*简谐振动的三个相互等价的定义：

$$F=-kx \Rightarrow \frac{d^2x}{dt^2}+\omega^2 x=0 \Rightarrow x=A\cos(\omega t+\varphi)$$

其中：x——相对于平衡位置（坐标原点）的位移。

*谐振动的特征量

弹簧振子 $\omega=\sqrt{\frac{k}{m}}$ 单摆 $\omega=\sqrt{\frac{g}{l}}$ 复摆 $\omega=\sqrt{\frac{mgh}{J}}$

ω（T、ν）由振动体系内部性质决定。 $\omega=2\pi\nu=\frac{2\pi}{T}$

A，φ由初始条件确定

$$t=0\begin{cases} x_0=A\cos\varphi \\ v_0=-\omega A\sin\varphi \end{cases}$$

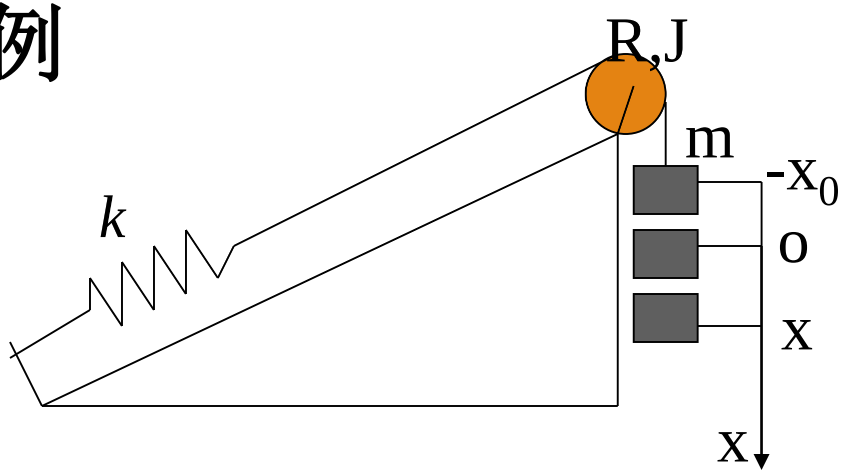
$$A=\sqrt{x_0^2+(v_0^2/\omega^2)}$$

$$\varphi=tg^{-1}\left(-\frac{v_0}{\omega x_0}\right)$$

约定：

φ ∈ (-π，π]

例



已知：初态时弹簧处于原长

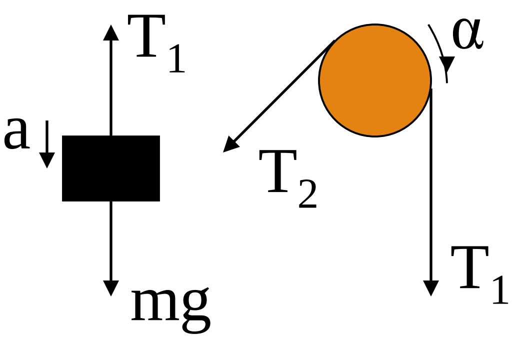
（1）证明物块作谐振动，

（2）写出振动表达式。

解:(1).确定平衡位置

$$mg = kx_0 \Rightarrow x_0 = \frac{mg}{k} \cdots\cdots(1)$$

(2).写出任意位置处物块的加速度



$$mg - T_1 = ma \cdots\cdots(2)$$

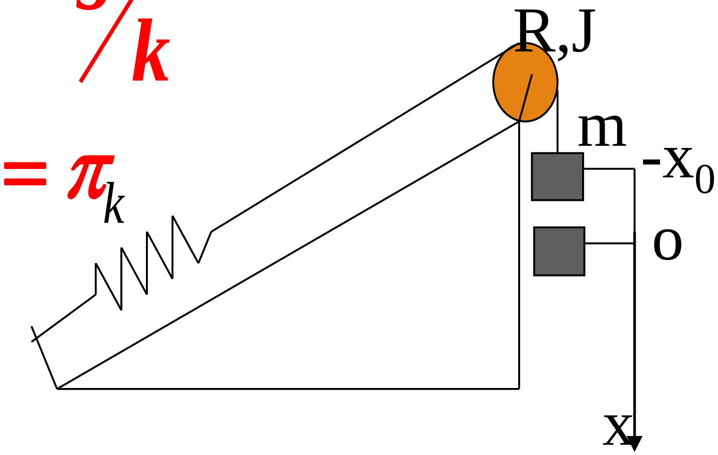
$$(T_1 - T_2)R = J\alpha = J\frac{a}{R} \cdots(3)$$

$$T_2 = k(x_0 + x\ ) \cdots\cdots(4)$$

$$\left.\begin{matrix}(1)\\(2)\\(3)\\(4)\end{matrix}\right\} \Rightarrow \left.\begin{matrix} a = -\dfrac{kR^2}{J + mR^2}x \\ a = -\omega^2 x \end{matrix}\right\} \omega = R\sqrt{\frac{k}{J + mR^2}}$$

——谐振动

$$*\text{初态为}\ t=0\begin{cases} x_{t=0} = -mg/k \\ v_{t=0} = 0 \end{cases} \Rightarrow \begin{cases} A = mg/k \\ \phi = \pi \end{cases}$$



$$x = \frac{mg}{k}\cos\left(R\sqrt{\frac{k}{J+mR^2}}t+\pi\right)$$

$$*\text{平衡位置为}\ t=0\text{，则：}x_{t=0}=0$$

$$mgx_0 = \frac{1}{2}kx_0{}^2 + \frac{1}{2}mv_{t=0}{}^2 + \frac{1}{2}J\left(\frac{v_{t=0}}{R}\right)^2 \Rightarrow v_{t=0}$$

$$\Rightarrow \begin{cases} A = mg/k \\ \varphi = -\dfrac{\pi}{2} \end{cases}$$

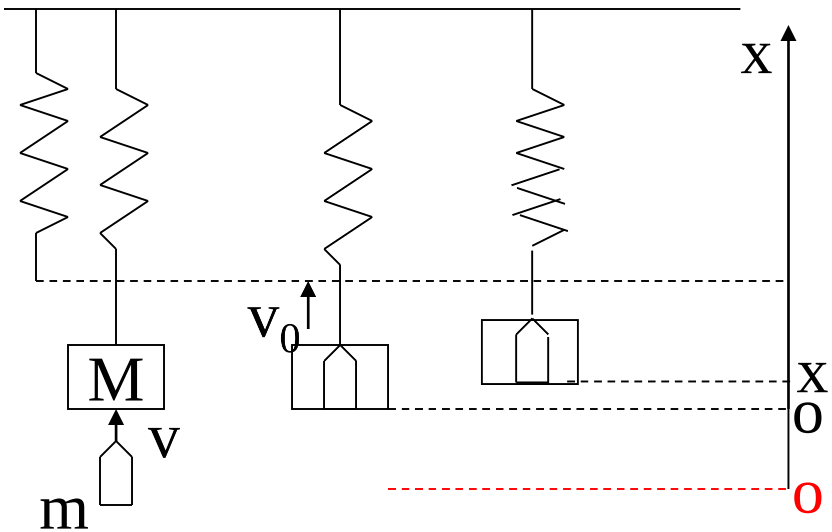
$$x = \frac{mg}{k}\cos\left(R\sqrt{\frac{k}{J+mR^2}}t-\frac{\pi}{2}\right)$$

$$A = \sqrt{x_0^2 + (v_0^2/\omega^2)}$$

$$\varphi = tg^{-1}\left(-\frac{v_0}{\omega\ x_0}\right)$$

【例】弹簧振子（M，k）竖直悬挂，处于平衡，子弹（m）以速度v由下而上射入物块并嵌入其内。求：(1).物块振动的T和A;

(2).物块从开始运动到最远处(上方）所需的时间。



解：(1).x处物块动力学方程

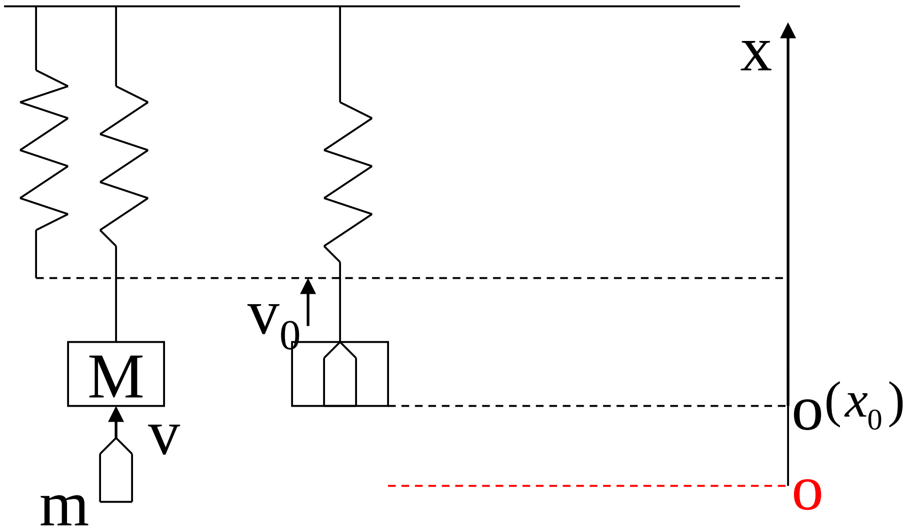
$$(m+M)\frac{d^2x}{dt^2}$$

$$=-(m+M)g+k(\frac{Mg}{k}-x)$$

$$=-mg-kx$$

正确解：$(m+M)\frac{d^2x}{dt^2}=-(m+M)g+k\left[\frac{(m+M)g}{k}-x\right]=-kx$

$$\therefore \omega=\sqrt{\frac{k}{m+M}} \qquad T=2\pi\sqrt{\frac{M+m}{k}}$$



$$*\text{初态为}t=0\begin{cases} x_0 = mg/k \\ v_0 = \dfrac{mv}{m+M} \end{cases}$$

（可由动量守恒得）

$$\omega = \sqrt{\frac{k}{m+M}}$$

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = \frac{mg}{k}\sqrt{1+\frac{kv^2}{(m+M)g^2}}$$

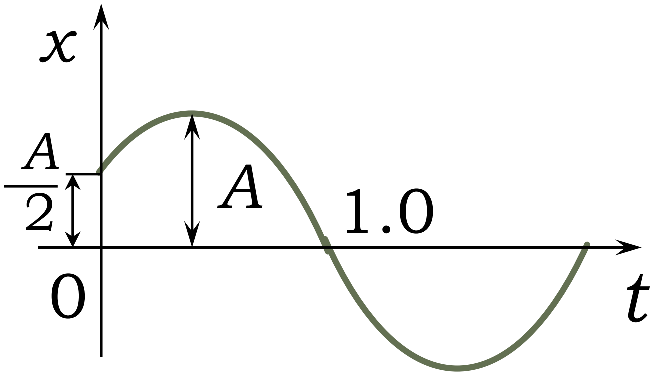
(2). $x = A\cos(\omega t + \varphi)$

最远点：$x = A$，　即 $\omega t + \varphi = 0 \Rightarrow t = -\dfrac{\varphi}{\omega}$

$$\left.\begin{array}{l} \because \varphi = tg^{-1}\left(-\dfrac{v_0}{\omega x_0}\right) \\ \begin{cases} \omega = \sqrt{\dfrac{k}{m+M}} \\ x_0 = mg/k \\ v_0 = \dfrac{mv}{m+M} \end{cases} \end{array}\right\} \varphi = -tg^{-1}\left(\frac{v}{g}\sqrt{\frac{k}{m+M}}\right)$$

$$\therefore t = \sqrt{\frac{m+M}{k}}\, tg^{-1}\left(\frac{v}{g}\sqrt{\frac{k}{m+M}}\right)$$

[例] 一谐振动的振动曲线如图所示，求:振动表达式x=Acos(ωt+φ)中的ω和φ。



1、解析法:

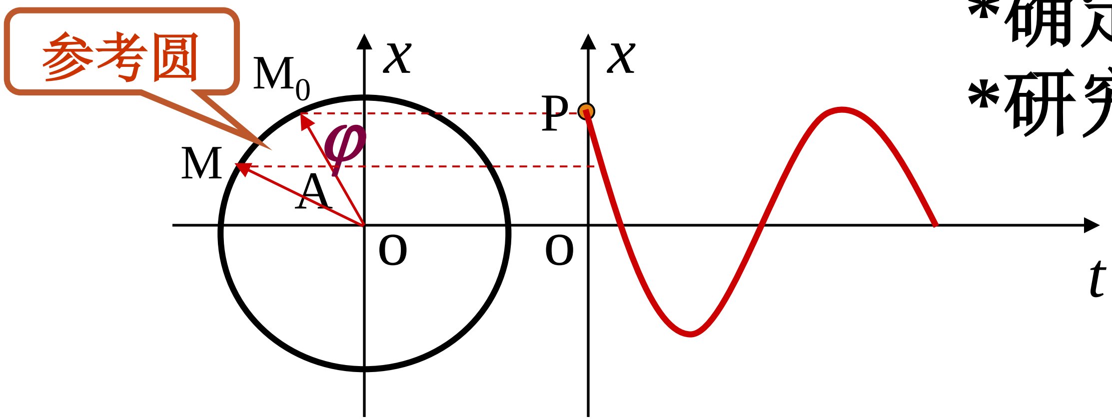
$$\left.\begin{array}{l} t=0:\ \dfrac{A}{2}=A\cos\varphi \Rightarrow \varphi=\pm\dfrac{\pi}{3} \\ v_0=-A\omega\sin\varphi>0 \end{array}\right\} \Rightarrow \varphi=-\frac{\pi}{3}$$

$$\left.\begin{array}{l} t=1:0=A\cos(\omega-\dfrac{\pi}{3}) \Rightarrow \omega-\dfrac{\pi}{3}=\pm\dfrac{\pi}{2} \\ v_1=-A\omega\sin(\omega-\dfrac{\pi}{3})<0 \end{array}\right\} \Rightarrow \omega-\frac{\pi}{3}=\frac{\pi}{2}$$

$$\Downarrow$$

$$\omega=\frac{5\pi}{6}$$

三、谐振动的旋转矢量表示法



*确定相位或初相；
*研究振动合成。

① $\vec{A}$ 的长度：振幅 A

② $\vec{A}$ 的旋转角速度：圆频率ω

③ $\vec{A}$ 的旋转的方向：逆时针向

④ 旋转矢量 $\vec{A}$ 与参考方向 x 的夹角：相位 $(\omega t+\varphi)$

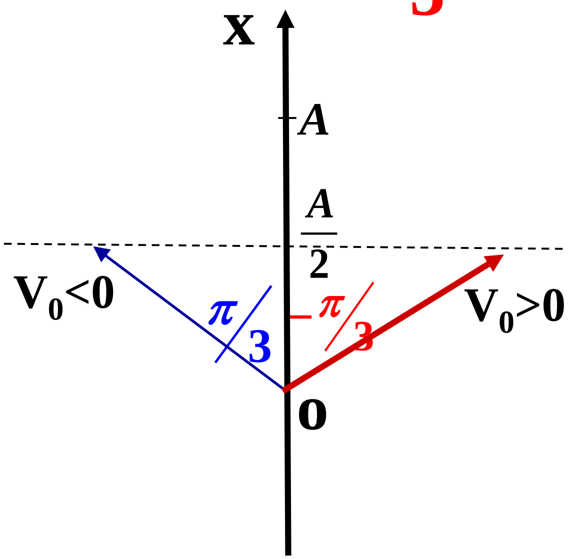
⑤ t=0时旋转矢量 $\vec{A}$ 与参考方向 x 的夹角：初相位 (φ)

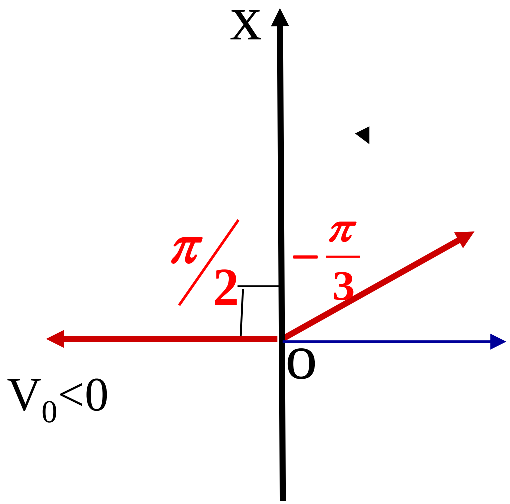
⑥ M 点在x 轴上投影点P 的运动规律：$x=A\cos(\omega t+\varphi)$

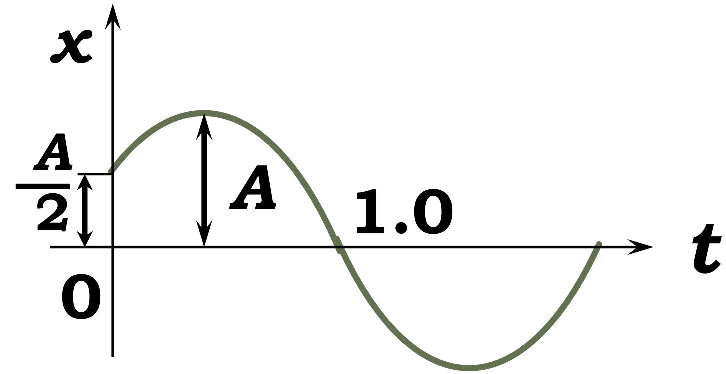
2.旋转矢量法

t=0: $\varphi=-\dfrac{\pi}{3}$

t=1: $\varphi_1=\dfrac{\pi}{2}$

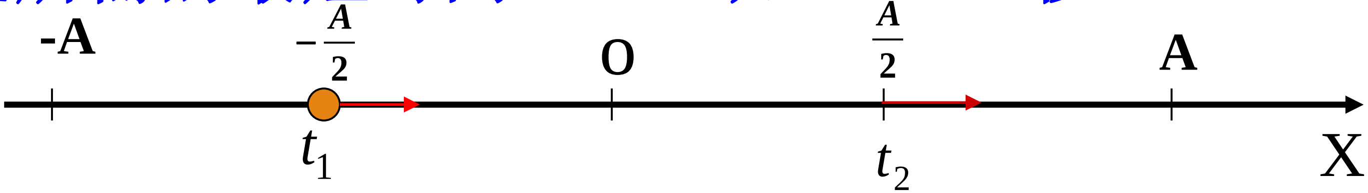






$$\left.\begin{aligned}\Delta\varphi&=\frac{\pi}{2}+\frac{\pi}{3}=\frac{5\pi}{6}\\ \Delta\varphi&=\omega t=\omega\end{aligned}\right\}\omega=\frac{5\pi}{6}$$

[例]　一弹簧振子由$-A$处释放，求振子从$-\dfrac{A}{2}$处向右运动到$\dfrac{A}{2}$处所需的最短时间。（已知：$T=2$秒）



t_1：　$x_1=-\dfrac{A}{2},\ v_1>0$　　　t_2：　$x_2=\dfrac{A}{2},v_2>0$

1、解析法：

$$t_{1:}\ -\frac{A}{2}=A\cos(\omega t_1+\varphi)\Rightarrow(\omega t_1+\varphi)=\pm\frac{2\pi}{3}$$

$$v_1=-A\omega\sin(\omega t_1+\varphi)>0$$

$$\left.\begin{matrix}\\ \\ \end{matrix}\right\}(\omega t_1+\varphi)=-\frac{2\pi}{3}$$

$$t_{2:}\ \frac{A}{2}=A\cos(\omega t_2+\varphi)\Rightarrow(\omega t_2+\varphi)=\pm\frac{\pi}{3}$$

$$v_2=-A\omega\sin(\omega t_2+\varphi)>0$$

$$\left.\begin{matrix}\\ \\ \end{matrix}\right\}(\omega t_2+\varphi)=-\frac{\pi}{3}$$

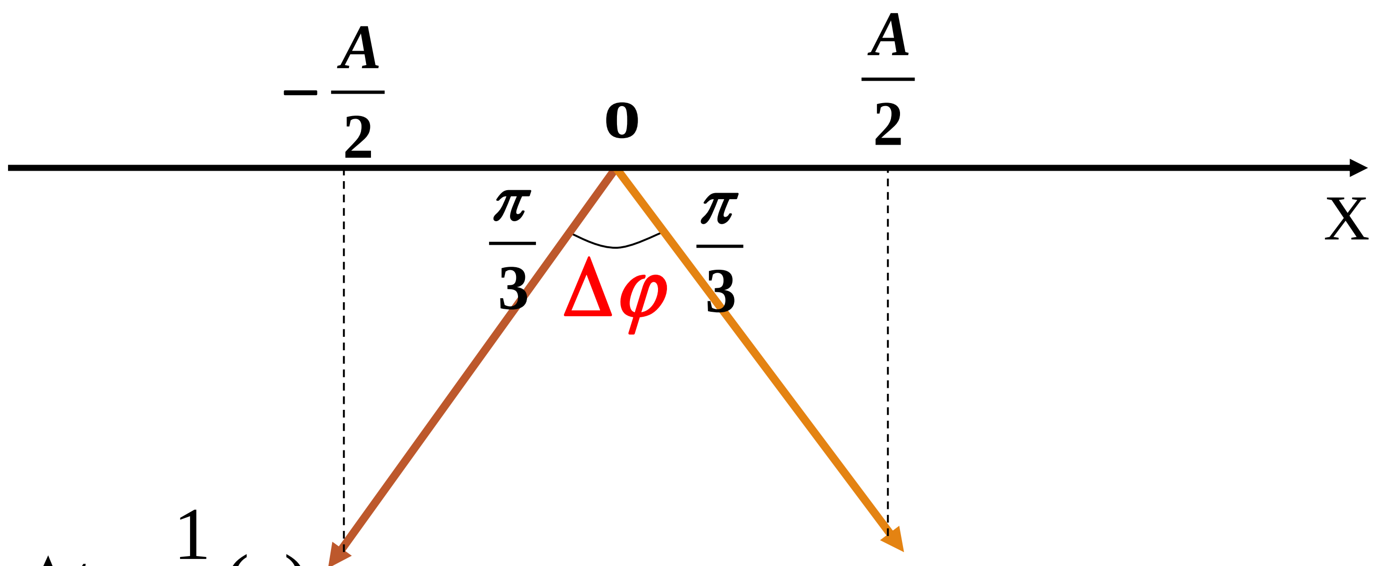
$$(\omega t_2+\varphi)-(\omega t_1+\varphi)=\omega(t_2-t_1)$$

$$(\omega t_1+\varphi)=-\frac{2\pi}{3} \qquad (\omega t_2+\varphi)=-\frac{\pi}{3}$$

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{2}=\pi$$

$$\Delta t=\frac{1}{3}(s)$$

2.旋转矢量法



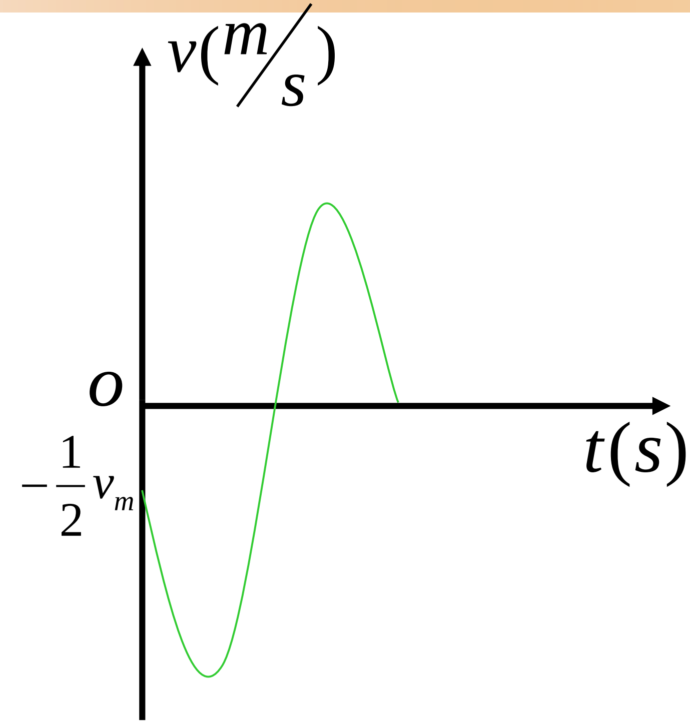
$$\Delta t=\frac{\Delta\varphi}{\omega}$$

$$\omega=\frac{2\pi}{T}=\pi$$

$$\Delta\varphi=\frac{\pi}{3}$$

$$\Delta t=\frac{1}{3}(s)$$

【例】.质点按余弦规律作谐振动，其v-t关系曲线如图所示，周期T=2。试求振动表达式。



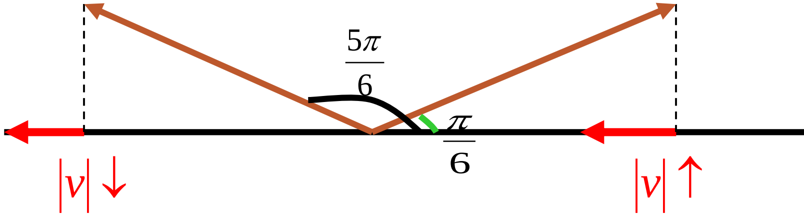
解： $x = A\cos(\omega t + \varphi)$

$$\left.\begin{array}{l} \omega = \frac{2\pi}{T} = \pi \\ v_m = \omega A \end{array}\right\} A = \frac{v_m}{\omega} = \frac{v_m}{\pi}$$

$$t = o: v_0 = -A\omega\sin\varphi = -\frac{1}{2}v_m$$

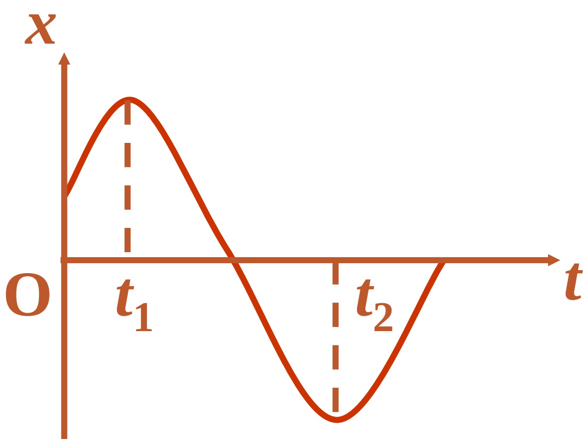
$$\varphi = \frac{\pi}{6}, \cancel{\frac{5\pi}{6}}$$

$$\therefore x = \frac{v_m}{\pi}\cos(\pi t + \frac{\pi}{6})$$



* 相位差

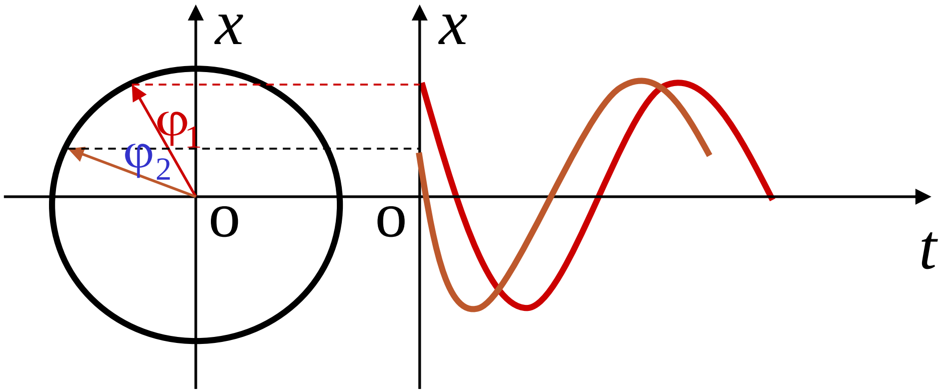
1). 对同一谐振动的两个不同时刻的态的比较



$$\Delta\varphi = (\omega t_2 + \varphi) - (\omega t_1 + \varphi)$$

$$= \omega(t_2 - t_1)$$

2). 对同一时刻两同频率的谐振动的比较



$$x_1 = A_1\cos(\omega t + \varphi_1)$$

$$x_2 = A_2\cos(\omega t + \varphi_2)$$

$$\Delta\varphi = (\omega t + \varphi_2) - (\omega t + \varphi_1) = \varphi_2 - \varphi_1$$ 初相差

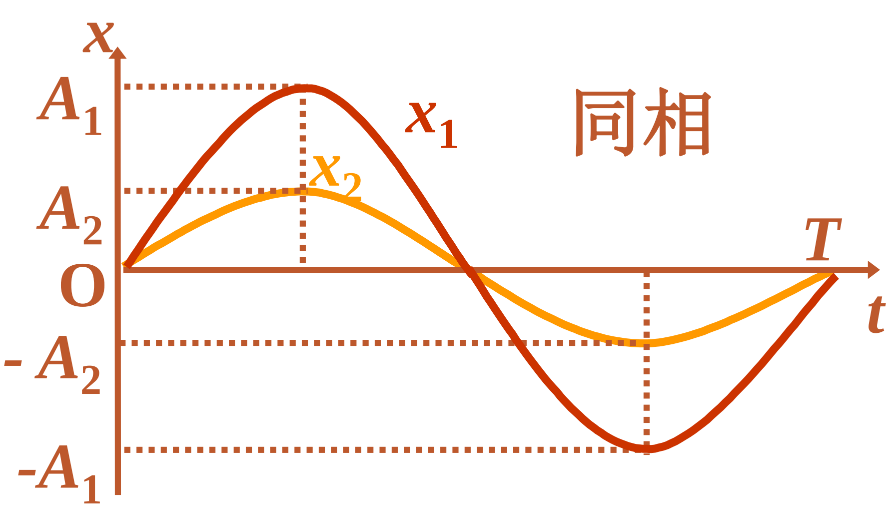
可用于比较两个谐振动的步调。

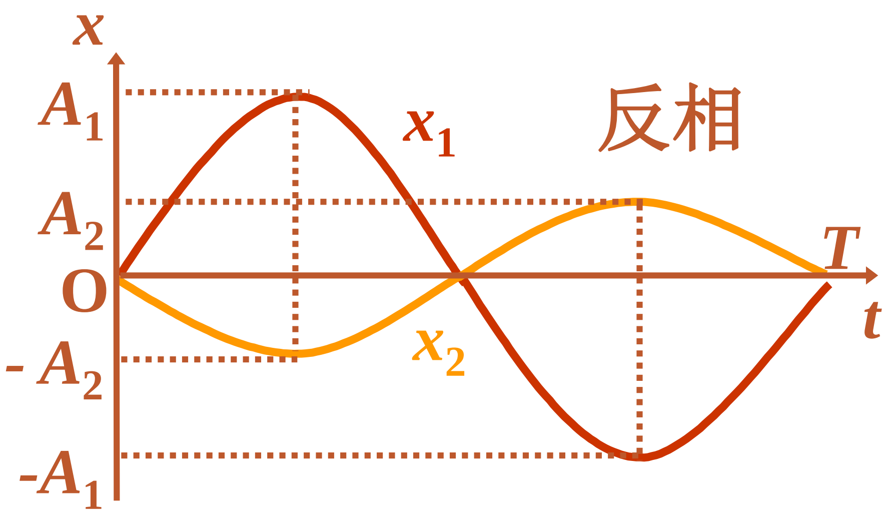
a 同相　两振动步调相同。

条件：$\Delta\varphi = \pm 2k\pi,\quad k = 0,1,2,\cdots$

b 反相　两振动步调相反。

条件：$\Delta\varphi = \pm(2k+1)\pi,\quad k = 0,1,2,\cdots$



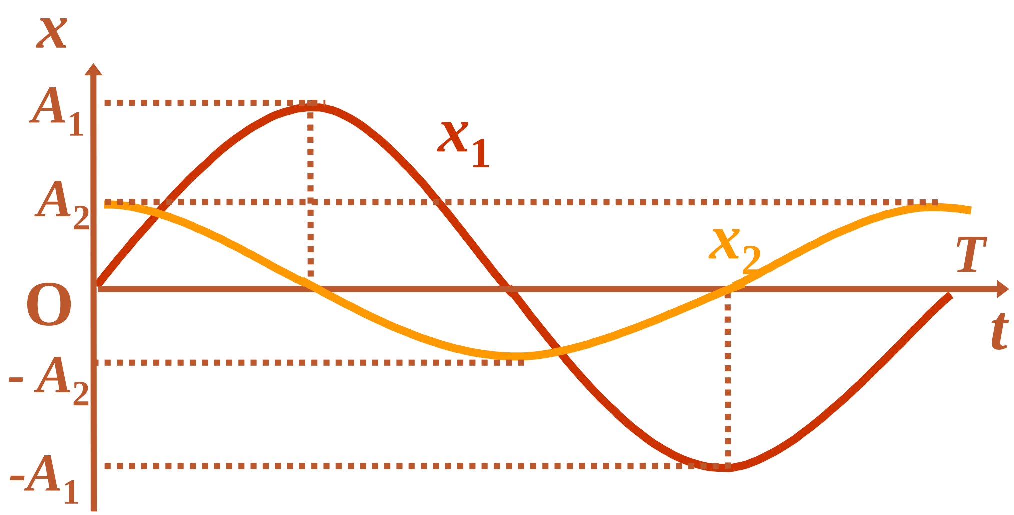


c 超前和落后　当$\Delta\varphi=\varphi_2-\varphi_1\neq\pm k\pi,\quad k=0,1,2,\cdots$

$$\begin{cases}\Delta\varphi>0，x_2\text{超前}x_1\text{振动}\Delta\varphi。\\ \Delta\varphi<0，x_2\text{落后}x_1\text{振动}|\Delta\varphi|。\end{cases}$$

约定：△φ ∈ (-π，π]

$$\Delta\varphi=-\frac{3}{2}\pi \longrightarrow \Delta\varphi=-\frac{3}{2}\pi+2\pi=\frac{1}{2}\pi$$



x_2超前x_1振动 $\dfrac{\pi}{2}$。

$$\Delta\varphi=\varphi_2-\varphi_1=0-(-\frac{1}{2}\pi)=\frac{1}{2}\pi$$