谐振子的完备性、

$\psi_n = N_n e^{-\frac{1}{2}\alpha^2 x^2} H_n(\alpha x)$

$\xi = \alpha x,\ \alpha = \sqrt{\frac{m\omega}{\hbar}}\quad N_n = \left(\frac{\alpha}{\sqrt{\pi}\, 2^n n!}\right)^{\frac{1}{2}}$

Hermit 递推关系，求证.

$$x\psi_n = \frac{1}{\alpha}\left[\sqrt{\frac{n+1}{2}}\psi_{n+1} + \sqrt{\frac{n}{2}}\psi_{n-1}\right] \quad ①$$

$$x^2\psi_n = \frac{1}{2\alpha^2}\left[\sqrt{(n+1)(n+2)}\,\psi_{n+2} + (2n+1)\psi_n + \sqrt{n(n-1)}\,\psi_{n-2}\right] \quad ②$$

并由此证明 在 ψ_n 下, $\bar{x}=0$, $\bar{V} = \frac{1}{2}E_n$

已知, $H_{n+1}(\alpha x) - 2\alpha x H_n(\alpha x) + 2nH_{n-1}(\alpha x) = 0$

$H_n = \frac{1}{N_n} e^{\frac{1}{2}\alpha^2 x^2}\psi_n$ 代入化简，即有 ①式

①式 再由 x算符作用.亦可得 ②式

根据.平均值的求法. 求 $\bar{x}_n = \int \psi_n^* x \psi_n\, dx$ $\quad V = \frac{1}{2}kx^2$

$\bar{V} = \int_{-\infty}^{+\infty} \psi_n^* \frac{1}{2}kx^2 \psi_n\, dx$

利用 $\frac{dH_n}{d\xi} = 2nH_{n-1}$ 和 $x\psi_n$ 证明

$$\frac{d}{dx}\psi_n = \alpha\left(\sqrt{\frac{n}{2}}\psi_{n-1} - \sqrt{\frac{n+1}{2}}\psi_{n+1}\right), \quad \frac{d^2}{dx^2}\psi_n = \cdots$$

以及 ψ_n 下, $\bar{V} = \bar{T} = \frac{1}{2}E_n$. $\bar{p} = 0$

eg. $V'(x) = V(x) + C$, 问粒子能量本征态、本征值如何改变.

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V\right)\psi = E\psi$$

$$\psi'' + \frac{2m}{\hbar^2}(E-V)\psi = 0.$$

$V \to V + C$.

$$\frac{2mE}{\hbar^2} - C = \frac{2mE'}{\hbar^2}$$

$$E' = E - \frac{\hbar^2}{2m}C$$

$$E = E' + C.$$