计算机图形学基础

华东理工大学计算机系·谢晓玲

习题7.4/P227

□7.4 将图7-40中物体ABCDEFGH进行如下变换的变换矩阵，写出复合变换后图形各顶点的规范化齐次坐标，并画出复合变换后的图形。

①平移，使点C与点$p(1,-1,0)$重合。

②绕z轴旋转$60°$。

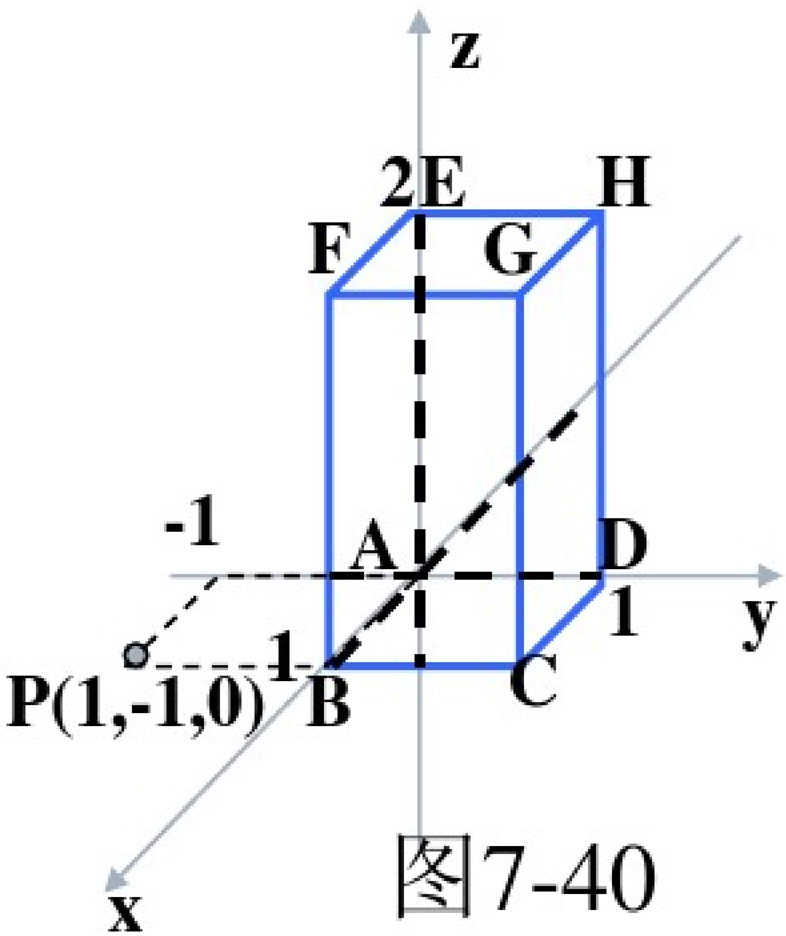


图7-40

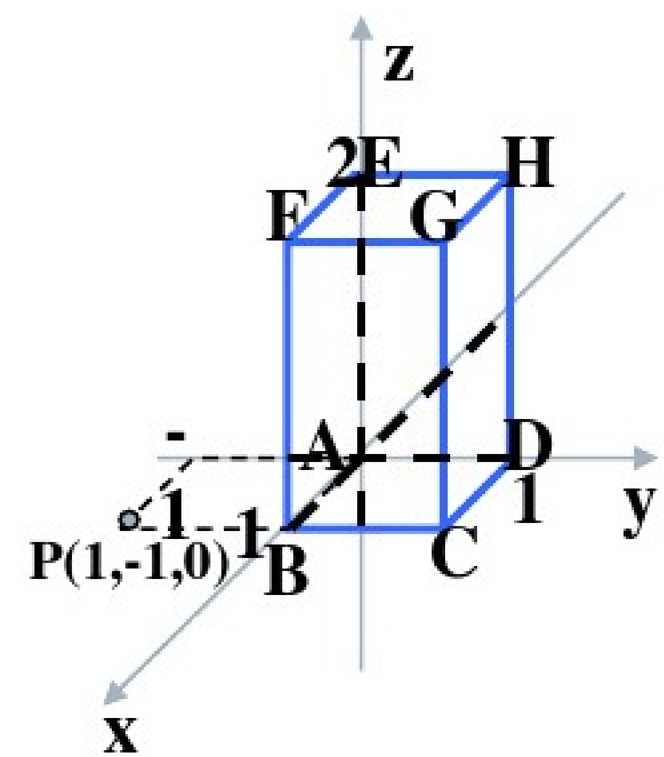
解：

①$T1=T(0,-2,0)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -2 & 0 & 1\end{pmatrix}$

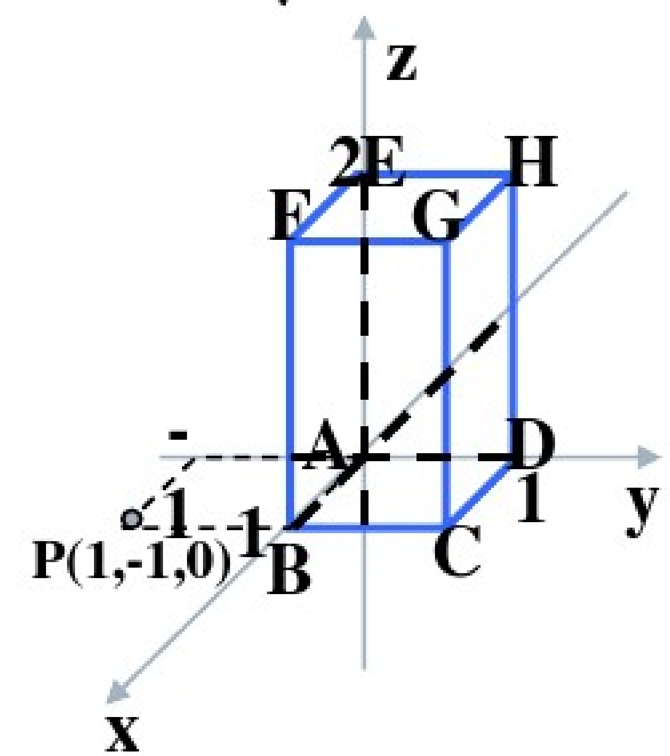
②$T2=R_z(60)=\begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\therefore T=T1\cdot T2=T(0,-2,0)R_z(60)$

$=\begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{3} & -1 & 0 & 1 \end{pmatrix}$



$$\begin{array}{c} A\\ B\\ C\\ D\\ E\\ F\\ G\\ H \end{array}\begin{pmatrix} 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 1\\ 1 & 1 & 0 & 1\\ 0 & 1 & 0 & 1\\ 0 & 0 & 2 & 1\\ 1 & 0 & 2 & 1\\ 1 & 1 & 2 & 1\\ 0 & 1 & 2 & 1 \end{pmatrix} \cdot T = \begin{pmatrix} \sqrt{3} & -1 & 0 & 1\\ 1/2+\sqrt{3} & \sqrt{3}/2-1 & 0 & 1\\ 1/2+\sqrt{3}/2 & \sqrt{3}/2-1/2 & 0 & 1\\ \sqrt{3}/2 & -1/2 & 0 & 1\\ \sqrt{3} & -1 & 2 & 1\\ 1/2+\sqrt{3} & \sqrt{3}/2-1 & 2 & 1\\ 1/2+\sqrt{3}/2 & \sqrt{3}/2-1/2 & 2 & 1\\ \sqrt{3}/2 & -1/2 & 2 & 1 \end{pmatrix}$$



四舍五入得：

$$
\begin{array}{c}
A' \\ B' \\ C' \\ D' \\ E' \\ F' \\ G' \\ H'
\end{array}
\begin{pmatrix}
2 & -1 & 0 & 1 \\
2 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & -1 & 0 & 1 \\
2 & -1 & 2 & 1 \\
2 & 0 & 2 & 1 \\
1 & 0 & 2 & 1 \\
1 & -1 & 2 & 1
\end{pmatrix}
$$

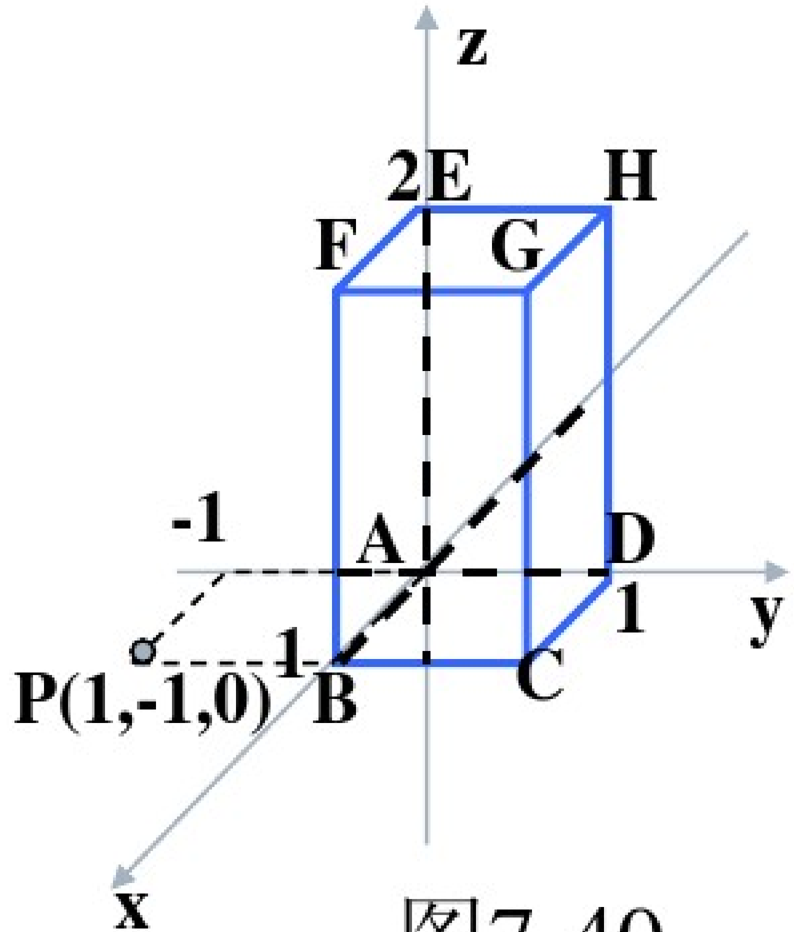
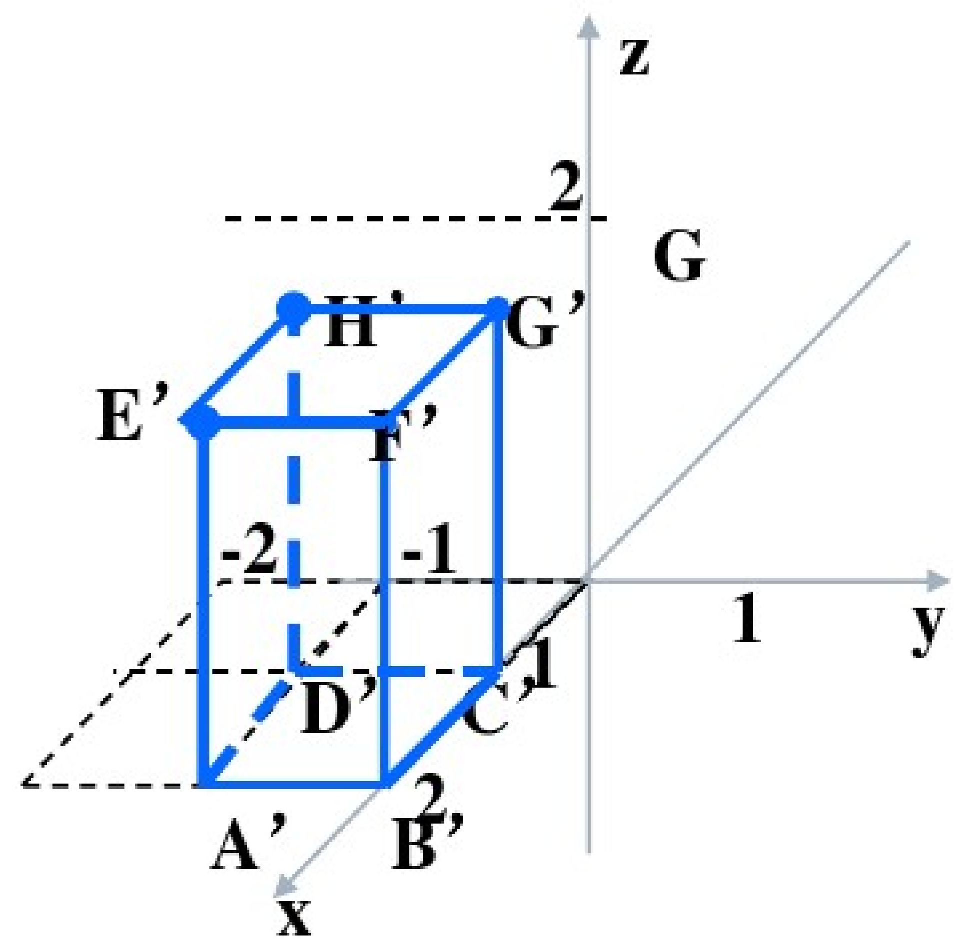


图7-40



习题7.4/P227

□7.4 将图7-40中物体ABCDEFGH进行如下变换的变换矩阵，写出复合变换后图形各顶点的规范化齐次坐标，并画出复合变换后的图形。

①平移，使点C与点$p(1,-1,0)$重合。

②相对于点$p(1,-1,0)$绕z轴旋转$60°$ 。

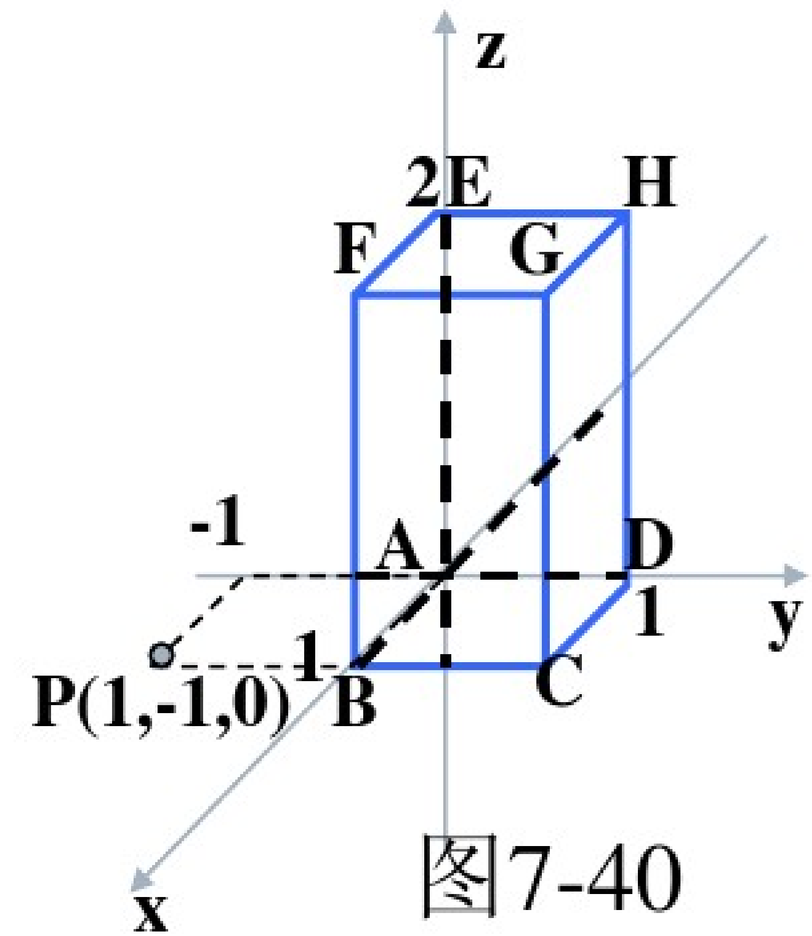


图7-40

解：

①$T1=T(0,-2,0)$

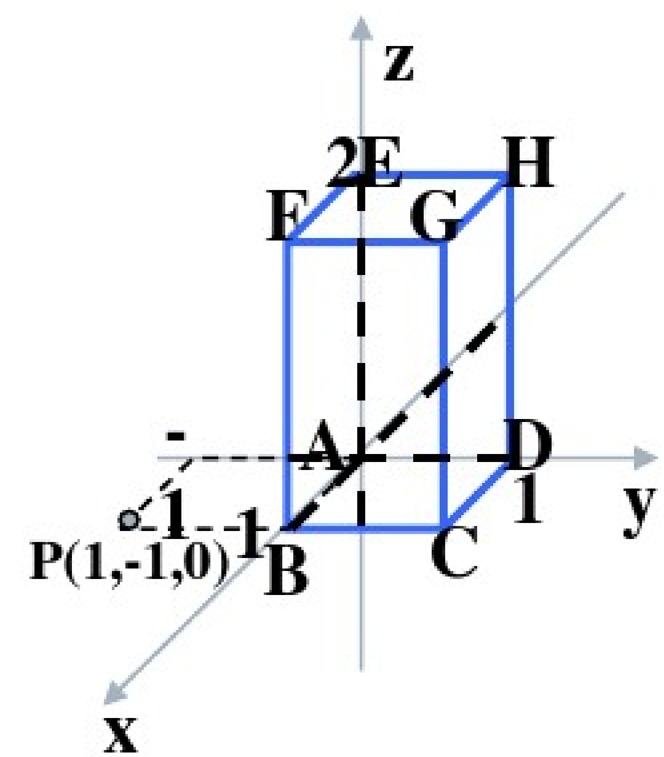
②$T2=T(-1,\ 1,0)R_z(60)T(1,-1,0)$

$\therefore T=T1\cdot T2=T(0,-2,0)T(-1,1,0)R_z(60)T(1,-1,0)$

$$T=T(0,-2,0)\,T(-1,1,0)\,R_z(60)\,T(1,-1,0)$$

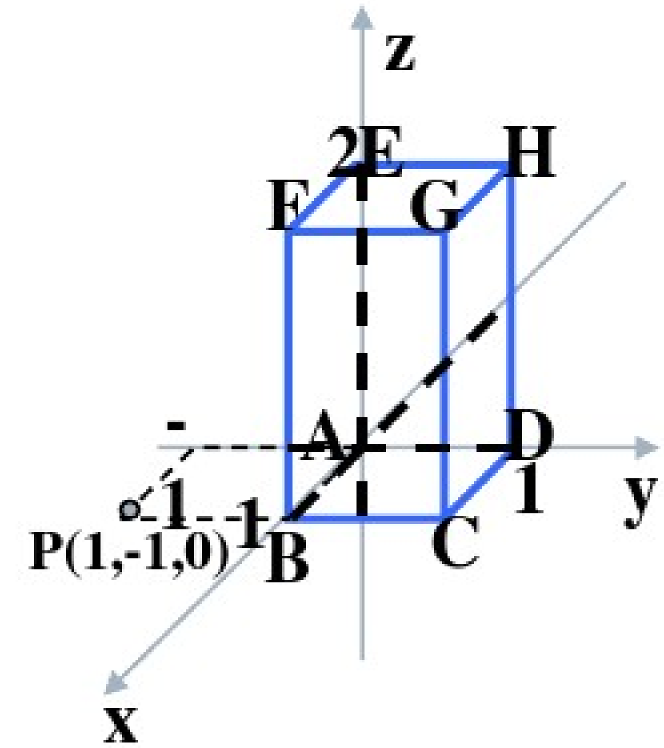
$$=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&-2&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-1&1&0&1\end{pmatrix}\begin{pmatrix}1/2&\sqrt{3}/2&0&0\\-\sqrt{3}/2&1/2&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&-1&0&1\end{pmatrix}$$

$$=\begin{pmatrix}1/2&\sqrt{3}/2&0&0\\-\sqrt{3}/2&1/2&0&0\\0&0&1&0\\(1+\sqrt{3})/2&-(3+\sqrt{3})/2&0&1\end{pmatrix}$$



$$T=T(0,-2,0)\,T(-1,1,0)\,R_z(60)\,T(1,-1,0)$$

$$=\begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (1+\sqrt{3})/2 & -(3+\sqrt{3})/2 & 0 & 1 \end{pmatrix}$$



$$\begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \cdot T = \begin{pmatrix} (1+\sqrt{3})/2 & -(3+\sqrt{3})/2 & 0 & 1 \\ 1+\sqrt{3}/2 & -3/2 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 1/2 & -1-\sqrt{3}/2 & 0 & 1 \\ (1+\sqrt{3})/2 & -(3+\sqrt{3})/2 & 2 & 1 \\ 1+\sqrt{3}/2 & -3/2 & 2 & 1 \\ 1 & -1 & 2 & 1 \\ 1/2 & -1-\sqrt{3}/2 & 2 & 1 \end{pmatrix}$$

四舍五入得：

$$\begin{array}{c} A' \\ B' \\ C' \\ D' \\ E' \\ F' \\ G' \\ H' \end{array} \begin{pmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -2 & 0 & 1 \\ 1 & -2 & 2 & 1 \\ 2 & -1 & 2 & 1 \\ 1 & -1 & 2 & 1 \\ 1 & -2 & 2 & 1 \end{pmatrix}$$

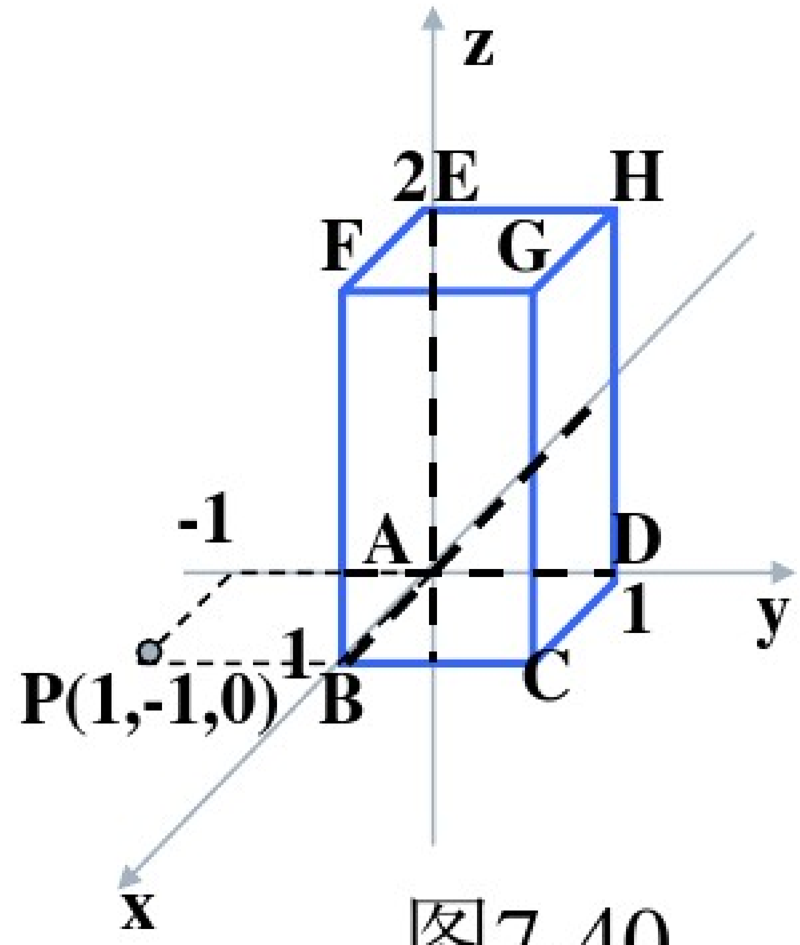
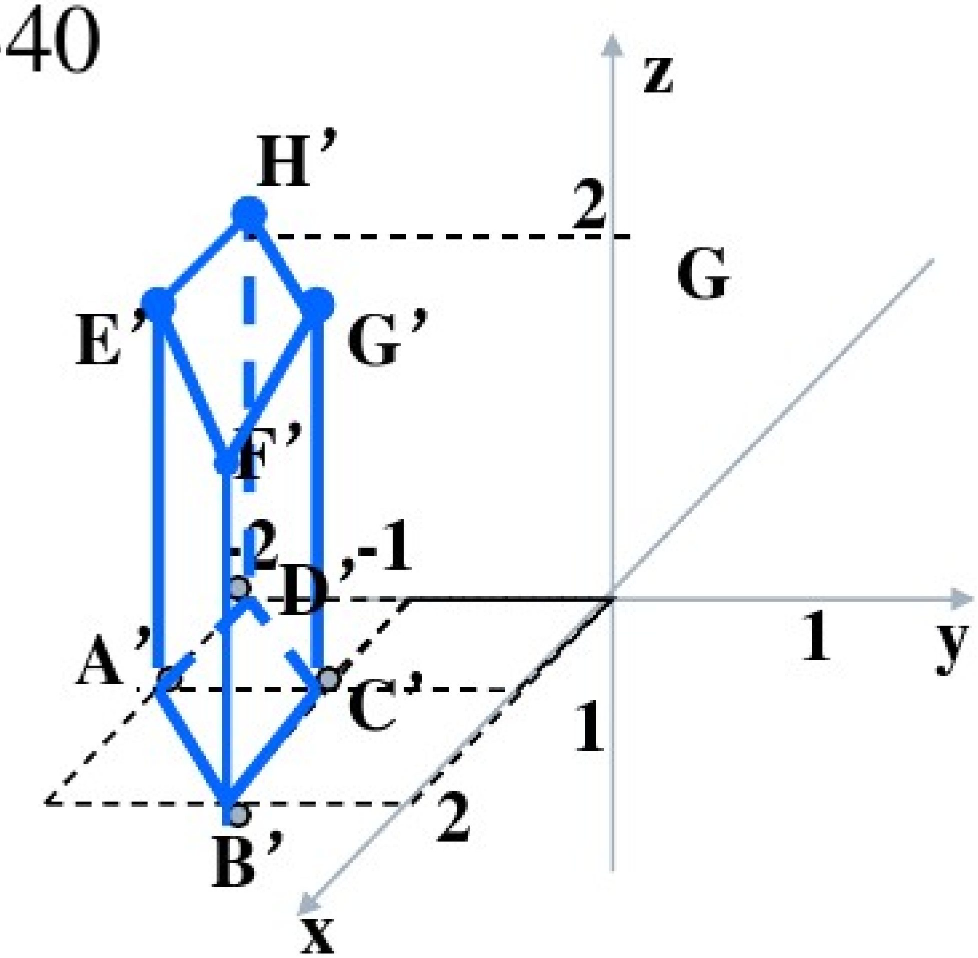


图7-40



习题7.5/P228

□7.5 将图7-41中四面体ABCD进行如下变换的变换矩阵，写出复合变换后图形各顶点的规范化齐次坐标，并画出复合变换后的图形。

①关于P点整体放大2倍。

②关于y轴进行对称变换。

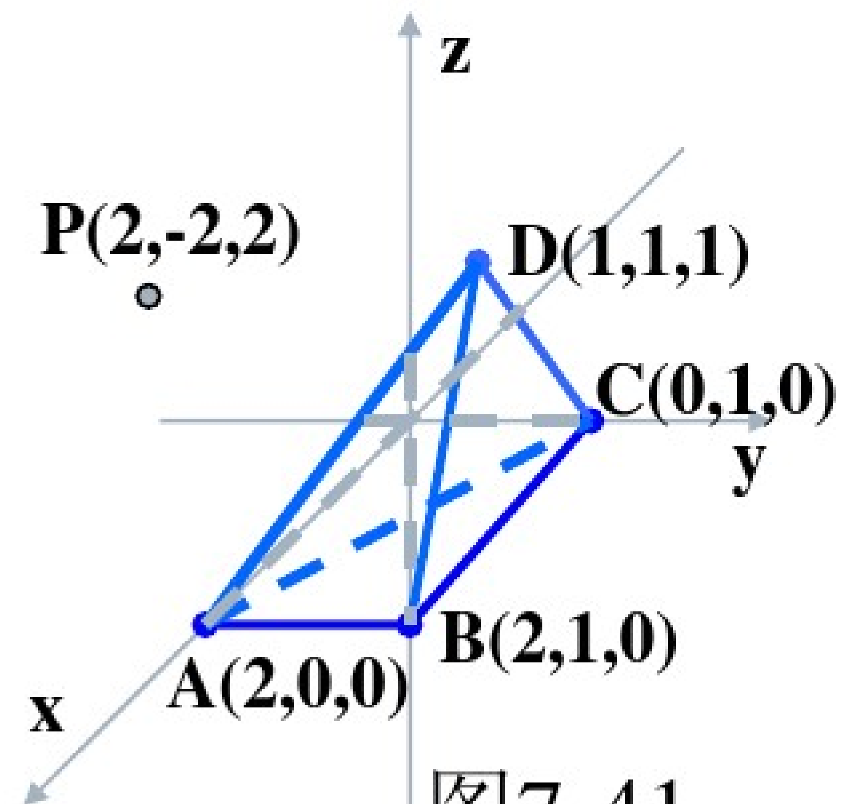


图7-41

解：

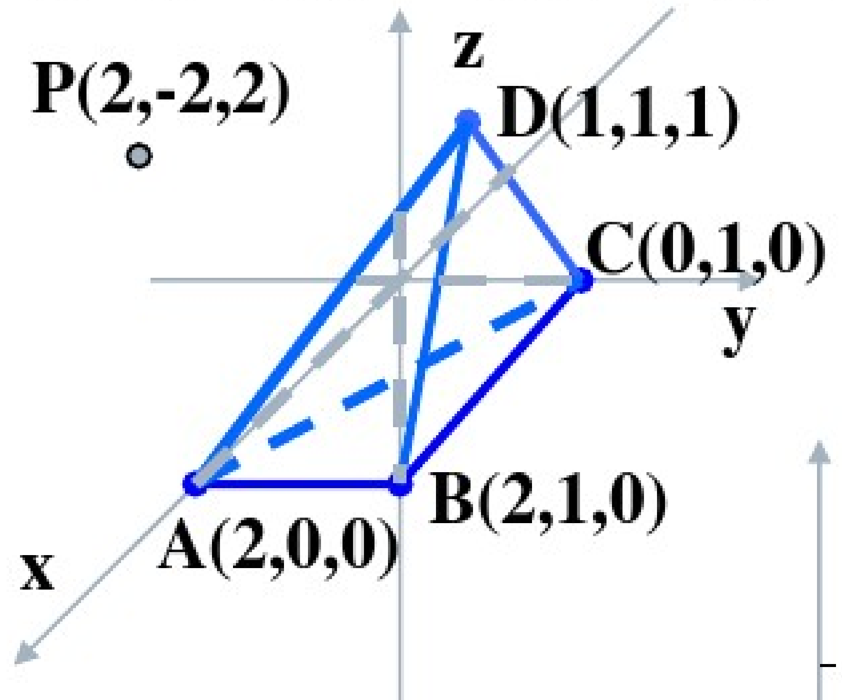
①$T1=T(-2,2,-2)S(2)T(2,-2,2)$

②$T2=T_{F_y}()$

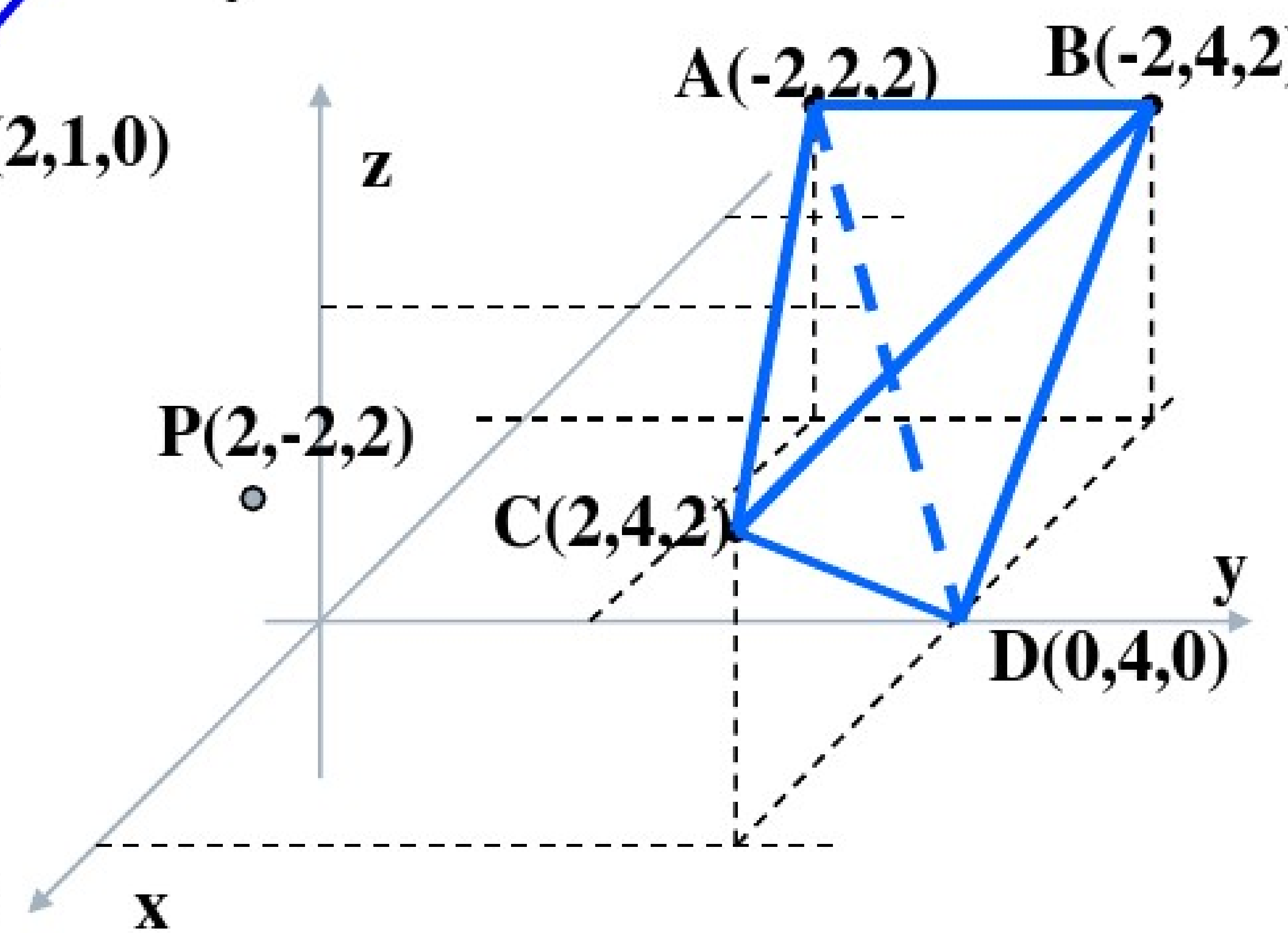
$\therefore T=T(-2,2,-2)S(2)T(2,-2,2)T_{F_y}()$

$$T=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-2&2&-2&1\end{pmatrix}\cdot\begin{pmatrix}2&0&0&0\\0&2&0&0\\0&0&2&0\\0&0&0&1\end{pmatrix}\cdot\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\2&-2&2&1\end{pmatrix}\cdot\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$$

$$=\begin{pmatrix}-2&0&0&0\\0&2&0&0\\0&0&-2&0\\2&2&2&1\end{pmatrix}$$



$$\begin{array}{c}A\\B\\C\\D\end{array}\begin{pmatrix}2&0&0&1\\2&1&0&1\\0&1&0&1\\1&1&1&1\end{pmatrix}\cdot T=\begin{pmatrix}-2&2&2&1\\-2&4&2&1\\2&4&2&1\\0&4&0&1\end{pmatrix}$$



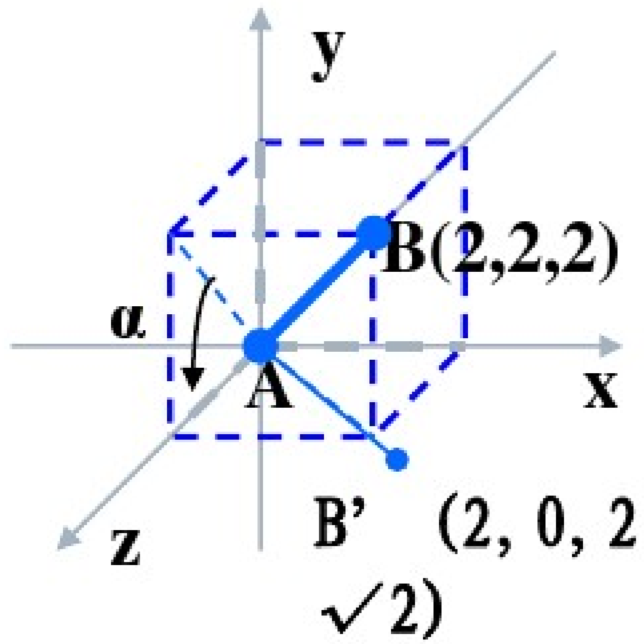
∴得A’(−2, 2, 2)、B’(−2, 4, 2)、C’(2, 4, 2)、D’(0, 4, 0)

习题7.6/P228

- 7.6 假定直线AB的两个端点为A(0,0,0)和B(2,2,2)，试写出绕AB旋转30°的三维复合变换矩阵。

第一步，使得AB与z轴重合

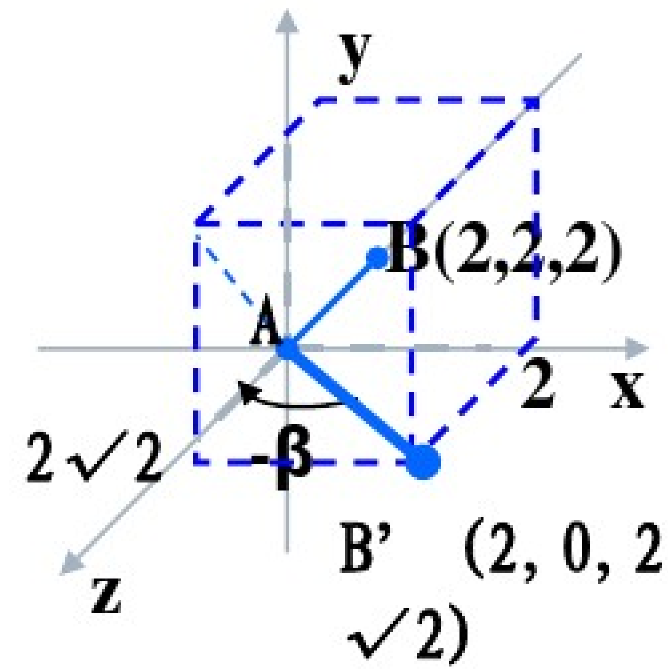
①T_{R_x}：绕x轴旋转α（45°）



$$T_{R_x}(45°)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0\\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$B\,[2\ \ 2\ \ 2\ \ 1]\cdot T_{R_x}(45)=[2\ \ 0\ \ 2\sqrt{2}\ \ 1]\ B'$$

②T_{R_y}：绕y轴旋转$-\boldsymbol{\beta}$



∵AB’ $=2\sqrt{3}$

∴$\sin(-\beta)=-\sin\beta=-1/\sqrt{3}$

$\cos(-\beta)=\cos\beta=\sqrt{2}/\sqrt{3}$

$$T_{R_y}(-\beta)=\begin{pmatrix} \sqrt{2}/\sqrt{3} & 0 & 1/\sqrt{3} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{3} & 0 & \sqrt{2}/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

第二步，绕z轴（AB”）旋转30°

$$T_{R_z}(30)=\begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 & 0 \\ -1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

第三步，绕y轴逆旋转$-\beta$，绕x轴逆旋转α

① $T_{r_y}^{-1}(-\beta) = T_{r_y}(\beta)$

$$= \begin{pmatrix} \sqrt{2}/\sqrt{3} & 0 & -1/\sqrt{3} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{3} & 0 & \sqrt{2}/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

② $T_{R_x}^{-1}(45) = T_{R_x}(-45)$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

第四步，$T=T_{R_x}(45)\,T_{R_y}(-\beta)\,T_{R_z}(30)\,T_{r_y}^{-1}(-\beta)\,T_{R_x}^{-1}(45)$

习题7.7/P228

□7.7 试作出图7-41中四面体的三视图，平移矢量均为1，要求写出变换矩阵。

①V主视图（xoz）

$$T_V=T_{xoz}=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{array}{c}A\\B\\C\\D\end{array}\begin{bmatrix}2&0&0&1\\2&1&0&1\\0&1&0&1\\1&1&1&1\end{bmatrix}\cdot T_{XOZ}=\begin{bmatrix}2&0&0&1\\2&0&0&1\\0&0&0&1\\1&0&1&1\end{bmatrix}\begin{array}{c}A'\\B'\\C'\\D'\end{array}$$

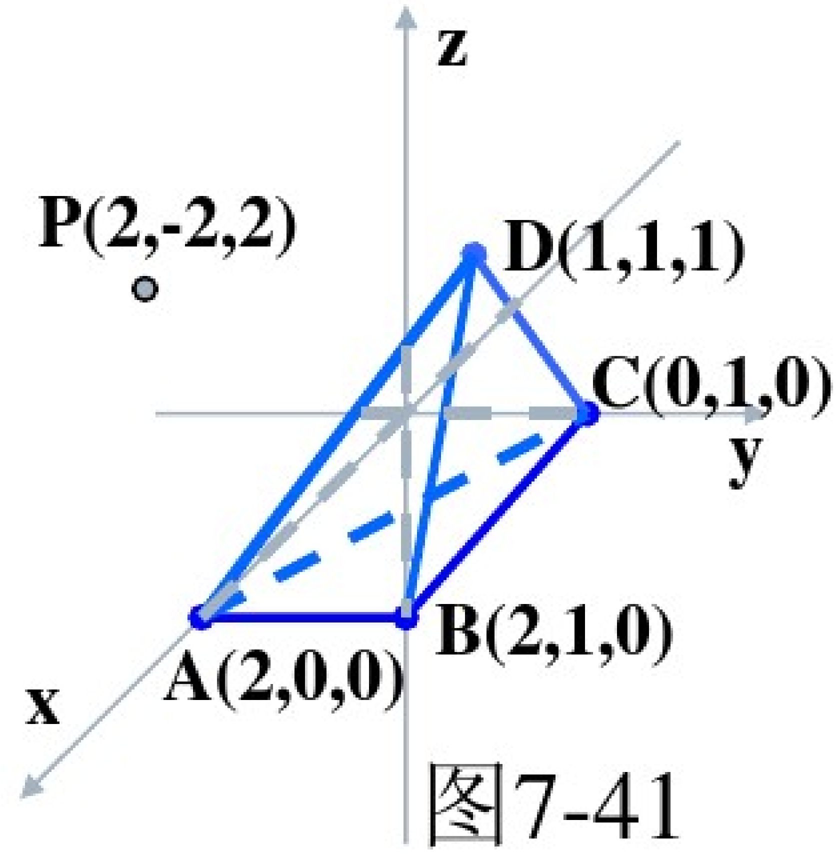
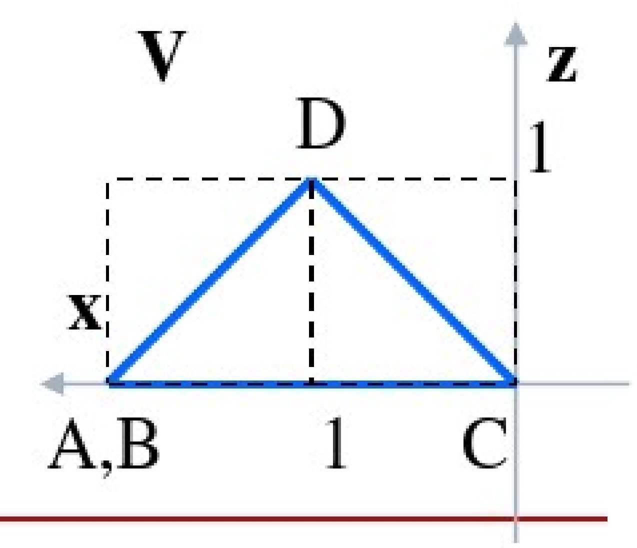


图7-41

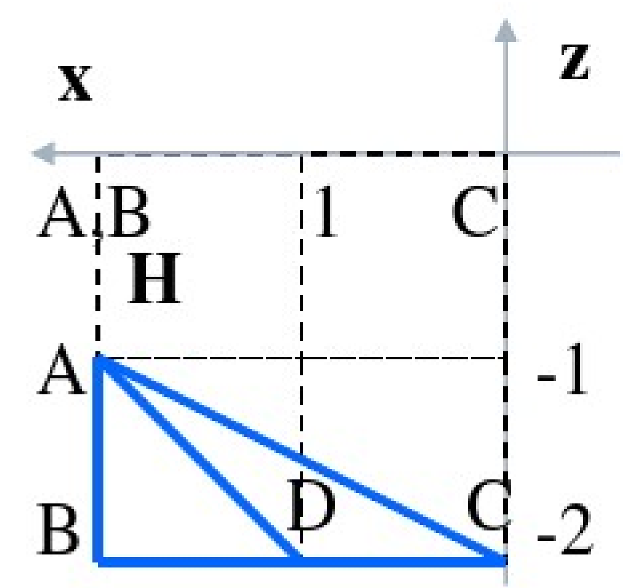


②H俯视图（xoy）

$$T_v = T_{xoy} T_{R_x}(-90)\, T(0,0,-1)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \bullet T_v = \begin{pmatrix} 2 & 0 & -1 & 1 \\ 2 & 0 & -2 & 1 \\ 0 & 0 & -2 & 1 \\ 1 & 0 & -2 & 1 \end{pmatrix} \begin{array}{c} A' \\ B' \\ C' \\ D' \end{array}$$

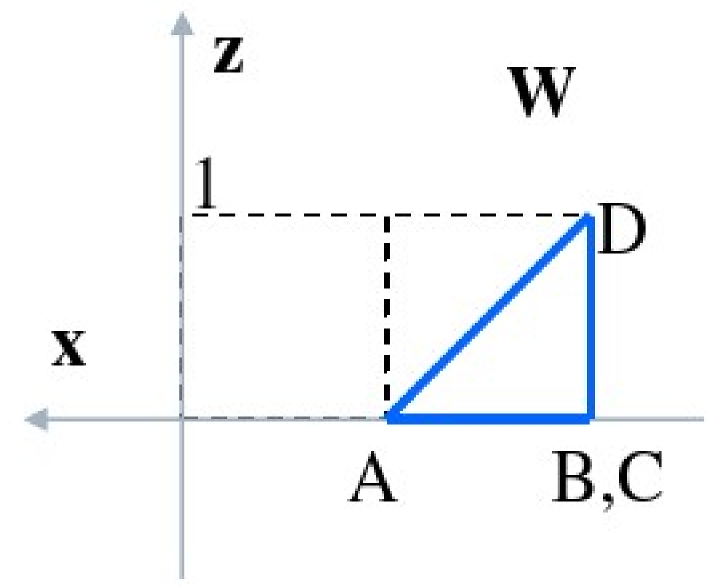


③W侧视图（yoz）

$$T_v=T_{yoz}T_{R_z}(90)\,T(-1,0,0)$$

$$=\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\-1&0&0&1\end{pmatrix}=\begin{pmatrix}0&0&0&0\\-1&0&0&0\\0&0&1&0\\-1&0&0&1\end{pmatrix}$$

$$\begin{array}{c}A\\B\\C\\D\end{array}\begin{pmatrix}2&0&0&1\\2&1&0&1\\0&1&0&1\\1&1&1&1\end{pmatrix}\bullet T_v=\begin{pmatrix}-1&0&0&1\\-2&0&0&1\\-2&0&0&1\\-2&0&1&1\end{pmatrix}\begin{array}{c}A'\\B'\\C'\\D'\end{array}$$



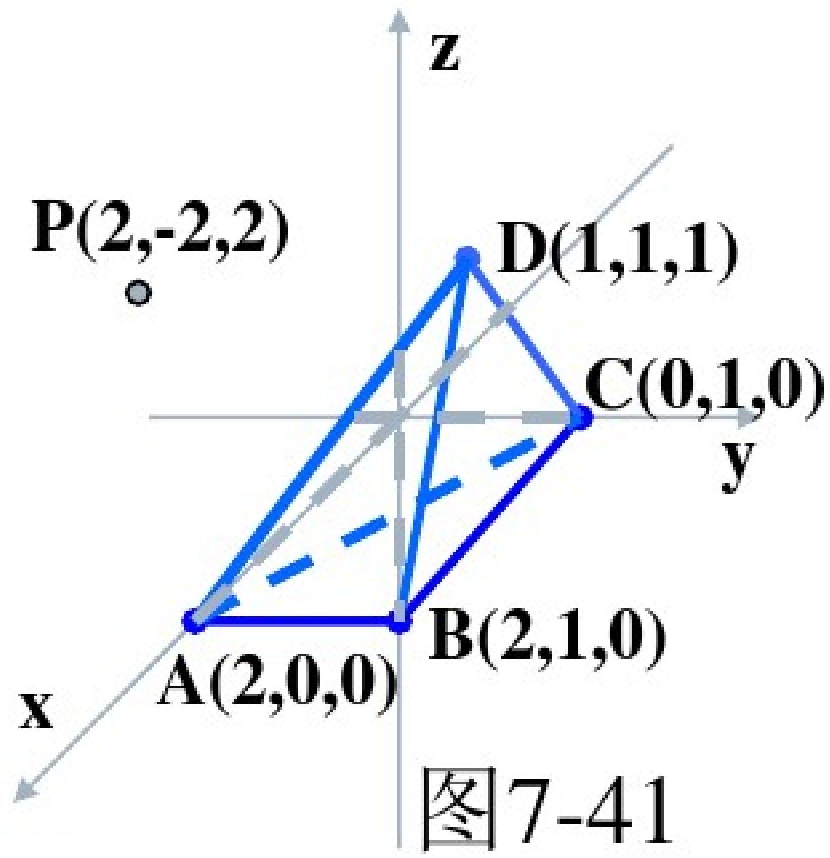
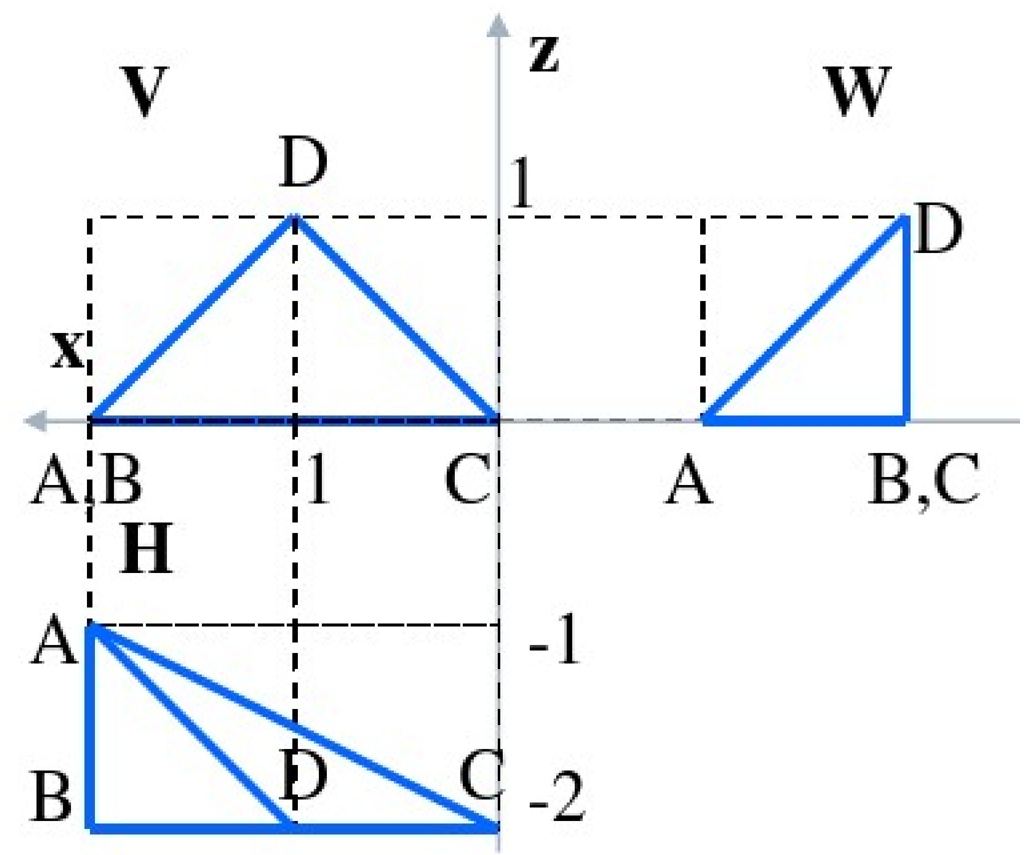


图7-41



习题7.8/P228

7.8 试推导正轴测图的投影矩阵，并写出图7-41四面体经过正等测变换或二测变换（ β=30° ）后各顶点的齐次坐标。

①正轴测图的投影矩阵

$T=T_{R_y}(-\alpha)T_{R_x}(\beta)T_{xoy}$

$$=\begin{pmatrix} \cos-\alpha & 0 & -\sin-\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \sin-\alpha & 0 & \cos-\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

①正轴测图的投影矩阵

$$T = T_{Ry} \cdot T_{Rx} \cdot T_p = \begin{bmatrix} \cos\alpha & -\sin\alpha \cdot \sin\beta & 0 & 0 \\ 0 & \cos\beta & 0 & 0 \\ -\sin\alpha & -\cos\alpha \cdot \sin\beta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

②正等测的投影矩阵

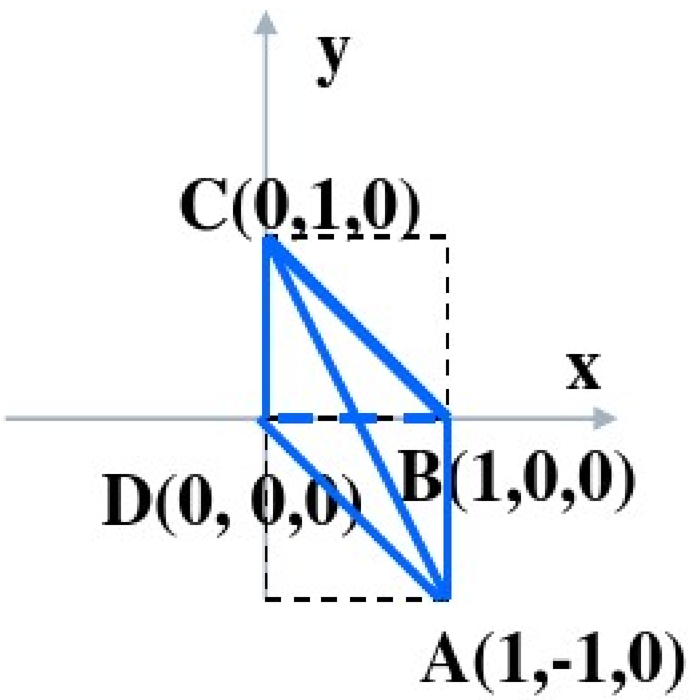
$\sin\alpha = \cos\alpha = 1/\sqrt{2}; \sin\beta = 1/\sqrt{3}, \cos\beta = \sqrt{2}/\sqrt{3}$

$$T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 0 & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 0 & 0 \\ -1/\sqrt{2} & -1/\sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

②正等测的投影矩阵

$$\begin{array}{c} A \\ B \\ C \\ D \end{array}\begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 0 & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 0 & 0 \\ -1/\sqrt{2} & -1/\sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{2} & -\sqrt{2}/\sqrt{3} & 0 & 1 \\ \sqrt{2} & 0 & 0 & 1 \\ 0 & \sqrt{2}/\sqrt{3} & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



四舍五入得：A’ (1, -1, 0)、

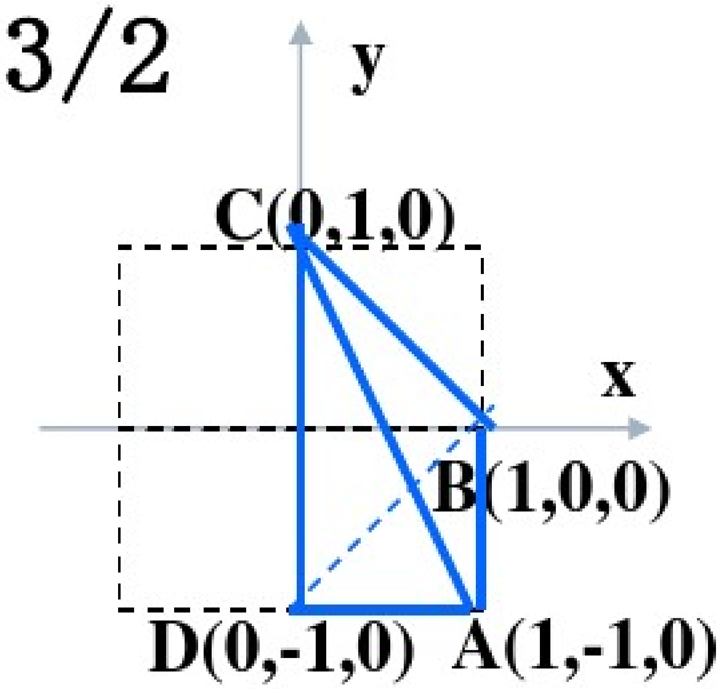
B’ (1, 0, 0)、

C’ (0, 1, 0)、

D’ (0, 0, 0)

③二测变换（ $\beta=30°$ ）的投影矩阵

$\sin\alpha=\cos\alpha=1/\sqrt{2};\sin\beta=1/2,\cos\beta=\sqrt{3}/2$



$$T=\begin{pmatrix} 1/\sqrt{2} & -1/2\sqrt{2} & 0 & 0 \\ 0 & \sqrt{3}/2 & 0 & 0 \\ -1/\sqrt{2} & -1/2\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{c} A \\ B \\ C \\ D \end{array}\begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}\cdot T=\begin{pmatrix} \sqrt{2} & -1/\sqrt{2} & 0 & 1 \\ \sqrt{2} & -1/\sqrt{2}+\sqrt{3}/2 & 0 & 1 \\ 0 & \sqrt{3}/2 & 0 & 1 \\ 0 & -1/\sqrt{2} & 0 & 1 \end{pmatrix}\begin{array}{c} A' \\ B' \\ C' \\ D' \end{array}$$

四舍五入得：A’（1,−1,0）,B’（1,0,0）,C’（0,1,0）,
D’（0,−1,0）

习题7.9/P228

□7.9 求图7-41四面体经过斜等测变换或斜二测变换（ $\beta=30°$ ）后各顶点的齐次坐标。

$$T=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \cot\alpha\cos\beta & \cot\alpha\sin\beta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

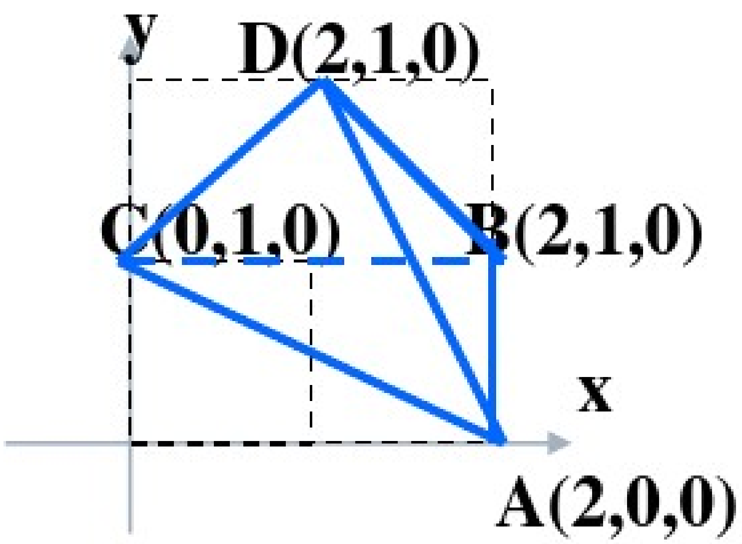
①斜等测（ $\cot\alpha=1$、 $\beta=30°$ ）的投影矩阵

$$T=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

①斜等测（ $\cot\alpha=1$、 $\beta=30°$ ）的投影矩阵

$$\begin{array}{c} A \\ B \\ C \\ D \end{array}\begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \surd 3/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1+\surd 3/2 & 3/2 & 0 & 1 \end{pmatrix}\begin{array}{c} A' \\ B' \\ C' \\ D' \end{array}$$

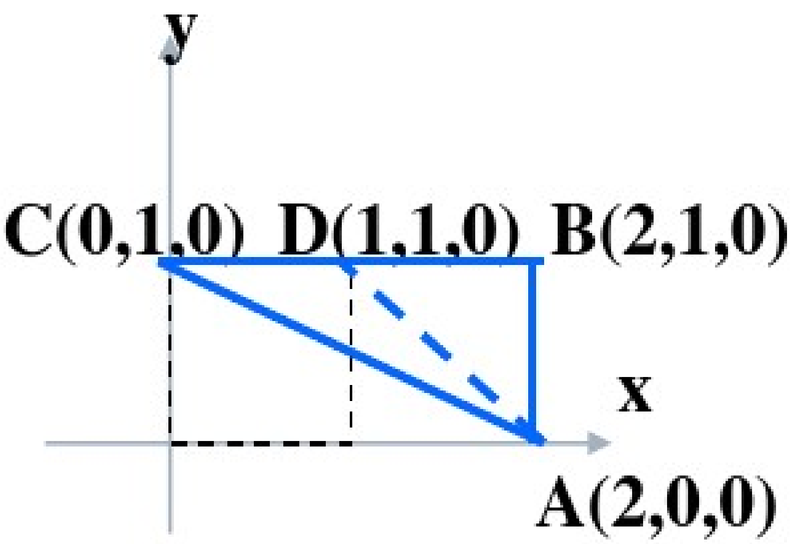
四舍五入得：A’（2, 0, 0）, B’（2, 1, 0）, C’（0, 1, 0）, D’（2, 1, 0）



②斜二测（$\cot\alpha=1/2$、$\beta=30°$）的投影矩阵

$$\begin{array}{c} A \\ B \\ C \\ D \end{array}\begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{3}/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1+\sqrt{3}/4 & 5/4 & 0 & 1 \end{pmatrix}\begin{array}{c} A' \\ B' \\ C' \\ D' \end{array}$$

四舍五入得：A’（2, 0, 0）, B’（2, 1, 0）, C’（0, 1, 0）, D’（1, 1, 0）



习题7.10/P228

- 7.10 求图7-40中平面多面体经过二点透视变换后各顶点的齐次坐标。给定$p=-1$，$q=1$，$r=-1$，$\varphi=30°$，$l=n=1$，$m=-1$。

①二点透视

1. 将顶点（0, 0, 0）平移（1, m, n)
2. 绕y轴旋转φ角度（φ<90°）

- 二点透视变换
- 向xoy平面(z=0)进行正投影变换

$$T=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ l & m & n & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\varphi & 0 & \sin\varphi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi & 0 & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} \cos\varphi & 0 & 0 & p\cos\varphi-r\sin\varphi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & p\sin\varphi+r\cos\varphi \\ l\cos\varphi+n\sin\varphi & m & 0 & p(l\cos\varphi+n\sin\varphi)+r(n\cos\varphi-l\sin\varphi)+1 \end{bmatrix}$$

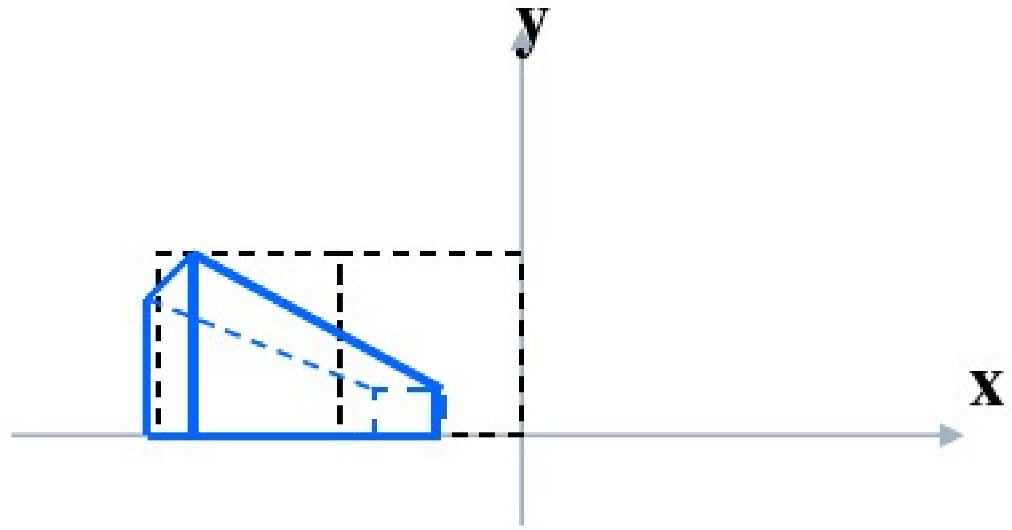
$$=\begin{pmatrix} \sqrt{3}/2 & 0 & 0 & (1-\sqrt{3})/2 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & -(1+\sqrt{3})/2 \\ (\sqrt{3}+1)/2 & -1 & 0 & 1-\sqrt{3} \end{pmatrix}$$

$$T=\begin{pmatrix} \sqrt{3}/2 & 0 & 0 & (1-\sqrt{3})/2 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & -(1+\sqrt{3})/2 \\ (\sqrt{3}+1)/2 & -1 & 0 & 1-\sqrt{3} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \cdot T = \begin{pmatrix} 1.366 & -1 & 0 & -0.732 \\ 2.232 & -1 & 0 & -1.098 \\ 2.232 & 0 & 0 & -1.098 \\ 1.366 & 0 & 0 & -0.732 \\ 2.366 & -1 & 0 & -3.464 \\ 3.232 & -1 & 0 & -3.830 \\ 3.232 & 0 & 0 & -3.830 \\ 2.366 & 0 & 0 & -3.464 \end{pmatrix}$$

得规范化坐标：　　　　　　　　四舍五入得：

$$
\begin{array}{c}
A' \\ B' \\ C' \\ D' \\ E' \\ F' \\ G' \\ H'
\end{array}
\begin{pmatrix}
-1.866 & 1.366 & 0 & 1 \\
-2.03 & 0.91 & 0 & 1 \\
-2.03 & 0 & 0 & 1 \\
-1.866 & 0 & 0 & 1 \\
-0.68 & 0.288 & 0 & 1 \\
-0.84 & 0.26 & 0 & 1 \\
-0.84 & 0 & 0 & 1 \\
-0.68 & 0 & 0 & 1
\end{pmatrix}
=
\begin{pmatrix}
-2 & 1 & 0 & 1 \\
-2 & 1 & 0 & 1 \\
-2 & 0 & 0 & 1 \\
-2 & 0 & 0 & 1 \\
-1 & 0 & 0 & 1 \\
-1 & 0 & 0 & 1 \\
-1 & 0 & 0 & 1 \\
-1 & 0 & 0 & 1
\end{pmatrix}
$$

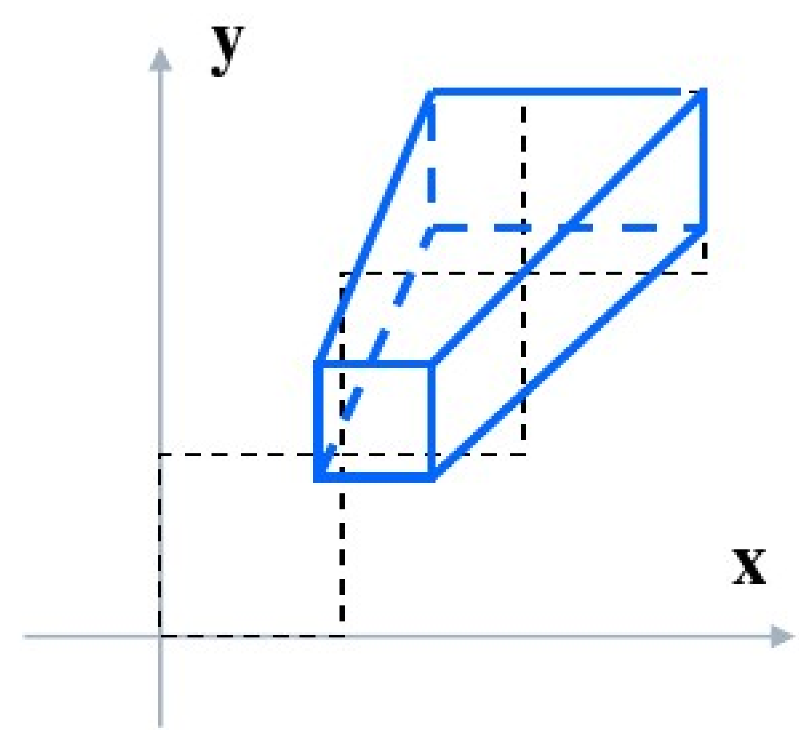


□7.10’ 求图7-40中平面多面体经过一点透视或二点透视变换后各顶点的齐次坐标。给定l=m=1，n= -1，d= -3，φ =30°。

①一点透视(l=m=1，n= -1，d= -3)

$$T=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \\ l & m & 0 & 1+n/d \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1/3 \\ 1 & 1 & 0 & 4/3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \bullet T = \begin{pmatrix} 1 & 1 & 0 & 4/3 \\ 2 & 1 & 0 & 4/3 \\ 2 & 2 & 0 & 4/3 \\ 1 & 2 & 0 & 4/3 \\ 1 & 1 & 0 & 2/3 \\ 2 & 1 & 0 & 2/3 \\ 2 & 2 & 0 & 2/3 \\ 1 & 2 & 0 & 2/3 \end{pmatrix} = \begin{pmatrix} 3/4 & 3/4 & 0 & 1 \\ 3/4 & 3/2 & 0 & 1 \\ 3/2 & 3/2 & 0 & 1 \\ 3/4 & 3/2 & 0 & 1 \\ 3/2 & 3/2 & 0 & 1 \\ 3 & 3/2 & 0 & 1 \\ 3 & 3 & 0 & 1 \\ 3/2 & 3 & 0 & 1 \end{pmatrix}$$



②两点透视另一种绘图方法：

1. 将形体绕y轴旋转α 角度（右手法则，α <90°）；
2. 将顶点（0, 0, 0）平移（l, m, n）；
3. 以(0, 0, d)为投影中心向xoy平面(z=0)进行透视投影变换

二点透视（$l=m=1$，$n=-1$，$d=-3$，$\varphi=30°$）

$$T=\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ l & m & n & 0 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} \cos\alpha & 0 & 0 & -\sin\alpha/d \\ 0 & 1 & 0 & 0 \\ \sin\alpha & 0 & 0 & \cos\alpha/d \\ l & m & 0 & n/d+1 \end{bmatrix}=\begin{pmatrix} \sqrt{3}/2 & 0 & 0 & 1/6 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & -\sqrt{3}/6 \\ 1 & -1 & 0 & 4/3 \end{pmatrix}$$

$$T=\begin{pmatrix} \sqrt{3}/2 & 0 & 0 & 1/6 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & -\sqrt{3}/6 \\ 1 & -1 & 0 & 4/3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \cdot T=\begin{pmatrix} 1 & -1 & 0 & 4/3 \\ \sqrt{3}/2+1 & -1 & 0 & 3/2 \\ \sqrt{3}/2+1 & 0 & 0 & 3/2 \\ 1 & 0 & 0 & 4/3 \\ 2 & -1 & 0 & 4/3-1/\sqrt{3} \\ \sqrt{3}/2+2 & -1 & 0 & 3/2-1/\sqrt{3} \\ \sqrt{3}/2+2 & 0 & 0 & 3/2-1/\sqrt{3} \\ 2 & 0 & 0 & 4/3-1/\sqrt{3} \end{pmatrix}$$

得规范化坐标：

$$
\begin{pmatrix}
3/4 & -3/4 & 0 & 1 \\
1/\sqrt{3}+2/3 & -2/3 & 0 & 1 \\
1/\sqrt{3}+2/3 & 0 & 0 & 1 \\
3/4 & 0 & 0 & 1 \\
6/(4-\sqrt{3}) & -1/(4/3-1/\sqrt{3}) & 0 & 1 \\
(\sqrt{3}/2+2)/(3/2-1/\sqrt{3}) & -1/(3/2-1/\sqrt{3}) & 0 & 1 \\
(\sqrt{3}/2+2)/(3/2-1/\sqrt{3}) & 0 & 0 & 1 \\
6/(4-\sqrt{3}) & 0 & 0 & 1
\end{pmatrix}
$$

得规范化坐标：

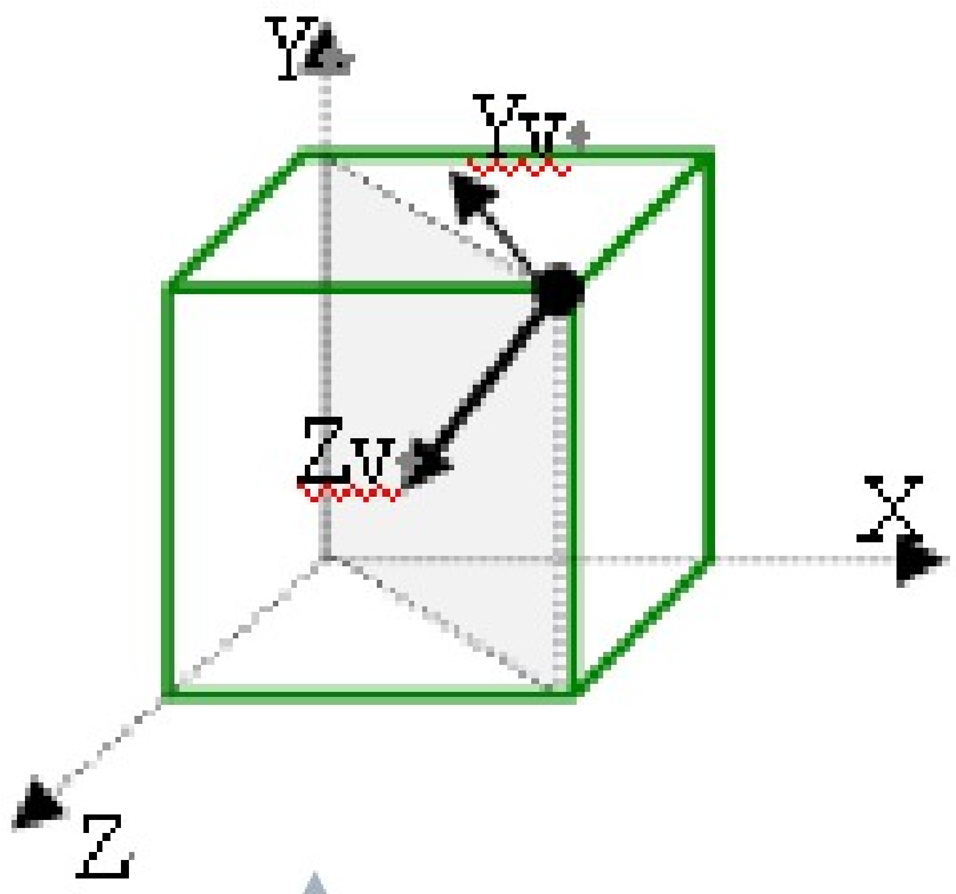
$$\begin{pmatrix} 0.75 & -0.75 & 0 & 1 \\ 2.088 & -1.5 & 0 & 1 \\ 2.088 & 0 & 0 & 1 \\ 0.75 & 0 & 0 & 1 \\ 2.6 & -1.34 & 0 & 1 \\ 3.125 & -1.096 & 0 & 1 \\ 3.125 & 0 & 0 & 1 \\ 2.6 & 0 & 0 & 1 \end{pmatrix}$$

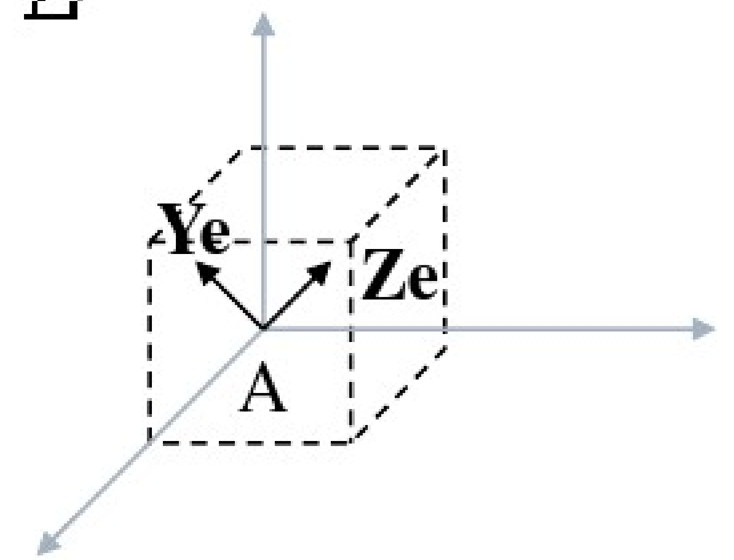
补充题：已知一个单位立方体，其中一个角点O在(0,0,0)，另一个对角点A在(1,1,1)，导出以主对角线OA（(0,0,0)到(1,1,1)）为Ze轴、 A点为观察参考点、Ye（UP方向）过主对角线面的观察坐标系的变换矩阵。

设Up轴过主对角面。

①T(-1,-1,-1)：平移使得A过原点

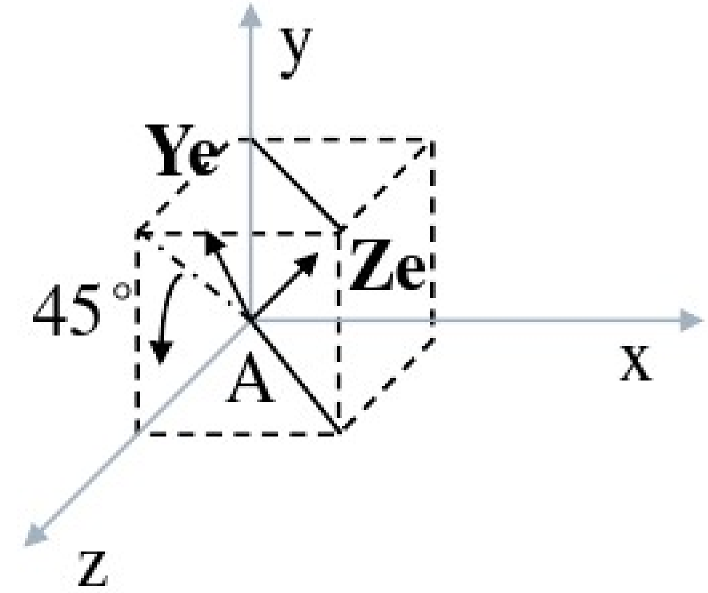
$$T(-1,-1,-1)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$





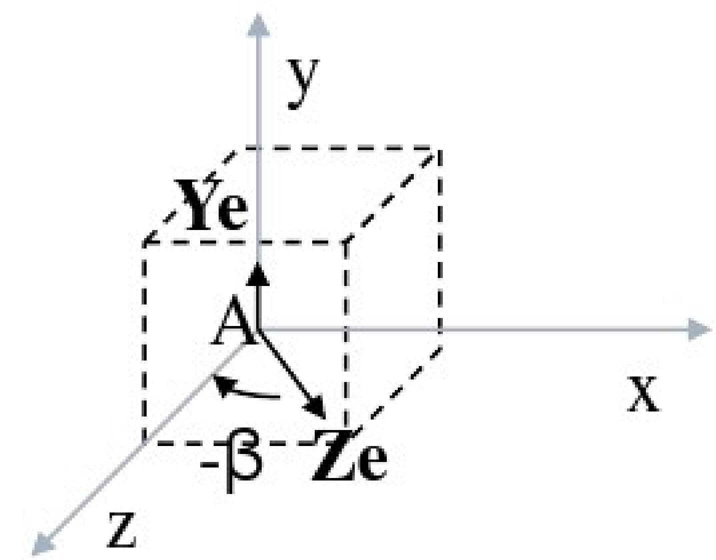
②$T_{R_x}(45°)$：绕x轴旋转45°，使Ze（OA）在XOZ平面上，Ye（Up轴）垂直XOZ平面并且与y轴重合。

$$T_{R_x}(45°)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



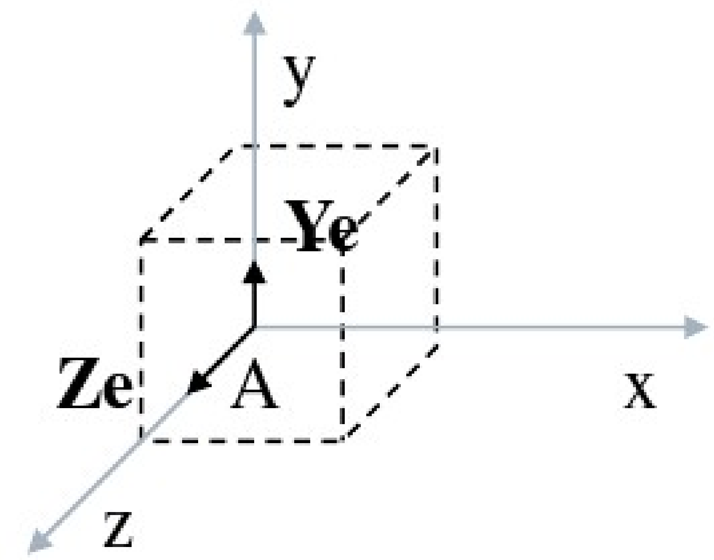
③$T_{R_y}(-\beta)$：绕y轴旋转$-\beta$，使Ze（OA）在z轴重合

$$T_{R_y}(-\beta)=\begin{pmatrix} \sqrt{2}/\sqrt{3} & 0 & 1/\sqrt{3} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{3} & 0 & \sqrt{2}/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



④$R_{F_{xy}}$：关于XOY平面的对称，使右手坐标系->左手坐标系。

$$R_{F_{xy}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



⑤$T= T(-1,-1,-1)\ T_{R_x}(45°)T_{R_y}(-\boldsymbol{\beta})\ R_{F_{xy}}$