思考练习：
1、一维弦振动方程的初值问题

$$\begin{cases} u_{tt} - a^2 u_{xx} = 0, & x \in R^1, t > 0 \\ u(x,0) = \phi(x), u_t(x,0) = \psi(x), & x \in R^1 \end{cases}$$

的d'Alembert 公式为

$$u(x,t) = \frac{1}{2}[\phi(x+at) + \phi(x-at)] + \frac{1}{2a}\int_{x-at}^{x+at} \psi(\xi)d\xi$$

2、已知定解问题$\begin{cases} u_{tt} = 4u_{xx}, & x \in R, t > 0 \\ u|_{t=0} = 0, u_t|_{t=0} = x, & x \in R \end{cases}$利用d'Alembert 公式可知其解$u(x,t) =$ xt

3、$\mathscr{F}^{-1}[\hat{\phi}(\lambda)\cos a\lambda t] =$ $\frac{1}{2}[\phi(x+at) + \phi(x-at)]$，

$\mathscr{F}^{-1}[\hat{\psi}(\lambda)\frac{\sin a\lambda t}{a\lambda}] =$ $\frac{1}{2a}\int_{x-at}^{x+at} \psi(y)dy$; $\mathscr{F}^{-1}[\hat{\psi}(\lambda)\frac{\sin a\lambda(t-\tau)}{a\lambda}] =$ $\frac{1}{2a}\int_{x-a(t-\tau)}^{x+a(t-\tau)} \psi(y)dy$

3.6、(2)$\begin{cases} u_{tt} - u_{xx} = t\sin x, & x \in R, t > 0 \\ u(x,0) = 0, u_t(x,0) = \cos x, & x \in R \end{cases}$

解：令$f(x,t) = t\sin x, \psi(x) = \cos x$，方程和初始条件关于$x$施行Fourier变换，记$\hat{u}(\lambda,t) = \mathscr{F}[u], \hat{\psi}(\lambda) = \mathscr{F}[\psi], \hat{f}(\lambda,t) = \mathscr{F}[f]$

$$\begin{cases} \frac{d^2}{dt^2}\hat{u}(\lambda,t) + (\lambda)^2\hat{u} = \hat{f}(\lambda,t), & t > 0 \\ \hat{u}(\lambda,0) = 0, \hat{u}_t(\lambda,0) = \hat{\psi}(\lambda) \end{cases}$$

解初值问题

$$\hat{u}(\lambda,t) = \hat{\psi}(\lambda)\frac{\sin\lambda t}{\lambda} + \int_0^t \hat{f}(\lambda,\tau)\frac{\sin\lambda(t-\tau)}{\lambda}d\tau$$

作Fourier逆变换(利用(3.2.17)的结论)

$$u(x,t) = \frac{1}{2}\int_{x-t}^{x+t}\cos\xi d\xi + \frac{1}{2}\int_0^t\int_{x-(t-\tau)}^{x+(t-\tau)}\tau\sin\xi d\xi d\tau$$

$$= \cos x\sin t + (t - \sin t)\sin x$$