

### 10.9题 动态规划模型

阶段: $k=1,2,3,4$

状态变量 $S_k$ : 第 $k$ 个月初的库存量

决策变量 $U_k$ : 第 $k$ 个月的产量

$g_k$ : 第 $k$ 个月的需求量

$y_k$ : 0--第 $k$ 个月不生产 1--第 $k$ 个月生产

状态转移方程:  $S_{k+1}=S_k+U_k-g_k$

指标函数:  $V_k=S_k+U_k-g_k+500y_k+U_k$

$f_k(S_k)$ : 当 $k$ 个月初的库存量为 $S_k$ 时, 从第 $k$ 个月到第 $n$ 个月的最小总费用

$f_k(S_k)=\min\{V_k+f_{k+1}(S_{k+1})\}$

$f_5(0)=0, f_5(S_5)=\infty \quad S_5 \neq 0$

状态变量 $S_k$ : 和决策变量可能的取值

$$S_4 : 0, 200$$

$$U_4 : 0, 200$$

$$S_3 : 0, 300, 500$$

$$U_3 : 0, 300, 500$$

$$S_2 : 0, 500, 800, 1000$$

$$U_2 : 0, 500, 800, 1000$$

$$S_1 : 100$$

$$U_1 : 300, 800, 1100, 1300$$

计算过程

$$f_k(S_k) = \min_{u_k} \{S_k + u_k - g_k + 500y_k + u_k y_k + f_{k+1}(S_k + u_k - g_k)\} \quad f_5(0) = 0$$

$$y_k \leq u_k, y_k = 0, 1$$

$$S_4 : 0, 200$$

$$f_5(S_5) = \infty, S_5 \neq 0$$

$$U_4 : 0, 200$$

$$f_4(0) = 0 + u_4 - g_4 + 500 + u_4 + f_5(0) = 700 \quad : u_4 = g_4 = 200, y_4 = 1$$

$$f_4(200) = 200 - g_4 + f_5(0) = 0 \quad : u_4 = 0, g_4 = 200, y_4 = 0$$

$$S_3 : 0, 300, 500$$

$$U_3 : 0, 300, 500$$

$$f_3(0) = \min\{u_3 - g_3 + 500 + u_3 + f_4(u_3 - g_3)\}$$

$$= \min\{300 - g_3 + 500 + 300 + f_4(0), 500 - g_3 + 500 + 500 + f_4(200)\}$$

$$= \min\{1500, 1200\} = 1200 \quad : u_3 = 500, g_3 = 300, y_3 = 1$$

$$f_3(300) = \min\{S_3 + u_3 - g_3 + 500y_3 + u_3 + f_4(u_3 - g_3)\}$$

$$= f_4(0) = 700 \quad : u_3 = 0, g_3 = 300, y_3 = 0$$

$$f_3(500) = \min\{S_3 + u_3 - g_3 + 500y_3 + u_3 + f_4(u_3 - g_3)\}$$

$$= 200 + f_4(200) = 200 \quad : u_3 = 0, g_3 = 300, y_3 = 0$$

$$f_k(S_k) = \min_{u_k} \{S_k + u_k - g_k + 500 y_k + u_k y_k\}$$

$$y_k \leq u_k, y_k = 0, 1$$

$$f_5(0) = 0$$

$$f_5(S_5) = \infty, S_5 \neq 0$$

$$S_2 : 0, 500, 800, 1000$$

$$U_2 : 0, 500, 800, 1000$$

$$f_2(0) = \min_{u_2} \{u_2 - g_2 + 500 + u_2 + f_3(u_2 - g_2)\} : u_2 = 500, 800, 1000, g_2 = 500, y_2 = 1$$

$$= \min\{1000 + f_3(0), 1600 + f_3(300), 2000 + f_3(500)\}$$

$$= \min\{2200, 2300, 2500\} = 2200 : u_2 = 500, y_3 = 1$$

$$f_2(500) = \min\{S_2 + u_2 - g_2 + 500 y_2 + u_2 + f_3(S_2 + u_2 - g_2)\}$$

$$= f_3(0) = 1200 : u_2 = 0, g_2 = 500, y_2 = 0$$

$$f_2(800) = \min\{S_2 + u_2 - g_2 + 500 y_2 + u_2 + f_3(S_2 + u_2 - g_2)\}$$

$$= 300 + f_3(300) = 1000 : u_2 = 0, g_2 = 500, y_2 = 0$$

$$f_2(1000) = \min\{S_2 + u_2 - g_2 + 500 y_2 + u_2 + f_3(S_2 + u_2 - g_2)\}$$

$$= 500 + f_3(500) = 700 : u_2 = 0, g_2 = 500, y_2 = 0$$

$$S_I : 100$$

$$U_I: 300, 800, 1100, 1300$$

$$\begin{aligned} f_1(100) &= \min_{u_1} \{S_1 + u_1 - g_1 + 500 + u_1 + f_2(S_1 + u_1 - g_1)\} \quad : u_1 = 300, 800, 1100, 1300, g_1 = 400, y_1 = 1 \\ &= \min\{800 + f_2(0), 1800 + f_2(500), 2400 + f_2(800), 2800 + f_2(1000)\} \\ &= \min\{3000, 3000, 3400, 3500\} = 3000 \quad : u_1 = 300 \text{ or } 800, y_1 = 1 \end{aligned}$$

可得两组最优生产策略

$$u_1 = 300 \rightarrow S_2 = 0, u_2 = 500 \rightarrow S_3 = 0, u_3 = 500 \rightarrow S_4 = 200, u_4 = 0$$

$$u_1 = 800 \rightarrow S_2 = 500, u_2 = 0 \rightarrow S_3 = 0, u_3 = 500 \rightarrow S_4 = 200, u_4 = 0$$