[讨　论] $\vec{r} = at^2\vec{i} + bt^2\vec{j}$($a,b$均常量)

质点作什么运动?

解: $\vec{r} = at^2\vec{i} + bt^2\vec{j}$ $\quad \begin{cases} x = at^2 \\ y = bt^2 \end{cases} \Longrightarrow y = \dfrac{b}{a}x \Longrightarrow$ 直线

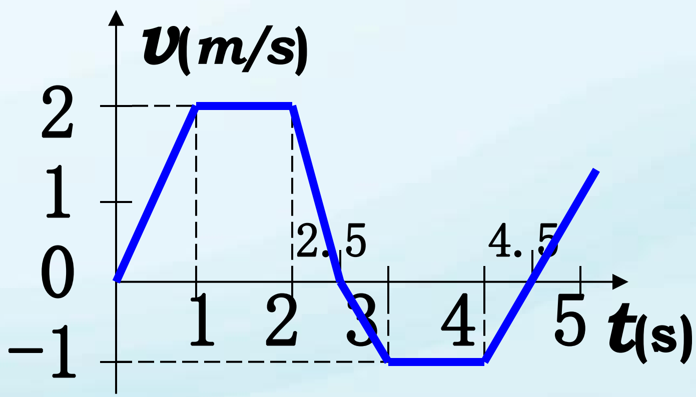
$\vec{v} = 2at\vec{i} + 2bt\vec{j} = \vec{v}_{(t)} \Rightarrow$ 变速　　变速直线

$\vec{a} = 2a\vec{i} + 2b\vec{j} = const. \Rightarrow$ 匀变速　　匀变速直线

FangYi

[讨 论]质点沿x运动，由图得t=4.5s位置坐标x

$(A)0 \quad (B)5m \quad (C)2m \quad (D)-2m \quad (E)-5m$



$t=0, x=0$

解：$v = dx/dt$

$$\Rightarrow x\Big|_{x_0}^{x} = \int_0^{4.5} v dt = \int_0^{2.5} v dt + \int_{2.5}^{4.5} v dt$$

$$\Rightarrow x - 0 = s_{上梯形} + [-s_{下梯形}]$$

$$\Rightarrow x = 3.5 - 1.5$$

$v \sim t \quad a \sim t \quad x \sim t \quad v \sim x \quad a \sim x \quad v \sim a$

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[例题1-2]物体由静止作匀变加速直线运动（a每秒增$2m/s^2$），a_0为$1m/s^2$．求:3秒末速度大小和4秒内位移。

解：$$\frac{da}{dt} = 2 \Rightarrow \int_1^a da = \int_0^t 2dt \Rightarrow a = 2t + 1$$

$$\frac{dv}{dt} = a \Rightarrow \int_0^v dv = \int_0^t (2t+1)dt$$

$$\Rightarrow v = t^2 + t \qquad \Rightarrow \underline{\underline{v\big|_{t=3} = 12m/s}}$$

$$\frac{dx}{dt} = v \Rightarrow \int_{x_0}^x dx = \int_0^t (t^2 + t)dt$$

$$\Rightarrow \Delta x = \frac{1}{3}t^3 + \frac{1}{2}t^2 \Rightarrow \underline{\underline{\Delta x\big|_{t=4} = 29.3m}}$$

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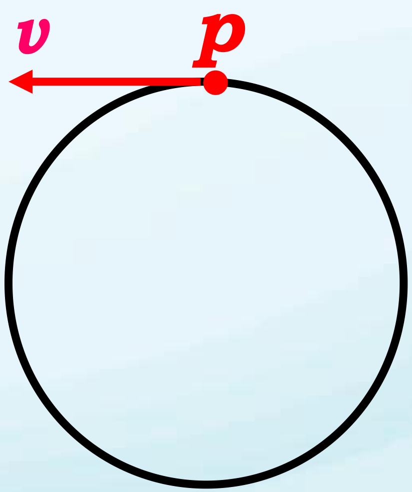
[例题1-3]质点沿R以$v = A + Bt$ 运动(AB均为常数)

求:从开始绕一周回到起点时a_t及a_n



解: $a_t = \dfrac{dv}{dt} = B$

$$a_n = \left.\frac{v^2}{R}\right|_{v=v^*} = \left.\frac{(A+Bt)^2}{R}\right|_{t=t^*} = 4\pi B + \frac{A^2}{R}$$

方法1:求v^*

$$v = \frac{ds}{dt}\cdot\frac{dv}{dv}$$

$$\Rightarrow \int_{v_0}^{v^*} v\,dv = \int_0^{2\pi R} B\,ds$$

$$\Rightarrow v^{*2} = 4\pi RB + v_0^2$$

方法2:求t*

$$\int_0^{2\pi R} ds = \int_0^{t^*} (A+Bt)\,dt$$

$$\Rightarrow 4\pi RB + A^2 = (A + Bt^*)^2$$

[例题1－4] $\vec{r}=t\vec{i}+2t^2\vec{j}$，求任$t$时$a_t$、$a_n$、$\rho$

解：$$\vec{v}=\frac{d\vec{r}}{dt}=\vec{i}+4t\vec{j} \Rightarrow v=\sqrt{1+16t^2}$$

$$\left.\begin{aligned} &\Rightarrow a_t=\frac{dv}{dt}=\underline{\underline{\frac{16t}{\sqrt{1+16t^2}}}} \\ &\vec{a}=\frac{d\vec{v}}{dt}=4\vec{j} \end{aligned}\right\} \Rightarrow a_n=\sqrt{a^2-a_t^2}=\underline{\underline{\frac{4}{\sqrt{1+16t^2}}}}$$

$$a_n=\frac{v^2}{\rho} \Rightarrow \rho=\frac{v^2}{a_n} \Rightarrow \rho=\underline{\underline{\frac{(1+16t^2)^{\frac{3}{2}}}{4}}}$$

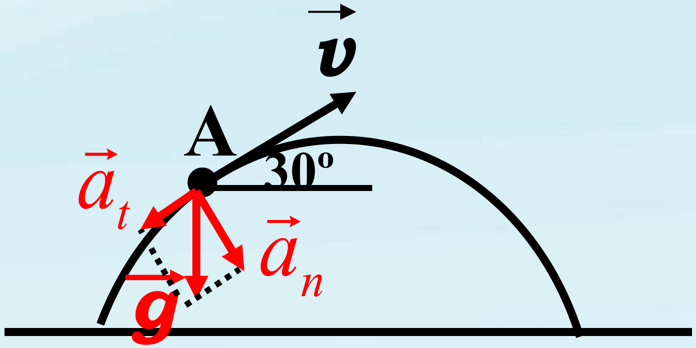
另解：$$y=2x^2 \Rightarrow \rho=\left|\frac{(1+y'^2)^{3/2}}{y''}\right|$$

$$a_n=\frac{v^2}{\rho}$$

[思 考]物体斜抛, 已知**A**点v的大小,求**A**点的a_t、a_n、ρ?

解： $$a_t = \frac{dv}{dt} = -g\sin 30^0 = -g/2$$

$$\left.\begin{aligned} a_n &= g\cos 30^0 = \sqrt{3}g/2 \\ a_n &= \frac{v^2}{\rho} \end{aligned}\right\}$$



$$\Rightarrow \rho = \frac{2\sqrt{3}v^2}{3g}$$

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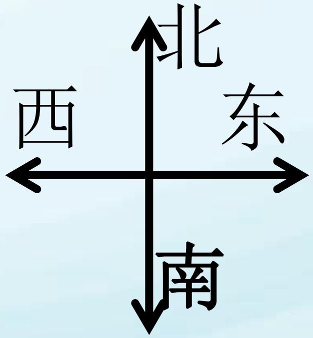
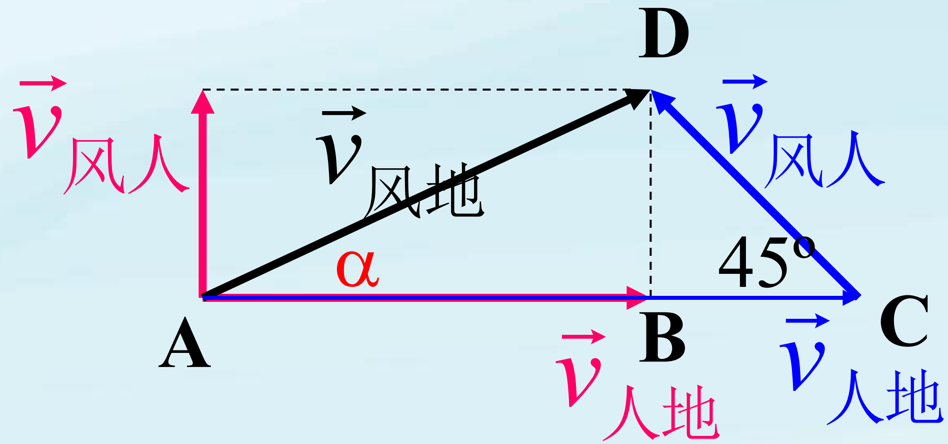
［例题1-5］某人东行，v=50m/min感觉有南风，v=75m/min感觉有东南风，求风速。

解：标方位



作矢量图



由图：$BC=75-50=25$　　$\therefore BD=BC=25$

$$AD=(AB^2+BD^2)^{1/2}=55.9$$

$$\alpha=\tan^{-1}(DB/AB)=27^{\circ}$$

风速大小：55.9m/min；方向：东偏北27°