



思考练习:

1、定解问题
$$\begin{cases} u_t - a^2 u_{xx} - bu_x - cu = 0, & x \in R, t > 0 \\ u(x, 0) = \phi(x), & x \in R \end{cases}$$

其中 a, b, c 是常数, 关于 x 施行Fourier变换, 原定解问题可转化为

$$\begin{cases} \hat{u}_t(\lambda, t) + (a^2 \lambda^2 - ib\lambda - c)\hat{u}(\lambda, t) = 0 \\ \hat{u}(\lambda, 0) = \hat{\phi}(\lambda) \end{cases}$$

2、对于积分 $\frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4a^2 t}} y dy$, 可令 $\frac{x-y}{2a\sqrt{t}} = \eta$, 积分可化为

$$\begin{aligned} \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4a^2 t}} y dy &= \frac{1}{2a\sqrt{\pi t}} \int_{+\infty}^{-\infty} e^{-\eta^2} (x - 2a\sqrt{t}\eta) (-2a\sqrt{t}) d\eta \\ &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\eta^2} (x - 2a\sqrt{t}\eta) d\eta = \frac{1}{\sqrt{\pi}} \left[\int_{-\infty}^{+\infty} e^{-\eta^2} x d\eta + \int_{-\infty}^{+\infty} e^{-\eta^2} (-2a\sqrt{t}\eta) d\eta \right] \\ &= x + 0 = x \end{aligned}$$

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