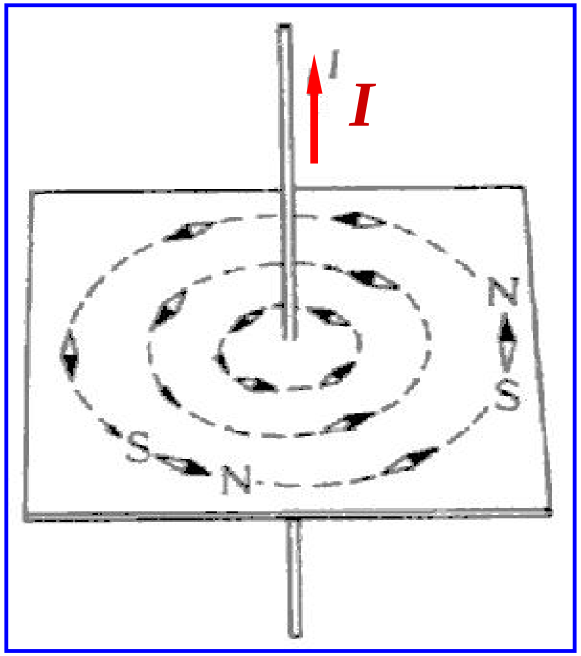
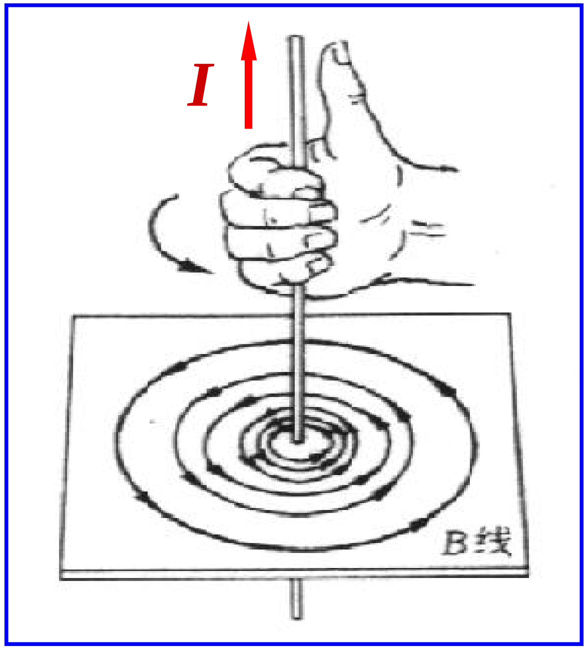
10.3 磁场的高斯定理

1.3.1 磁感应线

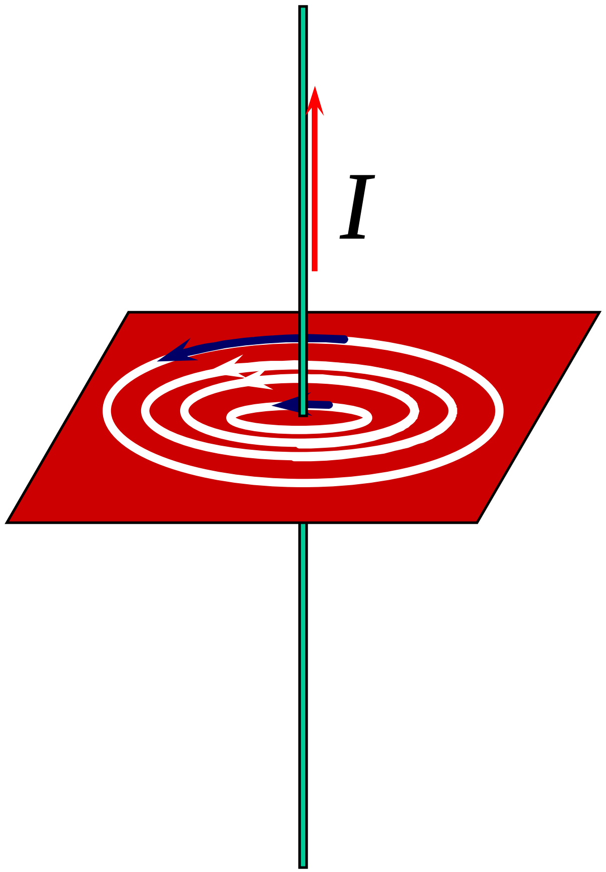
1.B 的方向:曲线上每一点的切线方向

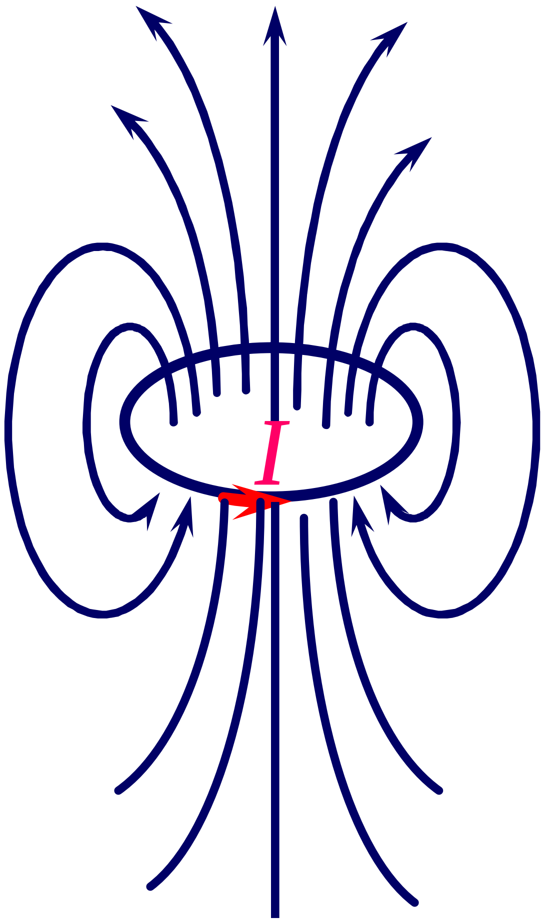
2.B 的大小:曲线的疏密程度. $B = \dfrac{dN}{dS_\perp}$

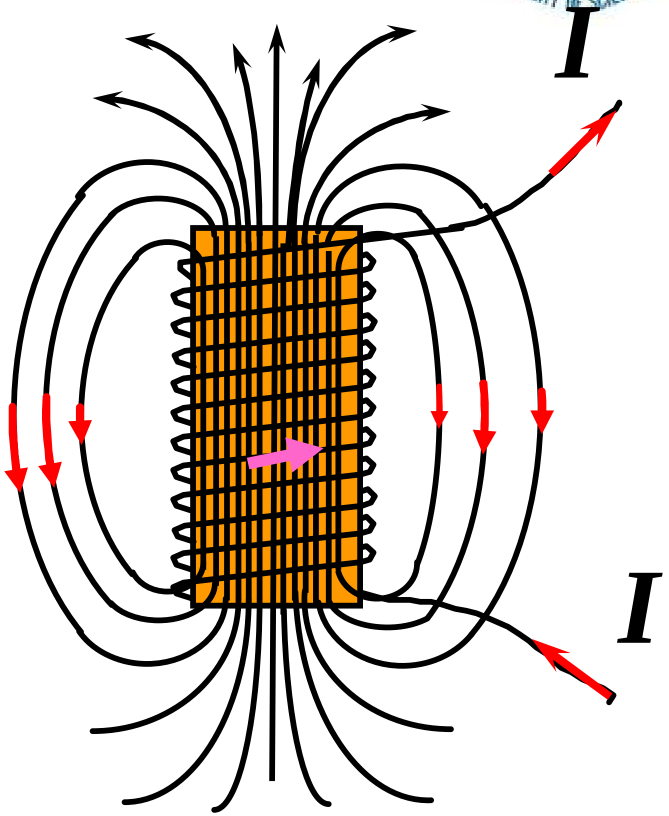




直线电流的磁力线 | 圆电流的磁力线 | 通电螺线管的磁力线







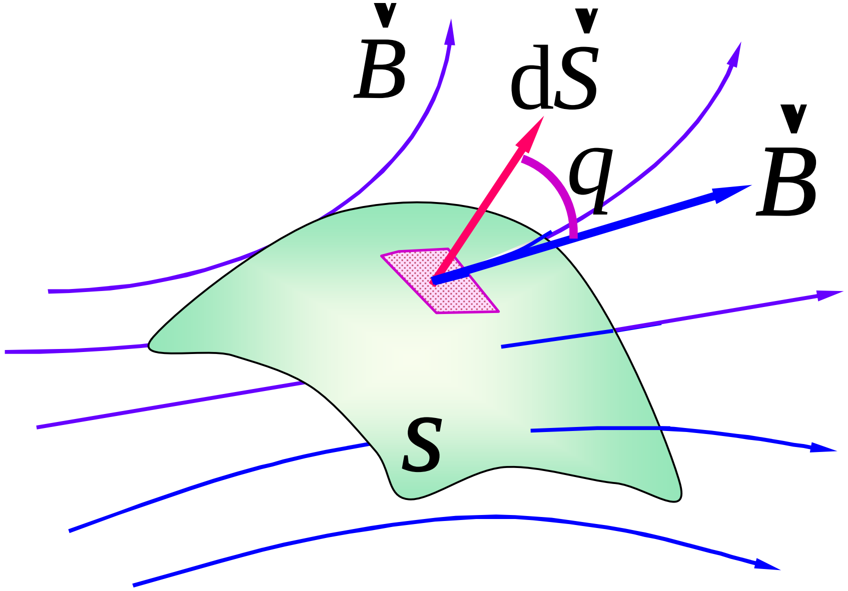
（与电流套连）方向与电流成右手螺旋关系　　互不相交

$\vec{B}$线是无头无尾的闭合线　- 无源场

10.3.2 磁通量Φ_m　磁场的高斯定理

一、磁通量 Φ_m

通过某一曲面的磁感线数.



dS：

$$d\Phi_m = \vec{B} \cdot d\vec{S}$$

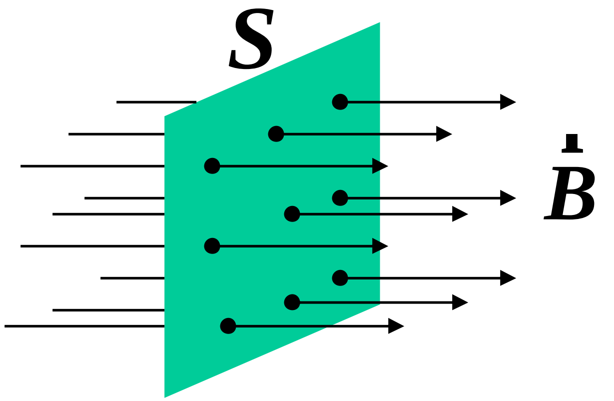
$$= BdS\cos\theta$$

S：

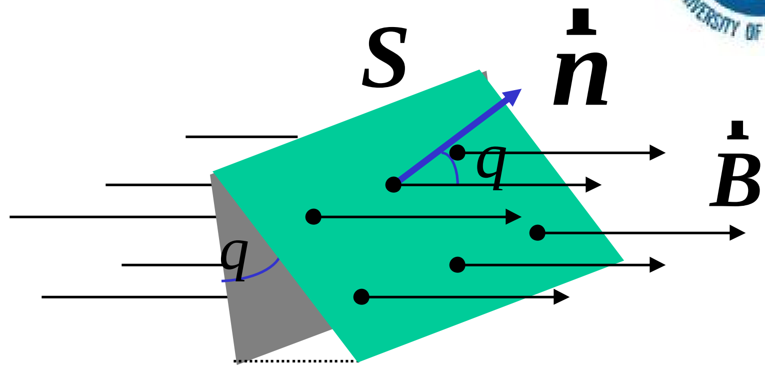
$$\boxed{\Phi_m = \int_S \vec{B} \cdot d\vec{S}}$$

单位　$1Wb = 1T \times 1m^2$

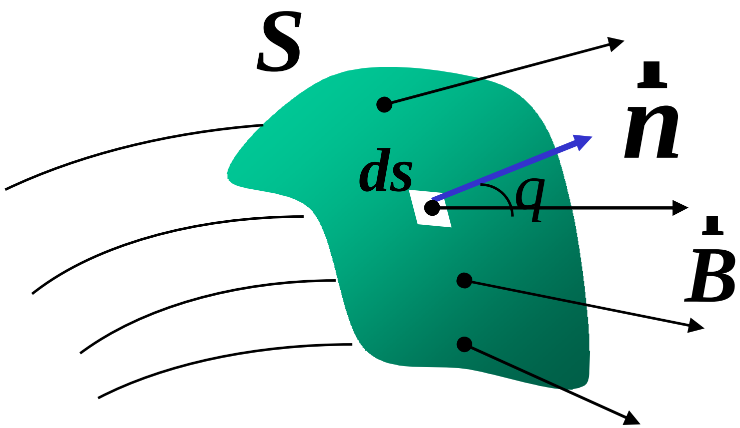
磁通量的计算



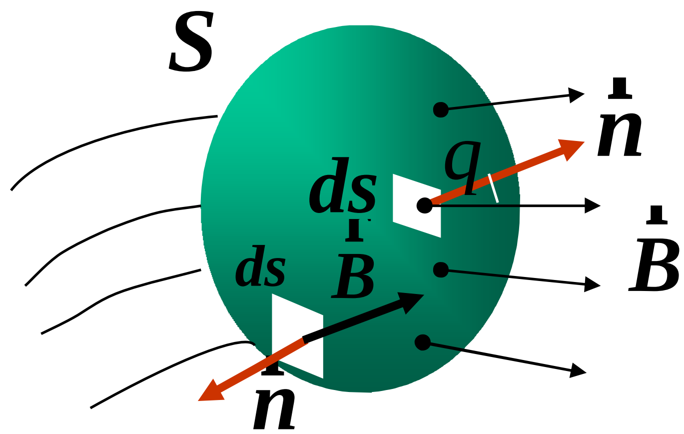
$$\Phi_B = BS$$



$$\Phi_B = \vec{B}\cdot\vec{S} = BS\cos\theta$$



$$\Phi_B = \iint \vec{B}\cdot d\vec{s} = \iint B\cos\theta\, ds$$



$$\Phi_B = \oint\!\!\oint \vec{B}\cdot d\vec{s} = \oint\!\!\oint B\cos\theta\, ds$$

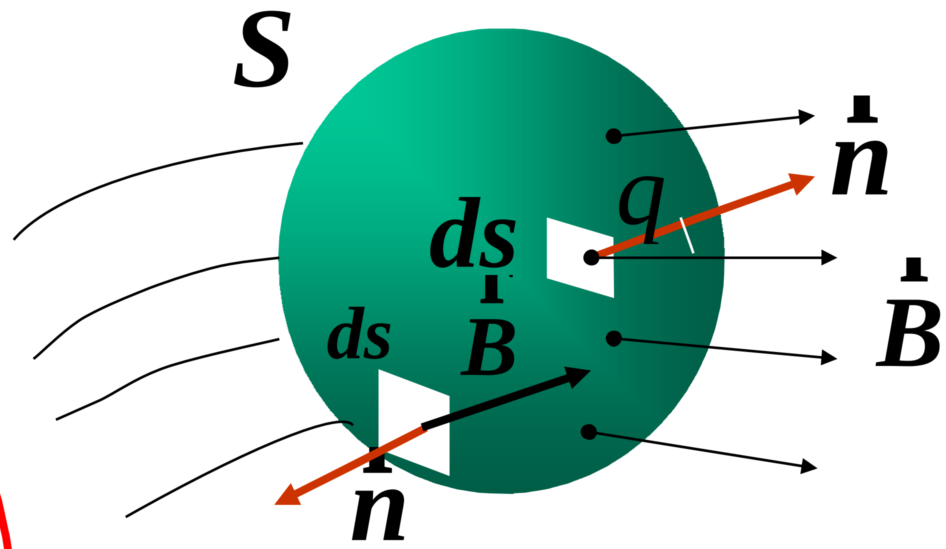
二、 磁场的高斯定理

$$\boxed{\oint \vec{B}\cdot d\vec{s} = 0}$$

穿过任一闭合曲面的磁通量为零

比较

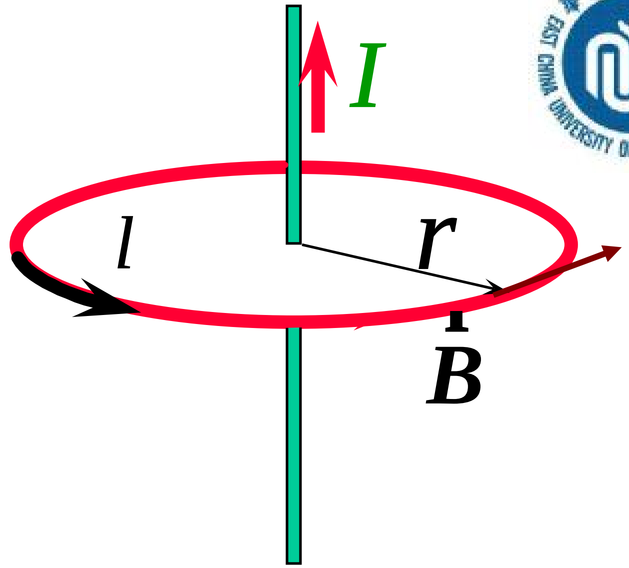
$$\boxed{\oint_s \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\sum q_i}$$



静电场	有源	电力线起于正电荷、止于负电荷
稳恒磁场	无源	磁力线闭合、无自由磁荷

10.4 安培环路定理及其应用

一、 安培环路定理



静电场 $\oint \vec{E}\cdot d\vec{l} = 0$

磁 场 $\oint \vec{B}\cdot d\vec{l} = ?$

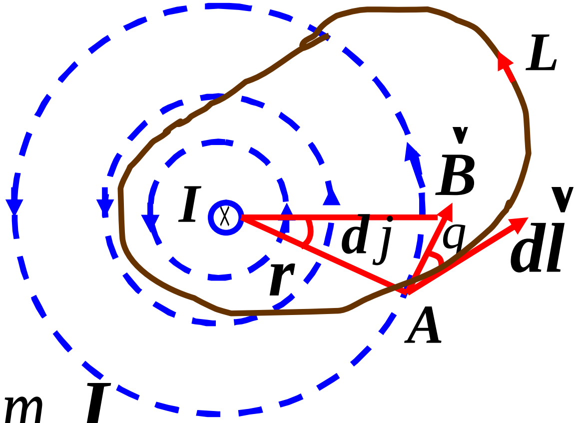
1. 圆形积分回路

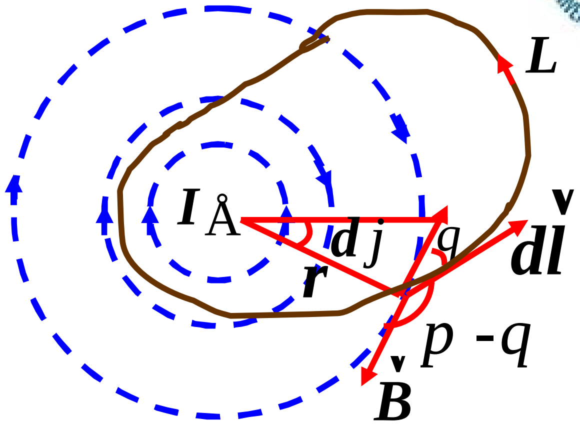
长直电流

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi r} dl = \frac{\mu_0 I}{2\pi r}\oint dl = \frac{\mu_0 I}{\cancel{2\pi r}}\cdot \cancel{2\pi r}$$

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$

2. 闭合回路包围电流，且在垂直导线的平面内





$$B=\frac{\mu_0 I}{2\pi r} \qquad dl\cos\theta = r d\varphi$$

$$\oint \vec{B}\cdot d\vec{l}=\oint B\cos\theta dl=\oint\frac{\mu_0 I}{2\pi r}\cdot r d\varphi=\mu_0 I$$

仅与闭合回路包围的电 流有关,与回路形状无关,

3.反向电流

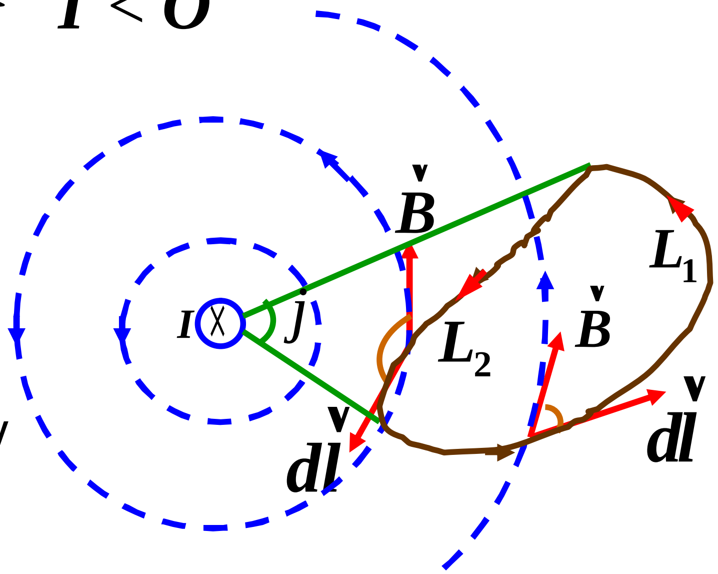
$$\because dl\cos(\pi-\theta)=-dl\cos\theta \quad \therefore dl\cos\theta=-r d\varphi$$

$$\oint \vec{B}\cdot d\vec{l}=\oint B\cos\theta dl=\oint\frac{\mu_0 I}{2\pi r}(-r d\varphi)=-\mu_0 I$$

电流正负与积分回路绕行方向有关

由右手螺旋法则决定 $\begin{cases} \text{符合} & I > O \\ \text{不符合} & I < O \end{cases}$

4. 闭合回路不包围电流



对 L_1 $\qquad dl\cos\theta = r\,d\varphi$

对 L_2 $\qquad dl\cos\theta = -r\,d\varphi$

$$\oint \vec{B}\cdot d\vec{l} = \int_{L_1} \vec{B}\cdot d\vec{l} + \int_{L_2} \vec{B}\cdot d\vec{l}$$

$$= \int_{L_1} \frac{\mu_0 I}{2\pi r}\cdot r\,d\varphi + \int_{L_2} \frac{\mu_0 I}{2\pi r}(-r\,d\varphi)$$

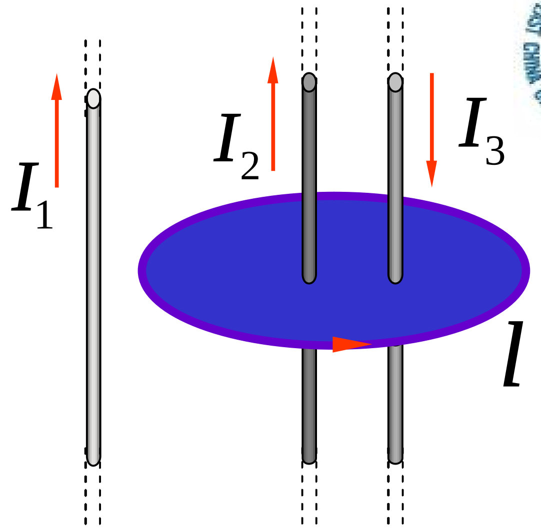
$$= \frac{\mu_0 I}{2\pi}\left[\int_{L_1} d\varphi - \int_{L_2} d\varphi\right] = O$$

5. 多根导线穿过闭合回路

$$\oint \vec{B}\cdot d\vec{l} = \oint(\vec{B}_1+\vec{B}_2+\cdots)\cdot d\vec{l}$$

$$=\oint \vec{B}_1\cdot d\vec{l}+\oint \vec{B}_2\cdot d\vec{l}+\cdots$$

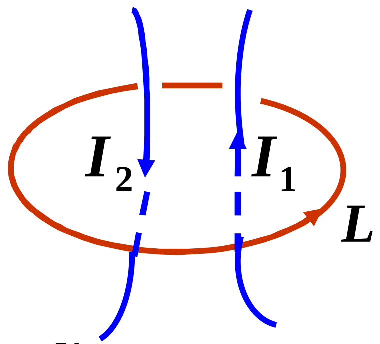
$$=\mu_0 I_1+\mu_0 I_2+\cdots=\mu_0\sum I$$



$$\boxed{\oint \vec{B}\cdot d\vec{l}=\mu_0\sum I}$$

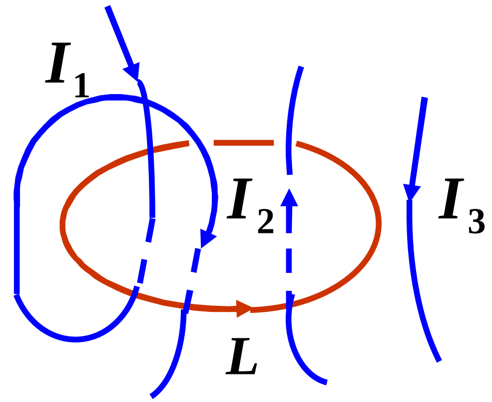
非保守场(涡旋场)

I 的正负由右手螺旋法则决定



$$\oint \vec{B}\cdot d\vec{l}=\mu_0(I_1-I_2)$$

若$I_1=I_2$，则$\oint \vec{B}\cdot d\vec{l}=0$



$$\oint \vec{B}\cdot d\vec{l}=\mu_0(I_2-2I_1)$$

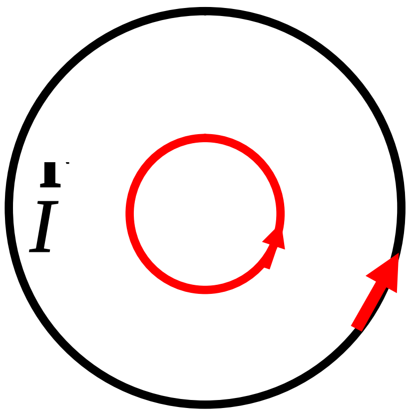
问题：1）$\vec{B}$ 是否与回路 L 外电流有关？

2）若 $\oint_L \vec{B}\cdot d\vec{l} = 0$，是否回路 L 上各处 $\vec{B} = 0$ ？

是否回路 L 内无电流穿过？

注意：1. 式中 $\vec{B}$ 是积分回路内、外电流共同产生

2. $\vec{B}$ 的环流仅与积分回路包围的电流有关



$$\oint \vec{B}\cdot d\vec{l} = ?$$

二、安培环路定理的应用

当场源分布具有高度对称性时，利用安培环路定理计算磁感应强度

例1．无限长载流圆柱导体 已知：I、R

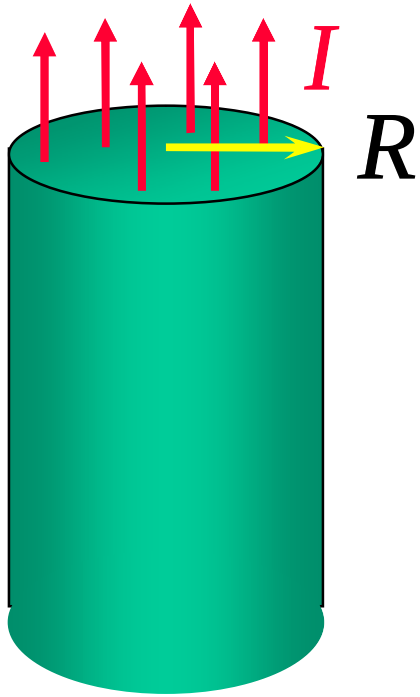
电流沿轴向，在截面上均匀分布

- 分析对称性

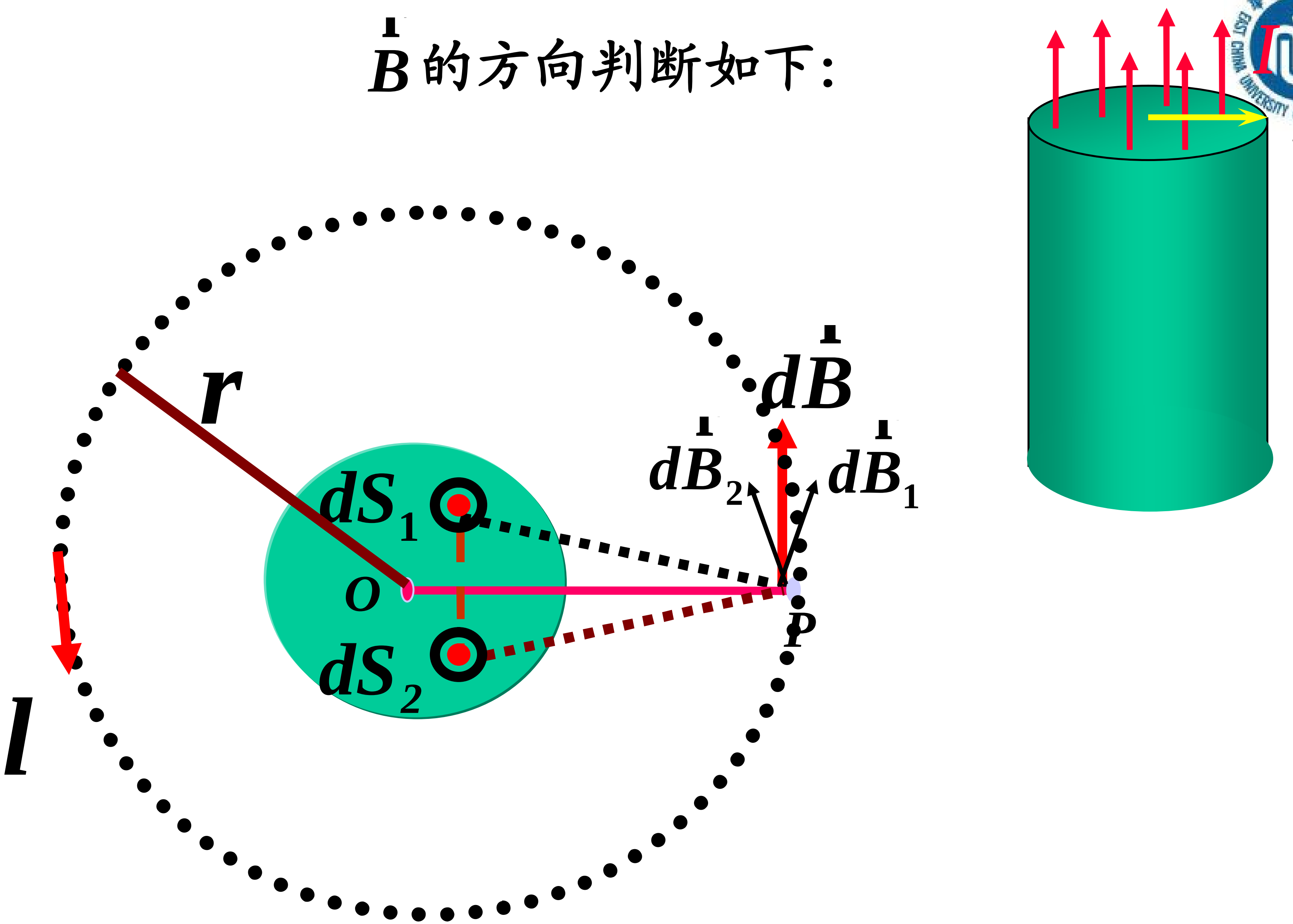
电流分布

磁场分布

轴对称



$\vec{B}$的方向判断如下：



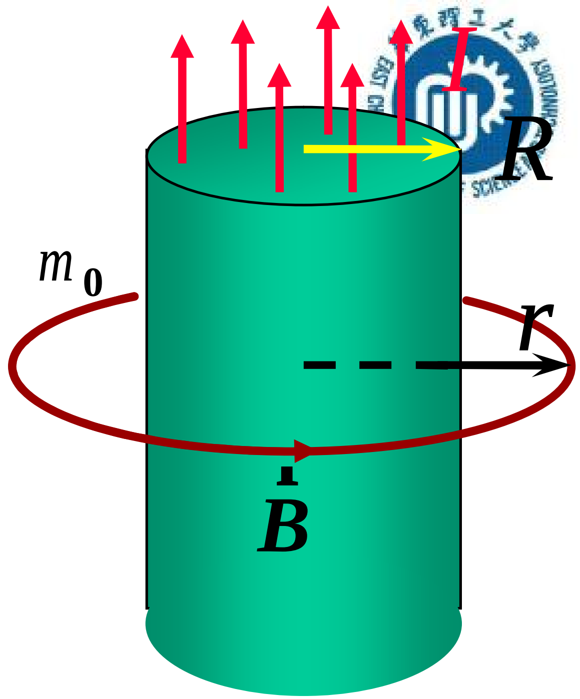
作积分环路并计算环流

$r > R \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = 2\pi r B$$

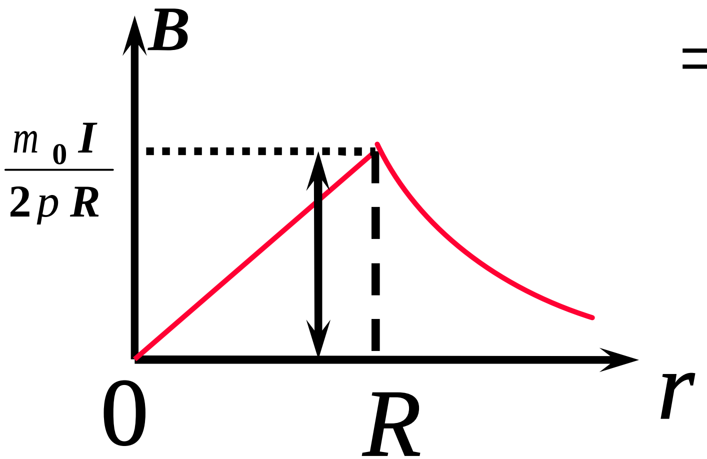
$$2\pi r B = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

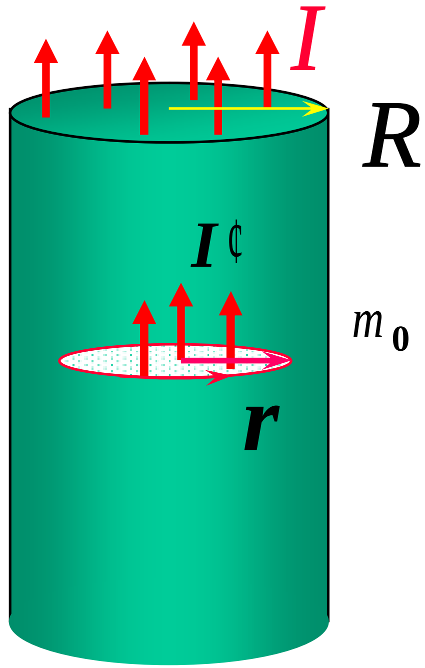


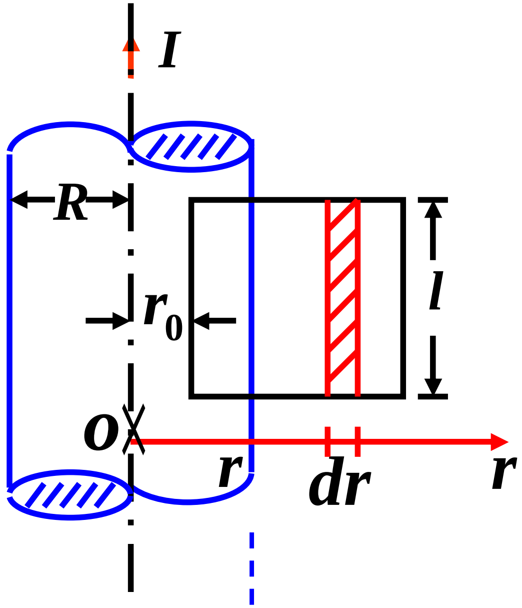
$\boxed{r < R} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I'$

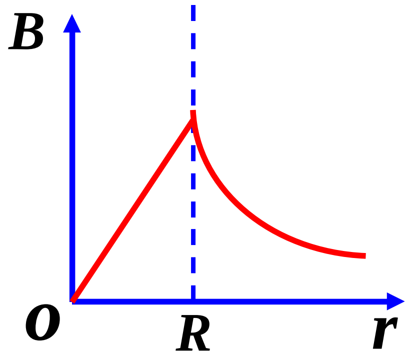
$$= \mu_0 \frac{I}{\cancel{\pi} R^2} \cancel{\pi} r^2$$



$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$







例2.求通过图正方形的磁通量

设 $l > R$，正方形与圆柱轴线共面

$$d\Phi_m = \vec{B}\cdot d\vec{S} = B\cdot l\,dr$$

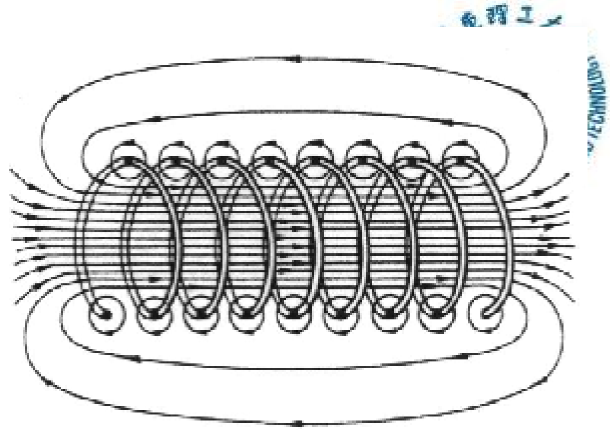
$$\Phi_m = \iint \vec{B}\cdot d\vec{S} = \int Bl\,dr$$

$$= \int_{r_0}^{R} B_{内} l\,dr + \int_{R}^{r_0+l} B_{外} l\,dr$$

$$= \int_{r_0}^{R} \frac{\mu_0 I r}{2\pi R^2} l\,dr + \int_{R}^{r_0+l} \frac{\mu_0 I}{2\pi r} l\,dr$$

$$= \frac{\mu_0 I l}{2\pi}\left\{\frac{1}{2}\left[1-\left(\frac{r_0}{R}\right)^2\right] + \ln\left(\frac{r_0+l}{R}\right)\right\}$$

例3. 长直载流螺线管 已知：I、n



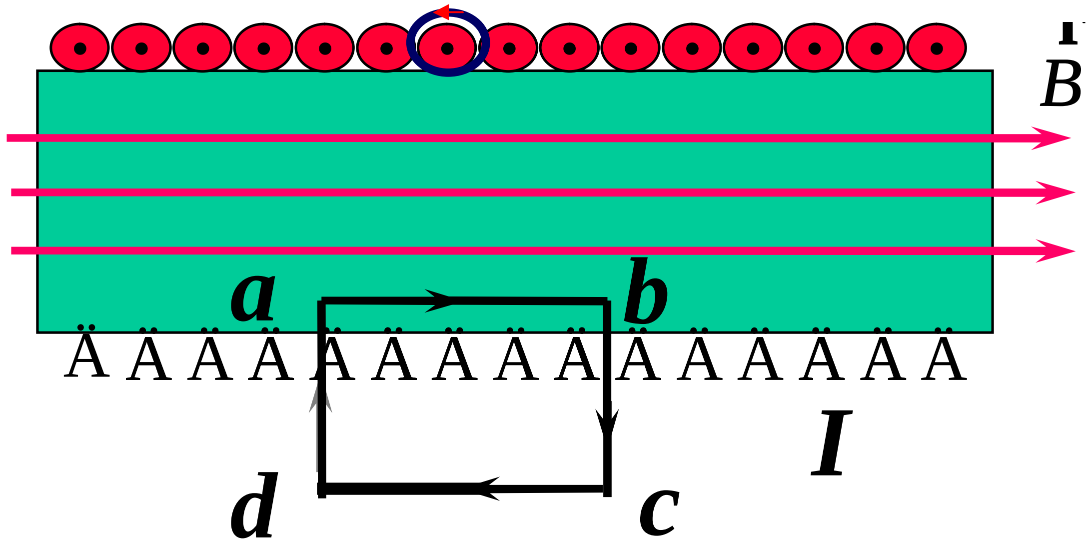
¶ 分析对称性

管内磁力线平行于管轴

管外磁场为零

作积分回路如图

方向 → 右手螺旋

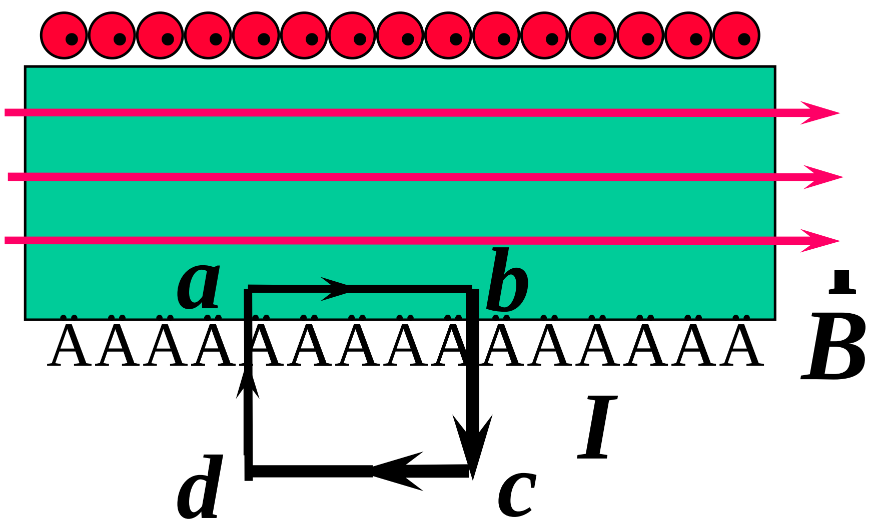


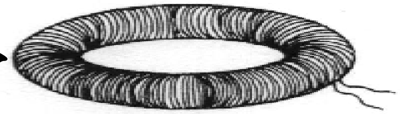
$$\oint \vec{B}\cdot d\vec{l} = \int_a^b Bdl\cos 0 + \int_b^c Bdl\cos\frac{\pi}{2} + \int_c^d Bdl\cos\pi + \int_d^a Bdl\cos\frac{\pi}{2}$$

$$= \boxed{B\cdot\overline{ab}}$$

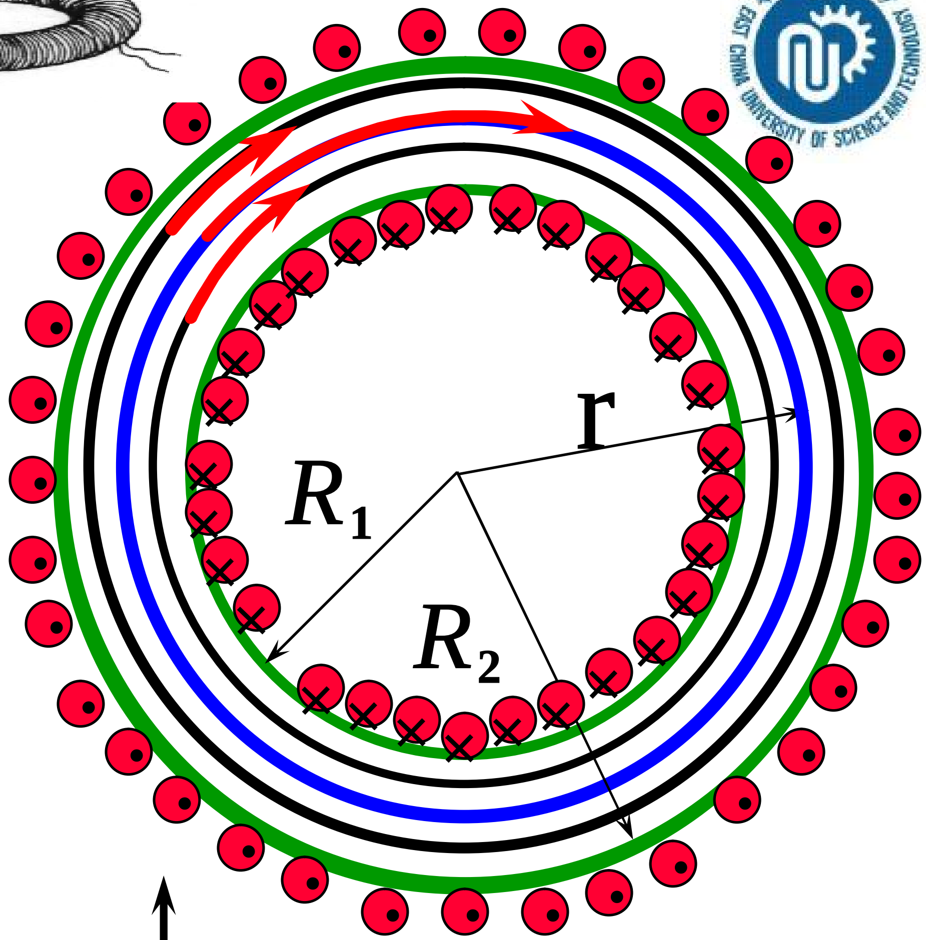
$$\oint \vec{B}\cdot d\vec{l} = \boxed{\mu_0 n\overline{ab}I}$$

$$B = \begin{cases} \mu_0 nI & \text{内} \\ 0 & \text{外} \end{cases}$$



例4. 环行载流螺线管 

已知：I N R_1 R_2



$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = 2\pi r B$$

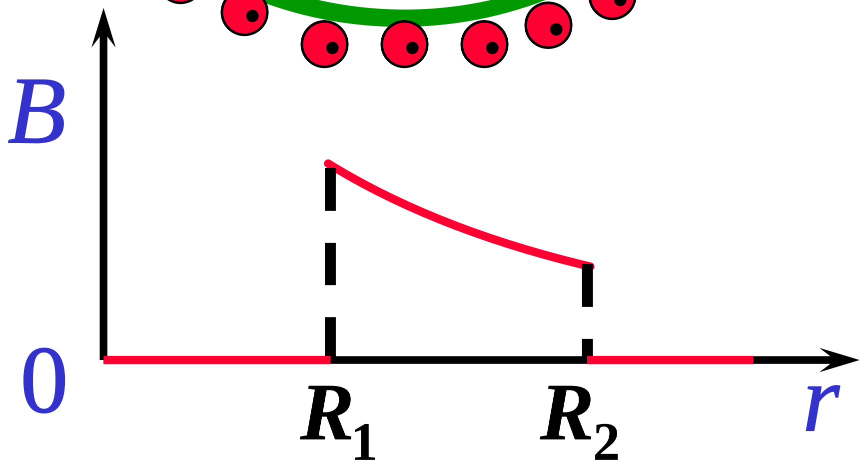
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 NI$$

$$\boxed{B = \begin{cases} \dfrac{\mu_0 NI}{2\pi r} & 内 \\ 0 & 外 \end{cases}}$$

R_1、$R_2 >> R_1 - R_2$

$$n = \frac{N}{2\pi R_1}$$

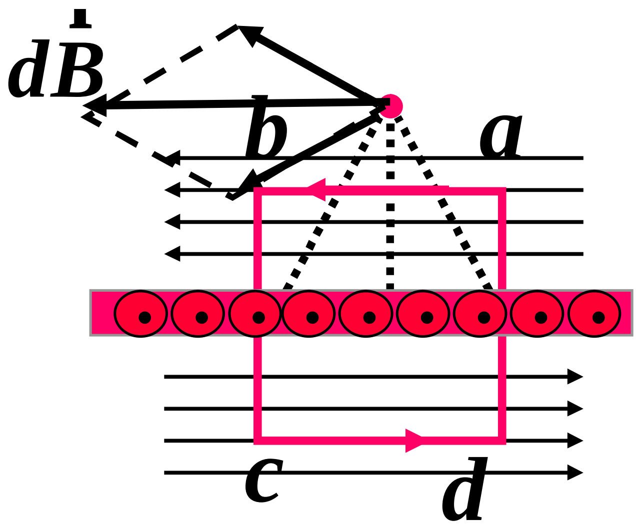
$$\boxed{B \approx \mu_0 n I}$$



例5.无限大载流导体薄板 面电流密度为i

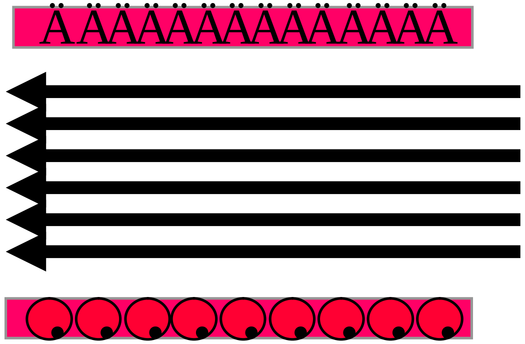
$$\oint \vec{B}\cdot d\vec{l} = \int_a^b Bdl\cos 0 + \int_b^c Bdl\cos \boxed{0} + \int_c^d Bdl\cos 0 + \int_d^a Bdl\cos \boxed{0}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \cdot \overline{ab} \cdot i$$



$$\boxed{B = \mu_0 i/2}$$

推论：一对无限大载流导体薄板



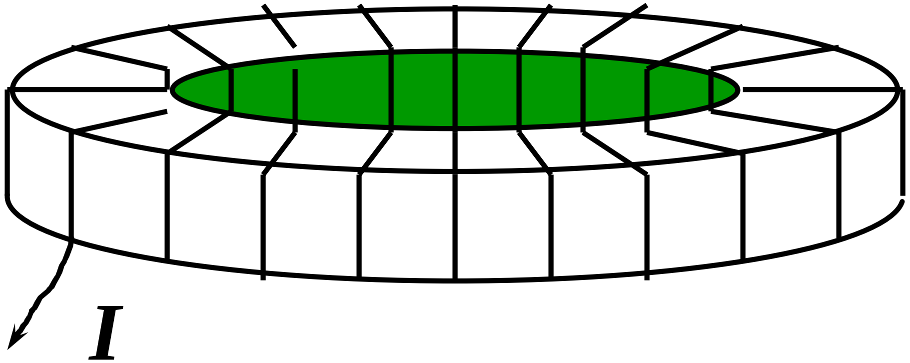
$$B = \begin{cases} 0 & \text{板外} \\ \mu_0 i & \text{板间} \end{cases}$$

练习

如图，螺绕环截面为矩形

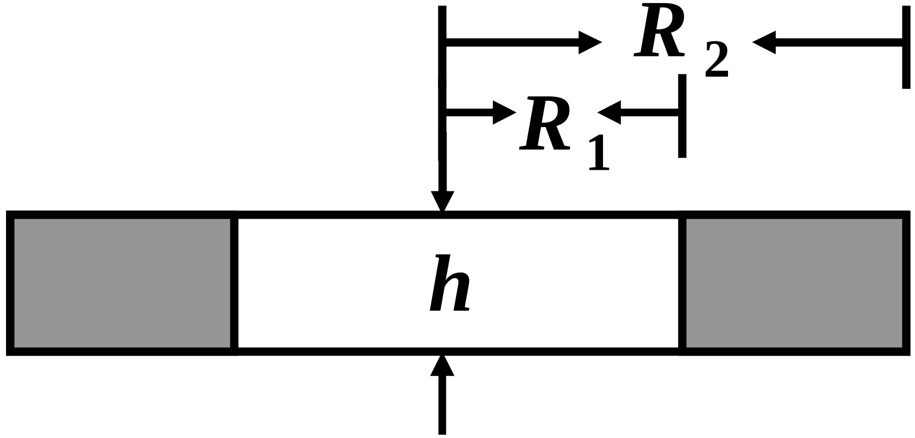
导线总匝数 N，高 h

外半径与内半径分别为 R_2, R_1



求：1. 磁感应强度的分布

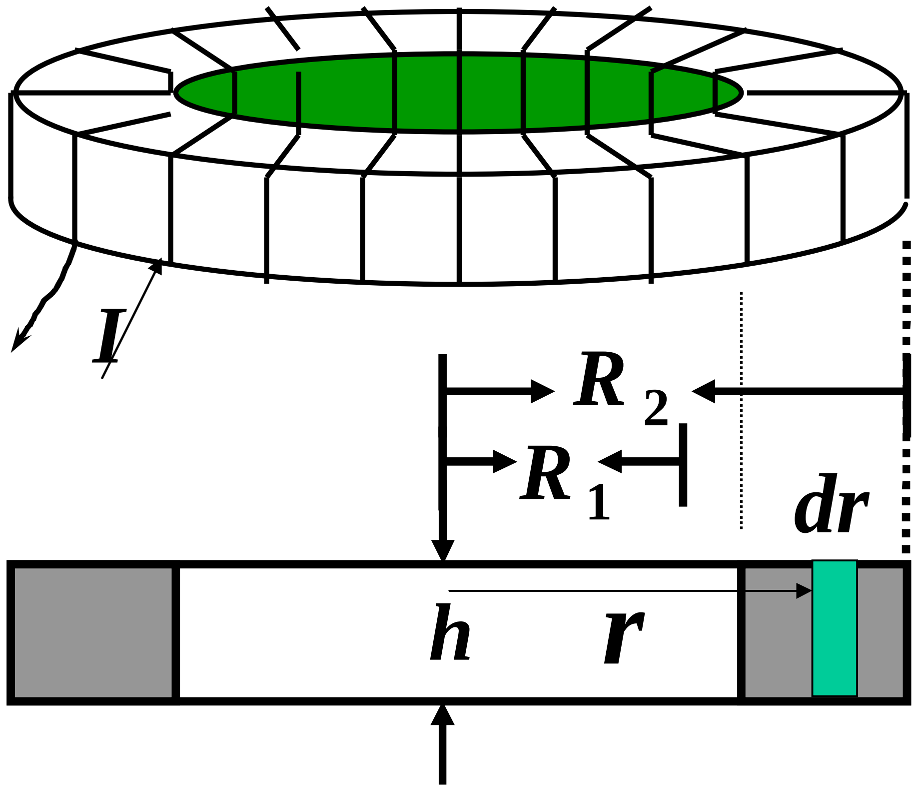
2. 通过截面的磁通量



解： 1. $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = 2\pi r B = \mu_0 N I$

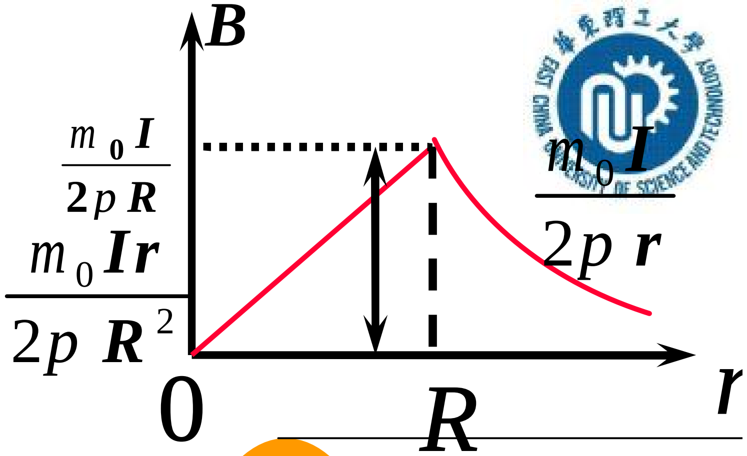
$$B = \mu_0 N I / 2\pi r$$

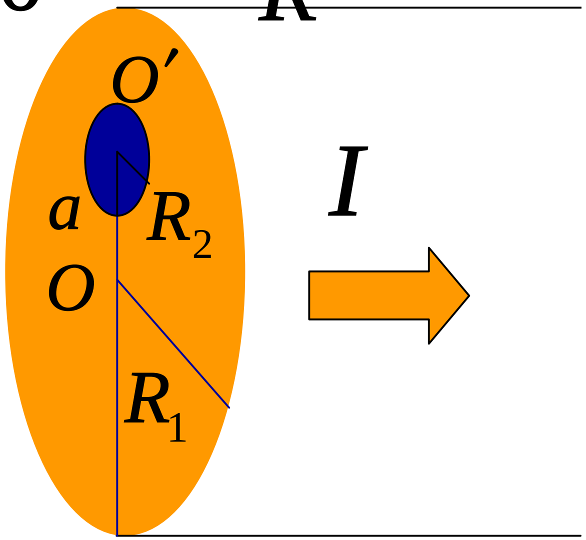
2. $\oint \vec{B}\cdot d\vec{s} = \int_{R_1}^{R_2} \frac{\mu_0 N I}{2\pi r} h\,dr = \frac{\mu_0 N I h}{2\pi r} \ln \frac{R_2}{R_1}$



思考:

一根外半径为R_1的无限长圆柱形导体管,管内空心部分的半径为R_2,空心部分的轴与圆柱的轴相平行但不重合，两轴间距离为a($a>R_2$),现有电流I沿导体管流动,电流均匀分布在管的横截面上,方向与管轴平行.





求:1)圆柱轴线上的磁感应强度的大小.

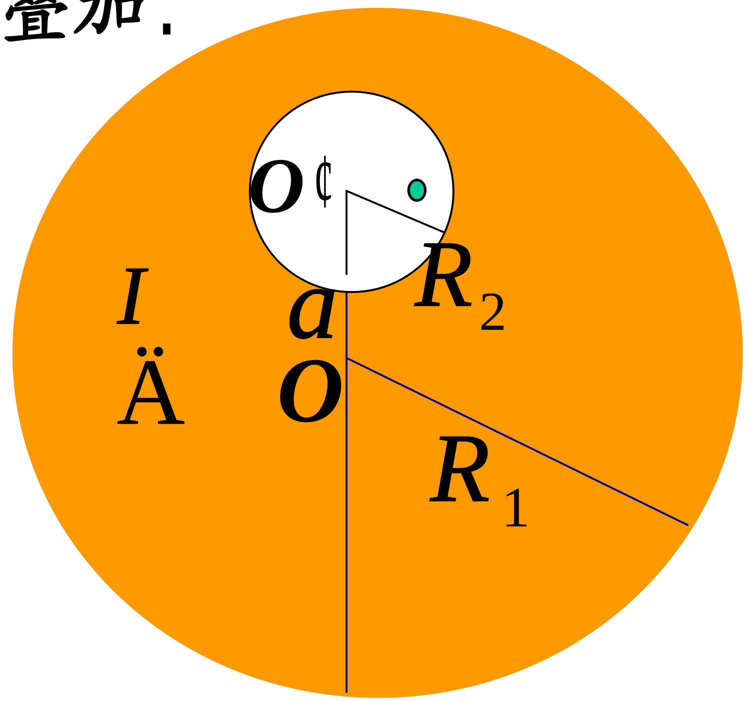
2)空心部分轴线上的磁感应强度的大小.

解:填补法　　(电流密度不变.)

以电流I'填满空心部分

$$I' = \frac{I}{\pi R_1^2 - \pi R_2^2}\pi R_2^2$$

整个磁场相当于与一个大的圆柱电流和一个半径为R_2的反向圆柱电流-I'产生的磁场的叠加.

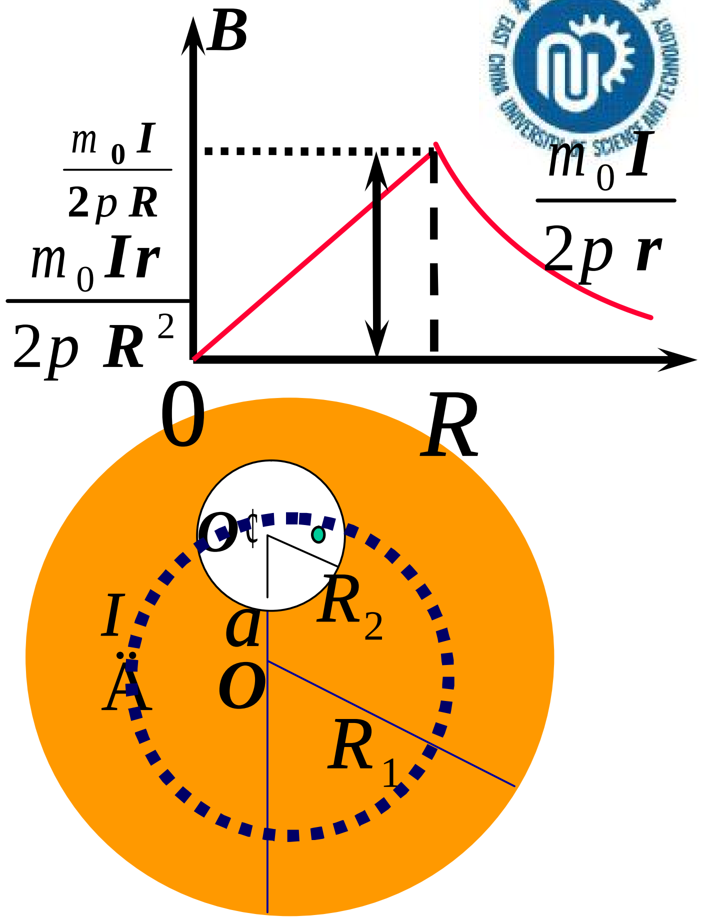


1)圆柱轴线上的磁感应强度B_0

$$B_{大O} = 0$$

$$B_0 = \frac{\mu_0 I R_2^{\,2}}{2\pi a\left(R_1^{\,2} - R_2^{\,2}\right)}$$

$$B_{小O} = \frac{\mu_0 I'}{2\pi a}$$



2)空心部分轴线上磁感应强度$B_{0'}$

$$B_{小O'} = 0$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I'$$

$$B 2\pi a = \mu_0 \frac{I}{\pi(R_1^{\,2} - R_2^{\,2})}\pi a^2$$

$$B_{0'} = \frac{\mu_0 I a}{2\pi\left(R_1^{\,2} - R_2^{\,2}\right)}$$