[讨论1]求微振动角频率

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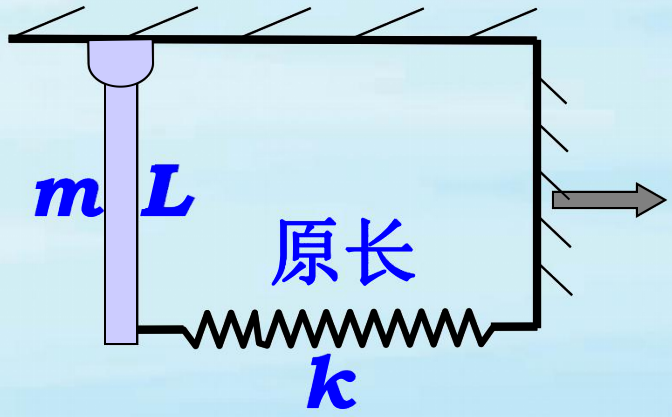

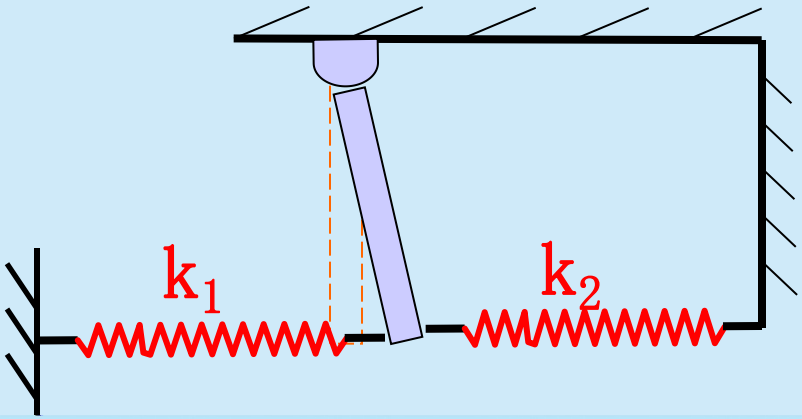

解：$M = -mg\frac{L}{2}\sin\theta - (kx)L\cos\theta$

$$\Rightarrow M \approx -(mg\frac{L}{2} + kL^2)\theta \qquad M = J\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} + \frac{mg(L/2) + kL^2}{J}\theta = 0$$

$$\Rightarrow \omega = \sqrt{\frac{mg(L/2) + kL^2}{mL^2/3}}$$

讨论：

$$\Rightarrow \frac{1}{k_{串}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\Rightarrow k_{并} = k_1 + k_2$$

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[例题4-1] $k=4\mathrm{N/m}$两弹簧并联，$m=2\mathrm{kg}$，$x_0=3\mathrm{m}$，$v_0=8\mathrm{m/s}$　求：ω，A，φ及振动方程

解：$$\omega=\sqrt{\frac{k_{并}}{m}}=\sqrt{\frac{2k}{m}}=2\mathrm{rad/s}$$

$$A=\sqrt{x_0^2+(\frac{v_0}{\omega})^2}=5m$$

$$tg\varphi=-\frac{v_0}{\omega x_0}=-\frac{4}{3}\Rightarrow\begin{cases}\varphi_1=-0.296\pi\\ \varphi_2=0.704\pi\end{cases}$$

$$\because\begin{cases}x_0=A\cos\varphi=3>0\\ v_0=-\omega A\sin\varphi=8>0\end{cases}$$

$$\Rightarrow x=5\cos(2t-0.296\pi)m$$

[例题4-2]压木块与水面平齐放手，

证：木块谐振，写谐振方程（不计阻力）

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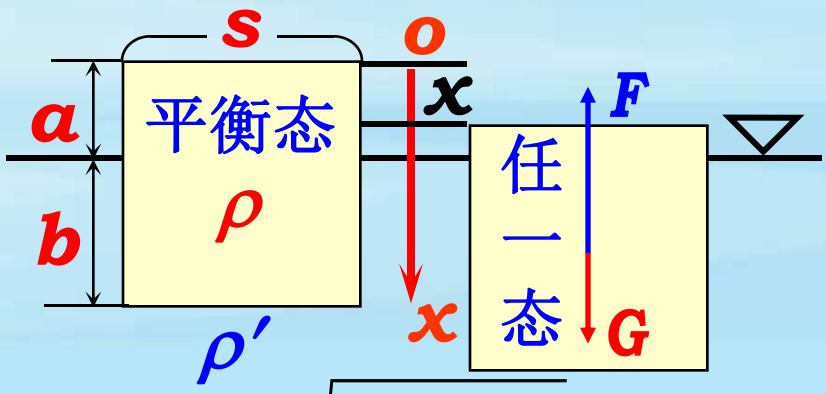
证：
$$\left.\begin{aligned}\sum F &= (a+b)s\rho g-(b+x)s\rho' g\\ \sum F &= \rho(a+b)s\ddot{x}\end{aligned}\right\}$$



$$\Rightarrow \ddot{x}+\frac{\rho' g}{\rho(a+b)}x=0 \quad \therefore \text{木块作谐振} \quad \Rightarrow \omega=\sqrt{\frac{\rho' g}{\rho(a+b)}}$$

$$\begin{cases}x_0 = a\\ v_0 = 0\end{cases}\Rightarrow\begin{cases}A=\sqrt{x_0^2+v_0^2/\omega^2}=a\\ \varphi=tg^{-1}(-v_0/\omega x_0)=0,\cancel{\pi}\end{cases}\because\begin{cases}x_0=A\cos\varphi>0\\ v_0=-\omega A\sin\varphi=0\end{cases}$$

$$\therefore \text{谐振动方程} x=a\cos\sqrt{\frac{\rho' g}{\rho(a+b)}}t$$

证谐振三步骤 1st定平衡　2nd给偏移　3rd证线恢

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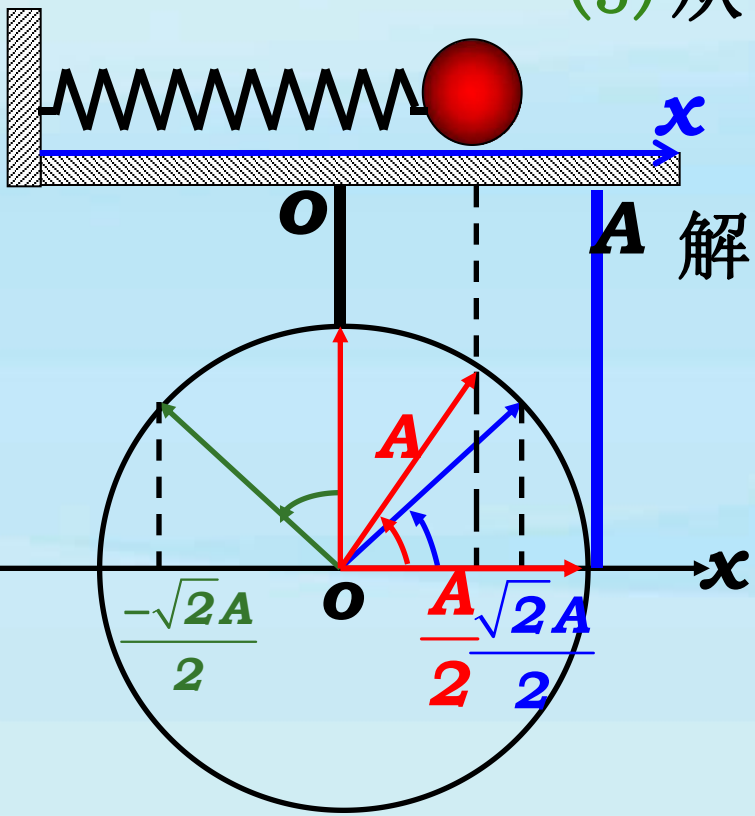
[讨论2]简谐振子 (1) 从最远点到平衡位置费时T/4，
求走完该距离一半费时？~~T/8~~

(2) 从最远点出发经T/8的位移大小？

(3) 从平衡位置出发经T/8的位移大小？



~~$\Delta x = A - \sqrt{2}A/2$~~

解 (1) $\omega t = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{3(2\pi/T)} = \frac{T}{6}$

(2) $\omega t = \frac{2\pi}{T} \times \frac{T}{8} = \frac{\pi}{4} \Rightarrow \Delta x = A - \frac{\sqrt{2}A}{2}$

(3) $\omega t = \frac{2\pi}{T} \times \frac{T}{8} = \frac{\pi}{4} \Rightarrow \Delta x = \frac{\sqrt{2}A}{2}$

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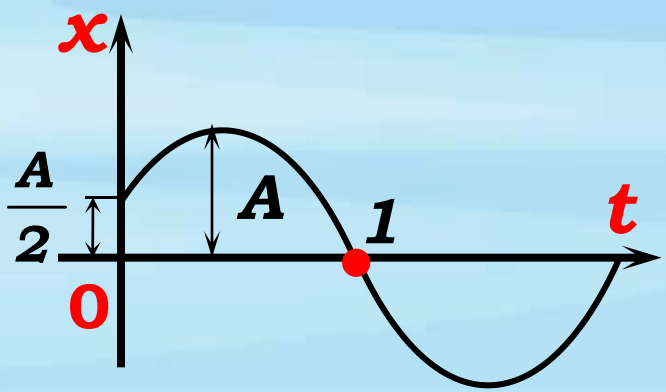
[例题4-3]已知谐振$x\sim t$曲线，

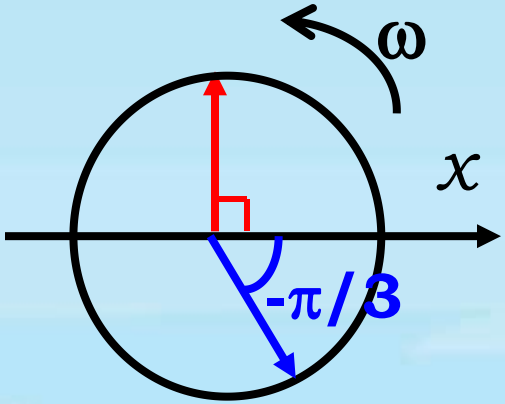
求:φ、ω及振动方程



解:

读初态 $\begin{cases} x_0=A/2=A\cos\varphi \\ v_0>0 \end{cases} \Longrightarrow \varphi=-\pi/3$

读t=1态 $\begin{cases} x_1=0 \\ v_1<0 \end{cases} \Longrightarrow \phi_1=\pi/2$



$\phi=\omega t+\varphi \rightarrow \omega=(\phi_1-\varphi)/1=5\pi/6$

∴振动方程为 $x=A\cos[(5\pi/6)t-\pi/3]$

x_0,v_0共同确定φ !!!

[讨论3]已知v-t曲线, 写振动方程

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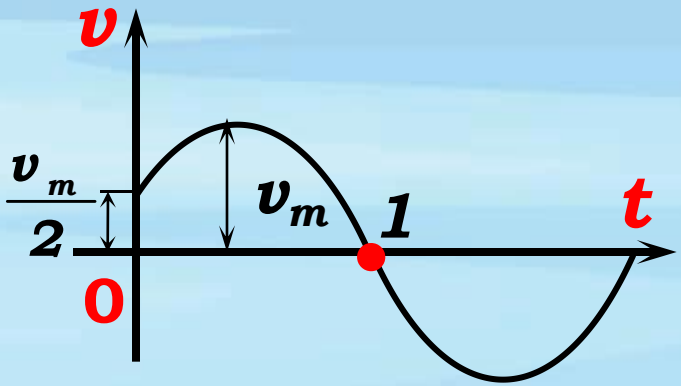
解: $x = A\cos(\omega t + \varphi)$

$\Rightarrow v = -v_m \sin(\omega t + \varphi)$

$v_0 = v_m / 2$

$\Rightarrow \sin\varphi = \dfrac{-1}{2}$



$\Rightarrow$ $\varphi = -5\pi/6$

$\varphi = -\pi/6$ $\rightarrow$舍去

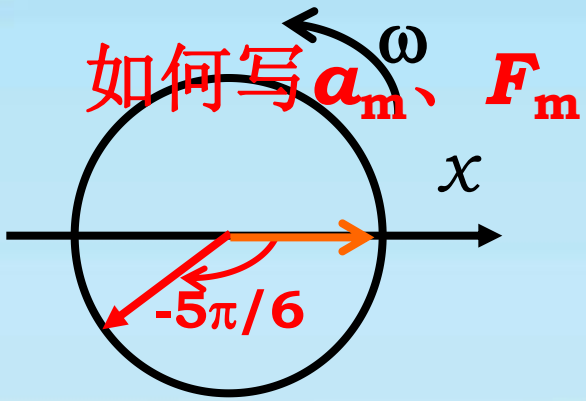
t从0↑$\rightarrow$ v↑

$v_1 = 0$

$\Rightarrow$ $\phi_1 = 0$

$\phi_1 = \pi$ $\rightarrow$舍去

t从1↑$\rightarrow$ v从0$\rightarrow$ -



$\Rightarrow \omega = (\phi_1 - \varphi)/1 = 5\pi/6$

$v_m = \omega A$

$\Rightarrow A = \dfrac{6v_m}{5\pi}$ $\therefore x = \dfrac{6v_m}{5\pi}\cos(\dfrac{5\pi}{6}t - \dfrac{5\pi}{6})$