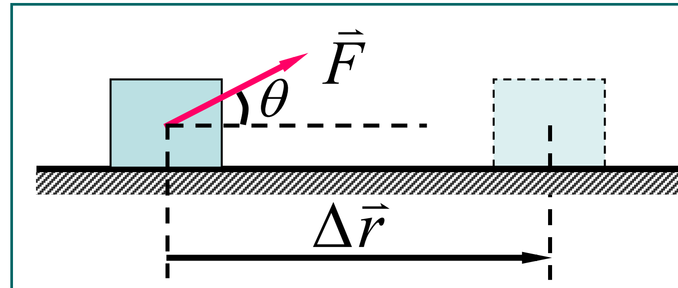
第二章　守恒定律

§2.1能量守恒

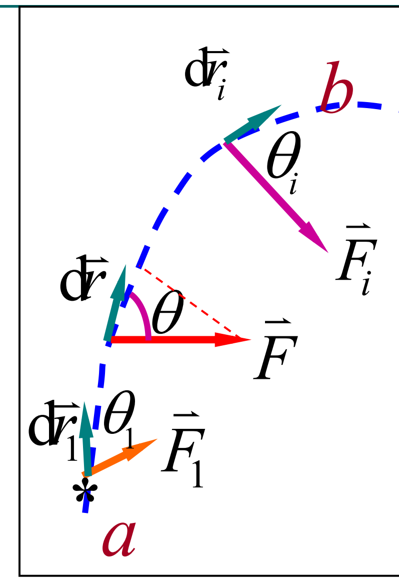
一、功

1. 恒力作用下的功



$$A = F\cos\theta \cdot \left|\Delta\vec{r}\right| = \vec{F}\cdot\Delta\vec{r}$$

2. 变力的功

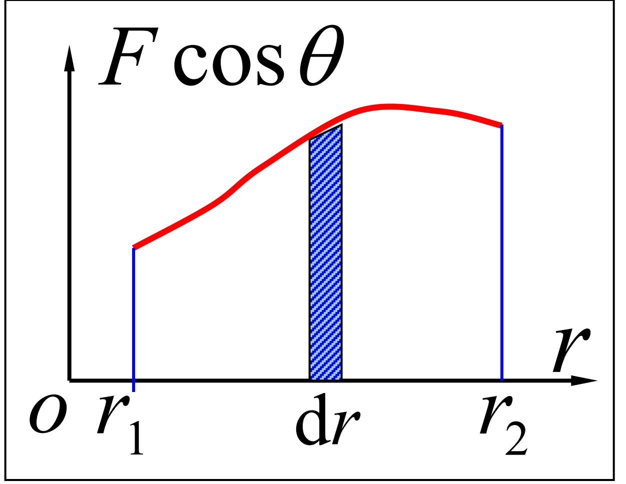


$$\boxed{A = \int_a^b \vec{F}\cdot\mathrm{d}\vec{r} = \int_a^b F\cos\theta\,\mathrm{d}r}$$

讨论 (1) 功的正、负

$$\begin{cases} 0^\circ < \theta < 90^\circ, \quad A > 0 \\ 90^\circ < \theta < 180^\circ, \quad A < 0 \\ \theta = 90^\circ \quad \vec{F}\perp\mathrm{d}\vec{r} \quad A = 0 \end{cases}$$

(2) 作功的图示 $A=\int_{r_1}^{r_2} F\cos\theta\,\mathrm{d}r$



(3) 功是一个过程量，与路径有关.

(4) 合力的功，等于各分力的功的代数和.

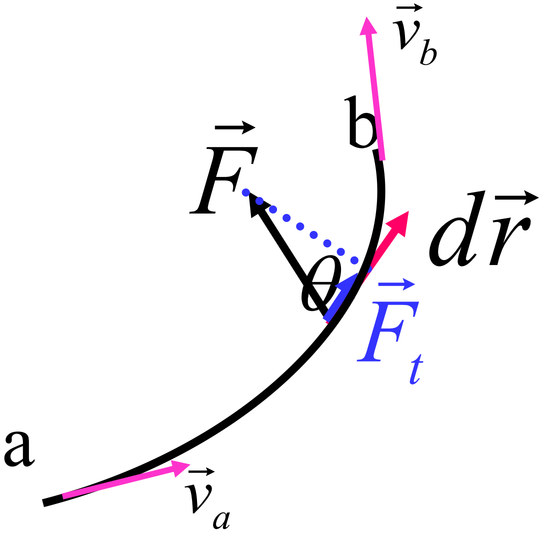
$$\vec{F}=F_x\vec{i}+F_y\vec{j}+F_z\vec{k} \qquad \mathrm{d}\vec{r}=\mathrm{d}x\vec{i}+\mathrm{d}y\vec{j}+\mathrm{d}z\vec{k}$$

$$A=\int_a^b \vec{F}\cdot\mathrm{d}\vec{r}=\int_a^b (F_x\mathrm{d}x+F_y\mathrm{d}y+F_z\mathrm{d}z)$$

$$A_x=\int_{x_a}^{x_b}F_x\mathrm{d}x \qquad A_y=\int_{y_a}^{y_b}F_y\mathrm{d}y \qquad A_z=\int_{z_a}^{z_b}F_z\mathrm{d}z$$

$$\boxed{A=A_x+A_y+A_z}$$

3. 动能定理



$$dA = \vec{F} \cdot d\vec{r} = F\cos\theta dr = F_t dr$$

$$= ma_t dr = m\frac{dv}{dt}dr = mvdv$$

$$\int_0^A dA = \int_{v_a}^{v_b} mvdv \Rightarrow A = \tfrac{1}{2}mv_b^2 - \tfrac{1}{2}mv_a^2 = \vec{E}_{Kb} - \vec{E}_{Ka}$$

$E_k = \frac{1}{2}mv^2$ 　　E_k是状态量,称为质点的平动动能。

合力对物体所做的功等于物体动能的增量

[例1]质点m=0.5Kg,运动方程x=5t,y=0.5t² (SI)，求从t=2s到t=4s这段时间内外力所作的功.

解法1: 用功的定义式

$$A = \int \vec{F} \cdot d\vec{r}$$

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \vec{j}$$

$$\vec{r} = 5t\vec{i} + 0.5t^2\vec{j}$$

$$\Downarrow$$

$$d\vec{r} = 5dt\vec{i} + tdt\vec{j}$$

$$\Rightarrow A = \int_2^4 0.5tdt$$

$$= 0.25t^2\Big|_2^4 = 3J$$

解法2: 用动能定理

$$A = \Delta E_k = \frac{1}{2}m(v_4^2 - v_2^2)$$

$$= \frac{1}{2} \times 0.5 \times (41 - 29)$$

$$= 3J$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = \frac{dx}{dt} = 5$$

$$v_y = \frac{dy}{dt} = t$$

$$v_2 = \sqrt{29}$$

$$v_4 = \sqrt{41}$$

二、势能

1. 重力(gravitation)作功

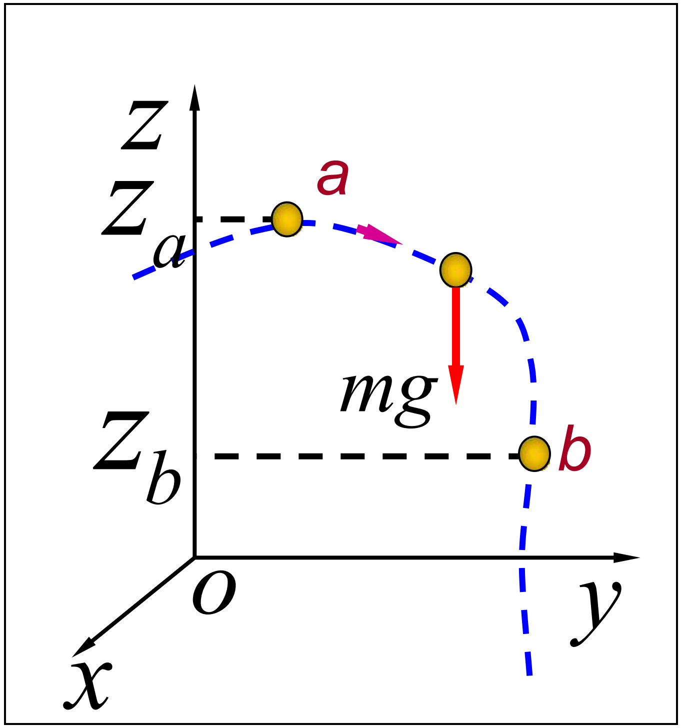
$$\boxed{\vec{G} = -mg\vec{k}}$$

$$\mathrm{d}\vec{r} = \mathrm{d}x\vec{i} + \mathrm{d}y\vec{j} + \mathrm{d}z\vec{k}$$

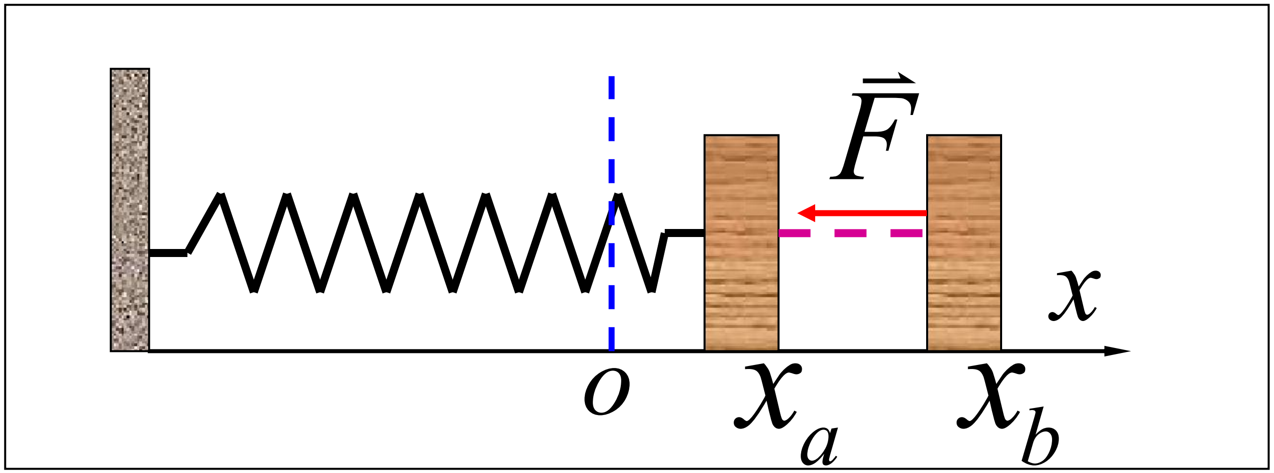
$$A = \int_a^b \vec{G}\cdot \mathrm{d}\vec{r} = \int_{z_a}^{z_b} -mg\mathrm{d}z$$

$$= -(mgz_b - mgz_a)$$

$$A = \oint -mg\mathrm{d}z = 0$$



2. 弹性力(elastic force)作功



$$\vec{F} = -kx\,\vec{i}$$

$$A = \int_{x_a}^{x_b} \vec{F} \cdot \mathrm{d}\vec{x} = \int_{x_a}^{x_b} -kx\mathrm{d}x$$

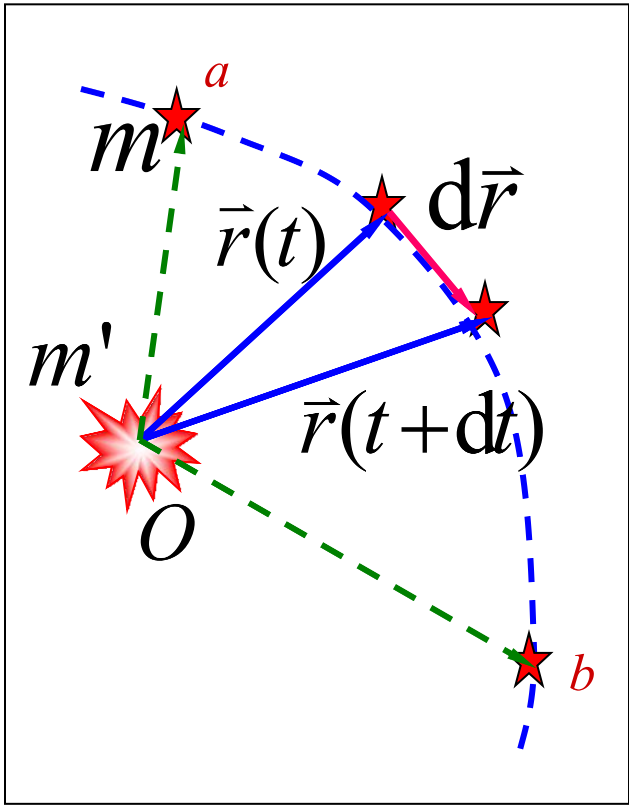
$$A = -\left(\frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2\right) \qquad A = \oint -kx\mathrm{d}x = 0$$

3. 万有引力(universal gravitation)作功

以m'为参考系，m 的位置矢量为 $\vec{r}$

m' 对 m 的万有引力为

$$\boxed{\vec{F} = -G\frac{m'm}{r^2}\vec{r}_0}$$



m 由a 点移动到 b 点时 $\vec{F}$ 作功为

$$A = \int \vec{F}\cdot\mathrm{d}\vec{r} = \int_{r_a}^{r_b} -G\frac{m'm}{r^2}\mathrm{d}r$$

$$= -\left[(-G\frac{m'm}{r_b}) - (-G\frac{m'm}{r_a})\right]$$

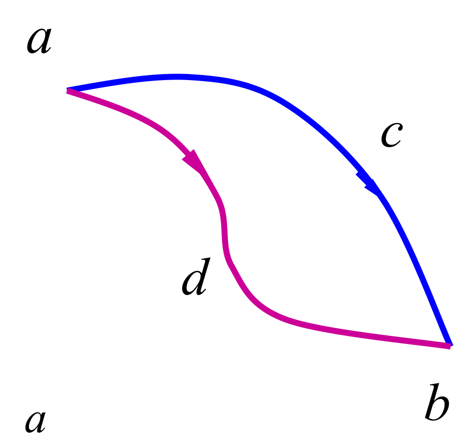
4. 保守力和非保守力(conservative force and non-conservative force)

保守力:力所作的功与路径无关，仅决定于相互作用质点的始末相对位置 .

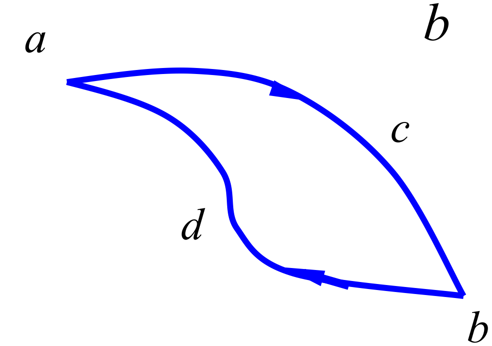
重力功 $$A = -(mgz_b - mgz_a)$$

弹力功 $$A = -(\frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2)$$

引力功 $$A = -\left[(-G\frac{m'm}{r_b}) - (-G\frac{m'm}{r_a})\right]$$



$$\int_{acb} \vec{F}\cdot\mathrm{d}\vec{r} = \int_{adb} \vec{F}\cdot\mathrm{d}\vec{r}$$



$$\oint_l \vec{F}\cdot\mathrm{d}\vec{r} = 0$$

非保守力:力所作的功与路径有关 .（例如摩擦力）

5. 势能(potential energy)

保守力作功的特点： $A=\int_{\vec{r}_a}^{\vec{r}_b}\vec{F}\cdot\mathrm{d}\vec{r}=G(\vec{r}_b)-G(\vec{r}_a)$

如果能找到$G(\vec{r})$，则$G(\vec{r})$ 不是唯一的。

定义势能 $E_P=-G(\vec{r})$

$$A=G(\vec{r}_b)-G(\vec{r}_a)$$

$$=-\left[E_P(\vec{r}_b)-E_P(\vec{r}_a)\right]=-\Delta E_P$$

◆ 势能：与物体间相互作用及相对位置有关的能量．

重力功

$$A=-(mgz_b-mgz_a)$$

弹力功

$$A=-(\frac{1}{2}kx_b^2-\frac{1}{2}kx_a^2)$$

引力功

$$A=-\left[(-G\frac{m'm}{r_b})-(-G\frac{m'm}{r_a})\right]$$

重力势能

$$E_{\mathrm{p}}=mgz$$

弹性势能

$$E_{\mathrm{p}}=\frac{1}{2}kx^2$$

引力势能

$$E_{\mathrm{p}}=-G\frac{m'm}{r}$$

◆ 保守力的功：

$$\boxed{A=-(E_{pb}-E_{\mathrm{pa}})=-\Delta E_{\mathrm{P}}}$$

讨论

- 势能是状态函数 $E_p = E_p(x, y, z)$

- 势能具有相对性，势能大小与势能零点的选取有关．

- 势能是属于系统的．

- 势能计算 $\boxed{A = -(E_{pb} - E_{pa}) = -\Delta E_p}$

令 $E_{pa} = 0$

$$\boxed{E_p(x, y, z) = \int_{(x,y,z)}^{E_{pa}=0} \vec{F} \cdot \mathrm{d}\vec{r}}$$

三、机械能守恒定律

1. 质点系的动能定理

考虑n个质点组成的质点系（系统）

对第i个质点 $A_i = \frac{1}{2}m_i\upsilon_i^2 - \frac{1}{2}m_i\upsilon_{i0}^2$

n 个质点 $\sum_{i=1}^{n} A_i = \sum_{i=1}^{n}\frac{1}{2}m_i\upsilon_i^2 - \sum_{i=1}^{n}\frac{1}{2}m_i\upsilon_{i0}^2 = E_k - E_{k0}$

$$A_{外} + A_{内} = E_k - E_{k0}$$

2. 系统的功能原理

$$A_{保内} + A_{非保内}$$

$$A_{保内} = -(E_p - E_{p0})$$

$$A_{外} + A_{非保内} = (E_k - E_{k0}) + (E_p - E_{p0}) = (E_k + E_p) - (E_{k0} + E_{p0})$$

$$A_{外} + A_{非保内} = E - E_0$$