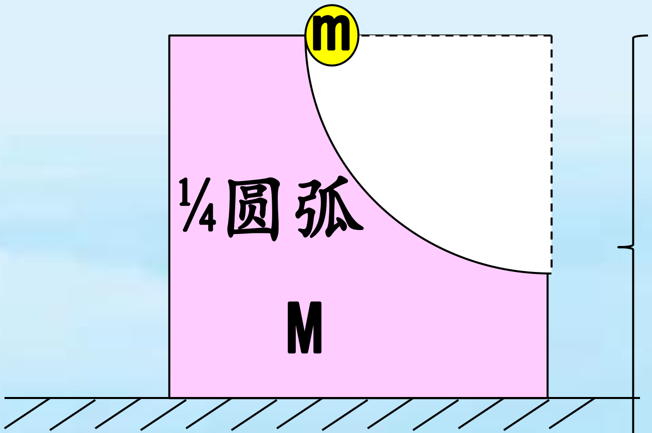
相对运动示例

[1]不计一切摩擦，求v_{mM}



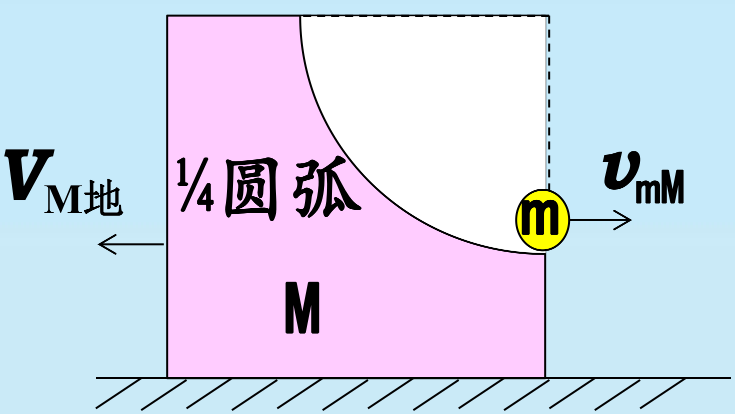
动量守恒

$$m(v_{mM}-V)-MV=0$$

机械能守恒

$$mgh=\frac{1}{2}MV^2+\frac{1}{2}m(v_{mM}-V)^2$$

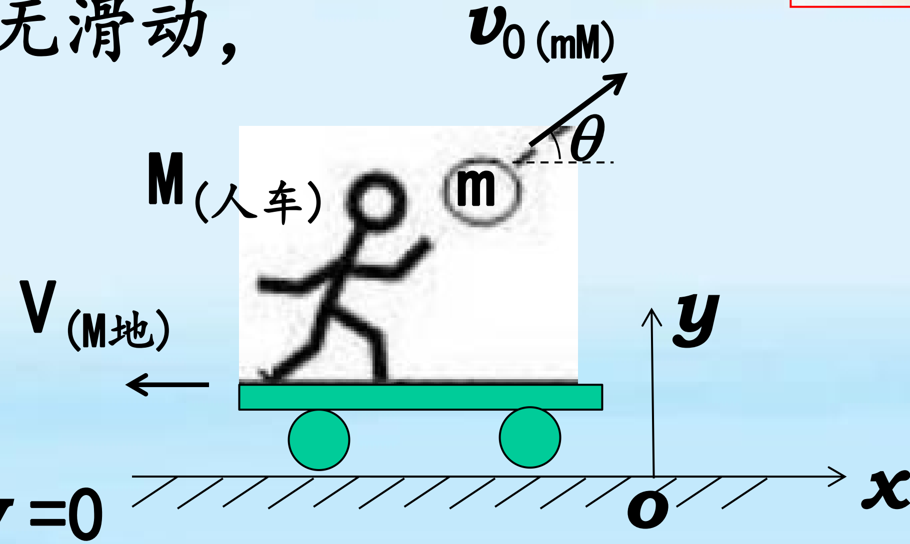
$\rightarrow v_{mM}$



$p_M = MV$　　$p_m = m(v_{mM}-V)$

[2]开始人车球静止，人车无滑动，

求$\vec{V}_{(车对地)}$、$\vec{v}_{(球对地)}$



动量守恒

$$m(v_{mM}\cos\theta - V) - MV = 0$$

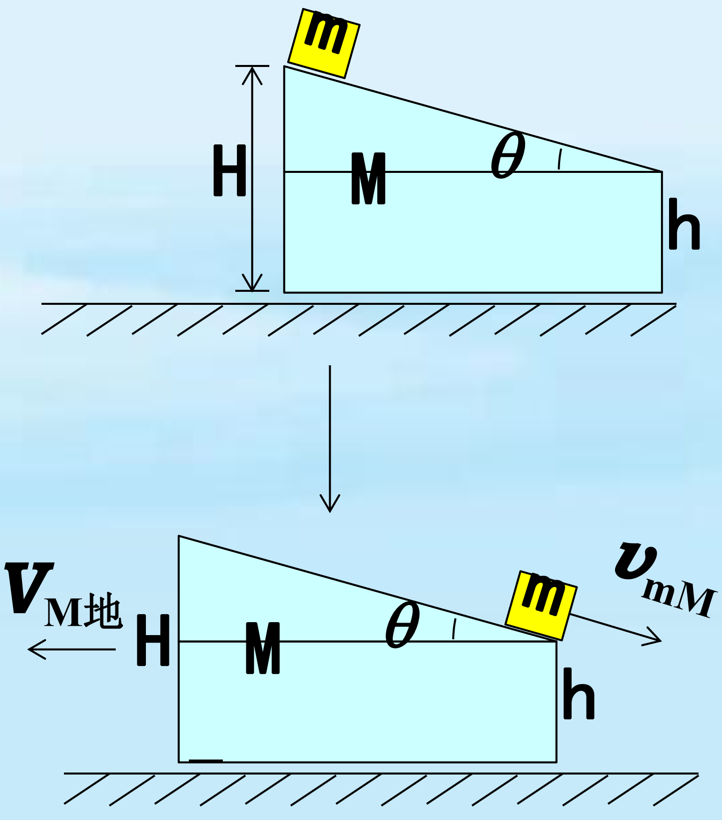
$$\to V = \frac{m v_{mM}\cos\theta}{M+m} \to \vec{V} = -\frac{m v_0 \cos\theta}{M+m}\vec{i}$$

$$\to \vec{v}_m = (v_{mM}\cos\theta - V)\vec{i} + (v_{mM}\sin\theta)\vec{j}$$

$$\to \vec{v}_m = v_0\cos\theta\,\frac{M}{M+m}\vec{i} + (v_0\sin\theta)\vec{j}$$

FangYi

[3]不计摩擦，M, m, H, h, θ均为已知, 从静止释放, 求V、v



动量守恒

$$m(v_{mM}\cos\theta - V) - MV = 0$$

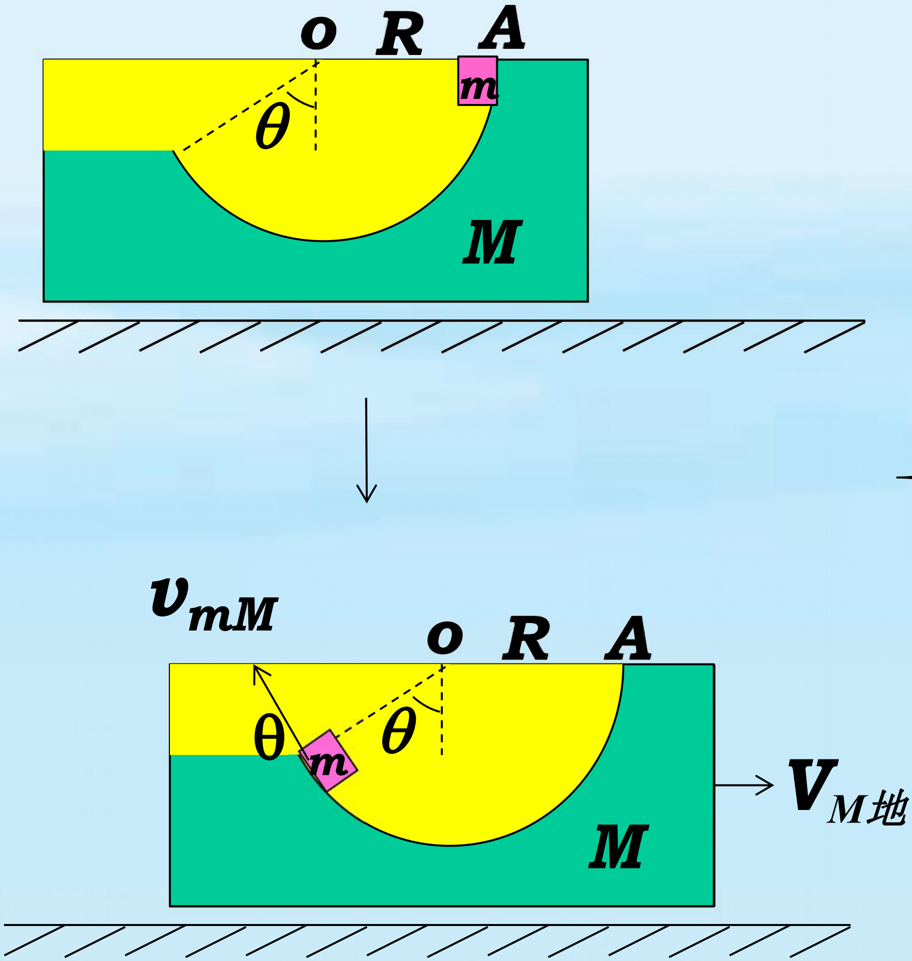
机械能守恒

$$mg(H-h) = \frac{1}{2}MV^2 + \frac{1}{2}m[(v_{mM}\cos\theta - V)^2 + (v_{mM}\sin\theta)^2]$$

$$\rightarrow V = m\sqrt{\frac{2g(H-h)\cos^2\theta}{(M+m)(M+m\sin^2\theta)}},\quad v = \sqrt{\frac{2g(H-h)(M+m)}{(M+m\sin^2\theta)}}$$

FangYi

[4]不计摩擦，M, m, R, θ均为已知, 从静止释放，求V、v



动量守恒

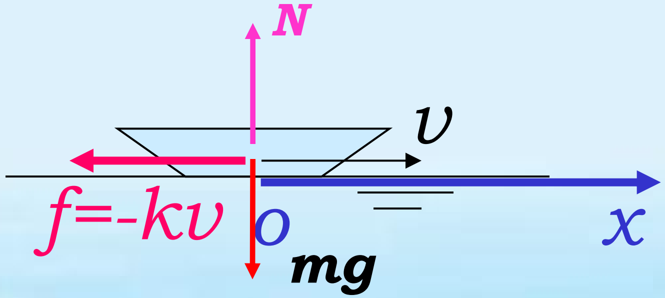
$$m(v_{mM}\cos\theta - V) - MV = 0$$

机械能守恒

$$mgR\cos\theta = \frac{1}{2}MV^2 + \frac{1}{2}m[(v_{mM}\cos\theta - V)^2 + (v_{mM}\sin\theta)^2]$$

$\rightarrow V,\ v$

[例题1-9] m, v_0 阻力 $f=-kv$，关引擎，求 $v_{(t)}$？行驶的 Δx_{max}？



解：选船　建系　受力　方程

(1) $$f = ma \Rightarrow -kv = m\frac{dv}{dt} \Rightarrow \int_{v_0}^{v}\frac{dv}{v} = -\int_0^t \frac{k}{m}dt$$

$$\Rightarrow \ln\frac{v}{v_0} = -\frac{k}{m}t \Rightarrow v = v_0 e^{-\frac{k}{m}t}$$

(2) $$\Delta x_{max} = \int_0^{\infty} v dt = \left.\frac{-v_0 m}{k} e^{\frac{-k}{m}t}\right|_0^{\infty} = \frac{mv_0}{k}$$

or $$f = ma \Rightarrow -k\cancel{v} = m\frac{dv}{dx}\cancel{\frac{dx}{dt}}$$

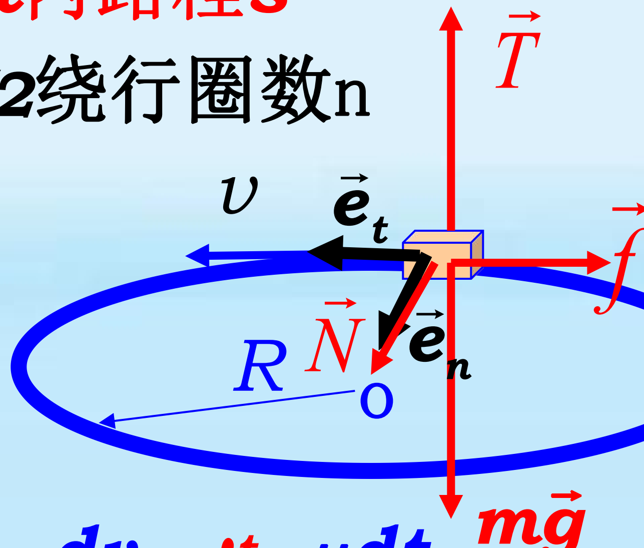
$$\Rightarrow \int_{x_0}^{x} dx = -\int_{v_0}^{0}\frac{m}{k}dv \Rightarrow \Delta x_{max} = \frac{mv_0}{k}$$

[课后思考] $f=-kx$ or $f=-kt$

[例题1-10]光滑水平面上固定圆环R, 物块与环μ,

初速v_0, 求:(1) $v_{(t)}$ (2) t内路程s

(3) 降为$v_0/2$绕行圈数n



解:(1)选块 建系

受力 方程

$$\left.\begin{aligned} &\vec{e}_t: -f = m\,a_t = m\frac{dv}{dt} \\ &\vec{e}_n: N = m\,a_n = mv^2/R \\ &f = \mu N \end{aligned}\right\} \Rightarrow \int_{v_0}^{v} -\frac{dv}{v^2} = \int_0^t \frac{\mu dt}{R}$$

$$\Rightarrow v = \frac{v_0}{1+\mu v_0 t/R}$$

(2) $ds = vdt$

$$\int_0^s ds = \int_0^t \frac{v_0}{1+\mu v_0 t/R}dt \Rightarrow s = \frac{R}{\mu}\ln(1+\frac{\mu v_0 t}{R})$$

[例题1-10]光滑水平面固定圆环R, 物块与环μ, 初速v_0, 求:(1) $v_{(t)}$ (2) t内路程s

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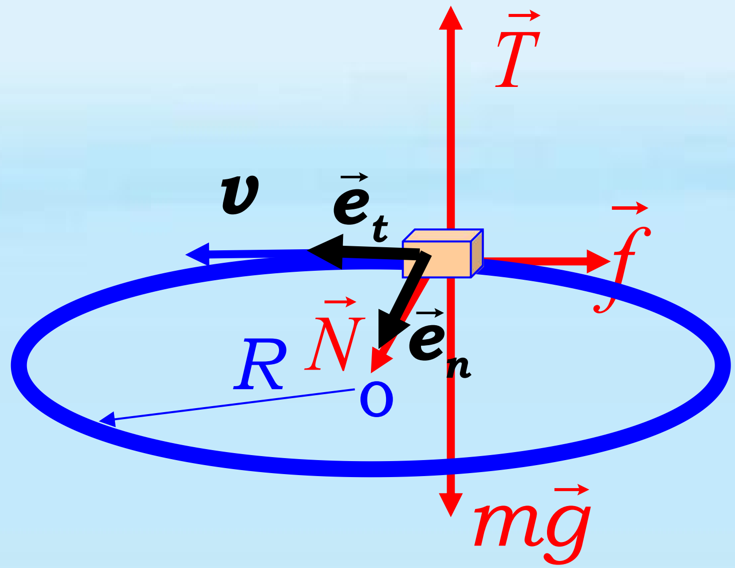
(3) 降为$v_0/2$绕行圈数n



解: (3) $\vec{e}_t$: $-f = m\dfrac{dv}{dt}$

$\vec{e}_n$: $N = mv^2/R$

$f = \mu N$

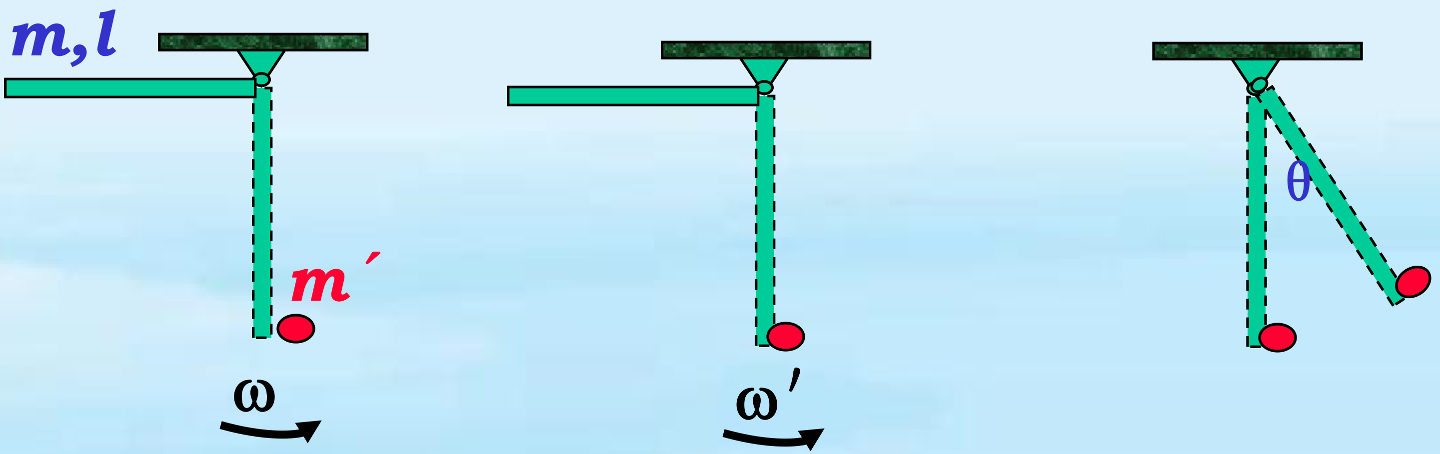
$$\Rightarrow -\frac{dv}{v^{\cancel{2}}} = \frac{\mu \cancel{dt}}{R}\frac{ds}{\cancel{ds}}$$

$$\Rightarrow -\int_{v_0}^{\frac{v_0}{2}}\frac{dv}{v} = \int_0^{n2\pi R}\frac{\mu}{R}ds \Rightarrow n = \frac{\ln 2}{2\pi\mu}$$

或: $$\left.\begin{aligned}\frac{v_0}{2} &= \frac{v_0}{1+\mu v_0 t_0/R}\\ n2\pi R &= \frac{R}{\mu}\ln\left(1+\frac{\mu v_0 t_0}{R}\right)\end{aligned}\right\}$$

刚体力学-动量矩定理、动量矩守恒、功能原理、机械能守恒

[课后练]轴承阻力矩恒M,水平均匀棒静止释放,完全非,求θ



下摆，未碰　　　完全非碰后　　　上摆至θ

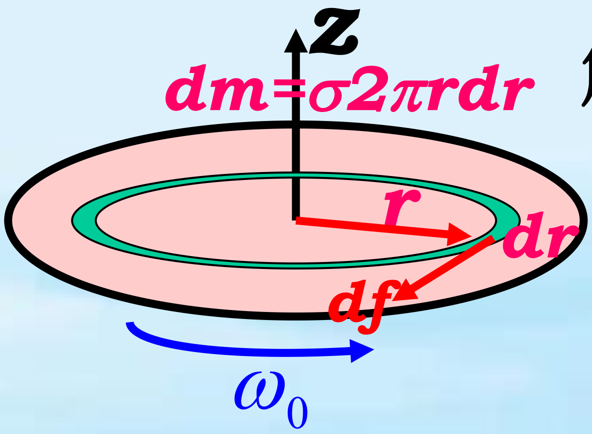
思考：完全弹性碰撞呢？

阻力矩**M=M**（θ）？

没有阻力矩呢？

[例题3-4]已知:园盘m,R, ω_0绕z轴转, μ,

求:(1)M_f (2)园盘停下需t



解: (1)

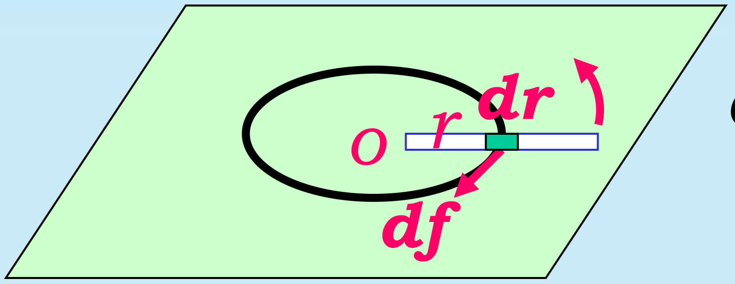
$$\left.\begin{aligned} df &= \mu g\,dm \\ dM_f &= -r\,df \end{aligned}\right\}$$

$$\Rightarrow \int_0^{M_f} dM_f = \int_0^R -\mu g\sigma 2\pi r^2 dr \Rightarrow M_f = -\frac{2}{3}\mu m g R$$

$$(2)\vec{J}_{冲} = \vec{L} - \vec{L}_0 \Rightarrow M_f t = 0 - L_0 \Rightarrow t = \frac{-J\omega_0}{M_f} = \frac{-\frac{1}{2}mR^2\omega_0}{-\frac{2}{3}\mu m g R} = \frac{3R\omega_0}{4\mu g}$$

$$or.\quad \alpha = \frac{M_f}{J} = -\frac{4\mu g}{3R} \qquad 0 - \omega_0 = \alpha t$$

[讨论3]圆盘换成棒

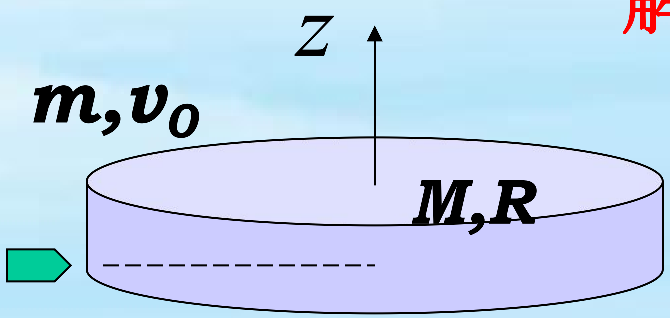


$$dM = r\,df \Rightarrow M = \int_0^l r\mu g\lambda\,dr = \mu m g l / 2$$

FangYi

[讨论7]静盘M,R可绕Z转, 子弹m,v_0(⊥半径)边缘射入,$\mu_{盘桌面}$,(嵌盘边，子弹重力$M_{摩擦}$不计)

求(1)碰后圆盘ω(2)停下需t(3)停下走过θ



解:(1){子弹,盘}Lz守恒

$$Rmv_0=(\frac{1}{2}MR^2+mR^2)\omega\Rightarrow\omega=\frac{mv_0}{(\frac{1}{2}M+m)R}$$

(2) 参见例题3-4

$$\left.\begin{aligned}M_f&=-\frac{2}{3}\mu MgR\\ M_f t&=0-J_{总}\omega\end{aligned}\right\}\Rightarrow t=-\frac{J_{总}\omega}{M_f}=\frac{3mv_0}{2\mu Mg}$$

如果考虑子弹重力

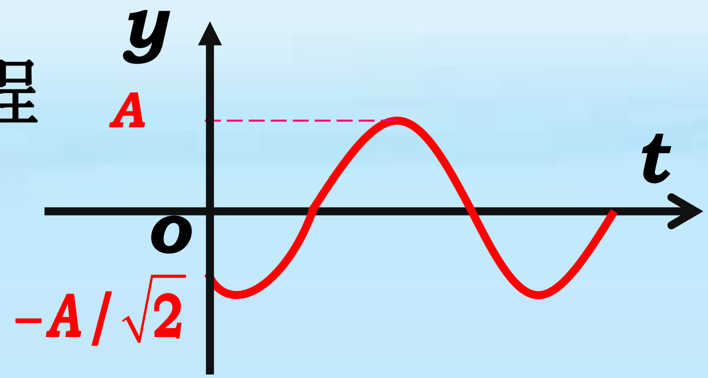
$$M_f=-\frac{2}{3}\mu MgR$$

$-R\mu mg$

(3) $M_f\theta=0-\frac{1}{2}J_{总}\omega^2\Rightarrow\theta=\frac{-J_{总}\omega^2}{2M_f}$

课后思考：如何用运动学公式求解

[例5-2]平面谐波$u=120m/s$,$\lambda=60m$,沿x负向,坐标原点o点振动曲线如图,写波动方程。



解:参考点o振动方程→波动方程

$$\left.\begin{array}{l}\omega = 2\pi / T \\ T = \lambda / u\end{array}\right\} \Rightarrow \omega = 4\pi$$

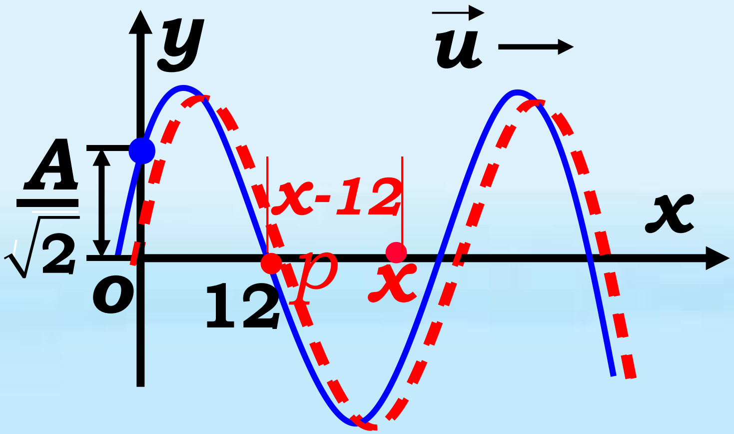
$$\left\{\begin{array}{l}y_0 = -A/\sqrt{2} \\ v_0 < 0\end{array}\right. \Rightarrow \varphi = 3\pi / 4$$

o振动方程:$y = A\cos(4\pi t + 3\pi / 4)\ m$

波动方程: $y=A\cos[4\pi(t+x/120)+3\pi/4]\ m$

[例5-3]平面谐波$u=8$, $t=0$波形如图

求[1]λ [2]波动方程



解:(1) $2\pi : \lambda = [\pi/4 - (-\pi/2)] : 12$

$\Rightarrow \lambda = 32$

(2) $\omega = 2\pi\upsilon = 2\pi u/\lambda = \pi/2$

o振动方程$y_0 = A\cos(\frac{\pi}{2}t + \frac{\pi}{4})$

$$\begin{cases} y_0 = \frac{A}{\sqrt{2}} \\ v_0 < 0 \end{cases} \Rightarrow \varphi_0 = \frac{\pi}{4}$$

波动方程$y = A\cos[\frac{\pi}{2}(t - \frac{x}{8}) + \frac{\pi}{4}]$

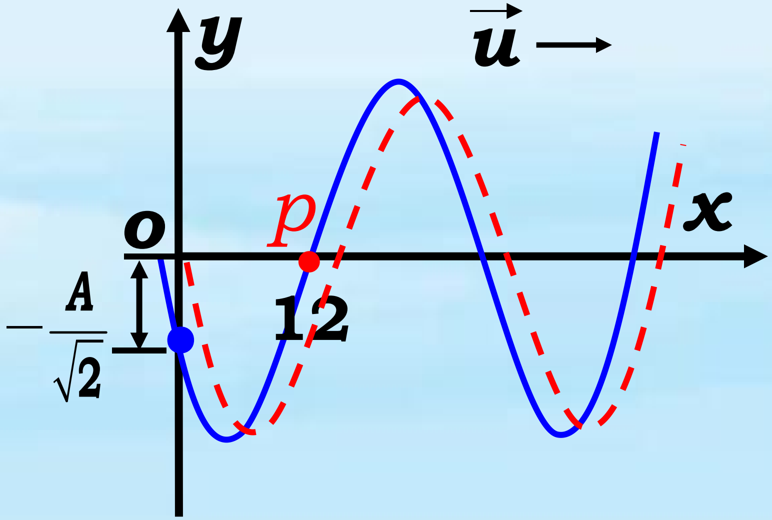
p振动方程$y_p = A\cos(\frac{\pi}{2}t - \frac{\pi}{2})$

$$\begin{cases} y_p = 0 \\ v_p > 0 \end{cases} \Rightarrow \varphi_p = -\frac{\pi}{2}$$

波动方程 $y = A\cos[\frac{\pi}{2}(t - \frac{x-12}{8}) - \frac{\pi}{2})]$

[例5-3]*平面谐波 $u=8$，$t=0$ 波形如图

求[1]λ [2]波动方程



$$\begin{cases} y_0 = -\dfrac{A}{\sqrt{2}} \\ v_0 > 0 \end{cases} \Rightarrow \varphi_0 = -\frac{3\pi}{4}$$

$$\begin{cases} y_p = 0 \\ v_p < 0 \end{cases} \Rightarrow \varphi_p = \frac{\pi}{2} \quad ×$$

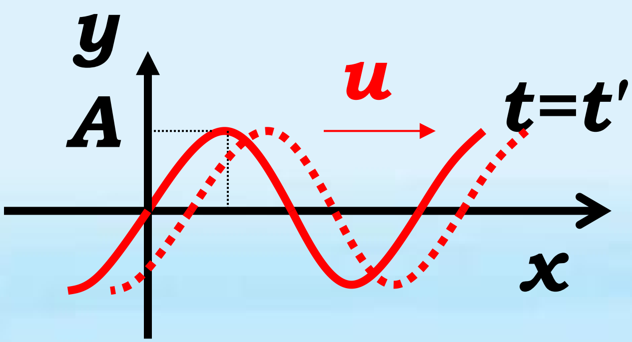
$$\Rightarrow \varphi_p = -\frac{3\pi}{2} \quad √$$

[讨论1]平面谐波沿x正向, A、υ、u已知,

$t=t'$ 波形如右,

求(1)原点振动方程

(2)波动方程



解:(1) $y = A\cos(\omega t + \varphi)$

原点$t=t'$ $\begin{cases} y = 0 \\ v < 0 \end{cases} \Rightarrow \quad = \dfrac{\pi}{2}$ 位相定初相

初相 $\varphi = \phi - \omega t' = \dfrac{\pi}{2} - 2\pi\upsilon t'$

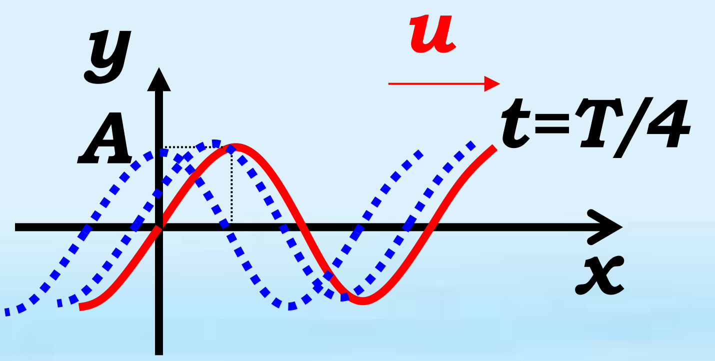
$\Rightarrow y = A\cos[2\pi\upsilon t + (\dfrac{\pi}{2} - 2\pi\upsilon t')]$

(2) 波动方程 $y = A\cos[2\pi\upsilon(t - \dfrac{x}{u}) + (\dfrac{\pi}{2} - 2\pi\upsilon t')]$

FangYi

[讨论2]已知t=T/4 波形,如何确定原点初相?

解：原点*t=0* $\begin{cases} y = A \\ v = 0 \end{cases} \Rightarrow \varphi = 0$



[例5-6] 设0点 $:y = A\cos\omega t$ 激发起波沿±x方向传播，确定驻波形成区域并确定波节及波腹位置. 写出另一区域的波动方程

FangYi

解：驻波形成区域：x>0

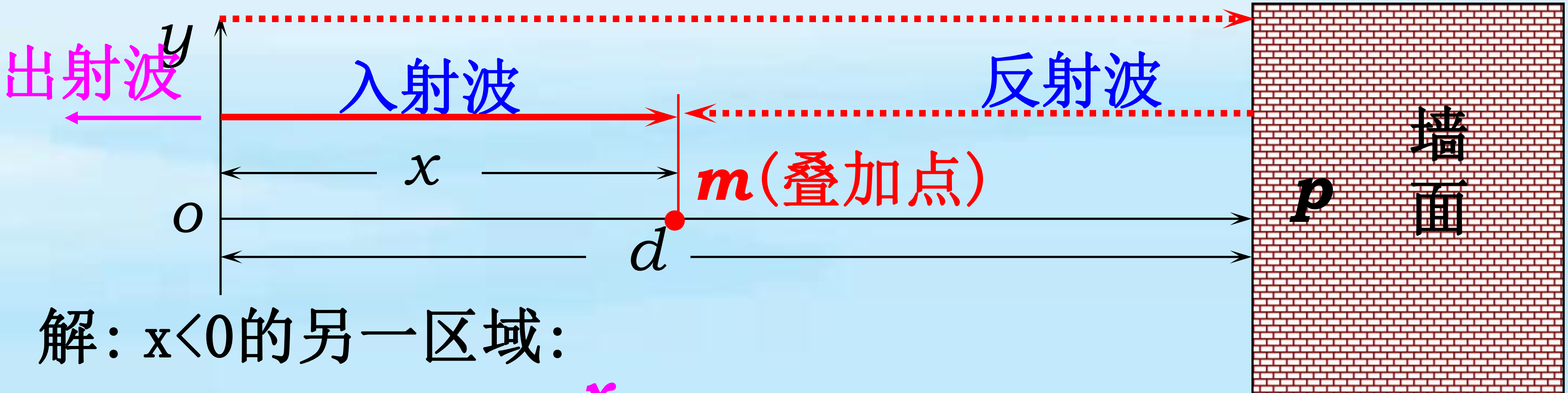
$$y_{入} = A\cos\omega\left(t - \frac{x}{u}\right) \longrightarrow y_{入p} = A\cos\omega\left(t - \frac{d}{u}\right)$$

$$y_{反} = A\cos\left[\omega\left(t - \frac{d}{u} - \frac{d-x}{u}\right) + \pi\right] \longleftarrow y_{p反} = A\cos\left[\omega\left(t - \frac{d}{u}\right) + \pi\right]$$

$$y_{驻} = y_{入} + y_{反} = 2A\cos\left[\frac{2\pi(d-x)}{\lambda} - \frac{\pi}{2}\right]\cos\left[\omega t - \frac{2\pi d}{\lambda} + \frac{\pi}{2}\right]$$

$$\frac{2\pi(d-x)}{\lambda} - \frac{\pi}{2} = \begin{cases} k\pi & \text{波腹}\ x = d - \frac{\lambda}{4}(2k+1) \quad (k = 0,1,2\ldots) \\ (2k+1)\frac{\pi}{2} & \text{波节}\ x = d - \frac{\lambda}{2}(k+1) \quad (k = -1,0,1\ldots) \end{cases}$$

[例5-6] 设0点 $y = A\cos\omega t$ 激发起波沿±x方向传播，确定驻波形成区域并确定波节及波腹位置.写出另一区域的波动方程



解: x<0的另一区域:

$$y_{出} = A\cos\omega\left(t+\frac{x}{u}\right)$$

$$y_{反} = A\cos\left[\omega\left(t-\frac{d}{u}-\frac{d-x}{u}\right)+\pi\right]$$

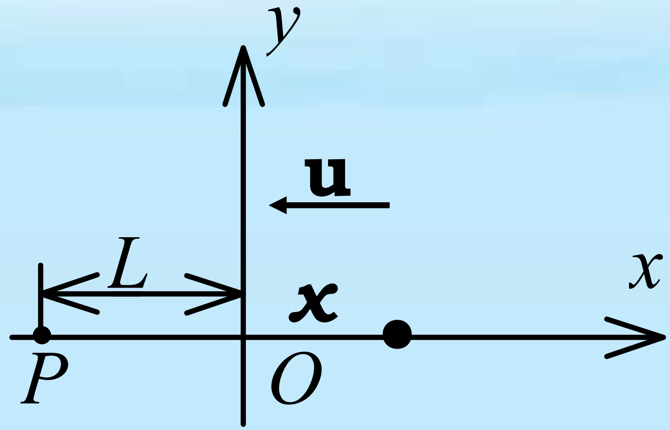
$$= A\cos\left[\omega t+\frac{\omega x}{u}+\left(\pi-\frac{2\omega d}{u}\right)\right]$$

$$y_{行} = y_{出}+y_{反} = 2A\sin\frac{\omega d}{u}\cos\left(\omega t+\frac{\omega x}{u}+\frac{\pi}{2}-\frac{\omega d}{u}\right)$$

习题课[讨论2]平面谐波λ沿x负向，P点 $y_P = A\cos(2\pi\upsilon t+\frac{1}{2}\pi)$

(1)波函数

(2)P点何时与0点t_1时的振动状态相同.　　08期末B



解：(1) $y = A\cos[2\pi\upsilon(t+\frac{x+L}{u})+\frac{\pi}{2}]$

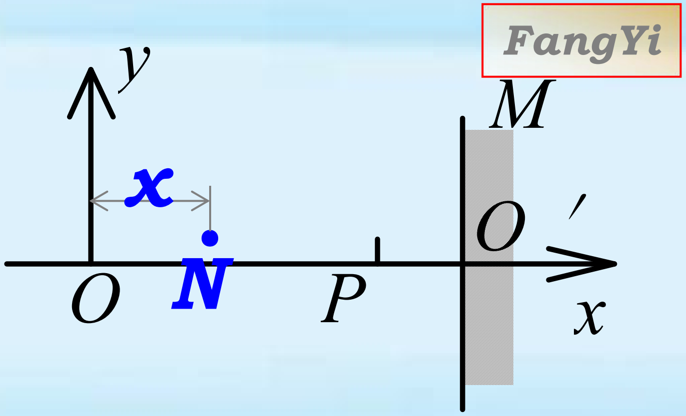
(2) $y_P = A\cos(2\pi\upsilon t+\frac{1}{2}\pi)$

$$y_o\Big|_{t_1} = A\cos[2\pi(\upsilon t_1+\frac{0+L}{\lambda})+\frac{\pi}{2}]$$

$$\begin{cases}\Delta\Phi=[2\pi(\upsilon t_1+\frac{L}{\lambda})+\frac{\pi}{2}]-(2\pi\upsilon t+\frac{1}{2}\pi)\\ \Delta\Phi=k2\pi\end{cases}$$

$$\rightarrow t = t_1+\frac{L}{\lambda\upsilon}+\frac{k}{\upsilon}\qquad (k=0,\pm1,\pm2,\ldots)$$

习题课[习题3] ω, A沿$+x$, $t=0$时0点向$-y$运动. 波密面$M \perp x$轴. $oo'=7\lambda/4$, $Po'=\lambda/4$. 求:(1)入射、反射波函数;
(2)P点的振动方程.



解: $t=0$: $y_0=0$, $v_0<0$, $\therefore \varphi = \dfrac{\pi}{2}$ $\rightarrow y_0 = A\cos(\omega t + \dfrac{\pi}{2})$

入射波 $$y_{入} = A\cos(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda}x) \rightarrow y_{入o'} = A\cos(\omega t + \frac{\pi}{2} - \frac{2\pi}{\lambda} \times \frac{7\lambda}{4})$$

$$y_{o'反} = A\cos(\omega t + \underbrace{\frac{\pi}{2} - \frac{2\pi}{\lambda} \times \frac{7\lambda}{4} + \pi}_{-2\pi})$$

反射波 $$y_{反} = A\cos[\omega t + \frac{2\pi}{\lambda}x + \frac{\pi}{2}]$$

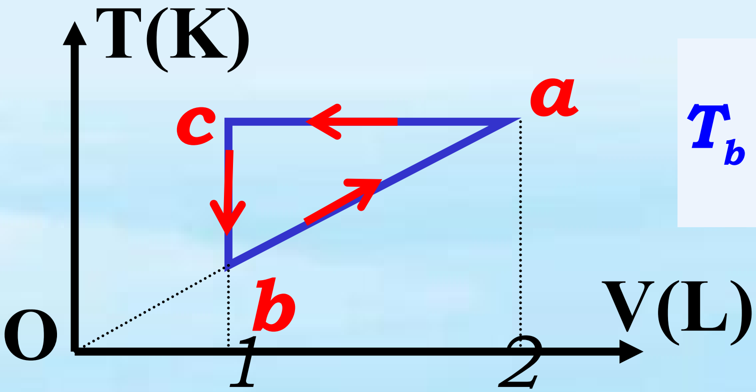
合成波 $$y = y_{入} + y_{反} = 2A\cos\frac{2\pi}{\lambda}x\cos(\omega t + \frac{\pi}{2})$$

$$y\Big|_{x_p=3\lambda/2} = -2A\cos(\omega t + \frac{\pi}{2})$$

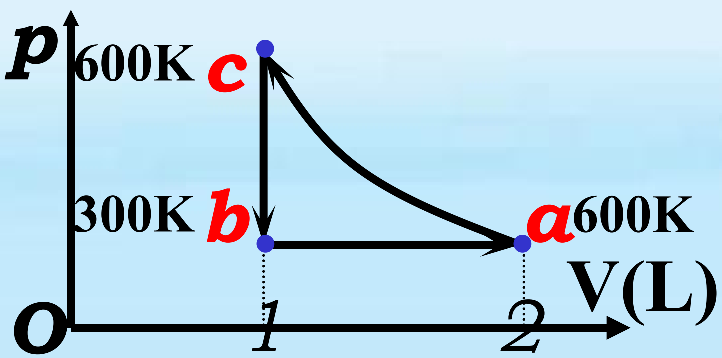
热一律用于循环-其他图转pV图

[例题7－4]1mol单原子分子理气循环如T－V图，T_c=600K.

试求(1) Q_{ac}, Q_{cb}, Q_{ba} (2)整个循环$A_净$ (3) η or ω



$$T_b = \frac{V_b}{V_a} T_a$$



解(1) ac等温压缩 $Q_{ac} = A_{ac} = \upsilon RT\ln\frac{V_c}{V_a} = R(600\ln\frac{1}{2}) = -416R$

cb等容降压(温) $Q_{cb} = \Delta E = \upsilon C_v \Delta T = 1\times\frac{3R}{2}(300-600) = -450R$

ba等压膨胀 $Q_{ba} = \upsilon C_p \Delta T = 1\times\frac{5R}{2}(600-300) = 750R$

(2) 由热一律 $A = Q_{ac} + Q_{cb} + Q_{ba} = -116R$

(3) $\omega = \frac{Q_吸}{|A_{net}|} = \frac{750R}{116R} = 6.47$

[课后练]1mol双原子分子理想气体循环如V—T图, $T_c=400K$.

试求(1) Q_{ac}, Q_{cb}, Q_{ba} (2) 整个循环$A_{净}$ (3) η0Rω