机械波的能量 $\Delta W_{\mathrm{k}} = \Delta W_{\mathrm{p}} = \frac{1}{2}\rho \Delta V A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$

$$\Delta W = \rho \Delta V A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$$

能量密度

$$w = \rho A^2 \omega^2 \sin^2 \omega (t - \frac{x}{u})$$

平均能量密度 $\overline{w} = \frac{1}{2}\rho \omega^2 A^2$

平均能流： $\boxed{\overline{P} = \overline{w} u S}$

能流密度 $\boxed{I = \frac{1}{2}\rho A^2 \omega^2 u}$

波的干涉

波源振动 $\begin{cases} y_1 = A_1 \cos(\omega t + \varphi_1) \\ y_2 = A_2 \cos(\omega t + \varphi_2) \end{cases}$

点P 的两个分振动 $\begin{cases} y_{1p} = A_1 \cos(\omega t + \varphi_1 - 2\pi \frac{r_1}{\lambda}) \\ y_{2p} = A_2 \cos(\omega t + \varphi_2 - 2\pi \frac{r_2}{\lambda}) \end{cases}$

$$y_p = y_{1p} + y_{2p} = A\cos(\omega t + \phi)$$

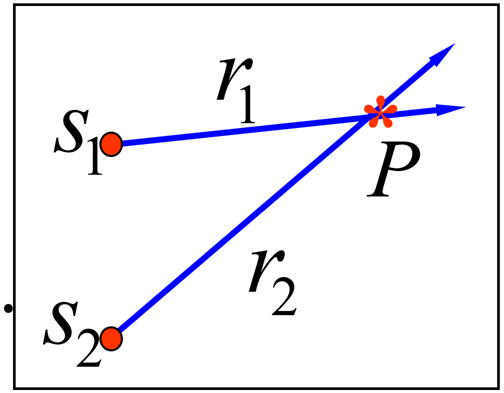
$$\tan\phi = \frac{A_1 \sin(\varphi_1 - \frac{2\pi r_1}{\lambda}) + A_2 \sin(\varphi_2 - \frac{2\pi r_2}{\lambda})}{A_1 \cos(\varphi_1 - \frac{2\pi r_1}{\lambda}) + A_2 \cos(\varphi_2 - \frac{2\pi r_2}{\lambda})}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\Delta\phi}$$

$$\Delta\phi = \varphi_2 - \varphi_1 - 2\pi\frac{r_2 - r_1}{\lambda}$$

常量

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\phi}$$



$$\Delta\phi=\varphi_2-\varphi_1-2\pi\frac{r_2-r_1}{\lambda}$$

常量

1）

$$\Delta\phi=\pm 2k\pi \quad k=0,1,2,\cdots$$

$$A=A_1+A_2$$ 振动始终加强

$$\Delta\phi=\pm(2k+1)\pi \quad k=0,1,2,\cdots$$

$$A=\left|A_1-A_2\right|$$ 振动始终减弱

$\Delta\phi=$ 其他 $\left|A_1-A_2\right|<A<A_1+A_2$

若 $\varphi_1=\varphi_2$ 则 $\Delta\phi=-2\pi\dfrac{\delta}{\lambda}$

波程差 $\delta=r_2-r_1$

2）

$$\delta=\pm k\lambda \quad k=0,1,2,\cdots$$

$$A=A_1+A_2$$ 振动始终加强

$$\delta=\pm(2k+1)\lambda/2 \quad k=0,1,2,\cdots$$

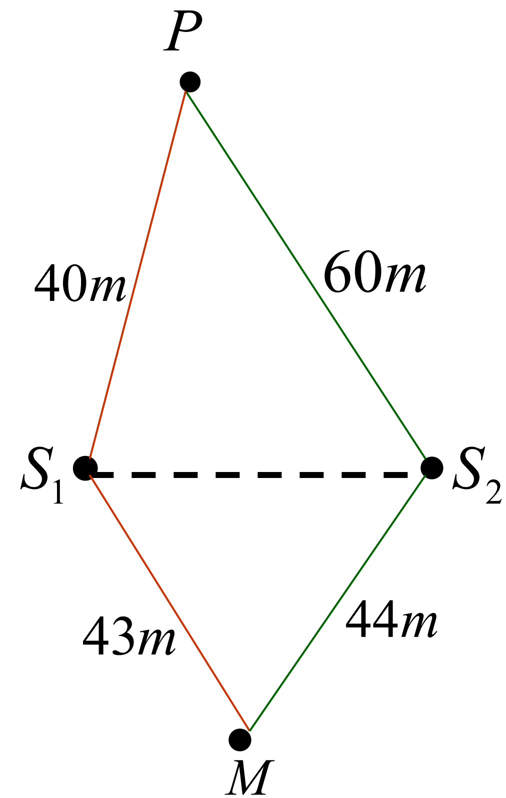
$$A=\left|A_1-A_2\right|$$ 振动始终减弱

$\delta=$ 其他 $\left|A_1-A_2\right|<A<A_1+A_2$

[例] 如图所示，两相干波源S_1、S_2相距$30m$，

$\nu_1 = \nu_2 = 100HZ$，$A_1 = A_2 = 1cm$，$u = 400\, m/s$，

$\varphi_1 = 0$，$\varphi_2 = \pi$



求（1）P点及M点的振动方程。

（2）S_1S_2连线上静止点的坐标。

解（1）$y_P = A_P \cos(\omega t + \phi_p)$

$$y_M = A_M \cos(\omega t + \phi_M)$$

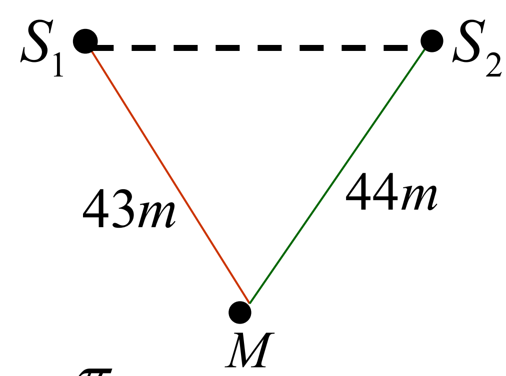
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi}$$

$$\left.\begin{array}{l} \Delta\phi = \varphi_2 - \varphi_1 - 2\pi(r_2 - r_1)/\lambda \\ \lambda = uT = \dfrac{u}{\nu} = \dfrac{400}{100} = 4m \end{array}\right\} \begin{array}{l} \Delta\phi_p = -9\pi \\ \Delta\phi_M = \dfrac{\pi}{2} \end{array}$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\phi}$$

$$\because \Delta\phi_p=-9\pi \qquad \therefore A_P=\left|A_1-A_2\right|=0 \Longrightarrow y_P=0$$

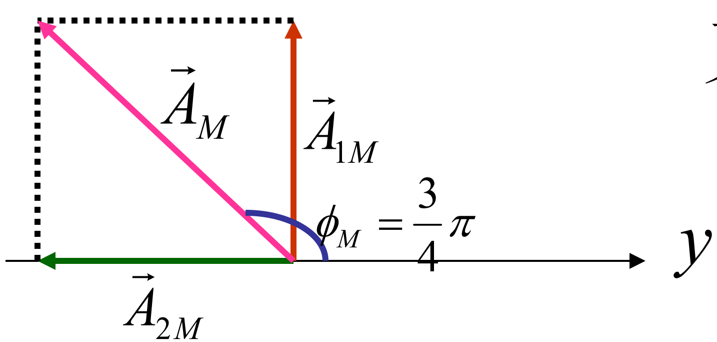
$$\because \Delta\phi_M=\frac{\pi}{2} \qquad \therefore A_M=\sqrt{2}A_1=\sqrt{2}cm$$



$$y_{\mathrm{M}}=A_{\mathrm{M}}\cos(\omega t+\phi_{\mathrm{M}}) \qquad \phi_M=?$$

S_1在M点振动初相：$\phi_1=\varphi_1-\dfrac{2\pi}{\lambda}r_1=-\dfrac{43}{2}\pi$（或$\dfrac{\pi}{2}$）

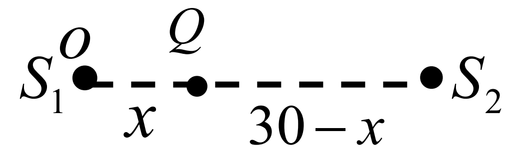
S_2在M点振动初相：$\phi_2=\varphi_2-\dfrac{2\pi}{\lambda}r_2=-21\pi$（或$\pi$）



又：$\omega=2\pi\nu=200\pi$

$$\therefore y_M=\sqrt{2}\cos(200\pi t+\frac{3}{4}\pi)cm$$

（2）S_1S_2连线上静止点的坐标。



解：令S_1为坐标原点，静止点Q离S_1为x

$$\left.\begin{array}{l} \Delta\phi_Q = \varphi_2 - \varphi_1 - 2\pi(\mathrm{r}_2 - \mathrm{r}_1)/\lambda \\ \varphi_1 = 0,\quad \varphi_2 = \pi \qquad \lambda = 4m \\ r_1 = x \qquad r_2 = 30 - x \end{array}\right\} \quad \begin{aligned} \Delta\phi_Q &= \pi - \frac{2\pi}{4}\left[(30-x)-x\right] \\ &= \pm(2k+1)\pi \end{aligned}$$

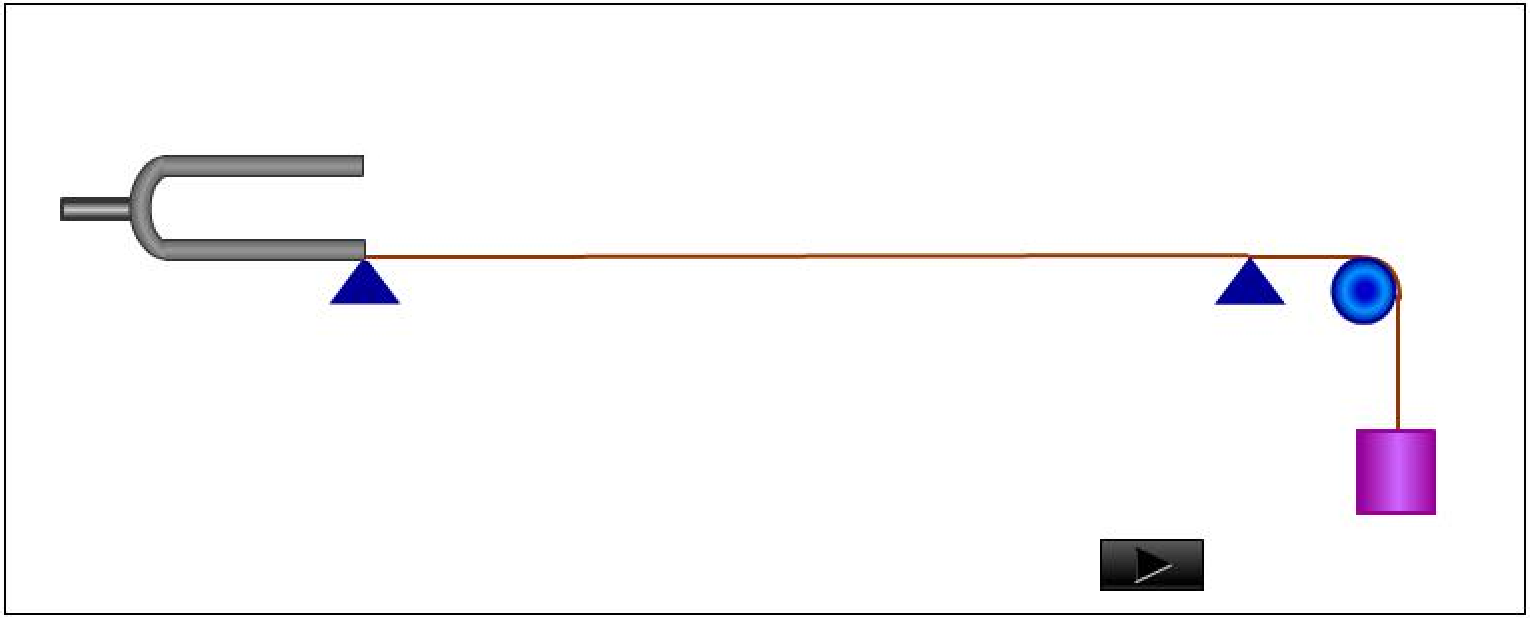
(合振动减弱条件)

得：$x = 15 \pm 2k$ （$k = 0,1,2,\cdots 7$）

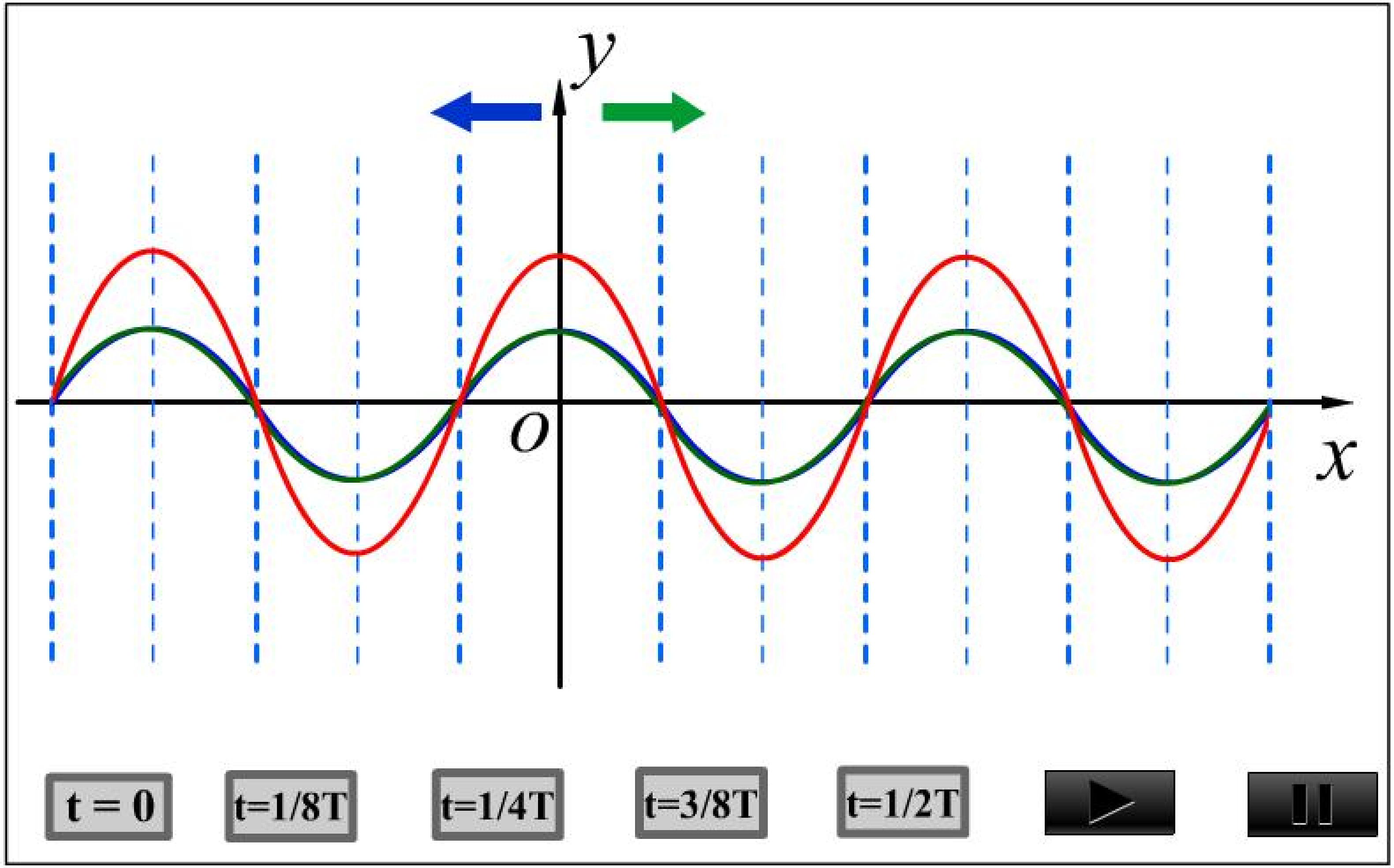
5.7 驻波

一 驻波的产生

振幅、频率、传播速度都相同的两列相干波，在同一直线上沿相反方向传播时叠加而形成的一种特殊的干涉现象.



驻 波 的 形 成



二、驻波表达式

$$y_1 = A\cos 2\pi(\frac{t}{T} - \frac{x}{\lambda}) \ , \quad y_2 = A\cos 2\pi(\frac{t}{T} + \frac{x}{\lambda})$$

$$y = y_1 + y_2 = (2A\cos\frac{2\pi}{\lambda}x)\cdot\cos\frac{2\pi}{T}t$$

合振幅 A' 随**x** 作周期性变化 $\quad A' = \left|2A\cos\frac{2\pi}{\lambda}x\right|$

1. $\frac{2\pi}{\lambda}x = \pm k\pi \quad k = 0, 1, 2, \cdots \quad A' = 2A$

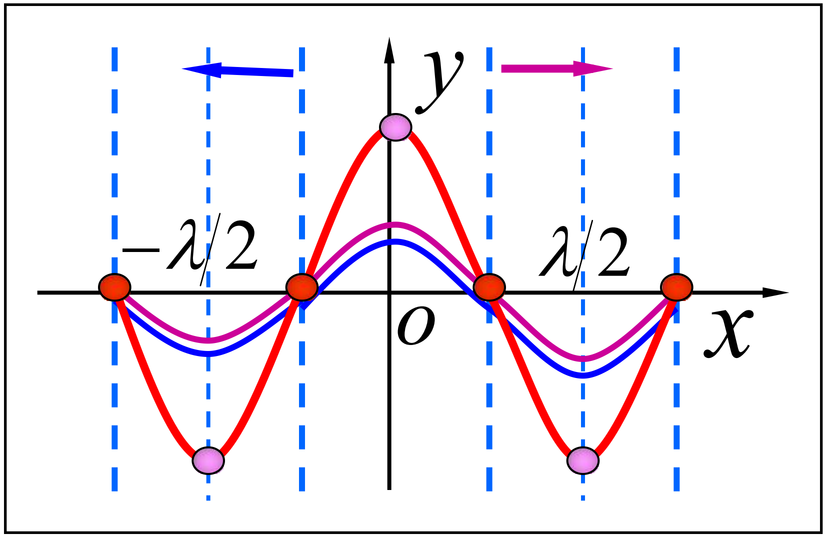
波腹 $x = \pm k\frac{\lambda}{2} \quad x = 0, \ \pm\frac{\lambda}{2}, \ \pm\lambda\cdots$

2. $\frac{2\pi}{\lambda}x = \pm(2k+1)\frac{\pi}{2} \quad k = 0, 1, 2, \cdots \quad A' = O$

波节 $x = \pm(2k+1)\frac{\lambda}{4} \quad x = \pm\frac{\lambda}{4}, \ \pm\frac{3\lambda}{4}\cdots$

3. 相邻两波节之间质点振动同相位，任一波节两侧振动相位相反，在**波节**处产生π的**相位跃变**．（与行波不同，无相位的传播）．

$$y = 2A\cos\frac{2\pi}{\lambda}x\cos\frac{2\pi}{T}t$$



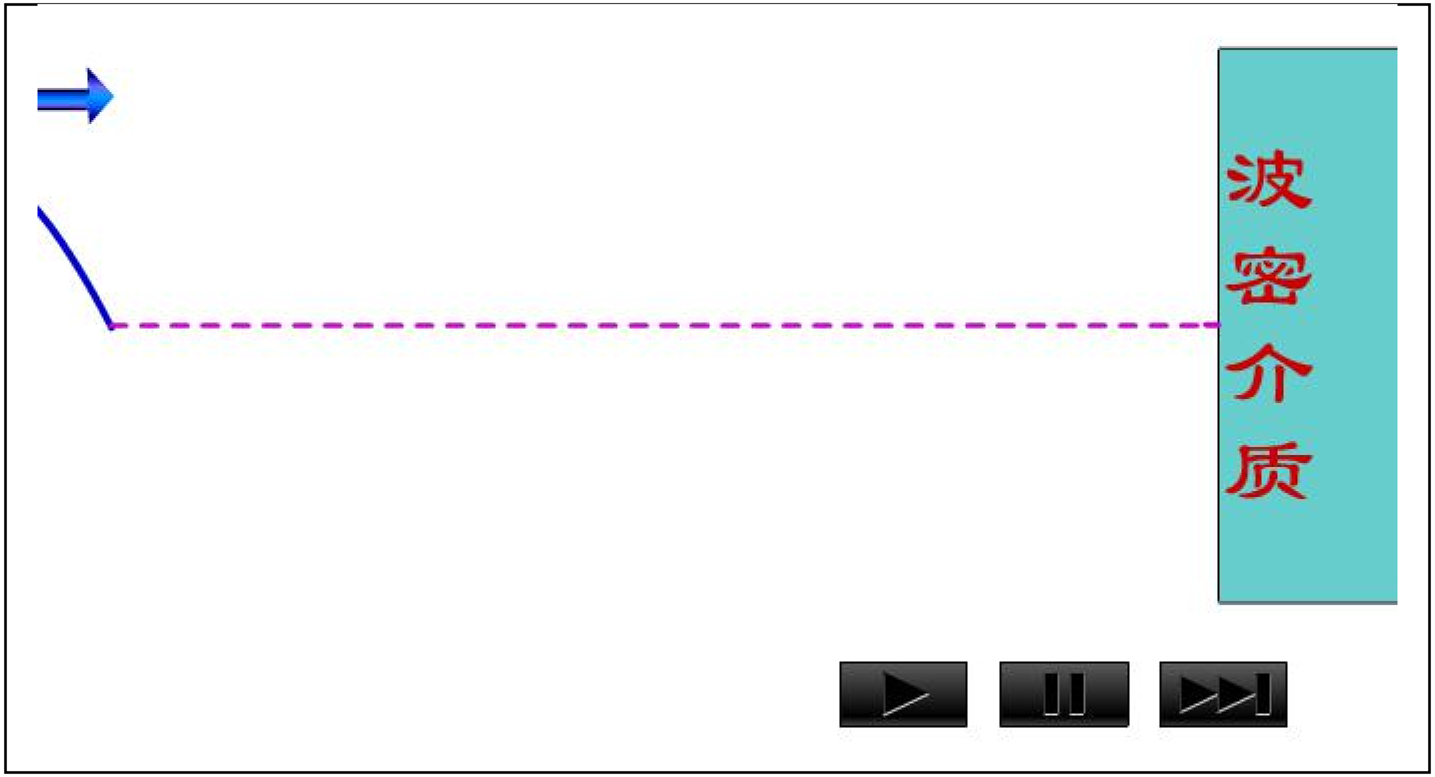
例 $x = \dfrac{\lambda}{4}$ 为**波节**

$$\boxed{\cos\frac{2\pi}{\lambda}x > 0}, \quad -\frac{\lambda}{4} < x < \frac{\lambda}{4}, \qquad y = 2A\cos\frac{2\pi}{\lambda}x\cos\frac{2\pi}{T}t$$

$$\boxed{\cos\frac{2\pi}{\lambda}x < 0}, \quad \frac{\lambda}{4} < x < \frac{3\lambda}{4}, \qquad y = \left|2A\cos\frac{2\pi}{\lambda}x\right|\cos(\frac{2\pi}{T}t + \pi)$$

三 相位跃变（半波损失）

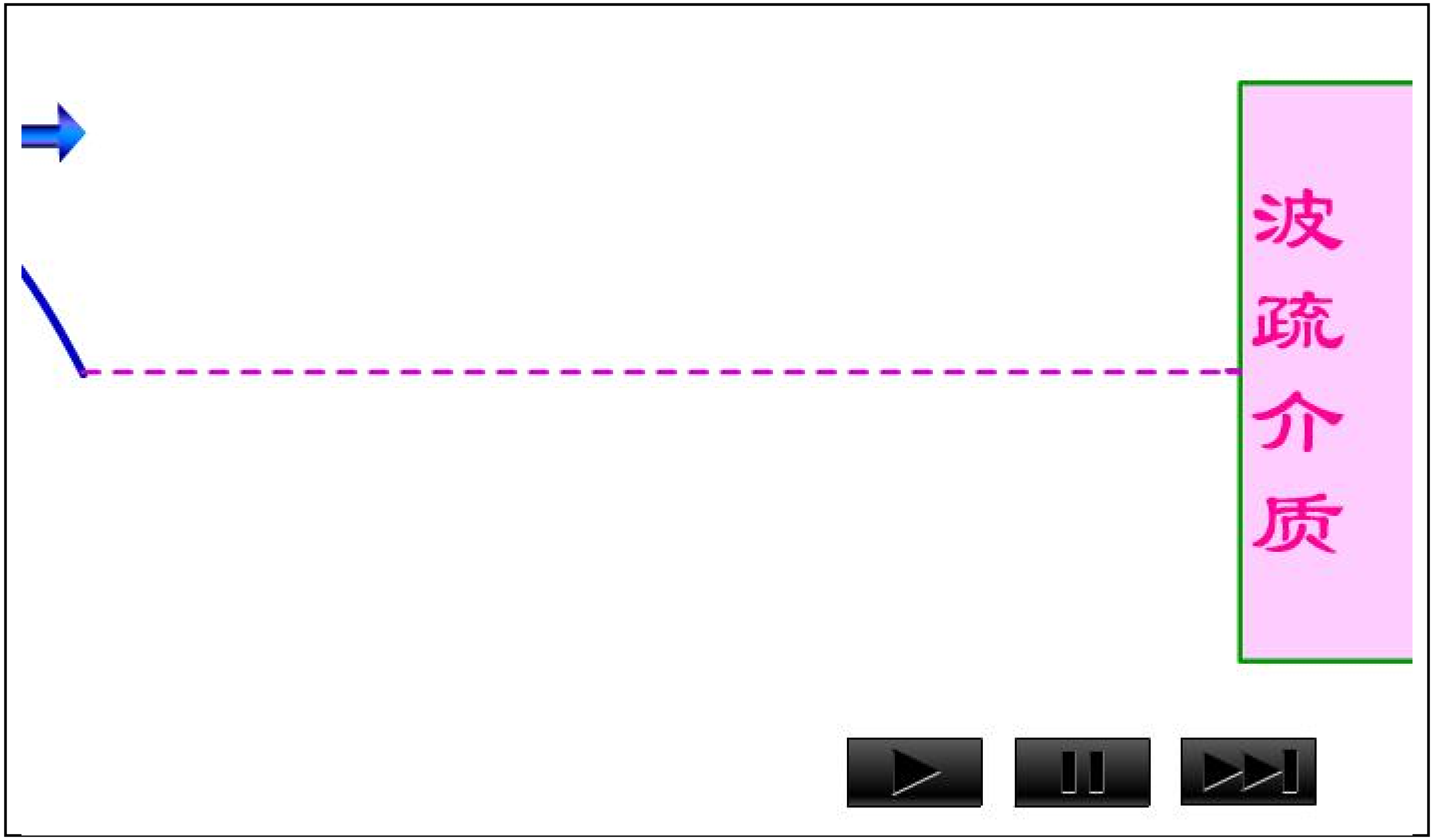
波疏介质 ρu 较小



波密介质 ρu 较大

当波从波疏介质垂直入射到波密介质被反射到波疏介质时形成波节. 入射波与反射波在此处的相位时时相反，即反射波在分界处产生 π 的相位跃变，相当于出现了半个波长的波程差，称半波损失.

波密介质



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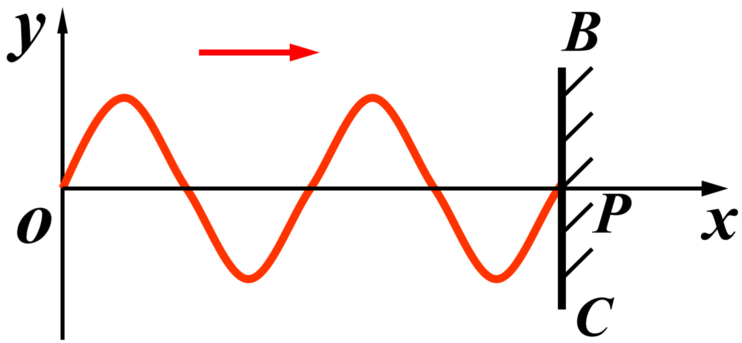
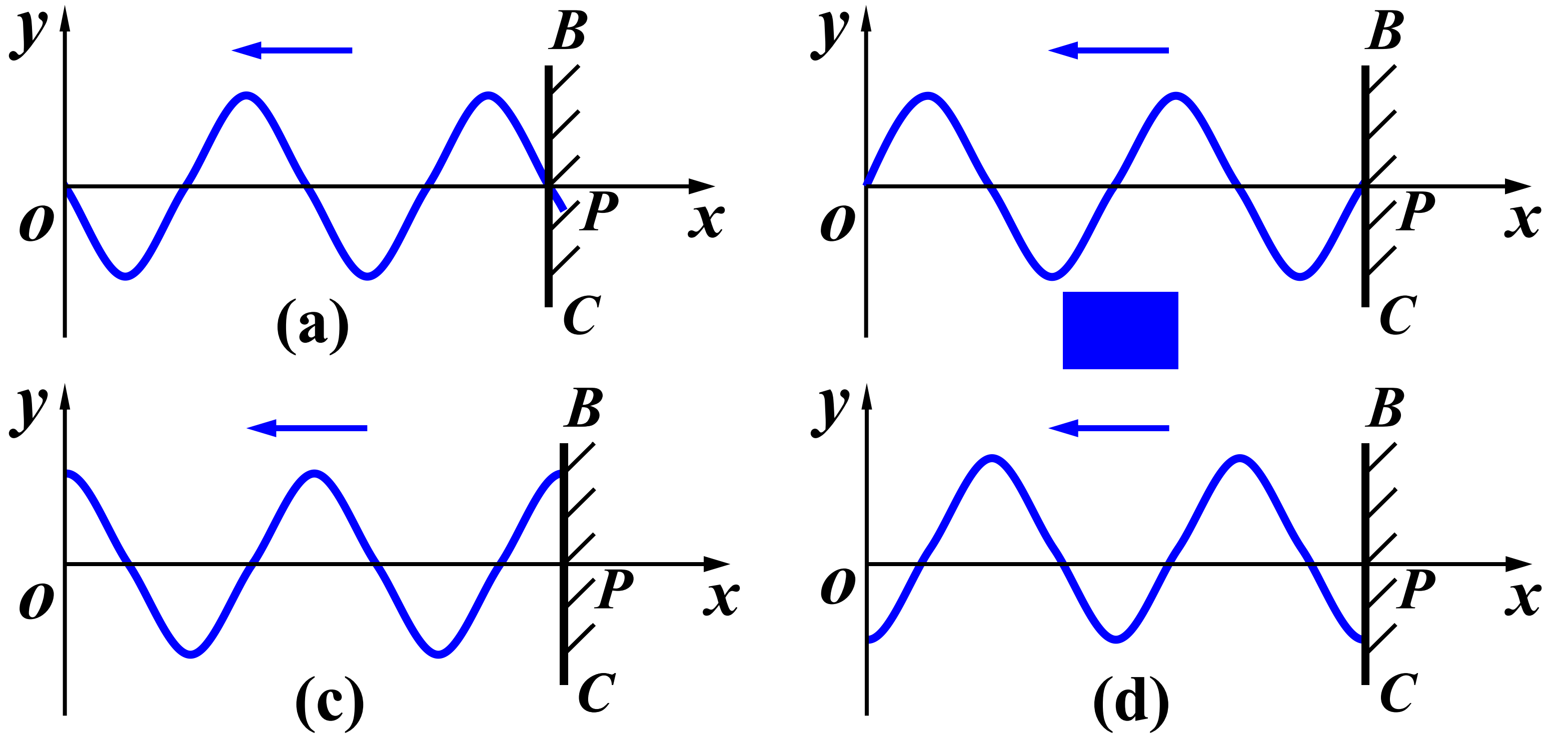
当波从波密介质垂直入射到波疏介质， 被反射到波密介质时形成波腹. 入射波与反射波在此处的相位时时相同，即反射波在分界处不产生相位跃变.

例**1**.如图为一向右传播的简谐波在 ***t*** 时刻的波形图, ***BC*** 为波密介质的反射面, 波由***P***点反射, 则反射波在***t*** 时刻的波形图为

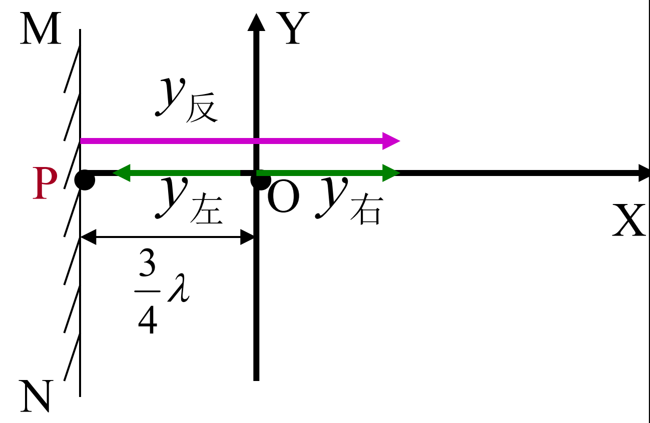




例2. 已知$x=0$处有一$y_o = A\cos\omega t$的振源，产生的波沿x轴正、负方向传播。波长为λ，MN为一波密反射面。

求：合成波

解：先求反射波方程：　方法一



$$y_入^P = A\cos[\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x]_{x=-\frac{3}{4}\lambda}$$

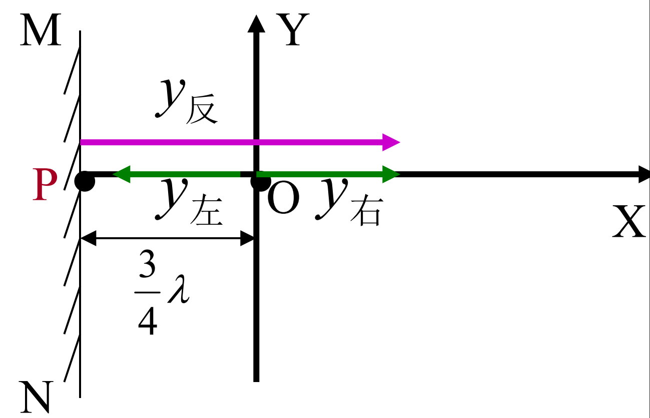
$$= A\cos[\frac{2\pi}{T}t - \frac{3}{2}\pi]$$

$$y_反^P = A\cos[(\frac{2\pi}{T}t - \frac{3}{2}\pi) + \pi] = A\cos[\frac{2\pi}{T}t - \frac{1}{2}\pi]$$

$$y_反 = A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x' - \frac{1}{2}\pi]$$

$$\xlongequal{x' = x + \frac{3}{4}\lambda} A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x]$$

法二：$y_{反} = A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \alpha]$



其中：α为反射波在$x=0$处的初相

振源o处初相$\varphi = 0$

入射波（$y_{左}$）在P点位相落后o的位相为：

$$\Delta\varphi' = -\frac{3}{2}\pi$$

反射波（$y_{右}$）在o点位相落后P的位相为：

$$\Delta\varphi'' = -\frac{3}{2}\pi$$

且在P点存在半波损失，

故反射波在o点位相较振源o点的位相落后：

$$2\times(-\frac{3}{2}\pi) - \pi = -4\pi$$

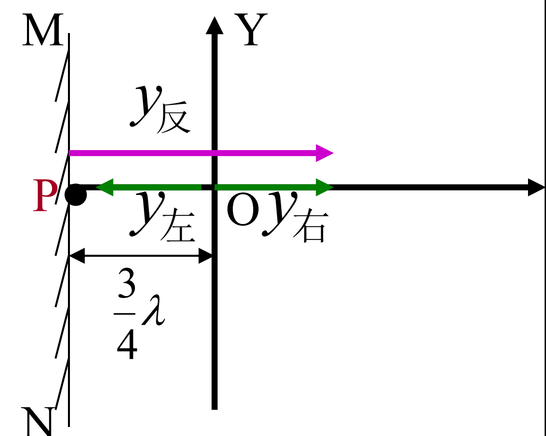
即：$\alpha = -4\pi$　　$\therefore y_{反} = A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x]$

OMN区域内的合成波：

$$y_{合} = y_{左} + y_{反}$$

$$= A\cos[\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x] + A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x]$$

$$= 2A\cos\frac{2\pi}{\lambda}x\cos\frac{2\pi}{T}t$$ —— 驻波



$x > 0$区域内的合成波：

$$y'_{合} = y_{右} + y_{反}$$

$$= A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x] + A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x]$$

$$= 2A\cos[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x]$$ —— 行波