静电场是无旋的，电势

$\varphi(P_2)-\varphi(P_1)=-\int_{P_1}^{P_2}\vec{E}\cdot d\vec{l}$

$\oint \vec{E}d\vec{l}=0$ ，与路径无关.

$d\varphi=-\vec{E}\cdot d\vec{l}$ ，

$\left(d\varphi=\frac{\partial\varphi}{\partial x}dx+\frac{\partial\varphi}{\partial y}dy+\frac{\partial\varphi}{\partial z}dz=\nabla\varphi\cdot d\vec{l}\right)$

$\Rightarrow \nabla\varphi=-\vec{E}$ ， $\vec{E}=-\nabla\varphi$

$\varphi(P)=\int_P^{\infty}\vec{E}\cdot d\vec{l}$

φ 与 $\vec{E}$ 在研究问题中是等价的.

$\vec{E}=\frac{Q\vec{r}}{4\pi\varepsilon_0 r^3}=-\frac{Q}{4\pi\varepsilon_0}\nabla\frac{1}{r}$

$\Rightarrow \vec{E}=-\nabla\varphi$ ， $\varphi=\frac{Q}{4\pi\varepsilon_0 r}$

静电学核心问题

$\rho\longrightarrow\varphi\longrightarrow\vec{E}$

eg. 均匀电场 $\longrightarrow E$

$\longrightarrow x$

取 $x=0$ 为电势0点.

$\varphi(x)=\int_P^0\vec{E}\cdot d\vec{l}=\vec{E}\cdot(-\vec{x})$

能量

线性介质 静电场无磁场.

$W=\frac{1}{2}\int_{V_\infty}\vec{E}\cdot\vec{D}\,dV$ （全空间）

$\vec{E}\cdot\vec{D}=-\nabla\varphi\cdot\vec{D}=-\nabla\cdot(\varphi\vec{D})+\varphi\nabla\cdot\vec{D}=\rho\varphi-\nabla\cdot(\varphi\vec{D})$

$\int\nabla\cdot(\varphi\vec{D})dV=\int\varphi\cdot\vec{D}\cdot d\vec{S}$ 在 $r\to\infty$ 时，积分为0.

$\frac{1}{r}\quad\frac{1}{r^2}\quad r^2$

$\oint D\,dS=\rho$

故 $W=\frac{1}{2}\int\rho\varphi\,dV$ $\qquad\varphi=\int\frac{\rho'(\vec{r})}{4\pi\varepsilon_0 r}dV'$

静电势的微分方程和边值关系.

$$\varphi = \int \frac{\rho \, dV}{4\pi\varepsilon_0 r}$$

$$\begin{cases} \vec{E} = -\nabla\varphi \\ \vec{D} = \varepsilon\vec{E} \\ \nabla\cdot\vec{D} = \rho_f \end{cases} \Rightarrow \nabla^2\varphi = -\frac{\rho_f}{\varepsilon}$$ 泊松方程

边值关系,

$$\begin{cases} \vec{n}\cdot(\vec{D}_2-\vec{D}_1) = \sigma \\ \vec{D} = \varepsilon\vec{E} \end{cases} \Rightarrow -\left(\varepsilon_2\frac{\partial\varphi_2}{\partial n} - \varepsilon_1\frac{\partial\varphi_1}{\partial n}\right) = \sigma$$

$$E_{2t} = E_{1t} \quad \varphi = \int_r^\infty E_t \, dl_t \Rightarrow \varphi_2 = \varphi_1$$

导体内部无 $\vec{E}$ φ是常量. $\vec{n}$是导体面法向, 从导体指向空气.

而且$\vec{n}$由介质1指向介质2. $\longrightarrow$ 故 φ_1是导体电势 $\frac{\partial\varphi_1}{\partial n} = 0$. $\varepsilon\frac{\partial\varphi_2}{\partial n} - 0 = -\sigma$. $\frac{\partial\varphi_2}{\partial n} = -\frac{\sigma}{\varepsilon}$

边界条件

$\varphi|_{边界} = \varphi_0$ (第一类边值问题, 狄利克雷边界条件)

边值关系: 求解区域内, 分界面的影响.

边界条件: 体系边界.