A Recent Development of Particle Filter

Inquiry towards a Continuous Time Limit and Scalability

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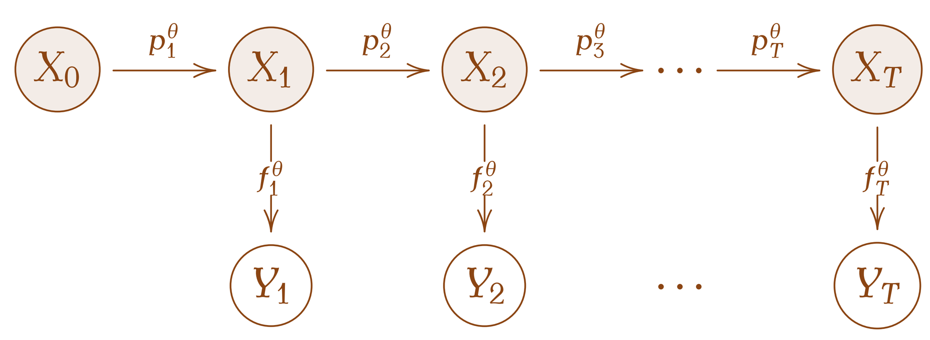
What Is a Particle Filter?

Particle Filter = Sequential Monte Carlo (SMC)

- an simulation-based algorithm which performs **filtering** even in **non-Gaussian** and **non-linear** state space models
  → **overcoming the weeknesses** of then-standard Kalman-based filtering methods (e.g. EKF).
- a filtering distribution is approximated by **a cloud of weighted samples**, hence giving rise to the term 'particle filter'.
- The samples are propagated to approximate the next distribution
  → leading to efficient sequential estimation in **dynamic settings**

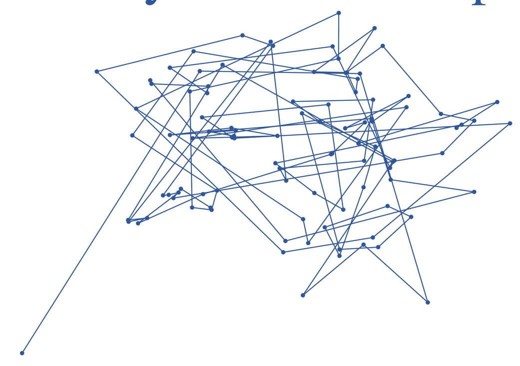


MCMC vs. SMC

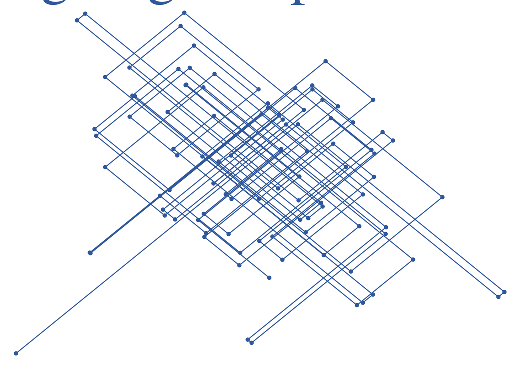
MCMC has evolved into PDMP. How about SMC?

- PDMPs (Piecewise Deterministic Markov Processes) have shown great potential for developing scalable sampling methods, notably in creating **continuous-time versions of MCMCs**.
- In 2012, a PDMP was **identified through the continuous limit of the MCMC**, Metropolis-Hastings algorithm.
- Empirical evidence suggests that **continuous-time MCMCs are more efficient than their discrete-time counterparts**.

Bouncy Particle Sampler



Zig-Zag Sampler

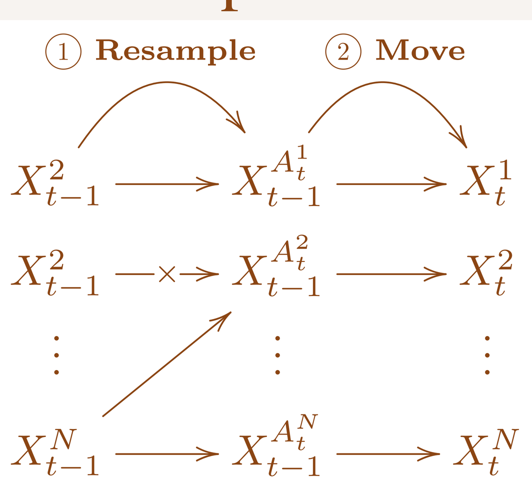


Inquiry for Continuous-time SMC

MCMC has now taken a step ahead; it is time for SMC to explore its continuous-time limit!

A Generic Particle Filter: An Algorithmic Description

Procedure of a generic step of a Particle Filter at time t



① Resampling Step

Particles with high weights are **duplicated**, and those with the lowest weights are **discarded**.

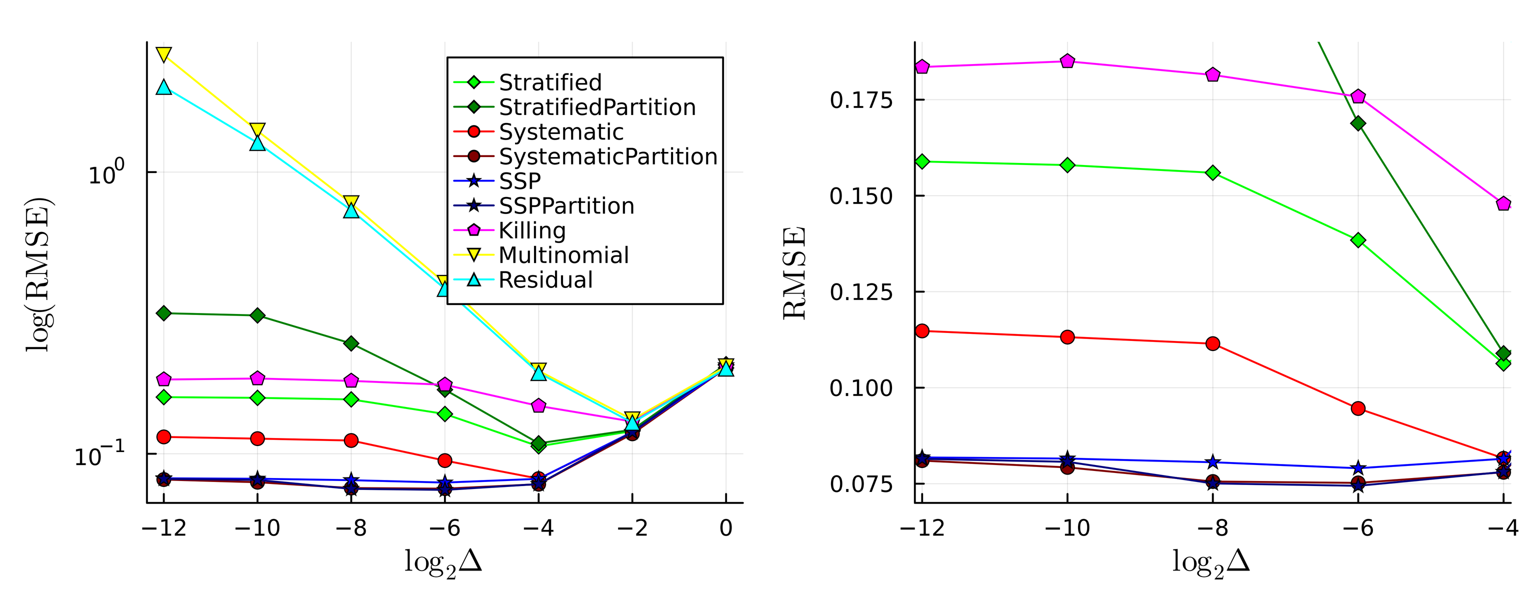
② Movement Step

Subsequently, a **MCMC move** is executed from the resampled particles.

A Necessary Condition: Resampling Stability

- In order to have a time-step $\Delta \to 0$ limit, resampling events must occur with (at most linearly) **decreasing frequency**.
- Only the most **efficient resampling schemes** satisfy this property.

Root mean squared errors of marginal likelihood estimates [Chopin et al., 2022]



The Continuous-time Limit Process

The continuous-time limit process, if it exists, is characterized by a **Feller-Dynkin process**, whose infinitesimal generator is given by:

$$
\begin{aligned}
\mathcal{L}f(x) := &\sum_{n=1}^N \sum_{i=1}^d b_i(x^n)\frac{\partial f}{\partial x_i^n}(x) \\
&+ \sum_{n=1}^N \frac{1}{2}\sum_{i,j=1}^d (\sigma\sigma^\top)_{ij}(x^n)\frac{\partial^2 f}{\partial x_i^n \partial x_j^n}(x) \\
&+ \sum_{a\neq 1:N} \bar{\iota}(V(x),a)\left(f(x^{a(1:N)}) - f(x^{1:N})\right)
\end{aligned}
$$

when the latent process (X_t) is an **Itô process** given by the generator:

$$
Lf(x) = \sum_{i=1}^d b_i(x)\frac{\partial f}{\partial x_i}(x) + \frac{1}{2}\sum_{i,j=1}^d (\sigma\sigma^\top)_{ij}(x)\frac{\partial^2 f}{\partial x_i \partial x_j}(x)
$$

Conclusions

SMC with efficient resampling schemes possess a continuous-time limit $\Delta \to 0$, which turns out to be a Feller-Dynkin process, a diffusion process with jumps, when (X_t) is a diffusion.

Forthcoming Research

- What are the **properties of this limit jump process**, and how do they change with modifications to the underlying latent process?
- How does the **timing of resampling** affect overall efficiency? Can insights be gained from the perspective of continuous-time limits?
- Does the continuous-time limit process improve SMC efficiency when used for **particle propagation**?

References

[Chopin et al., 2022] Chopin, N., Singh, S. S., Soto, T., and Vihola, M. (2022). On resampling schemes for particle filter with weakly informative observations. *The Annals of Statistics*, 50(6):3197–3222.

Acknowledgements

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