华东理工大学 2016 - 2017 学年第二学期

《专业阅读》课程期末考试试卷 A 2017. 6

开课学院：理学院 专业：应用数学、信息与计算 考试形式：闭卷 所需时间：120分钟

考生姓名：__________ 学号：__________ 班级：__________ 任课教师：杨勤民

题序	I	II	III	IV	总 分
得分					
评卷人	杨 勤 民				

I. Spell the English words according to the following Chinese meaning ($1' \times 12 = 12'$).

测地线	__________	子集	__________
非线性的	__________	可微的	__________
挠率	__________	半圆	__________
基	__________	半径	__________
上界	__________	双曲线	__________
无穷大的	__________	斜率	__________

II. Give the Chinese meaning for each of the following English words ($1' \times 12 = 12'$).

plus	__________	unbounded	__________
lemma	__________	mapping	__________
isomorphic	__________	integer	__________
invariant	__________	odevity	__________
minimal	__________	rank	__________
inequality	__________	exponent	__________

III. Translate the following passages into Chinese ($12' \times 3 = 36'$).

Passage 3.1 We shall often refer somewhat loosely to the n-tuples $(x^1, \dots, x^n)$ themselves as the points of the space. The simplest example of a Cartesian space is the real number line. Here each point has just one coordinate x^1, so that $n = 1$, i.e. it is a 1-dimensional Cartesian coordinate space. Other examples, familiar from analytic geometry, are provided by Cartesian coordinatizations of the plane (which is then a 2-dimensional Cartesian space), and of ordinary (i.e. 3-dimensional) space. These Cartesian spaces are completely adequate for solving the problems of school geometry.

Passage 3.2 Various identities are satisfied by the matrix operation, such as the distributive laws

$$A(B + B') = AB + AB' \quad \text{and} \quad (A + A')B = AB + A'B \tag{1}$$

and the associative law

$$(AB)C = A(BC). \tag{2}$$

These laws hold whenever the matrices involved have suitable sizes, so that the products are defined. For the associative law, for example, the sizes should be $A = l \times m$, $B = m \times n$, and $C = n \times p$, for some l, m, n, p. Since the two products (2) are equal, the parentheses are not required, and we will denote them by ABC. The triple product ABC is then an $l \times p$ matrix.

Passage 3.3 This relative frequency interpretation of probability is said to be objective because it rests on a property of the experiment rather than on any particular individual concerned with the experiment. For example, two different observers of a sequence of coin tosses should both use the same probability assignments since the observers have nothing to do with limiting relative frequency. In practice, this interpretation is not as objective as it might seem, since the limiting relative frequency of an event will not be known. Thus we will have to assign probabilities based on our beliefs about the limiting relative frequency of events under study. Fortunately, there are many experiments for which there will be a consensus with respect to probability assignments. When we speak of a fair coin, we shall mean $P(H) = P(T) = .5$, and a fair die is one for which limiting relative frequencies of the six outcomes are all $\frac{1}{6}$, suggesting probability assignments $P(1) = \cdots = P(6) = \frac{1}{6}$.

考生姓名: ___________　　　　　　　　　　　　学号: ___________

IV. Read the following passages and translate them into Chinese ($10' \times 4 = 40'$).

Passage 4.1 The definition says that f is continuous at a if $f(x)$ approaches $f(a)$ as x approaches a. Thus a continuous function f has the property that a small change in x produces only a small change in $f(x)$. In fact, the change in $f(x)$ can be kept as small as we please by keeping the change in x sufficiently small. If f is defined near a (in other words, f is defined on an open interval containing a, except perhaps at a), we say that f is discontinuous at a (or f has a discontinuity at a) if f is not continuous at a.

Passage 4.2 When you differentiate a function in which the independent variable shows up in several places, how do you carry out the derivative? For example, what is the derivative with respect to x of x^x? The answer is that you treat each instance of x one at a time, ignoring the others; differentiate with respect to that x and add the results. For a proof, use the definition of a derivative and differentiate the function $f(x, x)$. Start with the finite difference quotient:

$$\frac{f(x+\Delta x, x+\Delta x) - f(x, x)}{\Delta x} = \frac{f(x+\Delta x, x+\Delta x) - f(x, x+\Delta x)}{\Delta x} + \frac{f(x, x+\Delta x) - f(x, x)}{\Delta x}$$

The first quotient in the last equation is, in the limit that $\Delta x \to 0$, the derivative of f with respect to its first argument(自变量). The second quotient becomes the derivative with respect to the second argument.

Passage 4.3 In this chapter we discuss methods for solving the unconstrained optimization problem

$$\text{minimize} \quad f(x)$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is convex and twice continuously differentiable (which implies that the domain of f is open). We will assume that the problem is solvable, *i.e.*, there exists an optimal point $x^\star$. (More precisely, the assumptions later in the chapter will imply that $x^\star$ exists and is unique.) We denote the optimal value, $\inf_x f(x) = f(x^\star)$, as $p^\star$.

Passage 4.4 The nascent(新兴的,新生的) field of discrete differential geometry deals with discrete geometric objects (such as polygons) which act as analogues to continuous geometric objects (such as curves). The discrete objects can be measured (length, area) and can interact with other discrete objects (collision(冲突)/response). From a computational standpoint, the discrete objects are attractive(有吸引力的), because they have been designed from the ground up with data-structures and algorithms in mind. From a mathematical standpoint, they present a great challenge: the discrete objects should have properties which are analogues of the properties of continuous objects. One important property of curves and surfaces is their curvature, which plays a significant role in many application areas. In the continuous domain there are remarkable theorems dealing with curvature; a key requirement for a discrete curve with discrete curvature is that it satisfies analogous theorems. In this chapter we examine the curvature of continuous and discrete curves on the plane.

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《专业阅读》课程期末考试标准答案　A　2017. 6

I. Spell the English words according to the following Chinese meaning ($1' \times 12 = 12'$).

测地线	geodesic	子集	subset
非线性的	nonlinear	可微的	differentiable
挠率	torsion	半圆	semicircle
基	base	半径	radius
上界	upper bound	双曲线	hyperbola
无穷大的	infinity	斜率	slope

II. Give the Chinese meaning for each of the following English words ($1' \times 12 = 12'$).

plus	加	unbounded	无界的
lemma	引理	mapping	映射
isomorphic	同构	integer	整数
invariant	不变的	odevity	奇偶性
minimal	极小的, 最小的	rank	秩
inequality	不等式	exponent	指数

III. Translate the following passages into Chinese ($12' \times 3 = 36'$).

(评分标准: 在内容完整的前提下, 每个小错扣0.5分, 扣完为止)

Passage 3.1　我们经常会有些不严格的把n-元组$(x^1, \ldots, x^n)$ 本身称为空间中的点. 笛卡尔空间的最简单例子是实数轴. 这里每个点刚好有一个坐标x^1, 因此$n = 1$, 即, 它是1维的笛卡尔空间. 其他的例子, 有在解析几何中比较熟悉的, 平面(则它是一个2维的笛卡尔空间)和通常(即3维)空间的笛卡尔坐标系. 这些笛卡尔空间对于求解中学几何问题是完全足够的.

Passage 3.2　矩阵运算满足各种恒等式, 比如分配律

$$A(B + B') = AB + AB' \quad 和 \quad (A + A')B = AB + A'B$$

以及结合律

$$(AB)C = A(BC). \tag{2}$$

只要当所涉及的矩阵有合适的大小使得这些乘积有意义时, 这些定律就成立. 比如结合律, 对于某些l, m, n, p, 大小应该为$A = l \times m$, $B = m \times n$ 和$C = n \times p$. 因为两个乘积 (2)是相等的, 那里的括号就不必要了, 我们将把它们记作ABC. 则三元乘积ABC是一个$l \times p$的矩阵.

Passage 3.3　概率的相对频率的解释依赖于试验的性质, 而不是依赖于对于试验的个别观点, 因此它是客观的. 比如, 一个硬币投掷序列的两个不同的观察者与极限相对频率是无关的, 因此他们都应使用相同的概率指派. 在实际中, 这个解释不像它看上去的

那么客观, 这是因为事件的极限相对频率是未知的. 因此我们不得不基于我们对于所研究事件的极限相对频率的认识去指定概率. 幸运的是, 许多试验中都有针对概率指派的一致意见. 当我们说一个硬币是均匀的, 我们的意思是指$P(H) = P(T) = .5$; 当我们说一颗骰子是均匀的, 我们的意思是指六种结果的极限相对频率都是$\frac{1}{6}$, 使用概率指派$P(1) = \cdots = P(6) = \frac{1}{6}$.

IV. Read the following passages and translate them into Chinese ($10' \times 4 = 40'$)**.**

(评分标准: 在内容完整的前提下, 每个小错扣0.5分, 扣完为止)

Passage 4.1 这个定义表明如果当x趋近于a时, $f(x)$趋近于$f(a)$, 则f在点a连续. 因此连续函数f具有这样一个性质：x的微小改变量只会导致$f(x)$的微小改变量. 事实上, 只要让x的改变量充分地小, f的改变量可以任意地小.

如果f在a附近有定义(换句话说, f定义在一个含有a的开区间内, 可能除a点外), 当f在点a不连续时, 我们称f在点a间断(或f有一个间断点a).

Passage 4.2 当一个函数的自变量出现在多个位置时, 怎样求导? 比如, 求x^x 关于x的导数. 方法是每次处理一个x, 同时忽略其他位置的x; 对各个位置的x求导后, 然后将结果相加. 证明如下. 使用导数的定义对函数$f(x, x)$求导. 从有限差商:

$$\frac{f(x+\Delta x, x+\Delta x) - f(x, x)}{\Delta x} = \frac{f(x+\Delta x, x+\Delta x) - f(x, x+\Delta x)}{\Delta x} + \frac{f(x, x+\Delta x) - f(x, x)}{\Delta x}$$

开始. 在$\Delta x \to 0$时, 右端的第一个商式的极限为f关于第一个自变量的导数, 第二个商式的极限为关于第二个自变量的导数.

Passage 4.3 这一章我们讨论无约束的优化问题

$$\text{求 } f(x) \text{ 的最小值}$$

的求解方法, 其中$f : \mathbb{R}^n \to \mathbb{R}$ 是凸的, 并且二阶连续可微(这表明f 的定义域是开的). 我们假设这个问题是有解的, 即, 存在最优点$x^\star$. (更准确地说, 后面章节会说明在这些前提下能得到$x^\star$ 是存在的且唯一的.) 我们将这个最优值 $\inf_x f(x) = f(x^\star)$ 记为$p^\star$.

Passage 4.4 离散微分几何的新兴领域处理离散的几何对象(比如多边形), 这些对象的行为和连续的几何对象(比如曲线)类似. 这些对象能够被度量(长度,面积), 能与其他离散对象交互(冲突/反应). 从计算的观点看, 离散的对象是有吸引力的, 因为已经对它们从底层设计好了期望的数据结构和算法. 从数学观点看, 它们呈现了一个重大挑战: 离散对象应该具有与连续对象类似的性质. 曲线和曲面的一个重要特征是它们的曲率, 是许多应用领域中的重要角色. 在连续的领域里面, 有大量的处理曲率的定理. 一个重要要求是具有离散曲率的离散曲线要满足类似的定理. 这一章, 我们探讨平面中连续曲线和离散曲线的曲率.

华东理工大学 2016 - 2017 学年第二学期

《专业阅读》课程期末考试试卷　B　2017. 6

开课学院：理学院　专业：应用数学、信息与计算　考试形式：闭卷　所需时间：120分钟

考生姓名：__________　学号：__________　班级：__________　任课教师：杨勤民

题序	I	II	III	IV	总　分
得分					
评卷人	杨	勤	民		

I. Spell the English words according to the following Chinese meaning ($1' \times 12 = 12'$)**.**

数值分析	__________	极坐标	__________
多项式	__________	交错级数	__________
离散的	__________	球面	__________
行列式	__________	多边形	__________
矩形	__________	测度	__________
中点	__________	维数	__________

II. Give the Chinese meaning for each of the following English words ($1' \times 12 = 12'$)**.**

nonsingular	__________	composite	__________
negative	__________	supremum	__________
sparse	__________	period	__________
ordinate	__________	perpendicular	__________
hexagon	__________	trace	__________
transformation	__________	quadrant	__________

III. Translate the following passages into Chinese ($12' \times 3 = 36'$)**.**

Passage 3.1　Newton's second law of motion says that if a force $F(t)$ acts on the particle and is directed along its line of motion, then $ma(t) = F(t)$; that is, $F = ma$, where m is the mass of the particle. If the force F is known, then the equation $x''(t) = F(t)/m$ can be integrated twice to find position function $x(t)$ in terms of two constants of integration. These two arbitrary constants are frequently determined by the initial position $x_0 = x(0)$ and the initial velocity $v_0 = v(0)$ of the particle.

Passage 3.2 Let us consider the problem of evaluating the integral $\int_\Gamma f(z)\,\mathrm{d}\,z$, where Γ is a simple closed positively oriented contour and $f(z)$ is analytic on and inside Γ except for a single isolated singularity z_0 lying interior to Γ. As we know, the function $f(z)$ has a Laurent series expansion

$$f(z) = \sum_{j=-\infty}^{\infty} a_j(z - z_0)^j, \tag{3.2}$$

converging in some punctured circular neighborhood of z_0; in particular Eq. (3.2) is valid for all z on the small positively oriented circle C indicated in Fig. 1. By the methods of Sec. 4. 4, integration over Γ can be converted to integration over C without changing the integral $\int_\Gamma f(z)\,\mathrm{d}\,z = \int_C f(z)\,\mathrm{d}\,z$.

Passage 3.3 Suppose we have a change of coordinate systems where, as above, the old coordinates are expressed in terms of the new by $x^i = x^i(z)$, $i = 1, \ldots, n$, and let $x_0^i = x^i(z_0^1, \ldots, z_0^n)$, $i = 1, \ldots, n$, be the coordinates of some point with the property that $J = \det\left(\frac{\partial x}{\partial z}\right) \neq 0$ at $z^1 = z_0^1, \ldots, z^n = z_0^n$. Then for some sufficiently small neighborhood of (i.e. region about) the point $(x_0^1, \ldots, x_0^n)$ we shall have that: the coordinates $z^1, \ldots, z^n$ of points of that neighborhood are expressible in terms of $x^1, \ldots, x^n$, say $z^i = z^i(x)$, where, in particular, $z_0^i = z^i(x_0^1, \ldots, x_0^n)$, $i = 1, \ldots, n$; and at each point of the neighborhood the matrix $(b^i_j) = \left(\frac{\partial z^i}{\partial x^j}\right)$ (the Jacobian matrix of the inverse transformation) is the inverse of the matrix $(a^k_l) = \left(\frac{\partial x^k}{\partial z^l}\right)$; i.e. $\frac{\partial z^i}{\partial x^j}\frac{\partial x^j}{\partial z^k} = \delta^i_k$.

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IV. Read the following passages and translate them into Chinese ($10' \times 4 = 40'$)**.**

Passage 4.1 A mathematical optimization problem, or just optimization problem, has the form

$$\text{minimize } f_0(x), \qquad \text{subject to } f_i(x) \le b_i, \ i = 1, \dots, m. \tag{4.1}$$

Here the vector $x = (x_1, \dots, x_n)$ is the optimization variable of the problem, the function $f_0 : \mathbb{R}^n \to \mathbb{R}$ is the objective function, the functions $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, m$, are the (inequality) constraint functions, and the constants $b_1, \dots, b_m$ are the limits, or bounds, for the constraints. A vector $x^\star$ is called optimal, or a solution of the problem (4.1), if it has the smallest objective value among all vectors that satisfy the constraints: for any z with $f_1(z) \le b_1, \dots, f_m(z) \le b_m$, we have $f_0(z) \ge f_0(x^\star)$.

Passage 4.2 The concept of set is basic to all of mathematics and mathematical applications. A set is simply a collection of objects. The objects are sometimes referred to as elements or members. If a set is finite and not too large, we can describe it by listing the elements in it. For example, the equation $A = \{1, 2, 3, 4\}$ describes a set A made up of the four elements 1, 2, 3, and 4. A set is determined by its elements and not by any particular order in which the elements might be listed. Thus the set A might just as well be specified as $A = \{1, 3, 4, 2\}$. The elements making up a set are assumed to be distinct, and although for some reason we may have duplicates(重复) in our list, only one occurrence of each elements is in the set. For this reason we may also describe the set A as $A = \{1, 2, 2, 3, 4\}$.

Passage 4.3 Consider the following sequence: 1.79, 1.799, 1.7999, $\cdots$. We could observe that the numbers are approaching 1.8, the limit of the sequence.

Formally, suppose $x_1, x_2, \cdots$, is a sequence of real numbers. We say that the real number L is the limit of this sequence and we write $\lim_{n\to\infty} x_n = L$ to mean: For every real number $\varepsilon > 0$, there exists a natural number N such that for all $n > N$, $|x_n - L| < \varepsilon$.

Intuitively, this means that eventually all elements of the sequence get as close as we want to the limit, since the absolute value $|x_n - L|$ is the distance between x_n and L. Not every sequence has a limit; if it does, we call it convergent, otherwise divergent. One can show that a convergent sequence has only one limit.

Passage 4.4 Continuity is a fundamental concept in the theory of functions. Loosely speaking, a function is continuous if it has 'no jumps', but this naive definition is only adequate for simple situations. We begin by considering continuity at a specific point of the domain of a function. An informal — yet accurate — characterisation of continuity is to say that a function is continuous at a point if its value there can be inferred unequivocally(明确地, 不含糊的) from the values at neighbouring points. Thus neighbourhoods come into play.

For example, the value of the sign function at the origin cannot be inferred from the surrounding environment; even if we defined, say, sign(0) = 1, the ambiguity would remain. Things can get a lot worse. Consider the real function $x \mapsto \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Given that in any neighbourhood of any real number there are both rational and irrational numbers, there is no way of inferring the value of this function at any point by considering how the function behaves in the surrounding region.

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《专业阅读》课程期末考试标准答案　B　2017. 6

I. Spell the English words according to the following Chinese meaning ($1' \times 12 = 12'$).

数值分析	numerical analysis	极坐标	polar coordinates
多项式	polynomial	交错级数	alternate series
离散的	discrete	球面	sphere
行列式	determinant	多边形	polygon
矩形	rectangle	测度	measure
中点	midpoint	维数	dimension

II. Give the Chinese meaning for each of the following English words ($1' \times 12 = 12'$).

nonsingular	非奇异的	composite	复合的
negative	负的	supremum	上确界
sparse	稀疏的	period	周期
ordinate	纵坐标	perpendicular	垂直的
hexagon	六边形	trace	迹
transformation	变换	quadrant	象限

III. Translate the following passages into Chinese ($12' \times 3 = 36'$).

Passage 3.1 牛顿第二运动定律表明如果一个力$F(t)$作用在质点上, 方向指向质点运动的直线方向, 则有$ma(t) = F(t)$, 即$F = ma$, 其中m是质点的质量. 如果已知力F, 则能将式子$x''(t) = F(t)/m$积分两次得到用两个积分常数表示的位移函数$x(t)$. 这两个积分常数通常由质点的初始位置$x_0 = x(0)$和初始速度$v_0 = v(0)$确定.

Passage 3.2 考虑计算积分$\int_\Gamma f(z)\,\mathrm{d}z$, 的问题, 其中$\Gamma$是一个简单的封闭的正向的积分曲线, $f(z)$在Γ内部和上面除了位于Γ内部的一个单一孤立的奇点z_0之外是解析的. 如我们所知道的, 函数$f(z)$有一个在z_0的某去心圆形邻域内收敛的劳伦级数展开式

$$f(z) = \sum_{j=-\infty}^{\infty} a_j (z - z_0)^j; \tag{3.2}$$

特别是式子(3.2)对于图1中显示的正向小圆C上的所有z是成立的. 由4.4节中的方法, Γ上的积分能够转化为C上的积分, 而不会改变积分值: $\int_\Gamma f(z)\,\mathrm{d}z = \int_C f(z)\,\mathrm{d}z$.

Passage 3.3 假设有一个坐标变换, 其中, 与上面一样, 老坐标用新坐标表示为$x^i = x^i(z)$, $i = 1, \dots, n$. 令$x_0^i = x^i(z_0^1, \dots, z_0^n)$, $i = 1, \dots, n$, 是使得在$z^1 = z_0^1, \dots, z^n = z_0^n$处$J = \det\left(\frac{\partial x}{\partial z}\right) \neq 0$的某个点的坐标. 则对于点$(x_0^1, \dots, x_0^n)$的某个足够小的邻域（即周围区域）有：该邻域内的点的坐标$z^1, \dots, z^n$能用$x^1, \dots, x^n$表示, 比如$z^i = z^i(x)$, 其中, 特别是$z_0^i = z^i(x_0^1, \dots, x_0^n)$, $i = 1, \dots, n$. 并且在该邻域的每个点处, 矩阵$(b_j^i) = \left(\frac{\partial z^i}{\partial x^j}\right)$ (逆变换的雅可比矩阵) 是矩阵$(a_l^k) = \left(\frac{\partial x^k}{\partial z^l}\right)$的逆, 也就是$\frac{\partial z^i}{\partial x^j}\frac{\partial x^j}{\partial z^k} = \delta_k^i$.

IV. Read the following passages and translate them into Chinese ($10' \times 4 = 40'$).

Passage 4.1 一个数学优化问题或者就是优化问题具有如下形式：

$$\text{求 } f_0(x) \text{在条件 } f_i(x) \leq b_i,\ i = 1, \ldots, m \text{ 的限制的最小值} \tag{4.1}$$

这里，向量$x = (x_1, \ldots, x_n)$是问题的优化变量，函数$f_0 : \mathbb{R}^n \to \mathbb{R}$ 是目标函数，函数$f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$是(不等式)约束函数，常数$b_1, \ldots, b_m$是约束的临界值或边界值. 如果一个向量$x^\star$在所有满足约束条件的向量中具有最小的目标值，即对满足$f_1(z) \leq b_1, \ldots, f_m(z) \leq b_m$的任意的$z$有$f_0(z) \geq f_0(x^\star)$，则称该向量为是最优的，或者称为问题(4.1)的解.

Passage 4.2 集合的概念是所有数学和数学应用的基础. 一个集合只不过是对象的聚集. 对象有时称为元素或成员. 如果一个集合是有限的，且不是太大，我们可通过将它里面的元素列举出来来表示这个集合. 比如，式子$A = \{1, 2, 3, 4\}$描述了由四个元素1, 2, 3, 和4组成的一个集合A. 集合由它的元素确定，而不是列举元素时采用的任何特定的顺序. 因此集合A也能表示为$A = \{1, 3, 4, 2\}$. 组成集合的元素是互不相同的，尽管因为某种原因，在列举元素时会有重复，但是在集合中的每个元素只有一个是有效的. 由此，也能将集合A表示为$A = \{1, 2, 2, 3, 4\}$.

Passage 4.3 考虑如下序列：1.79, 1.799, 1.7999, $\cdots$. 我们能够看出这些数趋近于这个序列的极限1.8.

正式地，设$x_1, x_2, \cdots$, 是一个实数序列. 我们说实数L是这个序列的极限，并写为$\lim_{n\to\infty} x_n = L$，表示的是：对于每个实数$\varepsilon > 0$，都存在一个自然数$N$使得对于所有$n > N$，有$|x_n - L| < \varepsilon$.

直观上，它表示序列中的所有元素最终都任意接近这个极限，这是因为绝对值$|x_n - L|$表示x_n 与L之间的距离. 不是所有序列都有极限；如果一个序列有极限，我们称它收敛，否则称它发散. 能够证明一个收敛的序列只能有一个极限.

Passage 4.4 连续是函数论中的基本概念. 宽泛地说，如果一个函数‘没有跳跃’，则它就是连续的. 但这个粗略的定义只适用于简单情形. 我们开始考虑函数在定义域中的一个指定点处的连续性. 连续的一个非正式（但却是精确）的描述是：如果函数的值能由邻近点处的函数值明确地推断出来，则函数是连续的. 因此邻域起到了作用.

例如，符号函数在原点处的值不能由周围点处的函数值推断出来，即使定义了，比如，$\mathrm{sign}(0) = 1$，仍然存在不确定性. 还有更糟的情形. 考虑实函数$x \mapsto \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. 由于任意实数的任意邻域内都含有有理数和无理数，因而无法通过函数在某点的邻近点处的函数值去推断该点处的函数值.