3.16、(2) $\begin{cases} u_{xy} = x^2y^2, & x > 1, y > 0 \\ u(x,0) = x^2, & x > 0 \\ u(1,y) = \cos y, y \geq 0 \end{cases}$

解：关于y作Laplace变换，原定解问题可化为

$$\begin{cases} \frac{\partial \tilde{u}}{\partial x} = \frac{2x}{p} + \frac{x^2}{p^3} \\ \tilde{u}(1,p) = \frac{p}{1+p^2} \end{cases}$$

解初值问题

$$\tilde{u}(x,p) = \frac{x^2}{p} + \frac{x^3}{3p^3} + \frac{p}{p^2+1} - \frac{1}{p} - \frac{1}{3p^2}$$

作Laplace逆变换

$$u(x,y) = x^2 + \frac{x^3y^2}{6} + \cos y - 1 - \frac{y^2}{6}$$

3.16、(3) $\begin{cases} u_{tt} - a^2 u_{xx} = 0, & x > 0, t > 0 \\ u(0,t) = \sin t, & t > 0 \\ u(x,0) = u_t(x,0) = 0, & x > 0 \\ u(x,t)\text{有界} \end{cases}$

解：关于t实行Laplace变换

$$\begin{cases} \tilde{u}_{xx}(x,p) = \frac{p^2}{a^2}\tilde{u}(x,p) \\ \tilde{u}(0,p) = \frac{1}{p^2+1} \end{cases}$$

解初值问题可得

$$\tilde{u}(x,p) = c_1(p)e^{\frac{p}{a}x} + c_2(p)e^{-\frac{p}{a}x}$$

由$u(x,t)$有界$\Rightarrow c_1(p) = 0$,由$\tilde{u}(0,p) = \frac{1}{p^2+1} \Rightarrow c_2(p) = \frac{1}{p^2+1}$,所以

$$\tilde{u}(x,p) = \frac{1}{p^2+1}e^{-\frac{p}{a}x}$$

由延迟性质

$$u(x,t) = \sin(t - \frac{x}{a})H(t - \frac{x}{a})$$