#### **Revision Paper**

#### Question 1 (25 marks)

A restaurant offers pizza delivery service to a campus dormitory. Delivery times follow a normal distribution with mean 20 minutes and standard deviation 5 minutes.

a. For a single delivery, state in which of the following ranges (expressed in minutes) delivery time is most likely to lie. Briefly explain your choice without doing any calculation.

18-20 19-21 20-22 21-23 (3 marks)

- b. The restaurant does not charge for the pizza if delivery takes more than 30 minutes. What is the probability of getting a free pizza from a single order? (3 marks)
- During examination period, a student plans to order pizza five consecutive evenings. Assume that
  these delivery times are independent of each other. What is the probability that the student will
  get at least one free pizza? (4 marks)
- d. Find the range of times symmetrically distributed around the population mean that includes 60% of all deliveries from this service. (5 marks)
- e. The restaurant selected a random sample of 26 deliveries, what is the probability that the sample mean delivery time is between 18 and 23 minutes? (5 marks)
- f. Suppose the restaurant plans to expand its delivery network, and offers service to commercial areas as well. A random sample of delivery times is required to estimate the new population mean of delivery time. How large a sample is needed to ensure the probability that the sample mean is larger than the new population mean by more than 2 minutes is less than 0.05? Assumed the population standard deviation is 10 minutes. (5 marks)

#### Question 2 (25 marks)

Many food products contain small quantities of substances that would give an undesirable taste or smell if they were present in large amounts. An example is the "off-odors" caused by sulfur compounds in wine. Oenologists (wine experts) have determined the odor threshold, the lowest concentration of a compound that the human nose can detect. For example, in the oenology literature, the odor threshold for dimethyl sulfide (DMS) is given as 25 micrograms per liter of wine ( $\mu$ g/l). Here are the DMS odor thresholds for 10 randomly selected oenologists:

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Assume that the standard deviation of the odor threshold is known to be 7µg/l.

a. Sketch a box plot and comment on the shape of this sample data set.

(7 marks)

- b. In order to say the sampling distribution of sample means is Normal, what assumption is required? Why? Should Z or t distribution be used for conducting inferential analysis? (3 marks)
- c. Based on your answer in part (b), give a 95% confidence interval for the mean DMS odor threshold among all oenologists. (5 marks)
- d. Are you convinced that the mean odor threshold for oenologists is higher than the published threshold,  $25\mu g/l$ ? At 5% level of significance, carry out a hypothesis test using critical value approach to justify your answer. (8 marks)
- e. Compute the p-value for part (d).

(2 marks)

### Question 3 (25 marks)

A health club randomly selected 500 of its members and revealed that 34% of them are overweighted. 76% of the selected members were male. 35% of the selected males were overweighted.

- a. What is the probability that a randomly selected member is female and overweighted? (3 marks)
- b. Suppose a randomly selected member is overweighted, what is the chance that the member is male? (3 marks)
- c. Are "overweighted" and "gender" independent? Why or why not? (4 marks)
- d. Construct an 85% confidence interval estimate for the population proportion of overweighted members. (7 marks)
- e. Using the confidence interval constructed in part (d), what conclusion will be drawn for a hypothesis test,  $H_0$ :  $\pi=0.3$  against  $H_1$ :  $\pi\neq0.3$ , at 15% level of significance? Explain. (4 marks)
- f. If you want to be 90% confidence of estimating the proportion of overweighted members in part (d) to within  $\pm 2\%$ , what sample size is needed? (4 marks)

#### Question 4 (25 marks)

The City Office Equipment Corporation sells an imported copier on a franchised basis and performs maintenance service on this copier. The service manager randomly selected 45 recent requests on performing maintenance service. The information being collected include the total time in minutes (Y) spent by the service person, and the number of copiers serviced (X). The service manager carried out a simple linear regression analysis on the data, and part of the Excel output is given as below:

Regression Statistics	
R Square	0.9575

	Coefficients	Standard Error	t Stat	P-value
Intercept	-0.5802	2.8039	???	???
Χ	15.0352	0.4831	???	???

- a. State the estimated simple linear regression equation. Interpret the meaning of the estimated slope coefficient. (4 marks)
- b. Determine the correlation coefficient. Describe the relationship between Y and X. (4 marks)
- c. Why correlation coefficient is preferred over covariance? (3 marks)
- d. Interpret the meaning of R-Square. (2 marks)
- e. At 5% level of significance, is there any positive relationship between Y and X? (8 marks)
- f. Predict the total time needed for maintaining
  - (i) 1 copier, and
  - (ii) 5 copiers. (2 marks)
- g. Given that the observed X values ranged between 2 to 8, of the two predictions obtained for (i) and (ii) in part (f), which is more justifiable? Explain why. (2 marks)

## **Formulae Sheet**

1. 
$$\mu = \frac{1}{N} \sum X_i \quad ; \qquad \qquad \sigma^2 = \frac{1}{N} \sum (X_i - \mu)^2$$
$$\overline{X} = \frac{1}{n} \sum X_i \quad ; \qquad \qquad s^2 = \frac{1}{n-1} \sum (X_i - \overline{X})^2$$

2. 
$$\mu = E(X) = \sum X_i P(X_i)$$
;  $\sigma^2 = Var(X) = \sum (X_i - \mu)^2 P(X_i)$ 

3. If 
$$X \sim Bin(n, p)$$
, then (a)  $P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$ 

(b) 
$$\mu = E(X) = np$$
;  $Var(X) = np(1-p)$ 

4. 
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
;

$$P(A \text{ and } B) = P(B \mid A)P(A)$$
;  $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$ 

5. If 
$$X \sim N(\mu, \sigma^2)$$
, then  $Z = \frac{X - \mu}{\sigma} \sim N(0, 1^2)$ 

6. If 
$$\overline{X} \sim N(\mu, (\frac{\sigma}{\sqrt{n}})^2)$$
, then  $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1^2)$ 

7. 
$$\overline{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$
;  $\overline{X} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}}$ ;  $e = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ 

8. 
$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$
;  $t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$ 

9. If 
$$p \sim N(\pi, \sqrt{\frac{\pi(1-\pi)}{n}}^2)$$
, then  $Z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \sim N(0,1^2)$ ;

$$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$$
 ;  $e = Z_{\frac{\alpha}{2}} \sqrt{\frac{\pi(1-\pi)}{n}}$ 

10. 
$$s_{xy} = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{n-1}$$
;  $r_{xy} = \frac{S_{XY}}{S_X S_Y}$ ;

For 
$$Y = \beta_0 + \beta_1 X + \varepsilon$$
,  $b_1 = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2}$ ;  $b_0 = \overline{Y} - b_1 \overline{X}$ 

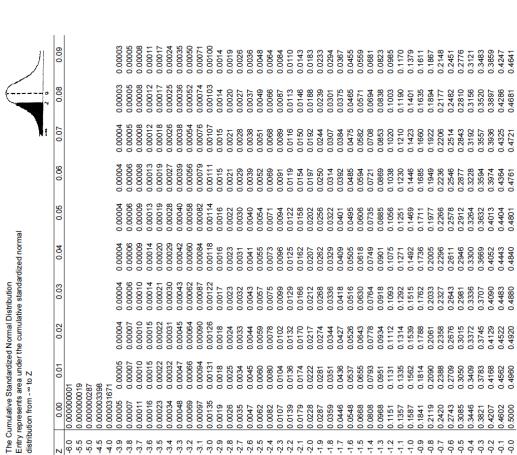
## The Cumulative Standardized Normal Distribution



Entry represents area under the cumulative standardized normal distribution from - $^{\circ}$  to Z The Cumulative Standardized Normal Distribution (Continued)

0.000         0.011         0.02         0.03         0.04         0.05         0.06         0.07           0.5000         0.5040         0.5580         0.5160         0.5169         0.5239         0.5279           0.5000         0.5040         0.5580         0.5410         0.5510         0.5519         0.5529           0.5732         0.5832         0.5478         0.5517         0.6598         0.6406         0.6072           0.6732         0.6253         0.6233         0.6538         0.6406         0.6072         0.6698           0.6544         0.6564         0.6700         0.6738         0.6732         0.6698         0.6440         0.6698           0.6547         0.6256         0.6628         0.6628         0.6608         0.6772         0.7577         0.7784         0.7									ţ	,	N:
0.5000         0.5040         0.5080         0.5120         0.5160         0.5296         0.5239         0.5275           0.5398         0.5438         0.5478         0.5257         0.5296         0.5636         0.5636         0.5675           0.6179         0.6217         0.6256         0.6256         0.6256         0.6639         0.6431         0.6677         0.6004           0.6179         0.6217         0.6256         0.6624         0.6710         0.7734         0.7734         0.7734         0.7457         0.7643           0.6241         0.6250         0.6624         0.6700         0.7734         0.7452         0.7452         0.6608           0.7251         0.7261         0.7734         0.7734         0.7734         0.7742         0.7454         0.7754           0.7261         0.7761         0.7767         0.7734         0.7734         0.7742         0.7742         0.7754         0.7742         0.7754	Z	0.00	0.01	0.02	0.03	0.04	0.05	90.0	0.07	0.08	0.09
0.5388         0.5478         0.5517         0.5557         0.5689         0.5675         0.5697         0.5698         0.5698         0.5678         0.5678         0.5678         0.5689         0.5678         0.5698         0.5689         0.5689         0.6689         0.6684         0.6679         0.6599         0.6598         0.6598         0.6684         0.6670         0.6402         0.6604         0.6684         0.6709         0.7689         0.6789         0.6789         0.6789         0.6408         0.6686         0.7019         0.7024<	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.5793         0.5871         0.5910         0.5948         0.5987         0.6024         0.6044         0.6024         0.7034<	0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.6179         0.6217         0.6255         0.6239         0.6331         0.6368         0.6406         0.6443           0.6554         0.65541         0.6228         0.6664         0.7004         0.7034         0.7123         0.7157           0.6554         0.6561         0.6560         0.7044         0.7734         0.7123         0.7157           0.7527         0.7261         0.7684         0.7744         0.7744         0.7745         0.7466           0.7580         0.7612         0.7642         0.7754         0.7744         0.7744         0.7746           0.7581         0.7816         0.7816         0.7744         0.7744         0.7744         0.7746           0.8159         0.7816         0.7826         0.8264         0.8707         0.7424         0.7446         0.7744           0.8164         0.7810         0.7826         0.8264         0.8704         0.7824         0.7846         0.8717           0.9025         0.8643         0.8665         0.8686         0.8773         0.9744         0.8944         0.8954         0.8727           0.9025         0.9644         0.9647         0.9486         0.9474         0.9486         0.9474         0.9486      <	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.6554         0.6591         0.6628         0.6664         0.6704         0.6773         0.6772         0.6904           0.6915         0.6950         0.7204         0.7389         0.7422         0.7424         0.7424         0.7424         0.7424         0.7424         0.7424         0.7444         0.7424         0.7444         0.8747         0.8444	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
(6915         (6650         (6686         (77014         (77054         (77084         (77123         (77124 <td>4.0</td> <td>0.6554</td> <td>0.6591</td> <td>0.6628</td> <td>0.6664</td> <td>0.6700</td> <td>0.6736</td> <td>0.6772</td> <td>0.6808</td> <td>0.6844</td> <td>0.6879</td>	4.0	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.7257         0.7291         0.7324         0.7324         0.7324         0.7324         0.7324         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7486         0.7784         0.8787         0.8873         0.8784         0.8787         0.8874         0.8787         0.8874         0.8784         0.8787         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987         0.8987<	0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.7580         0.77612         0.77642         0.77673         0.77794         0.77764         0.77794         0.8770         0.9770 <t< td=""><td>9.0</td><td>0.7257</td><td>0.7291</td><td>0.7324</td><td>0.7357</td><td>0.7389</td><td>0.7422</td><td>0.7454</td><td>0.7486</td><td>0.7518</td><td>0.7549</td></t<>	9.0	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7881         0.7910         0.7939         0.7967         0.7995         0.8023         0.8024         0.8030         0.8041         0.8048         0.8186         0.8212         0.8238         0.8264         0.8299         0.81416         0.8148         0.8148         0.8441         0.8543         0.8440         0.8170         0.8770           0.8443         0.8464         0.8465         0.8668         0.8704         0.8774         0.8770         0.8771         0.8770         0.8771         0.8771         0.8770         0.8771         0.8771         0.8771         0.8771         0.8771         0.8771         0.8771         0.8772         0.8772         0.9784         0.8772         0.8772         0.9784         0.9786         0.9787         0.9782         0.9784         0.9782         0.9874         0.9787         0.9782         0.9784         0.9786         0.9984         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986         0.9967         0.9986	0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8155         0.8186         0.8212         0.8238         0.8264         0.8289         0.8315         0.8440           0.8643         0.8648         0.8729         0.8749         0.8749         0.8749         0.8749         0.8749         0.8770         0.8749         0.8770         0.8749         0.8770         0.8749         0.8770         0.8749         0.8770         0.8749         0.8770         0.8770         0.8749         0.8770         0.8770         0.8770         0.8770         0.8770         0.8770         0.8770         0.8770         0.8770         0.8770         0.8771         0.9760         0.9062         0.9069         0.9164         0.8960         0.9171         0.9171         0.9171         0.9171         0.9171         0.9171         0.9171         0.9171         0.9171         0.9171         0.9174         0.9466         0.9060	0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.8413         0.8428         0.8461         0.8485         0.8508         0.8572         0.8531         0.8554         0.8577           0.8843         0.88665         0.88686         0.8708         0.8729         0.8749         0.8770         0.8790           0.8853         0.88686         0.88078         0.8904         0.9086         0.9082         0.9145         0.9131         0.9147           0.9132         0.9246         0.9082         0.9322         0.9326         0.9496         0.9066         0.9496	6.0	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
0.8643         0.8665         0.8886         0.8708         0.8729         0.8749         0.8770         0.8792           0.8849         0.8869         0.8888         0.8070         0.8925         0.8944         0.8970         0.8926           0.9032         0.9046         0.9062         0.9052         0.9052         0.9141         0.9147           0.9132         0.9445         0.9062         0.9256         0.9266         0.9496         0.9406         0.9418           0.9532         0.9463         0.9444         0.9496         0.9666         0.9496         0.9506         0.9418           0.9542         0.9563         0.9573         0.9592         0.9506         0.9418         0.9418           0.9544         0.9664         0.9664         0.9674         0.9606         0.9418	1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
0.8849         0.8869         0.8888         0.8907         0.8925         0.8944         0.8962         0.8888         0.8907         0.8925         0.8944         0.8962         0.8988         0.8907         0.9936         0.99415         0.9949         0.99066         0.9908         0.9929         0.9718         0.9718         0.9719         0.9729         0.9739         0.9739         0.9739         0.9739         0.9747         0.9452         0.9463         0.9474         0.9484         0.9495         0.9506         0.9418         0.9418         0.9484         0.9495         0.9506         0.9418         0.9472         0.9472         0.9573         0.9569         0.9668         0.9668         0.9676         0.9418         0.9472         0.9773         0.9573         0.9578         0.9773         0.9773         0.9773         0.9773         0.9773         0.9773         0.9774         0.9774         0.9750         0.9774         0.977	1.	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
0.9032         0.9049         0.9066         0.9082         0.9049         0.9115         0.9131         0.9147           0.9132         0.92077         0.9222         0.9236         0.9246         0.9406         0.9418           0.9132         0.92077         0.9322         0.9326         0.9566         0.9406         0.9418           0.9454         0.9446         0.9486         0.9464         0.9487         0.9490         0.9418           0.9554         0.9644         0.9482         0.9568         0.9668         0	1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
0.9192         0.9207         0.9222         0.9236         0.9264         0.9266         0.9278           0.9332         0.9346         0.9347         0.9482         0.9466         0.9418         0.9406         0.9418           0.9454         0.9484         0.9464         0.9604         0.9616         0.9418           0.9544         0.9644         0.9673         0.9682         0.9690         0.9606         0.9616           0.9713         0.9718         0.9728         0.9793         0.9798         0.9606         0.9616           0.9713         0.9718         0.9726         0.9793         0.9798         0.9603         0.9616           0.9821         0.9786         0.9793         0.9798         0.9607         0.9608         0.9617           0.9821         0.9826         0.9868         0.9871         0.9878         0.9867         0.9867         0.9867         0.9867         0.9867         0.9867         0.9867         0.9966         0.9960         0.9911         0.9967         0.9966         0.9960         0.9911         0.9967         0.9966         0.9960         0.9961         0.9960         0.9961         0.9966         0.9960         0.9961         0.9966         0.9966	1.3	0.9032	0.9049	9906.0	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
0.9332         0.9445         0.9557         0.9370         0.9382         0.9345         0.9457         0.9484         0.9496         0.9418         0.9496         0.9418         0.9456         0.9606         0.9515         0.9516         0.9516         0.9516         0.9517         0.9596         0.9517         0.9596         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9617         0.9698         0.9608         0.9618<	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
0.9452         0.9443         0.9474         0.9484         0.9495         0.9604         0.9625         0.9645         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9654         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9666         0.9672         0.9732         0.9733         0.9732         0.9732         0.9742         0.9746         0.9677         0.9676         0.9676         0.9676         0.9676         0.9676         0.9676         0.9676         0.9677         0.9676         0.9676         0.9676         0.9676         0.9676         0.9676         0.9676<	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
0.9554         0.9664         0.9573         0.9582         0.9594         0.9504         0.9664         0.9673         0.9589         0.9608         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9616         0.9617         0.9718         0.9728         0.9738         0.9738         0.9738         0.9738         0.9738         0.9738         0.9748         0.9668         0.9608         0.9608         0.9608         0.9608         0.9608         0.9608         0.9608         0.9748         0.9748         0.9608         0.9608         0.9608         0.9608         0.9608         0.9608         0.9748         0.9608         0.9608         0.9608         0.9608         0.9608         0.9608         0.9608         0.9748         0.9748         0.9608         0.9608         0.9608         0.9608         0.9608         0.9608         0.9808         0.9808         0.9808         0.9808         0.9808         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908         0.9908<	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
0.9641         0.9649         0.9656         0.9664         0.9671         0.9678         0.9686         0.9693           0.9713         0.9719         0.9778         0.9738         0.9784         0.9780         0.9786           0.9871         0.9778         0.9788         0.9798         0.9803         0.9808           0.9821         0.9868         0.9871         0.9838         0.9847         0.9808         0.9871           0.9861         0.9808         0.9971         0.9906         0.9901         0.9906         0.9911           0.9863         0.9806         0.9907         0.9904         0.9906         0.9911         0.9806         0.9906         0.9911           0.9938         0.9906         0.9917         0.9904         0.9906         0.9911         0.9906         0.9911           0.9938         0.9906         0.9917         0.9966         0.9967         0.9960         0.9911         0.9960           0.9946         0.9966         0.9967         0.9960         0.9917         0.9960         0.9917         0.9960           0.9941         0.9982         0.9968         0.9960         0.9917         0.9968         0.9960         0.9917         0.9968 <t< td=""><td>1.7</td><td>0.9554</td><td>0.9564</td><td>0.9573</td><td>0.9582</td><td>0.9591</td><td>0.9599</td><td>0.9608</td><td>0.9616</td><td>0.9625</td><td>0.9633</td></t<>	1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
0.9713         0.9719         0.9726         0.9732         0.9738         0.9744         0.9750         0.9756           0.9772         0.9778         0.9788         0.9733         0.9796         0.9978         0.9978         0.9978         0.9978         0.9978         0.9978         0.9980         0.9908         0.9908         0.9908         0.9909         0.9978         0.9987         0.9988         0.9997         0.9987         0.9988         0.9996         0.9987         0.9988         0.9996         0.9997         0.9988         0.9996         0.9988         0.9996         0.9988         0.9996         0.9988         0.9996         0.9988         0.9996         0.9988         0.9996         0.9988         0.9996         0.9998	1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
0.9772         0.9778         0.9783         0.9788         0.9730         0.9788         0.9980         0.9803         0.9898           0.9821         0.9826         0.9834         0.8838         0.8842         0.9864         0.9866           0.9861         0.9826         0.9834         0.8842         0.9846         0.9864         0.9868           0.9861         0.9864         0.9868         0.9901         0.9908         0.9911         0.9908         0.9911           0.9918         0.9920         0.9941         0.9942         0.9945         0.9948         0.9	1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
0.9821         0.9826         0.9830         0.9834         0.9838         0.9840         0.9850           0.9861         0.9864         0.9868         0.9871         0.8975         0.9908         0.9908         0.9908         0.9911           0.9838         0.9904         0.9904         0.9904         0.9904         0.9901         0.9911           0.9938         0.9940         0.9941         0.9942         0.9946         0.9948         0.9941           0.9958         0.9940         0.9941         0.9947         0.9946         0.9948         0.9949           0.9958         0.9966         0.9967         0.9969         0.9940         0.9941         0.9949           0.9967         0.9966         0.9967         0.9960         0.9971         0.9972         0.9972           0.9967         0.9967         0.9967         0.9974         0.9978         0.9978         0.9978           0.9968         0.9968         0.9987         0.9988         0.9988         0.9988         0.9988           0.9969         0.9996         0.9998         0.9998         0.9998         0.9998         0.9998           0.9996         0.9996         0.9998         0.9998         0.9998 <td>2.0</td> <td>0.9772</td> <td>0.9778</td> <td>0.9783</td> <td>0.9788</td> <td>0.9793</td> <td>0.9798</td> <td>0.9803</td> <td>0.9808</td> <td>0.9812</td> <td>0.9817</td>	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
0.9861         0.9864         0.9868         0.9871         0.9875         0.9878         0.9881         0.9884           0.8833         0.9866         0.9868         0.9901         0.9906         0.9906         0.9901         0.9906         0.9901         0.9902         0.9902         0.9901         0.9902	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
0.9893         0.9896         0.9898         0.9901         0.9906         0.9909         0.9911           0.9918         0.9922         0.9925         0.9927         0.9927         0.9927         0.9927         0.9927         0.9927         0.9927         0.9927         0.9927         0.9927         0.9927         0.9929         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9949         0.9947	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
0.99418         0.99220         0.99322         0.99425         0.99427         0.99430         0.99430         0.99441         0.99446         0.99468         0.99448 <t< td=""><td>2.3</td><td>0.9893</td><td>0.9896</td><td>0.9898</td><td>0.9901</td><td>0.9904</td><td>0.9906</td><td>0.9909</td><td>0.9911</td><td>0.9913</td><td>0.9916</td></t<>	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
0.9938         0.9940         0.99441         0.9943         0.9946         0.9949         0.9972         0.9974         0.9972         0.9972         0.9972         0.9972         0.9972         0.9972         0.9972         0.9974         0.9972         0.9972         0.9972         0.9972         0.9972         0.9972         0.9972         0.9972	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
0.9955         0.9956         0.9956         0.9956         0.9966         0.9967         0.9969         0.9960         0.9967         0.9969         0.9969         0.9969         0.9969         0.9972         0.9969         0.9977         0.9978         0.9979         0.9977         0.9986         0.9986         0.9986         0.9986         0.9986         0.9986         0.9986         0.9987         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988<	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
0.9965         0.9966         0.9967         0.9968         0.9970         0.9971         0.9971         0.9971         0.9971         0.9971         0.9972         0.9972         0.9973         0.9978         0.9979         0.9987         0.9987         0.9987         0.9987         0.9987         0.9987         0.9987         0.9987         0.9987         0.9987         0.9994<	5.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
0.9974         0.9975         0.9976         0.9977         0.9977         0.9979         0.9979         0.9979           0.9981         0.9982         0.9983         0.9984         0.9984         0.9985         0.9985           0.99861         0.99862         0.9983         0.9984         0.99885         0.99885         0.99885           0.99862         0.99864         0.99866         0.99846         0.99886         0.99889         0.99883           0.99831         0.99846         0.99846         0.99846         0.99847         0.99842         0.99844         0.99844         0.99944         0.99946           0.99852         0.99946         0.99946         0.99947         0.99947         0.99947         0.99947         0.99947         0.99946         0.99946         0.99946         0.99946         0.99946         0.99947	2.7	0.9965	9966.0	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
0.9981         0.9982         0.9983         0.9984         0.9984         0.9985         0.9986         0.9986         0.9986         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9988         0.9989         0.9989         0.9989         0.9989         0.9989         0.9989         0.9984         0.9998         0.9998         0.9998         0.9998         0.9998         0.9998         0.9998         0.9998         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999         0.9999<	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
0.99685         0.998669         0.998874         0.998878         0.998886         0.998899         0.998899         0.998899         0.998899         0.998899         0.998899         0.998899         0.998899         0.998899         0.998899         0.998899         0.9989924         0.998924         0.999924         0.999924         0.999924         0.999940         0.999996         0.999940         0.999940         0.999996	5.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
0.99903         0.999046         0.999913         0.999918         0.999921         0.999921         0.999924         0.999924         0.999924         0.999924         0.999924         0.999924         0.999924         0.999924         0.999944         0.999944         0.999944         0.999944         0.999944         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999946         0.999947         0.99994         0.999947         <	3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99897	0.99900
0.99931         0.99934         0.99936         0.99936         0.99940         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99944         0.99946         0.99946         0.99946         0.99966         0.99966         0.99966         0.99967         0.99966         0.99972         0.99973         0.99973         0.99974         0.99973         0.99974         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99987         0.99988         0.99987         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99988         0.99998         0.99998         0.99998         0.99998         0.99998         0.99998         0.99998         0.99998         0.99998         0.99999         0.99998         0.99998         0.999998         0.99999         0.99998         0.99999         <	3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
0.99952 0.99953 0.99955 0.99955 0.99956 0.99966 0.99966 1 0.99962 0.99966 0.99966 0.99966 0.99966 0.99968 0.99968 0.99968 0.99968 0.99968 0.99968 0.99974 0.99974 0.99974 0.99974 0.99974 0.9998 0.99986 0.99986 0.99986 0.99986 0.99986 0.99986 0.99987 0.99987 0.99987 0.99987 0.99988 0.99988 0.99988 0.99988 0.99998 0.99998 0.99998 0.99998 0.99998 0.99998 0.99998 0.99998 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.999998 0.99999 0.99999 0.999998 0.99999 0.999998 0.99999 0.999998 0.99999 0.99999 0.999998 0.99999 0.999998 0.99999 0.999998 0.999998 0.99999 0.999998 0.	3.5	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
0.99966 0.99968 0.99968 0.99977 0.99972 0.99972 0.99973 0.99974 0.99987 0.99974 0.99987 0.99987 0.99987 0.99987 0.99987 0.99987 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99988 0.99998 0.99998 0.99998 0.99998 0.99998 0.99998 0.99998 0.99998 0.999999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.99999 0.999	3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
0.999977 0.99978 0.99978 0.99979 0.99980 0.99981 0.99982 0.99982 0.99982 0.99982 0.99982 0.99982 0.99982 0.99982 0.99982 0.99982 0.99998 0.99998 0.99998 0.99998 0.99998 0.99999 0.999	3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
0.99984 0.99985 0.99986 0.99986 0.99986 0.99987 0.99988 0.99987 0.99988 0.99988 0.99988 0.99989 0.99998 0.99999 0.9999 0.9999 0.9999 0.9999 0.99999 0.	3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
0.99989 0.99990 0.99990 0.99990 0.99991 0.99992 0.99992 0.99993 0.99992 0.99993 0.99993 0.99999 0.9999	3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
0.99993 0.99993 0.99993 0.99994 0.99994 0.99994 0.99995 0.99995 0.99995 0.99995 0.99995 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.999966 0.99996729 0.99996602	3.7	0.99989	0.6666.0	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
0.99995 0.99995 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.99996 0.999966329 0.99996602	3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
	3.9	0.99995	0.99995	96666.0	96666.0	0.99996	0.99996	0.99996	96666.0	0.99997	0.99997
	4.0	0.999968	329								
	4.5	0.999996	602								
	2.0	0.999999713	713								

0.999999981



# Critical Values of t

			Inner-Tail Areas	90		
Degrees of			Opper-Lall Are	2		
Freedom	0.25	0.10	0.05	0.025	0.01	0.005
49	0.6795	1.2991	1.6766	2.0096	2.4049	2.6800
20	0.6794	1.2987	1.6759	2.0086	2.4033	2.6778
51	0.6793	1.2984	1.6753	2.0076	2.4017	2.6757
52	0.6792	1.2980	1.6747	2.0066	2.4002	2.6737
53	0.6791	1.2977	1.6741	2.0057	2.3988	2.6718
54	0.6791	1.2974	1.6736	2.0049	2.3974	2.6700
55	0.6790	1.2971	1.6730	2.0040	2.3961	2.6682
20	0.6789	1.2969	1.6725	2.0032	2.3948	2.6665
57	0.6788	1.2966	1.6720	2.0025	2.3936	2.6649
28	0.6787	1.2963	1.6716	2.0017	2.3924	2.6633
59	0.6787	1.2961	1.6711	2.0010	2.3912	2.6618
00	0.0 / 80	1.2938	00 / 0.1	2.0003	2.3901	2.0003
61	0.6785	1.2956	1.6702	1.9996	2.3890	2.6589
29	0.6785	1.2954	1.6698	1.9990		2.65/5
63	0.6784	1.2951	1.6694	1.9983	2.3870	2.6561
64	0.6783	1.2949	1.6690	1.9977	2.3860	2.6549
65	0.6783	1.2947	1.6686	1.9971	2.3851	2.6536
99	0.6782	1.2945	1.6683	1.9966	2.3842	2.6524
29	0.6782	1.2943	1.6679	1.9960		2.6512
88	0.6781	1.2941	1.6676	1.9955	2.3824	2.6501
9 6	0.6781	1.2939	1.6673	1.9949		2.6490
0,2	0.6780	1.2938	1.6669	1.9944	2.3808	2.6479
71	0.6780	1.2936	1.6666	1.9939	2.3800	2.6469
72	0.6779	1.2934	1.6663	1.9935	2.3793	2.6459
13	0.6779	1.2933	1.6660	1.9930	2.3785	2.6449
4 4	0.01/0	1.2931	1.0037	1.9923	2.3770	2.6439
76	0.0778	1.2929	1.0054	1.9921	2.377.1	2.6430
9 1	0.6777	1.2928	1,6632	1.9917	2.3764	2.0421
7.0	0.6776	1 2025	1 66.46	1 9008	2 3754	2.04.12
0 / 2	0.6776	1 2024	1.6644	1 9905	2.37.01	2.6403
2 0	0.6776	1 2922	1 6641	1 9901	2 3739	26387
8.0	0.6775	1 2921	1 6639	1 9897	2.3733	2.6379
82	0.6775	1 2920	1.6636	1.9893	2.3727	2.6371
83	0.6775	1.2918	1.6634	1.9890	2.3721	2.6364
84	0.6774	1.2917	1.6632	1.9886	2.3716	2.6356
85	0.6774	1.2916	1.6630	1.9883	2.3710	2.6349
98	0.6774	1.2915	1.6628	1.9879		2.6342
87	0.6773	1.2914	1.6626	1.9876	2.3700	2.6335
88	0.6773	1.2912	1.6624	1.9873		2.6329
S 0	0.6773	1.2911	1.6622	1.9870	2.3690	2.6322
06	0.6772	1.2910	1.6620	1.9867	2.3685	2.6316
91	0.6772	1.2909	1.6618	1.9864	2.3680	2.6309
92	0.6772	1.2908	1.6616	1.9861	2.3676	2.6303
93	0.6771	1.2907	1.6614	1.9858		2.6297
4,0	0.6771	1.2906	21 00.1	1.8000	2.3667	2.0291
90	0.6771	1 2904	1.66.1	1 9850	2.3062	2.6280
92	0.6770	1.2903	1.6607	1.9847	2.3654	2.6275
86	0.6770	1.2902	1.6606	1.9845	2.3650	2.6269
66	0.6770	1.2902	1.6604	1.9842	2.3646	2.6264
100	0.6770	1.2901	1.6602	1.9840	2.3642	2.6259
110	0.6767	1.2893	1.6588	1.9818	2.3607	2.6213
120	0.6765	1.2886	1.6577	1.9799	2.3578	2.6174
8	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758

			Upper-Tail Areas	as		
Degrees of Freedom	0.25	0.10	0.05	0.025	0.01	0 005
	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
	0.7267	1.4759	2.0150	2.5706	3.3649	2 7074
	0.7176	1 4149	1.8946	2.3646	3.1427 2.9980	3.7074
	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
0	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
_	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
2	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
8	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
4	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
5	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
1 0	0.6901	1.3368	1.7306	2.1199	2.5835	2.9208
- α	0.0092	1 3304	1 7341	2 1000	2 5524	2 8784
0 0	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
_	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
25	0.6844	1.3163	1 7081	2.0595	2 4851	2 7874
26	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
59	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
35	0.0010	1 3062	1 6896	2.0322	24411	2 7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
43	0.6802	1.3020	1.6811	2.0167	24163	2.6951
4	0.6801	1.3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896
16	0.6799	1.3002	1.6787	2.0129	2.4102	2.6870
47	0.6797	1.2998	1.6779	2.0117	2.4083	2.6846
~	0.6/96	1.2994	1.6//2	2.0106	2.4066	7.6822