

思考练习:

1、一维弦振动方程的初值问题

$$\begin{cases} u_{tt} - a^2 u_{xx} = 0, & x \in R^1, t > 0 \\ u(x, 0) = \phi(x), u_t(x, 0) = \psi(x), & x \in R^1 \end{cases}$$

的d'Alembert 公式为

$$u(x, t) = \frac{1}{2}[\phi(x + at) + \phi(x - at)] + \frac{1}{2a} \int_{x-at}^{x+at} \psi(\xi) d\xi$$

2、已知定解问题 $\begin{cases} u_{tt} = 4u_{xx}, & x \in R, t > 0 \\ u|_{t=0} = 0, u_t|_{t=0} = x, & x \in R \end{cases}$ 利

用d'Alembert 公式可知其解 $u(x, t) = xt$

3、 $\mathcal{F}^{-1}[\hat{\phi}(\lambda) \cos a\lambda t] = \frac{1}{2}[\phi(x + at) + \phi(x - at)],$

$$\mathcal{F}^{-1}\left[\hat{\psi}(\lambda) \frac{\sin a\lambda t}{a\lambda}\right] = \frac{1}{2a} \int_{x-at}^{x+at} \psi(y) dy; \mathcal{F}^{-1}\left[\hat{\psi}(\lambda) \frac{\sin a\lambda(t-\tau)}{a\lambda}\right] = \frac{1}{2a} \int_{x-a(t-\tau)}^{x+a(t-\tau)} \psi(y) dy$$

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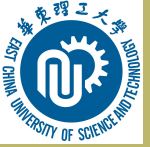
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$$3.6、(2) \begin{cases} u_{tt} - u_{xx} = t \sin x, & x \in R, t > 0 \\ u(x, 0) = 0, u_t(x, 0) = \cos x, & x \in R \end{cases}$$

解：令 $f(x, t) = t \sin x$, $\psi(x) = \cos x$, 方程和初始条件关于 x 施行Fourier变换, 记 $\hat{u}(\lambda, t) = \mathcal{F}[u]$, $\hat{\psi}(\lambda) = \mathcal{F}[\psi]$, $\hat{f}(\lambda, t) = \mathcal{F}[f]$ ■

$$\begin{cases} \frac{d^2}{dt^2} \hat{u}(\lambda, t) + (\lambda)^2 \hat{u} = \hat{f}(\lambda, t), & t > 0 \\ \hat{u}(\lambda, 0) = 0, \hat{u}_t(\lambda, 0) = \hat{\psi}(\lambda) \end{cases}$$

解初值问题

$$\hat{u}(\lambda, t) = \hat{\psi}(\lambda) \frac{\sin \lambda t}{\lambda} + \int_0^t \hat{f}(\lambda, \tau) \frac{\sin \lambda(t - \tau)}{\lambda} d\tau$$

作Fourier逆变换(利用(3.2.17)的结论)

$$\begin{aligned} u(x, t) &= \frac{1}{2} \int_{x-t}^{x+t} \cos \xi d\xi + \frac{1}{2} \int_0^t \int_{x-(t-\tau)}^{x+(t-\tau)} \tau \sin \xi d\xi d\tau \\ &= \cos x \sin t + (t - \sin t) \sin x \end{aligned}$$

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