

### 第三章答案 1

3.1  $\frac{1}{4}$ .

3.2  $\xi \setminus \eta$

	1	1.2	1.3	1.5
0.9	0	$\frac{1}{5}$	0	0
1	$\frac{1}{5}$	0	$\frac{1}{5}$	0
1.2	$\frac{1}{5}$	0	0	0
1.4	0	0	0	$\frac{1}{5}$

3.3  $P(\xi = m, \eta = n) = \frac{1}{6} \left( \frac{1}{3} \right)^{n-1}, \quad m = 1, 2, 3, 4, \quad n \geq 1$

$P(\xi = m, \eta = n) = 0, \quad m = 5, 6$

3.4 (1)

$\xi \setminus \eta$	0	1	$P(\xi = i)$
0	$\frac{a^2}{(a+b)^2}$	$\frac{ab}{(a+b)^2}$	$\frac{a}{a+b}$
1	$\frac{ab}{(a+b)^2}$	$\frac{b^2}{(a+b)^2}$	$\frac{a}{a+b}$
$P(\eta = j)$	$\frac{a}{a+b}$	$\frac{b}{a+b}$	1

(2)

$\xi \setminus \eta$	0	1	$P(\xi = i)$
0	$\frac{a(a-1)}{(a+b)(a+b-1)}$	$\frac{ab}{(a+b)(a+b-1)}$	$\frac{a}{a+b}$
1	$\frac{ab}{(a+b)(a+b-1)}$	$\frac{b(b-1)}{(a+b)(a+b-1)}$	$\frac{a}{a+b}$
$P(\eta = j)$	$\frac{a}{a+b}$	$\frac{b}{a+b}$	1

3.5  $\frac{15}{64}, 0, 0.5, 0.5$

$$F(x) = \begin{cases} 0 & x < 0 \text{ 或 } y < 0 \\ x^2 y^2 & 0 \leq x < 1, \quad 0 \leq y < 1 \\ x^2 & 0 \leq x < 1, \quad y \geq 1 \\ y^2 & x \geq 1, \quad 0 \leq y < 1 \\ 1 & x \geq 1, \quad y \geq 1 \end{cases}$$

$$3.6 \quad (1) A = \frac{1}{2}; \quad (2) p_{\xi}(x) = \begin{cases} \frac{1}{2}(\sin x + \cos x), & 0 < x < \frac{1}{2} \\ 0, & \text{其他} \end{cases}$$

$\xi$  与  $\eta$  同分布

$$3.7 \quad (1) \frac{3}{\pi R^3} \quad (2) \frac{3r^2}{R^2} \left(1 - \frac{2r}{3R}\right).$$

3.8 略.

$$3.9 \quad p(x, y) = \begin{cases} \frac{1}{(b-a)(d-c)} & a \leq x \leq b, \quad c \leq y \leq d \\ 0 & \text{其他} \end{cases}$$

$$p_{\xi}(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{其他} \end{cases}$$

$$p_{\eta}(y) = \begin{cases} \frac{1}{d-c} & c \leq y \leq d \\ 0 & \text{其他} \end{cases}$$

$\xi$  与  $\eta$  独立的

$$3.10 \quad \frac{5}{81}$$

3.11

$\xi \backslash \eta$	0	1
0	0.1	0.3
1	0.2	0.2
2	0.1	0.2

$\xi$	0	1
$P(\xi = x_i \setminus \eta_j \neq 1)$	0.5	0.5

$$3.12 \quad (1) P(\xi = n) = \frac{14^n e^{-14}}{n!} \quad n = 0, 1, 2, \dots$$

$$P(\eta = m) = \frac{7.14^m e^{-7.14}}{m!} \quad m = 0, 1, 2, \dots m$$

$$(2) \quad P(\xi = n | \eta = m) = \frac{(6.86)^{n-m}}{(n-m)!} e^{-6.86} \quad n = m, m+1, \dots$$

$$P(\eta = m | \xi = n) = \binom{n}{m} (0.51)^m (0.49)^{n-m}, \quad m = 0, 1, \dots, n$$

$$(3) \quad \xi = 20 \text{ 时}$$

$$P(\eta = m | \xi = 20) = \binom{20}{m} (0.51)^m (0.49)^{20-m} \quad m = 0, 1, \dots, 20$$

$$3.13 \quad \text{对于 } y > 0$$

$$p_{\xi|\eta}(x|y) = \begin{cases} \frac{1}{y}, & 0 < x < y \\ 0, & \text{其他} \end{cases}$$

$$\text{对于 } x > 0$$

$$p_{\eta|\xi}(y|x) = \begin{cases} e^{x-y}, & 0 < y < x \\ 0, & \text{其他 } \xi \setminus \xi \end{cases}$$

$$3.14 \quad \frac{7}{15}$$

$$3.15 \quad p_{\xi|\eta}(x|y) = (y+1)^2 x e^{-x(y+1)} \quad (x > 0)$$

$$p_{\eta|\xi}(y|x) = x e^{-xy} \quad (y > 0)$$

不独立

$$3.16 \quad 1.1, \quad 1.3, \quad -0.13$$

$$3.17 \quad 1.05, \quad 0.5, \quad 0.25, 1.2, \quad 0.36.$$

$$3.18 \quad \frac{3}{4} \sqrt{\pi}$$

$$3.19 \quad 7.$$

$$3.20 \quad (1) \frac{3}{4}, (2) \frac{y}{2}$$

$$3.21 \quad (1) \frac{1}{12} \quad (2) 17.8$$

$$3.22 \quad \frac{\alpha^2 - \beta^2}{\alpha^2 + \beta^2}$$

$$3.23 \quad \left(\frac{1}{2}\right)^{\sqrt[3]{k}}, \quad k = 1, 8, 27, 64, \dots$$

$$3.24 \quad p(x) = \begin{cases} \frac{1}{27}, & 0 < x < 27 \\ 0, & \text{其他} \end{cases}$$

$$3.25 \quad (1) \quad \begin{array}{ccc} \xi \setminus \eta & 0 & 1 \\ -1 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \\ 1 & \frac{1}{4} & 0 \end{array}$$

(2) 不独立 (3)

$\xi$	0	1
$P(\xi = x_i)$	$\frac{1}{4}$	$\frac{3}{4}$

$$3.26 \quad p_{\zeta}(z) = \begin{cases} (e-1)e^{-z}, & z \geq 1; \\ 1-e^{-z}, & 0 < z < 1; \\ 0, & z \leq 0 \end{cases}$$

$$3.27 \quad p_{\zeta}(z) = \frac{1}{\pi(1+z^2)} \quad -\infty < z < +\infty$$

$$3.28 \quad (1) \quad p_1(z) = \begin{cases} (\alpha + \beta)e^{-(\alpha+\beta)z}, & z > 0; \\ 0, & z \leq 0. \end{cases}$$

$$(2) \quad p_2(z) = \begin{cases} \alpha e^{-\alpha z} + \beta e^{-\beta z} - (\alpha + \beta)e^{-(\alpha+\beta)z}, & z > 0; \\ 0, & z \leq 0. \end{cases}$$

$$3.29 \quad \begin{array}{cccccc} \xi \setminus \eta & 0 & 1 & 2 & 3 & P(\xi = i) \\ 0 & \frac{1}{27} & \frac{1}{9} & \frac{1}{9} & \frac{1}{27} & \frac{8}{27} \\ 1 & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & 0 & \frac{4}{9} \\ 2 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{2}{9} \\ 3 & \frac{1}{27} & 0 & 0 & 0 & \frac{1}{27} \\ P(\eta = j) & \frac{8}{27} & \frac{4}{9} & \frac{2}{9} & \frac{1}{27} & 1 \end{array}$$

$$3.30 \quad P(\xi = i, \eta = j) = \frac{\binom{3}{i} \binom{2}{j} \binom{2}{4-i-j}}{\binom{7}{3}} \quad \begin{array}{l} i = 0, 1, 2, 3, \\ j = 0, 1, 2 \\ i + j \leq 4 \end{array}$$

或

$\xi \backslash \eta$	0	1	2
0	0	0	$\frac{1}{35}$
1	0	$\frac{6}{35}$	$\frac{6}{35}$
2	$\frac{3}{35}$	$\frac{12}{35}$	$\frac{3}{35}$
3	$\frac{2}{35}$	$\frac{2}{35}$	0

$$3.31 \quad (1) A = 3 \quad (2) \frac{37}{64} \quad (3) \frac{11}{16} \quad (4) \frac{1}{64} \quad (5) 0$$

$$3.32 \quad (1) \quad F_{\xi}(x) = \begin{cases} 0, & x < 0; \\ \frac{1}{7}(4x^3 + 3x^2), & 0 \leq x < 1; \\ 1, & x \geq 1; \end{cases} \quad (2) \frac{15}{56}$$

$$(3) 0.1540$$

$$3.33 \quad (1) \quad p_{\xi}(x) = \begin{cases} 1+x, & -1 < x < 0; \\ 1-x, & 0 \leq x < 1; \\ 0, & \text{其他}; \end{cases} \quad p_{\eta}(y) = \begin{cases} 2y & 0 < y < 1; \\ 0 & \text{其他}; \end{cases}$$

$$(2) \text{不独立}$$

$$3.34 \quad (1) \quad \begin{array}{c|ccc} \eta & 0 & 1 & 2 \\ \hline P(\eta = j) & \frac{7}{12} & \frac{7}{18} & \frac{1}{36} \end{array}$$

$$\begin{array}{c|ccc} \xi & 0 & 1 & 2 \\ \hline P(\xi = i) & \frac{5}{12} & \frac{1}{2} & \frac{1}{12} \end{array}$$

$$(2)$$

$$\begin{array}{c|cc} \eta & 0 & 1 \\ \hline P(\eta = j \mid \xi = 1) & \frac{2}{3} & \frac{1}{3} \end{array}$$

(3)

$\xi$	0	1	2
$P(\xi = j \mid \eta = 0)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

(4) 不独立

$$3.35 \quad (1) 5, \quad (2) 3, \quad (3) \frac{47}{64}, \quad (4) \frac{56}{125}.$$

$$3.36 \quad \frac{1}{\pi}; \quad D\xi, D\eta \text{ 都不存在}$$

$$3.37 \quad \frac{4}{3}, \quad \frac{1}{18}$$

$$3.38 \quad n[1 - \left(\frac{n-1}{n}\right)^N]$$

$$3.39 \quad \frac{n\lambda_1}{\lambda_1 + \lambda_2}$$

$$3.40 \quad \frac{2y+1}{3}$$

$$3.41 \quad \frac{\lambda+12}{2\lambda}$$

$$3.42 \quad E(\xi - \lambda)^3 = \lambda; \quad E(\xi - \lambda)^4 = \lambda(1 + 3\lambda)$$

$$3.43 \quad \rho_{\xi\eta} = \begin{cases} 0 & n \text{ 奇数} \\ \frac{n!!}{\sqrt{(2n-1)!!}} & n \text{ 偶数} \end{cases}$$

$$3.44 \quad \frac{3}{\sqrt{57}}$$

$$3.45 \quad 0; \quad \xi \text{ 与 } \eta \text{ 不独立}$$

$$3.46 \quad (1) \quad a = \frac{1}{15}$$

(2)

$\eta$	0	1	4	9
$P(\eta = j)$	$\frac{3}{15}$	$\frac{7}{30}$	$\frac{3}{15}$	$\frac{11}{30}$

$$3.47 \quad p_{\eta}(y) = \begin{cases} \frac{\beta}{\alpha} y^{\beta-1} e^{-y^{\beta}/\alpha}, & y > 0; \\ 0, & y \leq 0 \end{cases}$$

$$3.48 \quad p_{\eta}(y) = \begin{cases} \frac{2}{\pi\sqrt{1-y^2}}, & 0 < y < 1; \\ 0, & \text{其他.} \end{cases}$$

$$3.49 \quad (1) \quad p_{\eta}(y) = \begin{cases} \frac{1}{2\sqrt{\pi(y-1)}} e^{-(y-1)/4}, & y > 1; \\ 0, & \text{其他.} \end{cases}$$

$$(2) \quad p_{\eta}(y) = \begin{cases} \sqrt{\frac{2}{\pi}} e^{-\frac{y^2}{2}}, & y > 0; \\ 0, & \text{其他.} \end{cases}$$

$$3.50 \quad P(\xi + \eta = n) = \frac{n-1}{2^n}, \quad n = 2, 3, \dots$$

3.51

$\xi$	-1	0	1
$P(\xi = x_i)$	0.1344	0.7312	0.1344

$$3.52 \quad p_{\zeta}(z) = \begin{cases} \frac{1}{2} z^2 & 0 \leq z \leq 1 \\ -z^2 + 3z - \frac{3}{2} & 1 < z \leq 2 \\ \frac{1}{2} z^2 - 3z + \frac{9}{2} & 2 < z \leq 3 \\ 0 & \text{其他} \end{cases}$$

$$3.53 \quad F_{\zeta}(z) = \begin{cases} (1 - e^{-\frac{x^2}{8}})^5, & x \geq 0; \\ 0, & x < 0. \end{cases}$$

3.54 B

3.55 B

3.56 B

3.57 D

3.58 A

3.59  $\frac{5}{9}$

3.60 2.4

$$3.61 \quad p_{\eta}(y) = \begin{cases} \frac{3}{2} y^{\frac{1}{2}} & 0 < y < 1 \\ 0 & \text{其他} \end{cases}$$

$$3.62 \quad \frac{1}{5\sqrt{2\pi}} e^{-(x-5)^2/50}$$

$$3.63 \quad \begin{cases} 0, & z < 0 \\ \frac{1}{6}(1 - e^{-8z}), & 0 \leq z \leq 6 \\ 1 - e^{-8z} & z > 6 \end{cases}$$

$$3.64 \quad (1) \quad \frac{1}{9}, \frac{2}{9}, \frac{2}{9};$$

(2)

$\xi$	0	1	2
P	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

$\eta$	0	1	2
P	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

$\xi$  与  $\eta$  不独立;

$$(3) \quad F_{\xi}(z) = \begin{cases} 0, & z < 0 \\ \frac{1}{9}, & 0 \leq z < 1 \\ \frac{7}{9}, & 1 \leq z < 2 \\ 1, & z \geq 2 \end{cases};$$

$$(4) \quad -\frac{2}{9}.$$

$$3.65 \quad (1) \quad p(x, y) = \begin{cases} \frac{1}{2}, & (x, y) \in D \\ 0, & \text{其他} \end{cases}; (2) \quad p_{\xi}(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & \text{其他} \end{cases},$$

$$p_{\eta}(y) = \begin{cases} 1 - |y|, & |y| < 1 \\ 0, & \text{其他} \end{cases}; (3) \quad \xi \text{ 与 } \eta \text{ 不独立}; (4) \quad p_{\xi}(x) = \begin{cases} \frac{1}{2}, & |x| < 1 \\ 0, & \text{其他} \end{cases}$$

$$3.66 \quad (1) \quad 4; \quad (2) \quad \frac{2}{3}, \frac{2}{3}, \frac{1}{18}, \frac{1}{18} \quad (3) \quad 0, \quad 0.$$

$$3.67 \quad p_{\xi}(z) = \begin{cases} \frac{z^3}{6} e^{-z}, & z \geq 0 \\ 0, & z < 0 \end{cases}, \quad 4, \quad 4.$$