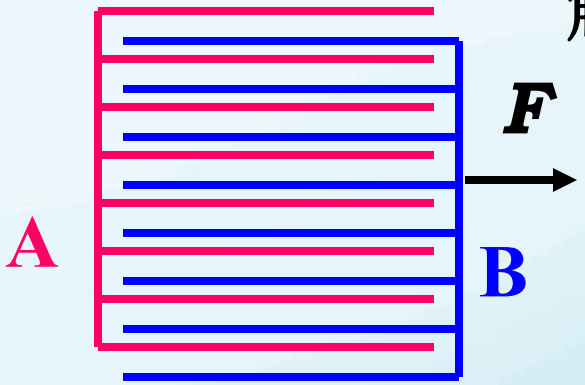
[例题1-7]各200页书A、B逐页交叉置水平面,每页 FangYi

m=5g,纸间、纸与桌面μ=0.1,A固定,求抽出B的F_{min}

解:选研究对象



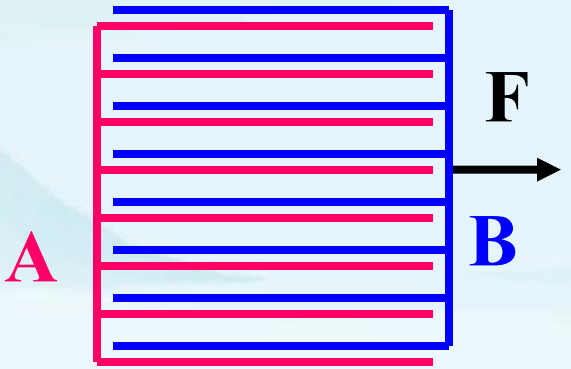
$$\left.\begin{aligned} f_i &= f_i' + f_i'' \\ f_i' &= (2i-1)mg\mu \\ f_i'' &= 2img\mu \end{aligned}\right\} \Rightarrow f_i = (4i-1)mg\mu$$

$$F - \sum_i f_i = ma$$

$$\Rightarrow F_{min} = \sum_i f_i = \sum_i (4i-1)mg\mu$$

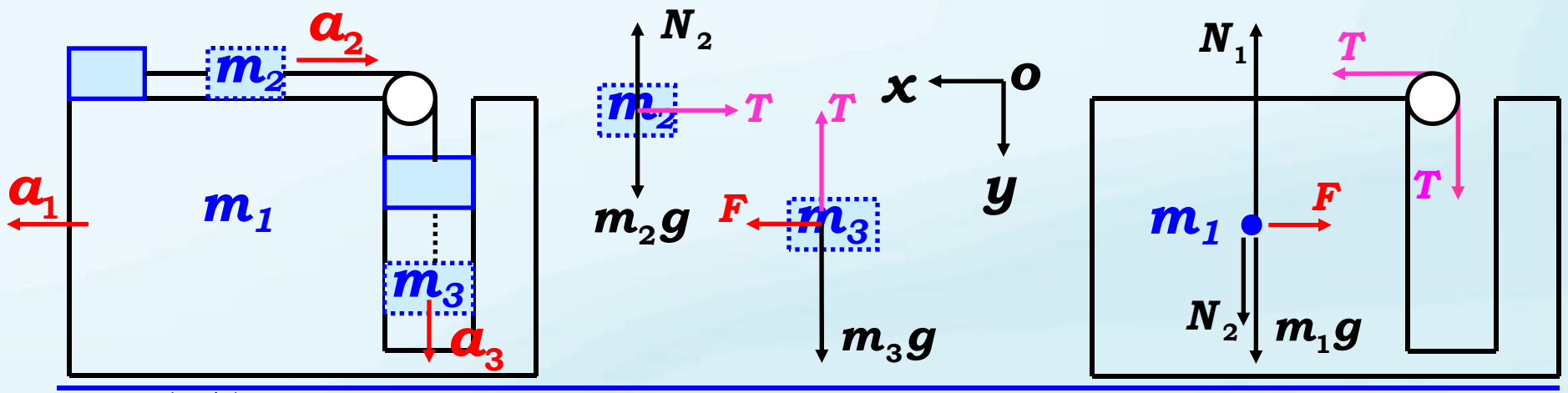
$$= \frac{3+799}{2} \times 200mg\mu = 393N$$

[课后思考]



A每页=5g, B每页=6g,纸间μ=0.1,
A固定,重求抽出B的F_{min}

[例题1-8]不计摩擦及$m_{滑轮}$，求(1) m_1,m_2,m_3加速度



解：选体

建系

受力

方程

$$\vec{a}_{21}=\vec{a}_{2地}-\vec{a}_{1地}\Rightarrow a_{21}=(-a_{2地})-(+a_{1地})$$

$$\begin{cases} m_1: T-F=m_1a_1 \\ m_2: T=m_2a_2 \\ m_3: F=m_3a_1 \\ m_3g-T=m_3a_3 \\ a_3=a_2+a_1 \end{cases}$$

$$\begin{cases} a_1=\dfrac{m_2m_3g}{m_1m_2+2m_2m_3+m_3m_1+m_3^2} \\ a_2=\dfrac{m_3(m_1+m_2)g}{m_1m_2+2m_2m_3+m_3m_1+m_3^2} \\ a_3=\dfrac{m_3(m_1+m_2+m_3)g}{m_1m_2+2m_2m_3+m_3m_1+m_3^2} \end{cases}$$

$$\vec{a}_{m1}=a_1\vec{i}\quad \vec{a}_{m2}=-a_2\vec{i}\quad \vec{a}_{m3}=a_1\vec{i}+a_3\vec{j}$$

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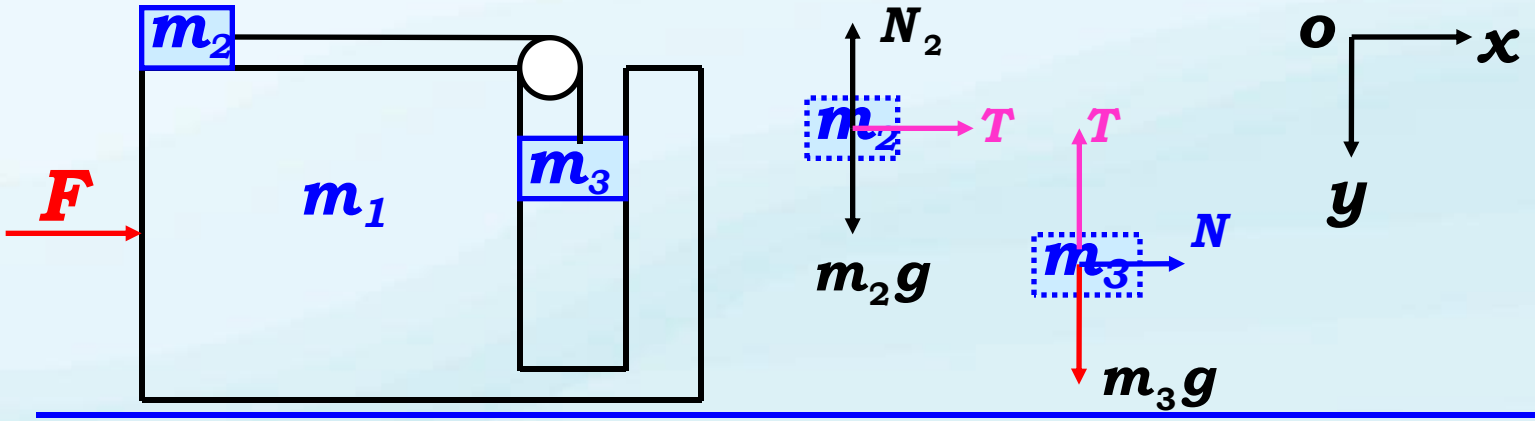
[例题1-8]不计摩擦及$m_{滑轮}$，求

(2) 使m_2, m_3相对静止的F .



解：选整体 选m_2, m_3为隔离体

建系

受力

方程

$$m_1 m_2 m_3 : F = (m_1 + m_2 + m_3)a$$

$$m_2 : \quad T = m_2 a$$

$$m_3 : \quad T = m_3 g$$

$$\Rightarrow F = \frac{m_3(m_1 + m_2 + m_3)g}{m_2}$$

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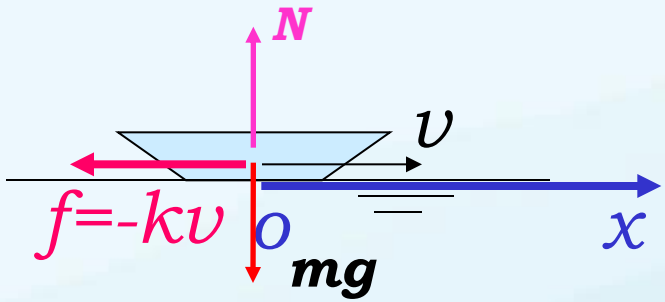
[例题1-9] m, v_0阻力$f=-kv$,关引擎,求$v_{(t)}$?行驶的Δx_{max}?



解：选船　建系　受力　方程

(1) $f = ma \Rightarrow -kv = m\dfrac{dv}{dt} \Rightarrow \int_{v_0}^{v}\dfrac{dv}{v} = -\int_0^t \dfrac{k}{m}dt$

$$\Rightarrow \ln\frac{v}{v_0} = -\frac{k}{m}t \Rightarrow v = v_0 e^{-\frac{k}{m}t}$$

(2) $\Delta x_{max} = \int_0^{\infty} v dt = \dfrac{-v_0 m}{k} e^{\frac{-k}{m}t}\Big|_0^{\infty} = \dfrac{mv_0}{k}$

or $f = ma \Rightarrow -kv = m\dfrac{dv}{dx}\dfrac{dx}{dt}$

$$\Rightarrow \int_{x_0}^{x} dx = -\int_{v_0}^{0}\frac{m}{k}dv \Rightarrow \Delta x_{max} = \frac{mv_0}{k}$$

[课后思考] $f=-kx$ or $f=-kt$

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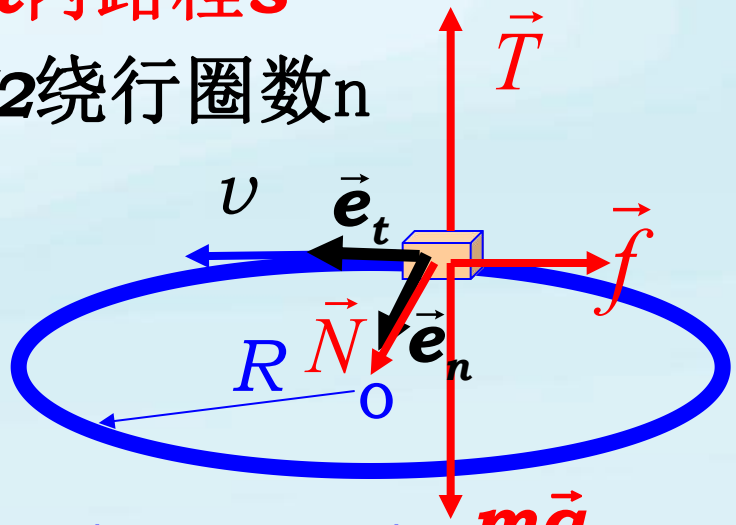
[例题1-10]光滑水平面上固定圆环R, 物块与环μ, 初速v_0, 求: (1) $v_{(t)}$ (2) t内路程s

(3) 降为$v_0/2$绕行圈数n



解: (1)选块　建系

受力　方程

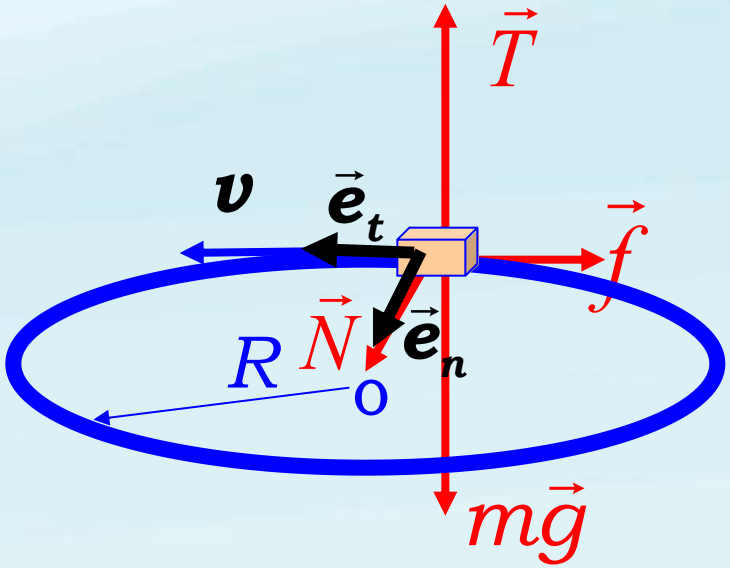
$$\left.\begin{aligned}\vec{e}_t: -f &= m a_t = m\frac{dv}{dt}\\ \vec{e}_n: N &= m a_n = mv^2/R\\ f &= \mu N\end{aligned}\right\} \Rightarrow \int_{v_0}^{v}-\frac{dv}{v^2}=\int_0^t\frac{\mu dt}{R} \Rightarrow v=\frac{v_0}{1+\mu v_0 t/R}$$

(2) $ds = vdt$

$$\int_0^s ds=\int_0^t\frac{v_0}{1+\mu v_0 t/R}dt \Rightarrow s=\frac{R}{\mu}\ln\left(1+\frac{\mu v_0 t}{R}\right)$$

[例题1-10]光滑水平面固定圆环R, 物块与环μ, 初速v_0, 求: (1) $v_{(t)}$ (2) t内路程s

(3) 降为$v_0/2$绕行圈数n



解: (3) $\vec{e}_t$: $-f = m\dfrac{dv}{dt}$

$\vec{e}_n$: $N = mv^2/R$

$f = \mu N$

$$\Rightarrow -\frac{dv}{v^2} = \frac{\mu dt}{R}\frac{ds}{ds}$$

$$\Rightarrow -\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v} = \int_0^{n2\pi R} \frac{\mu}{R} ds \Rightarrow n = \frac{\ln 2}{2\pi\mu}$$

或: $$\frac{v_0}{2} = \frac{v_0}{1+\mu v_0 t_0/R}$$

$$n2\pi R = \frac{R}{\mu}\ln\left(1+\frac{\mu v_0 t_0}{R}\right)$$

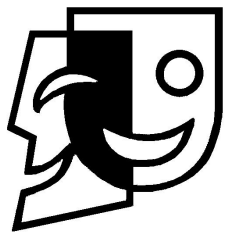
[讨　论] $m=1$质点受$\vec{F}=t\vec{i}$, 以初速$2\vec{j}$过坐标原点, 写出$\vec{r}(t)$ (均为SI制)

解: $\vec{a}=\dfrac{\vec{F}}{m}=t\vec{i}$

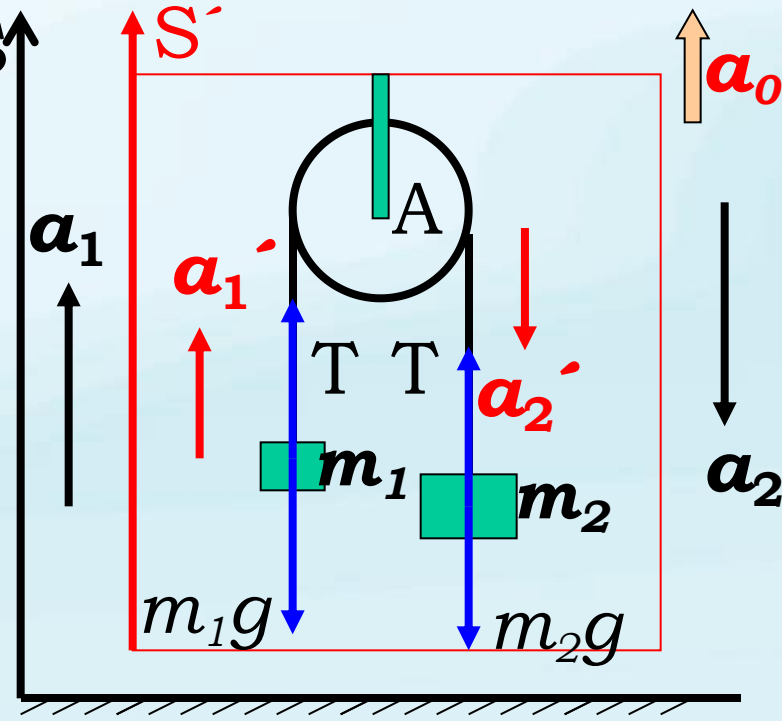
$$\int_{\vec{v}_0}^{\vec{v}} d\vec{v}=\int_0^t \vec{a}dt \Rightarrow \int_{2\vec{j}}^{\vec{v}} d\vec{v}=\int_0^t t\vec{i}dt \Rightarrow \vec{v}=2\vec{j}+\frac{1}{2}t^2\vec{i}$$

$$\int_0^{\vec{r}} d\vec{r}=\int_0^t \vec{v}dt=\int_0^t (2\vec{j}+\frac{1}{2}t^2\vec{i})dt \Rightarrow \vec{r}=2t\vec{j}+\frac{1}{6}t^3\vec{i}\ (m)$$

动力学与运动学：连接桥梁 $\vec{a}$



[例题1-12]轻滑轮A分别挂$m_1 m_2$，当A以a_0上升时，求$a_1 a_2 (m_2 > m_1)$



[解法1]地面惯性系S

选$m_1 m_2$

建系 受力 方程

a_1方向：$T - m_1 g = m_1 a_1$

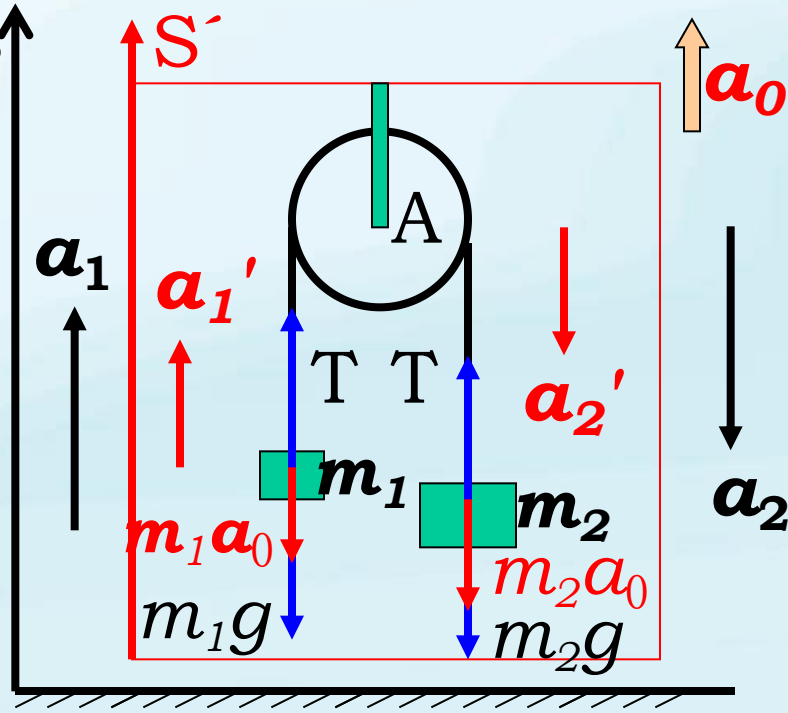
a_2方向：$m_2 g - T = m_2 a_2$

$a_1 \overset{?}{=} a_2$

加速度间关系 $a_1' = a_2' \Rightarrow a_1 - a_0 = a_2 + a_0$

$$\Rightarrow \begin{cases} a_1 = [(m_2 - m_1)g + 2m_2 a_0]/(m_2 + m_1) \\ a_2 = [(m_2 - m_1)g - 2m_1 a_0]/(m_2 + m_1) \end{cases}$$

[例题1-12]轻滑轮A分别挂$m_1 m_2$，S
当A以a_0上升时，求$a_1 a_2 (m_2 > m_1)$



[解法2]滑轮非惯性系S´

选$m_1 m_2$

建系 受力 方程

$$a_1'\text{方向}:\begin{cases} T - m_1 a_0 - m_1 g = m_1 a_1' \\ a_2'\text{方向}: \quad m_2 a_0 + m_2 g - T = m_2 a_2' \\ a_1' = a_2' = a' \end{cases} \Rightarrow a' = \frac{(m_2 - m_1)(g + a_0)}{m_1 + m_2}$$

$$\Rightarrow \begin{cases} a_1 = a_1' + a_0 = [(m_2 - m_1)g + 2m_2 a_0]/(m_2 + m_1) \\ a_2 = a_2' - a_0 = [(m_2 - m_1)g - 2m_1 a_0]/(m_2 + m_1) \end{cases}$$

[讨论]如果只求T，选哪种解法方便？