

$$3.11、(1) \begin{cases} u_t - a^2 u_{xx} = f(x, t), & x > 0, t > 0 \\ u(x, 0) = \phi(x), & x > 0 \\ u_x(0, t) = 0 \end{cases}$$

**解：** 我们做偶延拓,把 $\phi(x), f(x, t)$ 偶延拓成 $\Phi(x), F(x, t)$ ,

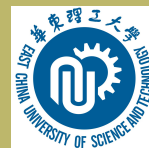
$$F(x, t) = \begin{cases} f(x, t), & x \geq 0, t \geq 0 \\ f(-x, t), & x < 0, t \geq 0 \end{cases} \quad \Phi(x) = \begin{cases} \phi(x), & x \geq 0 \\ \phi(-x), & x < 0 \end{cases}$$

**考虑初值问题**

$$\begin{cases} U_t - a^2 U_{xx} = F(x, t), & x \in R, t > 0 \\ U(x, 0) = \Phi(x), & x \in R \end{cases}$$

对于 $U(x, t)$

$$\begin{aligned} U(x, t) &= \frac{1}{2a\sqrt{\pi t}} \int_R \Phi(y) e^{-\frac{(x-y)^2}{4a^2 t}} dy + \int_0^t \frac{1}{2a\sqrt{\pi(t-\tau)}} \int_R F(y, \tau) e^{-\frac{(x-y)^2}{4a^2(t-\tau)}} dy d\tau \\ &= \frac{1}{2a\sqrt{\pi t}} \int_0^\infty \phi(y) e^{-\frac{(x-y)^2}{4a^2 t}} dy + \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^0 \Phi(y) e^{-\frac{(x-y)^2}{4a^2 t}} dy \\ &\quad + \int_0^t \frac{1}{2a\sqrt{\pi(t-\tau)}} \int_0^\infty f(y, \tau) e^{-\frac{(x-y)^2}{4a^2(t-\tau)}} dy d\tau + \int_0^t \frac{1}{2a\sqrt{\pi(t-\tau)}} \int_{-\infty}^0 F(y, \tau) e^{-\frac{(x-y)^2}{4a^2(t-\tau)}} dy d\tau \end{aligned}$$



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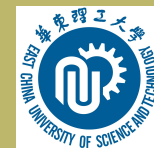
$\frac{(x-y)^2}{4a^2(t-\tau)}$

其中

$$\begin{aligned} & \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^0 \Phi(y) e^{-\frac{(x-y)^2}{4a^2 t}} dy \\ & + \int_0^t \frac{1}{2a\sqrt{\pi(t-\tau)}} \int_{-\infty}^0 F(y, \tau) e^{-\frac{(x-y)^2}{4a^2(t-\tau)}} dy d\tau \\ & \stackrel{y=-\eta}{=} \frac{1}{2a\sqrt{\pi t}} \int_0^{\infty} \phi(\eta) \exp\left(-\frac{(x+\eta)^2}{4a^2 t}\right) d\eta \\ & \int_0^t \frac{1}{2a\sqrt{\pi(t-\tau)}} \int_0^{+\infty} f(\eta, \tau) e^{-\frac{(x+\eta)^2}{4a^2(t-\tau)}} d\eta d\tau \end{aligned}$$

所以半无界区域上的解为

$$\begin{aligned} u(x, t) &= \frac{1}{2a\sqrt{\pi t}} \int_0^{\infty} \phi(y) \left( e^{-\frac{(x-y)^2}{4a^2 t}} + e^{-\frac{(x+y)^2}{4a^2 t}} \right) dy \\ &+ \int_0^t \frac{1}{2a\sqrt{\pi(t-\tau)}} \int_0^{\infty} f(y, \tau) \left( e^{-\frac{(x-y)^2}{4a^2(t-\tau)}} + e^{-\frac{(x+y)^2}{4a^2(t-\tau)}} \right) dy d\tau \end{aligned}$$

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