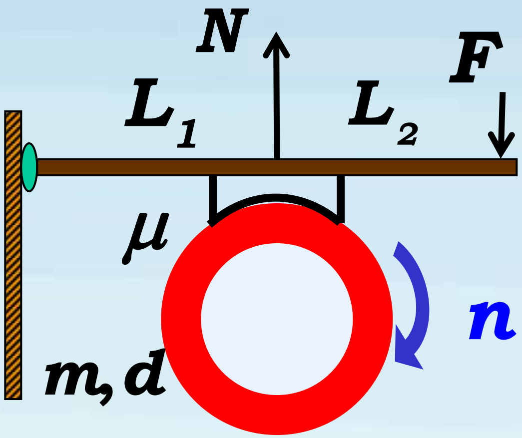
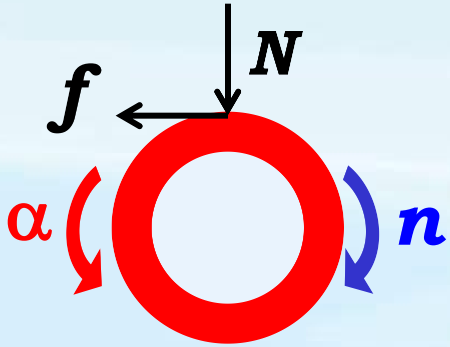
FangYi

[例题3-5]飞轮m分布于边缘，d、 n、μ已知，制动轮，t 秒停下．求(1) F？(2)转速降半转过θ？



解：(1)溯源法：$F \rightarrow N \rightarrow f \rightarrow M$

$$\begin{cases} NL_1 = F(L_1 + L_2) & (1) \\ f = \mu N & (2) \\ M = -f(d/2) & (3) \\ Mt = 0 - [m(d/2)^2][2\pi n] & (4) \end{cases}$$



$$\xrightarrow{(1)-(4)\text{解得}} F = \frac{mn\pi dL_1}{\mu t(L_1 + L_2)} \quad (5)$$

(2)由 $A = E_k - E_{k0}$

$$\Rightarrow \int_0^\theta M d\theta = \frac{1}{2}J\omega^2 - \frac{1}{2}J\omega_0^2 \Rightarrow \theta = \frac{3\pi nt}{4}$$

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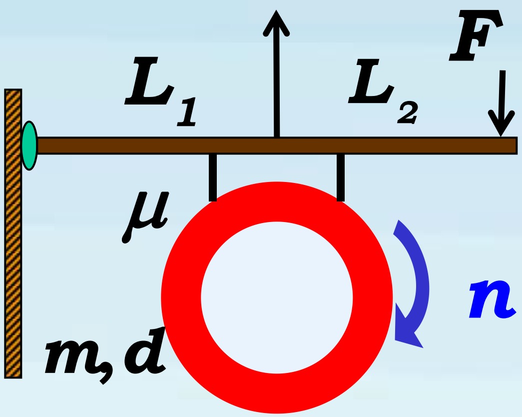
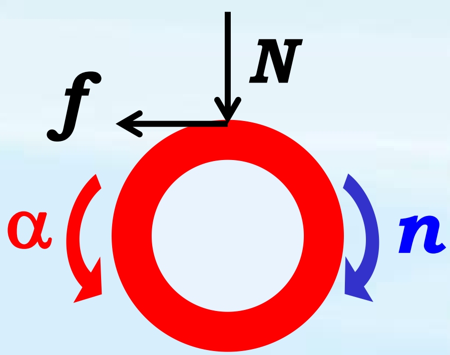
[例题3-5]飞轮**m分布于边缘，d**、 **n**、**μ**已知，

制动轮，*t* 秒停下. 求(1) ***F***？(2)转速降半转过 θ？

解：(1)溯源法：$F \rightarrow N \rightarrow f \rightarrow M \rightarrow \alpha$



$$\begin{cases} NL_1 = F(L_1 + L_2) & (1) \\ f = \mu N & (2) \\ M = -f(d/2) & (3) \\ \alpha = M/[m(d/2)^2] & (4) \\ -2\pi n = \alpha t & (5) \end{cases}$$



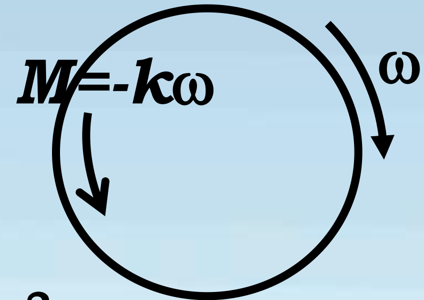
$$\xrightarrow{(1)-(5)\text{解得}} F = \frac{mn\pi dL_1}{\mu t(L_1 + L_2)} \qquad (6)$$

(1) (2) (3) (4) (5) (6)代入

(2) $(\pi n)^2 - (2\pi n)^2 = 2\alpha\theta \Rightarrow \theta = \dfrac{3\pi nt}{4}$

FangYi

[讨论6]飞轮J, ω_0,求$\omega_0 \to \omega_0/2$的θ?



解：由动能定理

$$A = E_k - E_{k0} \Rightarrow \int_0^\theta -k\omega\, d\theta = \frac{1}{2}J\left(\frac{\omega_0}{2}\right)^2 - \frac{1}{2}J\omega_0^2$$ Stop here

由转动定律 —基本方法

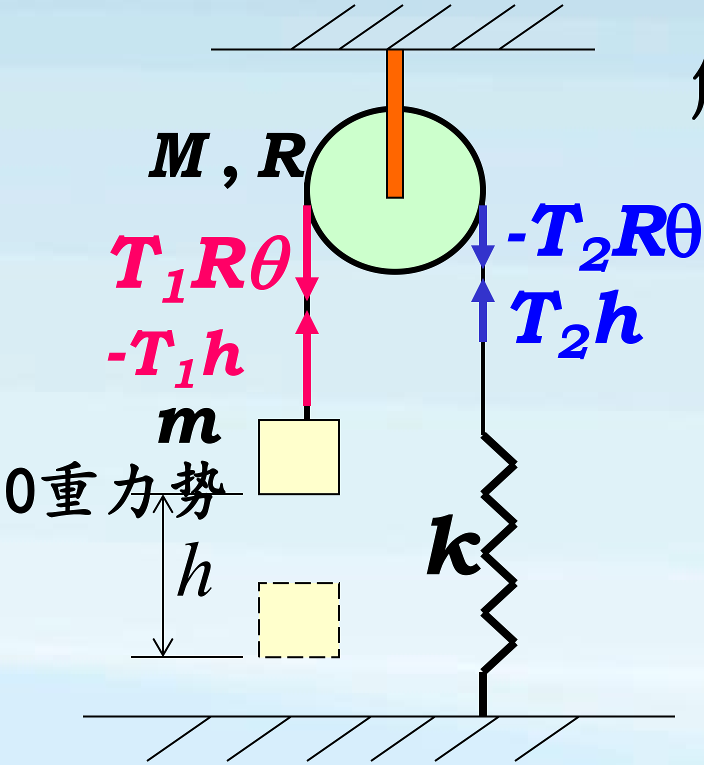
$$M = J\alpha \Rightarrow \cancel{-k\omega} = J\frac{d\omega}{\cancel{dt}}\frac{\cancel{d\theta}}{d\theta}$$

$$\Rightarrow \int_0^\theta d\theta = -\frac{J}{k}\int_{\omega_0}^{\frac{\omega_0}{2}} d\omega \Rightarrow \theta = \frac{J\omega_0}{2k}$$

与[讨论4]飞轮J, ω_0, 求使$\omega_0 \to \omega_0/2$的t? 比较

[例题3-6] 先托m 维持原长, 后由静止释放。

求:m下降h 时的v



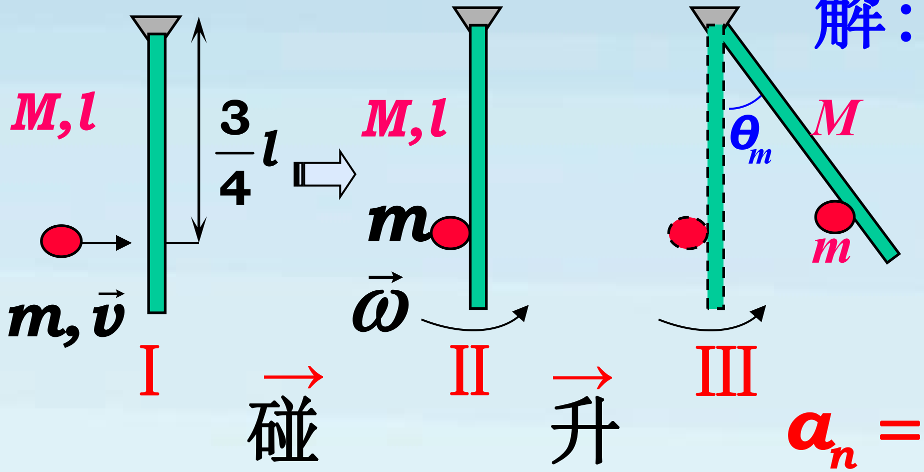
解： $M, m,$弹簧, 地球系统 $E = E_0$

$$\begin{cases} \frac{1}{2}kh^2 + (-mgh) + \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = 0 \\ v = \omega R \end{cases}$$

$$\Rightarrow v = \sqrt{\frac{2mgh - kh^2}{m + \frac{1}{2}M}}$$

[例题3-7] M、l均质杆. 橡皮泥m、v与杆完全非

求(1)碰后ω, 棒端a_n; (2)碰后杆摆θ_{max}



解: (1)棒泥碰 $\vec{L}$守恒

$$mv\frac{3}{4}l=[\frac{1}{3}Ml^2+m(\frac{3}{4}l)^2]\omega$$

$$\Rightarrow \omega=\frac{3}{4}mv/[\frac{9}{16}ml+\frac{1}{3}Ml]$$

$$a_n=\omega^2 l=\frac{9}{16}m^2v^2l/[\frac{9}{16}ml+\frac{1}{3}Ml]^2$$

讨论：棒作圆周运动的条件

$$\theta=\pi\ \rightarrow$$

$$\frac{1}{2}[\frac{1}{3}Ml^2+m(\frac{3}{4}l)^2]\omega^2$$

$$=mg\frac{3}{4}l\times 2+Mg\frac{l}{2}\times 2$$

(2)棒泥转E守恒

$$\frac{1}{2}[\frac{1}{3}Ml^2+m(\frac{3}{4}l)^2]\omega^2$$

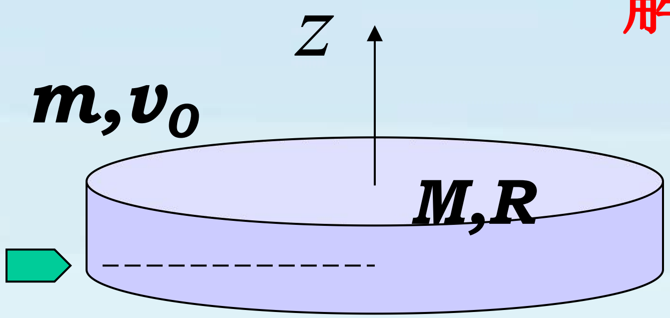
$$=mg\frac{3}{4}l(1-\cos\theta)+Mg\frac{l}{2}(1-\cos\theta)$$

$$\Rightarrow\theta_{max}=\cos^{-1}\frac{1-9m^2v^2/32}{(\frac{3}{4}m+\frac{1}{2}M)(\frac{9}{16}m+\frac{1}{3}M)gl}$$

FangYi

[讨论7]静盘M,R可绕Z转，子弹m,v_0(⊥半径)边缘射入，$\mu_{盘桌面}$，(嵌盘边，子弹重力$M_{摩擦}$不计)

求(1)碰后圆盘ω(2)停下需t(3)停下走过θ



解：(1){子弹，盘}Lz守恒

$$Rmv_0=(\frac{1}{2}MR^2+mR^2)\omega\Rightarrow\omega=\frac{mv_0}{(\frac{1}{2}M+m)R}$$

(2) 参见例题3-4

$$\left.\begin{aligned}M_f&=-\frac{2}{3}\mu MgR\\M_f t&=0-J_{总}\omega\end{aligned}\right\}\Rightarrow t=-\frac{J_{总}\omega}{M_f}=\frac{3mv_0}{2\mu Mg}$$

如果考虑子弹重力

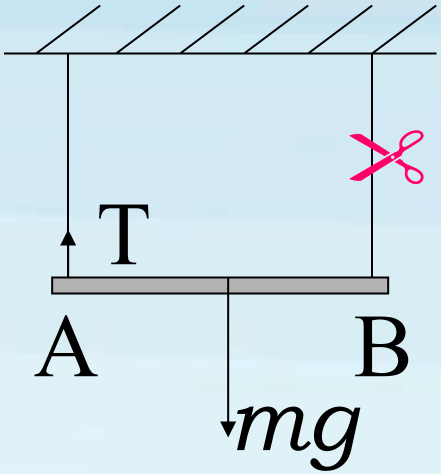
$$M_f=-\frac{2}{3}\mu MgR$$

$$-R\mu mg$$

(3) $M_f\theta=0-\frac{1}{2}J_{总}\omega^2\Rightarrow\theta=\frac{-J_{总}\omega^2}{2M_f}$

课后思考：如何用运动学公式求解

[例题3-8]均质棒m,L，两端用细绳悬挂，
求B端绳断开瞬间A端绳的拉力.



解：绳剪断瞬间，棒绕A点转动

棒（质心）平动 $$mg - T = ma_c \quad (1)$$

棒（绕A点）转动 $$mg\frac{L}{2} = (\frac{1}{3}mL^2)\alpha \quad (2)$$

棒（绕质心）转动 $$T\frac{L}{2} = (\frac{1}{12}mL^2)\alpha \quad (2)'$$

棒（绕A点）纯转动判剧 $$a_c = \alpha \cdot \frac{L}{2} \quad (3)$$

$$\xrightarrow[\text{或(1) (2)'(3)}]{\text{(1) (2) (3)}} T = \frac{1}{4}mg$$