方向导数. $\frac{\partial\varphi(\vec{x})}{\partial \vec{l}}\Big|_{\vec{x}=\vec{x}_0}$

梯度. 沿某一确定方向. $\varphi(\vec{x})$ 方向导数最大. $\nabla=(\frac{\partial}{\partial x}\vec{i}, \frac{\partial}{\partial y}\vec{j}, \frac{\partial}{\partial z}\vec{k})$ $\nabla\varphi = \frac{\partial}{\partial x}\varphi\vec{i} + \frac{\partial}{\partial y}\varphi\vec{j} + \frac{\partial}{\partial z}\varphi\vec{k}$

$\frac{\partial\varphi}{\partial l} = \frac{\partial\varphi}{\partial n}\cos\theta$ 方向导数是梯度的投影

通量

$\Phi = \int_S \vec{A}\, d\vec{S}$

$\Phi = 0$, 无源,

$\Phi < 0$, 负源.

$\Phi > 0$ 正源.

散度.

$\frac{\Phi}{\Delta V} \Rightarrow \vec{A}$ 在 ΔS 围成区域 单位体积.平均通量.

$\lim\limits_{\Delta V\to 0} \frac{\oint \vec{A}\, d\vec{S}}{\Delta V} = div\,\vec{A}.$

$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ (标量)

若 $\nabla\cdot\vec{A} = 0$, 无源.

>0. 正源

<0, 负源.

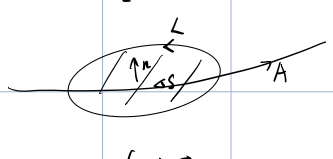
$\nabla\vec{A} = \rho$. 称 源密度

高斯公式.

$\Phi = \oint_S \vec{A}\cdot d\vec{S} = \oint_V \nabla\cdot\vec{A}\cdot dV$

环量

$\Gamma = \oint_L \vec{A}\cdot d\vec{l}$



$\lim\limits_{\Delta S\to 0} \frac{\oint_L \vec{A}\, d\vec{l}}{\Delta S} = \frac{d\Gamma}{dS}.$ 环量面密度.

路径.不同. 环量面密度不同

环量面密度 最大值为 旋度.

$rot\,\vec{A} = \nabla\times\vec{A}$

$rot\,\vec{A} = \vec{J} \neq 0$. A为旋度场. J为旋度源.

$rot\,\vec{A} \equiv 0$, 无旋场.

stokes 公式

$$\oint_L \vec{A}\cdot d\vec{l} = \oint_S \nabla\times\vec{A}\, d\vec{S}$$

∇在不同坐标系的表示.

$$\nabla = \vec{e}_x\frac{\partial}{\partial x} + \vec{e}_y\frac{\partial}{\partial y} + \vec{e}_z\frac{\partial}{\partial z}$$

$$= \vec{e}_r\frac{\partial}{\partial r} + \vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \vec{e}_z\frac{\partial}{\partial z}.$$

$$= \vec{e}_r\frac{\partial}{\partial r} + \vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \vec{e}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$$

$$\nabla\cdot A(r,\theta,\varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\nabla\times A(r,\theta,\varphi) = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(r\cdot A_\varphi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\cdot A_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\varphi}$$

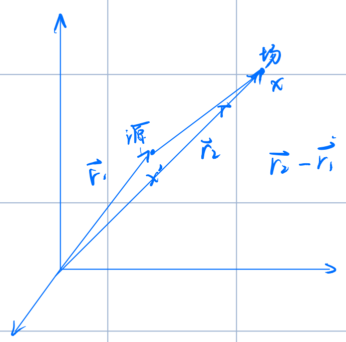
$\nabla\times(\nabla\varphi) = 0$ 梯度是无旋的

$$\nabla\cdot(\nabla\varphi) = \frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi = \nabla^2\varphi$$

$\nabla\cdot(\nabla\times\vec{A}) = 0$ 旋度是无散的

无散 ⟶ 矢量场的旋度

无旋 ⟶ 标量场的梯度.



$$\nabla\cdot\left(\frac{\vec{r}}{r^3}\right) \qquad \nabla\cdot\left(\nabla\frac{1}{r}\right)$$

$$\frac{\partial}{\partial x}\cdot\frac{x-x'}{(\cdots)^{\frac{3}{2}}}$$

$$= \frac{(\ )^{\frac{3}{2}} - (x-x')^2\frac{3}{2}r\cdot\cancel{2}(x-x')}{(\ )^3}$$

$$(a\vec{i}+b\vec{j}+c\vec{k})\cdot\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\vec{r}$$

$$\left(a\frac{\partial}{\partial x} + b\frac{\partial}{\partial y} + c\frac{\partial}{\partial z}\right)\vec{r}$$

$$\vec{a} + \vec{b} + \vec{c}.$$

$\vec{E}\times\vec{k}$

$$\begin{vmatrix} E_1 & E_2 & E_3 \\ k_1 & k_2 & k_3 \end{vmatrix}$$