



$$3.16、(2) \begin{cases} u_{xy} = x^2 y^2, & x > 1, y > 0 \\ u(x, 0) = x^2, & x > 0 \\ u(1, y) = \cos y, & y \geq 0 \end{cases}$$

解：关于 y 作Laplace变换，原定解问题可化为

$$\begin{cases} \frac{\partial \tilde{u}}{\partial x} = \frac{2x}{p} + \frac{x^2}{p^3} \\ \tilde{u}(1, p) = \frac{p}{1+p^2} \end{cases}$$

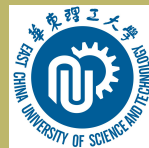
解初值问题

$$\tilde{u}(x, p) = \frac{x^2}{p} + \frac{x^3}{3p^3} + \frac{p}{p^2 + 1} - \frac{1}{p} - \frac{1}{3p^2}$$

作Laplace逆变换

$$u(x, y) = x^2 + \frac{x^3 y^2}{6} + \cos y - 1 - \frac{y^2}{6}$$

[Home Page](#)[Title Page](#)[◀](#) [▶](#)[◀](#) [▶](#)[Page 1 of 100](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)



$$3.16、(3) \begin{cases} u_{tt} - a^2 u_{xx} = 0, & x > 0, t > 0 \\ u(0, t) = \sin t, & t > 0 \\ u(x, 0) = u_t(x, 0) = 0, & x > 0 \\ u(x, t) \text{有界} \end{cases}$$

解：关于 t 实行Laplace变换

$$\begin{cases} \tilde{u}_{xx}(x, p) = \frac{p^2}{a^2} \tilde{u}(x, p) \\ \tilde{u}(0, p) = \frac{1}{p^2+1} \end{cases}$$

解初值问题可得

$$\tilde{u}(x, p) = c_1(p)e^{\frac{p}{a}x} + c_2(p)e^{-\frac{p}{a}x}$$

由 $u(x, t)$ 有界 $\Rightarrow c_1(p) = 0$, 由 $\tilde{u}(0, p) = \frac{1}{p^2+1} \Rightarrow c_2(p) = \frac{1}{p^2+1}$, 所以

$$\tilde{u}(x, p) = \frac{1}{p^2+1} e^{-\frac{p}{a}x}$$

由延迟性质

$$u(x, t) = \sin\left(t - \frac{x}{a}\right) H\left(t - \frac{x}{a}\right)$$

Contents

Home Page

Title Page

◀ ▶

◀ ▶

Page 2 of 100

Go Back

Full Screen

Close

Quit