波函数

$$y = A\cos[\omega(t - \frac{x}{u}) + \varphi] \quad u\text{沿 } x\text{ 轴正向}$$

$$y = A\cos[\omega(t + \frac{x}{u}) + \varphi] \quad u\text{ 沿}x\text{ 轴负向}$$

➢ 波动方程的其它形式

$$y(x,t) = A\cos[2\pi(\frac{t}{T} - \frac{x}{\lambda}) + \varphi]$$

$$v = \frac{\partial y}{\partial t} = -\omega A\sin[\omega(t - \frac{x}{u}) + \varphi]$$

$$a = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos[\omega(t - \frac{x}{u}) + \varphi]$$

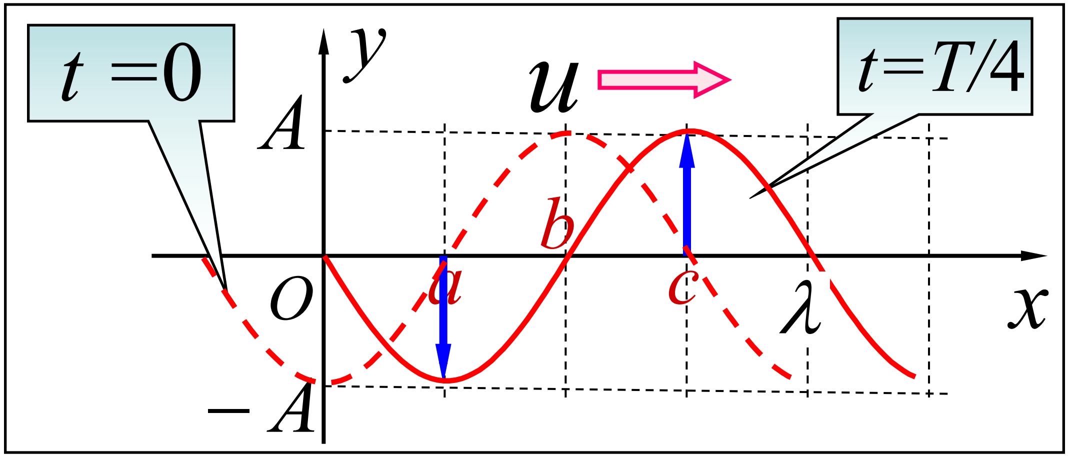
同一时刻相位法

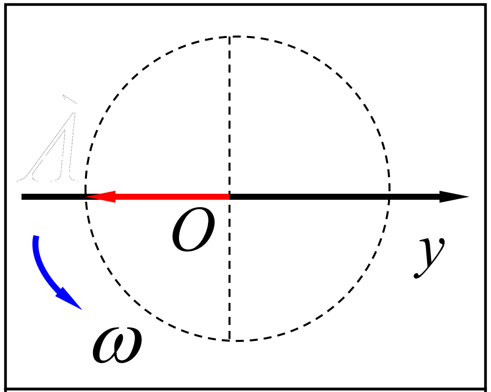
$$\Delta\phi_{12} = \phi_1 - \phi_2 = -2\pi\frac{x_1 - x_2}{\lambda} \in [-\pi, \pi] > 0, 1\text{超前}2;$$

$$< 0, 1\text{落后}2;$$

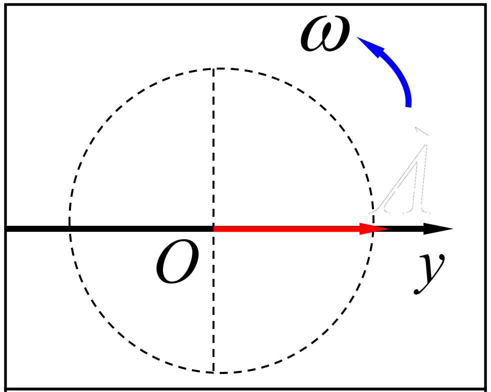
3） 如图简谐波以余弦函数表示，求 O、a、b、c 各点振动**初相位**.

$$\varphi(-\pi \sim \pi]$$

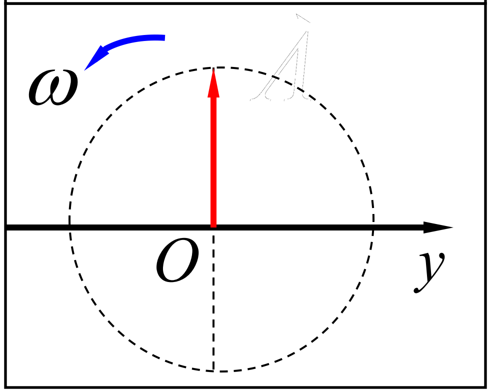




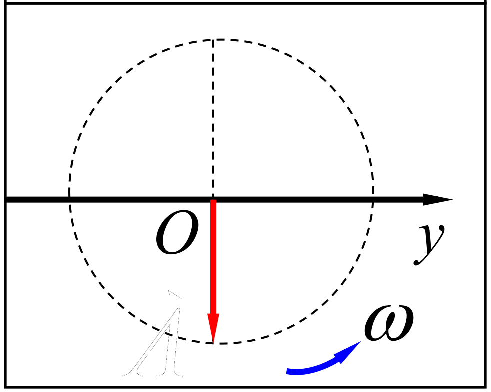
$$\varphi_o = \pi$$



$$\varphi_b = 0$$



$$\varphi_a = \frac{\pi}{2}$$



$$\varphi_c = -\frac{\pi}{2}$$

例1 已知波动方程如下，求波长、周期和波速.

$$y = 5\cos\pi(2.50t - 0.01x)$$

解：方法一（比较系数法）.

$$\boxed{y = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

把题中波动方程改写成

$$y = 5\cos 2\pi\left(\frac{2.50}{2}t - \frac{0.01}{2}x\right)$$

比较得

$$T = \frac{2}{2.5} = 0.8\,\text{s} \qquad \lambda = \frac{2}{0.01} = 200\,\text{cm} \qquad u = \frac{\lambda}{T} = 250\,\text{cm}\cdot\text{s}^{-1}$$

例1　已知波动方程如下，求波长、周期和波速.

$$y = 5\cos\pi(2.50t - 0.01x)$$

解：方法二（由各物理量的定义解之）.

波长是指同一时刻 t，波线上相位差为 2π 的两点间的距离.

$$\pi(2.50t - 0.01x_1) - \pi(2.50t - 0.01x_2) = 2\pi$$

$$\boxed{\lambda = x_2 - x_1 = 200\,\mathrm{cm}}$$

周期为相位传播一个波长所需的时间

$$\pi(2.50t_1 - 0.01x_1) = \pi(2.50t_2 - 0.01x_2)$$

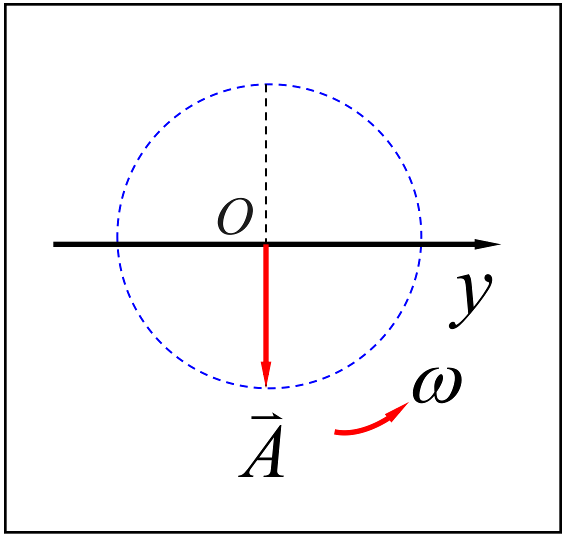
$$x_2 - x_1 = \lambda = 200\,\mathrm{cm}$$

$$T = t_2 - t_1 = 0.8\,\mathrm{s}$$

$$u = \frac{x_2 - x_1}{t_2 - t_1} = 250\,\mathrm{cm}\cdot\mathrm{s}^{-1}$$

例2 一平面简谐波沿 Ox 轴正方向传播，已知振幅 $A=1\text{m}$ ， $T=2\text{s}$ ， $\lambda=2\text{m}$. 在 $t=0$ 时坐标原点处的质点位于平衡位置沿 Oy 轴正方向运动 . 求

1）波动方程



解 写出波动方程的标准式

$$y = A\cos\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)+\boxed{\varphi}\right]$$

$$t=0 \quad x=0$$

$$y=0, v=\frac{\partial y}{\partial t}>0$$

$$\boxed{\varphi=-\frac{\pi}{2}}$$

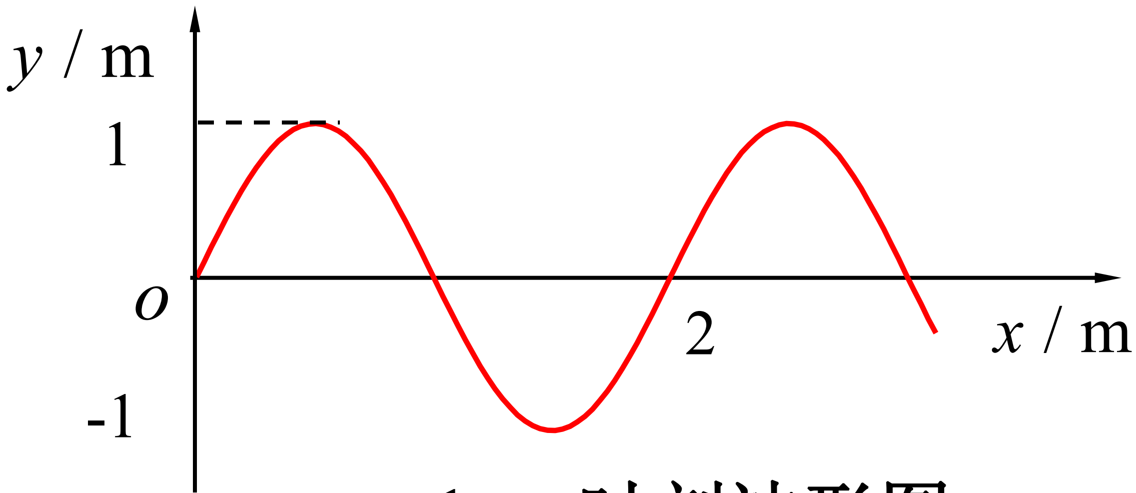
$$y=\cos\left[2\pi\left(\frac{t}{2}-\frac{x}{2}\right)-\frac{\pi}{2}\right](m)$$

2）求 $t=1\mathrm{s}$ 波形图.

$t=1\mathrm{s}$
波形方程

$$y=\cos[2\pi(\frac{t}{2}-\frac{x}{2})-\frac{\pi}{2}]$$

$$y=\cos[\frac{\pi}{2}-\pi x]=\sin(\pi x)(m)$$



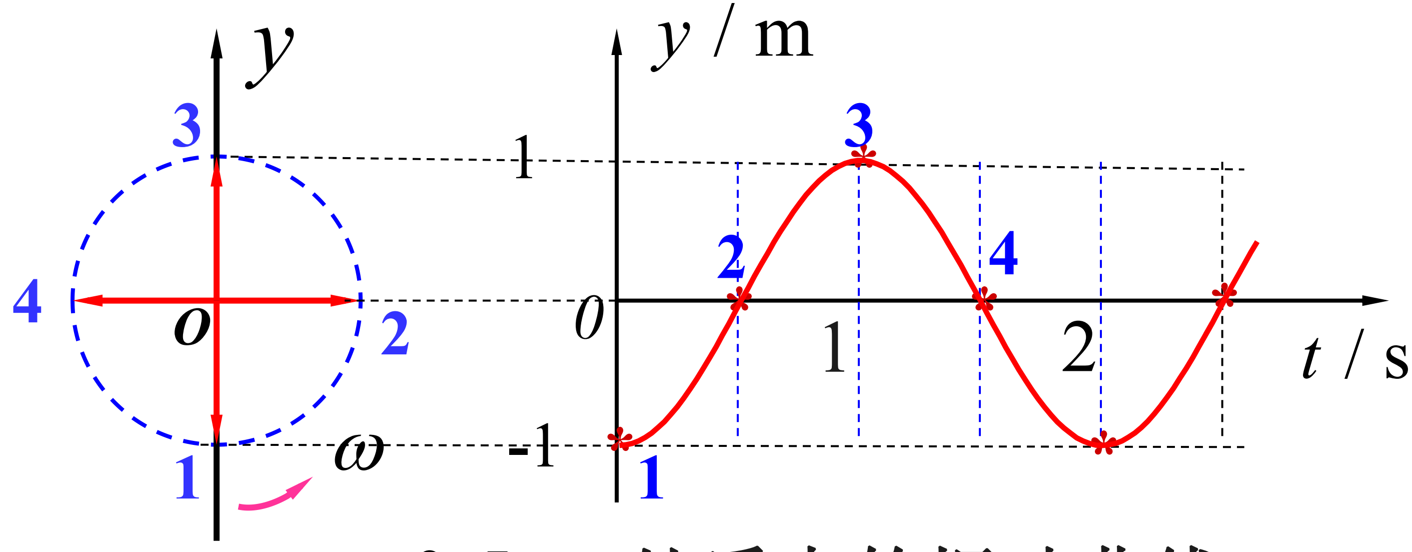
$t=1\ \mathrm{s}$ 时刻波形图

3） $x = 0.5\mathrm{m}$ 处质点的振动规律并做图 .

$$y = \cos[2\pi(\frac{t}{2} - \frac{x}{2}) - \frac{\pi}{2}]$$

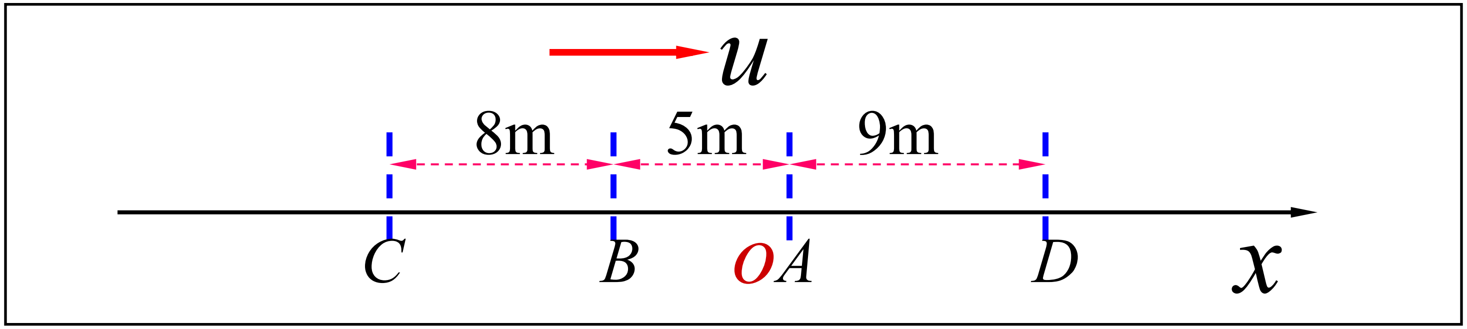
$x = 0.5\mathrm{m}$ 处质点的振动方程

$$y = \cos[\pi t - \pi](m)$$



$x = 0.5\ \mathrm{m}$ 处质点的振动曲线

例3　一平面简谐波以速度 $u=20\mathrm{m/s}$ 沿直线传播，波线上点 A 的简谐运动方程：$y_A=3\times10^{-2}\cos 4\pi t(m)$



1）以 A 为坐标原点，写出波动方程

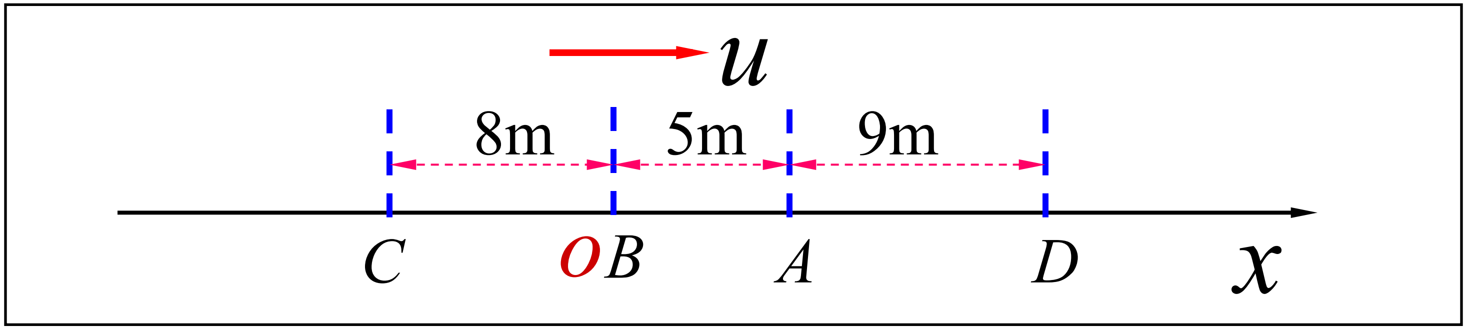
$$A=3\times10^{-2}\mathrm{m}\quad T=0.5\mathrm{s}\quad \varphi=0\qquad \lambda=uT=10\mathrm{m}$$

$$y=A\cos[\,2\pi\,(\frac{t}{T}-\frac{x}{\lambda})+\varphi\,]$$

$$y=3\times10^{-2}\cos 2\pi(\frac{t}{0.5}-\frac{x}{10})(m)$$

2）以 B 为坐标原点，写出波动方程

$$y_A = 3\times10^{-2}\cos(4\pi t)(m)$$

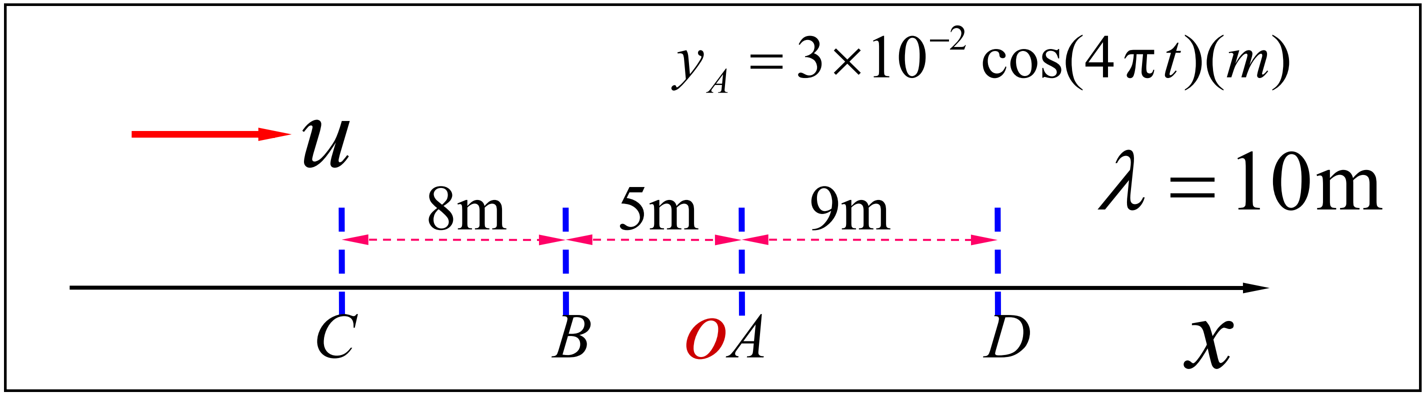


$$\varphi_B - \varphi_A = -2\pi\frac{x_B - x_A}{\lambda} = -2\pi\frac{-5}{10} = \pi$$

$$\varphi_B = \pi \qquad y_B = 3\times10^{-2}\cos(4\pi t + \pi)(m)$$

$$y = 3\times10^{-2}\cos[2\pi(\frac{t}{0.5} - \frac{x}{10}) + \pi](m)$$

3）写出传播方向上点C、点D 的简谐运动方程



点 C 的相位比点 A 超前

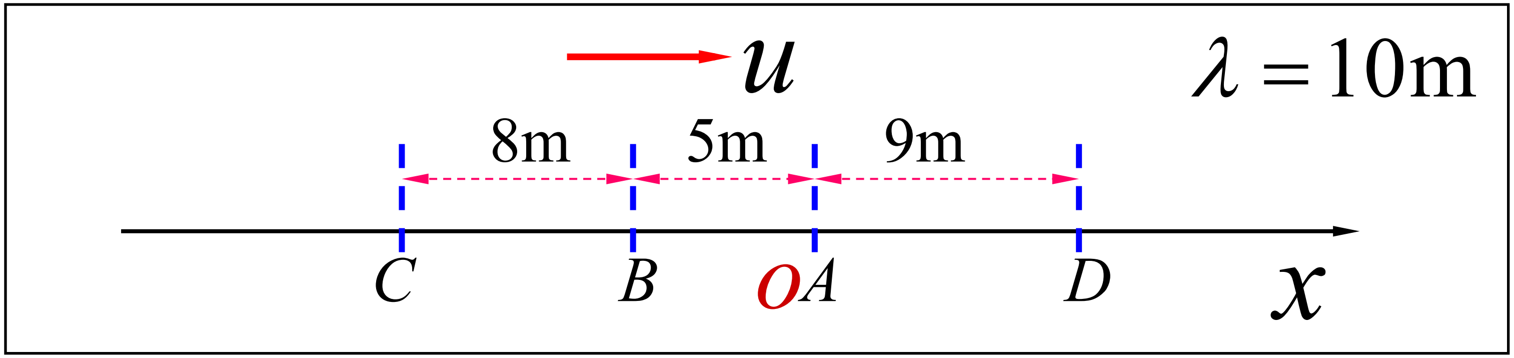
$$y_C = 3\times10^{-2}\cos(4\pi t + 2\pi\frac{AC}{\lambda})$$
$$= 3\times10^{-2}\cos(4\pi t + \frac{13}{5}\pi)(m)$$

点 D 的相位落后于点 A

$$y_D = 3\times10^{-2}\cos(4\pi t - 2\pi\frac{AD}{\lambda})$$
$$= 3\times10^{-2}\cos(4\pi t - \frac{9}{5}\pi)(m)$$

4）分别求出 BC ，CD 两点间的相位差

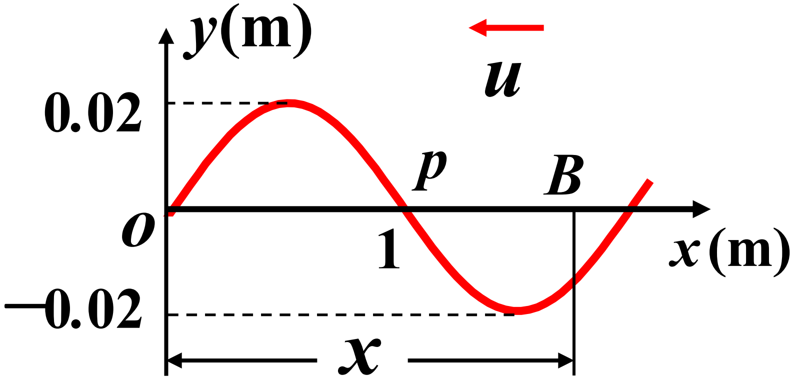
$$y_A = 3\times10^{-2}\cos(4\pi t)(m)$$

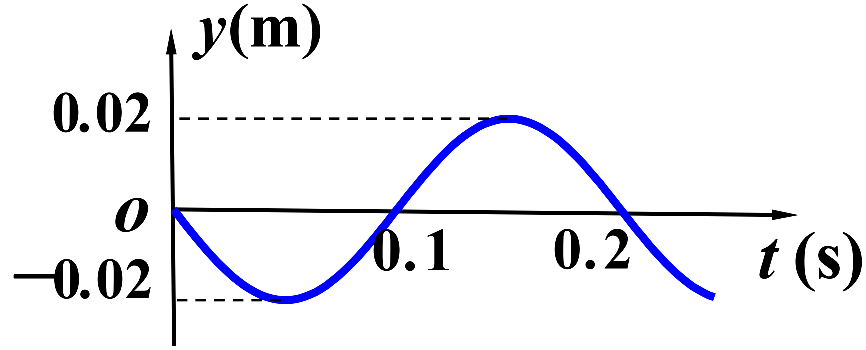


$$\varphi_B - \varphi_C = -2\pi\frac{x_B - x_C}{\lambda} = -2\pi\frac{8}{10} = -1.6\pi$$

$$\varphi_C - \varphi_D = -2\pi\frac{x_C - x_D}{\lambda} = -2\pi\frac{-22}{10} = 4.4\pi$$

例4. 已知 t = 0 时波形图和 p 处质点的振动曲线，求该平面波的波函数。





$$\omega = \frac{2\pi}{T} = 10\ \pi/\text{s} \qquad \lambda = 2\ \text{m} \qquad u = \frac{\lambda}{T} = 10\ \text{m/s}$$

$$y_p = A\cos(\omega t + \varphi) \qquad \varphi = ?$$

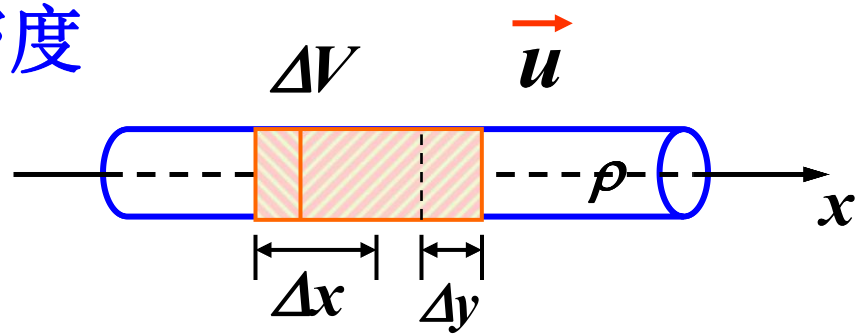
$$y_p = 0.02\cos(10\pi\ t + \frac{\pi}{2})$$

$\because$ t = 0 时 p 点向下振动，$\therefore$ 波沿x 负方向传播

$$y = 0.02\cos\left[10\pi(t + \frac{x-1}{10}) + \frac{\pi}{2}\right] = 0.02\cos\left[10\pi(t + \frac{x}{10}) - \frac{\pi}{2}\right]$$

5-4 机械波的能量

一、机械波的能量和能量密度



$$y = A\cos\omega(t - \frac{x}{u})$$

$$\upsilon = \frac{\partial y}{\partial t} = -A\omega\sin\omega(t - \frac{x}{u})$$

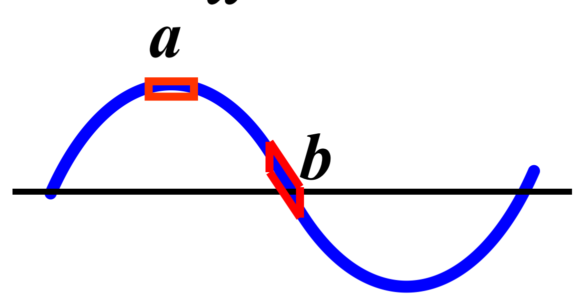
$$\Delta W_k = \frac{1}{2}(\Delta m)\upsilon^2 = \frac{1}{2}(\rho\Delta V)A^2\omega^2\sin^2\omega(t - \frac{x}{u})$$

$$\Delta W_p = \frac{1}{2}k(\Delta y)^2 = \frac{1}{2}(\rho\Delta V)A^2\omega^2\sin^2\omega(t - \frac{x}{u}) = \Delta W_k$$

$$\Delta W = \Delta W_k + \Delta W_P = \rho\Delta V A^2\omega^2\sin^2\omega(t - \frac{x}{u})$$

b点: υ最大, ΔW_k最大, ΔW_p最大

a点: $\upsilon = o$, $\Delta W_k = o$, $\Delta W_p = o$



能量密度 $w=\dfrac{\Delta W}{\Delta V}=\rho\omega^2A^2\sin^2\omega(t-\dfrac{x}{u})$

平均能量密度(一个周期内的平均值)

$$\overline{w}=\frac{1}{T}\int_0^T\rho A^2\omega^2\sin^2\omega(t-\frac{x}{u})dt=\boxed{\frac{1}{2}\rho A^2\omega^2}$$

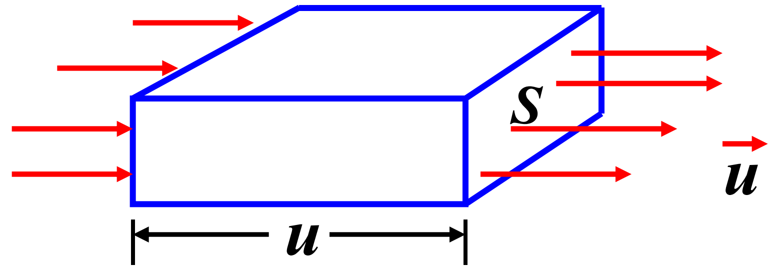
二、能流和能流密度

单位时间内通过介质中某面积的能量称为通过该面积的能流 (平均能流)

$$\boxed{\overline{P}=\overline{w}uS=\frac{1}{2}\rho A^2\omega^2uS}$$

通过垂直于波传播方向上单位面积的平均能流, 称为能流密度或波的强度

$$\boxed{I=\frac{\overline{P}}{S}=\overline{w}u=\frac{1}{2}\rho A^2\omega^2u}$$



例1一平面波在媒质中传播，在质元从最大位移处回到平衡位置的过程中

(A) 它的势能转换为动能

(B) 它的动能转换为势能

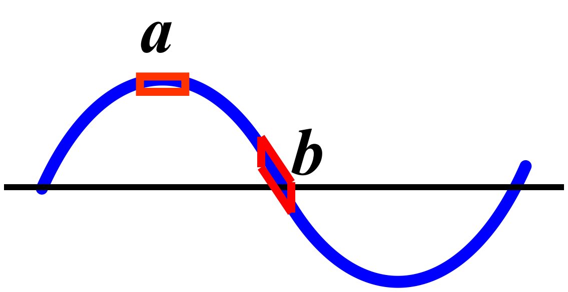
[redacted] 它从相邻的质元获得能量，并逐渐增加

(D) 它把自己的能量传给相邻的质元，能量逐渐减少

$$\Delta W_k = \Delta W_p$$

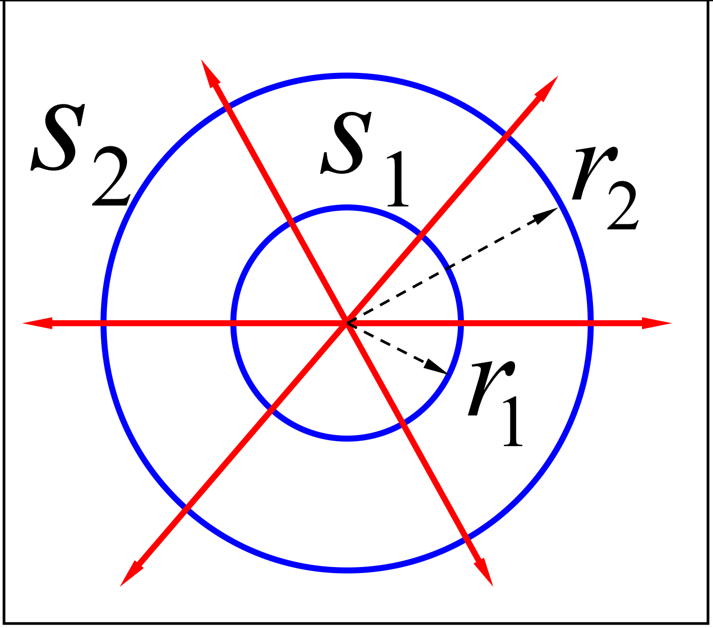
$$\Delta W = \Delta W_k + \Delta W_P = \rho \Delta V A^2 \omega^2 \sin^2 \omega\left(t - \frac{x}{u}\right)$$

a点: $\upsilon = o$, $\Delta W_k = o$, $\Delta W_p = o$



b点: υ最大, ΔW_k最大, ΔW_p最大

例2　证明球面波的振幅与离开其波源的距离成反比（介质无吸收）



证:通过两个球面的平均能流相等.

$$\overline{\omega}_1 u S_1 = \overline{\omega}_2 u S_2$$

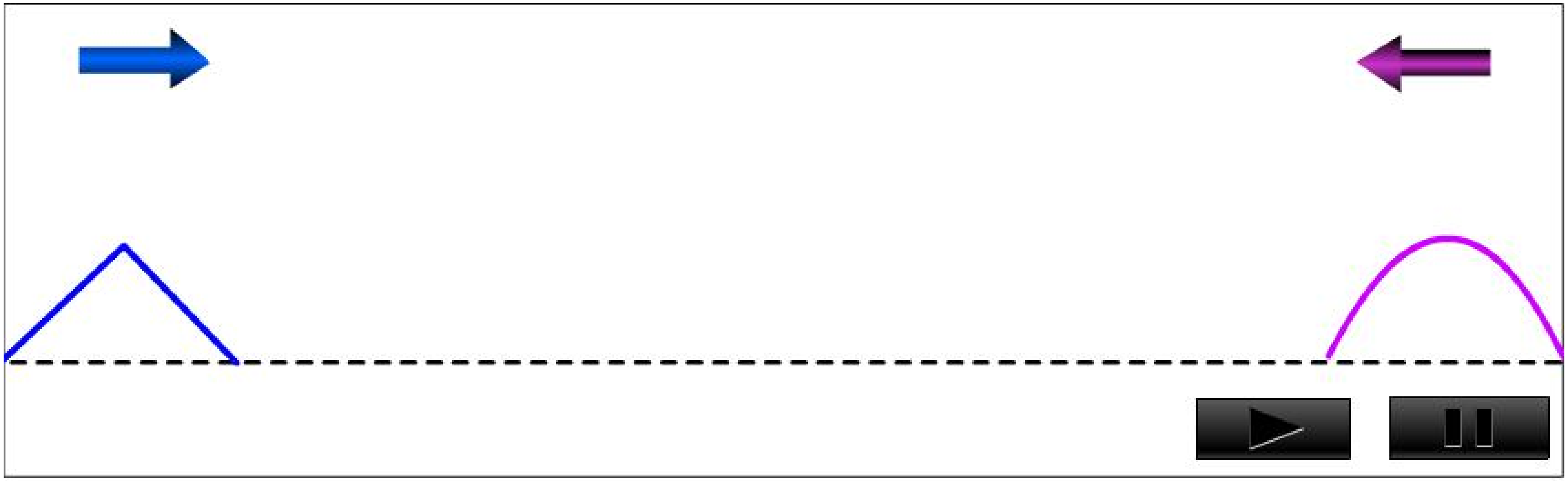
即

$$\frac{1}{2}\rho A_1^2 \omega^2 u 4\pi r_1^2 = \frac{1}{2}\rho A_2^2 \omega^2 u 4\pi r_2^2$$

$$\frac{A_1}{A_2} = \frac{r_2}{r_1}$$

5.6 波的干涉

一 波的叠加原理



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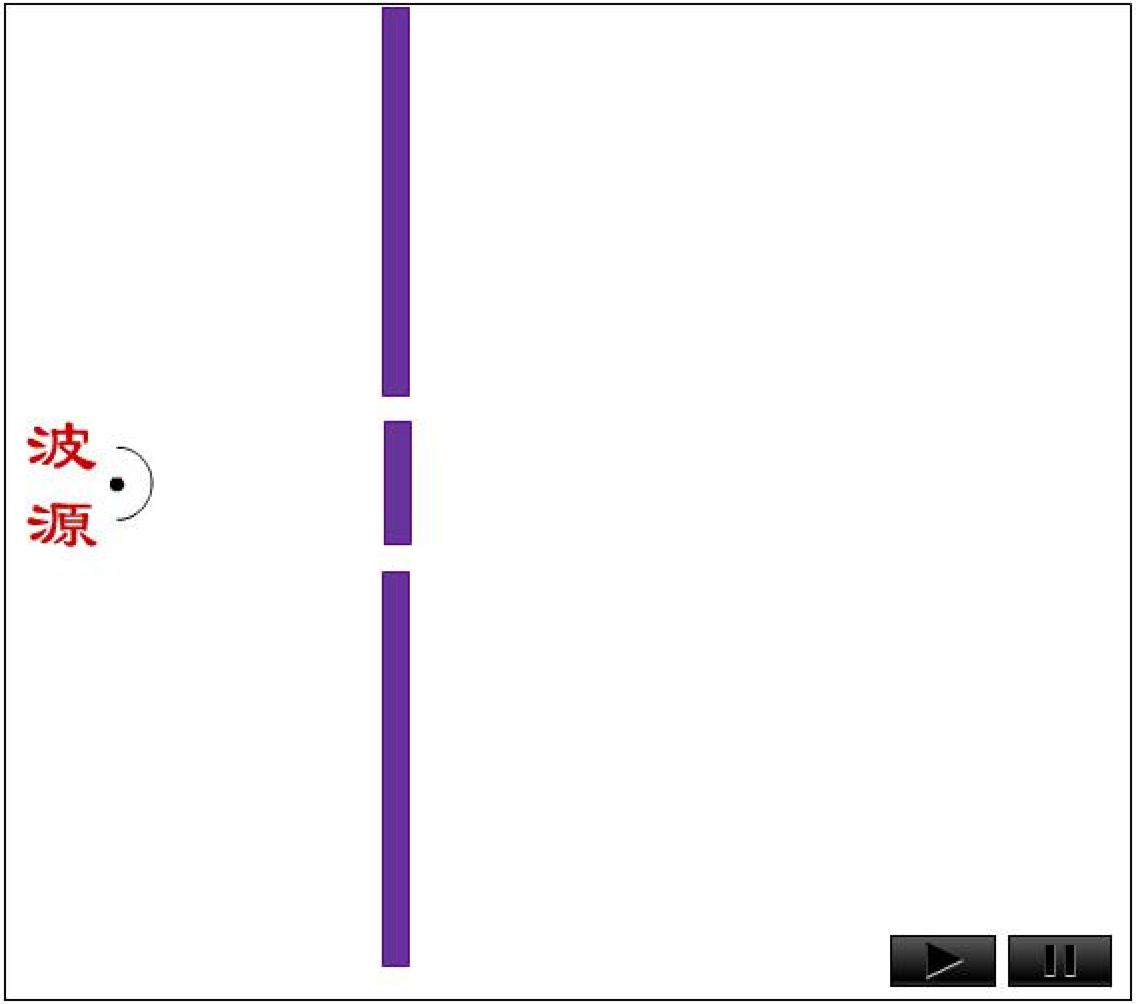
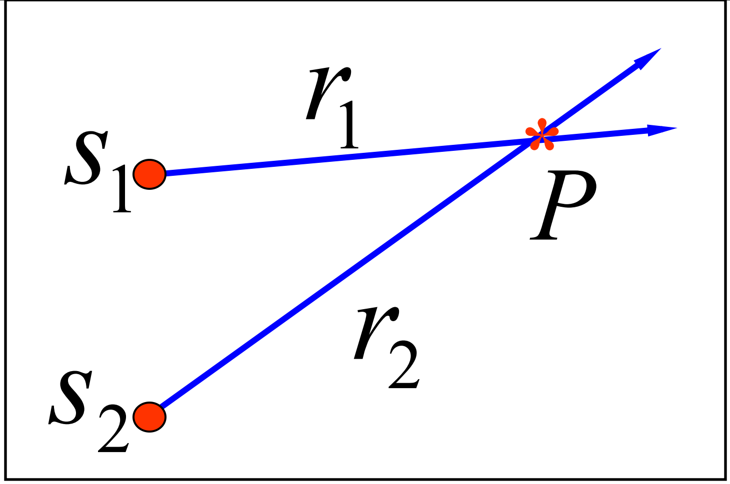
二 波的干涉



频率相同、振动方向平行、相位相同或相位差恒定的两列波相遇时，在空间形成稳定的加强或减弱—干涉



波源振动

$$\begin{cases} y_1 = A_1\cos(\omega t+\varphi_1) \\ y_2 = A_2\cos(\omega t+\varphi_2) \end{cases}$$

点*P* 的两个分振动

$$\begin{cases} y_{1p} = A_1\cos(\omega t + \boxed{\varphi_1 - 2\pi\frac{r_1}{\lambda}}) \\ y_{2p} = A_2\cos(\omega t + \boxed{\varphi_2 - 2\pi\frac{r_2}{\lambda}}) \end{cases}$$

$$y_p = y_{1p} + y_{2p} = A\cos(\omega t + \phi)$$

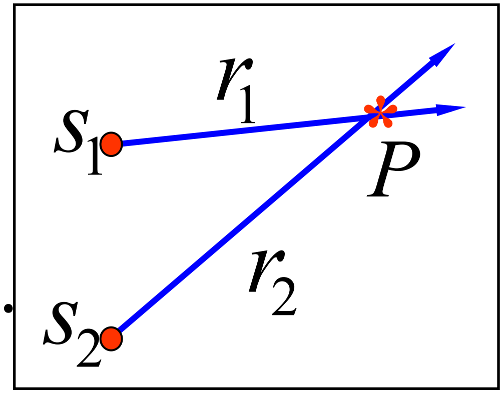
$$\tan\phi = \frac{A_1\sin(\varphi_1 - \frac{2\pi r_1}{\lambda}) + A_2\sin(\varphi_2 - \frac{2\pi r_2}{\lambda})}{A_1\cos(\varphi_1 - \frac{2\pi r_1}{\lambda}) + A_2\cos(\varphi_2 - \frac{2\pi r_2}{\lambda})}$$

$$A = ???$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\Delta\phi}$$

$$\Delta\phi=\varphi_2-\varphi_1-2\pi\frac{r_2-r_1}{\lambda}$$

常量



1）

$$\Delta\phi=\pm 2k\pi \quad k=0,1,2,\cdots$$

$$A=A_1+A_2$$ 振动始终加强

$$\Delta\phi=\pm(2k+1)\pi \quad k=0,1,2,\cdots$$

$$A=\left|A_1-A_2\right|$$ 振动始终减弱

$\Delta\phi=$ 其他 $\left|A_1-A_2\right|<A<A_1+A_2$

波程差 $\delta=r_2-r_1$

若 $\varphi_1=\varphi_2$ 则 $\Delta\phi=-2\pi\dfrac{\delta}{\lambda}$

2）

$$\delta=\pm k\lambda \quad k=0,1,2,\cdots$$

$$A=A_1+A_2$$ 振动始终加强

$$\delta=\pm(2k+1)\lambda/2 \quad k=0,1,2,\cdots$$

$$A=\left|A_1-A_2\right|$$ 振动始终减弱

$\delta=$ 其他 $\left|A_1-A_2\right|<A<A_1+A_2$