转置

$\int \psi^* \tilde{O} \varphi \, d\vec{x} = \int O \psi^* \varphi \, d\vec{x}$

$(\psi, \tilde{O}\varphi) = (\varphi^*, O\psi^*) = (O^*\psi, \varphi)$

$\widetilde{(AB)} = \tilde{B}\tilde{A}$

$(\psi, \widetilde{AB}\varphi) = ((AB)^*\psi, \varphi) = (A^* B^*\psi, \varphi) = (B^*\psi, \tilde{A}\varphi) = (\psi, \tilde{B}\tilde{A}\varphi)$

$\Rightarrow \widetilde{AB} = \tilde{B} \cdot \tilde{A}$.

$(AB)^\dagger = B^\dagger A^\dagger$

$(\widetilde{AB})^* = (\tilde{B}\tilde{A})^* = \tilde{B}^* \tilde{A}^* = B^\dagger A^\dagger$

A,B是线性算符，A·B也是线性算符。

A,B是厄密的，$A^\dagger = A$，$B^\dagger = B$.

$(A+B)^\dagger = A+B$. 也是厄密的

$(AB)^\dagger = B^\dagger A^\dagger = BA$.

当$[A,B]=0$，AB是厄密的

对应原理

$\vec{p}$ 在坐标表象下的表示 $\Rightarrow \hat{p}, -i\hbar\nabla$

其他力学量可用 $\vec{x}$与$\vec{p}$表示. $F(\vec{x},\vec{p}) \Rightarrow \hat{F}(\hat{\vec{x}},\hat{\vec{p}})$

$\hat{\vec{L}} = \vec{x} \times \hat{\vec{p}}$ $\qquad L_x = -i\hbar\, y\frac{d}{dz}$.

$\hat{H} = \frac{\hat{p}^2}{2m} + V(\vec{x})$

证明. $e^{a\frac{d}{dx}} f(x) = f(x+a)$ 平移算符.

$e^{au} = \sum \frac{a^n}{n!} u^n$

$e^{a\frac{d}{dx}} = \sum \frac{a^n}{n!} \frac{d^n}{dx^n}$

$e^{a\frac{d}{dx}} \cdot f(x) = \sum \frac{a^n}{n!} (f(x))^{(n)} = \sum \frac{[f(x)]^{(n)}}{n!} a^n = f(x+a)$

实际上，为了让求和有牢固收敛，a必须是小量.

$\left((AB)^* - A^* B^*\right)\psi$

$$\left[x\left(-i\hbar\frac{\partial}{\partial x}\right)\right]^{*} = x\, i\hbar\frac{\partial}{\partial x}$$

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = H(x)\,\psi(x,t)$$

$$\psi(x,t) = e^{-iHt/\hbar}\,\psi(x,0)$$

$$= e^{-iHt/\hbar}\sum c_n\varphi_n = \sum c_n e^{-iHt/\hbar}\varphi_n = \sum c_n e^{-iE_nt/\hbar}\varphi_n$$

$$e^{-iHt/\hbar}\varphi_n = \sum\frac{1}{m!}\left(-\frac{it}{\hbar}\right)^m H^m\varphi_n = \sum\frac{1}{m!}\left(-\frac{it}{\hbar}\right)^m E^m\varphi_n = \sum e^{-iE_nt/\hbar}\varphi_n.$$

eg. $\begin{cases}\psi \text{ 是常数}, & -a<x<a \\ 0, & \text{其他}\end{cases}$，用系统本征态表示以后时刻波函数。

$$\psi(x,0) = \sum c_n\varphi_n \qquad c_n = (\varphi_n, \psi(x,0)) = \frac{1}{\sqrt{2a}}\int_{-a}^{a}\varphi_n^{*}(x)\,dx$$

$$\psi(x,t) = \sum_n c_n e^{-iE_nt/\hbar}\varphi_n$$

eg. $\frac{1}{2}[\nabla^2, \vec{r}] = \nabla$

$$\left(\nabla^2\vec{r} - \vec{r}\nabla^2\right)\psi(\vec{r})$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(x\vec{i}+y\vec{j}+z\vec{k})\psi - (x\vec{i}+y\vec{j}+z\vec{k})\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi.$$

$$\frac{\partial^2}{\partial x^2}(x\psi) - x\left(\frac{\partial^2}{\partial x^2}\psi\right)$$
$$= \frac{\partial}{\partial x}\left(\psi + x\frac{\partial\psi}{\partial x}\right)$$
$$= \frac{\partial}{\partial x}\psi + x\frac{\partial^2\psi}{\partial x^2} + \frac{\partial\psi}{\partial x} = 2\frac{\partial\psi}{\partial x}.$$

$$[\nabla^2, \vec{r}] = [\nabla^2, x]\vec{i} + [\nabla^2, y]\vec{j} + [\nabla^2, z]\vec{k}$$
$$= \left[\frac{\partial^2}{\partial x^2}, x\right]\vec{i} + \cdots = \left(\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}, x\right] + \left[\frac{\partial}{\partial x}, x\right]\frac{\partial}{\partial x}\right)\vec{i} + \cdots$$
$$= 2\frac{\partial}{\partial x}\vec{i} + \cdots = 2\nabla.$$