Name: **Student No.:**
EID: **Tutorial Session Code:**

GE2262 Business Statistics, 2021/22 Semester B
Individual Assignment 2

Instructions:

1. Due on April 2, 5pm.
2. Fill in your particulars at the top of this page.
3. Answer all questions in the space provided below.
4. Show all calculations clearly.
5. Display all non-integer numeric values to 3 decimal places.
6. Late submission penalty: deduct 10% of the base score for late submission within 24 hours.

Question 1 (25 marks)

A random sample of 100 managers in a manager association was taken to study their working hours per week. The survey results showed that the sample mean $\bar{x}$ is 53 hours and the sample standard deviation *s* is 7.7 hours. The manager association has around 6000 members.

a. What are point estimators of population mean and population standard deviation? (2 marks)

Sample mean and sample standard deviation are point estimators of population mean and population standard deviation.

b. What is the sampling distribution of $\bar{x}$? Why? (3 marks)

Sample size is 100 > 30, by Central Limit Theorem, sample mean $\bar{x}$ follows a normal distribution approximately.

c. If the population mean was 51 hours, what is $P(\bar{x} \geq 53\ hours)$? (6 marks)

$$P(\bar{x} \geq 53\ hours) = P\left(z \geq \frac{53-51}{\frac{7.7}{\sqrt{100}}}\right)$$
$$= P(Z \geq 2.5974)$$
$$= 1 - 0.9953$$
$$= 0.0047$$

d. If the population mean was 51 hours, what is $P(49 \leq \bar{x} \leq 53\ hours)$? (5 marks)

$$P(49 \leq \bar{x} \leq 53\ hours) = P\left(\frac{49-51}{\frac{7.7}{\sqrt{100}}} \leq z \leq \frac{53-51}{\frac{7.7}{\sqrt{100}}}\right)$$
$$= P(-2.5974 \leq Z \leq 2.5974)$$
$$= 0.9953 - 0.0047$$
$$= 0.9906$$

e. If the population mean was 60 hours, what is $P(\bar{x} = 53\ hours)$? (2 marks)

$$P(\bar{x} = 53\ hours) = 0$$

f. If $P(\bar{x} \geq 53\ hours)$=0.5, what is the population mean? (4 marks)

$P(\bar{x} \geq 53\ hours)$=0.5 => $P(\bar{x} < 53\ hours)$=0.5

$=> \frac{53-\mu}{\frac{7.7}{\sqrt{100}}} = 0$

$=> \mu = 53\ hours$

g. If $P(\bar{x} \geq 53\ hours)$=0.75, without calculation, can you tell that the population mean is greater or less than 53 hours? Explain. (3 marks)

The population mean will be greater than 53 hours because for a normal distribution mean is equal to median. However, now we observe $P(\bar{x} \geq 53\ hours)$=0.75 > 0.5. Therefore, the population mean should be greater than 53 hours.

Question 2 (25 marks)

A consumer council investigates the battery life (in hundred hours) of a smart phone. A random sample of eight batteries was selected. The measurements of their battery life were recorded as follows:

72	83	78	65	69	77	81	71

a. What is the interval estimation of the population mean battery life with 90% level of confidence? To construct the confidence interval, what assumption about the population is needed? (10 marks)

We must assume that the population of battery life measurements is normally distributed, so that the sample mean follows normal distribution approximately.

90% confidence interval for μ:

$$\bar{X} \pm t_{\frac{0.10}{2},8-1} \frac{s}{\sqrt{n}}$$

$$= 74.5 \pm 1.8946 \frac{6.234}{\sqrt{8}} = [70.325,78.676]$$

We are 90% confident that the population mean battery life is between 70.325 and 78.676 hrs.

b. Describe how the width of the 90% confidence interval change if

i. the sample mean increases (2 marks)

The sample mean affects the location of the interval, no the width and thus the width of 90% confidence interval is unchanged.

ii. the sample standard deviation increases (2 marks)

Increase in sample standard deviation would increase the standard error, and thus the width of 90% confidence interval is increased.

iii. the standard error increases (2 marks)

Increase in standard error would increase the margin of error, and thus the width of 90% confidence interval is increased.

iv. the margin of error increases (2 marks)

Increase in the margin of error would increase the width of 90% confidence interval.

v. the sample size increases (2 marks)

Increase in sample size would decrease the standard error, and thus the width of 90% confidence interval is decreased.

c. How large a sample of the batteries would be needed in order to estimate the population mean battery life within ±2 hours with 85% confidence? (5 marks)

$\alpha = 1 - 85\% = 0.15$ => $Z_{\frac{0.15}{2}} = Z_{0.075}$ =1.44

$E = 2$

$s = 6.234$

$n = (\frac{Z_{\frac{\alpha}{2}}\sigma}{E})^2 = (\frac{1.44 \times 6.234}{2})^2 = 20.144 \cong 21$ (round up)