第三章、效用函数

- 效用（Utility)：人们从消费某种商品或服务中得到的满足程度。

- 效用是主观的。

- 基数效用和序数效用
- 边际效用和边际替代率
- 预算约束下的效用最大化

第一节、基数效用和序数效用

<u>基数效用</u>（cardinal utility）：是指用1，1.5，2，2.2，3等实数基数词来表示效用的大小。

<u>序数效用</u>（ordinal utility）：是指用第 1 ，第 2 ，第 3 等序数词来表示效用的大小。没有精确的大小，只有顺序的比较。

基数效用论

- 效用是可精确度量的；
- 效用是可比较的，可加减的；
- 总效用（total utility），是指消费者在一定时间内消费一定数量的商品或劳务所获得的总的满足程度。用TU表示。

序数效用论

- 效用值的大小次序表示满意程度的高低，而效用值的大小本身没有任何意义；

- 序数效用不能表示两消费组合之间相差的程度；

- 序数效用不能也不要求比较人与人之间的满意程度。

- *E.g.* if U(x) = 6 and U(y) = 2 then bundle x is strictly preferred to bundle y. But x is not preferred three times as much as is y.

- Consider the bundles (4,1), (2,3) and (2,2).
- Suppose $(2,3) \succ (4,1) \sim (2,2)$.
- Assign to these bundles any numbers that preserve the preference ordering; *e.g.* $U(2,3) = 6 > U(4,1) = U(2,2) = 4$.
- Call these numbers utility levels.

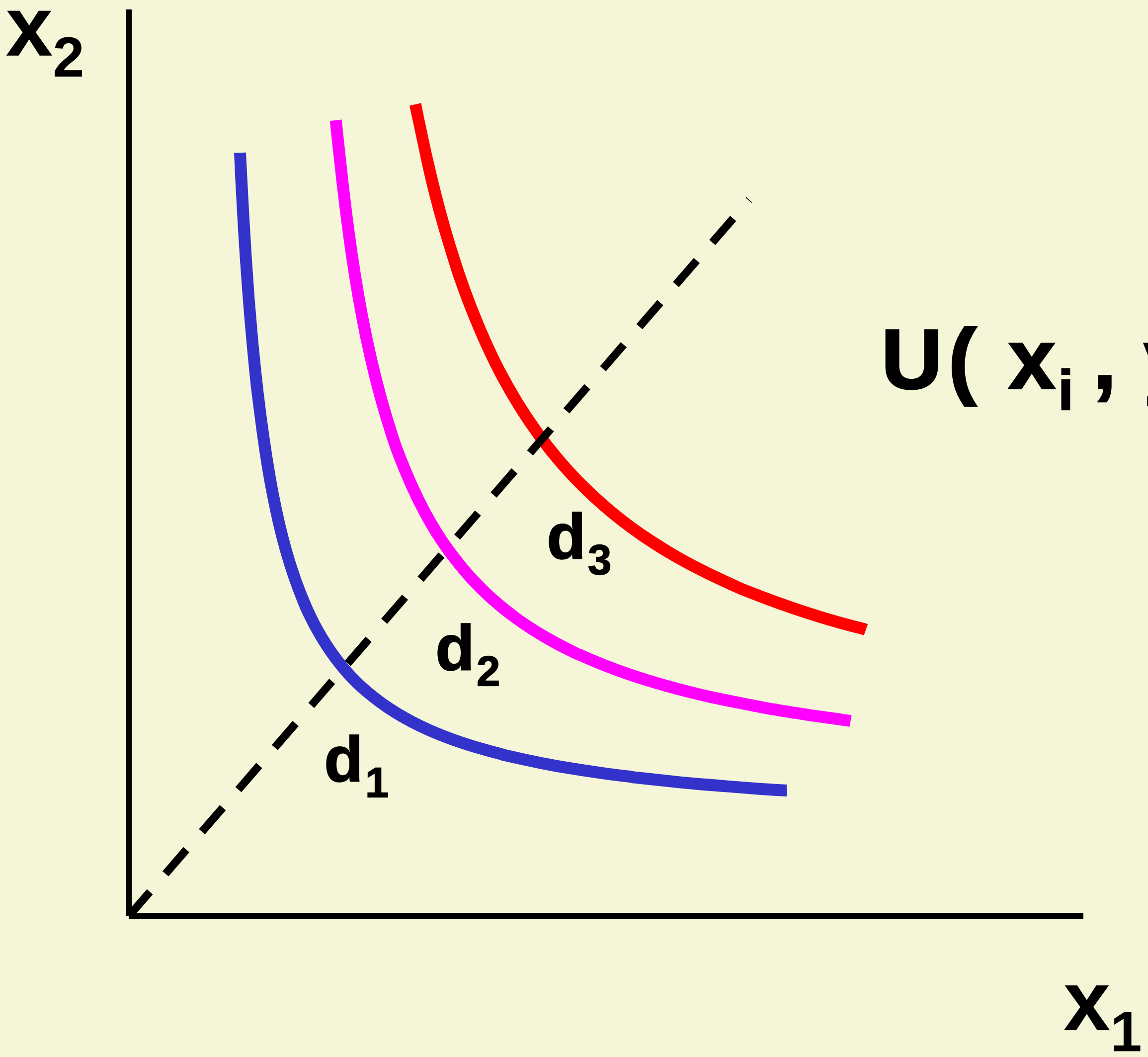
- 序数效用和偏好映照是等同的概念，它们是对同一理性偏好的不同表达形式。

- 要证明序数效用函数与偏好映照是描绘同一偏好的不同表达形式，需证明

$$(x,y) \succ (x',y') \iff U(x,y) > U(x',y')$$

$$(x,y) \sim (x',y') \iff U(x,y) = U(x',y')$$

i.由偏好映照构造效用函数



$$U(x_i, y_i) = d_i$$

- $U(x_1,x_2) = x_1x_2$, so

  $U(2,3) = 6 > U(4,1) = U(2,2) = 4;$

  that is, $(2,3) \succ (4,1) \sim (2,2)$.

- Define $V = U^2$.
- Then $V(x_1,x_2) = x_1^2 x_2^2$ and $V(2,3) = 36 > V(4,1) = V(2,2) = 16$ so again $(2,3) \succ (4,1) \sim (2,2)$.
- V preserves the same order as U and so represents the same preferences.

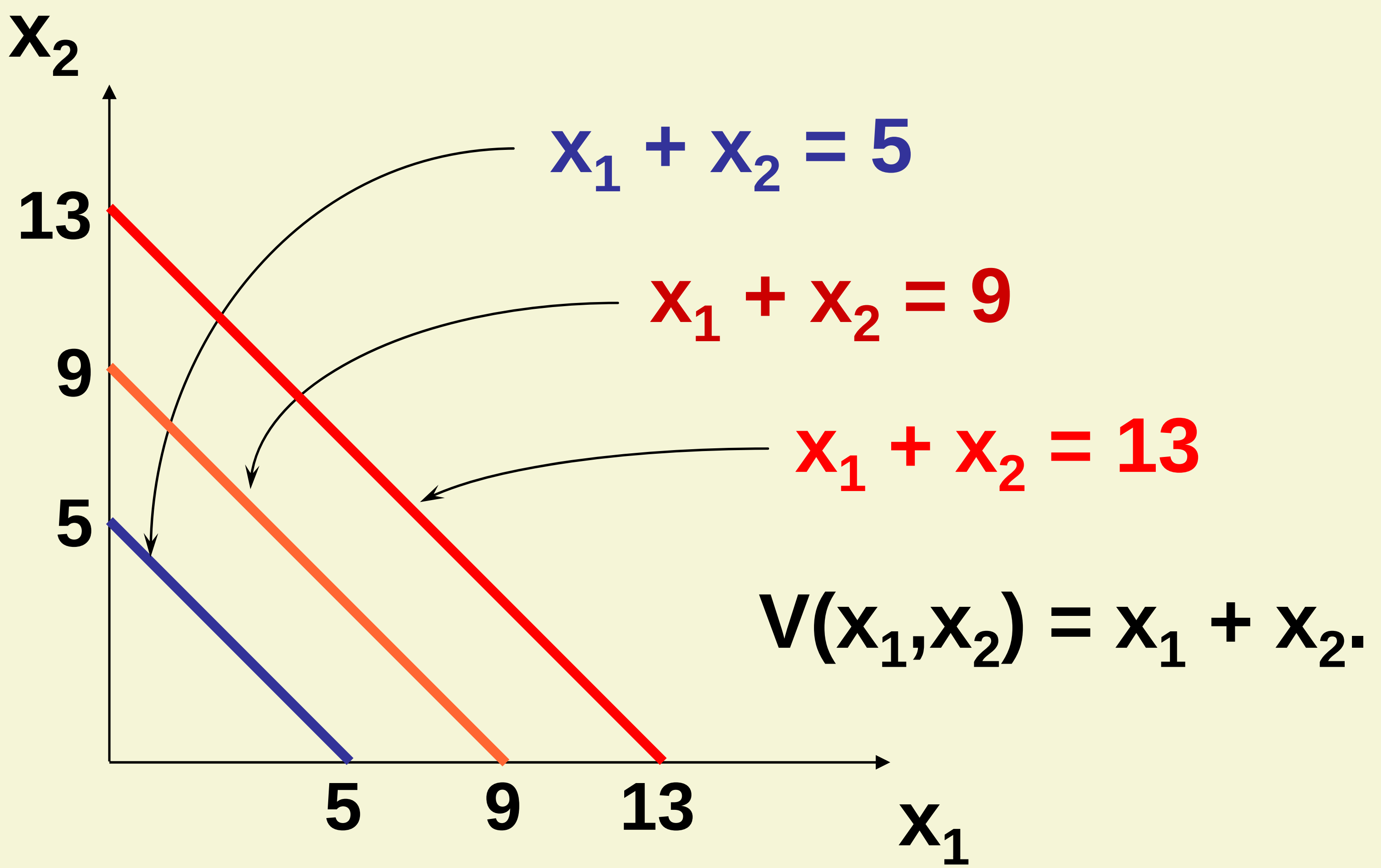
- Define W = 2U + 10.
- Then $W(x_1,x_2) = 2x_1x_2+10$ so $W(2,3) = 22 > W(4,1) = W(2,2) = 18$. Again, $(2,3) \succ (4,1) \sim (2,2)$.
- W preserves the same order as U and V and so represents the same preferences.

- 效用函数不是唯一的
- If
    - U is a utility function that represents a preference relation,
    - f is a strictly increasing function,
- then $V = f(U)$ is also a utility function.

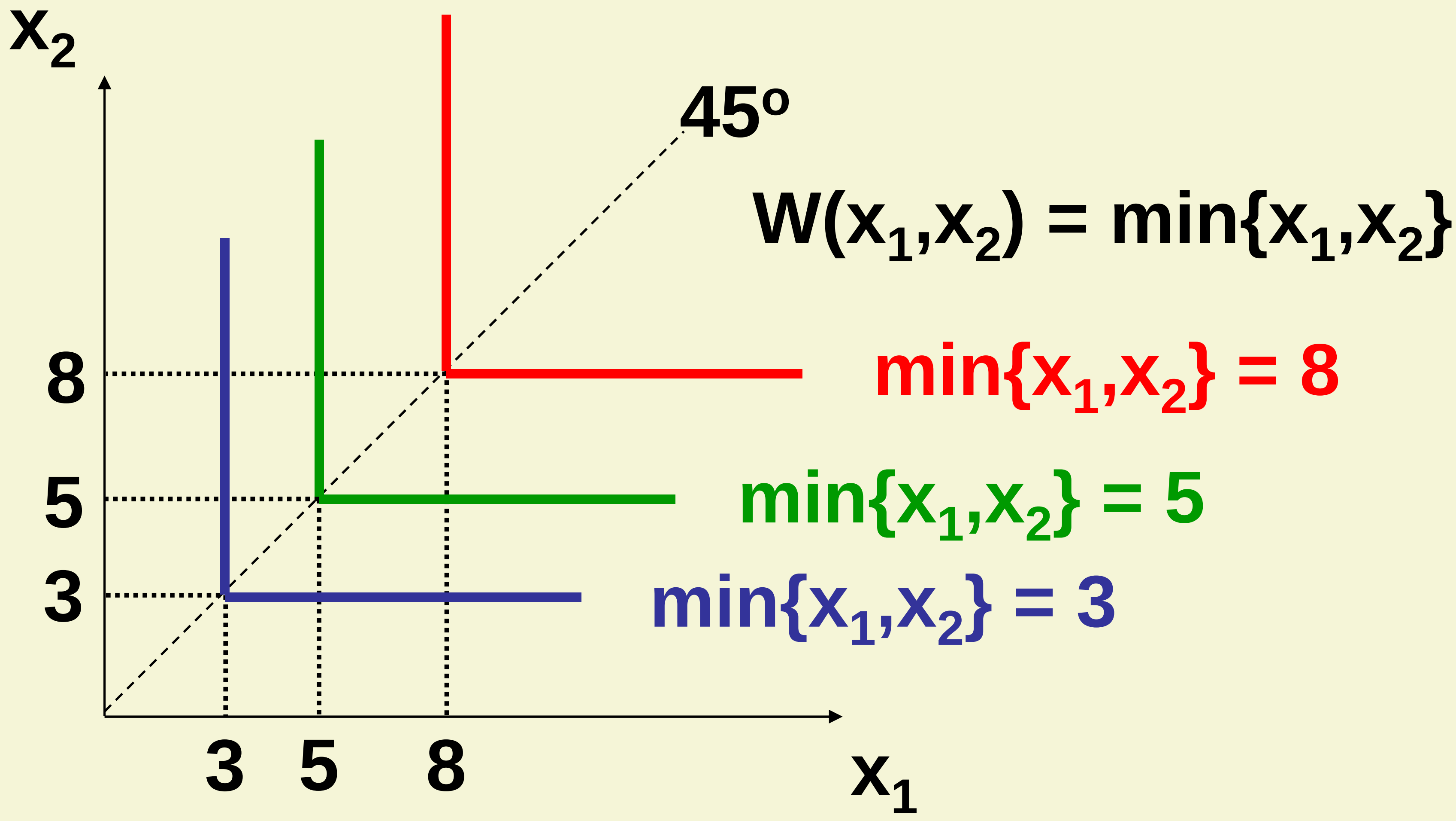
ii.由效用函数构造偏好映照

- Perfect substitute
  - $V(x_1,x_2) = x_1 + x_2.$
- Perfect complement
  - $W(x_1,x_2) = \min\{x_1,x_2\}$
- Cobb-Douglas Utility Function
  - $U(x_1,x_2) = x_1^a x_2^b$

Perfect Substitution Indifference Curves



Perfect Complementarity Indifference Curves



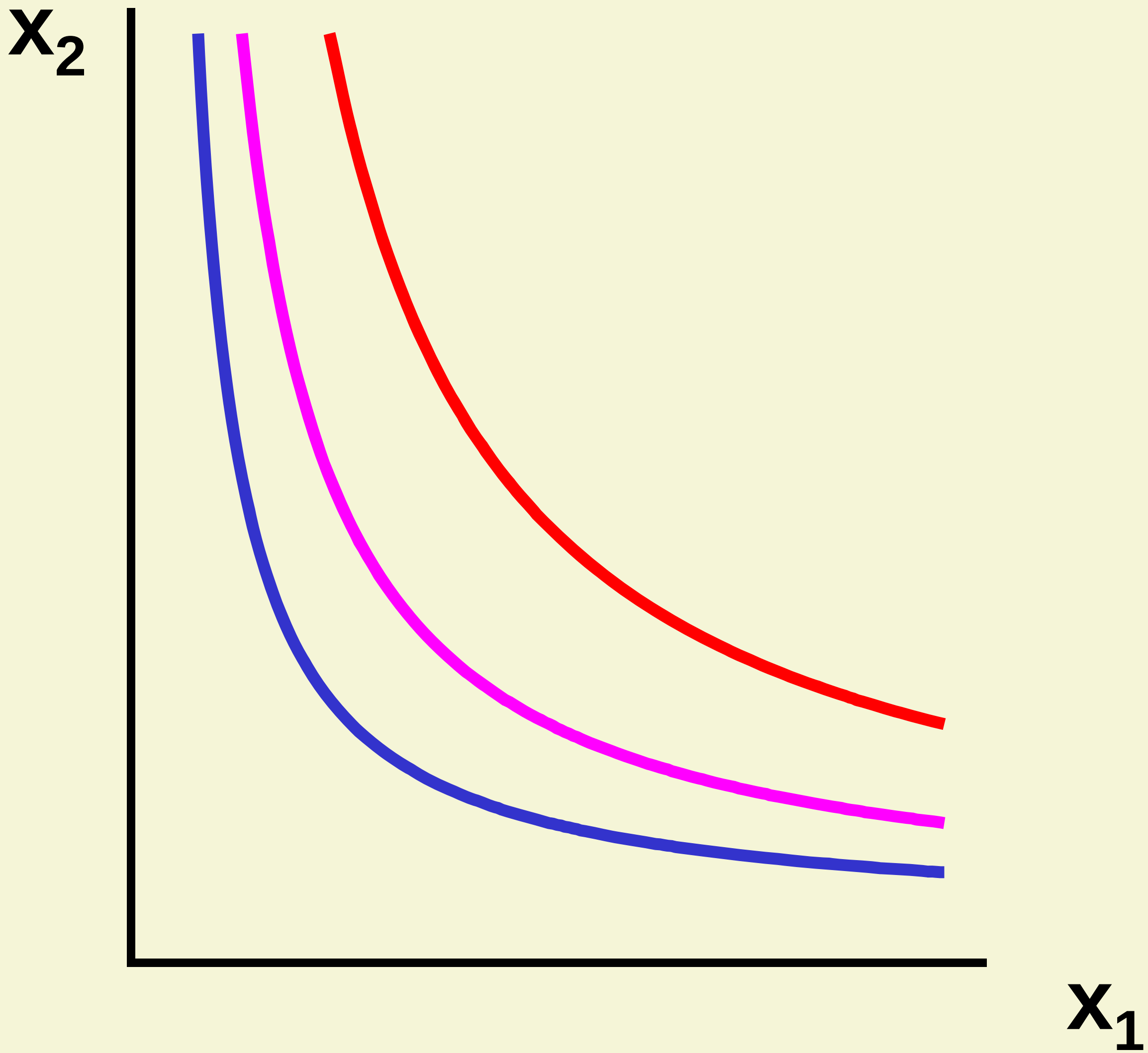
Cobb-Douglas Utility Function

- Any utility function of the form

$$U(x_1,x_2) = x_1^{a} x_2^{b}$$

  with a > 0 and b > 0 is called a Cobb-Douglas utility function.

- *E.g.* $U(x_1,x_2) = x_1^{1/2} x_2^{1/2}$ $(a = b = 1/2)$
  $V(x_1,x_2) = x_1 x_2^{3}$ $(a = 1, b = 3)$


x2
x1

第二节、边际效用和边际替代率

1、边际效用

- 商品的边际效用（Marginal Utilities）：某商品的消费量变化一单位，而其它商品的消费量保持不变，所引起的效用变化。
- $$\Delta U = U(x+\Delta x, y) - U(x, y)$$

$$MU_x = \frac{\Delta U}{\Delta x} = \frac{\partial U}{\partial x}$$

- If $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

$$MU_2 = \frac{\partial U}{\partial x_2} = 2 x_1^{1/2} x_2$$

2、边际效用和边际替代率

- The general equation for an indifference curve is
  $U(x,y) \equiv k$, a constant.

- Totally differentiating this identity gives

$$\frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy = 0$$

rearranged is

$$-\frac{dy}{dx} = \frac{\partial U / \partial x}{\partial U / \partial y}$$

$$MRS = \left| -\frac{dy}{dx} \right| = \frac{MU_x}{MU_y}$$

- 两商品之间的边际替代率等于它们的边际效用之比。

3、边际效用和消费者选择

$$MRS = \frac{P_x}{P_y} = \frac{MU_x}{MU_y}$$

rearranged is

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} = \cdots\cdots = \frac{MU_z}{P_z}$$

- 在最优消费条件下，商品的边际效用与其价格之比都互相相等。即：消费者花在各商品和服务上的最后一元钱所得到的边际效用全部相等。

第三节、预算约束下的效用最大化

$$\max_{x_1, x_2, \cdots, x_n} U(x_1, x_2, \cdots, x_n)$$

$$s.t. \quad P_1 x_1 + P_2 x_2 + \cdots + P_n x_n \leq I$$

$$x_1 \geq 0, x_2 \geq 0, \cdots, x_n \geq 0$$

- 假设商品越多越好，则用完所有收入

$$P_1 x_1 + P_2 x_2 + \cdots + P_n x_n = I$$

$$x_1 = \frac{I - (P_2 x_2 + \cdots + P_n x_n)}{p_1}$$

$$U\left(\frac{I - (P_2 x_2 + \cdots + P_n x_n)}{P_1}, x_2, \cdots, x_n\right)$$

- 由必要条件

$$\frac{\partial U}{\partial x_i} = -U_1 \frac{P_i}{P_1} + U_i = 0 \quad i = 2,3,\cdots,n$$

- 整理得

$$\frac{U_1}{P_1} = \frac{U_2}{P_2} = \cdots\cdots = \frac{U_n}{P_n}$$

- More generally, if V = F(U) where f is a strictly increasing function, then

$$\frac{V_1}{P_1} = \frac{V_2}{P_2} = \cdots\cdots = \frac{V_n}{P_n}$$

- 由偏导数

$$V_i = F'U_i$$

- 得到

$$\frac{U_1}{P_1} = \frac{U_2}{P_2} = \cdots\cdots = \frac{U_n}{P_n}$$

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