假定 II.

任意可观测力学量对应一个线性厄密算符;
从这些算符可计算力学量的平均值.

* operator. 算符. $\hat{O}$

线性算符. $\hat{O}(c_1\psi_1 + c_2\psi_2) = c_1\hat{O}\psi_1 + c_2\hat{O}\psi_2$.

算符和. $(\hat{A}+\hat{B})\psi = \hat{A}\psi + \hat{B}\psi$ ($\hat{A}$、$\hat{B}$ 是任意算符.)

算符积. $\hat{A}\hat{B}\psi = \hat{A}(\hat{B}\psi)$. B 先作用

通常不满足交换律. 对易关系. $[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$.

$[\hat{A},\hat{B}]=0$, 满足对易.

$[x, \frac{d}{dx}]\psi = -\psi$, 则 $[x, \frac{d}{dx}] = -1$

动量算符. $\hat{p} \Rightarrow -i\hbar\nabla$

$[x, \hat{p}_x] = -i\hbar[x, \frac{\partial}{\partial x}] = i\hbar$.

$[x, \hat{p}_y] = 0$.

$[x_\alpha, \hat{p}_\beta] = i\hbar\delta_{\alpha\beta}$.

算符的平均测量值.

$$\vec{F} \Rightarrow \langle F \rangle = \frac{\int \psi^* F \psi dx}{\int \psi^* \psi dx = 1} = \int \psi^* F \psi dx$$

* 厄密算符, 对 2 个任意状态. ψ, φ, F 满足.

$$\int \psi^* F \varphi dx = \int (F\psi)^* \varphi dx$$

作用.

内积 $(\psi, \varphi) = \int \psi^* \varphi dx$

$$\Rightarrow (\psi, F\varphi) = (F\psi, \varphi)$$

假定 III. 对某个体系, 测某一力学量, 测得确定的值.

则该力学量对应的算符. 有 $\hat{F}\psi = \lambda\psi$. 本征函数

假设 IV . 态叠加原理.

ψ_n 是可能的态, $\psi = \sum c_n \psi_n$ 也是体系的可能的态.

如果不是本征态, 有不确定的测量值.

V. schrodinger 方程

自由粒子. $E = \frac{p^2}{2m}$.

$\psi(x,t) \sim e^{i(\vec{k}\vec{x} - \omega t)} = e^{i(\vec{p}\cdot\vec{x} - Et)/\hbar}$

$$\begin{cases} i\hbar \frac{\partial}{\partial t} \psi = E \cdot \psi \\ -i\hbar \nabla \psi = p \cdot \psi \end{cases}$$

$\hat{E} = i\hbar \frac{\partial}{\partial t}$

$\hat{p} = -i\hbar \nabla$

$\hat{p}^2 = -\hbar^2 \nabla^2$

$$E = p^2/2m \Rightarrow \hat{E} \psi = \frac{\hat{p}^2}{2m} \psi$$

$$\Rightarrow \left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2\right) \psi = 0.$$

代入同样满足.

$$\psi(x,t) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}} \int \varphi(p)\, e^{i(\vec{p}\vec{x} - Et)/\hbar}\, d\vec{p}$$

$$E = T + V = \frac{p^2}{2m} + V$$

$$\Rightarrow E\psi = \frac{p^2}{2m} \psi + V \cdot \psi$$

$$\underbrace{\left(-\frac{\hbar^2}{2m} \nabla^2 + V\right)}_{\hat{H}} \psi = i\hbar \frac{\partial}{\partial t} \psi$$