1. 厄密算符的本征值为实数.

$F\psi = \lambda\psi$ $\Rightarrow \int \psi^* F\psi = \int \lambda \psi^*\psi \longrightarrow \lambda|\psi|^2$

$F^*\psi^* = \lambda^*\psi^*$ $\Rightarrow \int \psi F^*\psi^* = \int \psi \lambda^*\psi \longrightarrow \lambda^*|\psi|^2$

若 F 是厄密的.

$$\int \psi^* F\psi = \int \psi F^*\psi^*$$

推得. $\lambda = \lambda^*$. λ 是实数.

三要素：力学量、状态、观测值.
operator. ψ λ.

$\hat{O}\psi = \varphi$.
状态.

厄密算符的本征函数系，正交、归一、完备性.

正交.

$$\begin{cases} F\psi_n = \lambda_n\psi_n & \{\psi_n\} \text{ 本征函数系} \\ F\psi_m = \lambda_m\psi_m & \{\lambda_n\} \text{ 本征值 谱.} \end{cases}$$

$$F^*\psi_n^* = \lambda_n\psi_n^*$$

$$(F\psi_m, \psi_n) = \int \psi_m F^*\psi_n^* dx = \lambda_n \int \psi_n^*\psi_m dx = \lambda_n(\psi_m, \psi_n)$$

$$(\psi_n, F\psi_m) = \int \psi_n^* F\psi_m dx = \lambda_m \int \psi_m\psi_n^* dx = \lambda_n(\psi_n, \psi_m)$$

$\Rightarrow \lambda_n(\psi_m, \psi_n) = (\psi_n, \psi_m)\lambda_m$. $\Rightarrow (\psi_n, \psi_m) = 0$. 内积为0. 正交.

归一

又有 $\int |\psi|^2 dx = 1$ $\Rightarrow (\psi_n, \psi_n) = 1$

故 $(\psi_n, \psi_m) = \delta_{mn}$ $\Rightarrow$ 正交归一.

完备. eg: $\vec{i} = (1,0)$ $\vec{j} = (0,1)$. 任意二维向量 $\vec{A}$ 可用 $\vec{i}$ $\vec{j}$ 表示. $\vec{A} = x\vec{i} + y\vec{j}$

对某一体系的任何可能的态 ψ. 可表示为：

$$\psi = \sum_{i=1}^{n} C_i\psi_i$$

想证明. 只要求系数 C_n

$$(\psi_m, \psi) = (\psi_m, \Sigma C_i\psi_i)$$

$$\Rightarrow (\psi_m, \psi) = C_m.$$

力学量取各个可能值的几率.

$\begin{cases} F\psi_1 = \lambda_1\psi_1 \\ F\psi_2 = \lambda_2\psi_2 \end{cases}$

非简并 1个ψ对应λ.

简并. 多个ψ对应1个λ.

$\psi = C_1\psi_1 + C_2\psi_2$

$F\psi = C_1F\psi_1 + C_2F\psi_2$.

$= C_1\lambda_1\psi_1 + C_2\lambda_1\psi_2$ 对于简并情况可写成 $\lambda(C_1\psi_1 + C_2\psi_2)$.

非简并态, 不可以.

$\psi = \sum C_n\psi_n$ 下的 $\overline{F}$, $\overline{F} = \int \psi^* F \psi \, d\vec{x}$ $= \int \sum \psi_n^* C_n^* \, F \sum \psi_n C_n \, d\vec{x}$

$= \sum_n \int |C_n|^2 \, \psi_n^* \lambda_n \psi_n \, d\vec{x}$ ($F\psi_n = \lambda_n\psi_n$)

$= \sum |C_n|^2 \lambda_n$

$|C_n|^2$: P; λ_n: 测量值.

又由 $\int \psi^*\psi dx = 1$, $\sum |C_n|^2 = 1$