FangYi

[例题4-4]已知:谐振振幅**A**,

求:动能为(1)最大动能一半时x;

(2)势能一半时x .

解:(1)

$$\left.\begin{aligned} E_k &= \frac{1}{2}E_{kmax} \\ E_k + E_p &= E \end{aligned}\right\} \Rightarrow E_p = \frac{1}{2}E$$

$$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}\frac{1}{2}kA^2 \Rightarrow x = \pm\frac{\sqrt{2}}{2}A$$

(2)

$$\left.\begin{aligned} E_k &= \frac{1}{2}E_p \\ \boxed{E_k + E_p = E} \end{aligned}\right\} \Rightarrow \frac{3}{2}E_p = E$$

辅助关系式

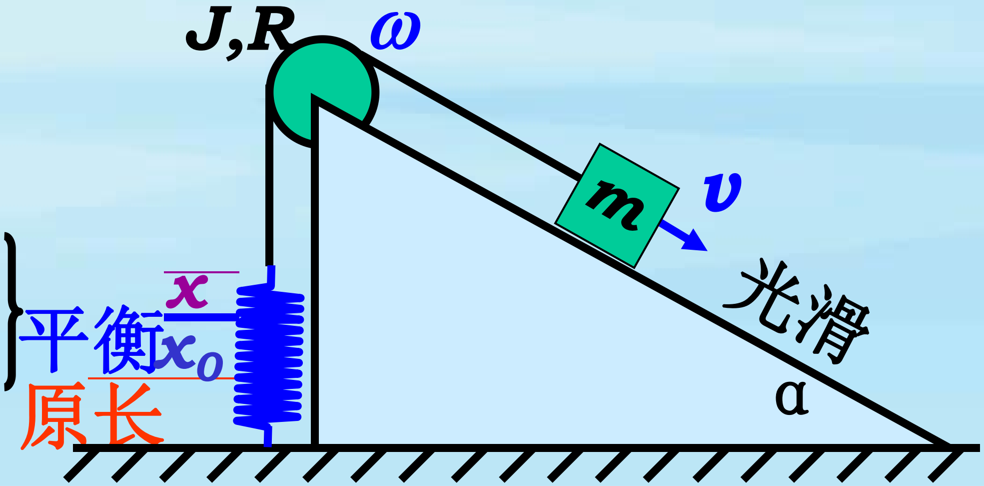
$$\Rightarrow \frac{3}{2}\frac{1}{2}kx^2 = \frac{1}{2}kA^2 \Rightarrow x = \pm\frac{\sqrt{6}}{3}A$$

[讨论4]弹簧k原长, 物体由静止释放,
证物体谐振, 求周期。

解:能量方法

$$\frac{1}{2}mv^2+\frac{1}{2}J\omega^2+\frac{1}{2}kx^2=\text{常量}$$

$$v=R\omega$$



两边对t求导

$$m\cancel{v}\frac{dv}{dt}+\frac{J}{R^2}\cancel{v}\frac{dv}{dt}+kx\cancel{\frac{dx}{dt}}=0$$

$$\Rightarrow\frac{d^2x}{dt^2}+\frac{k}{\frac{J}{R^2}+m}x=0\ \therefore \text{谐振}\Rightarrow T=\frac{2\pi}{\omega}=2\pi\Big/\sqrt{\frac{k}{\frac{J}{R^2}+m}}$$

问题:滑轮是否谐振? $x=R\theta$ 代入→ 滑轮角谐振

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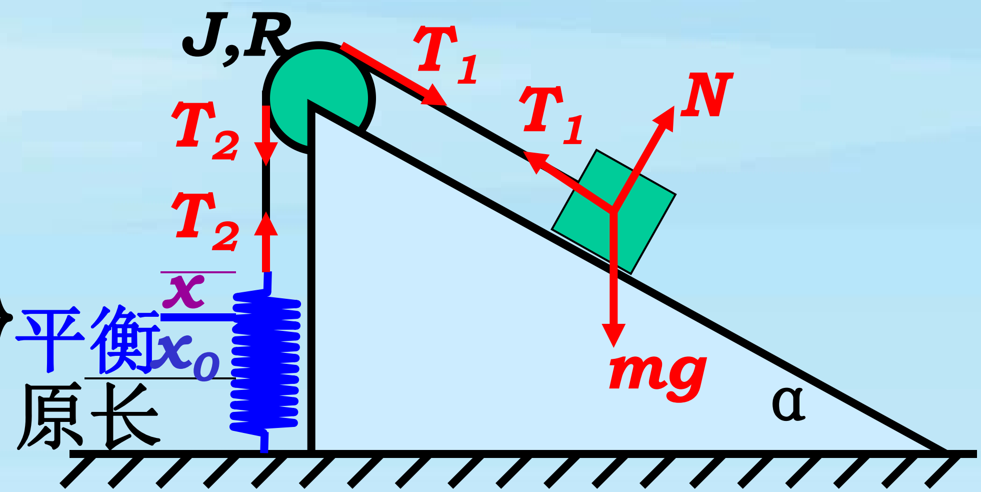
[讨论4]弹簧k原长, 物体由静止释放,
证物体谐振, 求周期。

解:动力学方法

$$\left.\begin{aligned} &\text{平}\quad mg\sin\theta - T_1 = ma && (1)\\ &\text{转}\quad (T_1 - T_2)R = J\alpha && (2)\\ &\qquad T_2 = k(x + x_0) && (3)\\ &\qquad mg\sin\theta = kx_0 && (4)\\ &\text{判}\quad a = R\alpha && (5) \end{aligned}\right\}$$



$$\rightarrow \frac{d^2x}{dt^2} + \frac{k}{\frac{J}{R^2} + m}x = 0 \quad \therefore \text{谐振} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi \Big/ \sqrt{\frac{k}{\frac{J}{R^2} + m}}$$

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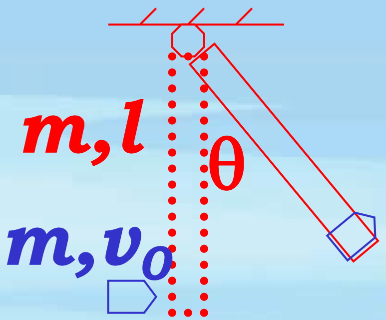
[讨论5]已知m、l 、v_0,小角振动,
证碰后谐振, 棒摆至θ_m需t



证明 能量方法

摆至任意位置θ

$$E=\frac{1}{2}J\Omega^2+\frac{3}{2}mgl(1-\cos\theta)=const.$$

$$(\frac{4}{3}ml^2)\Omega\frac{d\Omega}{dt}+\frac{3}{2}mgl\sin\theta\,\Omega=0$$

$$\Rightarrow\ \ddot{\theta}+\frac{9g}{8l}\theta=0$$ ⇒系统谐振

$$\left.\begin{aligned}t&=\frac{1}{4}T\\T&=2\pi/\omega\\\omega&=\sqrt{9g/(8l)}\end{aligned}\right\}\rightarrow t=\frac{\pi\sqrt{2l/g}}{3}$$

课后思考：棒摆至$\theta_m/2$需时

旋转矢量法如何求θ_m

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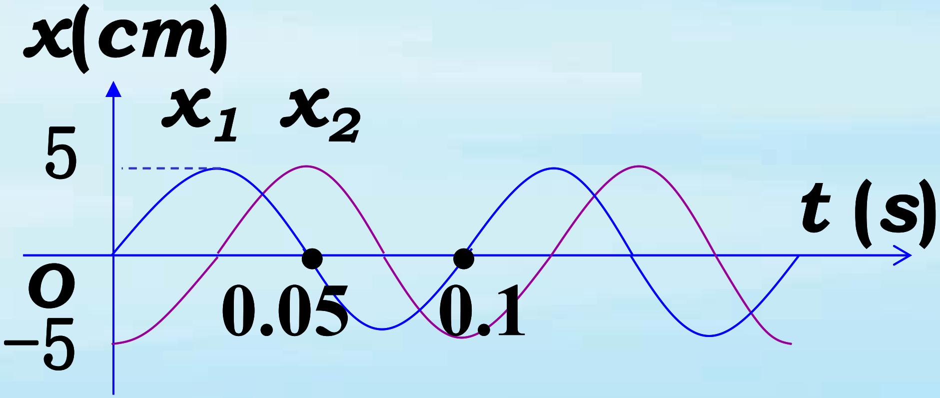
[例题4-5]两同频率谐振如图,求合振动方程



解1: (解析法) 由x~t图:

$A_1=A_2=5cm,\ T_1=T_2=0.1s$

$$\varphi_1 = -\frac{\pi}{2} \qquad \varphi_2 = \pi$$

$$x_1 = 5\cos(20\pi t - \pi/2)$$

$$x_2 = 5\cos(20\pi t + \pi)$$

$$A = \sqrt{5^2 + 5^2 + 2\times5\times5\cos\frac{3\pi}{2}} = 5\sqrt{2}$$

$$\varphi = \tan^{-1}\frac{5\sin\frac{-\pi}{2} + 5\sin\pi}{5\cos\frac{-\pi}{2} + 5\cos\pi} = \begin{cases} \cancel{\frac{\pi}{4}} \\ \frac{-3\pi}{4} \end{cases}$$

$$x = A\cos(\omega t + \varphi) \qquad x\big|_{t=0} < 0$$

$$x = x_1 + x_2$$

$$= 5\sqrt{2}\cos(20\pi t - 3\pi/4)cm$$

[例题4-5]两同频率谐振如图，求合振动方程

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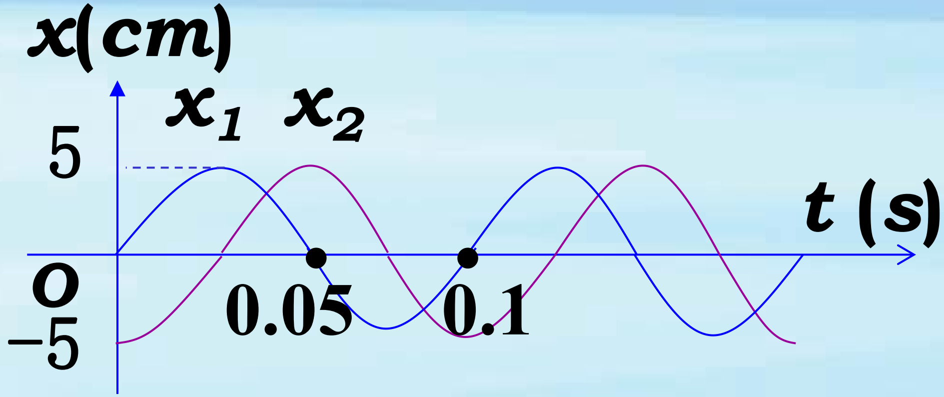
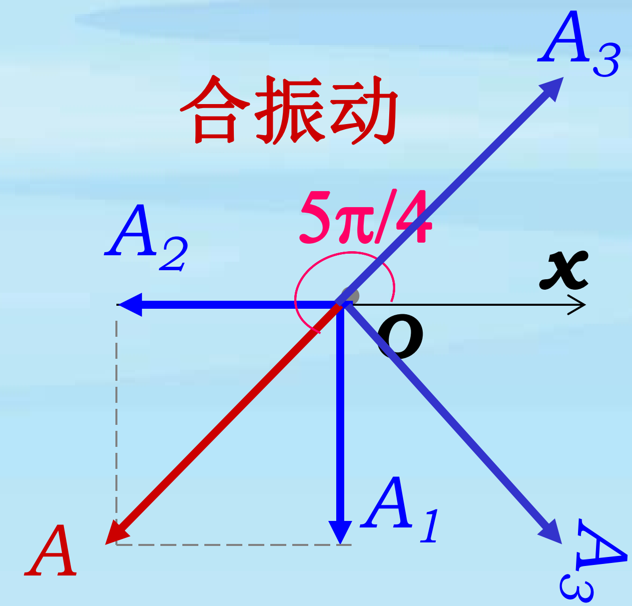




解2：(旋转矢量法)由x~t图：

$A_1=A_2=5cm$, $T_1=T_2=0.1s$

$\varphi_1=-\frac{\pi}{2}$ $\quad\varphi_2=\pi$ $\qquad A=5\sqrt{2}cm$ $\quad\varphi=-\frac{3}{4}\pi$

$x_1=5\cos(20\pi t-\pi/2)$ $\qquad x=x_1+x_2$

$x_2=5\cos(20\pi t+\pi)$ $\qquad =5\sqrt{2}\cos(20\pi t-3\pi/4)cm$

若 $x_3=5\sqrt{2}\cos(20\pi t+\pi/4)$ 则 $x=x_1+x_2+x_3=0$

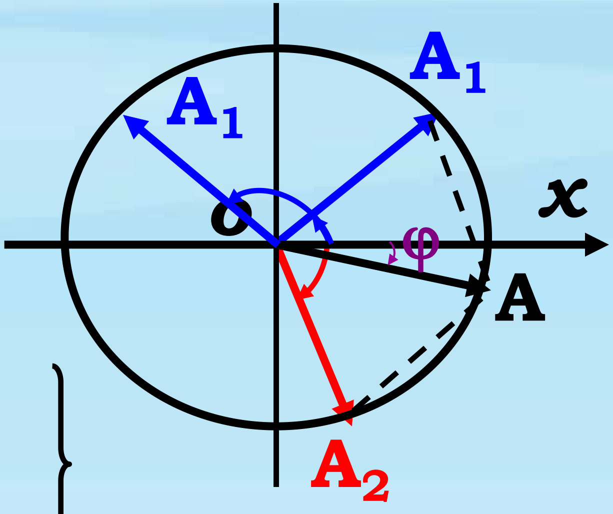
若 $x_3=5\sqrt{2}\cos(20\pi t-\pi/4)$ 则 $x=x_1+x_2+x_3=10\cos(20\pi t-\pi/2)cm$

[讨论6]2谐振方程

$$x_1 = 0.05\sin(\omega t + 3\pi/4) \quad \text{(SI)}$$

$$x_2 = 0.05\cos(\omega t - 5\pi/12) \quad \text{(SI)}$$

用旋转矢量法求合振动方程



解：

$$\left.\begin{aligned} \angle A_1OA_2 &= \frac{2\pi}{3} \\ \varphi &= -\left(\frac{1}{2}\angle A_1OA_2 - \frac{\pi}{4}\right) \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \varphi &= -\frac{\pi}{12} \\ A &= 0.05m \end{aligned}\right\}$$

$$\Rightarrow x = 0.05\cos(\omega t - \pi/12) \quad \text{(SI)}$$