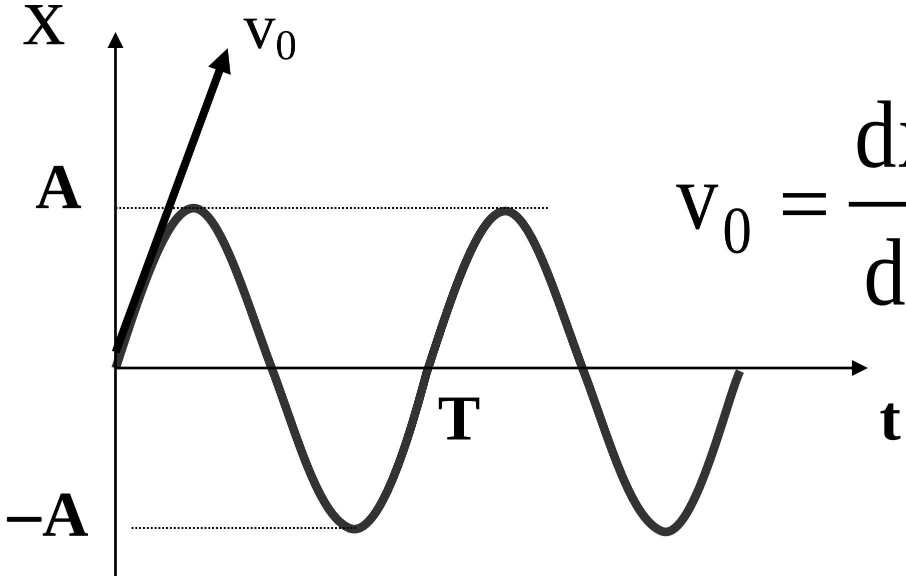
解析法: $x(t) = A\cos(\frac{2\pi}{T}t - \frac{\pi}{2})$

图示法:



$$v_0 = \frac{dx}{dt}\Big|_{t=0} > 0$$

$$x_0 = A\cos\alpha = 0$$

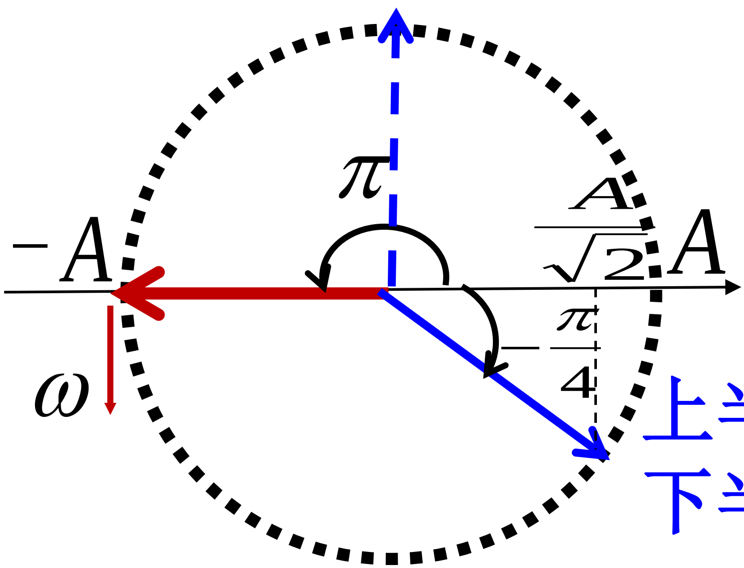
$$\cos\alpha = 0 \Rightarrow \begin{cases} \dfrac{\pi}{2} \\ \dfrac{3\pi}{2}(-\dfrac{\pi}{2}) \end{cases}$$

$$v_0 = -\omega A\sin\alpha > 0$$

$$\Rightarrow \sin\alpha < 0$$

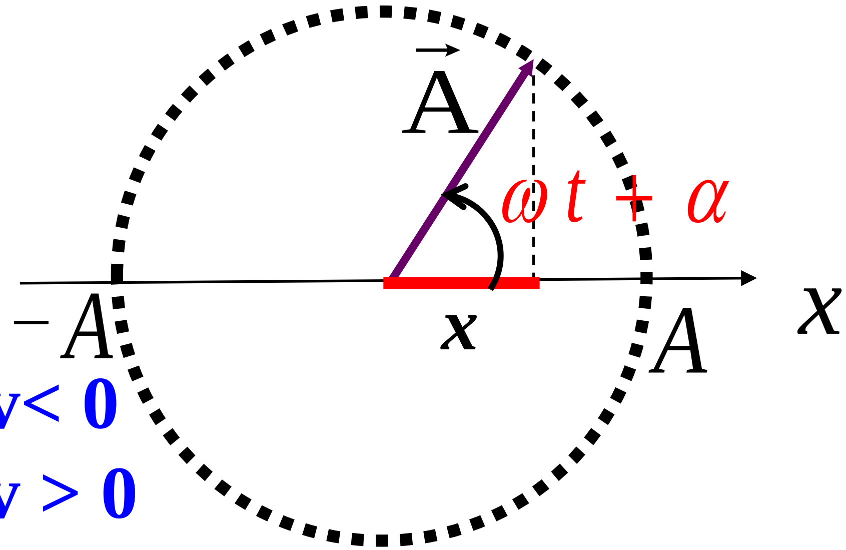
$$\Rightarrow \alpha = -\frac{\pi}{2}$$

旋转矢量法



上半圆v< 0

下半圆v > 0



$$x = A\cos(\omega t + \pi)$$

已知：$x_0 = \frac{A}{\sqrt{2}}, \quad v_0 > 0, T$

$$x_2 = A\cos\left(\frac{2\pi}{T}t - \frac{\pi}{4}\right)$$

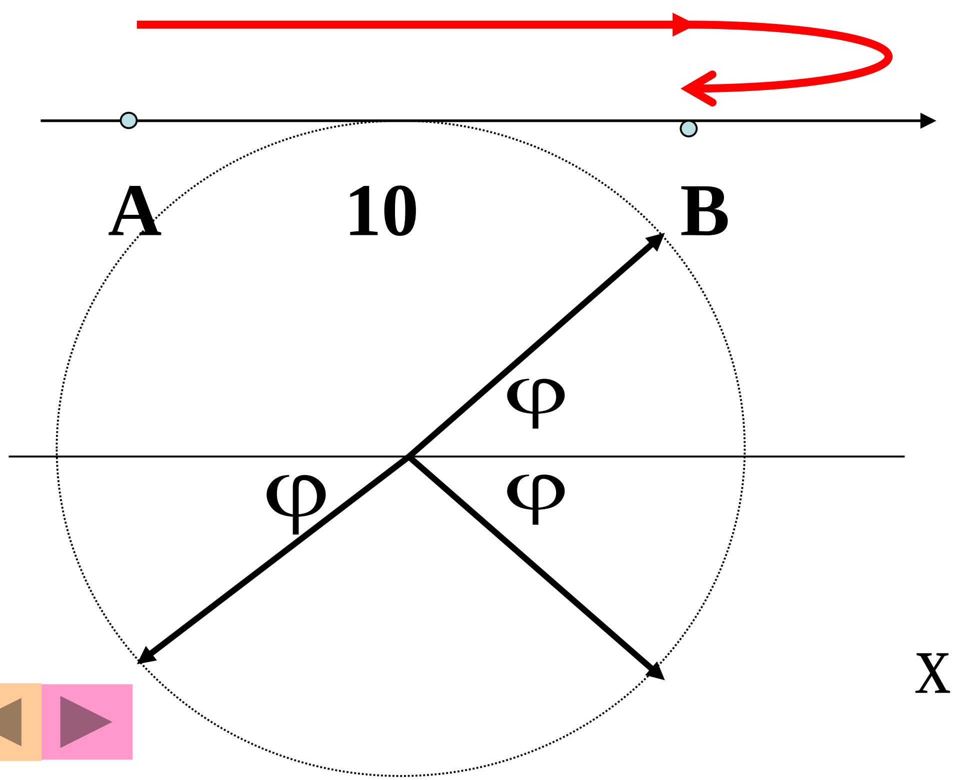
第一次通过平衡位置需要多少时间？

$$\Rightarrow \frac{\Delta\phi}{\Delta t} = \frac{2\pi}{T}$$

$$\frac{\frac{3}{4}\pi}{\Delta t} = \frac{2\pi}{T} \Rightarrow \Delta t = \frac{3}{8}T$$

例1、一质点在x轴上作简谐振动，选取质点向右运动通过A点时作为计时起点（t=0）。经过2秒后，质点第一次经过B点，再经过2秒后，质点第二次经过B点。若已知质点在A、B两点具有相同的速率，且AB=10cm，求质点的振动方程。

$$T = 8s \qquad \omega = \frac{2\pi}{T} = \frac{\pi}{4}$$

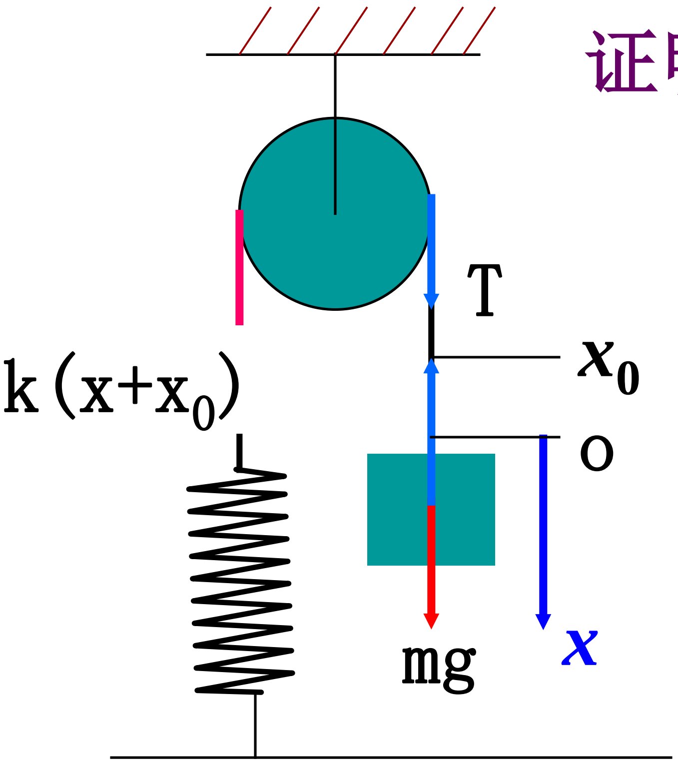


$$\frac{2\pi}{T} = \frac{2\varphi}{\Delta t(2s)} \Rightarrow \varphi = \frac{\pi}{4}$$

$$A = \frac{5}{\cos\frac{\pi}{4}} = 5\sqrt{2}$$

$$x = 5\sqrt{2}\cos\left(\frac{\pi}{4}t - \frac{3}{4}\pi\right)(cm)$$

例2、如图滑轮（J、R），弹簧 k、物体m，证明物体作简谐振动。设开始弹簧为原长，静止放手，求振动方程。



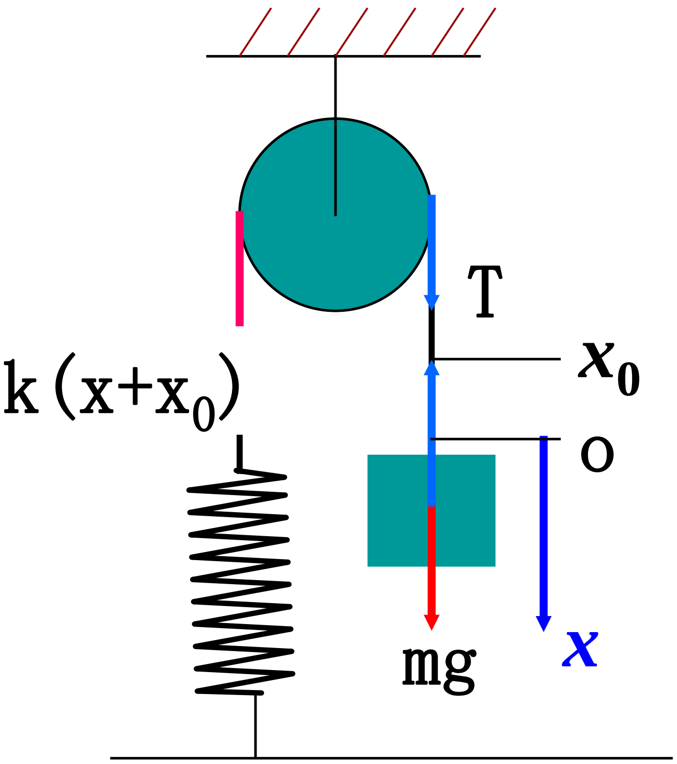
证明:

$$\begin{cases} mg - T = 0 \Rightarrow mg = kx_0 \\ mg - T = ma \\ TR - k(x + x_0)R = J\alpha \\ a = \alpha R \end{cases}$$

$$\frac{d^2x}{dt^2} + \boxed{\frac{k}{\frac{J}{R^2} + m}} x = 0$$

$$= \omega^2$$

当t = 0时 $\begin{cases} x_0 = -\dfrac{mg}{k} \\ v_0 = 0 \end{cases}$



$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = \frac{mg}{k}$$

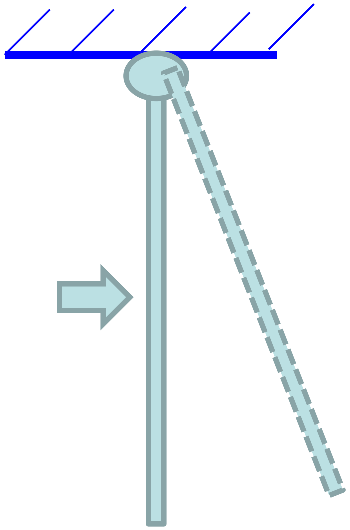
初始位置在负最大位移处

$$\Rightarrow \varphi = \pi$$

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

$$x = \frac{mg}{k}\cos\left(\sqrt{\frac{k}{m + \dfrac{J}{R^2}}}\,t + \pi\right)$$

例3 书P162 4-14



$$-(m+m_0)g\frac{l}{2}\sin\varphi = J\alpha$$

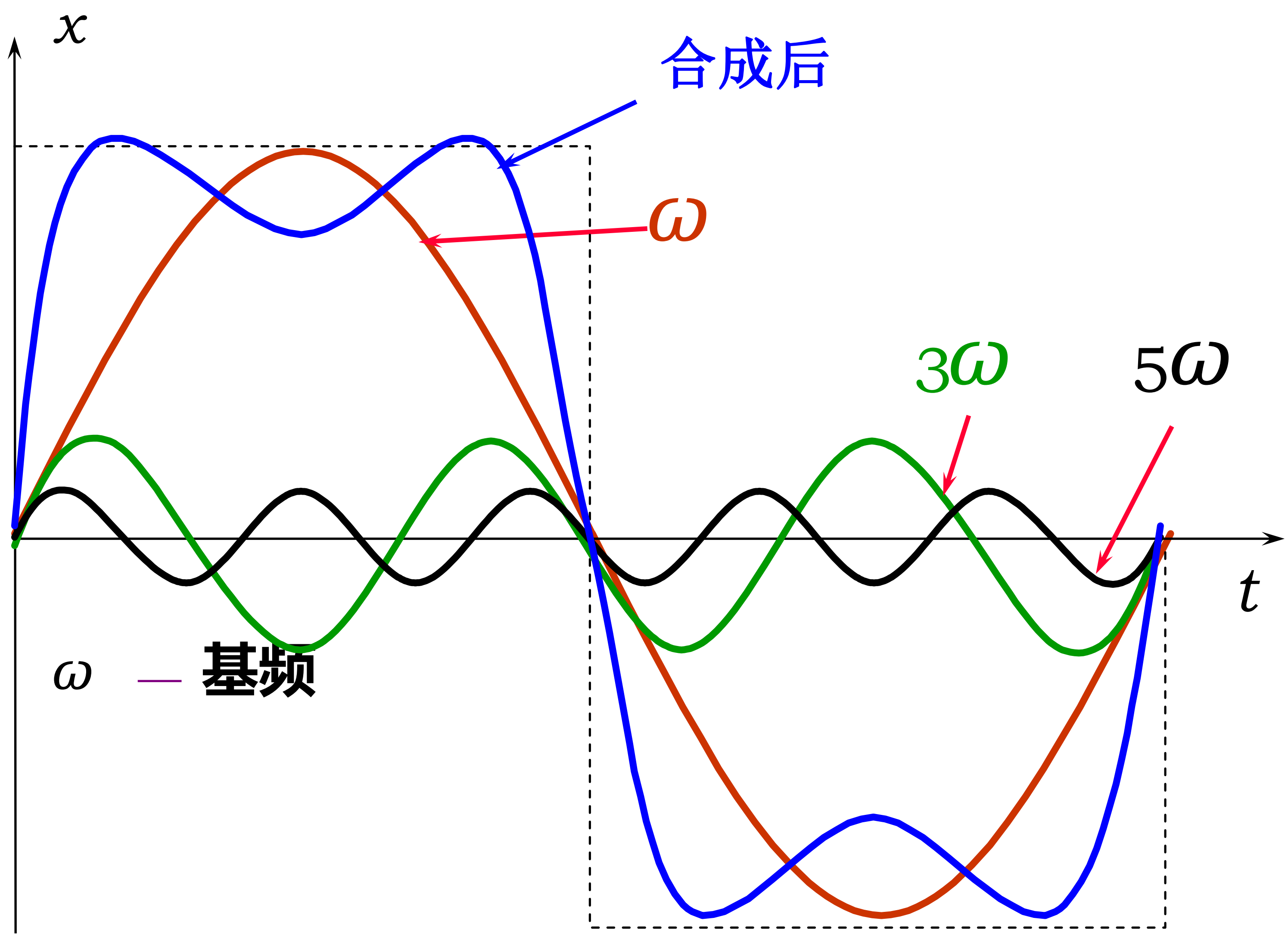
$$\Rightarrow -(m+m_0)g\frac{l}{2}\varphi = \left[\frac{1}{3}m_0 l^2 + m\left(\frac{l}{2}\right)^2\right]\frac{d^2\varphi}{dt^2}$$

$$\Rightarrow \frac{d^2\varphi}{dt^2} + \frac{6(m+m_0)\mathrm{g}}{(4m_0+3m)l}\varphi = 0$$

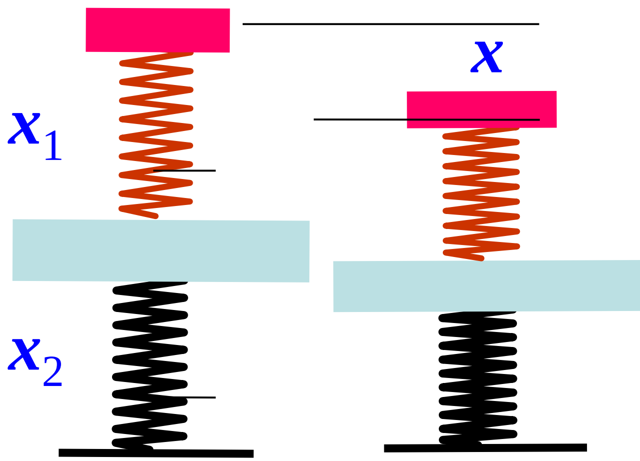
$$\omega^2 = \frac{6(m + m_0)g}{(4m_0 + 3m)l}$$

$$T = 2\pi\sqrt{\frac{(4m_0 + 3m)l}{6(m + m_0)g}}$$

$$x = \frac{4A}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t\right)$$



四、简谐振动的合成



$$x = x_1 + x_2$$

$$x = A\cos(\omega t + \alpha) \quad \begin{cases} x & y \\ \omega_1 & \omega_2 \end{cases}$$

同方向不同频率

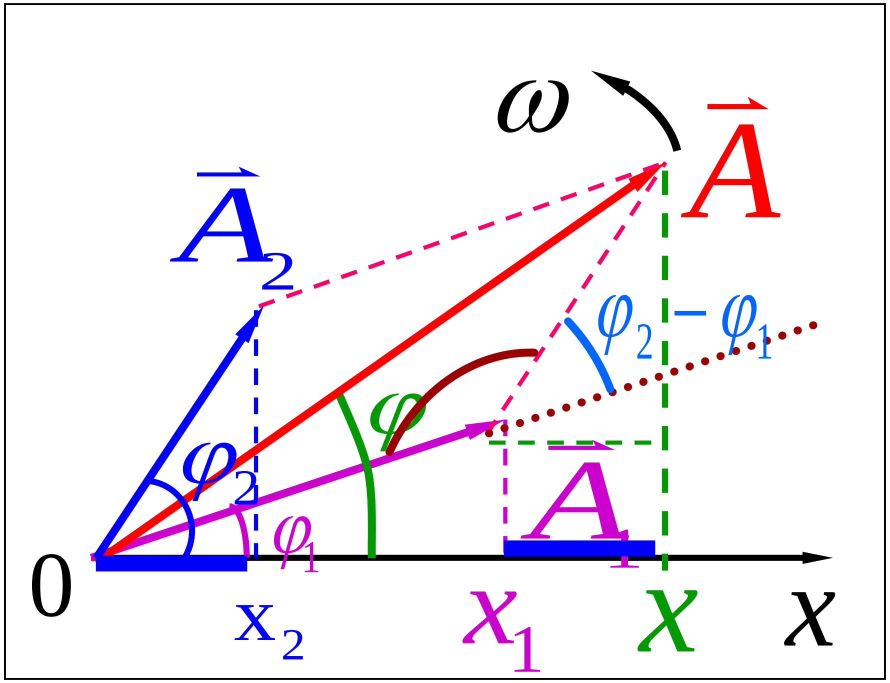
垂直方向不同频率

同方向同频率

垂直方向同频率

1、两个同方向同频率简谐运动的合成

$$\begin{cases} x_1 = A_1\cos(\omega t+\varphi_1) \\ x_2 = A_2\cos(\omega t+\varphi_2) \end{cases}$$



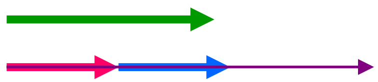
$$x = x_1 + x_2$$

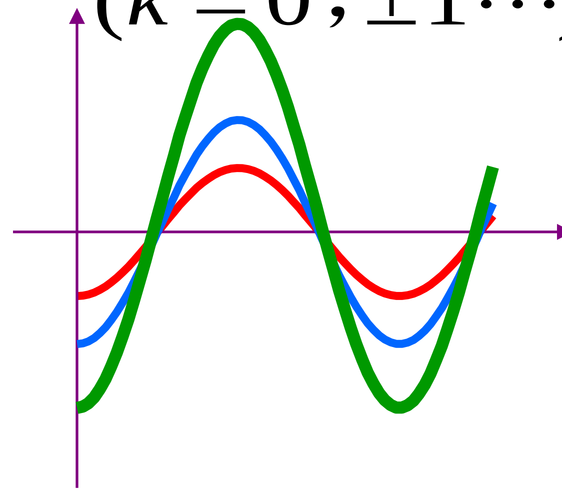
$$\boxed{x = A\cos(\omega t+\varphi)}$$

$$\begin{cases} A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2-\varphi_1)} \\ \tan\varphi = \dfrac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \end{cases}$$

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

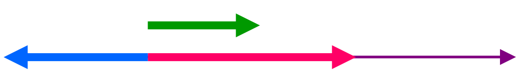
1）相位差 $\varphi_2-\varphi_1=2k\pi \qquad (k=0,\pm1\cdots)$

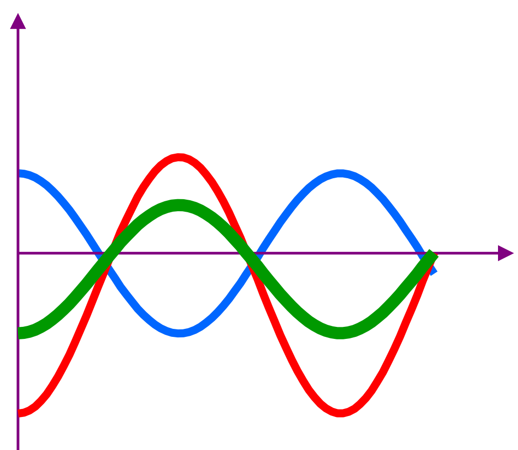




$$A=A_1+A_2$$ 相互加强

2）相位差 $\varphi_2-\varphi_1=(2k+1)\pi \qquad (k=0,\pm1\cdots)$





$$A=|A_1-A_2|$$ 相互削弱

3）一般情况

$$A_1+A_2>A>|A_1-A_2|$$

例7、 已知 $x_1 = 6\cos\left(5t + \dfrac{\pi}{2}\right) \quad x_2 = 2\sin(\pi - 5t)$

求：合振动的表达式

解法1： $x_2 = 2\sin(\pi - 5t) = 2\cos\left(5t - \dfrac{\pi}{2}\right)$

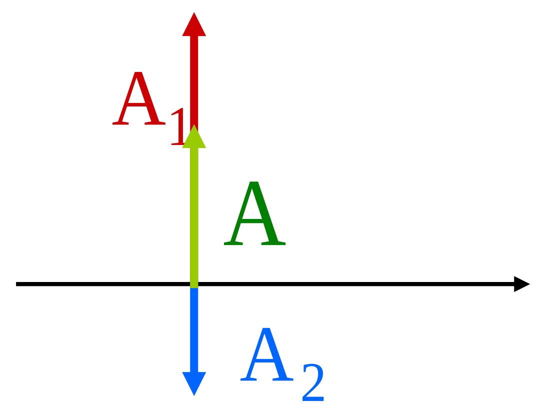
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 \boxed{\mp \varphi_1})}$$

$$= |A_1 - A_2| = 6 - 2 = 4$$

$$\tan\alpha = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} = \infty$$

$$\Rightarrow \varphi = \frac{\pi}{2} \qquad x = 4\cos\left(5t + \frac{\pi}{2}\right)$$

解法2：

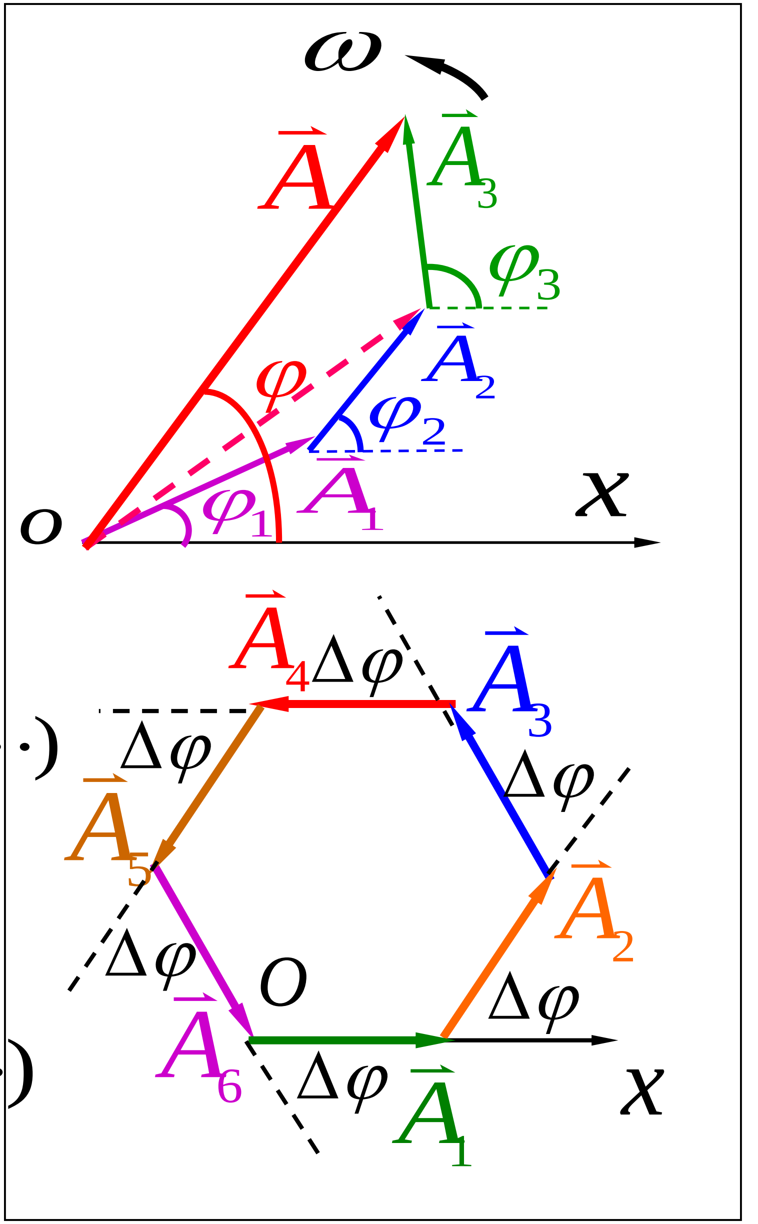


2、多个同方向同频率简谐运动合成

$$x = x_1 + x_2 + \cdots + x_n$$

若相互间的相位差相等

$$\begin{cases} x_1 = A_0 \cos\omega t \\ x_2 = A_0 \cos(\omega t + \Delta\varphi) \\ x_3 = A_0 \cos(\omega t + 2\Delta\varphi) \\ \cdots\cdots \\ x_N = A_0 \cos[\omega t + (N-1)\Delta\varphi] \end{cases}$$



1） $\Delta\varphi = 2k\pi \quad (k = 0,1,2,\cdots)$

$$A = \sum_i A_i = NA_0$$

2） $N\Delta\varphi = 2k'\pi$

$$(k' \neq kN, k' = \pm 1, \pm 2, \cdots)$$

$\Rightarrow A = 0$ （光栅多缝干涉）

3、两个同方向不同频率简谐运动合成

$$\boxed{x = x_1 + x_2}$$

$$\begin{cases} x_1 = A_1 \cos\omega_1 t = A_1 \cos 2\pi\,\nu_1 t \\ x_2 = A_2 \cos\omega_2 t = A_2 \cos 2\pi\,\nu_2 t \end{cases}$$

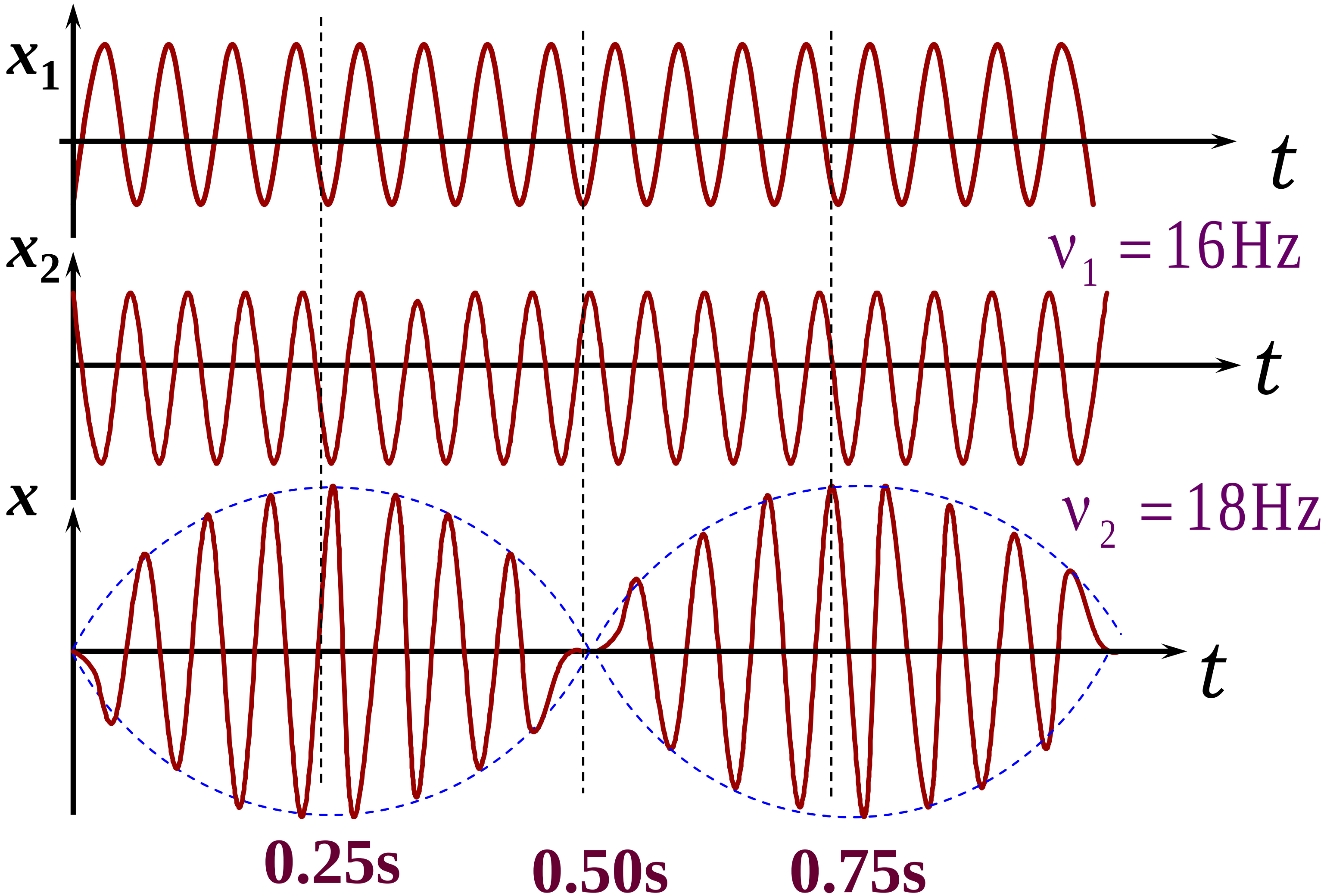
讨论 $A_1 = A_2$， $|\nu_2 - \nu_1| << \nu_1 + \nu_2$ 的情况

$$x = x_1 + x_2 = A_1 \cos 2\pi\,\nu_1 t + A_2 \cos 2\pi\,\nu_2 t$$

$$x = \underbrace{(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t)}_{\text{振幅部分}} \cos 2\pi \underbrace{\frac{\nu_2 + \nu_1}{2}}_{\text{合振动频率}} t$$

$$\boxed{\nu = \nu_2 - \nu_1}$$

拍频（振幅变化的频率）



频率较大而频率之差很小的两个同方向简谐运动的合成，其合振动的振幅时而加强时而减弱的现象叫拍.

3、垂直方向、同频率简谐振动的合成

设一个质点同时参与了两个振动方向相互垂直的同频率简谐振动，即

$$x = A_1 \cos(\omega t + \varphi_1)$$

$$y = A_2 \cos(\omega t + \varphi_2)$$

合成后质点的轨迹方程

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1 A_2}\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$$

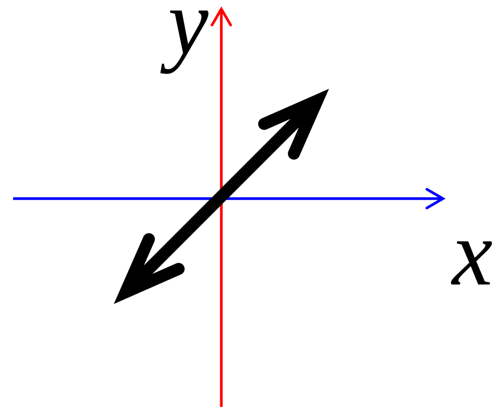
1）椭圆方程，具体形状由相位差（$\varphi_2 - \varphi_1$）决定

2）当 $A_1 = A_2$ 时，正椭圆退化为圆。

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

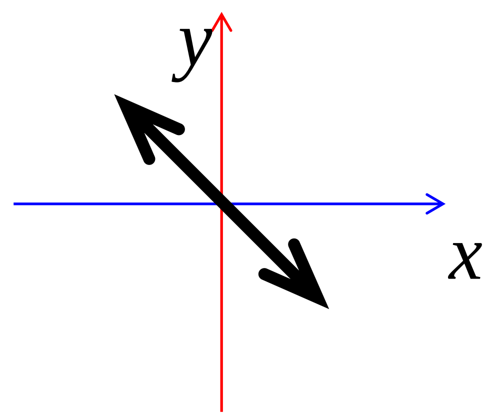
1） $\varphi_2-\varphi_1=0$

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}=0$$



$$\Rightarrow y=\frac{A_2}{A_1}x$$

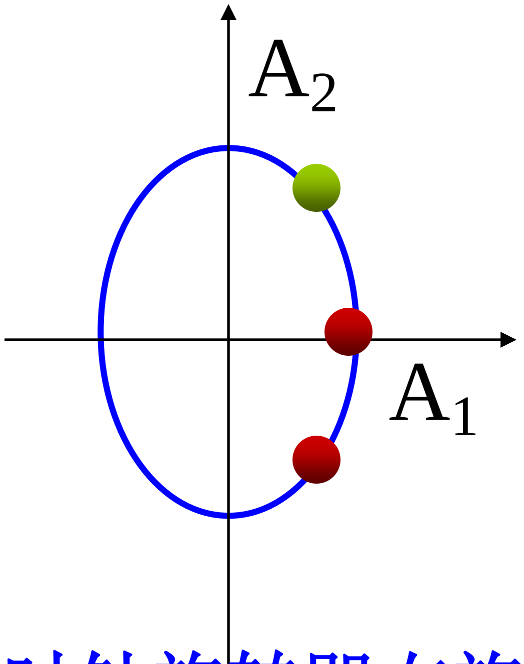
2） $\varphi_2-\varphi_1=\pi$



$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}+\frac{2xy}{A_1A_2}=0$$

$$\Rightarrow y=-\frac{A_2}{A_1}x$$

3） $\varphi_2 - \varphi_1 = \pm\frac{\pi}{2} \Rightarrow \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = \pm 1$



$a)\ \ x = A_1\cos(\omega t + \varphi)$

$$y = A_2\cos(\omega t + \varphi + \frac{\pi}{2})$$

$$\omega t + \varphi = 0 \Rightarrow x = A_1 \quad y = 0$$

$\frac{\pi}{2} > \omega t + \varphi > 0 \Rightarrow 0 < x < A_1 \quad y < 0$ 顺时针旋转即右旋

$b)\ \ x = A_1\cos(\omega t + \varphi)$

$$y = A_2\cos(\omega t + \varphi - \frac{\pi}{2})$$

$\frac{\pi}{2} > \omega t + \varphi > 0 \Rightarrow 0 < x < A_1 \quad y > 0$ 逆时针旋转即左旋

c) 其他位相差 **书p152图4-19**

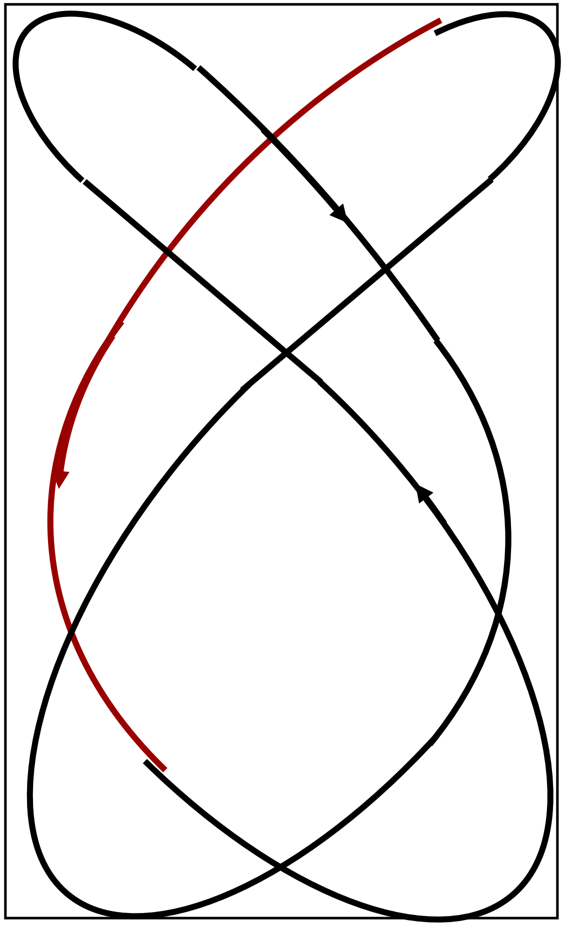
4、垂直方向、同（不同）频率简谐振动的合成

互相垂直、振动频率成整数比的谐振动的合成

合成运动的轨道是封闭曲线，运动也具有周期。这种运动轨迹的图形称为李萨如图形。

$$\alpha_1 - \alpha_2 = \frac{\pi}{8}$$

$$\frac{\nu_1}{\nu_2} = \frac{3}{2}$$



李萨如图形

用李萨如图形在无线电技术中可以测量频率