谐振动的能量：

$$E_k=\frac{1}{2}mv^2=\frac{1}{2}m[-A\omega\sin(\omega t+\varphi)]^2$$

$$E_p=\frac{1}{2}kx^2=\frac{1}{2}k[A\cos(\omega t+\varphi)]^2$$

$$E=\frac{1}{2}kA^2$$

谐振动是等幅振动，振动过程中机械能守恒

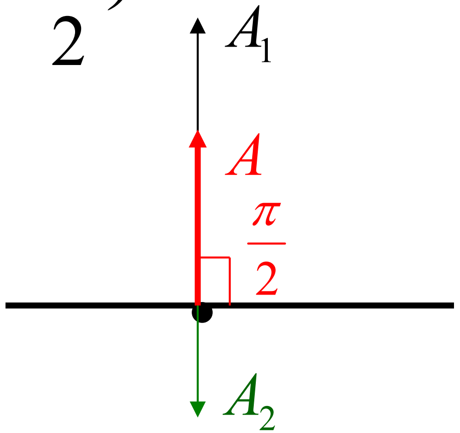
[例1]两个同方向同频率的简谐振动，其振动表达式分别为：$x_1 = 0.06\cos(5t+\frac{1}{2}\pi)\ m$ ，

$x_2 = 0.02\sin(\pi - 5t)\ m$,求：它们合振动的振动方程。

解：$x_2 = 0.02\sin[\frac{\pi}{2} - (5t - \frac{\pi}{2})] = 0.02\cos(5t - \frac{\pi}{2})$

$$= 0.02\cos(5t - \frac{\pi}{2})$$

$$\therefore x = x_1 + x_2 = 0.04\cos(5t + \frac{\pi}{2})m$$



[例2]

已知：同方向谐振动x_1=0.05cos(10t+3π/4)，

x_2=0.06cos(10t+π/4)，x_3=0.07cos(10t+φ_3)

求：(1) x_1、x_2合振动的A、φ

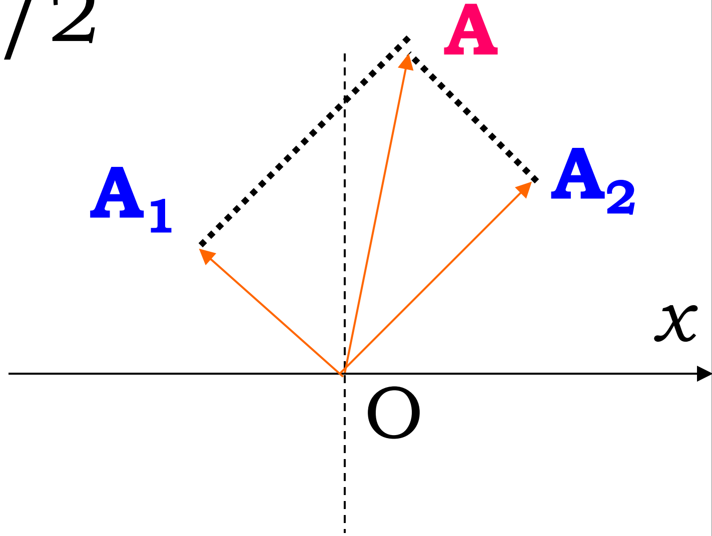
(2) φ_3为何值，x_1+x_3振幅最大？

(3) φ_3为何值，x_2+x_3振幅最小？

解：（1）$\angle A_1OA_2 = \varphi_1 - \varphi_2 = \pi/2$

$$\therefore\ A=\sqrt{A_1{}^2+A_2{}^2} = 0.078$$

$$\varphi = \pi/4+\mathrm{tg}^{-1}(A_1/A_2)$$



（2）$\Delta\phi_{13}=（\omega t+\varphi_1)-(\omega t+\varphi_3)=2k\pi$

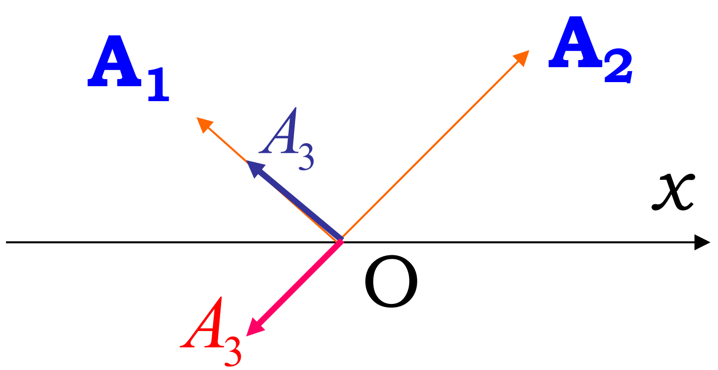
$(k=0,\pm1,\pm2\ldots)$

$\therefore \varphi_3=\varphi_1-2k\pi=3\pi/4-2k\pi\in[-\pi,\pi]\Rightarrow \varphi_3=3\pi/4$

（3）$\Delta\phi_{23}=(\omega t+\varphi_2)-(\omega t+\varphi_3)=(2k+1)\pi$

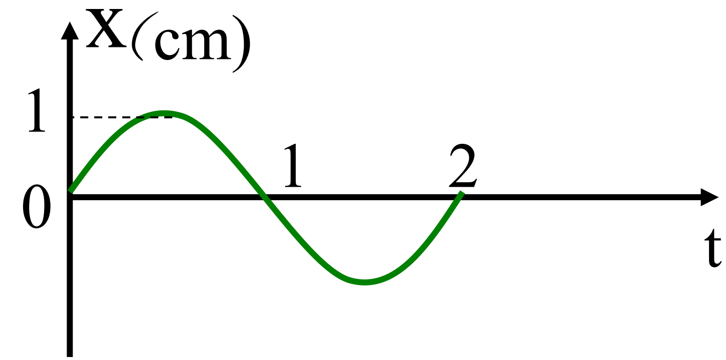
$(k=0,\pm1,\pm2\ldots)$

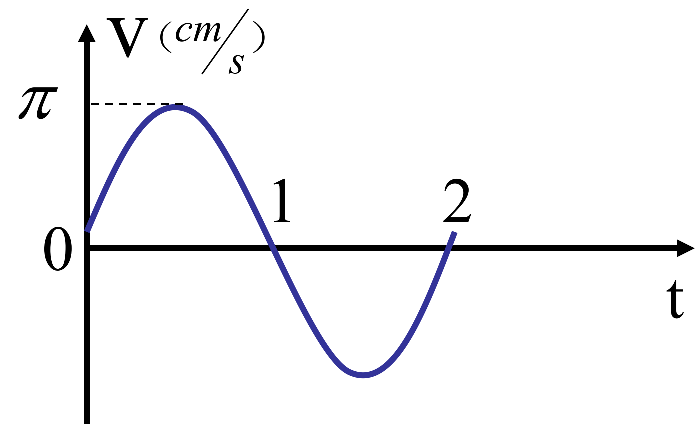
$\therefore \varphi_3=\varphi_2-(2k+1)\pi=\pi/4-(2k+1)\pi\in[-\pi,\pi]\Rightarrow \varphi_3=-3\pi/4$



[例3]:两同方向,同频率的简谐振动,振动1的$x \sim t$曲线及振动2的$v \sim t$曲线如图所示.

求:(1)$\varphi_2 - \varphi_1$　　(2)$A_{合}$





解(1).

$$\because t=0 \begin{cases} x_{10}=0 \\ v_{10}>0 \end{cases} \qquad \therefore \varphi_1 = -\frac{\pi}{2}$$

$$\because t=0 \begin{cases} v_{20}=0 \\ a_{20}>0 \end{cases} \qquad \therefore \varphi_2 = \pi$$

则:$\varphi_2 - \varphi_1 = \pi - (-\frac{\pi}{2}) = \frac{3}{2}\pi(\text{或} -\frac{\pi}{2})$

$(2)A_{合} = ?$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)}$$

$$A_1 = 1$$

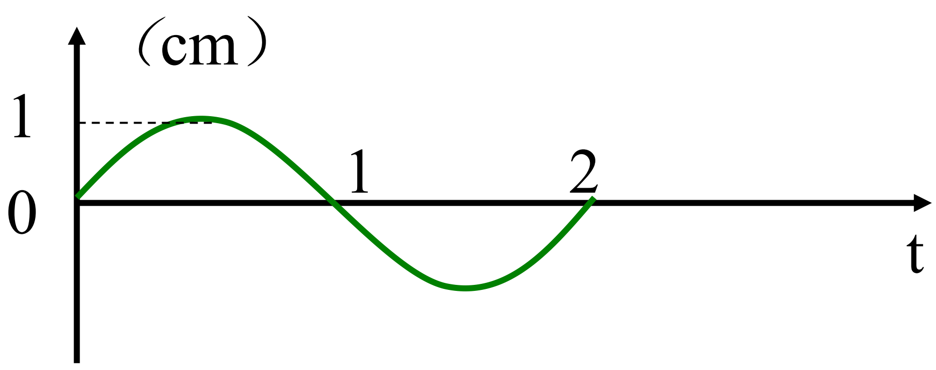
$$\varphi_2 - \varphi_1 = -\frac{\pi}{2}$$

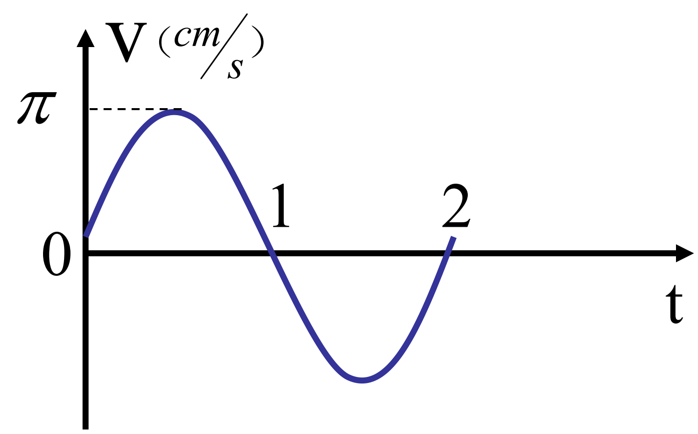
$$\because v_{2\max} = A_2\omega = \pi$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$$A_2 = 1$$

$$A = \sqrt{2}cm$$



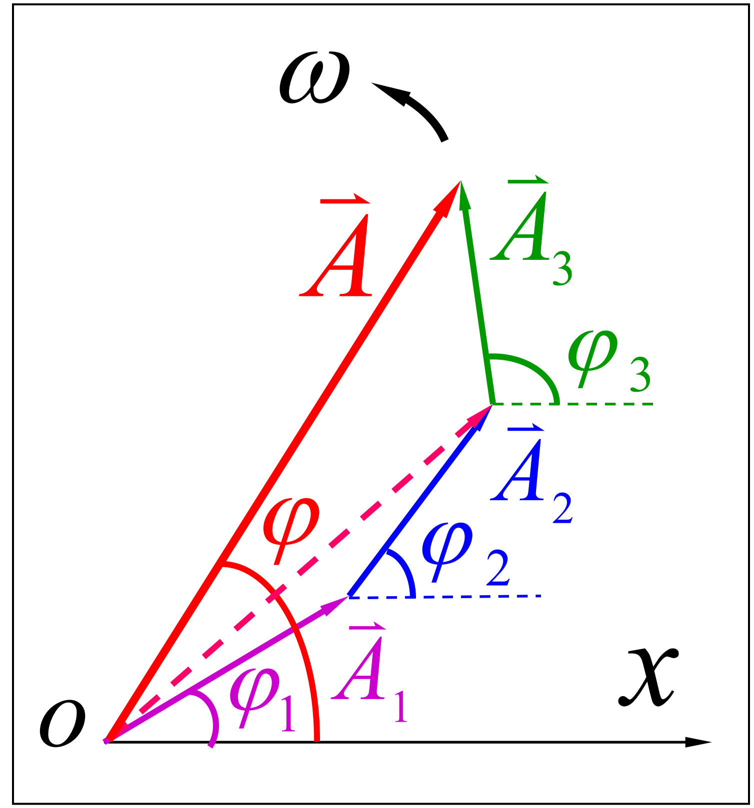


2. 多个同方向同频率简谐运动的合成

$$\begin{cases} x_1 = A_1\cos(\omega t+\varphi_1) \\ x_2 = A_2\cos(\omega t+\varphi_2) \\ \cdots\cdots \\ x_n = A_n\cos(\omega t+\varphi_n) \end{cases}$$

$$x = x_1 + x_2 + \cdots + x_n$$

$$\boxed{x = A\cos(\omega t+\varphi)}$$



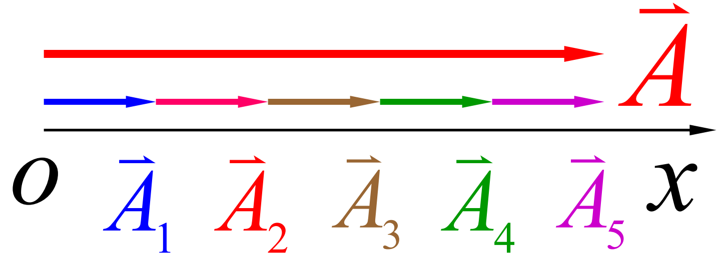
多个同方向同频率简谐运动合成仍为简谐运动

$$
\begin{cases}
x_1 = A_0 \cos \omega t \\
x_2 = A_0 \cos(\omega t + \Delta\varphi) \\
x_3 = A_0 \cos(\omega t + 2\Delta\varphi) \\
\quad \cdots\cdots \\
x_N = A_0 \cos[\omega t + (N-1)\Delta\varphi]
\end{cases}
$$

讨论

1） $\Delta\varphi = 2k\pi$

$(k = 0, \pm 1, \pm 2, \cdots)$

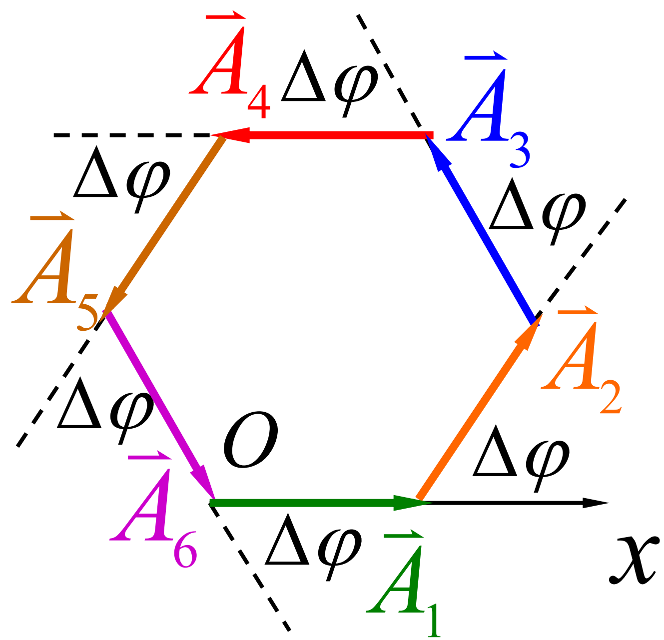


$$A = \sum_i A_i = NA_0$$

2） $N\Delta\varphi = 2k'\pi$

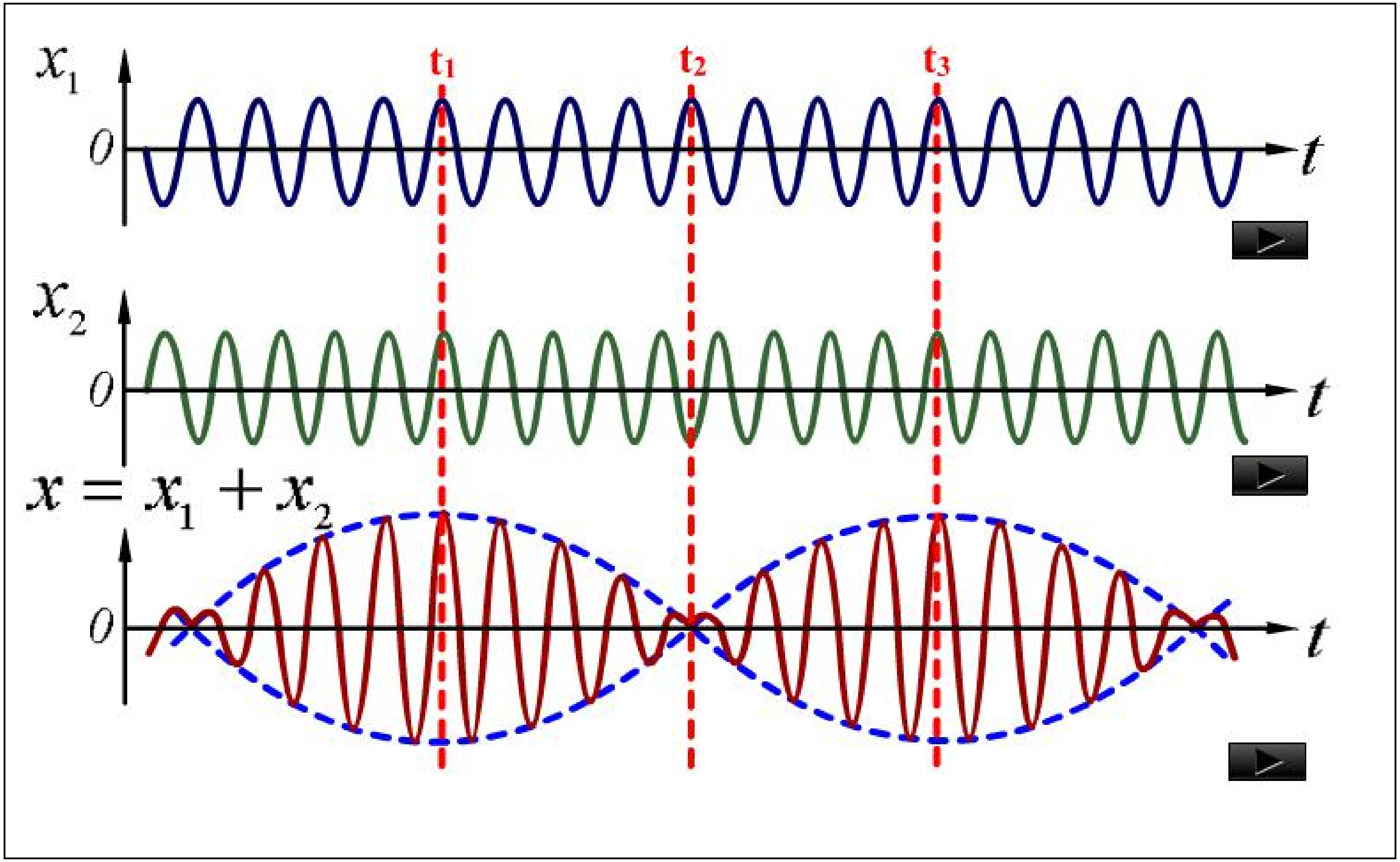
$(k' \neq kN, k' = \pm 1, \pm 2, \cdots)$

N个矢量依次相接构成一个**闭合**的多边形 .



$$A = 0$$

二．两个同方向不同频率简谐运动的合成



1

频率较大而频率之差很小的两个同方向简谐运动的合成，其合振动的振幅时而加强时而减弱的现象叫拍.

$$\varphi_1 = \varphi_2 = 0$$

$$\begin{cases} x_1 = A_1 \cos \omega_1 t = A_1 \cos 2\pi\, \nu_1 t \\ x_2 = A_2 \cos \omega_2 t = A_2 \cos 2\pi\, \nu_2 t \end{cases}$$

$$\boxed{x = x_1 + x_2}$$

◆ 讨 论

$A_1 = A_2 \quad |\nu_2 - \nu_1| << \nu_1 + \nu_2$ 的情况

$$x = x_1 + x_2 = A_1 \cos 2\pi\, \nu_1 t + A_2 \cos 2\pi\, \nu_2 t$$

$$x = \underbrace{\left(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t\right)}_{\text{振幅部分}} \cos 2\pi \underbrace{\frac{\nu_2 + \nu_1}{2}}_{\text{合振动频率}} t$$

$$x = \left(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t\right) \cos 2\pi \frac{\nu_2 + \nu_1}{2} t$$

振幅部分　　合振动频率

振动频率 $\nu = (\nu_1 + \nu_2)/2$

振幅 $A = \left| 2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t \right|$

$$\begin{cases} A_{\max} = 2A_1 \\ A_{\min} = 0 \end{cases}$$

$\boxed{\nu = \nu_2 - \nu_1}$ ⇐ 拍频（振幅变化的频率）

三. 两个相互垂直的同频率简谐运动的合成

$$\begin{cases} x = A_1 \cos(\omega t + \varphi_1) \\ y = A_2 \cos(\omega t + \varphi_2) \end{cases}$$

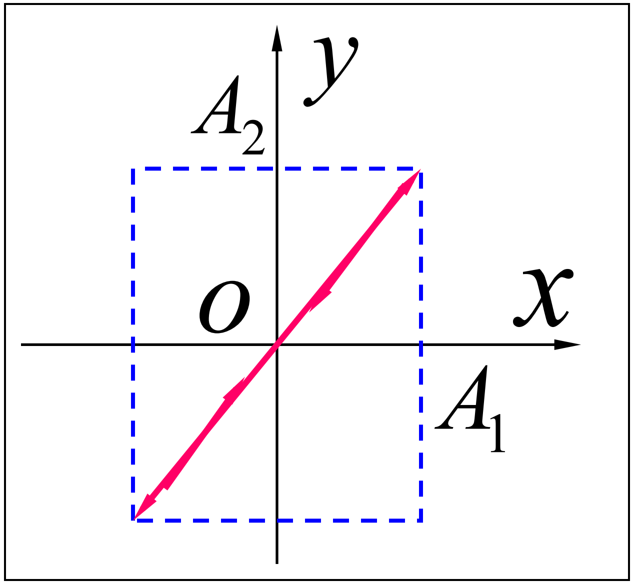
质点运动轨迹 （椭圆方程）

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1 A_2}\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$$

讨论

1） $\varphi_2 - \varphi_1 = 0$ 或 2π

$$y = \frac{A_2}{A_1} x$$



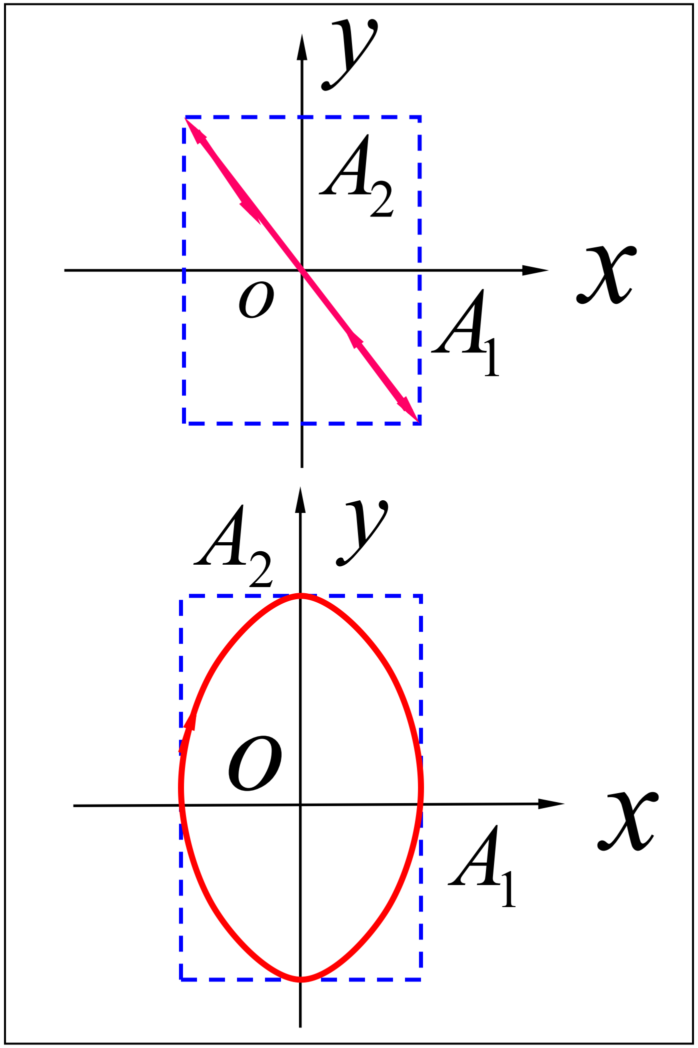
$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

2） $\varphi_2-\varphi_1=\pi$ $\quad y=-\frac{A_2}{A_1}x$

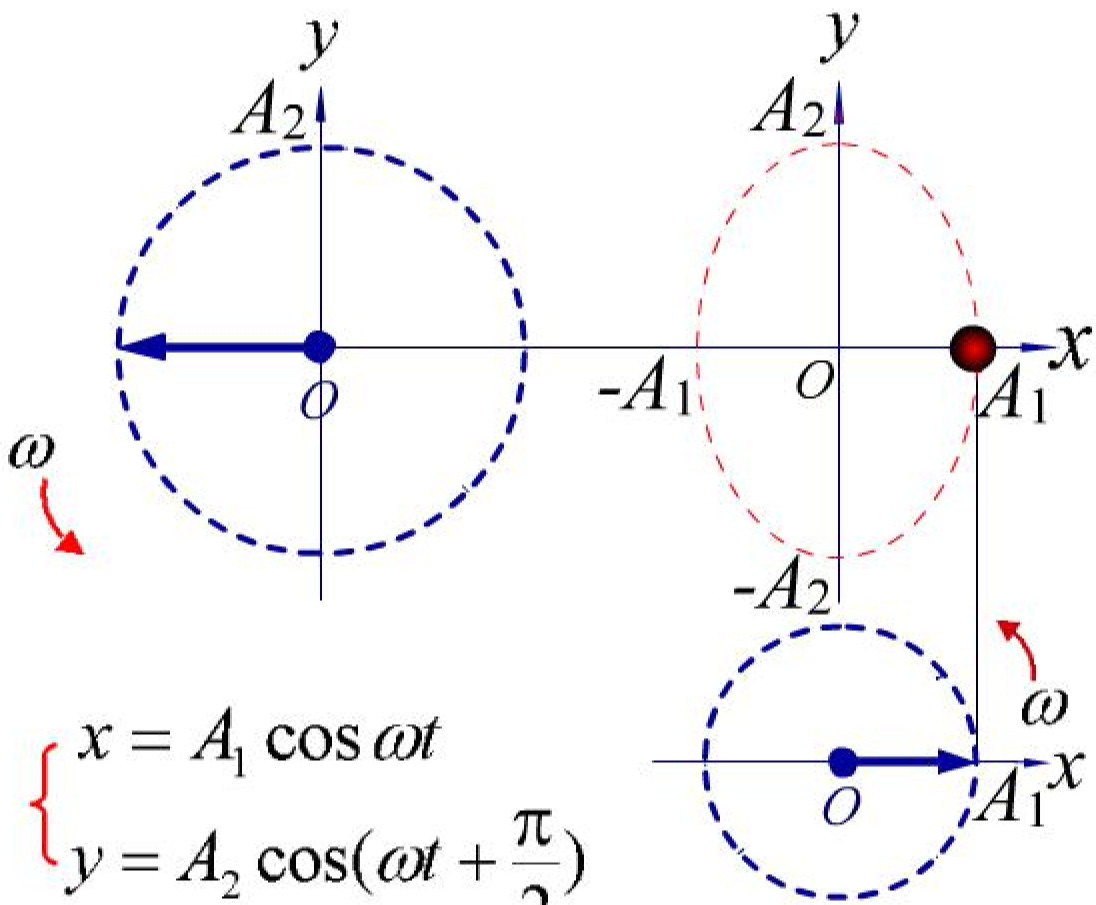
3） $\varphi_2-\varphi_1=\pm\pi/2$

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1$$

$$\begin{cases} x=A_1\cos\omega t \\ y=A_2\cos(\omega t+\frac{\pi}{2}) \end{cases}$$



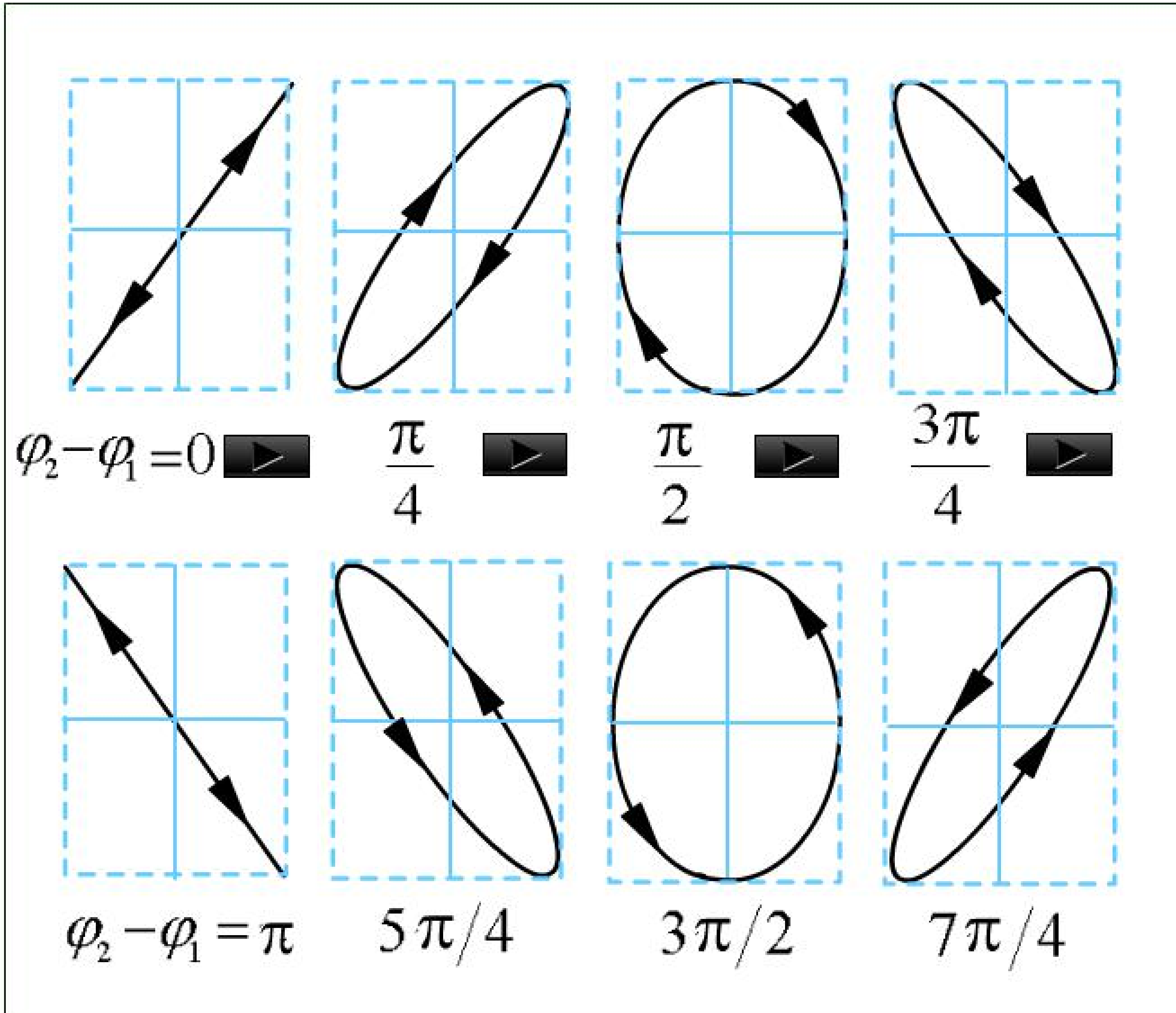
用旋转矢量描绘振动合成图



$$\begin{cases} x = A_1 \cos \omega t \\ y = A_2 \cos(\omega t + \frac{\pi}{2}) \end{cases}$$

两相互垂直同频率不同相位差

简谐运动的合成图



四. 两相互垂直不同频率的简谐运动的合成

$$\begin{cases} x = A_1 \cos(\omega_1 t + \varphi_1) \\ y = A_2 \cos(\omega_2 t + \varphi_2) \end{cases}$$

李 萨 如 图

$$\varphi_1 = 0$$

$$\varphi_2 = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$$

$$\frac{\omega_1}{\omega_2} = \frac{T_y}{T_x} = \frac{m_y}{n_x}$$

测量振动频率和相位的方法