$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{1}{i\hbar}[A,H] = 0$$

守恒量的性质. ① $\frac{dA}{dt}=0$, ② 几率分布不随时间改变.

$$\begin{cases} \psi(t) = \sum c_n(t)\psi_n = \sum_n c_n(0)\, e^{-iE_n t/\hbar}\psi_n. & \text{概率分布不变,} \\ \psi(0) = \sum c_n(0)\psi_n. & \end{cases}$$

模为1

若A不守恒,则 A,H无共同本征态. A的几率分布不定.

特例:

- $\psi(0) = \psi_k$.

  则 $\psi(t) = e^{-i\frac{E_k t}{\hbar}}\psi_k = \psi_k$, 状态不变.

  好量子数. $|k\rangle$

  - 力学量完全集是一组守恒量, 包括 H

    用共同本征态表示系统状态.

- $\psi(0) = c_1\psi_1 + c_2\psi_2$.

  $\psi(t) = c_1 e^{-iE_1 t/\hbar}\psi_1 + c_2 e^{-iE_2 t/\hbar}\psi_2$

- 若 H 不显含 t. $[H,H]=0$, $\frac{d}{dt}H = 0$. 能量守恒.

$$H = \frac{p^2}{2m} + V(\vec{x}) = H(\vec{x})$$

$$i\hbar\frac{\partial\psi(\vec{x},t)}{\partial t} = H(\vec{x})\psi(\vec{x},t)$$

分离变量 $\psi(\vec{x},t) = \varphi(\vec{x})f(t)$

$$\frac{i\hbar\frac{\partial f}{\partial t}\varphi(\vec{x})}{\varphi(\vec{x})f(t)} = \frac{H(\vec{x})\varphi(\vec{x})\cdot f(t)}{\varphi(\vec{x})f(t)}$$

$$i\hbar\frac{\partial f}{\partial t}\cdot\frac{1}{f} = \frac{H(\vec{x})\varphi(\vec{x})}{\varphi(\vec{x})} = E$$

$$\Rightarrow \begin{cases} H(\vec{x})\varphi(\vec{x}) = E\varphi(\vec{x}) & \text{(定态 Schrodinger 方程)} \\ i\hbar\frac{df}{dt} = Ef \Rightarrow f = Ce^{-\frac{iEt}{\hbar}}. & \end{cases}$$

$$\psi(x,t) = \varphi(\vec{x})\, e^{-\frac{iEt}{\hbar}}$$