Topic 3: Discrete and Continuous Probability Distributions Solutions

Q1

a) $\alpha = 1 - 0.02 - 0.09 - 0.09 - 0.01 - 0.02 - 0.06 - 0.15 - 0.05 - 0.03$
$\quad - 0.02 - 0.06 - 0.11 - 0.04 - 0.05$
$= 0.2$

b) Probability distribution of weight

Weight	100	110	120	130	140
Probability	0.1	0.3	0.4	0.1	0.1

c) Expected Weight $= 100 \times 0.1 + 110 \times 0.3 + 120 \times 0.4 + 130 \times 0.1 + 140 \times 0.1 = 118$ lb

d) Standard deviation $= \sqrt{(100-118)^2 \times 0.1 + ... + (140-118)^2 \times 0.1} = 10.77$lb

e) Take Age= 21 and Weight= 110 lb as example,
P (Age=21 and Weight = 110) =0.06
P (Age=21) = 0.02+0.06+0.11+0.04+0.05 = 0.28
P (Weight=110) = 0.3
P (Weight=110) $\times$ P (Age=21) $= 0.3 \times 0.28 = 0.084 \neq 0.06$
$\therefore$ Age and Weight are not independent.

Q2

a) Mean $= 0 \times 0.07 + 1 \times 0.15 + 2 \times 0.1 + 3 \times 0.05 = 0.5$
Standard Deviation = $\sqrt{0.7(0-0.5)^2 + 0.15(1-0.5)^2 + 0.1(2-0.5)^2 + 0.05(3-0.5)^2}$
$= 0.8660$

b) Expected total number $= n \times p = 100 \times 0.5 = 50$

Q3

a) Distribution A:

$$\mu = E(X) = \sum_{i=1}^{N} X_i P(X_i)$$

$$= (0)(0.5) + (1)(0.2) + (2)(0.15) + (3)(0.1) + (4)(0.05) = 1$$

Distribution B:

$$\mu = E(X) = \sum_{i=1}^{N} X_i P(X_i) = (0)(0.05) + (1)(0.1) + (2)(0.15) + (3)(0.2) + (4)(0.5) = 3$$

b) Distribution A:

$$\sigma^2 = \sum_{i=1}^{N} \left[X_i - E(X)\right]^2 P(X_i)$$

$$= (0-1)^2(0.5) + (1-1)^2(0.2) + (2-1)^2(0.15) + (3-1)^2(0.1) + (4-1)^2(0.05) = 1.5$$

$$\sigma = 1.2247$$

Distribution B:

$$\sigma^2 = \sum_{i=1}^{N} \left[X_i - E(X)\right]^2 P(X_i)$$

$$= (0-3)^2(0.05) + (1-3)^2(0.1) + (2-3)^2(0.15) + (3-3)^2(0.2) + (4-3)^2(0.5) = 1.5$$

$$\sigma = 1.2247$$

c) Distribution A and B has the same spread but locate at different position. The center of Distribution A is on the left-hand-side of that of Distribution B.

Q4

a) Stock X:

expected return = (-50)(0.1)+(20)(0.3)+(100)(0.4)+(150)(0.2)= 71

s.d.of return = $\sqrt{(-50-71)^2(0.1) + ... + (150-71)^2(0.2)}$ = 61.88

Stock Y:

expected return = (-100)(0.1)+…+(200)(0.2) = 97

s.d. of return = $\sqrt{((-100-97)^2(0.1) + ... + (200-97)^2(0.2)}$ = 84.27

b) Stock Y gives investor higher expected return than stock X., but also a higher standard deviation. Thus, a risk-averse investor should invest in stock X, while investor who is willing to take a higher risk can expect a higher return from stock Y.

Q5

a) X = no. of customers that the AIS detects as having exceeded their credit limit

p = success probability = 0.05

n = 20

∵ X is binomial distribution X ~B(n=20, p =0.05)

mean = n p = 20(0.05) = 1

variance = n p (1 - p) = 20(0.05)(0.95) = 0.95

standard deviation = $\sqrt{0.95}$ = 0.9747

b) P(X = 0) = $\dfrac{20!}{0!(20-0)!}(0.05)^0(0.95)^{20}$ = 0.3585

c) P(X = 1) = $\dfrac{20!}{1!(20-1)!}(0.05)^1(0.95)^{19}$ = 0.3774

d) $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - 0.3585 - 0.3774 = 0.2642$

Q6

Let X be the number of customers who will leave the site without completing a transaction.
X~B(20, 0.88)
$P(X=20) = {}_{20}C_{20}(0.88)^{20}(1\text{-}0.88)^{0} = 0.0776$

Q7

a) $P(\text{did well or went shopping}) = \dfrac{90+70+30}{200} = \dfrac{190}{200} = 0.95$

b) Binomial distribution is used since
1. no. of trials is fixed
2. two mutually exclusive outcomes
3. independent trials
4. probability of success is constant

Let Y be the no. of students did well on mid-term test and studied for mid-term test the weekend before the mid-term test out of the selected 10 students
Y ~ B(n = 10, p = 0.45)

$$P(Y=2) = \frac{10!}{2!(10-2)!}0.45^2(1-0.45)^{10-2} = 0.0763$$

Q8

a) P(wine) = 1 – 0.7 = 0.3

b) P(beer and male) = 0.8 *0.6 = 0.48
=> P(wine and male)= 0.6- 0.48 = 0.12
=> P(wine and female) = 0.3-0.12 = 0.18

c) P(male | wine) = P(male and wine) / P(wine) = 0.12/0.3 = 0.4

d) Define X be the number of patrons prefer beer in the 5 selected patrons,

X~B(5, 0.7) Binomial: n=5, p=0.7

P (at least 4 patrons) = P (x=4) + P(x=5)
= $5!/(4!\ 1!)\ 0.7^4 0.3^1 + 5!/(5!\ 0!)\ 0.7^5 0.3^0$
= 0.36015 + 0.16807
= 0.52822

Q9

X is the number of passengers responded to the survey
p is the population proportion of passenger responded to the survey
p = 1 - 0.87 = 0.13
X ~ B(15, 0.13)

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$
$$= 1 - \frac{15!}{0!15!}(0.13)^0(0.87)^{15} - \frac{15!}{1!14!}(0.13)^1(0.87)^{14}$$
$$=1 - 0.1238 - 0.2275$$
$$=0.5987$$

Q10

a) P(x<2) = 0.0102+0.0768 = 0.087

b) E(X) = 0*0.0102+1*0.0768+2*0.2304+3*0.3456+4*0.2592+5*0.0778 = 3.0002
It means, on average, among 5 dentists 3 of them will use "laughing gas".

c) $V(X) = (0-3)^2*0.0102+(1-3)^2*0.0768+(2-3)^2*0.2304++(3-3)^2*0.3456$
$+(4-3)^2*0.2592+(5-3)^2*0.0778$
$= 1.1998$
Thus, $SD(x) = \sqrt{1.1998} = 1.0954$

d) Define "success" = use laughing gas, "failure" = not use laughing gas, then X represents the number of "success" in n independent trials and the probability of success in each trial is p. According to results of b and c, we have np = 3 and np (1-p) = 1.1998. Solve the equations, we have n = 5 and p= 0.6.