Conditional Propositions and Logical Equivalence

Section 1.2

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Contents

1 Conditional Propositions

1.1 Conditional Propositions

Conditional Propositions

- "If it rains this afternoon, then I will carry an umbrella" is a proposition
- Is the proposition true or false?
  - *True* if it rains and I carry an umbrella.
  - *False* if it rains and I don't carry an umbrella.
  - What if it doesn't rain?
- If p and q are propositions, the proposition *"if p then q"* is a *conditional proposition*.
  - Denoted $p \to q$
  - p is the *hypothesis* or *antecedent*.
  - p is also called a *sufficient condition*.
  - q is the *conclusion* or *consequent*.
  - q is also called a *necessary condition*.
  - $p \to q$ is another binary operator.

1.2 Truth Table of Conditional Propositions

Truth Table of $p \to q$

- When is $p \to q$ true?
  - *True* if both p and q are true.
  - *False* if p is true, but q is false.
  - *True* if p is false.
  - Referred to as either:
    * *True by default*,
    * *Vacuously true*, or

* *Trivially true*

p	q	$p \to q$
T	T	
T	F	
F	T	
F	F	

Truth Table of $p \to q$

Example. "If Brett Favre is the starting quarterback for the Packers, then $2+2 = 5$" is a *true* proposition.

Problem. Complete the truth table.

p	q	$\neg(p \to q)$
T	T	
T	F	
F	T	
F	F	

1.3 Conditional Propositions in Computing

The "`if-then`" Statement

- There is no direct analog to $p \to q$ in Java.
- Java does have an "`if-then`" statement

```
if (condition){
   statement
}
```

- If *condition* is true, then *statement* executes
- If *condition* is false, then *statement* is irrelevant.

1.4 The Converse

The Converse

Problem. Complete the truth table.

p	q	$q \to p$
T	T	
T	F	
F	T	
F	F	

- Let $p \to q$ be a conditional proposition. The *converse* of $p \to q$ is $q \to p$
- The converse is not the same as the original proposition.

The Converse

Example.

- "If Brett Favre is the starting quarterback for the Packers, then $2 + 2 = 4$" is a *true* proposition.
- "If $2 + 2 = 4$, then Brett Favre is the starting quarterback for the Packers" is a *false* proposition.
- These propositions are converses of each other.

1.5 Biconditional Proposition

Biconditional Proposition

- If p and q are propositions, then *"p if and only if q* or *"p iff q"* is a *biconditional proposition*.
- We denote it by $p \leftrightarrow q$
- $p \leftrightarrow q$ is true precisely when p and q have the same truth values

p	q	$p \leftrightarrow q$
T	T	
T	F	
F	T	
F	F	

2 Logical Equivalence

2.1 Logical Equivalence

Logically Equivalent Propositions

- If P and Q are compound propositions built from $p_1, p_2, \ldots, p_n$, then P and Q are *logically equivalent* provided that P and Q have the same truth values, no matter what truth values $p_1, p_2, \ldots, p_n$ have.
  - We denote this $P \equiv Q$
  - This is the same as $P \leftrightarrow Q$ being a tautology.

Example. • $\neg(p \to q) \equiv p \wedge \neg q$

- $p \to q \not\equiv q \to p$

Logically Equivalent Propositions

Example. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

p	q	$p \leftrightarrow q$
T	T	
T	F	
F	T	
F	F	

p	q	$(p \to q) \wedge (q \to p)$
T	T	
T	F	
F	T	
F	F	

2.2 De Morgan's Laws

De Morgan's Laws

Theorem 1. *De Morgan's Laws*

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Augustus De Morgan was a 19th century British mathematician, born in India.
- Gives a way of negating conjunctions and disjunctions.

p	q	$\neg(p \vee q)$
T	T	
T	F	
F	T	
F	F	

p	q	$\neg p \wedge \neg q$
T	T	
T	F	
F	T	
F	F	

2.3 The Contrapositive

The Contrapositive

- The *contrapositive* of a proposition $p \to q$ is the proposition $\neg q \to \neg p$

Theorem 2. *The conditional proposition $p \to q$ is logically equivalent to its contrapositive $\neg q \to \neg p$.*

p	q	$p \to q$
T	T	
T	F	
F	T	
F	F	

p	q	$\neg q \to \neg p$
T	T	
T	F	
F	T	
F	F	

Summary

Summary

You should be able to:

- Use *conditional* and *biconditional* propositions.
- Use the *converse* and *contrapositive* of a statement.
- Identify *logically equivalent* propositions.
- Use *De Morgan's Laws*.