Topic 7: Inference for the Proportion Solutions

Q1

a) n=500 , $n\hat{p}$=500(0.27) =135 >5, n(1-$\hat{p}$)=500(1-0.27)=365>5

$\therefore$ Sampling distribution of $\hat{p}$ is approximately normal.

$\hat{p} = 0.27$, $\alpha = 0.05$, $Z_{0.05/2} = 1.96$

95% confidence interval for p:

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.27 \pm 1.96\sqrt{\frac{0.27(1-0.27)}{500}} = [0.2311, 0.3089]$$

We are 95% confident that the population proportion of small business owners who never check in with the office when on vacation is estimated to be between 0.2311 and 0.3089.

b) n=1000 , $n\hat{p}$=1000(0.27) =270 >5, n(1-$\hat{p}$)=1000(1-0.27)=730 >5

$\therefore$ sampling distribution of $\hat{p}$ is approximately normal.

$\hat{p} = 0.27$, $\alpha = 0.05$, $Z_{0.05/2} = 1.96$

95% confidence interval for p:

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.27 \pm 1.96\sqrt{\frac{0.27(1-0.27)}{1000}} = [0.2425, 0.2975]$$

We are 95% confident that the population proportion of small business owners who never check in with the office when on vacation is estimated to be between 0.2425 and 0.2975.

c) The larger the sample size, the narrower is the confidence interval holding everything constant.

Q2

a) n=658 , $n\hat{p}$=658(0.3799) =250 >5, n(1-$\hat{p}$)= 658(1-0. 3799)=408 >5

$\therefore$ sampling distribution of $\hat{p}$ is approximately normal.

$\hat{p} = \frac{X}{n} = \frac{250}{658} = 0.3799$, $\alpha = 0.05$, $Z_{0.05/2} = 1.96$

95% confidence interval for p:

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.3799 \pm 1.96\sqrt{\frac{0.3799(1-0.3799)}{658}} = [0.3429, 0.4170]$$

We are 95% confident that the population proportion of CEOs whose greatest concern was sustained and steady top-line growth is estimated to be between 0.3429 and 0.4170.

b) Replace p by $\hat{p}$

$$n = \frac{Z_{\frac{0.05}{2}}^2\hat{p}(1-\hat{p})}{E^2} = \frac{1.96^2(0.3799)(1-0.3799)}{0.01^2} = 9049.92 \approx 9050 \text{ (round up)}$$

Q3

$$n = \frac{Z_{\frac{\alpha}{2}}^{2}\hat{p}(1-\hat{p})}{E^2} = \frac{(1.96)^2(\frac{288}{400})(\frac{112}{400})}{(0.03)^2} = 860.5 \cong 861$$

If information is not available, let $\hat{p}=0.5$

$$n' = \frac{(1.96)^2(0.5)(0.5)}{(0.03)^2} = 1067.1 \cong 1068$$

Q4

a) $H_0: p \geq 0.95$

$H_1: p < 0.95$

$$Z = \frac{0.92-0.95}{\sqrt{\frac{(0.95)(0.05)}{150}}} = -1.69$$

$\therefore$ p-value=P(Z $\leq$ -1.69)=0.0455

Since p-value=0.0455 < 0.1= α

Ho is rejected, and we conclude that there is not sufficient evidence to support that manager's claim. i.e. Less than 95% of Pizza Delight's orders are delivered within 15 minutes of the time when order is placed at the 0.1 level of significance.

b) The decision Rule is given as follows:

Ho is rejected if the given significance level $\alpha > 0.0455$;

Ho is not rejected if the given significance level $\alpha \leq 0.0455$

Q5

a) Let p be the population proportion of customers who select products and then cancel their transaction

H_0: $p \geq 0.5$

H_1: $p < 0.5$

$\because$ n =500 $n\hat{p}$=210>5, n(1-$\hat{p}$)=290>5

$\therefore$ Sampling distribution of $\hat{p}$ is approximately normal

Reject H_0 if $Z < -2.33$

$$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{210/500-0.5}{\sqrt{\frac{0.5(1-0.5)}{500}}} = -3.577$$

Since Z =-3.577 < -2.33

Reject H_0 at $\alpha = 0.01$

There is sufficient evidence to conclude that the proportion of customers who selected products and then cancelled their transaction is less than 0.50 with the new system.

b) Let p be the population proportion of customers who select products and then cancel their transaction

H_0: $p \geq 0.5$
H_1: $p < 0.5$

$\because$ n =100 $\quad n\hat{p}$=42>5, n(1-$\hat{p}$)=58>5
$\therefore$ Sampling distribution of $\hat{p}$ is approximately normal
$\therefore$ Z-test (Lower Tail)
With $\alpha = 0.01$, $-Z_{0.01}$ = -2.33
Reject H_0 if $Z < -2.33$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0\,(1-p_0)}{n}}} = \frac{42/100 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{100}}} = -1.6$$

Since Z =-1.6 > -2.33
Do not Reject H_0 at $\alpha = 0.01$
There is insufficient evidence to conclude that the proportion of customers who selected products and then cancelled their transaction is less than 0.50 with the new system.

c) The larger the sample size, the smaller is the standard error. Even though the sample proportion is the same value at 0.42 in (a) and (b), the test statistic is more negative while the p-value is smaller in (a) compared to (b) because of the larger sample size in (a).