4. 力学量同时有确定值的条件.(共同本征函数问题)

若. F. G 有共同本征函数.

$$\begin{cases} F\psi = \lambda\psi \\ G\psi = \mu\psi \end{cases} \qquad \begin{aligned} GF\psi &= \lambda G\psi = \lambda\mu\psi \\ FG\psi &= \mu F\psi = \mu\lambda\psi \end{aligned}$$

得 $(FG - GF)\psi = 0.$

记 $[F, G] = 0$

逆定理 若 $[F, G]=0$, 则 F. G 有共同本征态, 同时有确定测量值.

5. 测不准关系, uncertainty principle.

$$I(\xi) = \int |\xi A\psi + iB\psi|^2 d\tau \qquad A, B \text{是力学量}. \ \xi\text{是实参数}.$$

$$= (\xi A\psi + iB\psi,\ \xi A\psi + iB\psi)$$

$$= (\xi A\psi, \xi A\psi) + (\xi A\psi, iB\psi) + (iB\psi, \xi A\psi) + (iB\psi, iB\psi)$$

复共轭. 取出来加负号

$$= \xi^2 (A\psi, A\psi) + \xi i (A\psi, B\psi) - i\xi (B\psi, A\psi) + (B\psi, B\psi)$$

$$= \xi^2 (\psi, A^2\psi) + \xi i (\psi, AB\psi) - i\xi (\psi, BA\psi) + (\psi, B^2\psi)$$

$$= \xi^2 (\psi, A^2\psi) + i\xi (\psi, [A,B]\psi) + (\psi, B^2\psi)$$

$$= \xi^2 \overline{A^2} + i\xi \overline{[A,B]} + \overline{B^2}$$

定义 $[A,B] = iC.$ $\quad = \xi^2 \overline{A^2} - \xi \overline{C} + \overline{B^2} \geqslant 0$ (平方积分一定不为负)

$$b^2 - 4ac \leq 0 \qquad (\overline{C})^2 - 4\overline{A^2}\,\overline{B^2} \leq 0 \qquad (\overline{C})^2 \leq \sqrt{4\overline{A^2}\,\overline{B^2}}$$

$$\frac{1}{2}|\overline{C}| \leq (\overline{A^2}\,\overline{B^2})^{\frac{1}{2}} \qquad \frac{1}{2}\left|\overline{[A,B]}\right| \leq (\overline{A^2}\,\overline{B^2})^{\frac{1}{2}}$$

$$\begin{cases} \Delta A = A - \overline{A} \\ \Delta B = B - \overline{B} \end{cases} \qquad [\Delta A, \Delta B] = [A, B].$$

A, B, ΔA ΔB 都是厄密的.

将 A, B. 换成 ΔA, ΔB.

$$\frac{1}{2}\left|\overline{[\Delta A, \Delta B]}\right| \leq \left(\overline{\Delta A^2}\ \overline{\Delta B^2}\right)^{\frac{1}{2}}$$

若 $[\Delta A, \Delta B] = 0$, 则. A, B 可同时确定.

$$\begin{aligned} A &= x \\ B &= p_x \end{aligned} \qquad [x, p_x] = i\hbar \ \Rightarrow \ \Delta x \Delta p_x \geqslant \frac{1}{2}\hbar$$

6 力学量随t的变化，守恒量

$\psi(t)$ 可测 $\overline{A(t)} = (\psi(t), A\psi(t))$

$$\frac{d\overline{A(t)}}{dt} = \left(\frac{\partial\psi}{\partial t}, A\psi(t)\right) + \left(\psi, \frac{\partial A}{\partial t}\psi\right) + \left(\psi, A\frac{\partial\psi}{\partial t}\right)$$

又，$i\hbar\frac{\partial}{\partial t}\psi = H\psi$

$$\Rightarrow = \left(\frac{1}{i\hbar}H\psi, A\psi\right) + \left(\psi, \frac{\partial A}{\partial t}\psi\right) + \left(\psi, A\cdot\frac{1}{i\hbar}H\psi\right)$$

$$= -\frac{1}{i\hbar}(H\psi, A\psi) + \left(\psi, \frac{\partial A}{\partial t}\psi\right) + \frac{1}{i\hbar}(\psi, AH\psi)$$

$$= \frac{1}{i\hbar}(\psi, [A,H]\psi) + \left(\psi, \frac{\partial A}{\partial t}\psi\right)$$

$$\frac{d\overline{A}}{dt} = \overline{\frac{\partial A}{\partial t}} + \frac{1}{i\hbar}\overline{[A,H]}$$

定义，守恒量. ① $\frac{\partial A}{\partial t} = 0$ ② $[A,H] = 0$. 则 A是守恒量.

守恒量的性质. ① $\frac{d\overline{A}}{dt} = 0$ ② 力学量在任意状态下测量值的概率分布不发生变化，

假设 $\{A,H\}$ 的共同本征态用 $\{\psi_k\}$ 表示.

$$\begin{cases} A\psi_k = \lambda_k\psi_k, \\ H\psi_k = E_k\psi_k, \end{cases}$$

$$\psi(t) = \sum_k C_k(t)\psi_k.$$

$$C_k(t) = (\psi_k, \psi(t))$$

$$\frac{d}{dt}C_k(t) = \left(\psi_k, \frac{1}{i\hbar}H\psi(t)\right)$$ H是 Hermit 算符.

$$= \left(H\psi_k, \frac{1}{i\hbar}\psi(t)\right)$$ 利用 $H\psi_k = E_k\psi_k$.

$$= \left(E_k\psi_k, \frac{1}{i\hbar}\psi(t)\right) = \frac{E_k}{i\hbar}(\psi_k, \psi(t)) = \frac{E_k}{i\hbar}C_k(t)$$

$$C_k(t) = C_k(0)\, e^{-iE_kt/\hbar}.$$

$$\psi(t) = \sum_k C_k(t)\psi_k = \sum_k C_k(0)e^{-iE_kt/\hbar}\psi_k$$

$$\psi(0) = \sum_k C_k(0)\psi_k$$