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1. Misc

1.1. Contest

1.1.1. Makefile

```
1 .PRECIOUS: ./p%
3 %: p%
   ulimit -s unlimited && ./p%
5 p%: p%.cpp
   g++ -o $@ $< -std=c++17 -Wall -Wextra -Wshadow \
7   -fsanitize=address,undefined
```

1.1.2. Default Code

```
1 #include <bits/stdc++.h>
3 #define pb      push_back
4 #define eb      emplace_back
5 #define F      first
6 #define S      second
7 #define SZ(v)   ((int)(v).size())
8 #define ALL(v)  (v).begin(), (v).end()
9 #define MEM(a, b) memset(a, b, sizeof a)
10 #define unpair(p) (p).F[(p).S]
12 using namespace std;
13 using ll = long long;
14 using ld = long double;
15 using LL = __int128;
16 using pii = pair<int, int>;
17 using pll = pair<ll, ll>;
19 int main() { ios::sync_with_stdio(0); cin.tie(0); }
```

1.2. How Did We Get Here?

1.2.1. Macros

Use vectorizations and math optimizations at your own peril.
For gcc ≥ 9 , there are `[[likely]]` and `[[unlikely]]` attributes.
Call gcc with `-fopt-info-optimized-missed-optall` for optimization info.

```
1 #define _GLIBCXX_DEBUG      1 // for debug mode
2 #define _GLIBCXX_SANITIZE_VECTOR 1 // for asan on vectors
3 #pragma GCC optimize("O3", "unroll-loops")
4 #pragma GCC optimize("fast-math")
5 #pragma GCC target("avx,avx2,abm,bmi,bmi2") // tip: `lscpu`
6 // before a loop
7 #pragma GCC unroll 16 // 0 or 1 -> no unrolling
8 #pragma GCC ivdep
```

1.2.2. constexpr

Some default limits in gcc (7.x - trunk):

- constexpr recursion depth: 512
- constexpr loop iteration per function: 262144
- constexpr operation count per function: 33554432
- template recursion depth: 900 (gcc *might* segfault first)

```
1 constexpr array<int, 10> fibonacci{[] {
2     array<int, 10> a{};
3     a[0] = a[1] = 1;
4     for (int i = 2; i < 10; i++) a[i] = a[i - 1] + a[i - 2];
5     return a;
6 }()};
7 static_assert(fibonacci[9] == 55, "CE");
9 template <typename F, typename INT, INT... S>
10 constexpr void for_constexpr(integer_sequence<INT, S...>,
11                             F &&func) {
12     int _[] = {(func(integral_constant<INT, S>{}), 0)...};
13 }
14 // example
15 template <typename... T> void print_tuple(tuple<T...> t) {
16     for_constexpr(make_index_sequence<sizeof...(T)>{}),
17     [&](auto i) { cout << get<i>(t) << '\n'; };
18 }
```

1.2.3. Bump Allocator

```
1 // global bump allocator
2 char mem[256 << 20]; // 256 MB
3 size_t rsp = sizeof mem;
4 void *operator new(size_t s) {
5     assert(s < rsp); // MLE
6     return (void *)mem[rsp - s];
7 }
8 void operator delete(void *) {}
9
10 // bump allocator for STL / pbds containers
11 char mem[256 << 20];
12 size_t rsp = sizeof mem;
13 template <typename T> struct bump {
14     typedef T value_type;
15     bump() {}
16     template <typename U> bump(U, ...) {}
17     T *allocate(size_t n) {
18         rsp -= n * sizeof(T);
19         return (T *)mem[rsp];
20     }
21     void deallocate(T *, size_t n) {}
22 };
23
```

1.3. Tools

1.3.1. Floating Point Binary Search

```
1 union di {
2     double d;
3     ull i;
4 };
5 bool check(double);
6 // binary search in [L, R) with relative error 2^-eps
7 double binary_search(double L, double R, int eps) {
8     di l = {L}, r = {R}, m;
9     while (r.i - l.i > 1LL << (52 - eps)) {
10         m.i = (l.i + r.i) >> 1;
11         if (check(m.d)) r = m;
12         else l = m;
13     }
14     return l.d;
15 }
```

1.3.2. SplitMix64

```
1 using ull = unsigned long long;
2 inline ull splitmix64(ull x) {
3     // change to `static ull x = SEED;` for DRBG
4     ull z = (x += 0x9E3779B97F4A7C15);
5     z = (z ^ (z >> 30)) * 0xBF58476D1CE4E5B9;
6     z = (z ^ (z >> 27)) * 0x94D049BB133111EB;
7     return z ^ (z >> 31);
8 }
```

1.3.3. <random>

```
1 #ifdef __unix__
  random_device rd;
3 mt19937_64 RNG(rd());
  #else
5 const auto SEED = chrono::high_resolution_clock::now()
  .time_since_epoch()
  .count();
7 mt19937_64 RNG(SEED);
9 #endif
  // random uint_fast64_t: RNG();
11 // uniform random of type T (int, double, ...) in [l, r]:
  // uniform_int_distribution<T> dist(l, r); dist(RNG);
```

1.4. Algorithms

1.4.1. Bit Hacks

```
1 // next permutation of x as a bit sequence
  ull next_bits_permutation(ull x) {
3   ull c = __builtin_ctzll(x), r = x + (1 << c);
   return (r ^ x) >> (c + 2) | r;
5 }
  // iterate over all (proper) subsets of bitset s
7 void subsets(ull s) {
   for (ull x = s; x;) { --x &= s; /* do stuff */ }
9 }
```

1.4.2. Aliens Trick

```
1 // min dp[i] value and its i (smallest one)
  pll get_dp(int cost);
3 ll aliens(int k, int l, int r) {
   while (l != r) {
5     int m = (l + r) / 2;
     auto [f, s] = get_dp(m);
7     if (s == k) return f - m * k;
     if (s < k) r = m;
9     else l = m + 1;
   }
11  return get_dp(l).first - l * k;
}
```

1.4.3. Hilbert Curve

```
1 ll hilbert(ll n, int x, int y) {
  ll res = 0;
3   for (ll s = n / 2; s; s >>= 1) {
     int rx = !(x & s), ry = !(y & s);
5     res += s * s * ((3 * rx) ^ ry);
     if (ry == 0) {
7       if (rx == 1) x = s - 1 - x, y = s - 1 - y;
       swap(x, y);
9     }
   }
11  return res;
}
```

2. Data Structures

2.1. GNU PBDS

```
1 #include <ext/pb_ds/assoc_container.hpp>
  #include <ext/pb_ds/priority_queue.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
  using namespace __gnu_pbds;

  // most of std::map + order_of_key, find_by_order
7 template <typename T, typename U = null_type>
  using ordered_map = tree<T, U, std::less<>, rb_tree_tag,
9   tree_order_statistics_node_update>;

  // useful tags: rb_tree_tag, splay_tree_tag
11

  template <typename T> struct myhash {
13   size_t operator()(T x) const; // splitmix, bswap(x*R), ...
  };

  // most of std::unordered_map, but faster (needs good hash)
15 template <typename T, typename U = null_type>
  using hash_table = gp_hash_table<T, U, myhash<T>>;
17

  // most of std::priority_queue + merge
19 using heap = priority_queue<int, std::less<>>;
21 // useful tags: pairing_heap_tag, binary_heap_tag,
  // binomial_heap_tag
```

2.2. Segment Tree (ZKW)

```
1 struct segtree {
  using T = int;
3   T f(T a, T b) { return a + b; } // any monoid operation
  static constexpr T ID = 0; // identity element
5   int n;
  vector<T> v;
7   segtree(int n_) : n(n_), v(2 * n, ID) {}
  segtree(vector<T> &a) : n(a.size()), v(2 * n, ID) {
9     copy_n(a.begin(), n, v.begin() + n);
     for (int i = n - 1; i > 0; i--)
11       v[i] = f(v[i << 1], v[i << 1 | 1]);
   }
13   void update(int i, T x) {
     for (v[i += n] = x; i /= 2;)
15       v[i] = f(v[i << 1], v[i << 1 | 1]);
   }
17   T query(int l, int r) {
     T tl = ID, tr = ID;
19     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
       if (l & 1) tl = f(tl, v[l++]);
21       if (r & 1) tr = f(v[--r], tr);
     }
23     return f(tl, tr);
25   };
}
```

2.3. Wavelet Matrix

```
1 #pragma GCC target("popcnt,bmi2")
  #include <immintrin.h>

  // T is unsigned. You might want to compress values first
5 template <typename T> struct wavelet_matrix {
  static_assert(is_unsigned_v<T>, "only unsigned T");
7   struct bit_vector {
     static constexpr uint W = 64;
9     uint n, cnt0;
     vector<ull> bits;
     vector<uint> sum;
     bit_vector(uint n_)
11       : n(n_), bits(n / W + 1), sum(n / W + 1) {}
     void build() {
13       for (uint j = 0; j != n / W; ++j)
         sum[j + 1] = sum[j] + _mm_popcnt_u64(bits[j]);
15       cnt0 = rank0(n);
     }
17     void set_bit(uint i) { bits[i / W] |= 1ULL << i % W; }
     bool operator[](uint i) const {
21       return !(bits[i / W] & 1ULL << i % W);
     }
23     uint rank1(uint i) const {
       return sum[i / W] +
25       _mm_popcnt_u64(_bzhi_u64(bits[i / W], i % W));
     }
27     uint rank0(uint i) const { return i - rank1(i); }
  };
29   uint n, lg;
  vector<bit_vector> b;
31   wavelet_matrix(uint _n = 0) : n(_n) {}
  wavelet_matrix(const vector<T> &a) : n(a.size()) {
33     lg =
      __lg(max(*max_element(a.begin(), a.end()), T(1))) + 1;
35     b.assign(lg, n);
     vector<T> cur = a, nxt(n);
37     for (int h = lg; h--;) {
       for (uint i = 0; i < n; ++i)
39         if (cur[i] & (T(1) << h)) b[h].set_bit(i);
       b[h].build();
41       int il = 0, ir = b[h].cnt0;
       for (uint i = 0; i < n; ++i)
43         nxt[(b[h][i] ? ir : il)++] = cur[i];
       swap(cur, nxt);
45     }
   }
47   T operator[](uint i) const {
     T res = 0;
49     for (int h = lg; h--;)
       if (b[h][i])
51       i += b[h].cnt0 - b[h].rank0(i), res |= T(1) << h;
     return res;
53   }

  // query k-th smallest (0-based) in a[l, r]
55   T kth(uint l, uint r, uint k) const {
     T res = 0;
57     for (int h = lg; h--;) {
       uint tl = b[h].rank0(l), tr = b[h].rank0(r);
59     if (k >= tr - tl) {
```

```

61     k -= tr - tl;
62     l += b[h].cnt0 - tl;
63     r += b[h].cnt0 - tr;
64     res |= T(1) << h;
65 } else l = tl, r = tr;
66 }
67 return res;
68 }
69 // count of i in [l, r] with a[i] < u
70 uint count(uint l, uint r, T u) const {
71     if (u >= T(1) << lg) return r - l;
72     uint res = 0;
73     for (int h = lg; h--;) {
74         uint tl = b[h].rank0(l), tr = b[h].rank0(r);
75         if (u & (T(1) << h)) {
76             l += b[h].cnt0 - tl;
77             r += b[h].cnt0 - tr;
78             res += tr - tl;
79         } else l = tl, r = tr;
80     }
81     return res;
82 }
83 };

```

3. Math

3.1. Number Theory

3.1.1. Mod Struct

A list of safe primes: 26003, 27767, 28319, 28979, 29243, 29759, 30467, 910927547, 919012223, 947326223, 990669467, 1007939579, 1019126699, 929760389146037459, 975500632317046523, 989312547895528379

NTT prime p	$p - 1$	primitive root
65537	$1 \ll 16$	3
998244353	$119 \ll 23$	3
2748779069441	$5 \ll 39$	3
194555039024054273	$27 \ll 56$	5

```

1 template <typename T> struct M {
2     static T MOD; // change to constexpr if already known
3     T v;
4     M() : v(0) {}
5     M(T x) {
6         v = (-MOD <= x && x < MOD) ? x : x % MOD;
7         if (v < 0) v += MOD;
8     }
9     explicit operator T() const { return v; }
10    bool operator==(const M &b) const { return v == b.v; }
11    bool operator!=(const M &b) const { return v != b.v; }
12    M operator-() { return M(-v); }
13    M operator+(M b) { return M(v + b.v); }
14    M operator-(M b) { return M(v - b.v); }
15    M operator*(M b) { return M((__int128)v * b.v % MOD); }
16    M operator/(M b) { return *this * (b ^ (MOD - 2)); }
17    friend M operator^(M a, ll b) {
18        M ans(1);
19        for (; b >= 1; a *= a)
20            if (b & 1) ans *= a;
21        return ans;
22    }
23    friend M &operator+=(M &a, M b) { return a = a + b; }
24    friend M &operator-=(M &a, M b) { return a = a - b; }
25    friend M &operator*=(M &a, M b) { return a = a * b; }
26    friend M &operator/=(M &a, M b) { return a = a / b; }
27 };
28 using Mod = M<int>;
29 template <> int Mod::MOD = 1'000'000'007;
30 int &MOD = Mod::MOD;

```

3.1.2. Miller-Rabin

Requires: Mod Struct

```

1 // checks if Mod::MOD is prime
2 bool is_prime() {
3     if (MOD < 2 || MOD % 2 == 0) return MOD == 2;
4     Mod A[] = {2, 7, 61}; // for int values (< 2^31)
5     // ll: 2, 325, 9375, 28178, 450775, 9780504, 1795265022
6     int s = __builtin_ctzll(MOD - 1), i;
7     for (Mod a : A) {
8         Mod x = a ^ (MOD >> s);
9         for (i = 0; i < s && (x + 1).v > 2; i++) x *= x;
10        if (i && x != -1) return 0;
11    }
12    return 1;
13 }

```

3.1.3. Extended GCD

```

1 // returns (p, q, g): p * a + q * b == g == gcd(a, b)
2 // g is not guaranteed to be positive when a < 0 or b < 0
3 tuple<ll, ll, ll> extgcd(ll a, ll b) {
4     ll s = 1, t = 0, u = 0, v = 1;
5     while (b) {
6         ll q = a / b;
7         swap(a -= q * b, b);
8         swap(s -= q * t, t);
9         swap(u -= q * v, v);
10    }
11    return {s, u, a};
12 }

```

3.1.4. Chinese Remainder Theorem

Requires: Extended GCD

```

1 // for 0 <= a < m, 0 <= b < n, returns the smallest x >= 0
2 // such that x % m == a and x % n == b
3 ll crt(ll a, ll m, ll b, ll n) {
4     if (n > m) swap(a, b), swap(m, n);
5     auto [x, y, g] = extgcd(m, n);
6     assert((a - b) % g == 0); // no solution
7     x = ((b - a) / g * x) % (n / g) * m + a;
8     return x < 0 ? x + m / g * n : x;
9 }

```

3.2. Combinatorics

3.2.1. Matroid Intersection

This template assumes 2 weighted matroids of the same type, and that removing an element is much more expensive than checking if one can be added. **Remember to change the implementation details.**

The ground set is $0, 1, \dots, n - 1$, where element i has weight $w[i]$. For the unweighted version, remove weights and change BF/SPFA to BFS.

```

1 constexpr int N = 100;
2 constexpr int INF = 1e9;
3
4 struct Matroid { // represents an independent set
5     Matroid(bitset<N>); // initialize from an independent set
6     bool can_add(int); // if adding will break independence
7     Matroid remove(int); // removing from the set
8 };
9
10 auto matroid_intersection(int n, const vector<int> &w) {
11     bitset<N> S;
12     for (int sz = 1; sz <= n; sz++) {
13         Matroid M1(S), M2(S);
14
15         vector<vector<pii>> e(n + 2);
16         for (int j = 0; j < n; j++)
17             if (!S[j]) {
18                 if (M1.can_add(j)) e[n].emplace_back(j, -w[j]);
19                 if (M2.can_add(j)) e[j].emplace_back(n + 1, 0);
20             }
21         for (int i = 0; i < n; i++)
22             if (S[i]) {
23                 Matroid T1 = M1.remove(i), T2 = M2.remove(i);
24                 for (int j = 0; j < n; j++)
25                     if (!S[j]) {
26                         if (T1.can_add(j)) e[i].emplace_back(j, -w[j]);
27                         if (T2.can_add(j)) e[j].emplace_back(i, w[i]);
28                     }
29             }
30
31         vector<pii> dis(n + 2, {INF, 0});
32         vector<int> prev(n + 2, -1);
33         dis[n] = {0, 0};
34         // change to SPFA for more speed, if necessary
35         bool upd = 1;
36         while (upd) {
37             upd = 0;
38             for (int u = 0; u < n + 2; u++)
39                 for (auto [v, c] : e[u]) {
40                     pii x(dis[u].first + c, dis[u].second + 1);
41                     if (x < dis[v]) dis[v] = x, prev[v] = u, upd = 1;
42                 }
43         }
44
45         if (dis[n + 1].first < INF)
46             for (int x = prev[n + 1]; x != n; x = prev[x])
47                 S.flip(x);
48         else break;
49
50         // S is the max-weighted independent set with size sz
51     }
52     return S;
53 }

```

4. Geometry

59 }
};

4.1. Point

```

1 template <typename T> struct P {
    T x, y;
3 P(T x = 0, T y = 0) : x(x), y(y) {}
    bool operator<(const P &p) const {
5         return tie(x, y) < tie(p.x, p.y);
    }
7    bool operator==(const P &p) const {
        return tie(x, y) == tie(p.x, p.y);
9    }
    P operator-() const { return {-x, -y}; }
11   P operator+(P p) const { return {x + p.x, y + p.y}; }
    P operator-(P p) const { return {x - p.x, y - p.y}; }
13   P operator*(T d) const { return {x * d, y * d}; }
    P operator/(T d) const { return {x / d, y / d}; }
15   T dist2() const { return x * x + y * y; }
    double len() const { return sqrt(dist2()); }
17   P unit() const { return *this / len(); }
    friend T dot(P a, P b) { return a.x * b.x + a.y * b.y; }
19   friend T cross(P a, P b) { return a.x * b.y - a.y * b.x; }
    friend T cross(P a, P b, P o) {
21         return cross(a - o, b - o);
    }
23 };
using pt = P<ll>;

```

4.1.1. Quaternion

```

1 constexpr double PI = 3.141592653589793;
constexpr double EPS = 1e-7;
3 struct Q {
    using T = double;
5    T x, y, z, r;
    Q(T r = 0) : x(0), y(0), z(0), r(r) {}
7    Q(T x, T y, T z, T r = 0) : x(x), y(y), z(z), r(r) {}
    friend bool operator==(const Q &a, const Q &b) {
9        return (a - b).abs2() <= EPS;
    }
11   friend bool operator!=(const Q &a, const Q &b) {
        return !(a == b);
13   }
    Q operator-() { return Q(-x, -y, -z, -r); }
15   Q operator+(const Q &b) const {
        return Q(x + b.x, y + b.y, z + b.z, r + b.r);
17   }
    Q operator-(const Q &b) const {
19         return Q(x - b.x, y - b.y, z - b.z, r - b.r);
    }
21   Q operator*(const T &t) const {
        return Q(x * t, y * t, z * t, r * t);
23   }
    Q operator*(const Q &b) const {
25         return Q(r * b.x + x * b.r + y * b.z - z * b.y,
                r * b.y - x * b.z + y * b.r + z * b.x,
27         r * b.z + x * b.y - y * b.x + z * b.r,
                r * b.r - x * b.x - y * b.y - z * b.z);
29   }
    Q operator/(const Q &b) const { return *this * b.inv(); }
31   T abs2() const { return r * r + x * x + y * y + z * z; }
    T len() const { return sqrt(abs2()); }
33   Q conj() const { return Q(-x, -y, -z, r); }
    Q unit() const { return *this * (1.0 / len()); }
35   Q inv() const { return conj() * (1.0 / abs2()); }
    friend T dot(Q a, Q b) {
37         return a.x * b.x + a.y * b.y + a.z * b.z;
    }
39   friend Q cross(Q a, Q b) {
        return Q(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z,
41         a.x * b.y - a.y * b.x);
    }
43   friend Q rotation_around(Q axis, T angle) {
        return axis.unit() * sin(angle / 2) + cos(angle / 2);
45   }
    Q rotated_around(Q axis, T angle) {
47         Q u = rotation_around(axis, angle);
        return u * *this / u;
49   }
    friend Q rotation_between(Q a, Q b) {
51         a = a.unit(), b = b.unit();
        if (a == -b) {
53             // degenerate case
            Q ortho = abs(a.y) > EPS ? cross(a, Q(1, 0, 0))
55             : cross(a, Q(0, 1, 0));
            return rotation_around(ortho, PI);
57   }
    return (a * (a + b)).conj();

```