Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
B		H421

An offensive and defensive game model based on linear programming

Summary

This question is based on the offense and defense of the USV USV, and discusses the offense and defense of the blue (offensive) and red (defender) sides in a two-dimensional situation. Based on the game theory, this paper establishes a dynamic linear programming model to analyze and simulate the offensive and defensive process of interception and penetration.

For the first question, we first propose a model of the red square movement strategy, and transform the problem into a dynamic linear programming model with movable boundary regions. In this model, we discussed the game between the movement of the point P controlled by the red side on the plane and the movement of the constraint condition controlled by the blue side, and finally the set of points in the rectangular area that satisfy the blue side must be able to penetrate.

For the second question, in the dynamic linear programming model based on the red square movement strategy, we discussed the dynamic change process of the red square USV cluster and the blue square USV boundary conditions on the x1-x3 plane based on the same principle, and found the red square optimal The placement position of, the upper and lower limits of the channel bandwidth M are calculated. Then, the shortest time penetration strategy of the blue USV and the interception strategy of the red USV cluster are given.

For the third question, similar to question two, in the dynamic linear programming model, we discussed the dynamic change process of the red USV cluster based on the same principle, and obtained the release time and position of the red USV cluster and the channel bandwidth M The upper limit.

Finally, the red and blue penetration and interception strategies proposed in this paper are simulated. Through simulation results, it is found that the penetration and interception strategies have certain practical value.

Keywords:zero-sum game, linear programming, penetration and interception, two-dimensional

Contents

1 Introduction

1.1 Problem Background

The Red and Blue USVs clusters conduct a confrontation of break and interception. The Blue team has only one USV, but has an advantage in maneuvering speed; So the Red side considers making up for the disadvantage through the cooperation of multi-USVs.

The Blue USV can only move inside the rectangular area $ABCD$, and needs to cross the boundary CD. While the Red USVs need to intercept the Blue USV.

1.2 Restatement of the Problem

Three major problems are discussed in this paper, which are:

- **Question 1**

  The initial position of the red USV and the initial formation are determined; the initial position of the blue USV can be any point in the $ABCD$ area. Find out all the points in the $ABCD$ area that enable the blue USV to successfully break through.

- **Question 2**

  The initial position of the red USV is at any point on the boundary CD, and the formation is a circle with a radius of 50m; the initial position of the blue USV is at the midpoint of AB, and the lower limit M_{min} and the upper limit M_{max} of the channel bandwidth M need to be found. When the actual $M > M_{min}$, the blue USV can break through the red USV interception and give the shortest time penetration strategy; when the actual $M < M_{max}$, the red USV can intercept the blue USV and give the interception strategy.

- **Question 3**

  The red side USV vehicle can release 10 USVs in two times. The number of releases each time is not less than 3, and the initial center position of the released USV is $200m$ away from the vehicle; the initial position of the USV released by the first wave of red is at any point on the boundary CD, and the USV released by the second wave of red The initial position and time can be determined as needed to obtain the best interception effect, and only need to meet the constraints of the aircraft carrier and the first wave of USV clusters. Determine whether the upper limit M_{max} of the channel bandwidth M exists. When $M < M_{max}$, the red USV can always intercept the blue USV and give an interception strategy.

2 Problem Analysis

2.1 Introduction to Game Theory and Interception

Game theory, also known as game theory, is a theoretical method for studying problems of confrontation, conflict, competition and cooperation. It is not only a new subject of modern mathematics, but also an important theory within the scope of operations research. It is widely used in Economic, military and other fields. In game theory, the participants in the game process have their own different goals and interests. In order to enable each participant to achieve the expected benefits, all participants must choose the strategy or plan that maximizes their own benefits under the premise of considering the strategy of the other party, forming a dynamic game process.

According to whether the choice of strategy is related to time, it can be divided into static game and dynamic game; according to the information mastery in the game process, it is divided into complete information game and incomplete information game; according to the sequence of actions of both parties, it is divided into sequential action And act at the same time. Zero-sum game, also known as zero-sum game, is opposite to non-zero-sum game. It is a concept of game theory and belongs to non-cooperative game. It refers to the parties participating in the game. Under strict competition, the gains of one party inevitably mean the losses of the other party. The sum of the gains and losses of all parties in the game is always "zero", and there is no possibility of cooperation between the two parties.

In the issue of interception and penetration, the threat of penetration against offensive targets is often used in reality. Multi-layer defense interception is currently a more effective solution. In order to ensure the safety of the protected targets, a multi-objective nonlinear programming mathematical model is established for the interception method of different types of offensive targets, and further solved by the theory of operations research.

This paper proposes a linear programming model to simulate the offensive and defensive process of interception and penetration, and uses this model to find the blue's optimal penetration strategy and the red's optimal interception strategy, and the guarantee The maximum value of the communication bandwidth under the premise of penetration or interception.

2.2 Problem classification and discussion

This question is based on the offense and defense of the USV USV, and discusses the offense and defense of the blue (offensive) and red (defender) sides in a two-dimensional situation. Based on the game theory, this paper establishes a dynamic linear programming model to analyze and simulate the offensive and defensive process of interception and penetration.

1. For question one, we first propose a model of the red square movement strategy, and transform the problem into a dynamic linear programming model with movable boundary regions. In this model, we discussed the game between the movement of the point P controlled by the red side on the plane and the movement of the constraint condition controlled by the blue side, and finally the set of points in the rectangular area that satisfy the blue side must be able to penetrate.

2. For question two, in the dynamic linear programming model based on the red square movement strategy, we discussed the dynamic change process of the red square USV cluster and the blue square USV boundary conditions on the x1-x3 plane based on the same principle, and found the red square optimal The placement position of, the upper and lower limits of the channel bandwidth M are calculated. Then, the shortest time penetration strategy of the blue USV and the interception strategy of the red USV cluster are given.

3. For question three, similar to question two, in the dynamic linear programming model, we discussed the dynamic change process of the red USV cluster based on the same

principle, and obtained the release time and position of the red USV cluster and the channel bandwidth M The upper limit.

Finally, the red and blue penetration and interception strategies proposed in this paper are simulated. Through simulation results, it is found that the penetration and interception strategies have certain practical value.

3 Assumptions and Notations

3.1 Assumptions

1. Ignore the time and space impact of the blue and red USV turns.

2. The single USV formation of the red side is circular, and the centroid of the USV cluster

3. Since the blue team's roundabout tactics are not considered, the blue team's time is very short, and this time is not enough to make the carrier and the red USV almost separate beyond the limited distance, so the restriction on the red USV by the position of the carrier is not considered . At the same time, due to the large distance between the blue side and the red side, this period of time is sufficient for the red side USV carrier to avoid the blue side USV, so the situation where the carrier and the blue side USV are close is not considered.

4. The form of the blue team's breakthrough strategy is fixed, and both sides agree that the blue team will adopt the penetration strategy discussed in the third.

5. . The red party adopts the strategy of moving side by side to intercept.

6. The blue party and the red party are smart enough to implement the best plan as long as there is an optimal plan under the constraints of the above assumptions.

7. Ignore the information propagation delay of the detection equipment and data link of both parties.

3.2 Notations

The primary notations used in this paper are listed in Table 1.

4 The Models

4.1 The USV of the red side cluster centroid model

Simplify the USV cluster of the red side into a centroid model, and use a weighting method to calculate its weighted average effective defense area.

Establish the following rectangular coordinate system, and set the coordinates of the quintile points as $(R_1\cos(\frac{2n\pi}{5}), R_1\sin(\frac{2n\pi}{5}))$, Among them, $n=0,1,2,3,4$, so the equation of

Table 1: Notations

Symbol	Definition
R_1	USV cluster radius
R_2	USV defense radius
Q	The breakthrough point of the competition between red and blue USV
P	Point corresponding to red square state in P x1-x3 coordinate system
d	horizontal distance between blue USV and red USV
d	at a certain moment Maximum turning distance of y-blue deception
y	Maximum turning distance of y-blue deception
dis	Effective average interception radius of dis red USV cluster
$x_i, i = 1,2,3,4,5$	the distance between red USV group and boundary / other USVs
F	The objective function of the game between red and blue
(x_0, y_0)	blue initial position
t_{min}	the shortest time for red and blue to reach the optimal position

five circles is $(x-R_1\cos(\frac{2n\pi}{5}))^2+(y-R_1\ sin(\frac{2n\pi}{5}))^2=R_2^2$, that is, the parametric equation is

$$\begin{cases} x = R_1\cos(\frac{2n\pi}{5}) + R_2\cos\theta \\ y = R_1\sin(\frac{2n\pi}{5}) + R_2\sin\theta \end{cases} \tag{1}$$

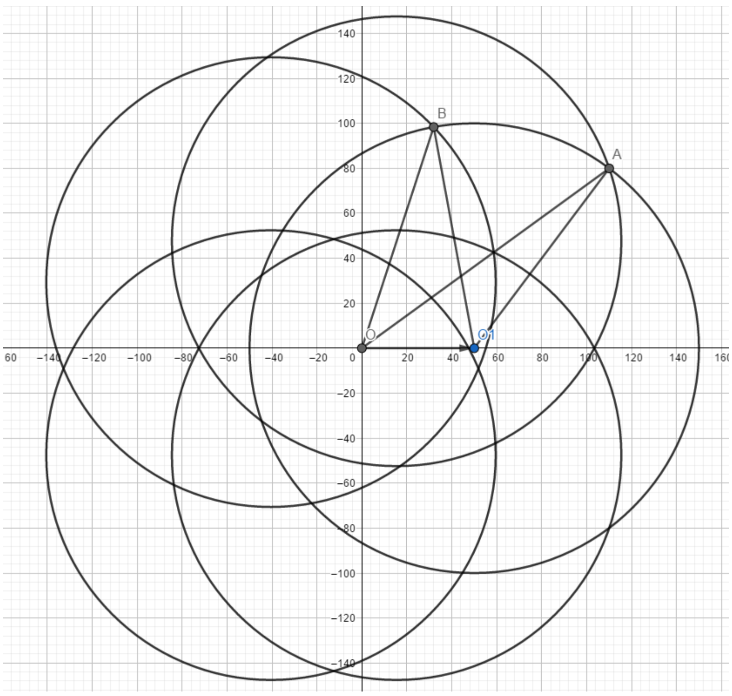

Figure 1: Red side's USV cluster diagram

Due to symmetry, only the intersection of the three circles $n = 0,1,2$ in the first quadrant is considered, which is located on the circle $(x-R_1)^2+y^2=R_2$. Let the intersection point correspond to the coordinates respectively As θ_1, θ_2. Take the arc derivative, get, and the average distance on the arc is

$$\begin{aligned} dis &= \frac{\int_{\overset{\frown}{AB}} \sqrt{x^2+y^2}\mathrm{d}s}{\int_{\overset{\frown}{AB}} \mathrm{d}s} \\ &= \frac{\int_{\theta_1}^{\theta_2} \sqrt{R_1^2+R_2^2+2R_1R_2\cos\theta}R_2\mathrm{d}\theta}{\int_{\theta_1}^{\theta_2} R_2\mathrm{d}\theta} \\ &= \frac{\int_{\theta_1}^{\theta_2} \sqrt{R_1^2+R_2^2+2R_1R_2\cos\theta}R_2\mathrm{d}\theta}{R_2(\theta_2-\theta_1)} \end{aligned} \tag{2}$$

In this question, in a USV cluster on the red side, each drone is located on a circle with a radius of $50m$, that is, $R_1 = 50$. It also requires that at least two drones are less than $100m$ away from the blue side. Therefore, the effective defense area of each USV is $100m$, so $R_2 = 100$, and the weighted defense area of the particle model is obtained as

$$dis = \frac{\int_{\theta_1}^{\theta_2} \sqrt{R_1^2+R_2^2+2R_1R_2\cos\theta}R_2\mathrm{d}\theta}{R_1(\theta_2-\theta_1)} = 128.2708m \tag{3}$$

Among them, the coordinates of the boundary of the USV defense are $(110.0544, 79.9592)$, $(31.9588, 98.3591)$, and the corresponding parameter equation coordinates are 0.9266, 1.3894

4.2 Determination of the objective function

Below we discuss how the red and blue sides play a decision game at any time. The distances between the two USV clusters on the red side to their nearest borders are x1 and x3 respectively. The vertical distance between the two clusters is x2. The specific penetration method of the blue USV is as follows.

As shown in the 2, when the blue side USV is on BC, its speed v1 is horizontal to the right, and the horizontal position of the red side fleet is d.

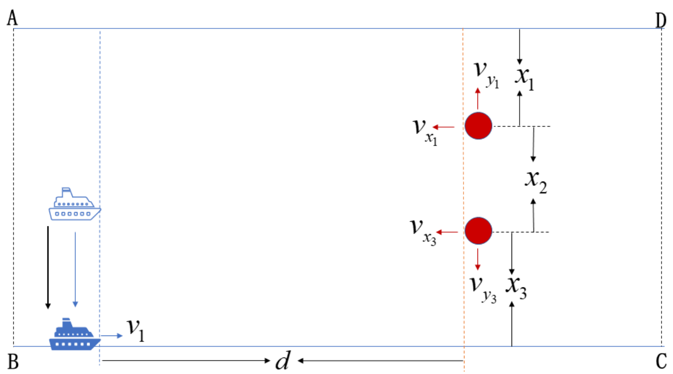

Figure 2: The first stage of the blue side movement

As shown in the 3 The second stage of the blue side movement , it is assumed that the blue side USV sailing direction changes to vertical upward sailing at a certain moment, and it is expected to turn to the y position and turn horizontally to the right sailing.

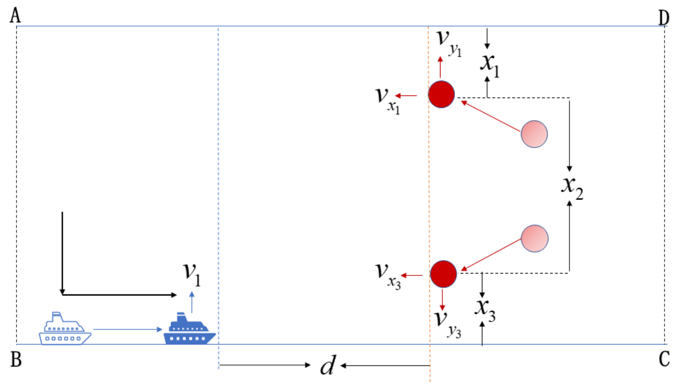

Figure 3: The second stage of the blue side movement

As shown in the 4, the blue USV sails upward to turn at y, and then always sails horizontally to the right. The Red USV continued to sail according to its judgment. If the red USV can reach the penetration path of the blue USV trajectory first, the red USV intercepts successfully, otherwise the blue USV breaks through.

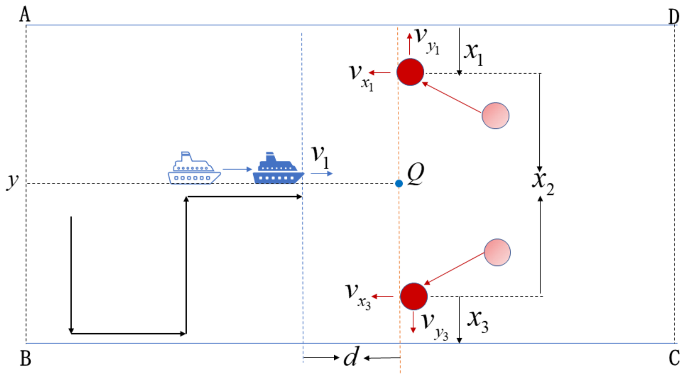

Figure 4: The third stage of the blue side movement

Take the intersection of the blue USV's final stage of penetration route and the vertical line where the red USV cluster centroid is located as Q, which is defined as the penetration point. Then the following theorem holds.

Theorem If the blue team reaches the penetration point Q first in the final stage, the blue team can definitely break through the interception of the red team's USV cluster.

proof Since we stipulate that the blue USV must move in a horizontal straight line or a vertical straight line, the red side needs to reach the blue side's final horizontal penetration path in the shortest time, that is, the red side is close to the corresponding one in the vertical direction. Penetrate the route to reach the penetration point Q. If the blue side reaches the point Q first, the blue side can rely on the speed advantage to completely throw off the red side's pursuit. If the red side reaches the point Q first, the red side can "stay" at the point Q by circling in place , And then wait until the arrival of the blue USV to intercept.

In summary, we will transform this question into the following game problem under the constraints of the title and hypothesis.

Participants:Blue USV, Red USV cluster. Define the decision set: Blue $\{(d,y)|0 <= d <= L-x0, 0 <= y <= M\}$,,red $\{(x1,x3)|0 <= x1 <= M/2, 0 <= x2 <= M/2\}$.

The core of the game between the blue side and the red side is that both sides hope to reach the penetration point Q before the other side reaches the designated point, so the**Decision function** is defined as

$$F(d,x_1,x_3) = \min_{y\in(0,M)} \{\frac{|x_3-y|-dis}{v_2}, \frac{|M-x_1-y|-dis}{v_2}\} - \frac{y+d}{v_1} \tag{4}$$

Among them, d and y represent the decision set of the blue party's USV, and x1 and x3 represent the decision set of the red party. The objective function of the two sides is

$$\begin{aligned} z_{blue} &= F(d,x1,x3) \\ z_{red} &= -F(d,x1,x3) \end{aligned} \tag{5}$$

This process is a zero-sum game. The blue party wants the F function as large as possible, and the red party wants the F function as small as possible. In the F function, the enumerated variable y of the max function reflects the blue party's process of finding the optimal breakthrough height. Each item in the min function represents the time for each USV of the red party to reach the penetration point Q, and the min function value represents the red By selecting a suitable USV cluster, the red party minimizes the time required to move to point Q. The blue side can change the distance d from the vertical line where the red side USV cluster is located by flying horizontally, so as to increase the F function as much as possible. The red party can reduce the F function by adjusting the size relationship between x1 and x3. For a given blue USV initial position (x0, y0), the initial flight time to the boundary is a constant, that is, $t = y0/v1$, so it is not included in the discussion of the objective function.

4.3 Red side interception strategy movement model

4.3.1 Red Square Symmetrical Movement Strategy Model-For Question 1

Because of assumption 6, when the red side's two USV clusters fly side by side, their horizontal velocity components are always the same, so $vx_1 = vx_2$. It is also stipulated by the title that the motion speed of all USVs is the same at all times, so $|vx_1| = |vx_2|$.In order to simplify the problem, in this scenario, consider first that the red party's mobile strategy is

$$\int_0^T (v_{y1}+v_{y2})\mathrm{d}t = x_1(T) - x_1(0) - x_3(T) + x_3(0) = 0 \tag{6}$$

We get that the distance between the two USV clusters on the red side to their nearest border is equal, that is,$x_1(T) = x_3(T)$

There are absolute value and min function in the objective function, so we need to discuss the value of F function.

The first case is

$$v_1x_3 < (v_1+v_2)y+v_2d+v_1dis \tag{7}$$

In order to maximize the objective function, the blue side will select y1=0. The second case is

$$\begin{cases} y-x_3 = M-x_1-y \\ \frac{y-x_3-dis}{v_2} < \frac{y+d+y_0}{v_1} \end{cases} \tag{8}$$

In order to maximize the objective function, the blue party will choose y2=M/2

The third case is

$$\frac{y+x_1-M-dis}{v_2} < \frac{y+d+y_0}{v_1} \tag{9}$$

In order to maximize the objective function, the blue side will choose y3=M

4.3.2 Red side asymmetric sports strategy model-for question 1

Considering the case of x1!=x3, and discussing the value of the objective function in the same way as 4.2.1, we get three sets of inequalities:

$$\forall y \in (0,M), \min\{\frac{|x_3-y|-dis}{v_2}, \frac{|M-x_1-y|-dis}{v_2}\} < \frac{y+d}{v_1} \tag{10}$$

Simplify to get

$$\begin{cases} 0 < x_1 < \frac{4(M+d)}{5}+dis \\ 0 < x_3 < \frac{4d}{5}+dis \\ x_1+9x_3 > M-10dis-8d \end{cases} \tag{11}$$

For the obtained constraint conditions, we can establish the x1-x3 coordinate system according to the knowledge of linear programming, and each constraint condition represents a half plane on the x1-x3 plane. **Definition of feasible set:** As shown in graph 4,In the x1-x3 plane, the set of points that satisfy all constraints. Constraints are expressed as half planes corresponding to inequalities, so the feasible set can also be expressed as the set of intersections of half planes corresponding to all constraints.

For the initial state of the red square, it can be represented as point P (2000, 2000) on the x1-x3 plane. The red side needs to adjust the position of point P according to the strategy of the strategy set, so that when the blue side adopts a strategy with y being the optimal value and d being the fastest decreasing strategy, it can be guaranteed to be in the feasible region at all times. Once the red side is outside the feasible zone, it means that there is a constraint that is not satisfied, and the blue side must find a corresponding penetration strategy to penetrate the defense.

In order to be within the region as much as possible, the red party must move toward the nearest point of the final feasible set. This process corresponds to the process of the red party x1 or x3 sailing at full speed in a vertical direction. By definition, we can calculate the shortest time for the red side to move to point Q. At the same time, for

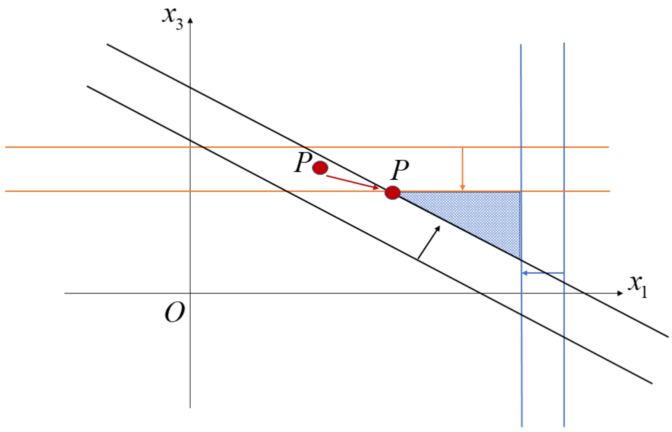

Figure 5: Application of Asymmetric Sports Strategy Model in Problem 1

a given starting position of a blue USV, we can also calculate the shortest time for the feasible set to reach the end state. By comparing the two, it can be concluded whether the starting position is a point that must be able to penetrate.

4.3.3 Red side asymmetric sports strategy model-for question 2

Combining the meaning of the question, the initial position of the red square G1G2 is not fixed. Therefore, the red party needs to select the initial position of the red USV in the x1-x3 plane as needed.

According to the dynamic law of the feasible set, for a given communication bandwidth M, the feasible set will surely bring all the constraints closer to each other as the horizontal distance d between the blue party and the red USV cluster decreases, thereby reducing the area of the area. Given a suitable M, the feasible set is still non-empty even if the horizontal distance d is the minimum value. Therefore, for the red party, (x1, x3) must always be within the final feasible set range to ensure The blue party cannot break through the red party's interception through the strategy defined in this article.

Theorem When the final state corresponding to d=0, if a certain point state $(x1_0,x3_0)$ belongs to the feasible set, the point $(x1_0,x3_0)$ must belong to the feasible set during the whole dynamic process that changes with d.

Prove that if $(x1_0,x3_0)$ does not belong to the feasible set at a certain moment, that is, there is a half-plane boundary passing through this point. In the final state, the point $(x1_0,x3_0)$ belongs to the feasible set, that is, there is no half-plane boundary passing through the point. It can be seen that must move in the opposite direction of the corresponding half-plane as d decreases. And all the constraints solved in this problem do not have a half-plane that moves in the opposite direction of the half-plane as d decreases. Therefore, there is a contradiction and the theorem is proved.

According to the theorem, the strategy of the red party is to initially deploy the two USV clusters to the state where the corresponding final state still belongs to the feasible set, so as to ensure that the blue party cannot pass the interception of the red party.

Because of the theory of linear programming, as M increases, the area of the feasible set of the final state will decrease accordingly. The process of each constraint condition changing with the change of M is shown in 6.

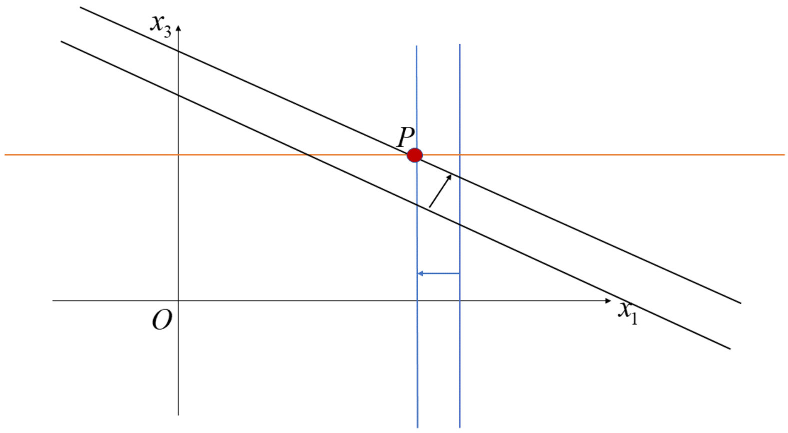

Figure 6: Application of Asymmetric Sports Strategy Model in Problem 2

Therefore, the analysis shows that when the feasible set of the final state is a single point, the area of the feasible set is 0 at this time, and the boundary lines of the corresponding half-planes intersect at one point by the three constraints. At this time, M takes the maximum value.

4.3.4 Red side asymmetric sports strategy model-for question 3

To simplify the problem, suppose that the red USVs still choose the same formation strategy, that is, five USVs form a circle to form a team. At this time, there are four USV clusters in the same formation. In order to use model two, we make a similar assumption for the situation corresponding to problem three, that is, the four USV clusters are always on the same vertical line. And x2=x3=x4=(M-x1-x5)/3

At this time, the same objective function is considered, that is, both parties hope to reach the designated point faster with the other party. The form of the objective function at this time is as follows.For this objective function, we still adopt a similar approach, namely discussing the moment when the positive and negative values of the absolute value and the two items of different sizes in the min function are equal in change.The decision variables of the blue side are d and y, and the decision variables of the red side are x1, x5.Examples of the five cases and the corresponding objective function expressions are shown below.

$$\forall y \in (0, M)$$
$$\frac{\min\{|y-x_5|, |y-x_4-x_5|, |y-x_3-x_4-x_5|, |y-x_2-x_3-x_4-x_5|\}-dis}{v_2} < \frac{y_0+y+d}{v_1} \tag{12}$$

Case 1:When d becomes 0, x1 and x5 need to satisfy the following inequality at all times

$$\frac{x_5-y-dis}{v_2} < \frac{y_0+y+d}{v_1} \tag{13}$$

For the blue side to maximize the objective function, take y1=0,

$$y_1 = 0 \tag{14}$$

Case 2:When d becomes 0, x1 and x5 need to satisfy the following inequality at all times

$$\begin{cases} x_4 + x_5 - y = y - x_5 \\ \frac{y - x_5 - dis}{v_2} < \frac{y_0 + y + d}{v_1} \end{cases} \tag{15}$$

In order to maximize the objective function, the blue side takes

$$y_2 = x_5 + \frac{x_4}{2} = \frac{M}{6} - \frac{x_1}{6} + \frac{5x_2}{6} \tag{16}$$

Case 3: When d becomes 0, x1 and x5 need to satisfy the following inequality at all times

$$\begin{cases} x_3 + x_4 + x_5 - y = y - x_4 - x_5 \\ \frac{y - x_4 - x_5 - dis}{v_2} < \frac{y_0 + y + d}{v_1} \end{cases} \tag{17}$$

In order to maximize the objective function, the blue side takes

$$y_3 = x_5 + x_4 + \frac{x_3}{2} = \frac{M}{2} - \frac{x_1}{2} + \frac{x_5}{2} \tag{18}$$

Case 4: When d becomes 0, x1 and x5 need to satisfy the following inequality at all times

$$\begin{cases} x_2 + x_3 + x_4 + x_5 - y = y - x_3 - x_4 - x_5 \\ \frac{y - x_3 - x_4 - x_5 - dis}{v_2} < \frac{y_0 + y + d}{v_1} \end{cases} \tag{19}$$

In order to maximize the objective function, the blue side takes

$$y_4 = x_5 + x_4 + x_3 + \frac{x_2}{2} = \frac{5M}{6} - \frac{5x_1}{6} + \frac{x_5}{6} \tag{20}$$

Case 5: When d becomes 0, x1 and x5 need to satisfy the following inequality at all times

$$\frac{y - x_2 - x_3 - x_4 - x_5 - dis}{v_2} < \frac{y_0 + y + d}{v_1} \tag{21}$$

In order to maximize the objective function, the blue side takes

$$y_5 = M \tag{22}$$

As shown in Figure 7, after obtaining five boundary constraints, similar to the second problem, the problem is transformed into a static game problem on the x1-x5 plane.

When M increases, it is easy to see that the restriction 1 2 5 firstly reduces the feasible region to the minimum, that is, when the three half-plane boundaries intersect at a point, M takes the maximum value.

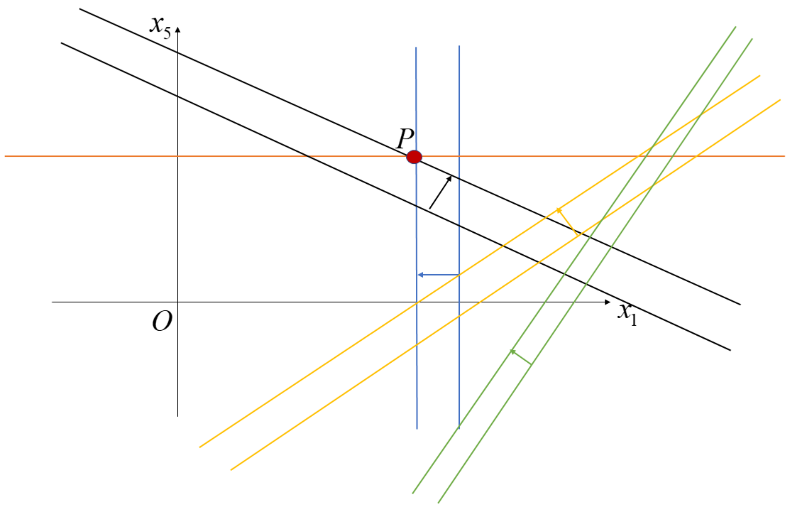

Figure 7: Application of Asymmetric Sports Strategy Model in Problem Three

5 Problem solving process

5.1 Problem one

5.1.1 Solve with model one

According to symmetry, the position where the blue team can break through is symmetrical about the midline, so we will only discuss the area below the midline, according to the red team's symmetrical movement strategy model

Case 1, obtained by simplification

$$x_3 < \frac{4d}{5} + dis \tag{23}$$

Case two, obtained by simplification

$$x_3 > \frac{\frac{1}{2}M - 4d}{5} + dis \tag{24}$$

Case three, obtained by simplification

$$x_3 = x_1 < \frac{4(M+d)}{5} + dis \tag{25}$$

As shown in Figure 8, according to the above results, ABCD can be divided into 3 regions.

In area ①, the blue party finally advances along y=0; in area ②, the blue party finally advances along y=M/2; in area ③, the blue party finally advances along y=M.The union of three cases

$$\frac{M-8d}{10} - dis < x_3 < \min\{\frac{4d}{5}, \frac{4(d+M)}{5}\} + dis = \frac{4d}{5} + dis \tag{26}$$

Let the left interval be equal to the right interval, at this time the feasible set is the smallest, and the maximum value of M is 20dis=2560m

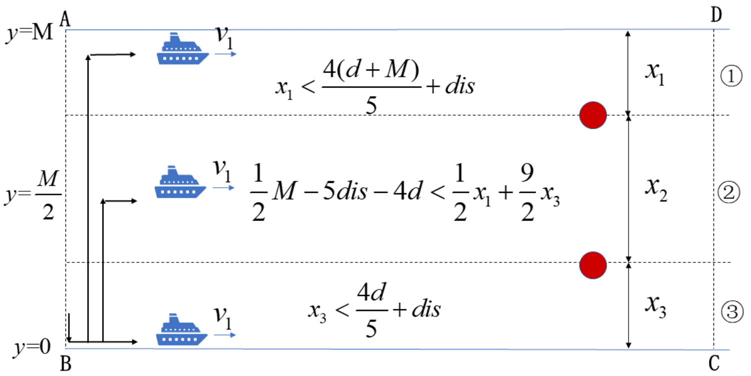

Figure 8: Three divisions of area ABCD

5.1.2 Solve with model two

5.1.2.1 Determination of v_x

From the restriction conditions, combining the red and blue sports strategies, we can see

$$d = L - \int_0^t (v_1 + |v_x|)\mathrm{d}t \tag{27}$$

For the feasible set in question 1, since the initial state of the red side is fixed, the red side needs to move as far as possible to the feasible set area of the final state as the feasible set is reduced. Combining (the expression of d) shows that, d The change of is determined by the horizontal velocity components of the blue and red sides at the same time. The blue side wants the red side P to leave the feasible set, so the blue side needs to move to the right at the maximum speed, so that the feasible set shrinks fastest. The red side wants the feasible set to shrink as slowly as possible, so the red side needs to be as far away from the blue side as possible, and since the initial position of the red side and the CD boundary are still, the red side only needs to have a horizontal velocity component of 0. In addition, due to the assumptions and assumptions, the speed of the USV is constant, and the time and space effects of the USV's steering are not considered. Therefore, only the red USV needs to change the direction of the vertical velocity component multiple times in a short time to maintain the horizontal static. And the movement rate in the horizontal direction becomes $[0, v_2]$. At this time, d is exclusively determined by the blue USV.

5.1.2.2 Judgment that the blue team cannot break through the interception of the red team

$v_x = 0$, the constraint condition of the final solution set is

$$\begin{cases} 0 < x_1 < \frac{4(M+d)}{5} + dis \\ 0 < x_3 < \frac{4d}{5} + dis \\ x_1 + 9x_3 > M - 10dis - 8d \end{cases} \tag{28}$$

For the final state d=0, the area is

$$\begin{cases} 0 < x_1 < \frac{4M}{5} + dis \\ 0 < x_3 < dis \\ x_1 + 9x_3 > M - 10dis \end{cases} \tag{29}$$

Get the actual area according to the data obtained by the model one solution

$$\begin{cases} 0 < x_1 < 5728 \\ 0 < x_3 < 128 \\ x_1 + 9x_3 > 5720 \end{cases} \tag{30}$$

As shown in Figure 9

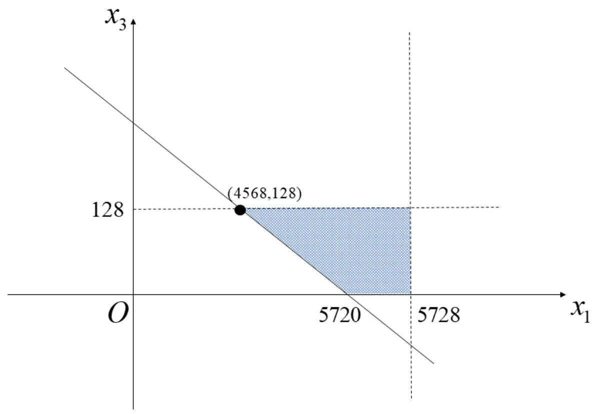

Figure 9: The optimal solution for the position of the red side in Problem 1

So the optimal solution for the position of the red side is (4568, 128), so

$$t_{minRed} = \frac{\min\{2000 - 128, 4568 - 2000\}}{20} = 93.6s \tag{31}$$

As shown in Figure 10, and for any point (x0, y0) of the blue side.

we have

$$t_{minBlue} = \frac{\min\{y_0 + L - x_0 + y\}}{v_1} = \frac{y_0 + L - x_0}{v_1} < 93.6s, y \in (0, M) \tag{32}$$

which is

$$x_0 - y_0 > 7660 \tag{33}$$

Therefore, the point where the blue team can break through is shown in Figure 11

When the blue side starts to act in this area, it will be able to break through successfully.

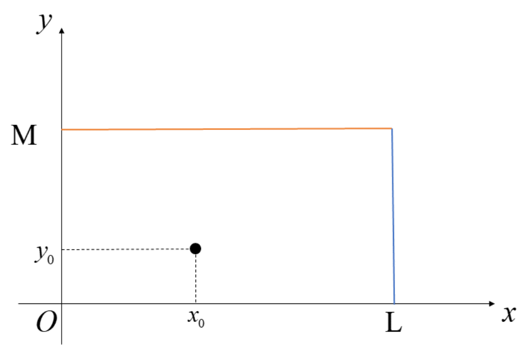

Figure 10: Schematic diagram of the location of any point on the blue side in Question 1

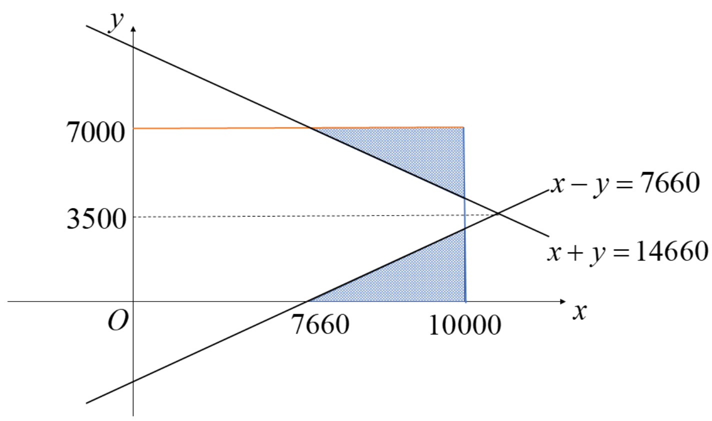

Figure 11: points that the blue side can break through in question 1

5.2 Problem two

Substituting the values corresponding to x1 and x3 into the constraint condition 3 in, where d=0, we can get

$$x_1 = \frac{4(M+d)}{5} + dis, x_3 = \frac{4d}{5} + dis \tag{34}$$

Solve it

$$M < 100dis = 12800m \tag{35}$$

Therefore, when $M < M_max$, when the blue party adopts the breakthrough strategy under the constraints of this article, it cannot ensure that it breaks through the interception of the red party. The red team only needs to adopt a defensive strategy of x1=dis, x2=M/5-2dis, x3=4M/5+dis.

When $M > M_min$, when the blue side adopts the breakthrough strategy under the constraints of this article, it will definitely break through the interception of the red side. At this time, the blue side adopts strategy①, and the time used is (12800/2+10000)/25=656s

5.3 Problem Three

As shown in Figure 11, substituting the values corresponding to x1 and x3 into the constraint condition 3 in , where d=0, we can get

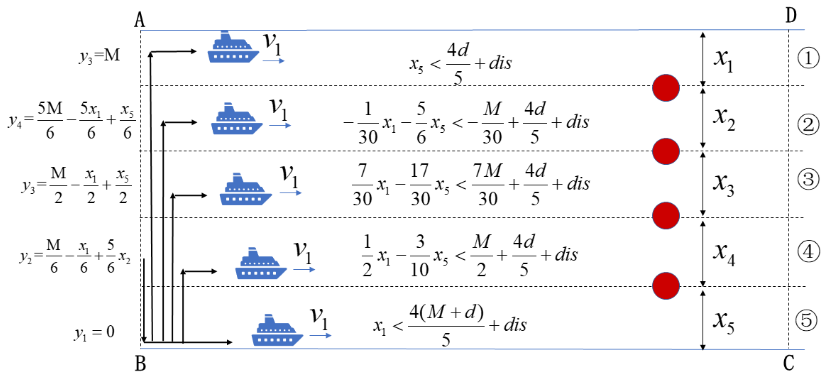

Figure 12: In question 1, the red party's movement process 1

After substituting the boundary conditions in 4.2.4.

$$x_1=\frac{4(M+d)}{5}+dis,x_5=\frac{4d}{5}+dis \tag{36}$$

he solution is $M_{min}=Min_{max}<280dis=35840m$

Therefore, when M_max, when the blue party adopts the breakthrough strategy under the constraints of this article, it cannot ensure that it breaks through the interception of the red party. The red party just takes $x1=dis,x2=M/15-2dis/3,x3=M/15-2dis/3,x4=M/15-2dis/3,x5=4M/5+dis$ defense strategy. At this time, when the red party satisfies the constraints, the four formations will be devolved as soon as possible. Therefore, it is calculated that when the strategy is used, the carrier will be at the distance between the boundary $M/15+dis/3$ and $2M/15-dis$ except for the initial time. After lowering the first two USV clusters in 3 places, drive to the horizontal boundary, and release the second wave of USV clusters at distances of dis and $4M/5+dis$ respectively. The required time is $\max\{x2,x4\}/v3=36.3s$

6 Simulation and verification

Considering the turning time and radius, the blue side uses the penetration strategy proposed in this article, and the red side uses the tracking strategy to simulate the situation in a simulation program written in matlab. As shown in the figure, the blue line represents the attacking route of the blue side, the red line represents the interception route of each USV of the red side, and the black line represents the movement route of the carrier. The two pictures respectively show the effect of the blue team's penetration strategy in the model and the red team's interception route in the model. It can be found that the penetration/interception strategy of the blue team and the red team are compared to no strategy, both achieve a certain penetration/interception effect.

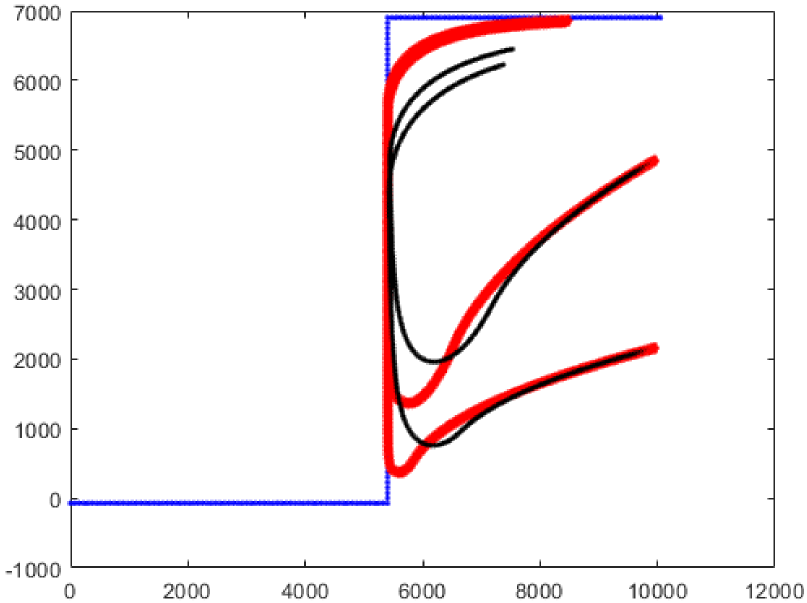

Figure 13: Simulation image 1

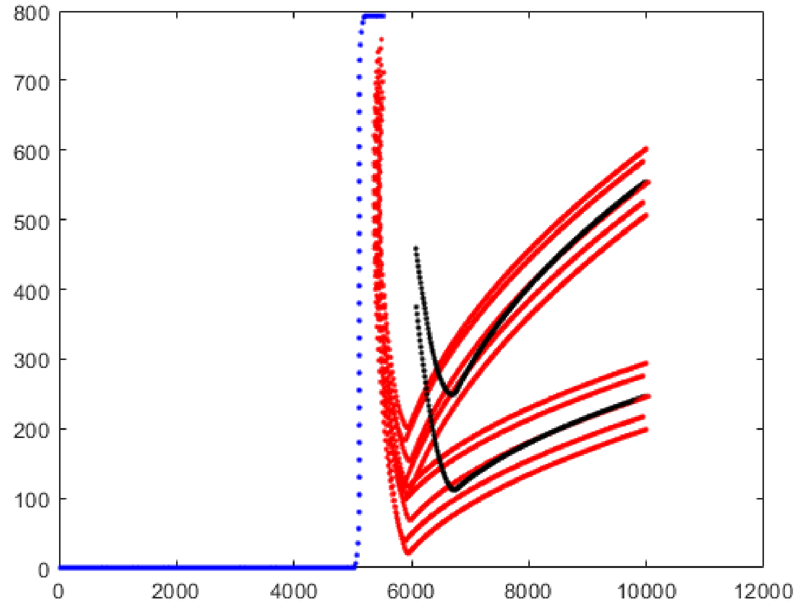

Figure 14: Simulation image 2

7 Model evaluation

7.1 Evaluation of Red and Blue SIDE Game Model

The blue team can choose the upper, middle and lower three routes, which reflects the blue team's mobile advantage. The red team chose to block side by side, reflecting the red team's quantitative advantage. On this basis, we set the objective function of the two parties according to the speed relationship between the red and blue parties reaching the penetration point Q, which means that the process is a zero-sum game, which is consistent with the facts. The blue party's decision set is the chosen penetration route and the degree of proximity to the red party, which maximizes its own benefits through decision-making. The red party's decision set is the relative distance of the USVs arranged side by side, which maximizes its own benefits through decision-making. By establishing this model, we have restored as much as possible the real offensive and defensive problems. However, due to the large gap between some hypotheses and the requirements of the title, and the large number of hypotheses, the results obtained by this model are quite different from reality.

7.2 The connection between model one and model two

For model 1: We assume x1=x3. Since d(t) decreases with time until it decreases to zero, the boundary of the region will continue to move closer to the center of the first quadrant, and the area of the region that satisfies the above inequality (*) becomes smaller and smaller. Suppose P(x1,x3), at any time, when point P is outside the line area, it means that the inequality (*) cannot be satisfied, that is, the red party fails to intercept; otherwise, if P is always in the line area until d=0 , The red side intercepts successfully. The blue side always hopes to break through the interception of the red side, which is reflected in the linear programming coordinate system, that is, the line area is constantly shrinking. The red side always wants to intercept the blue side, so it will constantly adjust its position in an attempt to accomplish the purpose of intercepting the blue side. The behavior of the red party is reflected in the linear programming coordinate system, that is, point P is constantly moving to keep point P in the area. As shown in Figure 15. As time increases, the feasible area continues to shrink. Since x1=x3, point P always moves on the straight line x1=x3.

As shown in Figure 16. As time increases, the feasible area shrinks again. Since x1=x3, point P always moves on the straight line x1=x3. After this move, P just moved to the boundary of the feasible area.

As shown in Figure 17. As time increases, the feasible area shrinks again. Since x1=x3, point P can only move on the straight line x1=x3, which results in point P being unable to enter the feasible area this time, and the red side's interception of the blue side fails.

Based on the above analysis, we can see that when x1=x3, the model misses some points where the red side could intercept the blue side. In fact, x1 x3 is a universal situation. Therefore, Model 1 is a special case of Model 2.

For the linear programming model, for the optimal value, M and d are equivalent, knowing that one can solve the other.

For question 1: M is determined, discuss the size of the initial d, the position of P is

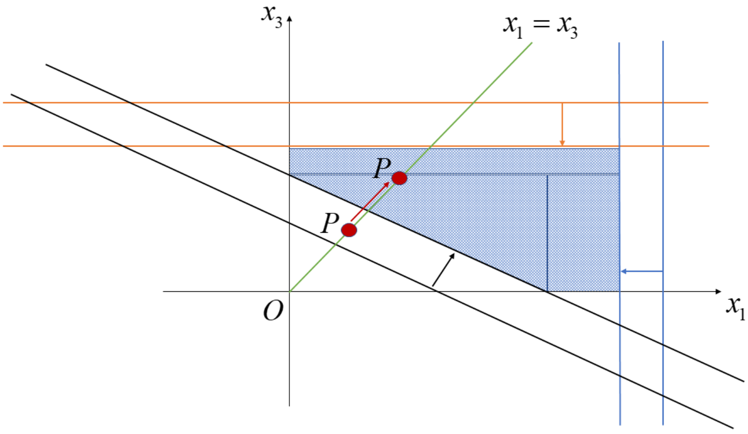

Figure 15: In question 1, the red party's movement process

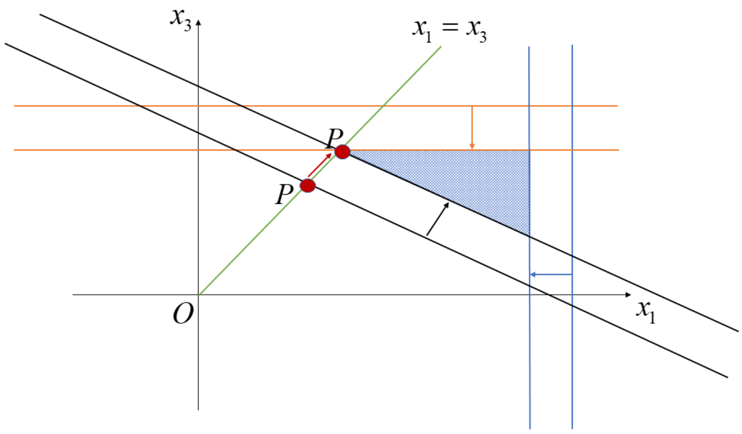

Figure 16: In question 1, the red party's movement process 2

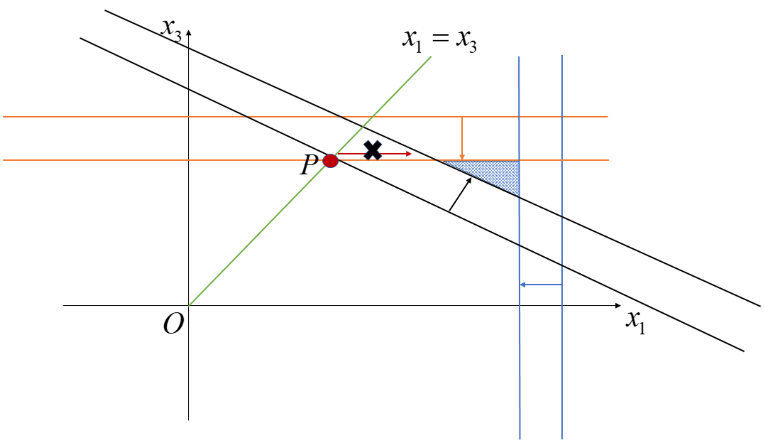

Figure 17: Limitations of the red party's movement in question 1

given, this is a dynamic linear programming problem about x1 and x3.

For question 2: M is uncertain, initial d is determined. And the initial position of point P can be selected, which is a static linear programming problem about x1 and x3.

8 Conclusion

Linear programming algorithm is an optimization algorithm that is easy to implement and can handle multiple variables at the same time. So far, this article uses the linear programming algorithm to simulate the USV game attack and defense. When the red and blue sides default to the blue side to adopt the breakthrough strategy under the constraints of this article, the offensive and defense strategies of the red and blue sides and the corresponding channel bandwidth have been determined.

References

[1] Lei zhang,Unmanned Surface Technology[M].Shanghai Jiao Tong University Press,2018

[2] Yumei Fanm,Mathematical Programming and Its Applications.China Machine Press,2018

[3] Antonios,Cooperative Path Planning of Unmanned Aerial Vehicles,National Defense Industry Press.

Appendix A: Verification procedure of linear programming results

```
dis=128;
M=10000;
for d=50:-10:0
        %plot(2000,2000,'o');
        hold on
        y=-4000:100:1000
        x=y*0+4*(M+d)/5+dis;
        plot(x,y);
        hold on
        x=8000:100:8200;
        y=x*0+4*(d)/5+dis;
        plot(x,y);
        hold on
        x=8000:100:8200;
        y=-6/5*(-M/30+4/5*d+dis+x/30);
        plot(x,y);
        hold on
        x=8000:100:8200;
        y=-30/17*(7*M/30+4/5*d+dis-7*x/30);
        plot(x,y);
        hold on
        x=8000:100:8200;
        y=-10/3*(M/2+4/5*d+dis-x/2);
        plot(x,y);
        hold on
        pause(0.1);
end
```

Appendix A:Blue Side USV adopts deception strategy simulation program

```

close all
clear all
clc
M=7000;L=10000;
deltatime=1;
Re=100;Ve=25*deltatime;Te=2*pi*Re/Ve;Alphae=deltatime*2*pi/Te;
Rp=80;Vp=20*deltatime;Tp=2*pi*Rp/Vp;Alphap=deltatime*2*pi/Tp;
Rc=500;Vc=16*deltatime;Tc=2*pi*Rc/Vc;Alphac=deltatime*2*pi/Tc;


mindisbr=100;brcnt=0;
mindisrr=30;
```

```
mindisbR=1000;
maxdisrr=100;rrcnt=0;
maxdisRr=2000;Rrcnt=0;
predtime=5;%the range of success is [4,15]
R0=50;Maxtime=700;timecnt=0;
blue.x=0;blue.y=M/2;
blue.v=[Ve,0];
size=5;size0=2;
red0(1).x=L;red0(1).y=5000;red0(1).v=[-Vc,0];
red0(2).x=L;red0(2).y=2000;red0(2).v=[-Vc,0];
for i=1:size0
        for j=1:size
                red(i,j).x=red0(i).x+R0*cos(2*pi/size*(j-1));
                red(i,j).y=red0(i).y+R0*sin(2*pi/size*(j-1));
                red(i,j).v=[-Vp,0];
        end
end


while timecnt<Maxtime && blue.x<L && brcnt<2
timecnt=timecnt+deltatime;
target.x=blue.x+predtime*blue.v(1);
target.y=blue.y+predtime*blue.v(2);
thetae=0;
%calculate angle
for i=1:size0
        vec=[(target.x-red0(i).x),(target.y-red0(i).y)];
        theta=acos(dot(red0(i).v,vec)/norm(red0(i).v)/norm(vec));
        if (red0(i).v(1)*vec(2)-red0(i).v(2)*vec(1))>0
                thetac(i)=max(-Alphac,-theta);
        else
                thetac(i)=min(Alphac,theta);
        end
        cen(1)=0;
        cen(2)=0;
        for j=1:size
                cen=[cen(1)+red(i,j).x,cen(2)+red(i,j).y];
        end
        cen=cen/size;
        vec=[(target.x-cen(1)),(target.y-cen(2))];
        theta=acos(dot(red(i,j).v,vec)/norm(red(i,j).v)/norm(vec));
        if (red(i,1).v(1)*vec(2)-red(i,1).v(2)*vec(1))>0
                thetap(i)=max(-Alphap,-theta);
        else
                thetap(i)=min(Alphap,theta);
        end
end

%calculate position
blue.v=blue.v*[cos(thetae) -sin(thetae);sin(thetae) cos(thetae)];
```

```
blue.x=blue.x+blue.v(1);
blue.y=blue.y+blue.v(2);
for i=1:size0
tmp_red0(i).v=red0(i).v*[cos(thetac(i)) -sin(thetac(i));sin(thetac(i)) co
tmp_red0(i).x=red0(i).x+red0(i).v(1);
tmp_red0(i).y=red0(i).y+red0(i).v(2);
for j=1:size
tmp_red(i,j).v=red(i,j).v*[cos(thetap(i)) -sin(thetap(i));sin(thetap(i))
tmp_red(i,j).x=red(i,j).x+red(i,j).v(1);
tmp_red(i,j).y=red(i,j).y+red(i,j).v(2);
end
end
subplot(1,2,2);
%check minrr
for p=1:size0
for i=1:size
for j=1:size
if i~=j
if norm([red(p,i).x-red(p,j).x,red(p,i).y-red(p,j).y])<mindisrr
plot(red(p,i).x,red(p,i).y,'k.');
hold on
end
end
end
end
end
%check maxrr
for p=1:size0
for i=1:size
rrcnt=0;
for j=1:size
if i~=j
if norm([red(p,i).x-red(p,j).x,red(p,i).y-red(p,j).y])<maxdisrr
rrcnt=rrcnt+1;
end
end
end
if rrcnt<2
plot(red(p,i).x,red(p,i).y,'r.');
hold on
end
end
end

%check maxrR
for p=1:size0
Rrcnt=0;
for i=1:size
if norm([red0(p).x-red(p,i).x,red0(p).y-red(p,i).y])<maxdisRr
Rrcnt=Rrcnt+1;
```

```
end
end
if Rrcnt==0
plot(red0(p).x,red0(p).y,'^');
hold on
end
end
%       %check minbR
%       for p=1:size0
%           if norm([red0(p).x-blue.x,red0(p).y-blue.y])<mindisbR
%               plot(red0(p).x,red0(p).y,'go');
%               hold on
%           end
%       end
%check minbr
brcnt=0;
for p=1:size0
for i=1:size
if norm([red(p,i).x-blue.x,red(p,i).y-blue.y])<mindisbr
brcnt=brcnt+1;
end
end
end
if brcnt>=2
plot(blue.x,blue.y,'ko');
hold on
end


for i=1:size0
red0(i)=tmp_red0(i);
for j=1:size
red(i,j)=tmp_red(i,j);
end
end



%plot
subplot(1,2,1);
plot(blue.x,blue.y,'b.');
hold on
for i=1:2
plot(red0(i).x,red0(i).y,'k.');
hold on
for j=1:size
plot(red(i,j).x,red(i,j).y,'r.');
hold on
end
end
```

```
pause(1e-9);
end
```