Faraday 电磁感应定律

$$\mathcal{E} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

$$\Rightarrow \oint_L \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} d\vec{S}$$

Stocks 公式. $\oint_L \vec{E}\cdot d\vec{l} = \int_S \nabla\times\vec{E}\, d\vec{S}$

$$\Rightarrow \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

同时取散度, $\nabla\cdot(\nabla\times\vec{E}) = 0$, $\nabla\cdot\left(\frac{\partial \vec{B}}{\partial t}\right) = 0$

$\Rightarrow \frac{\partial}{\partial t}(\nabla\cdot\vec{B}) = 0$ 偏导数为0为仍能推出常数.

$\nabla\cdot\vec{B} = C$.

只有恒定磁场时. $\nabla\cdot\vec{B} = 0$ B变化时. $\nabla\cdot B = 0$ 仍成立.

位移电流.

$$\nabla\times\vec{B} = \mu_0\vec{J}$$

静磁场下并不矛盾.

但 变化的磁场下,

$$\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\begin{cases}\nabla\cdot\vec{J} \neq 0\\ \nabla\cdot(\nabla\times\vec{B}) = 0\end{cases}$$ 矛盾.

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \rho = \nabla\cdot\varepsilon_0\vec{E}$$

$$\nabla\cdot\vec{J} + \nabla\cdot\varepsilon_0\frac{\partial \vec{E}}{\partial t} = 0$$

$$\nabla\cdot\left(\vec{J} + \underbrace{\varepsilon_0\frac{\partial \vec{E}}{\partial t}}_{\text{位移电流 } J_D}\right) = 0$$

原式改写为 $\nabla\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{\partial \vec{E}}{\partial t}\right)$

变化的电场产生磁场.

Maxwell 方程组. 真空.

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$ (静电场高斯定理)

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ (电磁感应)

$$\nabla\cdot\vec{B} = 0$$ ($\vec{B}$是一个量的旋度)

$$\nabla\times\vec{B} = \mu_0(\vec{J} + J_D) \qquad J_D = \varepsilon_0\frac{\partial \vec{E}}{\partial t}$$

(位移电流)

Lorentz Force.

$$d\vec{F} = \underbrace{\vec{J}dV}_{\text{磁场}}\times\vec{B} + \underbrace{\rho dV}_{\text{电场}}\vec{E}$$

力密度 $\vec{f} = \frac{d\vec{F}}{dV} = \vec{J}\times\vec{B} + \rho\vec{E}$

一个带电粒子：$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$