$f = -kx \qquad M = -mgl\theta$

力(矩)的大小与(相对于平衡位置)位移成正比,方向始终指向平衡位置———— 线性恢复力(矩)

物体在线性恢复力(矩)作用下的运动——谐振动

弹簧振子 $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ 谐振动动力学方程

单　　摆 $\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$

复　摆 $\frac{d^2\theta}{dt^2} + \frac{mgl}{J}\theta = 0$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

$$\left\{\begin{array}{l} \omega = \sqrt{\frac{k}{m}} \\ \omega = \sqrt{\frac{g}{l}} \\ \omega = \sqrt{\frac{mgl}{J}} \end{array}\right.$$

只与系统本身有关

谐振动的运动方程

$$\frac{d^2x}{dt^2}+\omega^2x=0 \longrightarrow x=A\cos(\omega t+\varphi)$$

$$v=\frac{dx}{dt}=-\omega A\sin(\omega t+\varphi)$$

$$a=\frac{dv}{dt}=-\omega^2 A\cos(\omega t+\varphi)$$

数学式

$$T=\frac{2\pi}{\omega}$$

数学式：$\nu=1/T$

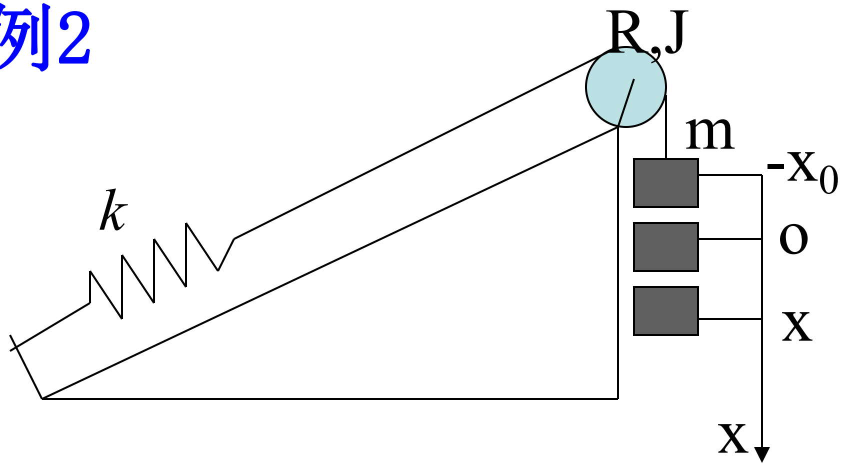
数学式$:\omega=2\pi\nu$

$$\begin{cases} A=\sqrt{x_0^2+v_0^2/\omega^2} \\ \phi=tg^{-1}(-v_0/\omega x_0)=[-\pi,+\pi) \end{cases}$$

证明系统作简谐振动的方法：

- 1. 选系统，找到平衡位置（系统所受合力为零）
- 2. 建立坐标系，（以平衡位置为原点）
- 3. 在任意位移处(x)进行受力分析
- 4. 写出合力的表达式 $F=-Kx$
- 动力学的微分方程 $\frac{d^2x}{dt^2}+\omega^2 x=0$
- 方程的解 $X=A\cos(\omega t+\varphi)$

例2



已知：初态时弹簧处于原长

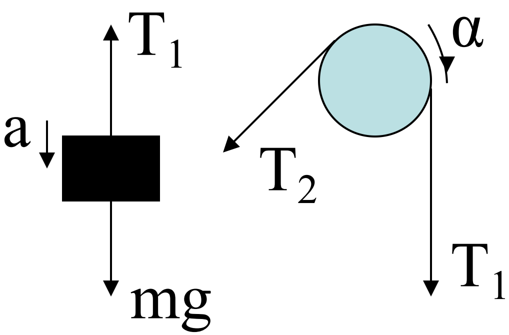
（1）证明物块作谐振动，

（2）写出振动表达式。

解:(1).确定平衡位置

$$mg = kx_0 \Rightarrow x_0 = \frac{mg}{k} \cdots\cdots(1)$$

(2).写出任意位置处物块的加速度



$$mg - T_1 = ma \cdots\cdots(2)$$

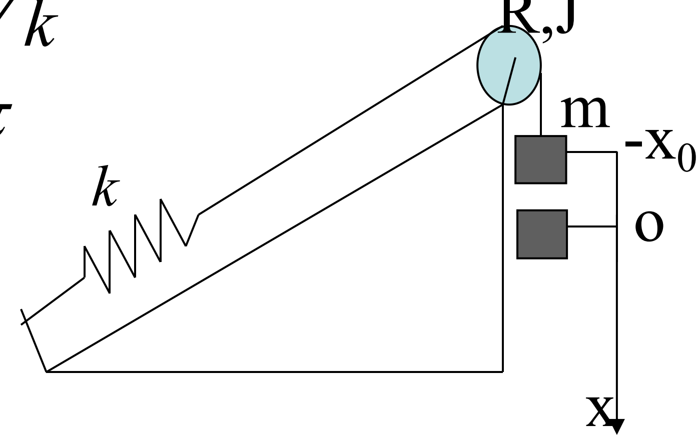
$$(T_1 - T_2)R = J\alpha = J\frac{a}{R} \cdots(3)$$

$$T_2 = k(x_0 + x\ ) \cdots\cdots(4)$$

$$\left.\begin{array}{l}(1)\\(2)\\(3)\\(4)\end{array}\right\} \Rightarrow a = -\frac{kR^2}{J + mR^2}x \text{——谐振动}$$

$$\left.\begin{array}{l}a = -\frac{kR^2}{J+mR^2}x\\ a = -\omega^2 x\end{array}\right\} \omega = R\sqrt{\frac{k}{J + mR^2}}$$

$*$初态为$t=0\begin{cases} x_0 = -{mg}/{k} \\ v_0 = 0 \end{cases} \Rightarrow \begin{cases} A = {mg}/{k} \\ \phi = \pi \end{cases}$



$$x = \frac{mg}{k}\cos(R\sqrt{\frac{k}{J+mR^2}}t+\pi)$$

$*$平衡位置为$t=0$，则：$x_0 = 0$

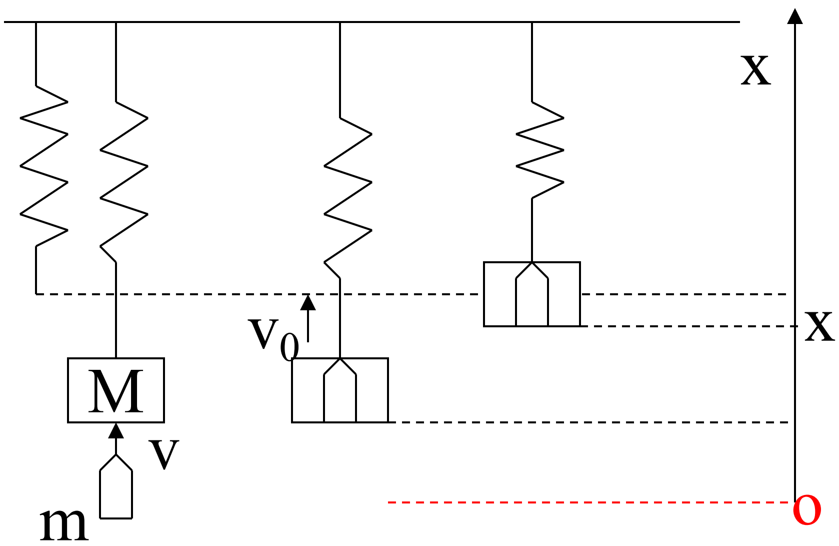
$$mgx_0 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv_o{}^2 + \frac{1}{2}J(\frac{v_0}{R})^2 \Rightarrow v_0 \qquad \Rightarrow \begin{cases} A = {mg}/{k} \\ \phi = -\dfrac{\pi}{2} \end{cases}$$

$$x = \frac{mg}{k}\cos(R\sqrt{\frac{k}{J+mR^2}}t-\frac{\pi}{2})$$

【例3】弹簧振子（M ，k）竖直悬挂，处于平衡，子弹（m）以速度v由下而上射入物块并嵌入其内。

求：(1).物块振动的T和A;

(2).物块从开始运动到最远处所需的时间。



解：(1).x处物块动力学方程

$$(m+M)\frac{d^2x}{dt^2} = -(m+M)g + k\left[\frac{(m+M)g}{k} - x\right] = -kx$$

$$\therefore \omega = \sqrt{\frac{k}{m+M}} \qquad T = 2\pi\sqrt{\frac{M+m}{k}}$$

$$* \text{初态为} t=0 \left\{ \begin{array}{l} x_0 = mg/k \\ v_0 = \dfrac{mv}{m+M} \end{array} \right.$$

（可由动量守恒得）

$$\omega = \sqrt{\frac{k}{m+M}}$$

$$\Rightarrow \quad A = \sqrt{x_0^2 + v_0^2/\omega^2}$$

$$(2). \quad x = A\cos(\omega t + \varphi)$$

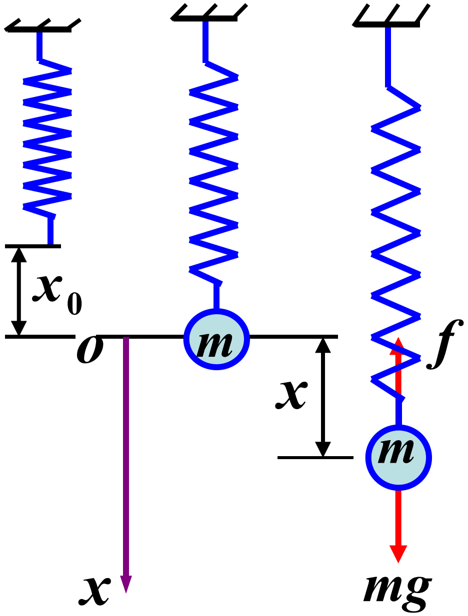
最远点：$x = A$，　即 $\omega t + \varphi = 0 \Rightarrow t = -\dfrac{\varphi}{\omega}$

$$\because \varphi = tg^{-1}(-\frac{v_0}{\omega x_0})$$

$$\left\{ \begin{array}{l} \omega = \sqrt{\dfrac{k}{m+M}} \\ x_0 = mg/k \\ v_0 = \dfrac{mv}{m+M} \end{array} \right.$$

$$\varphi \Rightarrow t$$

讨论：



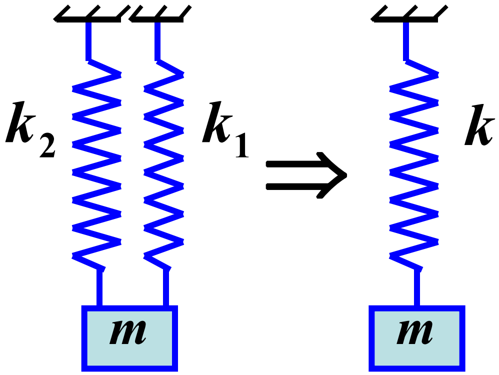
$$mg = kx_0$$

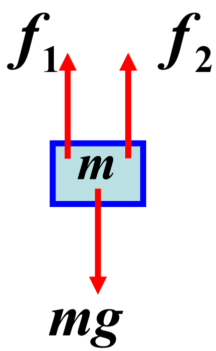
$$mg - f = m\frac{d^2x}{dt^2}$$

$$mg - k(x + x_0) = m\frac{d^2x}{dt^2}$$

$$-kx = m\frac{d^2x}{dt^2} \qquad \frac{d^2x}{dt^2} + \frac{k}{m}x = o$$

$$\omega^2 = \frac{k}{m} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$





并联：两弹簧伸长量相同(x)

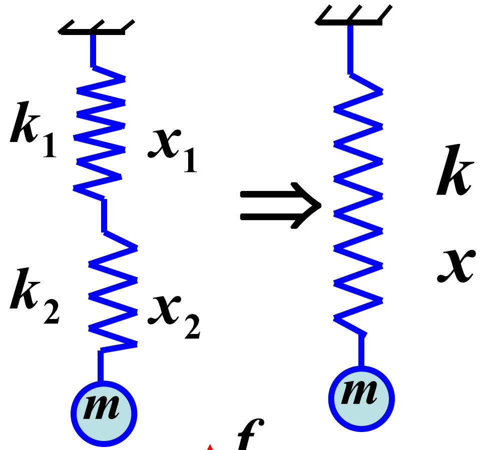
$$-k_1x-k_2x=m\frac{d^2x}{dt^2}$$

$$-(k_1+k_2)x=m\frac{d^2x}{dt^2}$$

$$-kx=m\frac{d^2x}{dt^2}$$

比较得 $k=k_1+k_2$

$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m}{k_1+k_2}}$$



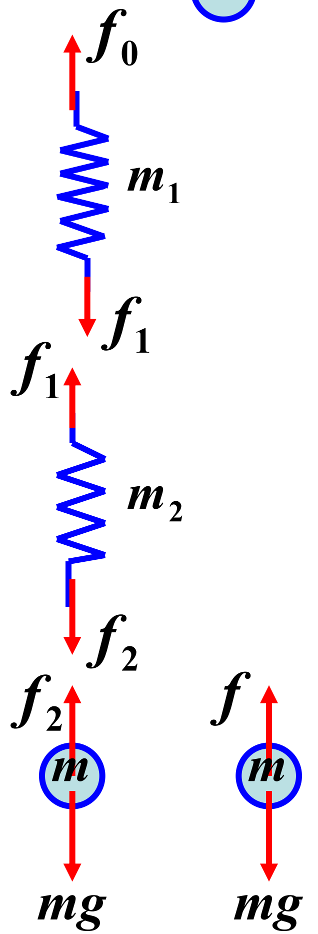
串联：两弹簧伸长量不同

$$x = x_1 + x_2$$

对m_1：$f_1 - f_0 = m_1 a = o \Rightarrow f_1 = f_0$

对m_2：$f_2 - f_1 = m_2 a = o \Rightarrow f_2 = f_1$

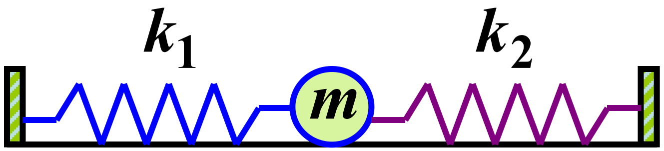
轻弹簧串联，受力相同

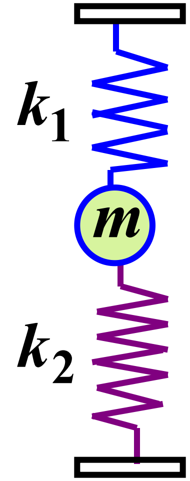


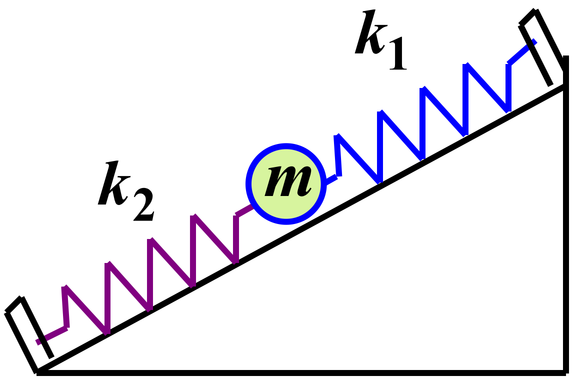
$$f = kx\,,\quad f = f_2 = k_2 x_2\,,\quad f = f_1 = k_1 x_1$$

$$\frac{f}{k} = \frac{f}{k_1} + \frac{f}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

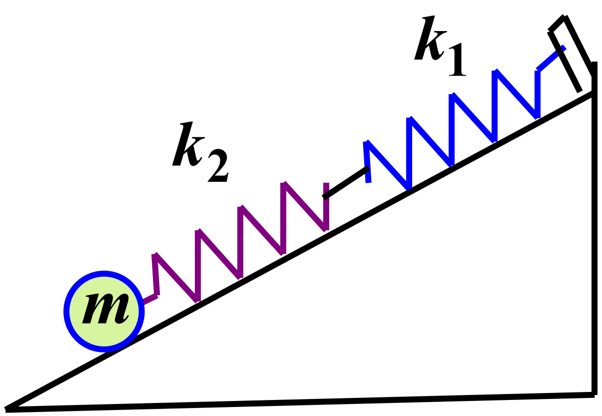
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}}$$





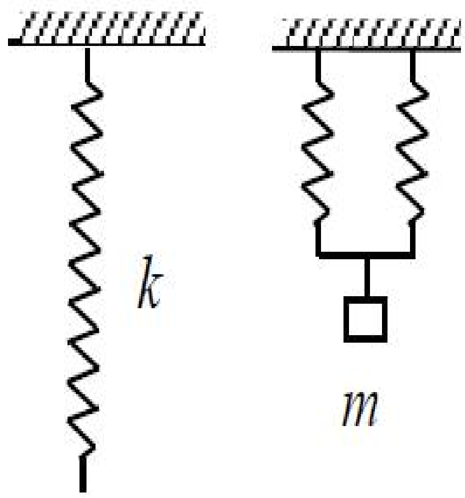


$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m}{k_1+k_2}}$$



$$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

一劲度系数为 k 的轻弹簧截成三等份，取出其中的两根，将它们并联，下面挂一质量为 m 的物体，如图所示。则振动系统的频率为

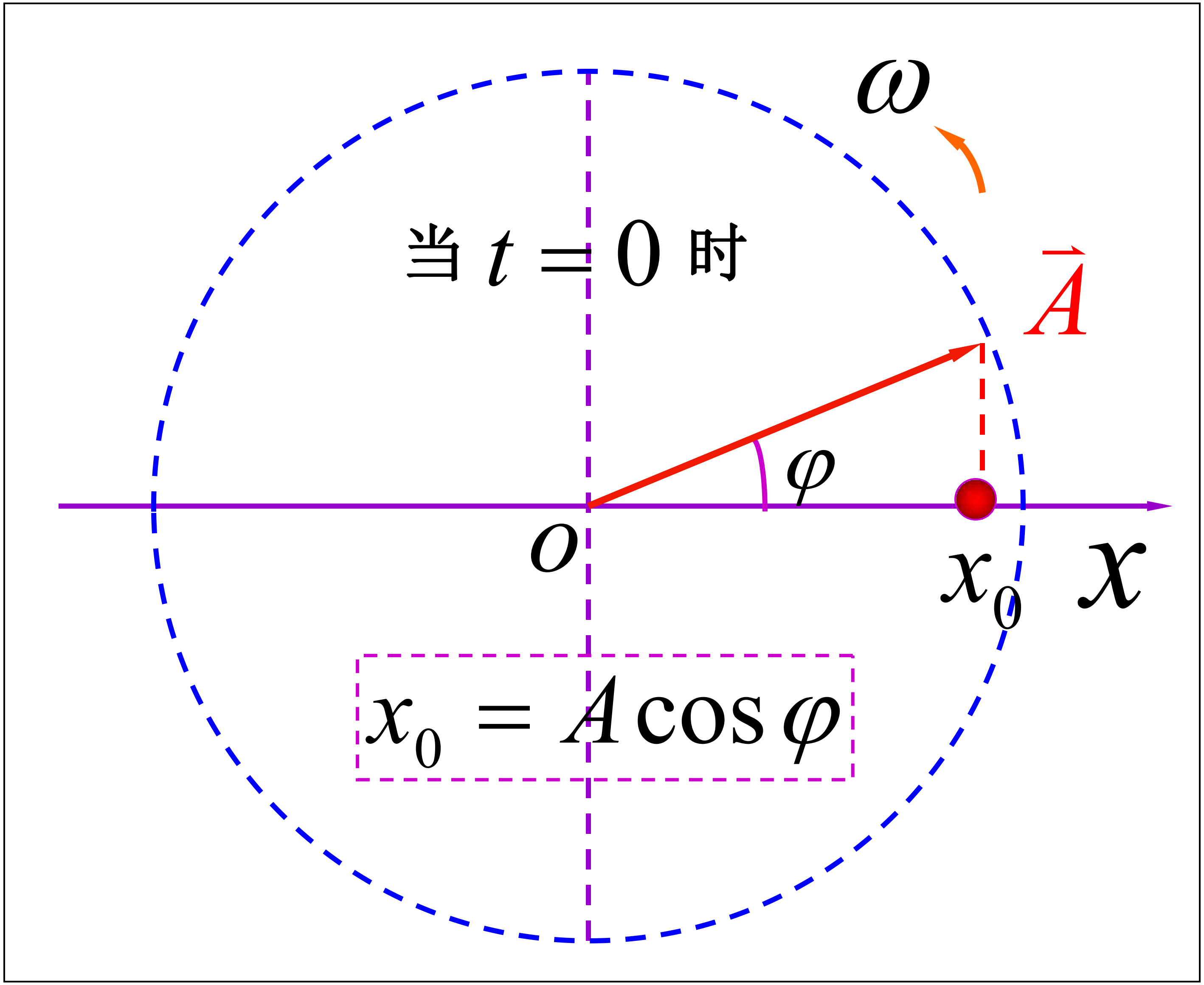


(A) $\frac{1}{2\pi}\sqrt{\frac{k}{3m}}$.　　(B) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

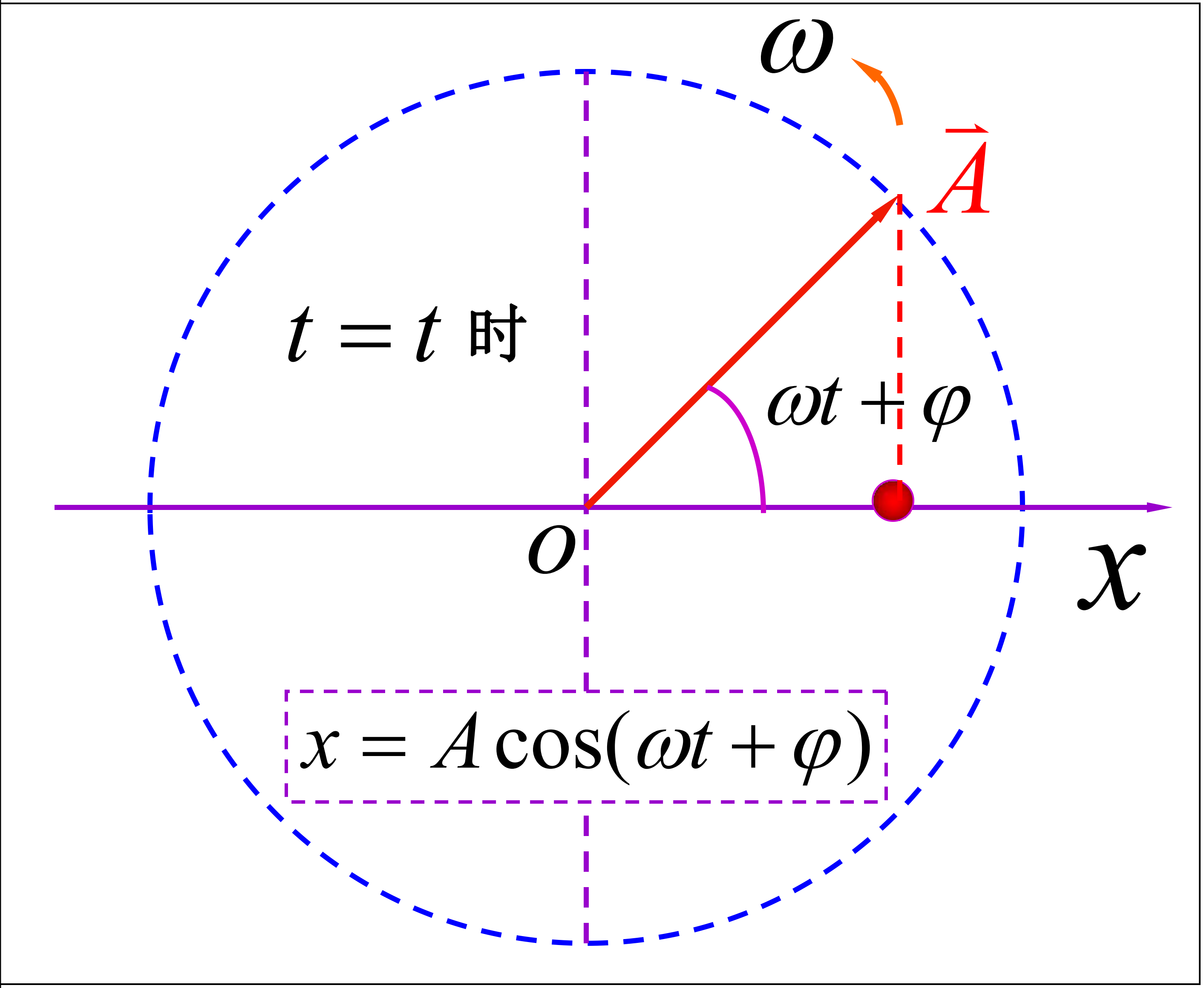
(C) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$.　　(D) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$.　　[　　]

答案：D

三. 谐振动的旋转矢量



以 o 为原点旋转矢量 $\vec{A}$ 的端点在 x 轴上的投影点的运动为简谐运动.



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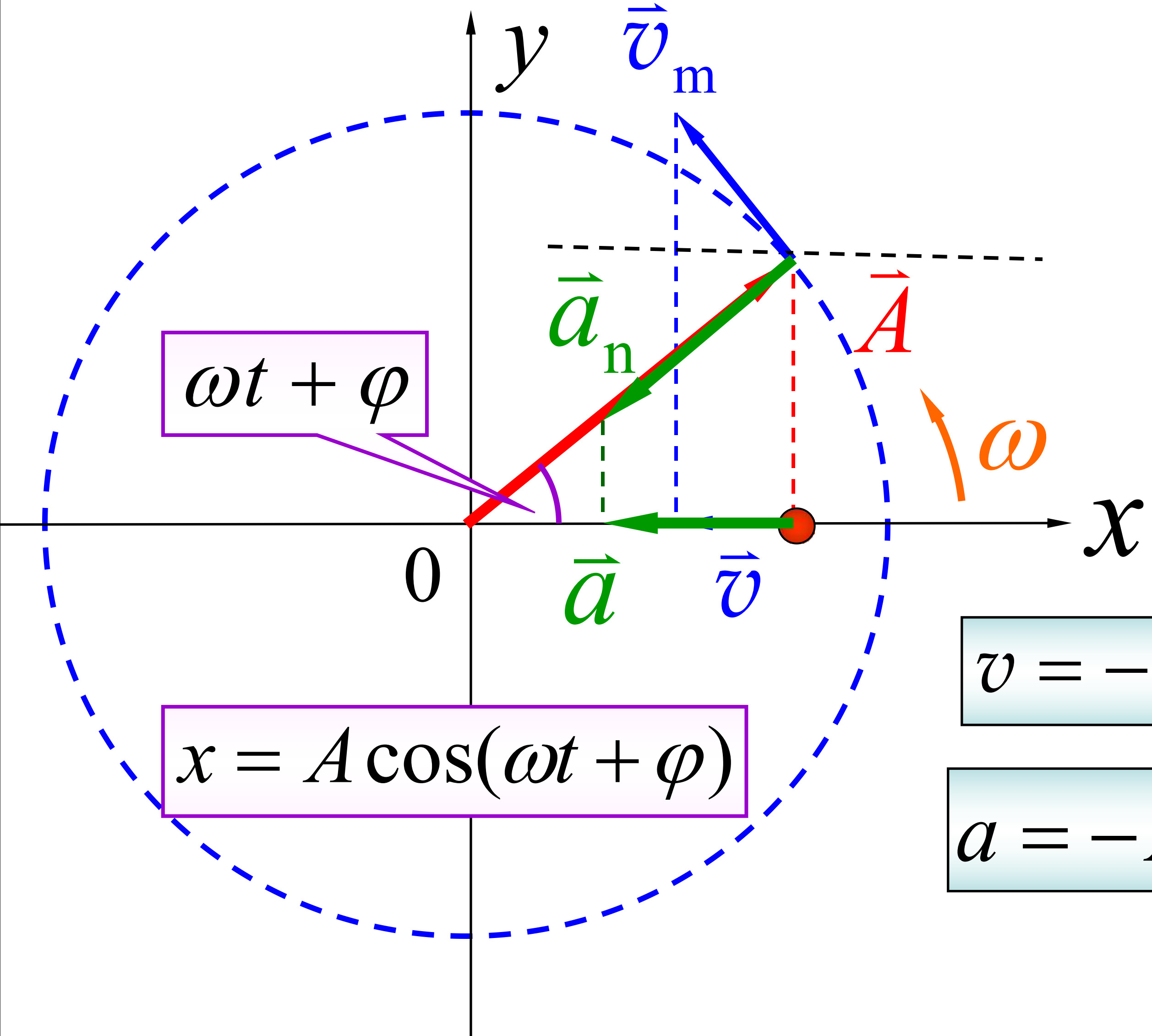


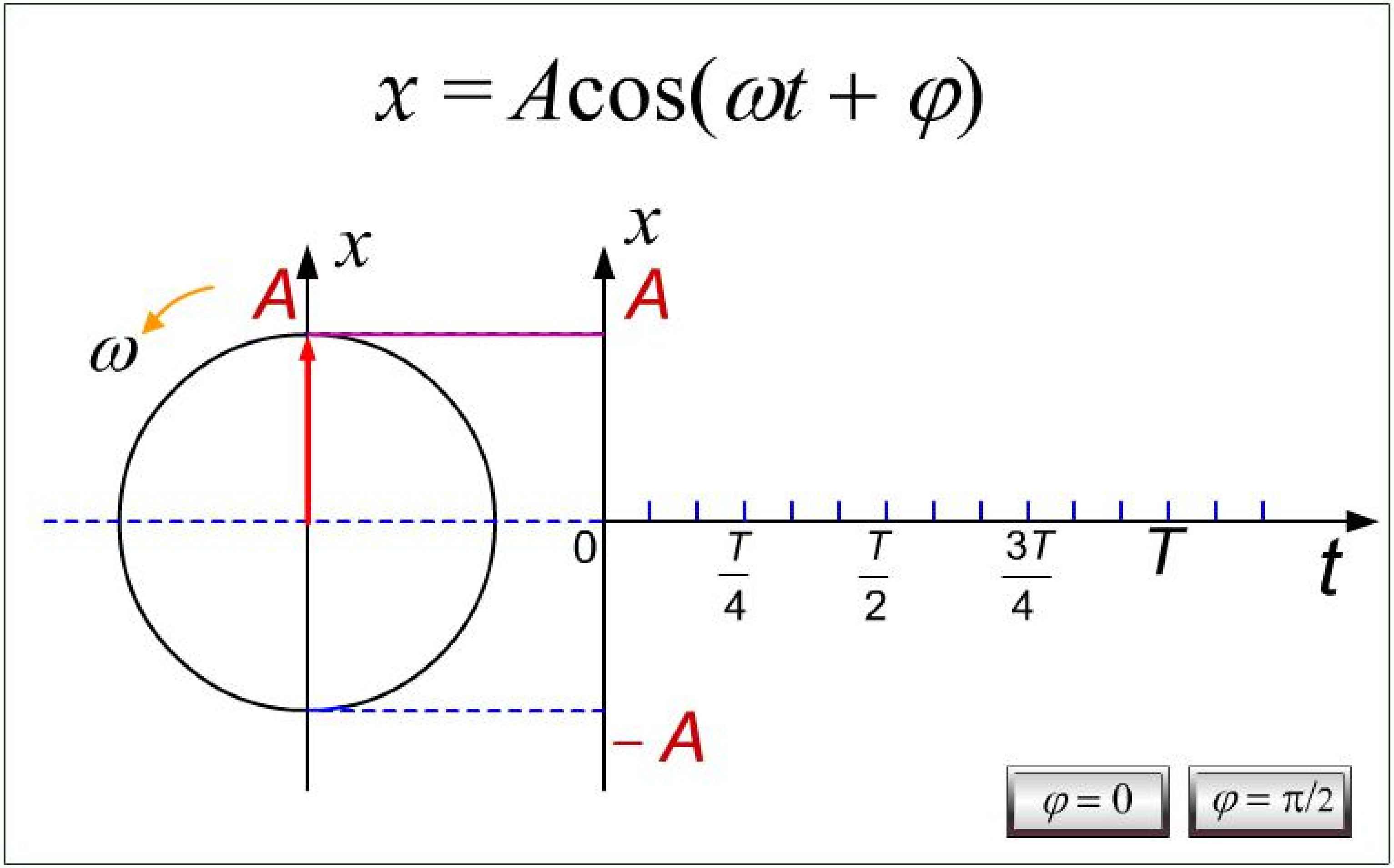
$$v_m = A\omega$$

$$a_n = A\omega^2$$

$$v = -A\omega\sin(\omega t+\varphi)$$

$$a = -A\omega^2\cos(\omega t+\varphi)$$

用旋转矢量图画简谐运动的 $x-t$ 图



$T = 2\pi/\omega$（旋转矢量旋转一周所需的时间）

讨论

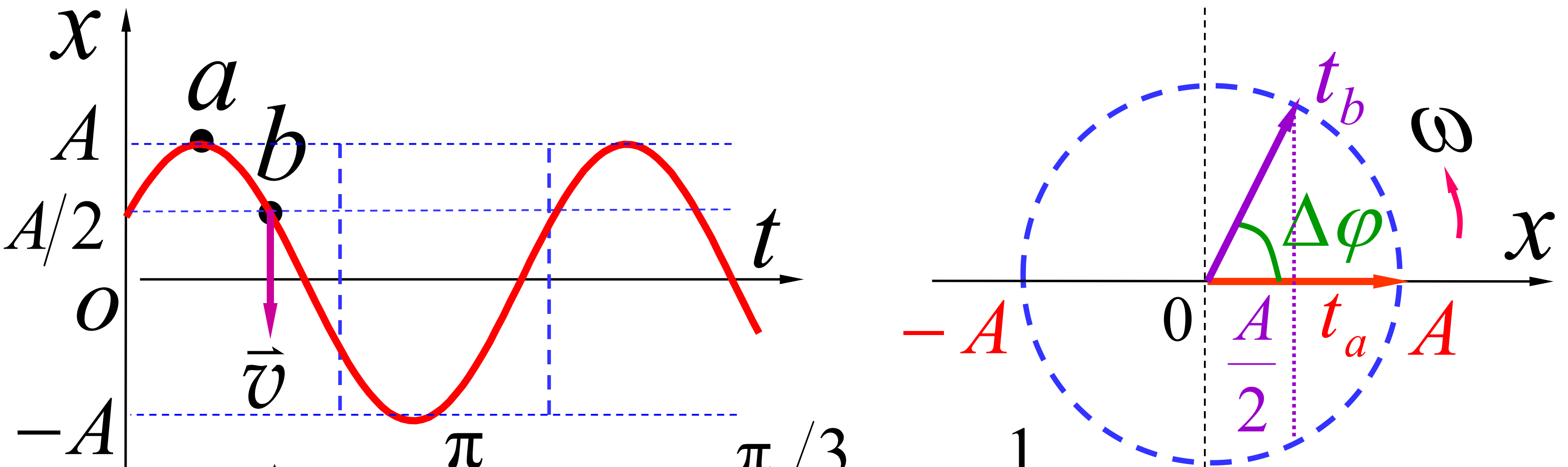
➢ 相位差：表示两个相位之差 .

1）对同一简谐运动，相位差可以给出两运动状态间变化所需的时间. $\Delta\varphi = (\omega t_2 + \varphi) - (\omega t_1 + \varphi)$

$$x = A\cos(\omega t_1 + \varphi)$$

$$x = A\cos(\omega t_2 + \varphi)$$

$$\Delta t = t_2 - t_1 = \frac{\Delta\varphi}{\omega}$$



$$\Delta\varphi = \frac{\pi}{3} \qquad \Delta t = \frac{\pi/3}{2\pi}T = \frac{1}{6}T$$

2）对于两个同频率的简谐运动，相位差表示它们间步调上的差异.（解决振动合成问题）

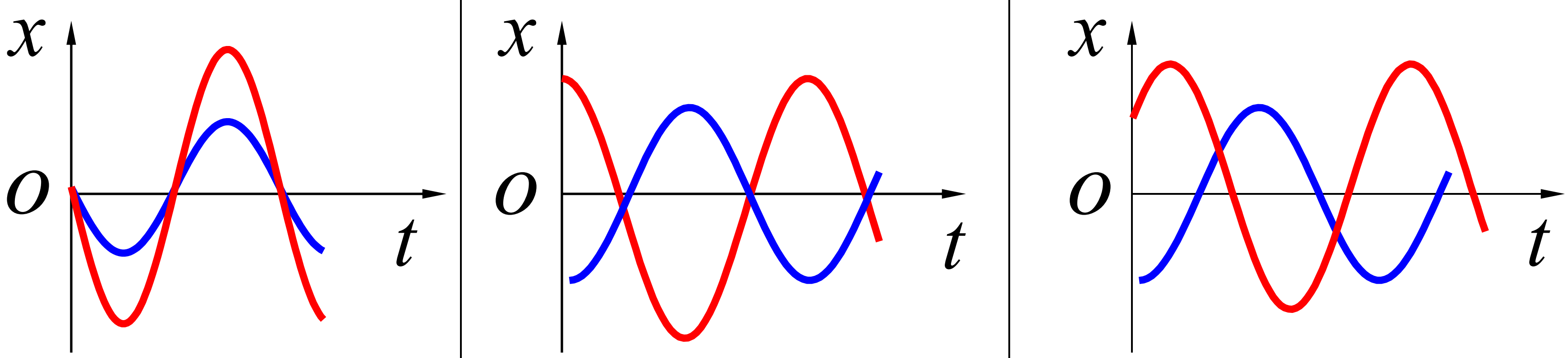
$$x_1 = A_1 \cos(\omega t + \varphi_1) \qquad x_2 = A_2 \cos(\omega t + \varphi_2)$$

$$\Delta\varphi = (\omega t + \varphi_2) - (\omega t + \varphi_1) \qquad \boxed{\Delta\varphi = \varphi_2 - \varphi_1}$$

$\Delta\varphi = 0$ 同步

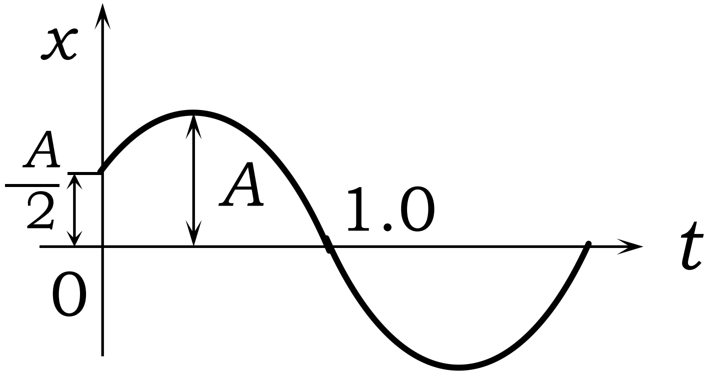
$\Delta\varphi = \pm\pi$ 反相

$\Delta\varphi$为其它 $\begin{cases} \text{超前} \\ \text{落后} \end{cases}$



［ 例4 ］　一谐振动的振动曲线如图所示，
求:振动表达式x=Acos(ωt+ φ)中的ω和 φ

解：由图形可知：A，



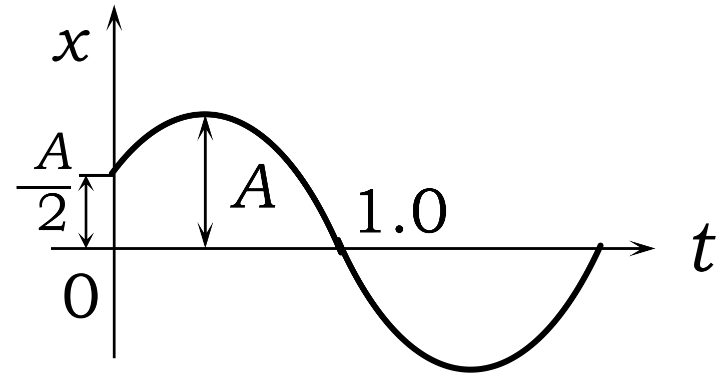
$t=0$:　　$x_0=\dfrac{A}{2}$，　$v_0>0$

$t=1$:　　$x_1=0, v_1<0$

a、解析法：

$$\left.\begin{aligned} t=0: \frac{A}{2}=A\cos\varphi \Rightarrow \varphi=\pm\frac{\pi}{3} \\ v_0=-A\omega\sin\varphi>0 \end{aligned}\right\} \Rightarrow \varphi=-\frac{\pi}{3}$$

$t=1: \quad x_1=0, v_1<0$



$$\varphi=-\frac{\pi}{3}$$

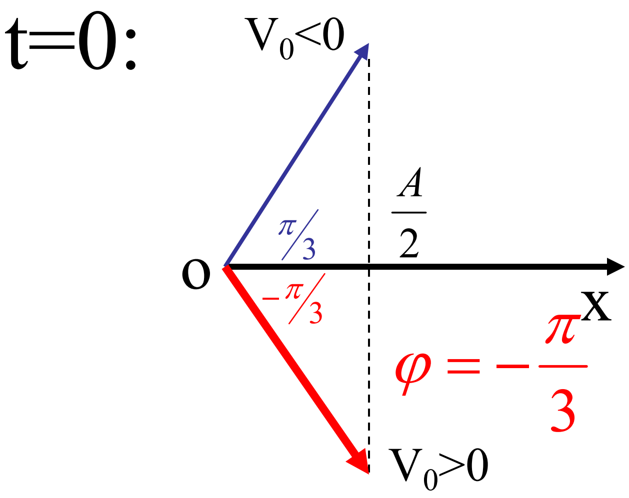
$$t=1: 0=A\cos(\omega-\frac{\pi}{3}) \Rightarrow \omega-\frac{\pi}{3}=\pm\frac{\pi}{2}$$

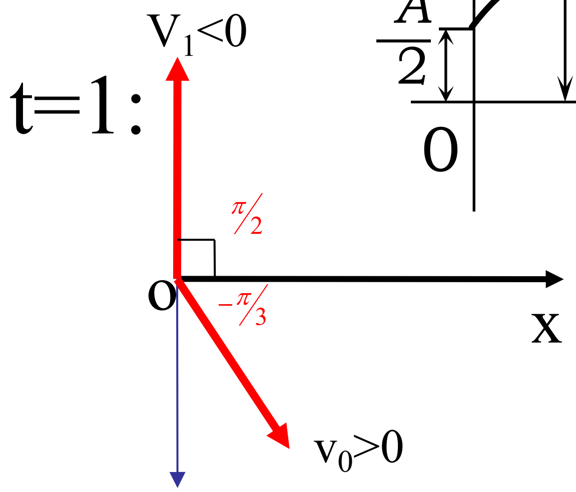
$$v_1=-A\omega\sin(\omega-\frac{\pi}{3})<0$$

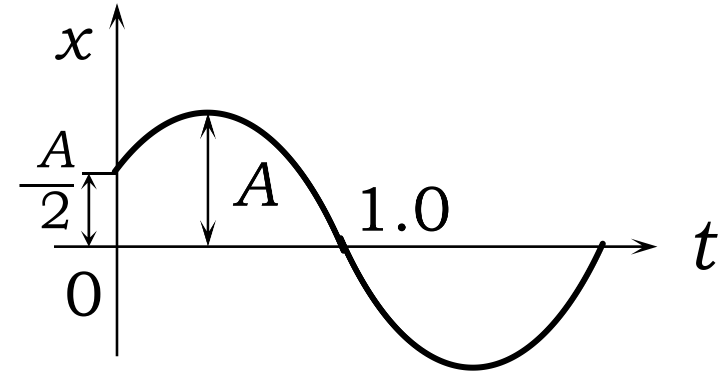
$$\Rightarrow \omega-\frac{\pi}{3}=\frac{\pi}{2}$$

$$\omega=\frac{5\pi}{6}$$

b. 旋转矢量法：







$$\left.\begin{aligned}\Delta\varphi&=\frac{\pi}{2}+\frac{\pi}{3}=\frac{5\pi}{6}\\ \Delta\varphi&=\omega\Delta t=\omega\end{aligned}\right\}\quad \omega=\frac{5\pi}{6}$$