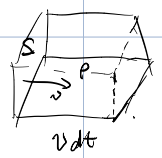
电荷守恒率的微分形式.

电流密度.

单位时间通过单位面积电荷量.

通过$d\vec S$的电流. $dI = \frac{dq}{dt} = \frac{\vec J\cdot d\vec S\cdot dt}{dt} = \vec J\cdot d\vec S$

$$I = \int_S \vec J\cdot d\vec S$$



$$dQ = \rho\cdot v\,dt\cdot S$$

$$J = \frac{dQ}{dt\cdot S} = \rho v$$

电荷守恒.

流过闭合表面的电流等于区域内Q减少率.

$$\oint_S \vec J\cdot d\vec S = -\int_V \frac{\partial\rho}{\partial t}\,dV$$

$$\Rightarrow \oint_S \vec J\cdot d\vec S = \int_V \nabla\cdot\vec J\,dV$$

$$\Rightarrow \nabla\cdot\vec J + \frac{\partial\rho}{\partial t} = 0$$

无限大空间 即无电流流出, $\int_V \frac{\partial\rho}{\partial t}\,dV = 0$

恒定电流, $\frac{\partial\rho}{\partial t} = 0$,$\Rightarrow \nabla\cdot\vec J = 0$ 恒电流是无源的.

毕萨定律 稳恒电流.

$$dF = I(x)\,d\vec l\times\vec B(x)$$

$$B(x) = \frac{\mu_0}{4\pi}\frac{\int I\,d\vec l\times\vec r}{r^3} \qquad I\,d\vec l = \vec J\,dV$$

场点. 源点.

$$B(x) = \frac{\mu_0}{4\pi}\frac{\int \vec J(x')\,dV\times\vec r}{r^3}$$

场点-源点距离.

$$dF = \vec J(x)\,dV\times B(x) = \vec J(x)\,dV\times\left[\frac{\mu_0}{4\pi}\int \vec J(x')\times\frac{\vec r}{r^3}\,dV\right]$$

磁场的环量

$\oint \vec B\cdot d\vec l = \mu_0 I$ 如何证明

$$\oint_l \vec{B}\cdot d\vec{l} = \oint_S \nabla\times\vec{B}\cdot d\vec{S} = \mu_0\int \vec{J}\cdot d\vec{S}$$

$$\Rightarrow \nabla\times B - \mu_0\vec{J} = 0$$

磁场的散度 $\nabla\cdot\vec{B}=0$（未证）

如何把 B 表示成一个量的旋度.

$$B(x) = \frac{\mu_0}{4\pi}\int \vec{J}\times\frac{\vec{r}}{r^3}\,dV = -\frac{\mu_0}{4\pi}\int \vec{J}\times\nabla\left(\frac{1}{r}\right)dV$$

$$\frac{\vec{r}}{r^3} = -\nabla\left(\frac{1}{r}\right)$$

$$\nabla\times(\varphi\vec{f}) = (\nabla\varphi)\times\vec{f} + \varphi\nabla\times\vec{f}$$

对比公式. 可令 $\begin{cases}\vec{f}=\vec{J}, & \vec{J}\text{是}x'\text{的函数, 对}x\text{求旋度为}0\\ \varphi=\frac{1}{r}\end{cases}$

$$B = \nabla\times\int\left(\frac{\mu_0}{4\pi}\cdot\frac{\vec{J}}{r}\right)dV = \nabla\times A.$$

可知. $\nabla\cdot\vec{B}=0$

B 的旋度.

$$\nabla\times B = \nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$$

$$\nabla\cdot(\varphi\cdot\vec{f}) = \nabla\varphi\cdot\vec{f} + \underbrace{\varphi\cdot\nabla\cdot\vec{f}}$$

$\vec{f}=\vec{J}$

$\varphi=\frac{1}{r}$ $\qquad \nabla(\nabla\cdot A) = \nabla\left(\frac{\mu_0}{4\pi}dV\cdot\nabla\left(\frac{1}{r}\right)\cdot\vec{J}\right)$

……

$$\nabla\times\vec{B} = \mu_0\vec{J}(\vec{x})$$