简谐振动

- 产生原因：$F \propto -x \ (\vartheta)$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

- 描述方法

解析法：

$$x(t) = A\cos(\omega t + \varphi)$$

ω（T）、A、φ

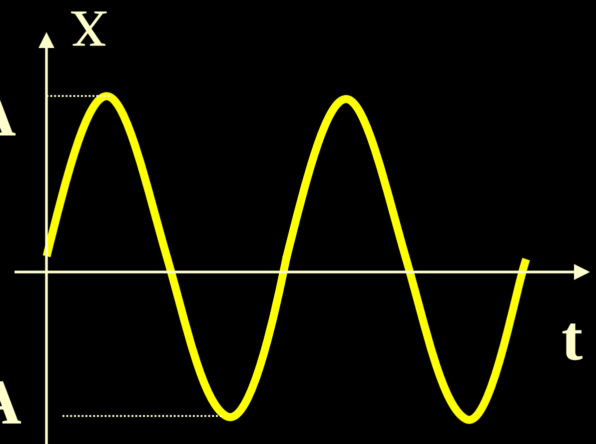
$$v(t) = -\boxed{A\omega}\sin(\omega t + \varphi)$$

$$a(t) = -\boxed{\omega^2 A}\cos(\omega t + \varphi) \sim F$$

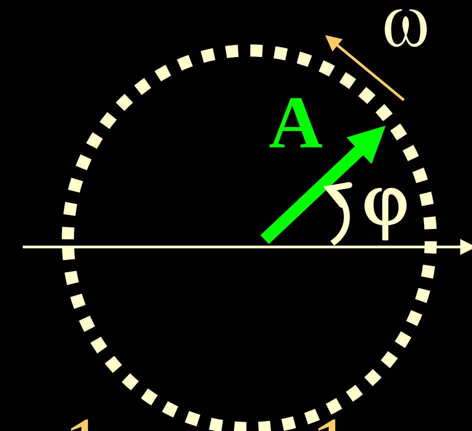
ω（T）——振动系统决定

A、φ——初始状态决定

图示法：



旋转矢量图



$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

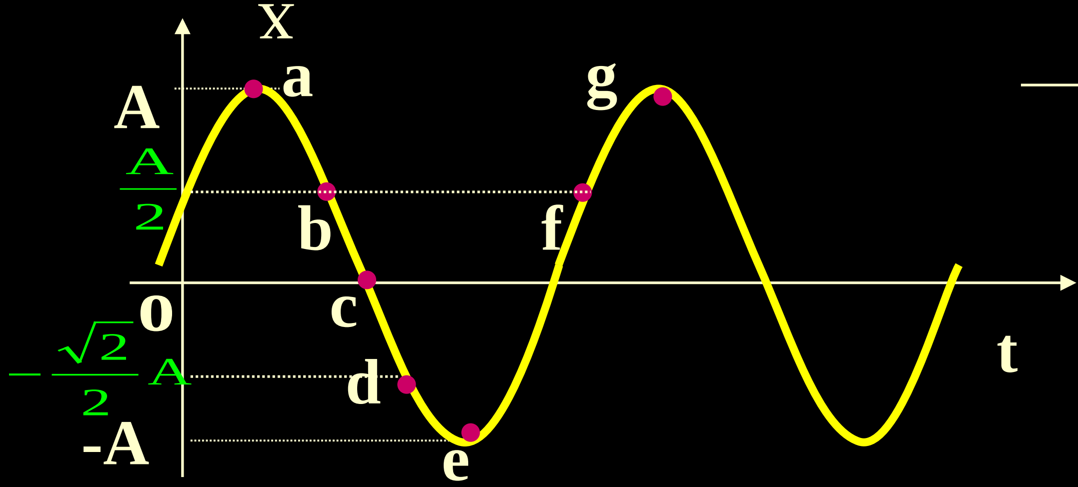
$$\mathrm{tg}\varphi = -\frac{v_0}{x_0\omega}$$

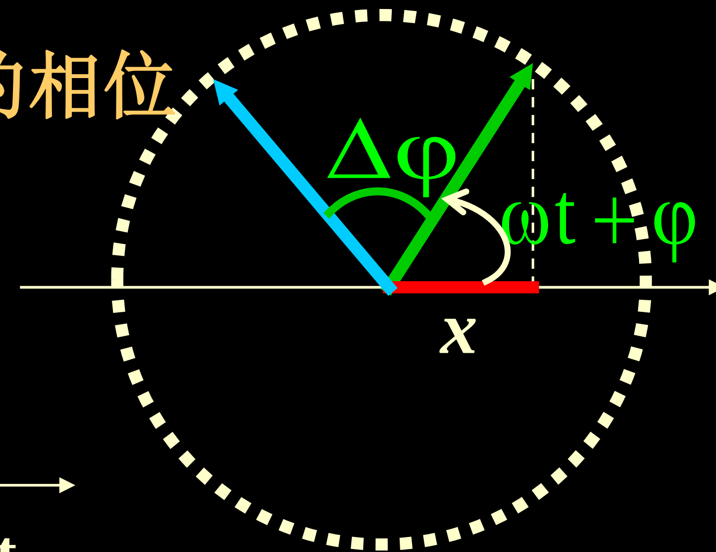
$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(\omega A)^2$$

振动曲线上那些点速度最大？加速度最大？动能最大？势能最大？

一、熟练掌握旋转矢量图法

（1）振动的表示（2）任意时刻的相位





$$\cos(\omega t+\varphi)=\frac{x}{A}$$

上半圆v< 0

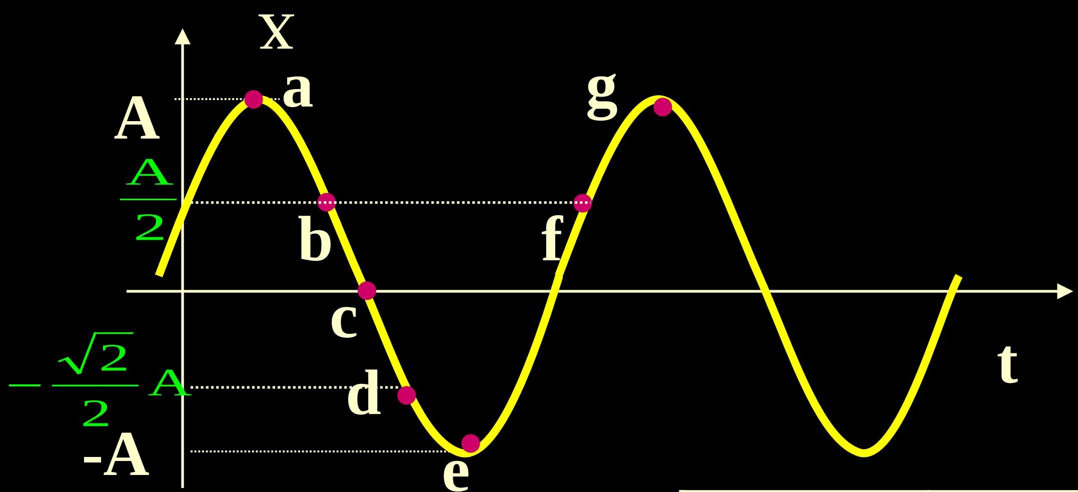
下半圆v > 0

例1（1）根据振动曲线填写下表,

且在旋转矢量图上标出相应的旋转矢量位置

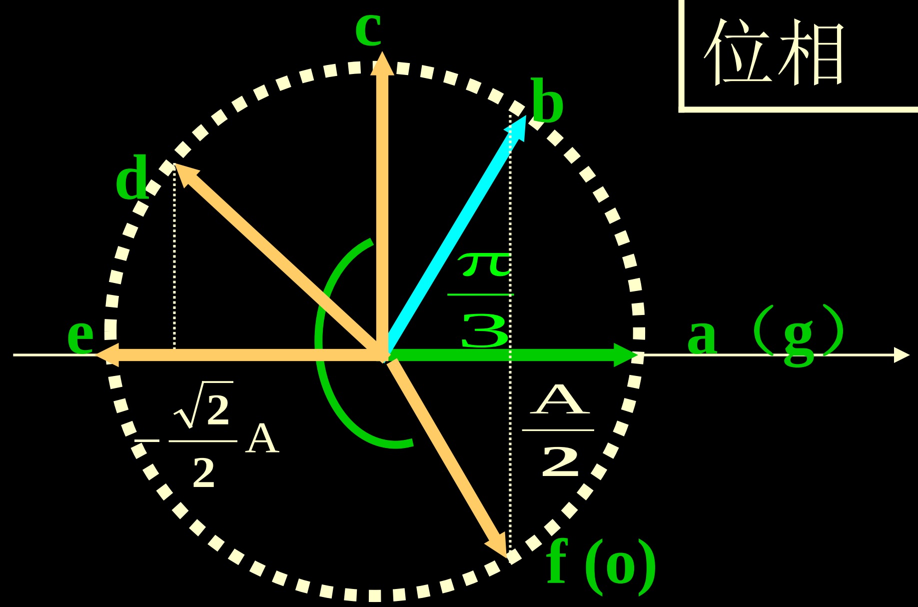
(2) a与b的振动状态的相位差是多少? c与f ?

（3）若 $\overline{Oc}=1\mathrm{s}$ ，则振动方程如何?



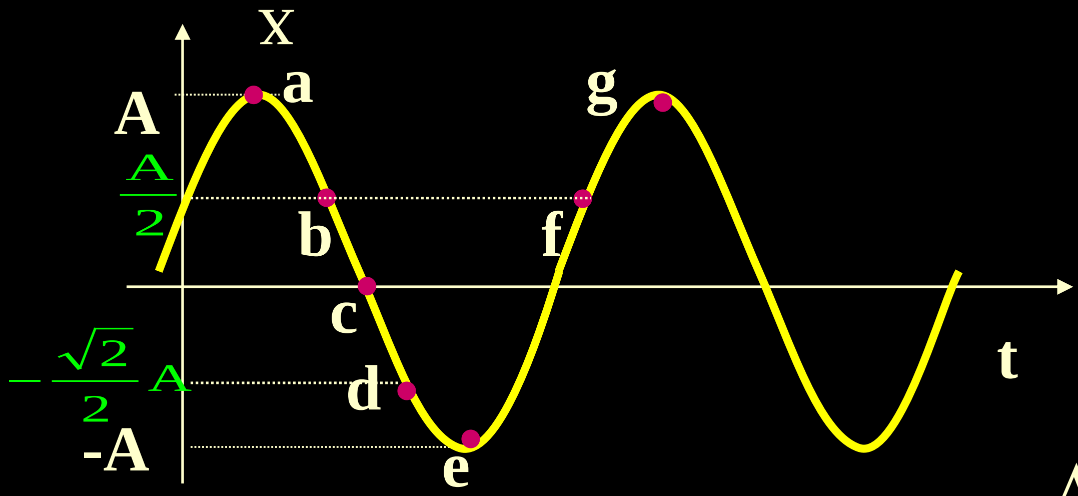
（1）

位置	a	b	c	d	e	f
位相	0	$\pi/3$	$\pi/2$	$3\pi/4$	π	$-\pi/3$



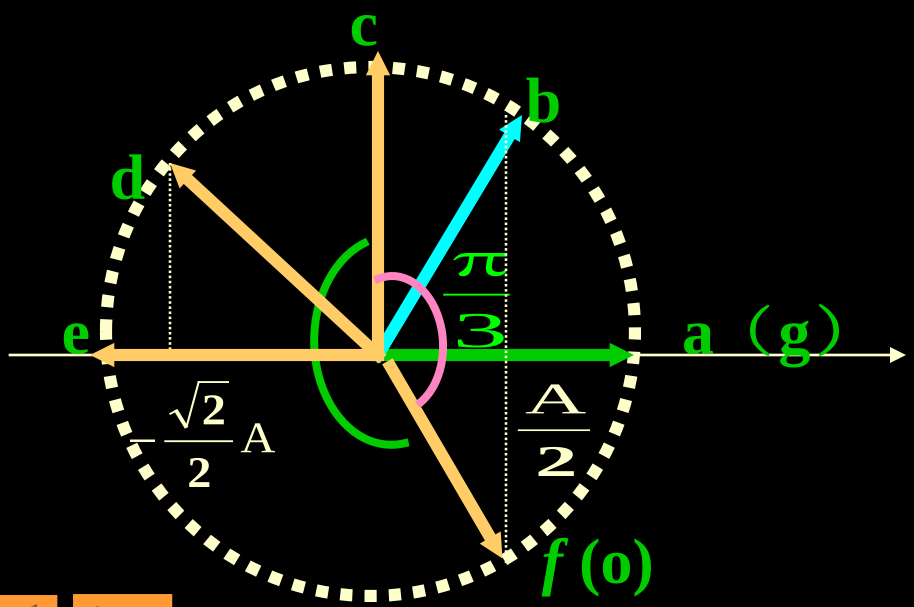
（2） $$\Delta\varphi_{ab}=\varphi_b-\varphi_a=\frac{\pi}{3}$$

$$\Delta\varphi_{cf}=\pi+\frac{\pi}{6}=\frac{7}{6}\pi$$



（3）

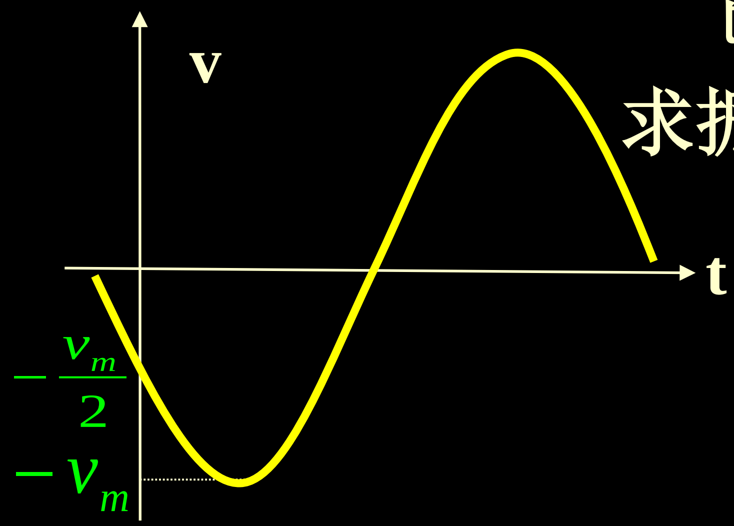
$$\Delta\varphi_{0c} = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5}{6}\pi$$



$$\omega = \frac{2\pi}{T} = \frac{\frac{5}{6}\pi}{1} = \frac{5}{6}\pi$$

$$x = A\cos\left(\frac{5}{6}\pi t - \frac{\pi}{3}\right)(SI)$$

已知简谐振动的v~t曲线及T=2s,求振动方程

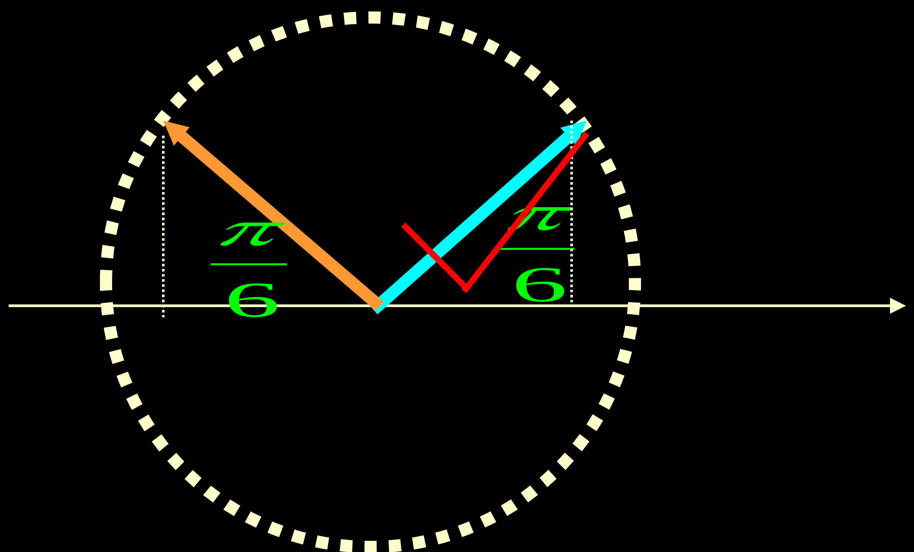


$$\omega = \frac{2\pi}{T} = \pi$$

$$v_m = \omega A \Rightarrow A = \frac{v_m}{\pi}$$

$$-\frac{v_m}{2} = -v_m \sin\alpha$$

$$\Rightarrow \sin\alpha = \frac{1}{2}$$



$$\alpha = \frac{\pi}{6} \quad or \quad \alpha = \frac{5}{6}\pi$$

$$x = \frac{v_m}{\pi}\cos(\pi t + \frac{\pi}{6})(SI)$$

例2、（自测练习p17 3）质量为0.25kg的物体与k=25N/m的弹簧组成弹簧振子，若初始时的动能和势能分别为0.02J和0.06J.而且物体的$x_0>0, v_0<0$, 试求:

(1)物体谐振动的表达式;

(2)物体从初位置回到平衡位置的最短时间;

(3)物体到达平衡位置时的速度.

解（1）

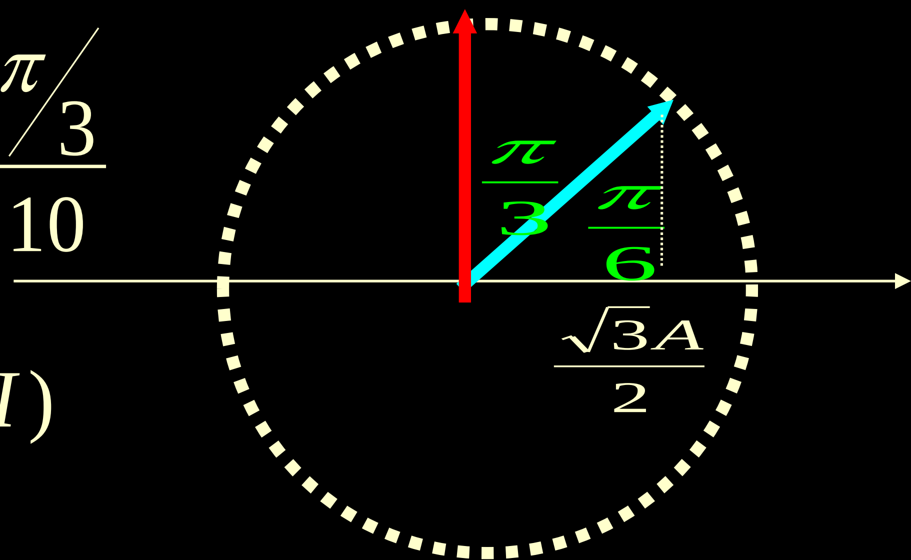
$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{25}{0.25}}=10s^{-1}$$

$$E=E_P+E_k=0.08=\frac{1}{2}kA^2 \Rightarrow A=0.08(m)$$

$$E_{P0}=0.06=\frac{1}{2}kx_0^2 \Rightarrow x_0=\frac{\sqrt{3}}{2}A \Rightarrow \alpha=\frac{\pi}{6}$$

$$x=0.08\cos(10t+\frac{\pi}{6})(SI)$$

（2） $\dfrac{2\pi}{T}=\dfrac{\Delta\varphi}{\Delta t}\Rightarrow \Delta t=\dfrac{\Delta\varphi}{\omega}=\dfrac{\pi/3}{10}$



别解：$x=0.08\cos(10t+\dfrac{\pi}{6})(SI)$

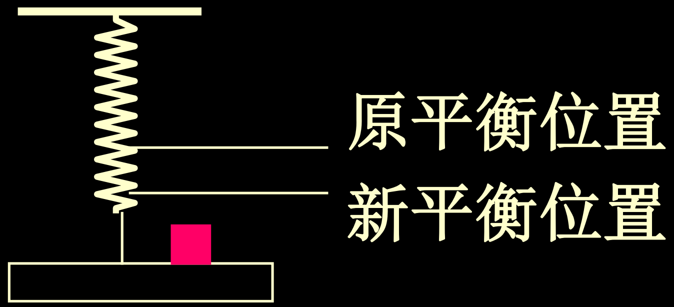
$$cos(10t+\frac{\pi}{6})=0\Rightarrow 10t+\frac{\pi}{6}=\left(2k+1\right)\frac{\pi}{2}$$

取**k=0** $\Rightarrow t$

（3） $$v=\frac{dx}{dt}=-0.08\sin(10t+\frac{\pi}{6})\times 10$$

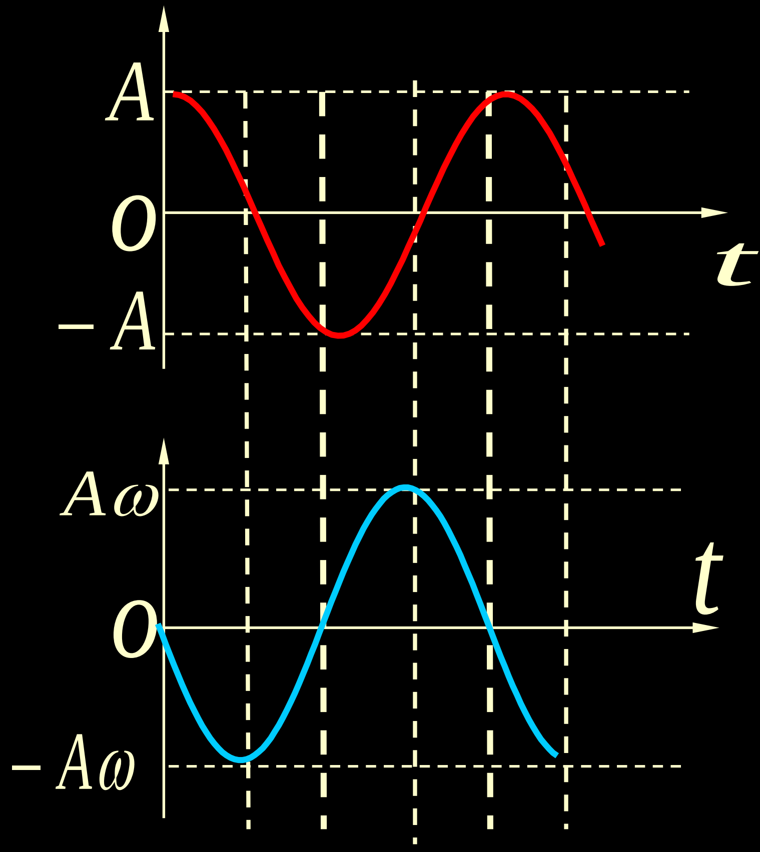
$$=-0.08\sin(\frac{\pi}{2})\times 10=-0.8m/s$$

练习：自测练习 **p13 1、3**

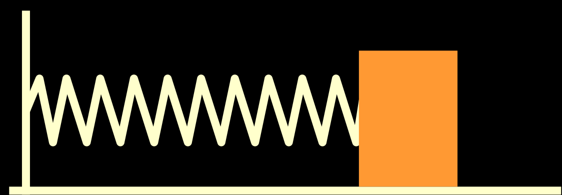


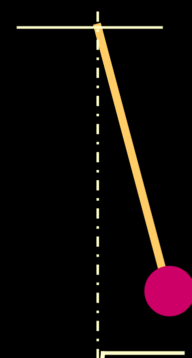
$x_0>0$, $v_0>0$

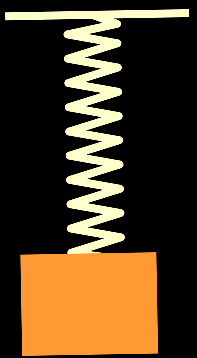
$$\varphi \sim \frac{3}{2}\pi \sim 2\pi$$

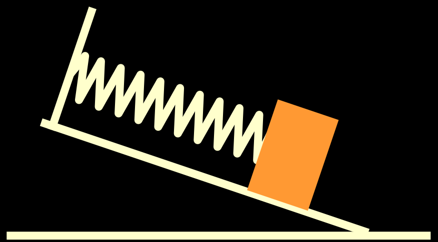


二、熟悉基本的振动系统的规律：弹簧、单摆、复摆



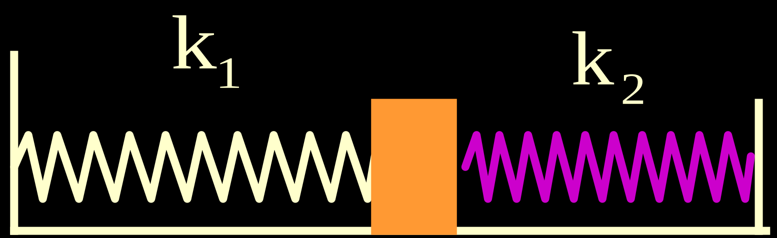


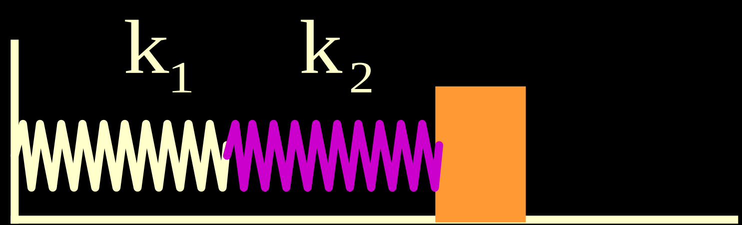




$$\omega = \sqrt{\frac{k}{m}}$$

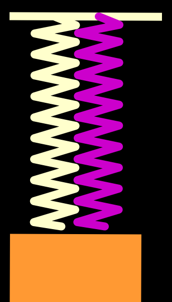
$$\omega = \sqrt{\frac{g}{l}}$$





$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = k_1 + k_2$$



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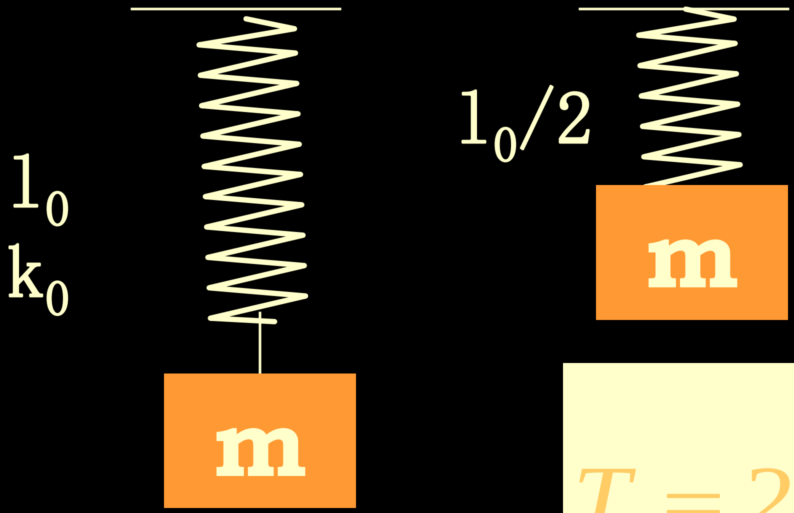
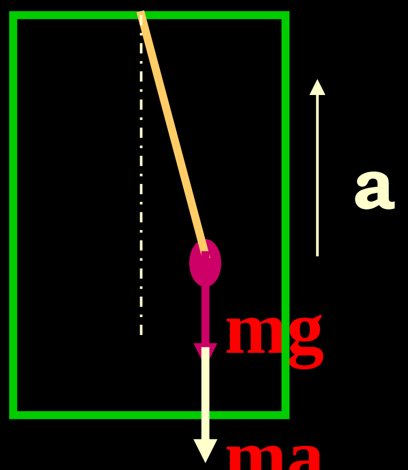
$$\frac{1}{k_o}=\frac{1}{k}+\frac{1}{k}=\frac{2}{k}$$

$$k=2k_0$$



$$k'=2k_0+2k_0=4k_0$$

$$T=2\pi\sqrt{\frac{m}{k}}\Rightarrow T:\ T'=\frac{\sqrt{4k_0}}{\sqrt{k_0}}=2:1$$



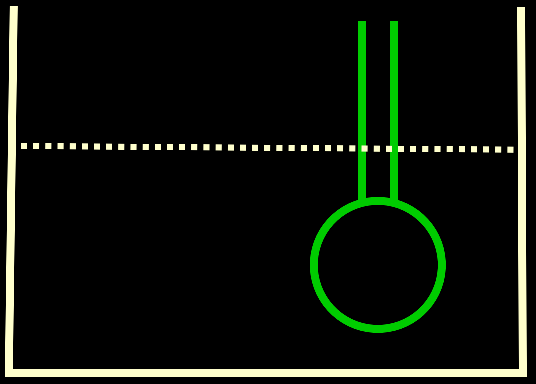
三、简谐振动的综合应用

1、证明物体作简谐振动

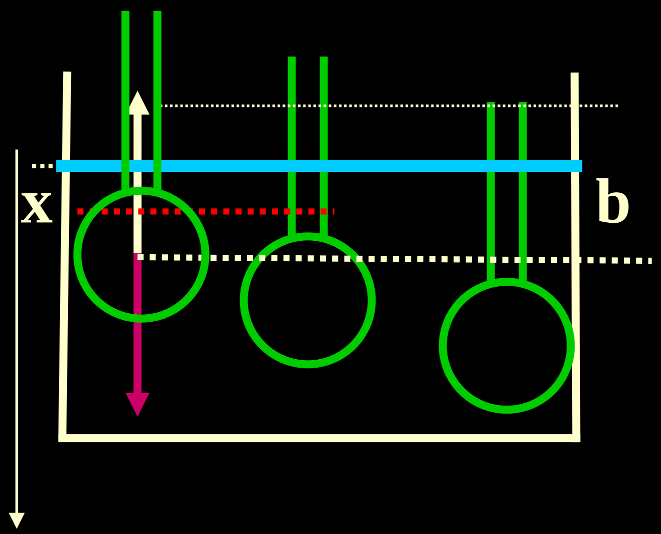
2、通过物理过程建立初始条件

3、通过物理过程研究力学问题

例3、已知：比重计m 、直径D， 液体密度ρ。若在平衡位置用力按下深度为b后，放手任其运动。求振动方程。



例5、已知：比重计m 、直径D， 液体密度ρ。若在平衡位置用力按下深度为b后，放手任其运动。求振动方程。



解：

$$mg = \rho V g$$

$$mg - \rho\left(V + \frac{1}{4}\pi D^2 x\right)g = ma$$

$$-\rho\frac{1}{4}\pi D^2 x g = ma$$

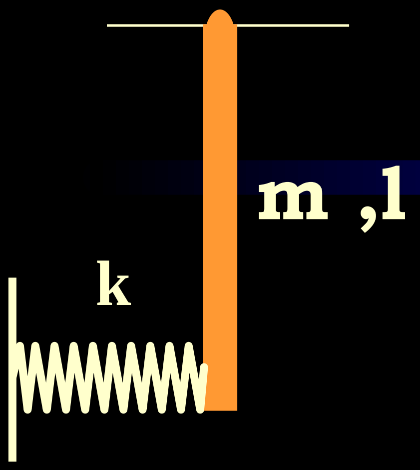
$$\frac{d^2x}{dt^2} + \frac{\rho\pi D^2 g}{4m}x = 0$$

初始条件： $x = b$ $v_0 = 0$

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = b$$

$$x = b\cos\left(\sqrt{\frac{\rho\pi D^2 g}{4m}}t\right)$$

例4、证明下列系统作简谐振动，并求其振动的周期。

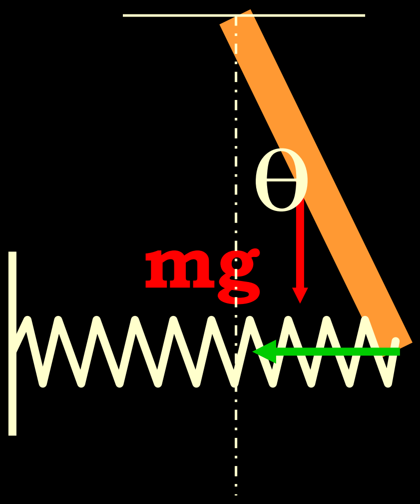


$$\boxed{x = l\sin\theta}$$

$$-mg\frac{l}{2}\sin\theta - k\boxed{x}l\cos\theta = J\frac{d^2\theta}{dt^2}$$

$$\sin\theta \sim \theta \qquad \cos\theta \sim 1$$

$$-mg\frac{l}{2}\theta - kl^2\theta \approx \frac{1}{3}ml^2\frac{d^2\theta}{dt^2}$$



$$\frac{d^2\theta}{dt^2} + \boxed{\frac{\frac{1}{2}mgl + kl^2}{\frac{1}{3}ml^2}}\theta = 0$$

$$\boxed{\omega^2}$$

$$T = \frac{2\pi}{\omega}$$

$$E=\frac{1}{2}mv^2+\frac{1}{2}kx^2 \qquad 0=m\cancel{v}\frac{dv}{dt}+kx\cancel{\frac{dx}{dt}}$$

$$\frac{dv}{dt}+\frac{k}{m}x=0 \qquad \left(\frac{d^2x}{dt^2}+\frac{k}{m}x=0\right)$$

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$$mgx=\frac{1}{2}mv^2+\frac{1}{2}J\omega^2+\frac{1}{2}kx^2$$

$$mgv=mv\frac{dv}{dt}+J\omega\frac{d\omega}{dt}+kxv$$

$$mg\cancel{v}=m\cancel{v}\frac{dv}{dt}+J\cancel{\frac{v}{R}}\frac{1}{R}\frac{dv}{dt}+kx\cancel{v}$$

$$\left(m+\frac{J}{R^2}\right)\frac{dv}{dt}+(kx-mg)=0$$

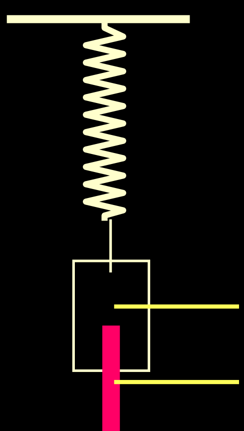
$$mg=kx_0$$

$$x=x_0+x'$$

习题册 5　一弹簧振子，由弹性系数为**k**的弹簧和质量为**M**的物块组成，将弹簧一端与顶板连接，如图所示。开始时物块静止，一质量为**m**、速度为v_0的子弹由下而上射入物块，并停留在物块中，试求：

（1）振子振动的振幅和周期；

（2）物块由初位置运动到最高点所需的时间；



$$\omega=\sqrt{\frac{k}{m+M}}\Rightarrow T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m+M}{k}}$$

$$mg=kx_0$$

$$mv_0=(M+m)V$$

$$A=\sqrt{(\frac{mg}{k})^2+\frac{(\frac{mv_0}{m+M})^2}{w^2}}$$

$$tg\varphi = -\frac{v_0}{x_0\omega} = -\frac{v_0}{g}\sqrt{\frac{k}{m+M}}$$

向上为正，最高点 $cos(\omega t+\varphi)=1$

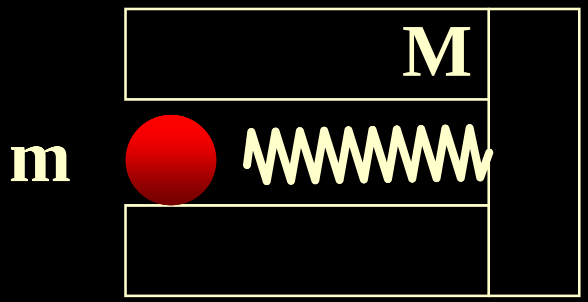
$$\frac{0-\varphi}{\Delta t} = \frac{2\pi}{T} = \omega$$

例8、一装有弹簧的框架静止地放在光滑的水平面上，框架的质量M，弹簧的弹性系数k。现有质量为m的小球以水平速度v_0射入框架与弹簧连接，并开始压缩弹簧，设小球与框架均光滑.

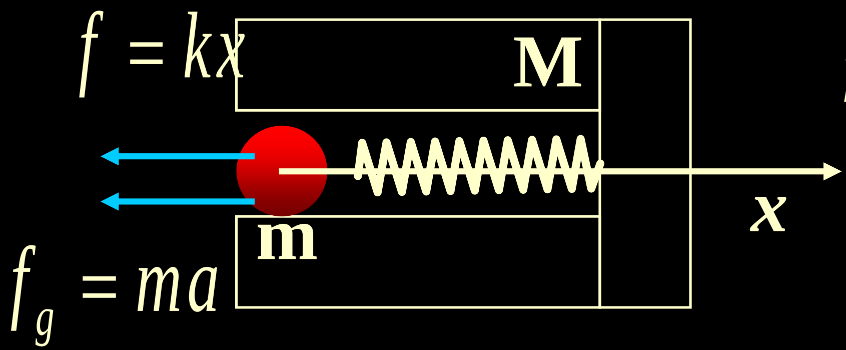
（1）证明小球在框架内作谐振动，并写出振动方程;

（2）试求弹簧的最大压缩量;

（3）从弹簧与小球接触到弹簧达到最大压缩量的时间。



$$kx = Ma_{M地}$$



$$m\frac{d^2x}{dt^2} = -kx - ma_{M\text{地}}$$

$$= -kx - m\frac{kx}{M} = -k\frac{m+M}{M}x$$

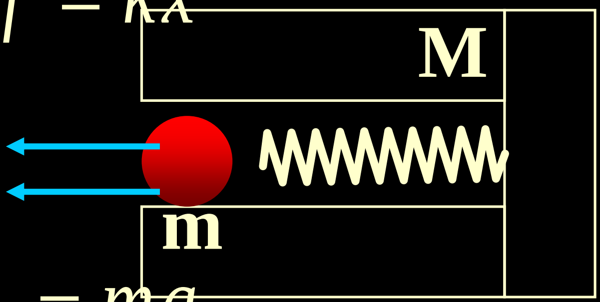
$$\frac{d^2x}{dt^2} + k\frac{m+M}{Mm}x = 0$$

$$x_0 = 0, \quad v_0 = v_0$$

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \left|\frac{v_0}{\omega}\right| \qquad \varphi = -\frac{\pi}{2}$$

$$x = \frac{v_0}{\sqrt{k\frac{m+M}{mM}}}\cos\left(\sqrt{k\frac{m+M}{mM}}\,t - \frac{\pi}{2}\right)$$

$$f = kx$$



$$f_g = ma$$

$$x_m = A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \left|\frac{v_0}{\omega}\right|$$

$$\Delta t = \frac{T\left(\dfrac{2\pi}{\omega}\right)}{4} = \frac{\pi}{2}\sqrt{\frac{Mm}{k(m+M)}}$$

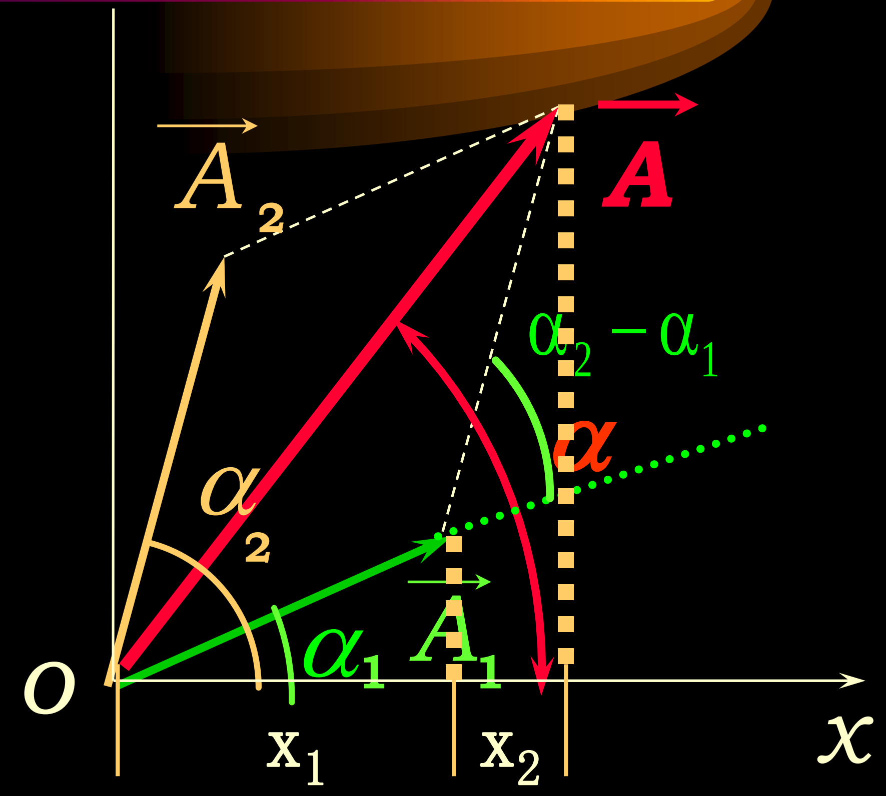
四、同方向同频率振动的合成

$$x_1 = A_1\cos(\omega t+\alpha_1)$$

$$x_2 = A_2\cos(\omega t+\alpha_2)$$

$$x = x_1 + x_2$$

$$= A\cos(\omega t+\alpha)$$



合振动的振幅为:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_2-\alpha_1)}$$

例7、已知 $\begin{cases} x_1 = \sqrt{3}\cos\left(3t + \dfrac{3\pi}{4}\right) \\ x_2 = \cos\left(3t + \dfrac{\pi}{4}\right) \end{cases}$

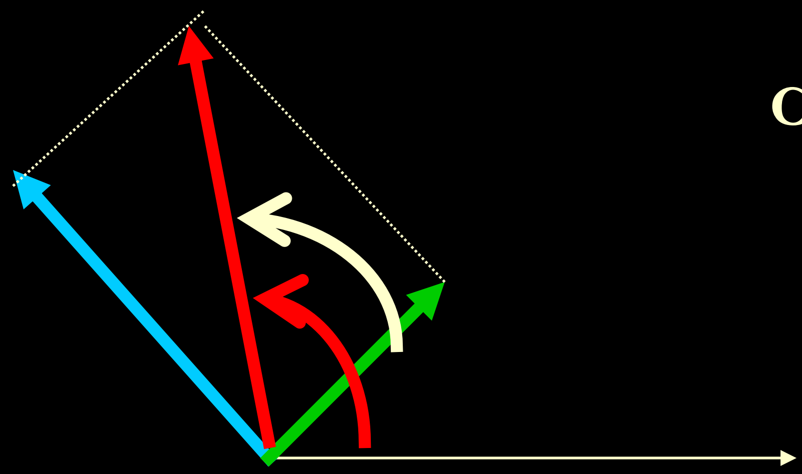
求：合振动的表达式

$$A = \sqrt{A_1^2 + A_2^2} = 2$$

$$\mathrm{tg}\beta = \frac{\sqrt{3}}{1}$$

$$\alpha = \frac{\pi}{4} + \frac{\pi}{3} = \frac{7}{12}\pi$$

$$x = 2\cos\left(3t + \frac{7\pi}{12}\right)$$



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$$\begin{cases} A = A_1 + A_2 \\ \varphi = \varphi_2 = \varphi_1 \end{cases}$$

$$x = (A_1 + A_2)\cos\left(\frac{2\pi}{T}t + \pi\right)$$

$$\begin{cases} A = |A_1 - A_2| \\ \varphi = \varphi_2 \end{cases}$$

$$x = (A_2 - A_1)\cos\left(\frac{2\pi}{T}t + \pi\right)$$