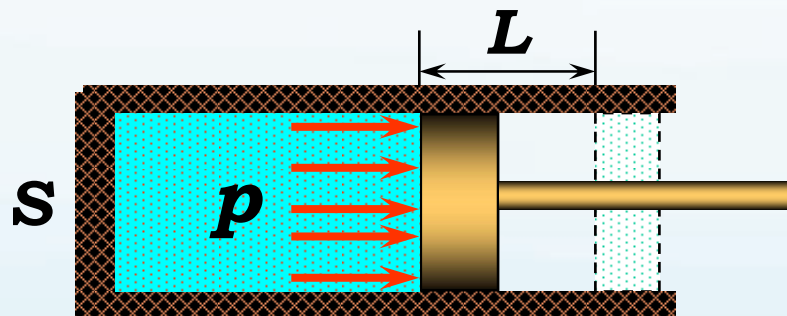


第四章 热力学第一定律

4.1 准静态过程

1°概念：每时每刻系统状态都无限接近平衡态

2°特点：(1) 过程进行缓慢
(2) 每个状态都可用确定的状态参量描述
(3) 可以用平滑的过程曲线来表示



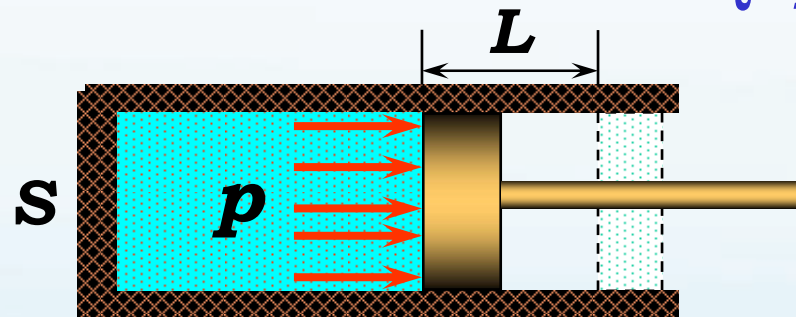
第四章 热力学第一定律

4.1 准静态过程

3°判剧

- (1) 弛豫时间 τ : 从原平衡态恢复到新平衡态需时
- (2) 准静态过程判剧: $\tau \text{恒} < t_i$ (每步进行的时间)
- (3) 示例: 内燃机汽缸活塞 $v=10\text{m/s}$, p 趋匀速率 300m/s

$$\tau = L/300 < t = L/10 \text{ 是!}$$



不作说明, 均为准静态过程!

4.2 功与热量

1° 广义功概念

- (1) 广义力: 破坏力学平衡的作用 (机械力 电磁力)
- (2) 广义位移: 广义力作用下的状态变化
- (3) 广义功: 广义力作用下产生广义位移所做的功

4.2 功与热量

2° 热力学典型的广义功

体积变化功

①数学表达式

$$A = \int_{V_1}^{V_2} p dV$$

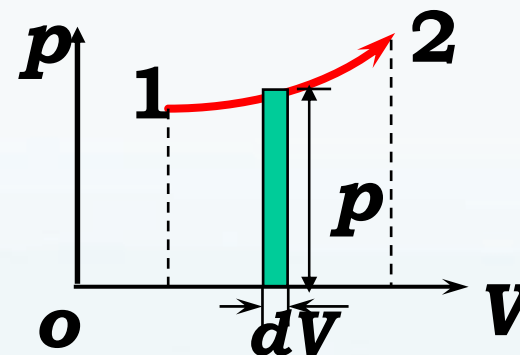
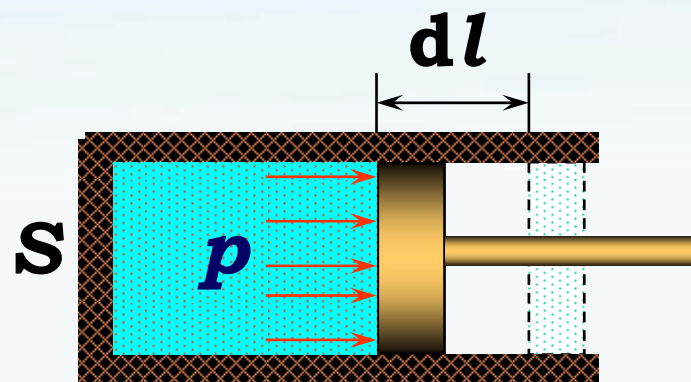
$$dA = \vec{F} \cdot d\vec{l} = pSdl = pdV$$

②本质：过程量 度量状态量变化

③功的正负：

系统做功取“+”，否则取“-”

电源功示例-电容器充电



3° 热量 Q

- (1) 本质: 是能量, 是过程量, 用来度量状态量变化.
- (2) 正负: 系统从外界吸热取“+”, 否则取“-”

4° 功、热量之比较

- (1) 联系: 都是过程量, 是系统内能变化仅有两条途径
- (2) 区别: 做功通过物体的宏观位移实现
传热通过分子间微观作用实现

4.3 热力学第一定律

1° 内能概念

微观视角 { 动能 (平转振) [不包括整体运动动能]
 势能 (分子间相互作用)
~~电子能量 原子核内能量~~

2° 内能定理

$$\overset{\text{def}}{A_{\text{绝热}}} = E_2 - E_1$$

系统从同一个初态变化为同一个末态的
 所有绝热过程中, 外界对系统做功 $A_{\text{绝热}}$ 为恒量

内能特点——态函数 关注其相对值

3°热力学第一定律（实验总结）

(1) 表述: 系统从外界获取热量, 一部分使系统内能增加, 另一部分使系统对外做功

(2) 数学表达式

有 限过程:	$Q = \Delta E + A$
无限小过程:	$dQ = dE + dA$
准静态过程:	$dQ = dE + pdV$

(3) 本质: 能量守恒定律

能量既不能产生又不能消失,
只能从一个物体传递给另一物体,
或从一种形式转化为另一种形式

4°热力学第一定律的应用

(1) 第一类永动机造不出

$$\begin{cases} Q = 0 \\ \Delta E = 0 \end{cases} \Rightarrow A = Q - \Delta E = 0$$

没有外界提供任何能量，
状态不断变化回到初态，
同时不断对外做功的机器。

(2) 科技发现的试金石

4.4 一定量理想气体对等值过程的应用

1° 摩尔热容

(1) 比热 (容) $c \stackrel{\text{def}}{=} dQ / mdT$ 单位质量物体变化1K与外界换热

(2) 热容 $C \stackrel{\text{def}}{=} dQ / dT = cm$ 物体变化1K与外界交换热量

(3) 摩尔热容 $C_{mol} \stackrel{\text{def}}{=} C / (m / M_m) = M_m c$ 1mol物体的热容
与过程有关

(4) 定容摩尔热容 $C_V \stackrel{\text{def}}{=} dQ_V / dT = \frac{i}{2} R$

等容过程 $C_{mol} \quad dQ_V = dE = \frac{i}{2} R dT$

(5) 定压摩尔热容 $C_P \stackrel{\text{def}}{=} dQ_P / dT = \frac{i+2}{2} R$

等压过程 $C_{mol} \quad dQ_P = \frac{i}{2} R dT + R dT$

$$C_P = C_V + R$$

迈尔公式

2° 理想气体内能

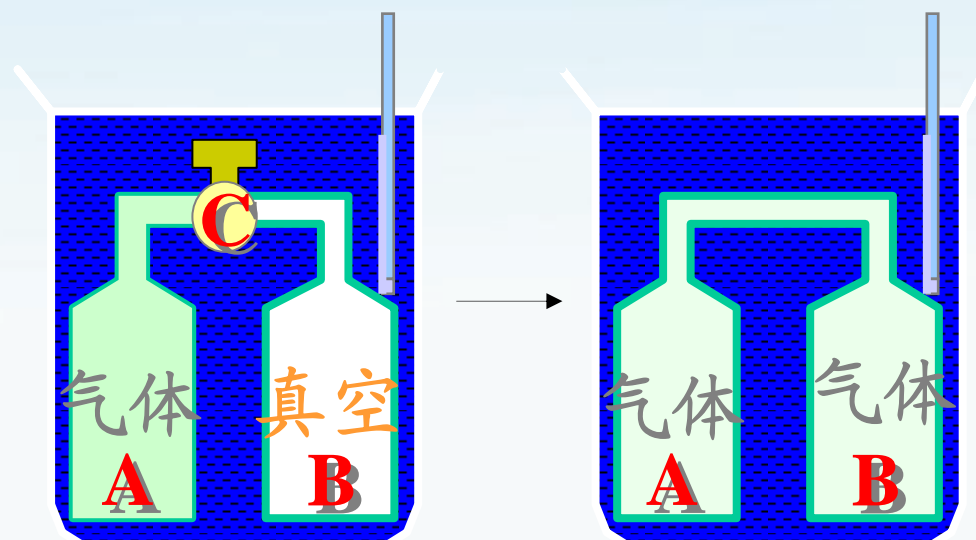
(1) 焦耳实验

(2) 焦耳定律

$$\begin{cases} A = 0 \\ Q = 0 \end{cases}$$

$$\Rightarrow \Delta E = Q - A = 0$$

$$\begin{cases} \Rightarrow E_2(T_2, V_2) = E_1(T_1, V_1) \\ T_2 = T_1 \end{cases}$$



打开阀门: 气体自由膨胀

实验结果: 水温不变!

→ 理想气体内能与体积无关，只是温度的函数

(3) 理想气体的定体热容与内能

$$C_V \stackrel{\text{def}}{=} \frac{dE}{\nu dT} \quad \Rightarrow \quad \Delta E = \int_{T_1}^{T_2} \nu C_V dT$$

(4) 理想气体的定压热容与焓

$$\left. \begin{aligned} C_p &\stackrel{\text{def}}{=} \frac{dQ_p}{\nu dT} \\ dQ_p &= d(E + pV) \\ H &\stackrel{\text{def}}{=} E + pV \end{aligned} \right\} \Rightarrow \Delta H = \int_{T_1}^{T_2} \nu C_p dT$$

3° 一定量理想气体对等值过程的应用

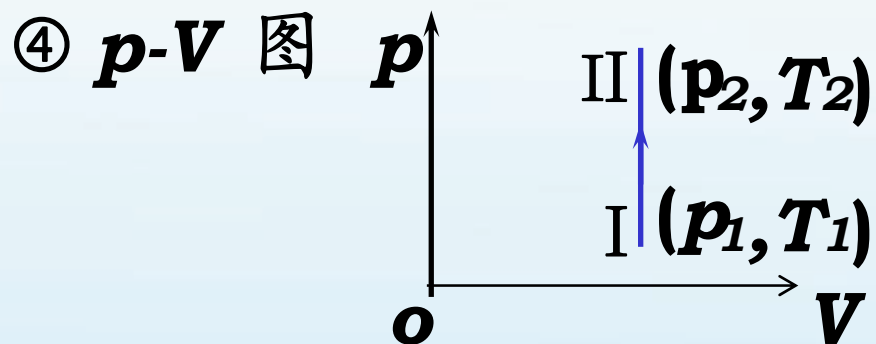
$$dQ = \nu C_V dT + p dV$$

(1) 等容过程

① 特征 $V = \text{const.}$

② 过程方程 $p_1/T_1 = p_2/T_2$

③ $Q, \Delta E, A \left\{ \begin{array}{l} dA = p dV \Rightarrow A = 0 \\ Q_V = \Delta E = \int_{T_1}^{T_2} \nu C_V dT \end{array} \right.$



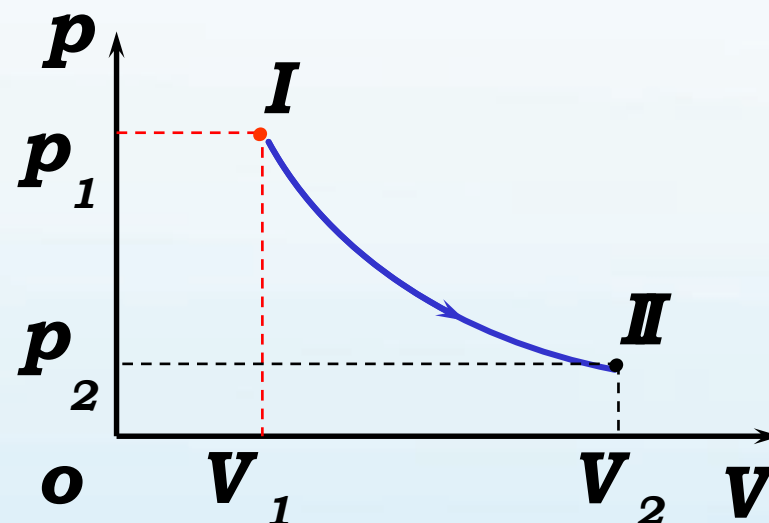
(2) 等温过程

① 特征 $T = \text{const.}$

② 过程方程 $p_1 V_1 = p_2 V_2$

③ $Q, \Delta E, A$ $\left\{ \begin{array}{l} \Delta E = \int_{T_1}^{T_2} \nu C_V dT \Rightarrow \Delta E = 0 \\ Q_T = A = \int_{V_1}^{V_2} p dV \xrightarrow{pV = \nu RT} \int_{V_1}^{V_2} \frac{\nu RT}{V} dV \\ = \nu RT \ln(V_2 / V_1) = \nu RT \ln(p_1 / p_2) \end{array} \right.$

④ p - V 图

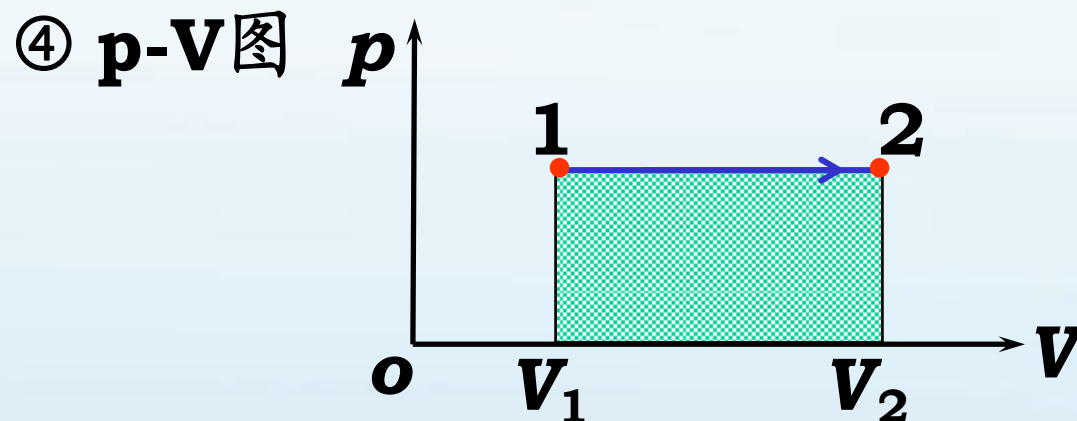


(3) 等压过程

① 特征 $p = \text{const.}$

② 过程方程 $V_1/T_1 = V_2/T_2$

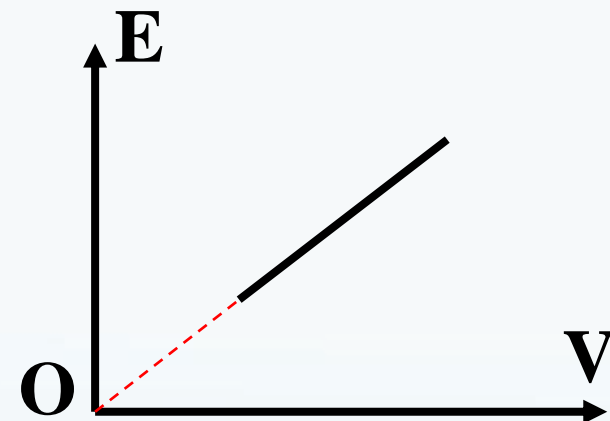
③ $Q, \Delta E, A$ $\left\{ \begin{array}{l} dA = p dV \Rightarrow A = p \Delta V \\ \quad \quad \quad \hookrightarrow \nu R dT \Rightarrow A = \nu R \Delta T \\ dQ_p = \nu C_p dT \Rightarrow Q_p = \int_{T_1}^{T_2} \nu C_p dT \\ dE = \nu C_V dT \Rightarrow \Delta E = \int_{T_1}^{T_2} \nu C_V dT \end{array} \right.$



[讨论1] 一定量理想气体E随V变化如图,
则直线表示的过程为:

(A) 等温 (B) 等压 (C) 等容 (D) 绝热

$$\begin{aligned}
 \text{解: } E = E(T) = \nu \frac{i}{2} RT & \\
 E = k_1 V & \\
 \Rightarrow \frac{V}{T} = k & \\
 \frac{pV}{T} = \text{const.} & \Rightarrow p \text{ 为常数}
 \end{aligned}$$



(4) 绝热过程

① 特征 $dQ=0$

$$\left. \begin{aligned} dQ = dE + dA &\Rightarrow p dV = -\nu C_V dT \\ pV = \nu RT &\Rightarrow p dV + V dp = \nu R dT \end{aligned} \right\} \Rightarrow \frac{p dV}{V dp} = \frac{-1}{\gamma}$$

$$\Rightarrow \frac{dp}{p} + \gamma \frac{dV}{V} = 0 \Rightarrow \ln p + \gamma \ln V = c'$$

② 过程方程

$$pV^\gamma = c$$

$$pV = \nu RT$$

消 p

消 V

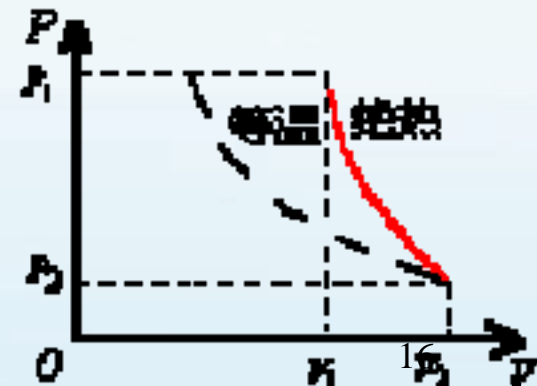
$$TV^{\gamma-1} = C / \nu R$$

$$p^{\gamma-1} T^{-\gamma} = (\nu R)^\gamma / C$$

③ $Q, \Delta E, A$ $\left\{ \begin{array}{l} Q=0 \end{array} \right.$

$$A = -\Delta E = -\int_{T_1}^{T_2} \nu C_V dT$$

④ p - V 图



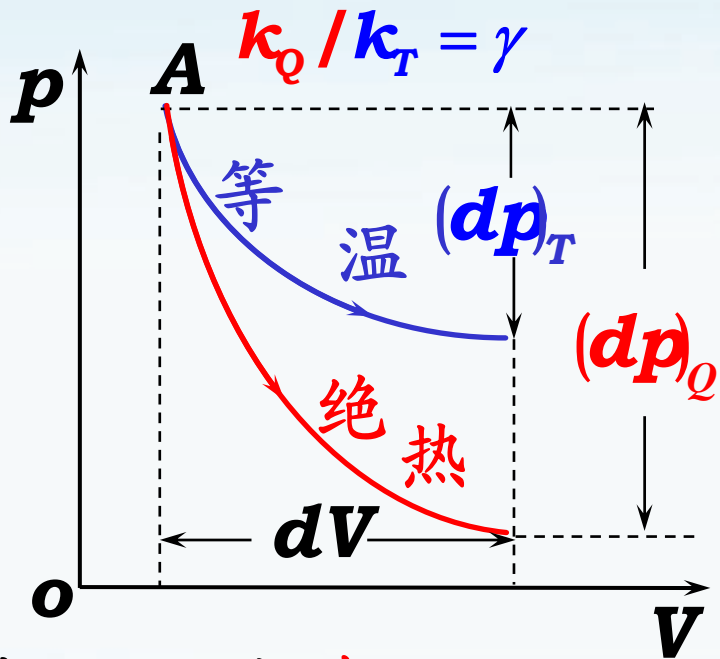
⑤p-V图上绝热线比等温线 陡

①数学解释

等温 $pV = c_1 \Rightarrow \frac{dp}{dV} = -\frac{c_1}{V^2} = -\frac{p}{V}$

绝热 $pV^\gamma = c_2 \Rightarrow \frac{dp}{dV} = -\frac{c_2\gamma}{V^{\gamma+1}} = -\gamma \frac{p}{V}$

对A: $\gamma > 1 \therefore |(dp/dV)_Q| > |(dp/dV)_T|$



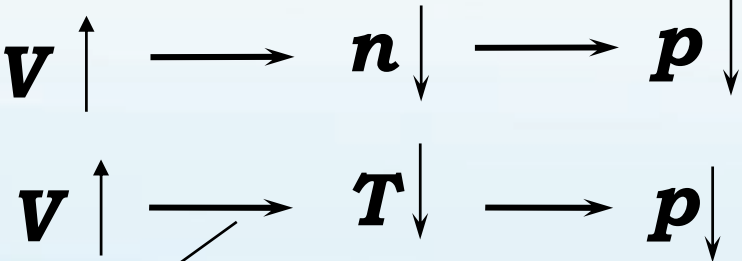
②物理意义: 膨胀相同体积绝热比等温p下降 多

$p = nkT$

等温



绝热



$TV^{\gamma-1} = C / \nu R$

[例题1] 一定量 N_2 , 绝热膨胀 $p \rightarrow 2p$, \bar{v} 变为原几倍?

$$\text{解: } \bar{v} = \sqrt{\frac{8RT}{\pi M_m}} \Rightarrow \frac{\bar{v}_2}{\bar{v}_1} = \sqrt{\frac{T_2}{T_1}} = 2^{1/7}$$

$$\left. \begin{array}{l} p^{\gamma-1} T^{-\gamma} = c_2 \\ \gamma \stackrel{\text{def}}{=} C_p / C_V = 1.4 \end{array} \right\} \Rightarrow \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{2/7} = 2^{2/7}$$

[讨论2] 若上题绝热压缩 $V \rightarrow V/2$, 压缩前后 \bar{z} 如何变化?

$$\begin{aligned} \text{解: } \bar{z} &= \sqrt{2n\pi d^2 \bar{v}} = \sqrt{2} \frac{N}{V} \pi d^2 \sqrt{\frac{8RT}{\pi M_m}} = c_3 \frac{\sqrt{T}}{V} \xrightarrow{TV^{\gamma-1} = c_4} \bar{z} = c_5 V^{-\frac{1}{2} - \frac{\gamma}{2}} \\ \Rightarrow \frac{\bar{z}_2}{\bar{z}_1} &= \left(\frac{V_2}{V_1} \right)^{-\frac{1}{2} - \frac{\gamma}{2}} = 2^{\frac{6}{5}} \end{aligned}$$

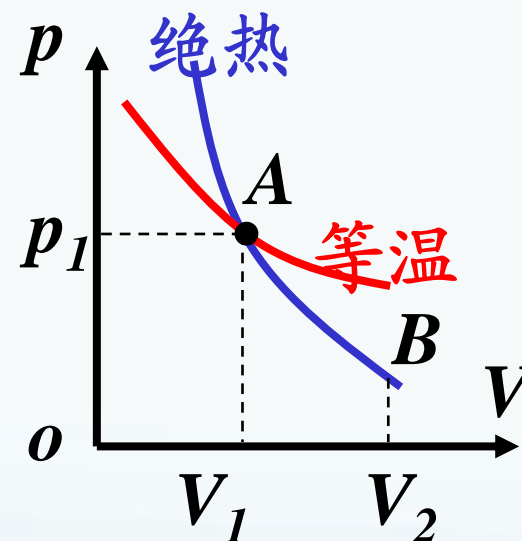
若为 H_2O 蒸汽, \bar{z} 如何变化? $\gamma = \frac{4}{3}$

[讨论3] 理想气体A点 $p_1 = 2 \times 10^5 \text{ Pa}$, $V_1 = 0.5 \times 10^{-3} \text{ m}^3$, 此处斜率比为 0.714, 从A绝热至B, 其体积 $V_2 = 1 \times 10^{-3} \text{ m}^3$. 求 (1) B点处 p_2 ; (2) 此过程气体对外做功.

$$\text{解: (1) } \frac{k_Q}{k_T} = \gamma = \frac{1}{0.714} \Rightarrow \gamma = 1.4$$

$$p_1 V_1^\gamma = p_2 V_2^\gamma \Rightarrow p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma$$

$$= 0.758 \times 10^5 p_a$$



$$(2) A = -\Delta E = -\nu \frac{5R}{2} (T_2 - T_1)$$

$$= -\frac{5}{2} (p_2 V_2 - p_1 V_1) = 60.5 \text{ J}$$

(5) 多方过程

等容	等温	等压	绝热	多方
$V = c_1$	$pV^1 = c_2$	$pV^0 = c_3$	$pV^\gamma = c_4$	$pV^n = c$

$\xrightarrow{\quad} p^{\frac{1}{n}} V = \text{const.} \quad (n = \infty) \quad \xleftarrow{\quad}$

①特征：满足 $pV^n = c$

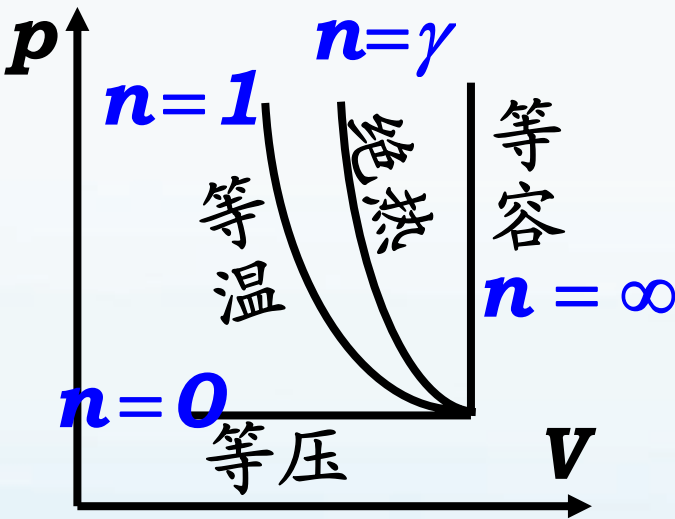
②过程方程

{

$pV^n = c$
 $pV = \nu RT$

消 p $V^{n-1} T = c / (\nu R)$

消 V $p^{n-1} T^{-n} = (\nu R)^n / c$



③ $Q, \Delta E, A$

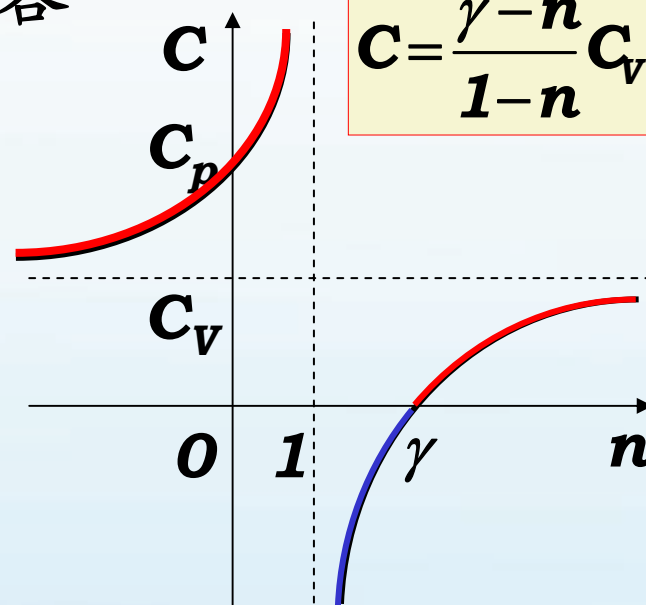
$$A = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} c dV / V^n = \frac{c}{1-n} \left(\frac{1}{V_2^{n-1}} - \frac{1}{V_1^{n-1}} \right)$$

$$\rightarrow \frac{\nu R \Delta T}{1-n} = \frac{p_2 V_2 - p_1 V_1}{1-n}$$

$$\Delta E = \nu C_V \Delta T$$

$$Q = \Delta E + A = \nu \left(\frac{C_V - n C_V}{1-n} + \frac{R}{1-n} \right) \Delta T = \nu \frac{\gamma - n}{1-n} C_V \Delta T$$

④ 摩尔热容



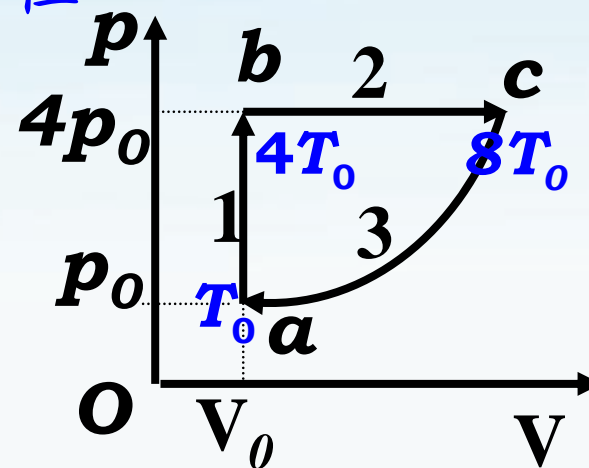
$$n > \gamma \text{ 或 } n < 1 \Rightarrow C > 0$$

$$1 < n < \gamma \Rightarrow C < 0$$

[例题2] 1mol 单原子理想气体pV图, **ca**方程

$$p = p_0 V^2 / V_0^2, \text{ a点温度为 } T_0,$$

以 T_0, R 表示1, 2, 3过程中 Q



解: $a \rightarrow b$ 等容 $\frac{p_0}{T_0} = \frac{4p_0}{T_b} \Rightarrow T_b = 4T_0$

$$b \rightarrow c \text{ 等压 } \frac{V_0}{4T_0} = \frac{V_c}{T_c}$$

$$c \rightarrow a \text{ 多方 } (4p_0)V_c^{-2} = p_0 V_0^{-2} \Rightarrow T_c = 8T_0$$

$$\Rightarrow V_c = 2V_0$$

$$A_{ca} = \int_{2V_0}^{V_0} p dV = \int_{2V_0}^{V_0} \frac{p_0}{V_0^2} V^2 dV$$

$$= \frac{p_0}{3V_0^2} V^3 \Big|_{2V_0}^{V_0} = -\frac{7RT_0}{3}$$

$$a \rightarrow b \text{ 等容 } Q_V = \Delta E = \nu C_V \Delta T = 1 \times (3R/2)(4 - 1)T_0 = 4.5RT_0$$

$$b \rightarrow c \text{ 等压 } Q_p = \nu C_p \Delta T = 1 \times (5R/2)(8 - 4)T_0 = 10RT_0$$

$$c \rightarrow a \text{ 多方 } Q_{\text{多方}} = \Delta E + A = (1 - 8)T_0 \cdot 3R/2 - 7RT_0/3 = -77RT_0/6$$

直接用 $C_{\text{多方}}$ 与 C_V 关系求 $Q_{\text{多方}}$, 判断哪种方法简洁? ²²

[讨论4] 1mol单原子理想气体作图示循环，
分析BC过程温度变化及吸放热情况

$$\text{解: } \begin{cases} p = -\frac{p_0}{V_0}V + 4p_0 \\ pV = \nu RT \end{cases} \Rightarrow T = \frac{1}{R} \left(4p_0V - \frac{p_0V^2}{V_0} \right)$$

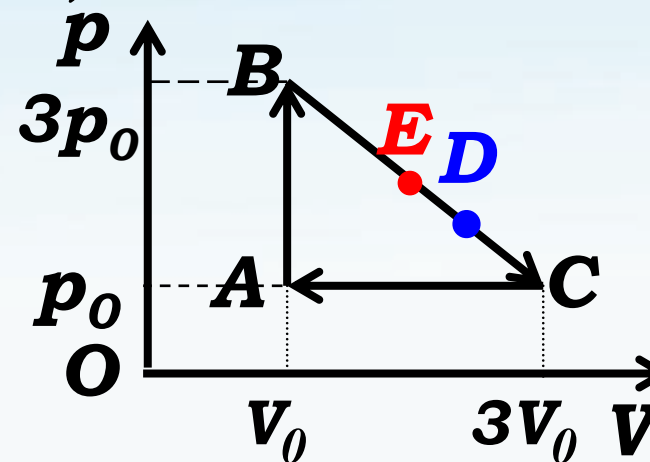
$$\text{令 } T' = 0 \Rightarrow V = 2.5V_0 \quad \begin{cases} \text{BE: } T \uparrow \\ \text{EC: } T \downarrow \end{cases}$$

$$dQ = dE + dA = \nu C_V \frac{p_0}{R} \left(4 - \frac{2V}{V_0} \right) dV + \left(-\frac{p_0}{V_0}V + 4p_0 \right) dV = 2p_0 \left(5 - \frac{2V}{V_0} \right) dV$$

$$\Rightarrow V = 2.5V_0 \quad V < 2.5V_0 \Rightarrow dQ > 0 \quad V > 2.5V_0 \Rightarrow dQ < 0$$

$$Q_{BD} = \int_{V_0}^{2.5V_0} 2p_0 \left(5 - \frac{2V}{V_0} \right) dV = 4.5p_0V_0 > 0$$

$$Q_{DC} = \int_{2.5V_0}^{3V_0} 2p_0 \left(5 - \frac{2V}{V_0} \right) dV = -0.5p_0V_0 < 0$$



[讨论5] 升温一定吸热，吸热一定升温？

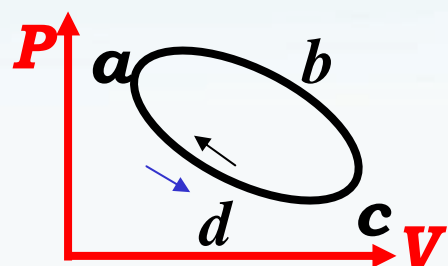
$T \uparrow$ 不一定吸热 $T \uparrow$ 等容等压 $Q = Q_{\text{吸}}$
 绝 热 $Q = 0$
 多方及其它 $C = \frac{\gamma - n}{1 - n} C_v$ 及 $Q \text{ 可能} = Q_{\text{放}}$

吸热不一定 $T \uparrow$ $Q_{\text{吸}}$ 等容等压 $T \uparrow$
 等 温 $T = \text{const}$
 多方及其它 $T \text{ 可能} \uparrow$ $T \text{ 可能} \downarrow$

4.5对循环过程的应用

1° 基本概念

(1) **循环过程**: 从初态出发最终回到原态的准静态过程



正(热)循环: p-V图上沿顺时针循环

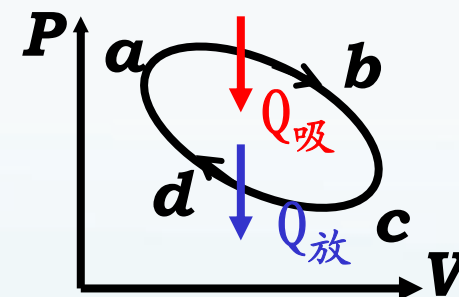
逆(致冷)循环: 反.....

(2) **热机**: 连续不断把热转换成功的装置

(3) **工质**: 循环系统的工作物质

(4) **热机效率** $\eta \stackrel{\text{def}}{=} \frac{A_{\text{对外净}}}{Q_{\text{吸}}} = \frac{Q_{\text{吸}} - |Q_{\text{放}}|}{Q_{\text{吸}}}$

物理意义: 热机从外界吸热
转化为对外有用净功百分数



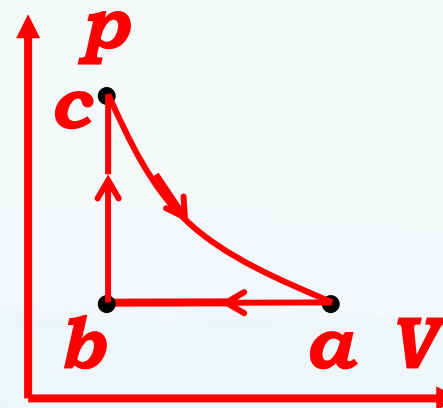
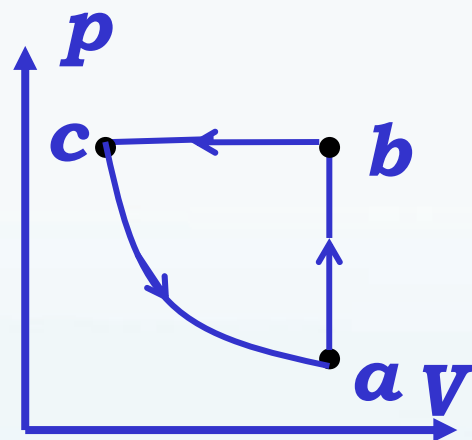
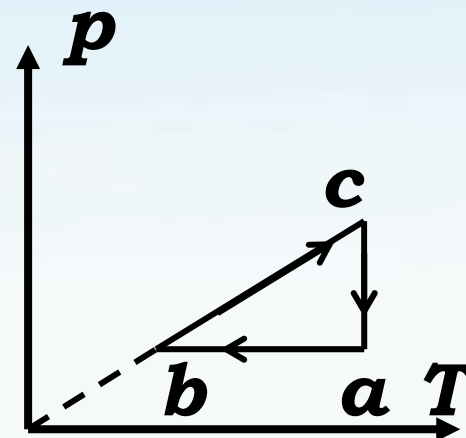
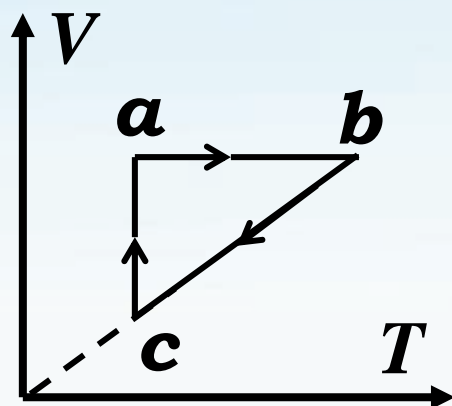
$abcda$ 循环

$$\Delta E = 0 \Rightarrow Q = A$$

$$Q_{\text{吸}} - |Q_{\text{放}}| = A_{\text{对外净}}$$

[例题3] 判断 (图形改画成p-V图)

FangYi

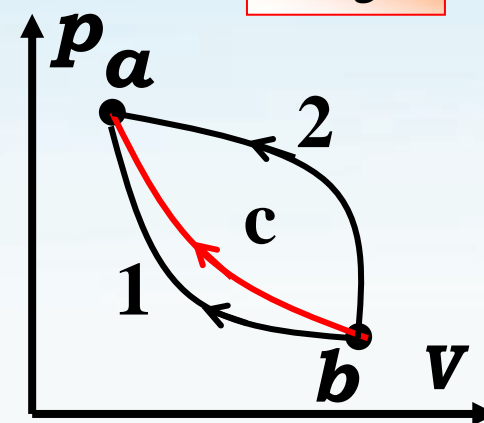


- | | |
|-----------------------|---|
| × (1) 该过程正循环 | √ |
| √ (2) ca 对外作正功 | √ |
| √ (3) ab 吸热 | × |
| √ (4) bc 等压 | × |

[讨论6] bca 绝热, $b1a, b2a$ 为任两过程

$b1a$ $b2a$

- (1) 放热,负功、 放热,负功
- (2) ☒ 吸热,负功、 放热,负功
- ~~(3) 吸热,正功、 吸热,负功~~
- ~~(4) 放热,正功、 吸热,正功~~



	循环	V单调
A	正>0	增>0
	逆<0	减<0

解: $A_{b1a} < 0$

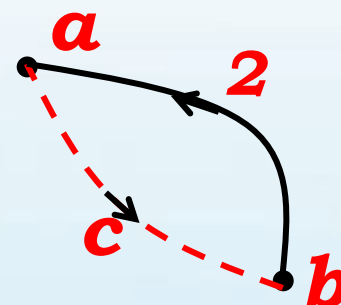
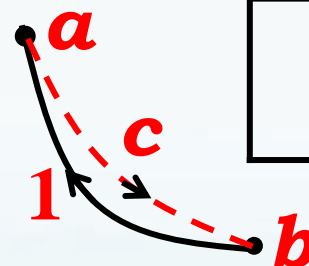
构造循环,由热一律

$$Q_{b1a} + 0 = A_{acb1a} > 0$$

$A_{b2a} < 0$

构造循环,由热一律

$$Q_{b2a} + 0 = A_{b2acb} < 0$$

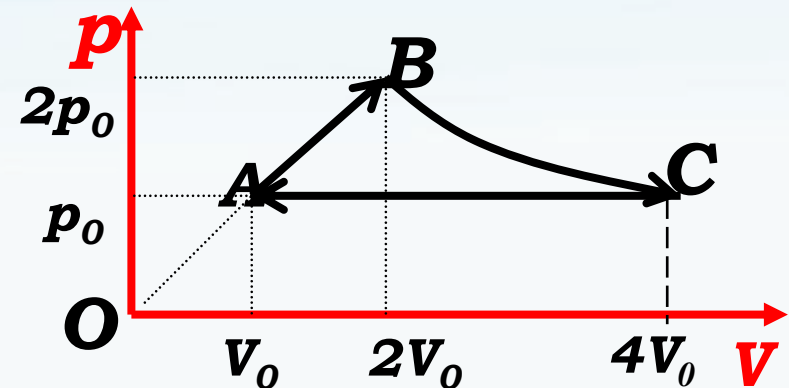


[例题4] **1mol He**循环: 试求 (1) **AB**过程的 n (2) ΔE_{AB}
 (3) 整个循环**A** (4) 整个循环**Q_净** (5) 该循环的 η

FangYi

解: (1) $p_0 V_0^n = 2p_0 (2V_0)^n \Rightarrow n = -1$

(2) $A \rightarrow B$ 多方 $\Delta E_{AB} = \nu \frac{3}{2} R(T_B - T_A)$
 $= \frac{3}{2} (4p_0 V_0 - p_0 V_0) = \frac{9}{2} p_0 V_0$



(3) $B \rightarrow C$ 等温 $2p_0 \times 2V_0 = p_0 \times V_C \Rightarrow V_C = 4V_0$

$$\left. \begin{aligned} A_{BC} &= \nu RT_B \ln(4V_0/2V_0) \\ A_{CA} &= p_0(V_0 - 4V_0) \\ A_{AB} &= (p_0 + 2p_0)V_0/2 \end{aligned} \right\} \Rightarrow A = A_{BC} + A_{CA} + A_{AB} = (4\ln 2 - 3/2)p_0 V_0$$

(4) 由热一律 $Q_{\text{净吸}} = A$

(5) $Q_{AB} = \Delta E_{AB} + A_{AB} = 6p_0 V_0 > 0$



$Q_{BC} = A_{BC} = 4p_0 V_0 \ln 2 > 0$

$Q_{CA} = \Delta E_{CA} + A_{CA} < 0$

$\eta = A / Q_{\text{吸}}$

$= \frac{4\ln 2 - 3/2}{4\ln 2 + 6}$

(5) 致冷机(逆循环)

① 致冷系数

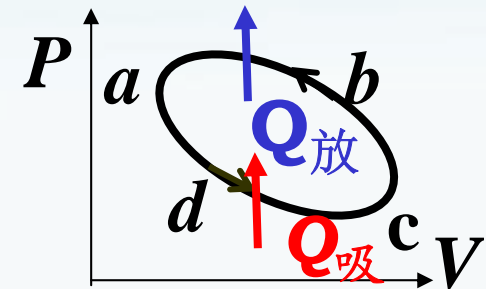
$$adcba: \Delta E = 0 \xrightarrow{\text{循环}} Q = A$$

$$Q_{\text{吸}} - |Q_{\text{放}}| = A_{\text{adc}} + A_{\text{cba}} = -|A_{\text{对系净}}|$$

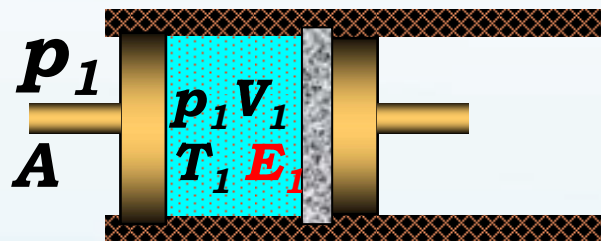
物理意义: 用系统从低温热源吸热 Q

与对系统作净功的比值表征致冷机吸热本领

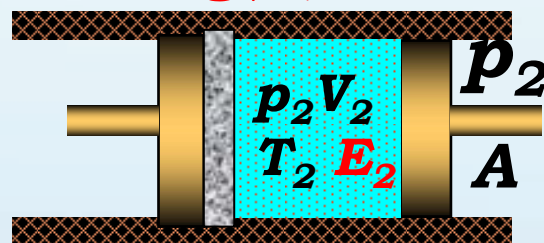
$$\omega \stackrel{\text{def}}{=} \frac{Q_{\text{吸}}}{|A_{\text{对系净}}|} = \frac{Q_{\text{吸}}}{|Q_{\text{放}}| - Q_{\text{吸}}}$$



② 焦耳汤姆孙效应



绝热



$$0 = E_2 - E_1 + p_2 V_2 - p_1 V_1 \Rightarrow H_2 = H_1$$

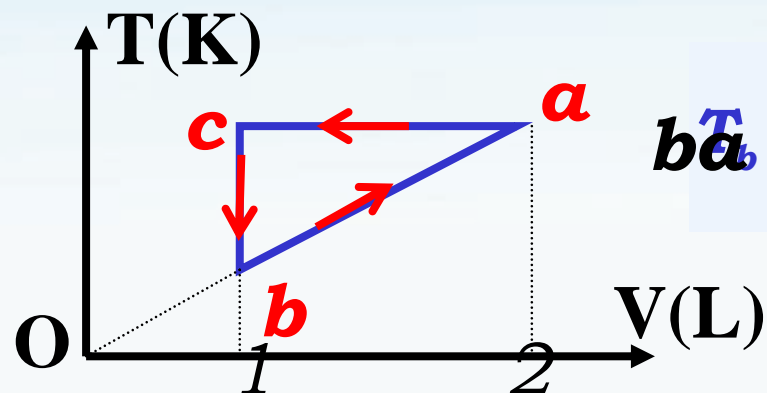
理想气体绝热节流过程焓不变 T

实际气体节流后

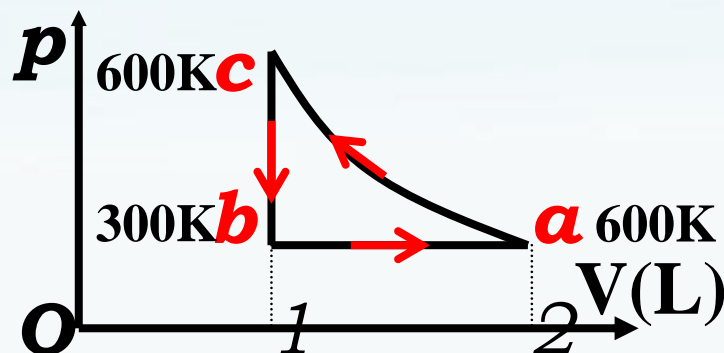
$T \downarrow \rightarrow$ 正节流 $T \uparrow \rightarrow$ 负节流

[讨论7] 1mol单原子分子理想气体循环如T-V图, $T_c=600K$.

试求 (1) Q_{ac}, Q_{cb}, Q_{ba} (2) 整个循环 $A_{\text{净}}$ (3) η OR ω



$$T_b = \frac{V_b}{V_a} T_a$$



解(1) **ac**等温压缩 $Q_{ac} = A_{ac} = \nu RT \ln \frac{V_c}{V_a} = (600 \ln \frac{1}{2})R = -416R$

cb等容降压 (温) $Q_{cb} = \Delta E = \nu C_v \Delta T = 1 \times \frac{3R}{2} (300 - 600) = -450R$

ba等压膨胀 $Q_{ba} = \nu C_p \Delta T = 1 \times \frac{5R}{2} (600 - 300) = 750R$

(2) 由热一律 $A = Q_{ac} + Q_{cb} + Q_{ba} = -116R$

(3) $\omega = \frac{Q_{\text{吸}}}{|Q_{\text{放}}| - Q_{\text{吸}}} = \frac{750R}{116R} = 6.47$

2° 卡诺循环

为了提高热机效率, 卡诺提出理想循环——卡诺循环

(1) 卡诺循环

工质只与两个恒温热库交换热量的准静态循环过程

(2) 卡诺机

按卡诺循环工作的热机

(3) 卡诺循环的组成

两等温, 两绝热共4个准静态过程

(4) 卡诺热机的效率 (工质为理想气体)

$a \rightarrow b$ (等温膨胀) $\Delta E_1 = 0$

$$Q_1 = A_1 = \nu RT_1 \ln(V_2/V_1) > 0, \text{吸热}$$

$b \rightarrow c$ (绝热膨胀)

$$Q = 0, A_2 = -\Delta E_2 = \nu C_V(T_1 - T_2)$$

$c \rightarrow d$ (等温压缩) $\Delta E_3 = 0$

$$Q_2 = A_3 = \nu RT_2 \ln(V_4/V_3) < 0, \text{放热}$$

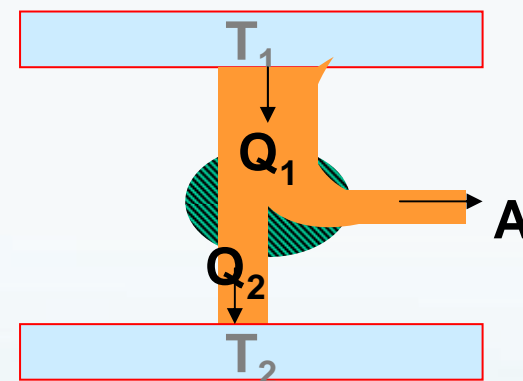
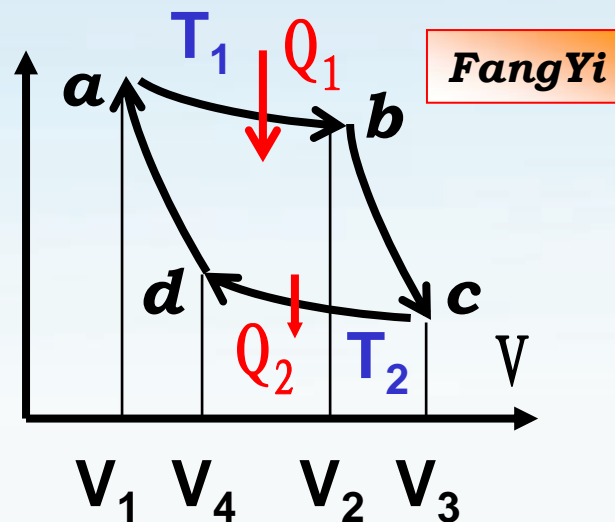
$d \rightarrow a$ (绝热压缩)

$$Q = 0, A_4 = -\Delta E_4 = \nu C_V(T_2 - T_1)$$

$$\therefore A = A_1 + \cancel{A_2} + A_3 + \cancel{A_4} = Q_1 + Q_2 = Q_1 - |Q_2|$$

$$\therefore \eta = \frac{A}{Q_1} = \frac{Q_1 - |Q_2|}{Q_1} = 1 - \frac{|Q_2|}{Q_1} = 1 - \nu RT_2 \ln(V_3/V_4) / \nu RT_1 \ln(V_2/V_1)$$

$$\left. \begin{array}{l} bc \text{ 绝热膨胀 } V_2^{\gamma-1} T_1 = V_3^{\gamma-1} T_2 \\ da \text{ 绝热压缩 } V_4^{\gamma-1} T_2 = V_1^{\gamma-1} T_1 \end{array} \right\} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4} \therefore \eta = \frac{T_1 - T_2}{T_1}$$



总结：完成一次卡诺循环，必须有高温、低温两个热源
 两热源温差越大，从高温热源吸热利用率越高
 不可获得 $T_1=\infty$ 或 $T_2=0\text{K}$ ，卡诺循环效率恒 <1 。

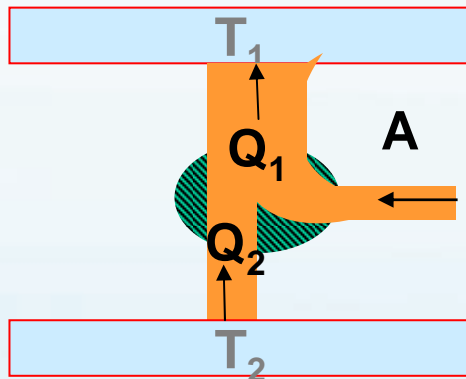
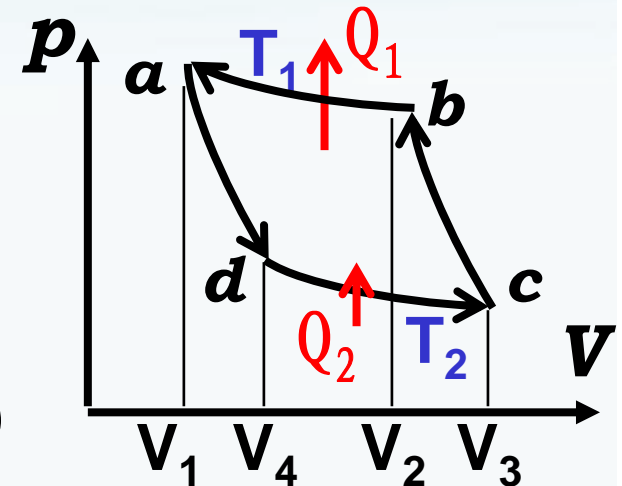
(5) 卡诺致冷机 [实例：冰箱、热泵]

$b \rightarrow a$ (等温压缩)

$$Q_1 = A_1 = \nu RT_1 \ln(V_1/V_2) < 0$$

$d \rightarrow c$ (等温膨胀)

$$Q_2 = A_3 = \nu RT_2 \ln(V_3/V_4) > 0$$



$$\omega = \frac{Q_2}{|Q_1| - Q_2} = \frac{\nu RT_2 \ln(V_3/V_4)}{\nu RT_1 \ln(V_2/V_1) - \nu RT_2 \ln(V_3/V_4)} = \frac{T_2}{T_1 - T_2}$$

显然, $T_2 \downarrow \omega \downarrow$,

即对系统作同样功, 低温越低, 从低温热源吸热越少;
 或者从低温热源吸取同样热量, 低温越低, 做功越多。

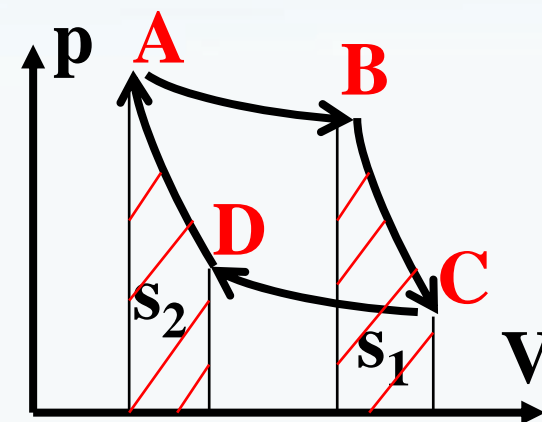
[讨论8] 理想气体卡诺循环两绝热线下面积 (阴影部分)
分别为 s_1 和 s_2 , 则

- (A) $s_1 > s_2$; ~~(B)~~ $s_1 = s_2$;
(C) $s_1 < s_2$; (D) 无法确定

解: $s_1 = A_{BC} = -\Delta E_{BC} = \nu C_V (T_B - T_C)$

$$s_2 = |A_{DA}| = \Delta E_{DA} = \nu C_V (T_A - T_D)$$

$$\because T_A = T_B, T_D = T_C \Rightarrow s_1 = s_2$$



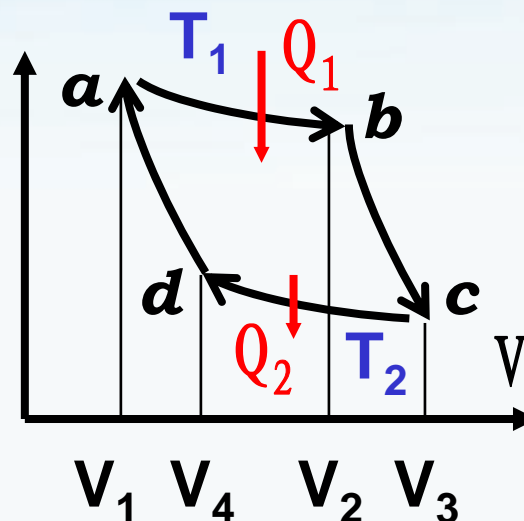
[例题5]

1mol 理气在 $T_1=400\text{K}$ 与 $T_2=300\text{K}$ 间作可逆卡诺循环。在 400K 线上起始 $V_1=0.001\text{m}^3$, 终止 $V_2=0.005\text{m}^3$, 求此气体在每一循环中

(1) 所作净功 A

(2) 从高温热源吸热 Q_1

(3) 向低温热源放热 Q_2



解 (1) $A = A_{ab} + \cancel{A_{bc}} + A_{cd} + \cancel{A_{da}}$

$$= \nu RT_1 \ln(V_2 / V_1) + \nu RT_2 \ln(V_4 / V_3)$$

$$\Rightarrow A = R(T_1 - T_2) \ln(V_2 / V_1) \Rightarrow A = 8.31 \times 100 \ln 5 = 1337 \text{ J}$$

$$(2) Q_1 = A_{ab} = \nu RT_1 \ln(V_2 / V_1) = 5348 \text{ J}$$

$$\text{or. } Q_1 = A / \eta = 1337 / 25\% = 5348 \text{ J} \quad \eta = 1 - T_2 / T_1 = 25\%$$

$$(3) Q_2 = A_{cd} = \nu RT_2 \ln(V_1 / V_2) = -4011 \text{ J}$$

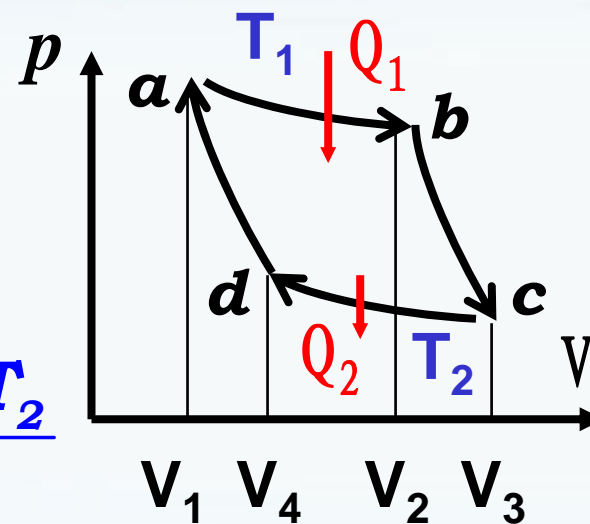
$$\text{or. } |Q_2| = Q_1 - A = 4011 \text{ J}$$

[讨论9]卡诺机高、低温热源分别为 27°C 、 -73°C ，工作物质为 10mol 的空气，若在等温膨胀过程中气缸体积增大到原来的 e 倍，确定此热机每一循环做功

解: $\frac{A}{A_{ab}} = \frac{A}{Q_{\text{吸}}} = \eta$

$$\rightarrow A = A_{ab}\eta = \nu RT_1 \ln(V_2 / V_1) \frac{T_1 - T_2}{T_1}$$

$$= 10 \times 8.31 \times 300 \ln e \times \frac{300 - 200}{300} = 8.31 \times 10^3 \text{J}$$



[讨论10] 某理气分别进行两卡诺循环: 1 ($abcda$) 和 2 ($a'b'c'd'a'$), 两循环曲线所围面积相等

设循环1效率 η , 每次循环从高温热源吸热 Q ;

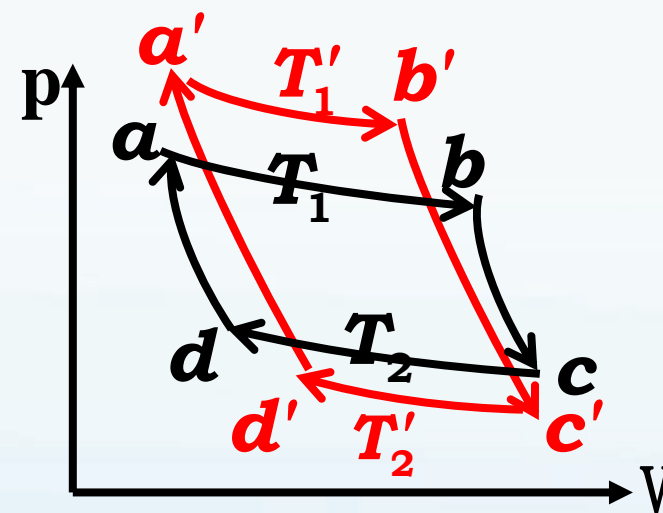
循环2效率 η' , 每次循环从高温热源吸热 Q' ,

则 (A) $\eta < \eta'$, $Q < Q'$; (B) $\eta < \eta'$, $Q > Q'$;

(C) $\eta > \eta'$, $Q < Q'$; (D) $\eta > \eta'$, $Q > Q'$.

$$\text{解: } \left. \begin{aligned} \eta &= 1 - \frac{T_2}{T_1} \\ \eta' &= 1 - \frac{T'_2}{T'_1} \end{aligned} \right\} \xrightarrow{T_2 > T'_2, T_1 < T'_1} \eta < \eta'$$

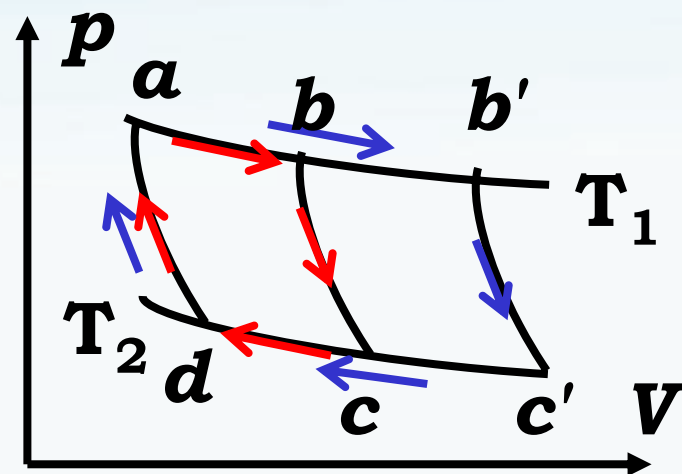
$$\left. \begin{aligned} Q &= \frac{A}{\eta} \\ Q' &= \frac{A'}{\eta'} \end{aligned} \right\} \xrightarrow{A = A', \eta < \eta'} Q > Q'$$



[讨论11] 卡诺热机循环曲线从 **abcda** 增为 **ab'c'da**,

则其A和 η 变化是

- (A) A增大, η 提高;
- (B) A增大, η 降低;
- (C) A和 η 均不变;
- (☒) (D) A增大, η 不变

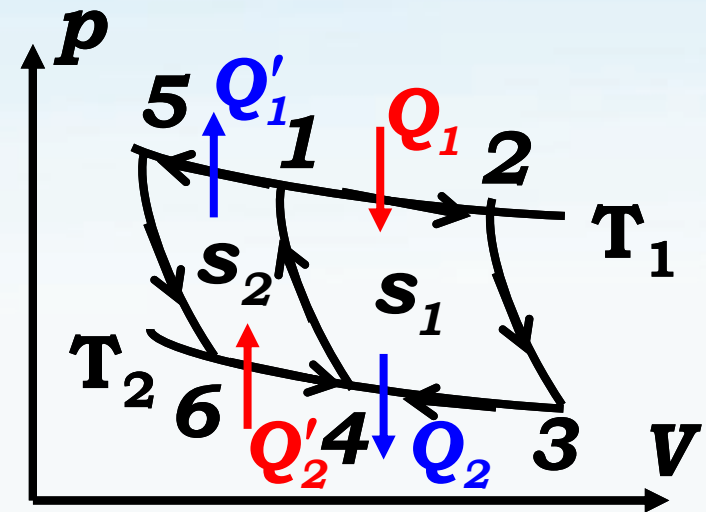


解: $S_{ab'c'da} > S_{abcda} \Rightarrow A_{ab'c'da} > A_{abcda}$

$$\left. \begin{aligned} \eta_{abcda} &= 1 - \frac{T_2}{T_1} \\ \eta_{ab'c'da} &= 1 - \frac{T_2}{T_1} \end{aligned} \right\} \Rightarrow \eta_{ab'c'da} = \eta_{abcda}$$

[思考12]*一定量某理气由一卡诺正与
逆组成循环, $T_1=4T_2, S_1=2S_2$
求 $\eta_{123415641}$

解: $\eta = 1 - \frac{|Q_{\text{放}}|}{Q_{\text{吸}}} = 1 - \frac{T_2}{T_1}$
 $\Rightarrow \eta = 1 - \frac{Q'_1 + Q_2}{Q_1 + Q'_2} = \frac{1}{3}$



$$\left. \begin{array}{l} \frac{Q_1}{Q_2} = \frac{T_1}{T_2} = 4 \\ Q_1 - Q_2 = S_1 \\ \frac{Q'_1}{Q'_2} = \frac{T_1}{T_2} = 4 \\ Q'_1 - Q'_2 = S_2 \\ S_1 = 2S_2 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} Q_1 = \frac{8}{3} S_2 \\ Q_2 = \frac{2}{3} S_2 \\ Q'_1 = \frac{4}{3} S_2 \\ Q'_2 = \frac{1}{3} S_2 \end{array} \right.$$

[讨论13] 热源 $T_{\text{高}}$ 是 $T_{\text{低}}$ 的 n 倍, 则理想气体一次卡诺循环, 传给低温热源热量是从高温热源吸取的

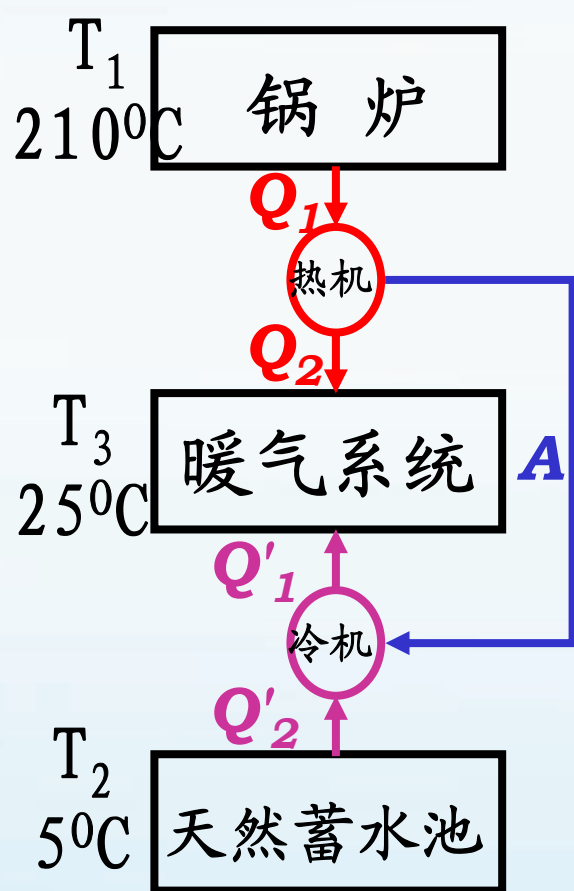
(A) n 倍 (B) $n-1$ 倍 (C) $1/n$ 倍 (D) $(n+1)/n$ 倍

解: 卡诺循环 $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{|Q_{\text{放}}|}{Q_{\text{吸}}} \Rightarrow |Q_{\text{放}}| = \frac{T_2}{T_1} Q_{\text{吸}} = \frac{1}{n} Q_{\text{吸}}$

[讨论14] 可逆卡诺热机 η , 其致冷机 $\omega = T_2 / (T_1 - T_2)$, 则 η 、 ω 的关系为

解: $\left. \begin{array}{l} \eta = \frac{T_1 - T_2}{T_1} \\ \omega = \frac{T_2}{T_1 - T_2} \end{array} \right\} \Rightarrow \eta\omega = \frac{T_2}{T_1}$

[例题6]动力暖气装置由一台卡诺热机与一台卡诺致冷机组成.热机靠燃料燃烧时释放热量工作并向暖气系统放热.同时热机带动致冷机从天然蓄水池中吸热并向暖气系统放热.计算暖气系统得到的热量(已知 $Q_1=2.1\times 10^7\text{J}$)



解: $\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{|Q_{\text{放}}|}{Q_{\text{吸}}}$

$$\Rightarrow |Q_2| = \frac{T_3}{T_1} Q_1$$

$$|Q'_1| = |Q'_2| + A$$

$$A = \eta Q_1 = \left(1 - \frac{T_3}{T_1}\right) Q_1$$

$$\omega = \frac{T_2}{T_3 - T_2} = \frac{Q'_2}{|Q'_1| - Q'_2}$$

$$\Rightarrow Q'_2 = \frac{T_2}{T_3} |Q'_1|$$

$$\Rightarrow |Q'_1| = \frac{T_3(T_1 - T_3)}{T_1(T_3 - T_2)} Q_1$$

$$\parallel$$

$$2.08 \times 10^8 \text{J}$$



奥托
1876



戴姆勒
1883



狄塞尔
1897

德国的骄傲

[讨论15] 求奥托循环效率

0进气1绝热压缩2点火3
绝热膨胀4排气1扫气0

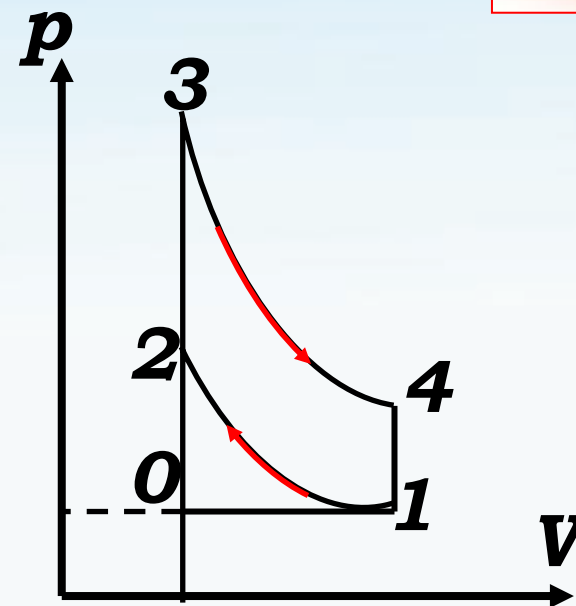
解: $Q_{\text{吸}} = \nu C_V (T_3 - T_2) > 0$

$Q_{\text{放}} = \nu C_V (T_1 - T_4) < 0$

$$\eta = 1 - \frac{|Q_{\text{放}}|}{Q_{\text{吸}}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2}$$

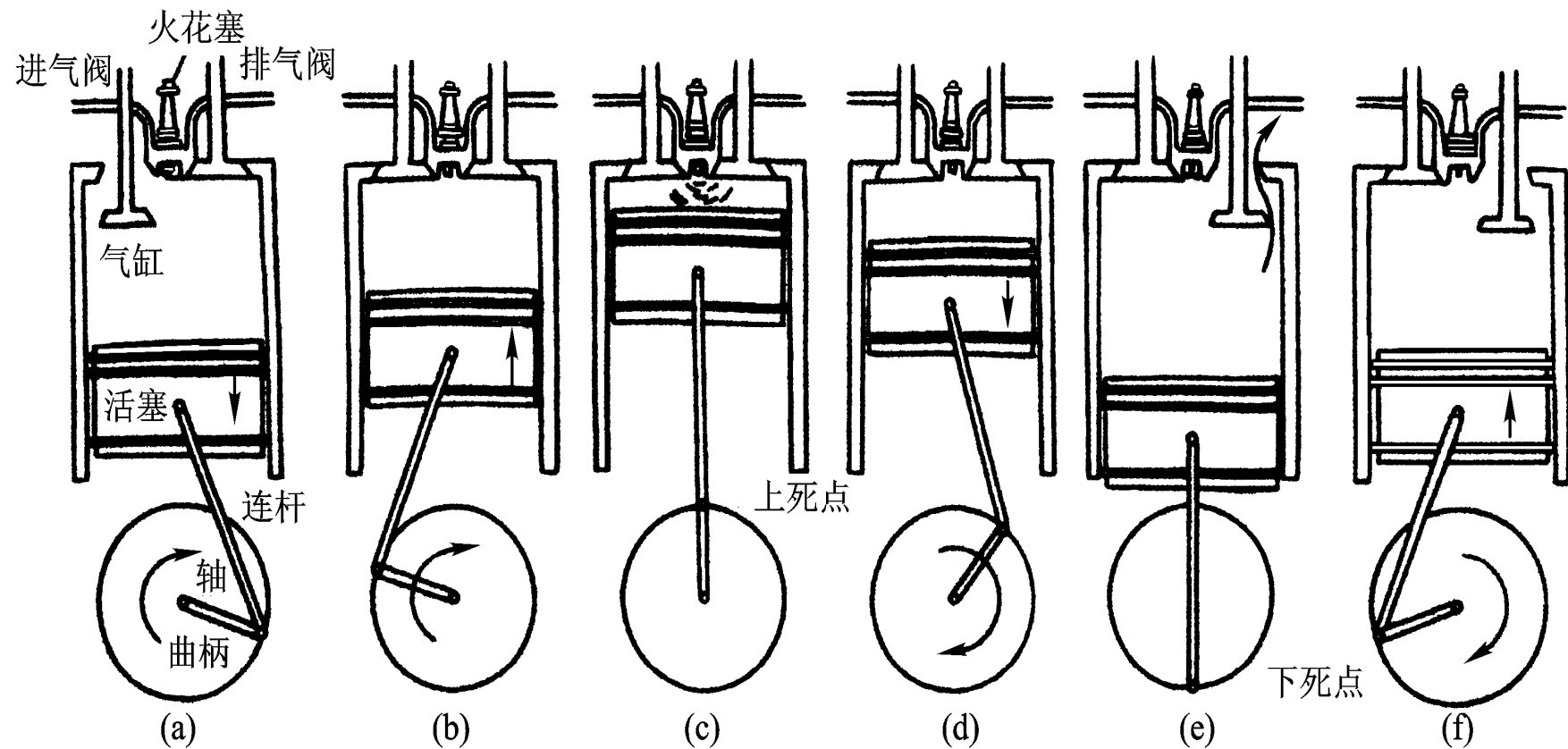
$$\Rightarrow \eta = 1 - K^{1-\gamma}$$

绝热容积压缩比



$$\left. \begin{array}{l} 12: V_1^{\gamma-1} T_1 = V_2^{\gamma-1} T_2 \\ 34: V_2^{\gamma-1} T_3 = V_1^{\gamma-1} T_4 \end{array} \right\}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{T_4}{T_3} = \frac{T_4 - T_1}{T_3 - T_2}$$



0-1 过程: 进气 1-2 过程: 压缩 2-3 过程: 加热 3-4 过程: 膨胀 4-1 过程: 排气 1-0 过程: 扫气

[讨论16] 求狄塞尔循环效率

0 进气 1 绝热压缩 2 等压点火
3 绝热膨胀 4 排气 1 扫气 0

解: $Q_{\text{吸}} = \nu C_p (T_3 - T_2) > 0$

$Q_{\text{放}} = \nu C_v (T_1 - T_4) < 0$

$$\eta = 1 - \frac{|Q_{\text{放}}|}{Q_{\text{吸}}} = 1 - \frac{(T_4 - T_1)}{\gamma (T_3 - T_2)}$$

$$12: \frac{T_2}{T_1} = K^{\gamma-1}$$

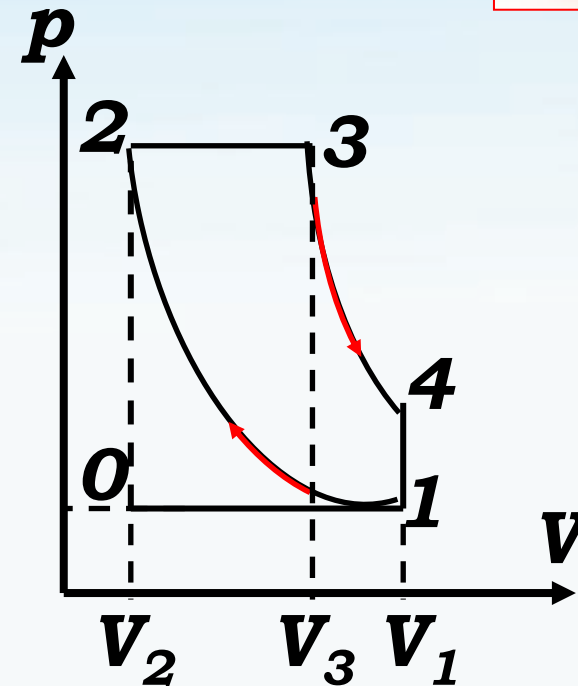
$$23: \frac{T_3}{T_2} = \frac{V_3}{V_2} = \rho$$

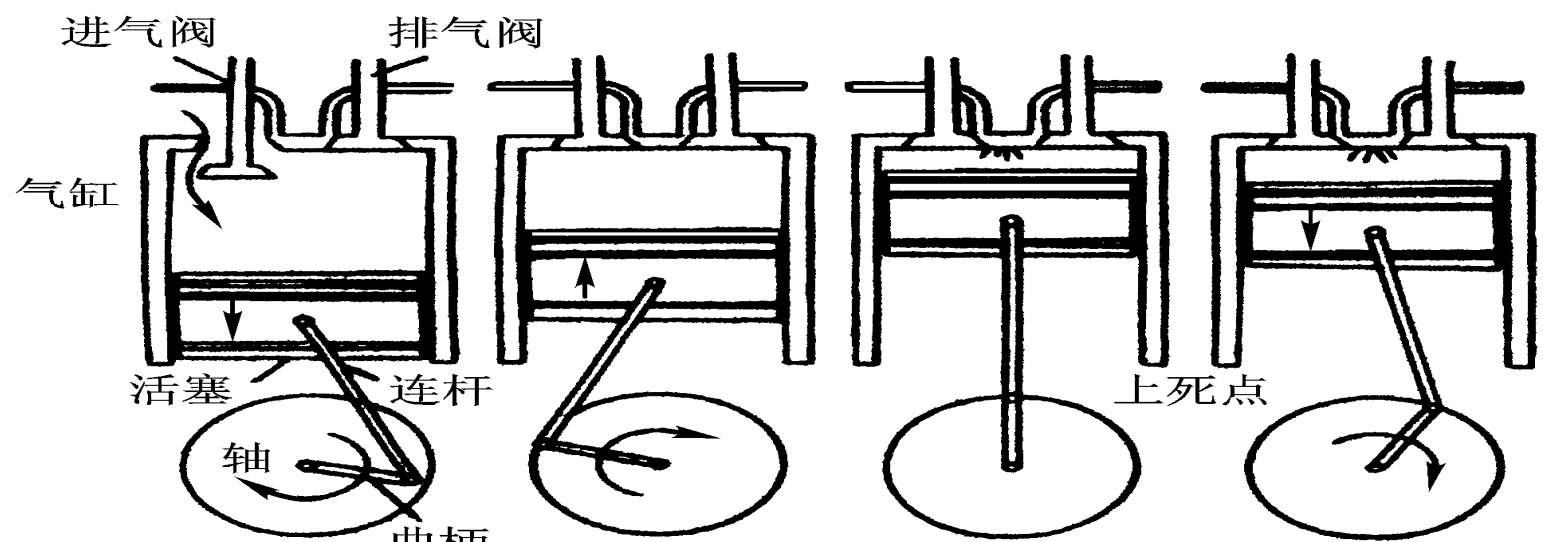
$$34: \frac{T_4}{T_3} = \left(\frac{\rho}{K}\right)^{\gamma-1}$$

K: 绝热容积压缩比

$$\Rightarrow \eta = 1 - \frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)K^{\gamma-1}}$$

ρ : 定压容积压缩比

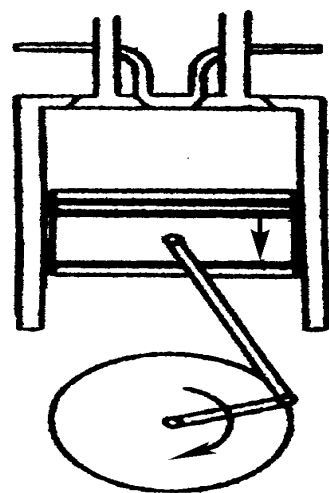




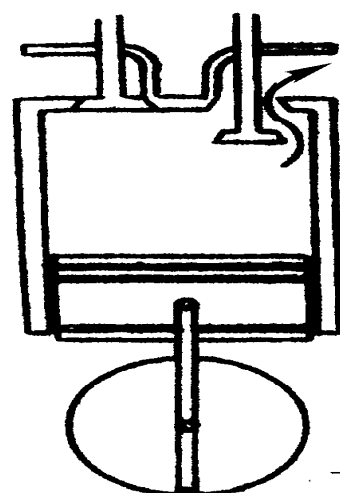
0-1 过程: 进气

1-2 过程: 压缩

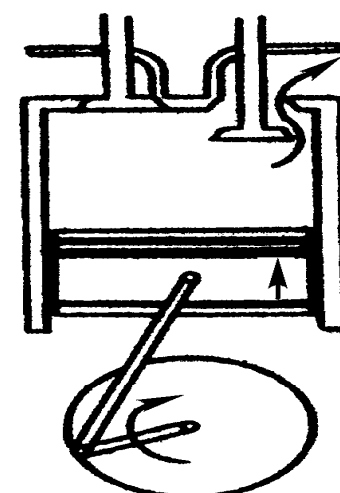
2-3 过程: 加热



3-4 过程: 工作



4-1 过程: 排气



1-0 过程: 扫气