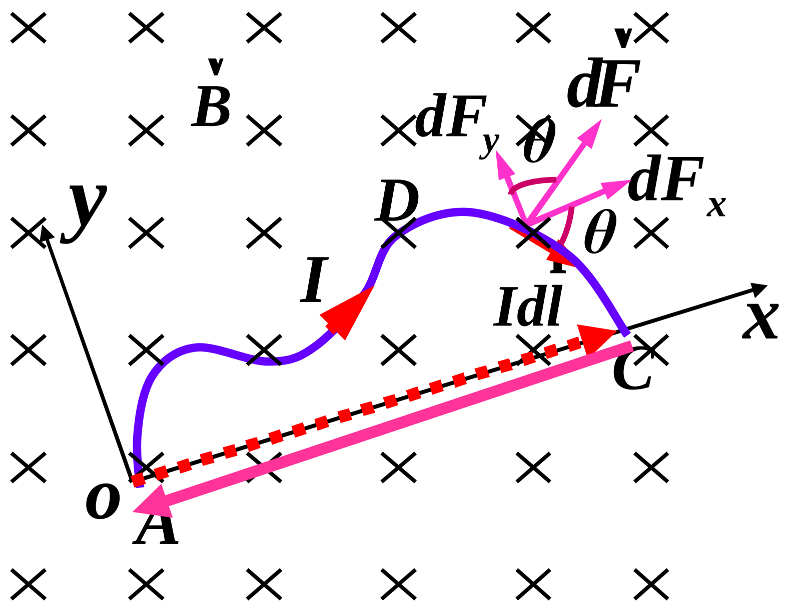
取长度为 $l$ 的一段

$$F_{21} = \int dF_{21} = \int_0^l \frac{\mu_0 I_1 I_2}{2\pi d} dl_2 = \frac{\mu_0 I_1 I_2}{2\pi d} l \quad \text{向左}$$

同理： $$F_{12} = \int dF_{12} = \int_0^l \frac{\mu_0 I_2}{2\pi d} I_1 dl_1 = \frac{\mu_0 I_1 I_2}{2\pi d} l \quad \text{向右}$$

$$\vec{F}_{21} = -\vec{F}_{12}$$ 电流同向相吸，反向相斥。

# 例7.任意通电导线在外磁场中的受力




$$d\vec{F} = Id\vec{l} \times \vec{B}$$

$$dF = IdlB\underbrace{\sin\alpha}_{=1}$$

$$dF_x = dF\sin\theta \qquad dF_y = dF\cos\theta$$

$$F_x = \int dF_x = \int BIdl\sin\theta = -\int_o^o BIdy = O$$

$$F_y = \int dF_x = \int BIdl\cos\theta = \int_A^c BIdx = BI\,\overline{AC}$$

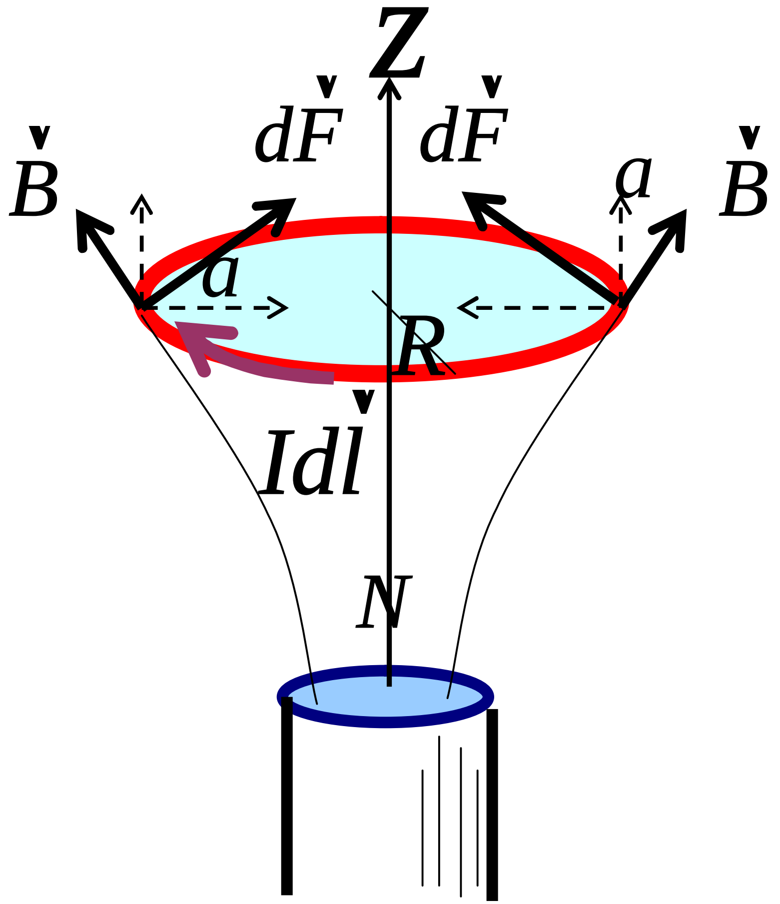
方向垂直$\overline{AC}$向上

相当于AC直导线

在均匀的磁场中闭合线圈所受的合力为0

例题8:圆柱形磁铁 N 极上方水平放置一个载流导线环，求其受力。

已知在导线所在处磁场B的方向与竖直方向成a角



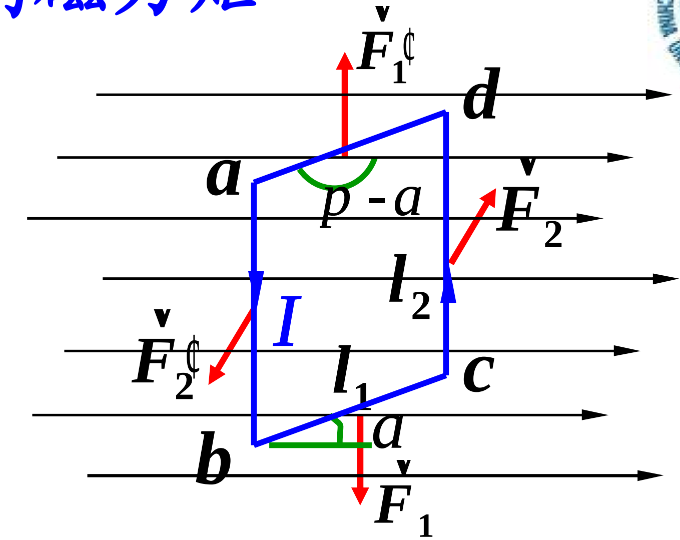

由图可知：圆环受的总磁力的方向在铅直方向，

$$F = F_z = \int dF \sin a$$

$$= \int_o^{2pR} IB \sin a \cdot dl$$

$$= 2pRIB \sin a$$

## 10.6.2 磁场对载流线图作用的磁力矩




$$F_1 = Il_1 B\sin\alpha$$

$$F_1' = Il_1 B\sin(\pi - \alpha)$$

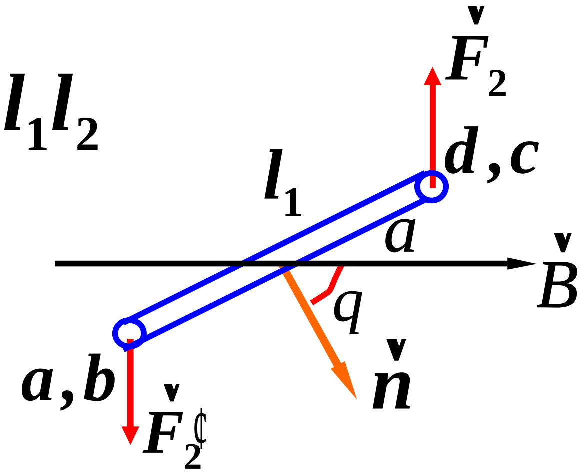
$$= Il_1 B\sin\alpha = F_1$$

$$F_2 = Il_2 B = F_2' \qquad \boxed{\sum \vec{F} = 0}$$

力矩 $M = F_2 \dfrac{l_1}{2}\cos\alpha + F_2' \dfrac{l_1}{2}\cos\alpha = IBl_2 l_1 \cos\alpha$

$\because \alpha + \theta = \dfrac{\pi}{2} \quad \therefore \cos\alpha = \sin\theta \quad$ 又$S = l_1 l_2$

则 $\quad M = ISB\sin\theta = P_m B\sin\theta$

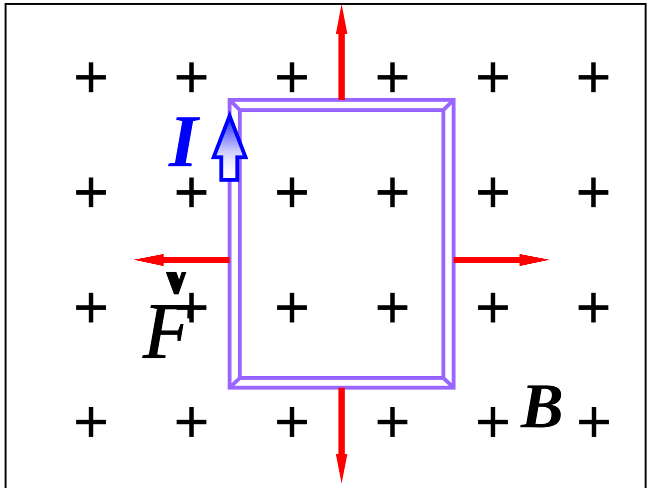



$$\vec{M} = \vec{P}_m \times \vec{B} \qquad (\vec{M}_e = \vec{P}_e \times \vec{E})$$

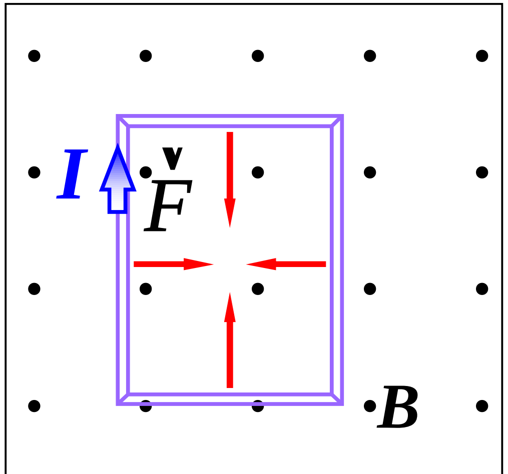
**讨论:** $M = P_m B\sin\theta$　　$\vec{M} = \vec{P}_m \times \vec{B}$

1）$\vec{n}$方向与 $\vec{B}$ 相同 2）方向相反　3）方向垂直

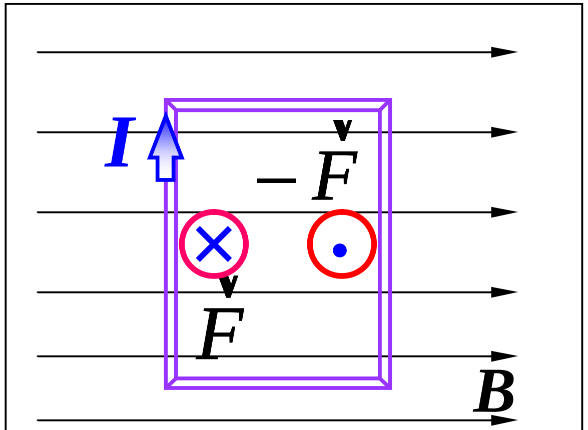
稳定平衡　　不稳定平衡　　力矩最大




$\theta = 0^0, M = 0$

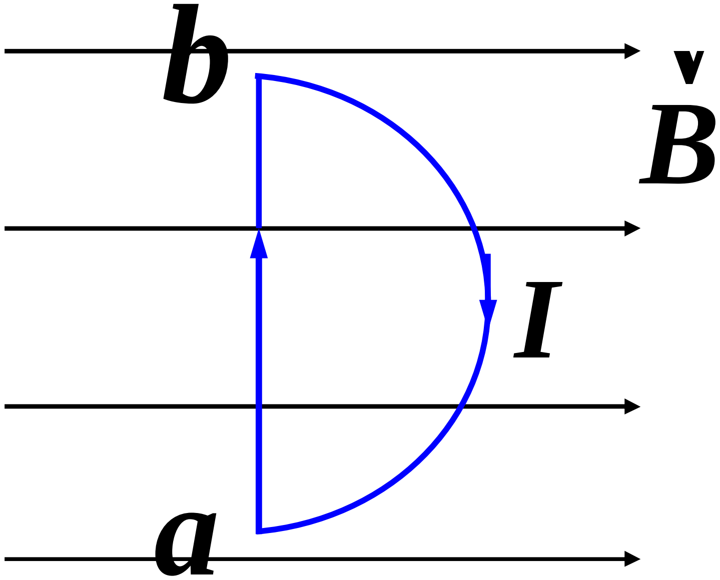



$\theta = \pi, M = 0$




$\theta = \dfrac{\pi}{2}, M = M_{\max}$

# 思考：半径R，电流I，M = ?

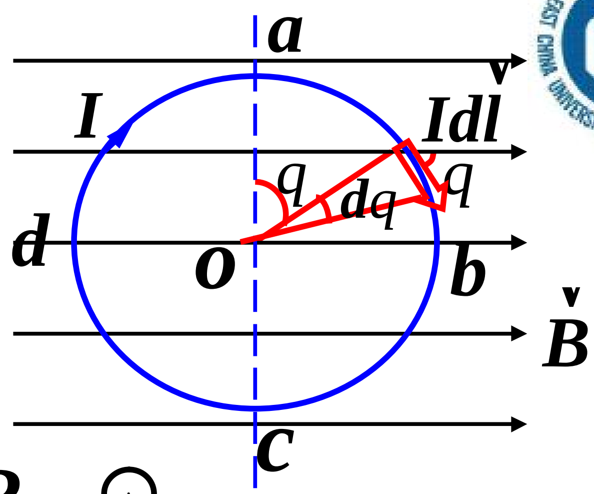



$$M = P_m B \sin\varphi = I\frac{\pi R^2}{2}B$$

方向沿纸面向下

例9.半径 R，电流 I，如图放置

求：abc和cda受力及线圈运动情况




$$abc: dF = IdlB\sin\theta$$

$$F = \int dF = \int_0^{\pi} IBR\sin\theta d\theta = 2IBR \quad \odot \qquad (dl = Rd\theta)$$

$$cda: F = \int_{\pi}^{0} IBR\sin\theta d\theta = -2IBR \quad \otimes$$

$$\sum F = 0$$

$$\vec{M} = \vec{P}_m \times \vec{B} \qquad M = P_m B\sin\varphi = I\pi R^2 \cdot B \ \downarrow$$

$$dM = x \cdot dF = R\sin\theta \cdot IBR\sin\theta d\theta$$

$$M = \int dM = \int_0^{2\pi} IBR^2 \sin^2\theta d\theta = IB\pi R^2 = P_m B$$

若磁场垂直纸面向里 ,再求上题

$abc：\quad dF = IdlB\sin\theta$

$$(\sin\theta = 1)$$

$$dF_x = dF\sin\alpha$$

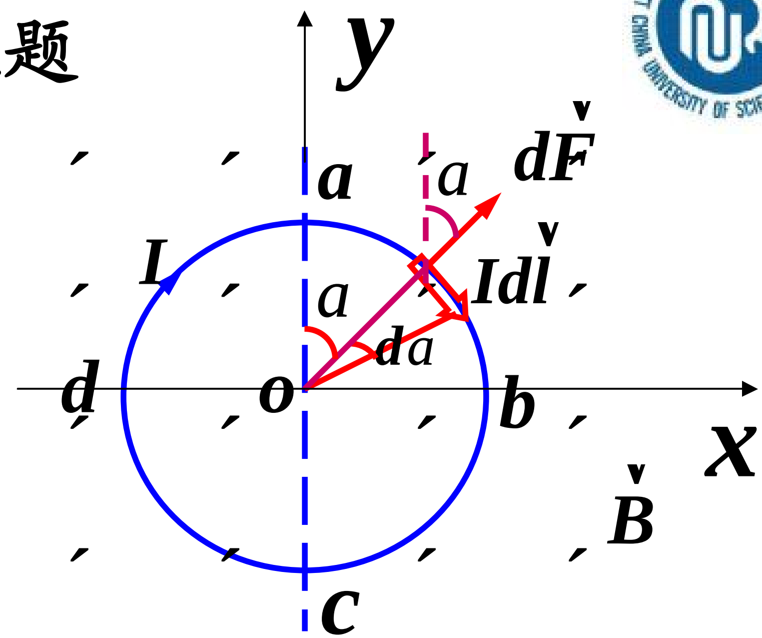
$$dF_y = dF\cos\alpha$$

$\because$ 对称性 $F_y = o$

$F = F_x = \int dF\sin\alpha = \int_o^{\pi} IB\sin\alpha \cdot Rd\alpha = 2IBR$ 向右

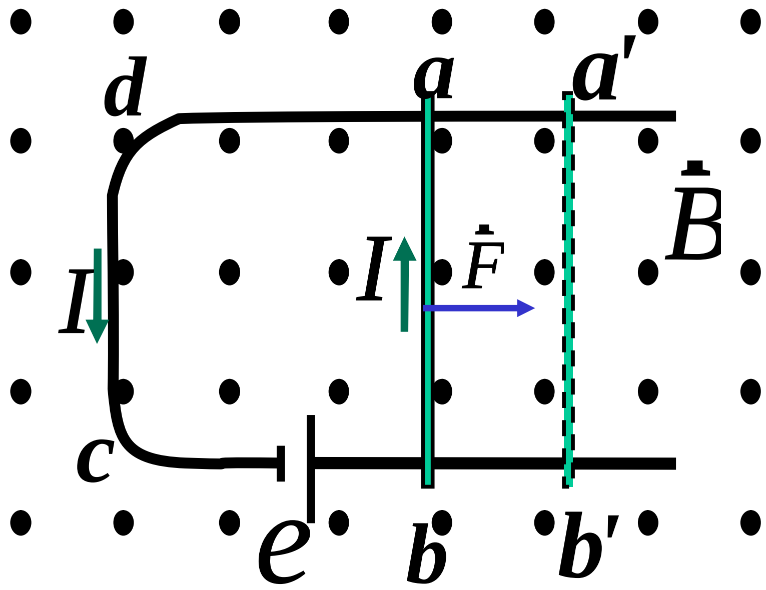
$cda \qquad F' = 2IBR$ 向左

$\sum F = O \qquad M = P_m B\sin\varphi = o$



# 10-7 磁力的功

## 10.7.1 载流导体在磁场中运动时磁力做的功




闭合回路 $abcda$

$I$ 不变, $ab$ 可滑动

$ab$ 受力 $F = IBl$

$$A = F \cdot \overline{aa'} = IBl\overline{aa'}$$

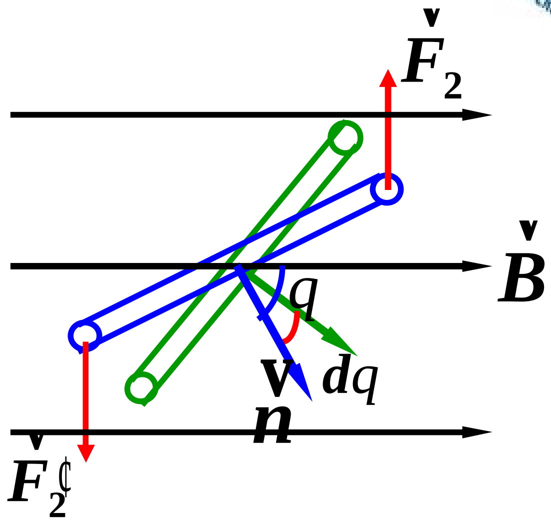
$\overline{ab}$ 移到 $\overline{a'b'}$, 磁通量变化

$$\Delta\Phi = \Phi - \Phi_0 = B\Delta S = Bl\overline{aa'}$$

$$\therefore \quad \boxed{A = I(\Phi - \Phi_0) = I\Delta\Phi}$$

## 10.7.2载流线圈在磁场中转动时磁力做的功

在磁力矩作用下转过 $d\theta$




$$dA = -Md\theta = -BIS\sin\theta d\theta$$

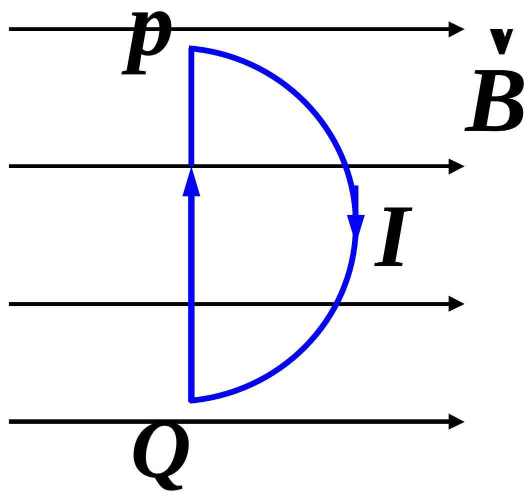
$$= BISd(\cos\theta) = Id(BS\cos\theta)$$

$$dA = Id\Phi$$

$\theta_1 \to \theta_2$ 总功 $A = \int_{\Phi_1}^{\Phi_2} Id\Phi = I(\Phi_2 - \Phi_1) = I\Delta\Phi$

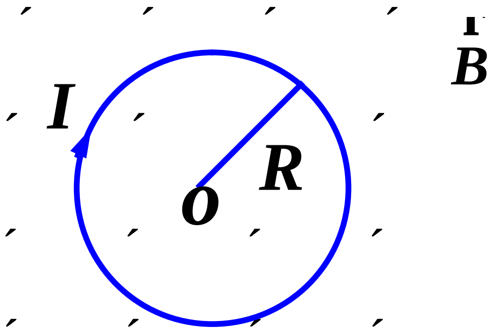
$$\left[A_{ab} = q_0(U_a - U_b)\right]$$

# 思考：如图通电线圈放入外磁场




1.绕 PQ 转$180^0$

$$A = I(\Phi_2 - \Phi_1) = 0$$




2.闭合圆电流线圈转 $90^0$

$$A = I(\Phi_2 - \Phi_1) = I(0 - B\pi R^2)$$

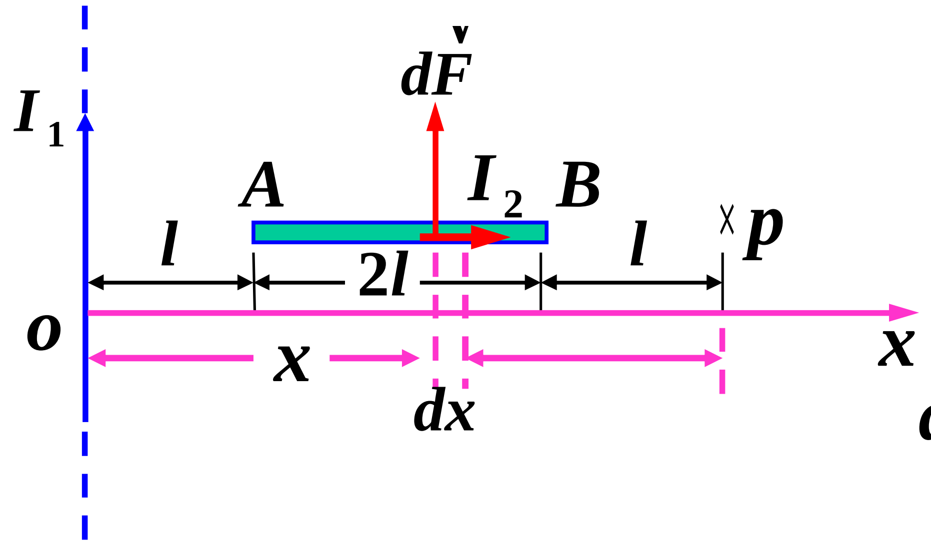
$$= -IB\pi R^2$$

3.转$180^0$

$$A = I(-B\pi R^2 - B\pi R^2)$$

$$= -2IB\pi R^2$$

# 例10.求AB导线对P点的磁力矩




$$d\vec{F} = Id\vec{l} \times \vec{B}$$

$$dF = I_2 dx B = I_2 \frac{\mu_0 I_1}{2\pi x} dx$$

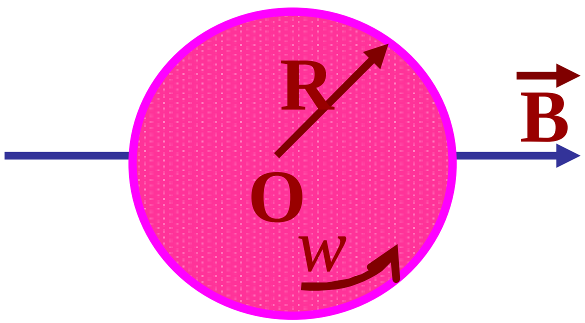
$$dM = r \cdot dF = (4l - x)\frac{\mu_0 I_1 I_2}{2\pi x} dx$$

$$M = \int dM = \int_l^{3l} \frac{\mu_0 I_1 I_2}{2\pi}(4l - x)\frac{dx}{x}$$

$$= \frac{\mu_0 I_1 I_2 l}{\pi}(2\ln 3 - 1)$$

例11.如图所示匀$\vec{B}$中圆盘R,带正电s=kr,
k为常数,盘以w旋转,求圆盘所受磁力矩

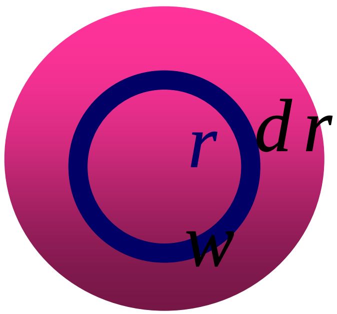
$$n=\omega/2\pi$$




解:(1)求旋转圆盘的等效磁矩

$$dI = ndq = n(\sigma 2\pi r dr)$$

$$= n(kr2\pi rdr)$$




$$dp_m = sdI$$

电流所围面积

$$= \pi r^2[n(kr2\pi rdr)]$$

$$\Rightarrow p_m = \int_0^{p_m} dp_m = \int_0^R 2kn\pi^2 r^4 dr$$

$$= \frac{2}{5}kn\pi^2 R^5$$

(2)求磁力矩

$$\vec{M} = \vec{p}_m \times \vec{B} \Rightarrow M = p_m B = \frac{2}{5}Bkn\pi^2 R^5$$

方向 $\uparrow$