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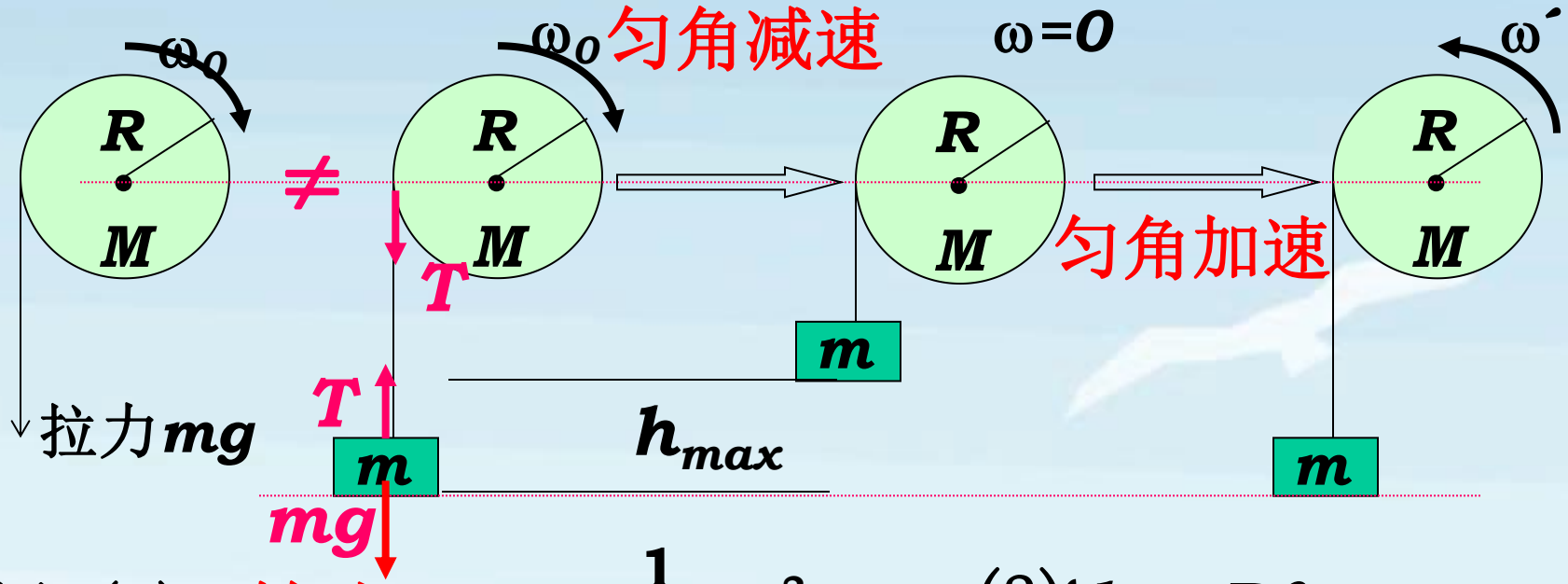
[例题3-3]关闭卷扬机$M=2,m=5,R=0.1,\omega_0=10$（SI）

求(1)α(2)m上升h_{max}(3)m回到原位置的ω'



解：(1) M转动 $mgR \times (\frac{1}{2}MR^2)\alpha$

平动 $mg - T = ma$

转动 $TR = (\frac{1}{2}MR^2)\alpha$

判据 $a = \alpha \cdot R$

$\Rightarrow \alpha = 81.7$

(2) $\begin{cases} h = R\theta \\ 0^2 - \omega_0^2 = -2\alpha\theta \end{cases}$

$\Rightarrow h = 6.12 \times 10^{-2}$

(3) $\omega'^2 - 0^2 = 2\alpha\theta$

$\Rightarrow \omega' = \sqrt{2\alpha\theta} = 10$

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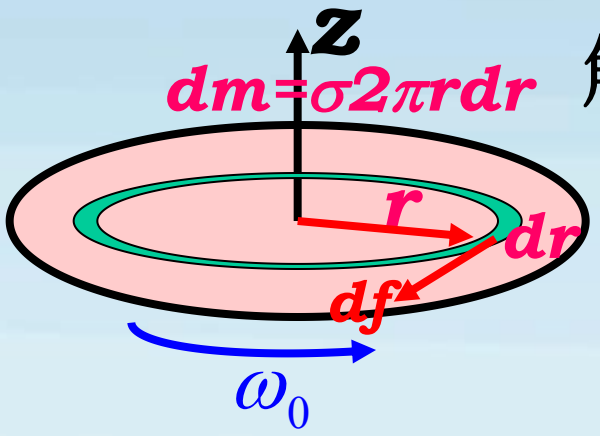
[例题3-4] 已知：园盘 m, R, ω_0 绕z轴转，μ，

求：(1) M_f (2) 园盘停下需 t



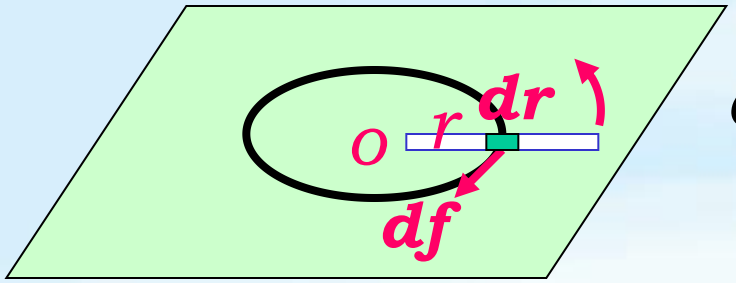
解：(1)
$$\left.\begin{aligned} df &= \mu g dm \\ dM_f &= -r df \end{aligned}\right\}$$

$$\Rightarrow \int_0^{M_f} dM_f = \int_0^R -\mu g \sigma 2\pi r^2 dr \Rightarrow M_f = -\frac{2}{3}\mu m g R$$

$$(2)\vec{J}_{冲} = \vec{L} - \vec{L}_0 \Rightarrow M_f t = 0 - L_0 \Rightarrow t = \frac{-J\omega_0}{M_f} = \frac{-\frac{1}{2}mR^2\omega_0}{-\frac{2}{3}\mu m g R} = \frac{3R\omega_0}{4\mu g}$$

or. $\alpha = \dfrac{M_f}{J} = -\dfrac{4\mu g}{3R}$ $\qquad 0 - \omega_0 = \alpha t$

[讨论3] 圆盘换成棒



$$dM = r df \Rightarrow M = \int_0^l r\mu g\lambda dr = \mu m g l / 2$$

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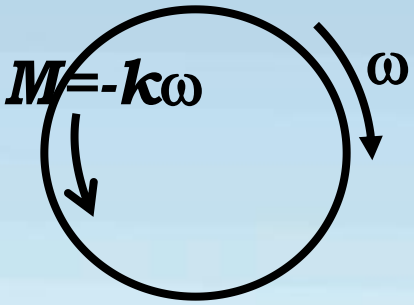
[讨论4] 飞轮 J, ω_0, 求 $\omega_0 \to \omega_0/2$ 的 t?



解: 由动量矩定理

$$\vec{J}_{冲} = \vec{L} - \vec{L}_0 \Rightarrow \int_0^t -k\omega\, dt = J\frac{\omega_0}{2} - J\omega_0$$

Stop here

由转动定律 -基本方法

$$M = J\alpha \Rightarrow -k\omega = J\frac{d\omega}{dt}$$

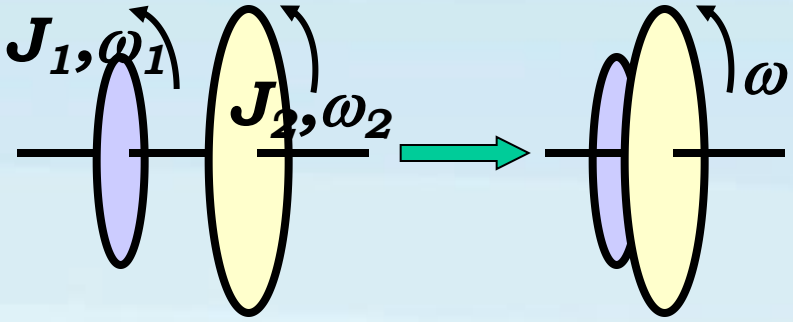
$$\Rightarrow \int_{\omega_0}^{\frac{\omega}{2}} \frac{d\omega}{\omega} = \int_0^t -\frac{k}{J}\, dt$$

$$\Rightarrow t = \frac{J}{k}\ln 2$$

(3)动量矩守恒示例

①飞轮啮合

A 同轴

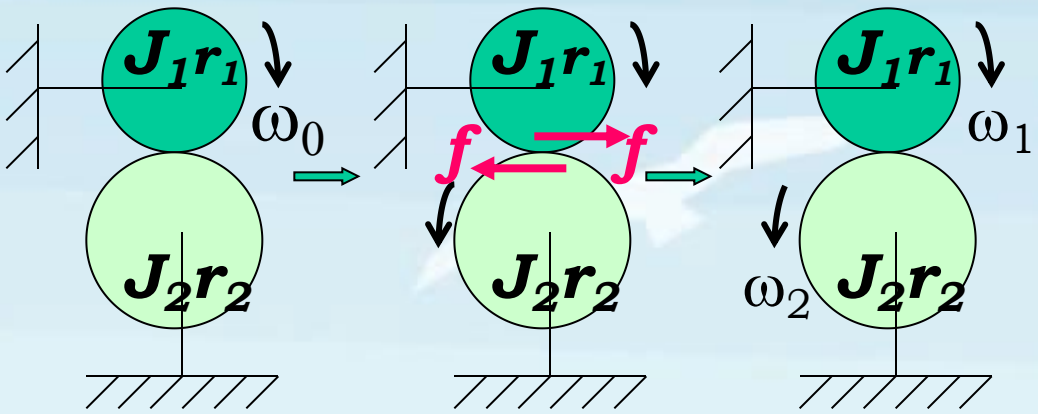


$$J_1\omega_1 + J_2\omega_2 = (J_1 + J_2)\omega$$

$$\Delta E = \frac{1}{2}(J_1 + J_2)\omega^2$$

$$-\frac{1}{2}J_1\omega_1^2 - \frac{1}{2}J_2\omega_2^2 \neq 0$$

B 不同轴



$$-fr_1\Delta t = J_1\omega_1 - J_1\omega_0$$

$$fr_2\Delta t = J_2\omega_2 - 0$$

$$v = \omega_1 r_1 = \omega_2 r_2$$

$$\Delta E = \frac{1}{2}J_1\omega_1^2 + \frac{1}{2}J_2\omega_2^2 - \frac{1}{2}J_1\omega_0^2 \neq 0$$

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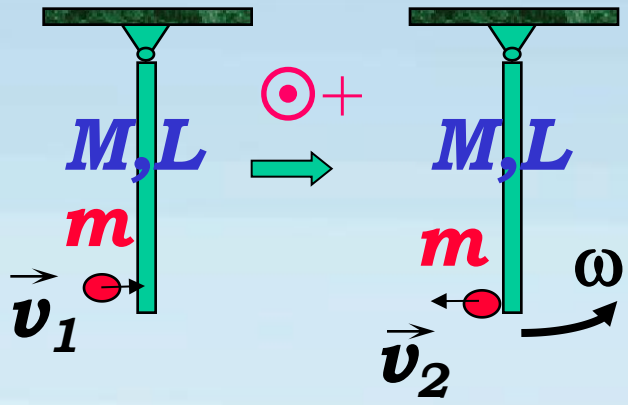
②质点碰棒、板、圆盘

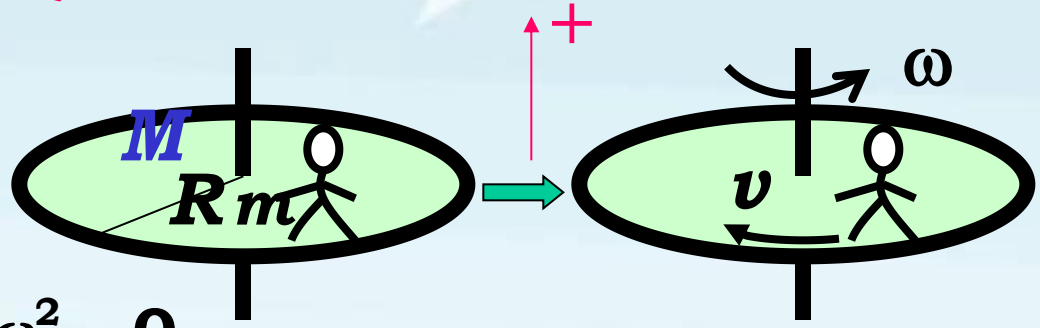


$$mv_1L=-mv_2L+\frac{1}{3}ML^2\omega$$

$$\Delta E=\frac{1}{2}mv_2^2+\frac{1}{2}(\frac{1}{3}ML^2)\omega^2-\frac{1}{2}mv_1^2$$

$\begin{cases}\neq 0 \text{——非(完全)弹性碰撞}\\ =0 \text{——(完全)弹性碰撞}\end{cases}$

③圆盘上行走（人对地v）



$$0=\frac{1}{2}MR^2\omega-Rmv$$

$$\Delta E=\frac{1}{2}mv^2+\frac{1}{2}(\frac{1}{2}MR^2)\omega^2-0$$

[思考]③中v为人对盘

$$0=\frac{1}{2}MR^2\omega-Rm(v-\omega R)$$

[课后思考]开始圆盘ω_0，人以v(对盘)向中心走到R/2时ω

[课后思考] 科学方法的启示-类比

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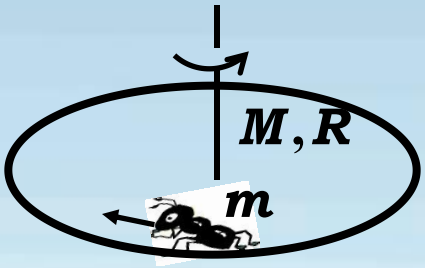

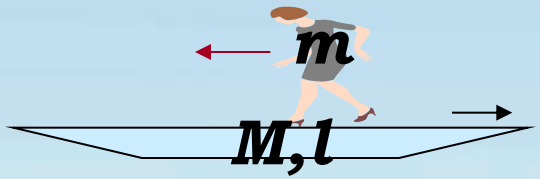

开始系统静止，
蚂蚁爬盘一圈，
盘对地的转角？

开始系统静止，
人船尾到船头，
船对地的位移？

[讨论5]运动员以$J\omega_0^2/2$的动能绕自身轴旋转，
突然收双臂将J变为$4J/9$,求此后ω？

$$\text{解:}\frac{1}{2}J\omega_0^2 = \frac{1}{2}\left(\frac{4}{9}J\right)\omega^2 \quad \Rightarrow \omega = \frac{3}{2}\omega_0$$

$$J\omega_0 = \frac{4}{9}J\omega \quad \Rightarrow \omega = \frac{9}{4}\omega_0$$