Topic 6 Hypothesis Testing

Tutorial agenda:

1. go through key concepts
2. tutorial questions
3. Q&A

Key concept

T9 L6

Hypothesis

A statistical hypothesis is a claim about the population parameter

Hypothesis Testing Procedure:

Step 1: Define hypotheses

Null hypothesis, H_0

- Always about a population parameter (μ)
- Always contains "="
- Always assumed to be true at start

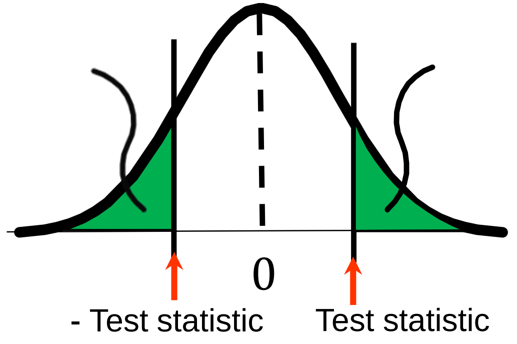
Alternative hypothesis, H_1

- The opposite of the null hypothesis
- X "="
- It is mutually exclusive and collectively exhaustive from the null hypothesis

There are three different sets of hypotheses to be tested

- Two-tail test: H_0: $\mu=\mu_0$ against H_1: $\mu\neq\mu_0$
- Lower-tail test: H_0: $\mu\geq\mu_0$ against H_1: $\mu<\mu_0$
- Upper-tail test: H_0: $\mu\leq\mu_0$ against H_1: $\mu>\mu_0$

Hypothesis



Step 2: Collect the data and identify the rejection region(s)

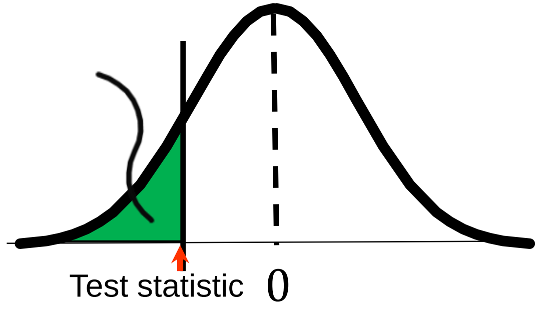
Representative sample

Rejection region/level of significance α

- Typical values: 0.01, 0.05, 0.1
- Probability of committing Type I error: The acceptable risk level for rejecting the null hypothesis wrongly

Location

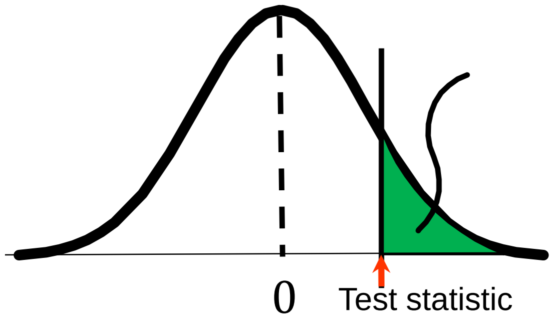
- Two-tail
- One-tail: lower-tail/upper-tail



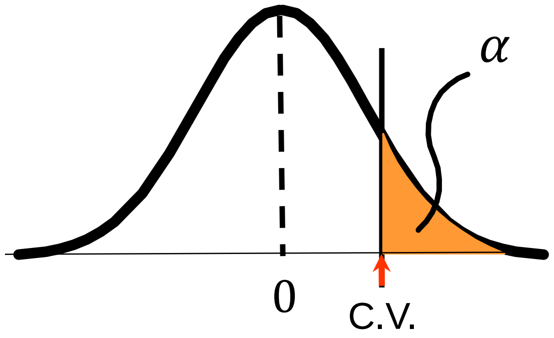
Step 3: Compute test statistic

σ is known: Z distribution $$Z = \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$$

σ is unknown: t distribution $$t = \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$$



Hypothesis



Step 4: Make statistical decision

Z/t statistic ***VS.*** Critical value (i.e. rejection region)

- If yes, then reject H_0
- Otherwise, do not reject H_0

p-value (converted from Z/t statistic) ***VS.*** level of significance (α)

- Two-tail: p-value = $P(Z \leq -|test\ statistic|) + P(Z \geq |test\ statistic|)$
- One-tail: p-value = $\mathrm{P}(Z \leq test\ statistic)$ or p-value = $P(Z \geq test\ statistic)$
- If p-value $< \alpha$, then reject H_0
- Otherwise, do not reject H_0

Risk of Making a Wrong Decision

Decision	The Truth	
	H_0 True	H_0 False
Do not reject H_0	Level of Confidence $(1-\alpha)$	Type II Error (β)
Reject H_0	Type I Error (α)	Power of the Test $(1-\beta)$

Type II error (β)

- depends on the true value of the parameter to be tested, often unknown if the null hypothesis is rejected
- ways to reduce the probability of making a Type II error:
  - By increasing *a*
  - By increasing the sample size for the test

Type I error (α)

- often pre-specified (e.g. *a* = 0.05)

Z Test for the Population Mean (σ Known)

Conditions: σ known + Population: normal, or large enough sample size (CLT)

Critical value(s): obtained from the Z-table, Z_α or $Z_{\alpha/2}$

Test statistic: obtained from sample, $\boldsymbol{Z} = \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$

p-value: obtained from test statistic,

- two-tail: P(Z < -| $\boldsymbol{Z}$ |) + P(Z > | $\boldsymbol{Z}$ |)
- lower-tail: P(Z < $\boldsymbol{Z}$)
- upper-tail: P(Z > $\boldsymbol{Z}$)

Reject / Do not reject H_0:

- Reject H_0 if $Z < C.V.$ or $Z > C.V.$ or p-value < α
- There is evidence that the true mean is not xxx

- Do not reject H_0 as p-value > α
- There is no evidence that the true mean is not xxx

t Test for the Population Mean (σ Unknown)

Conditions: σ unknown + Population: normal, or large enough sample size (CLT)

Critical value(s): obtained from the t-table, $t_{\alpha,df}$ or $t_{\alpha/2,df}$ (df=n-1)

Test statistic: obtained from sample, $\boldsymbol{t} = \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$

p-value: obtained from test statistic,

- two-tail: P(t < -| ***t*** |) + P(t > | ***t*** |)
- lower-tail: P(t < ***t***)
- upper-tail: P(t > ***t***)

* = find the range

p-value

$$= P(t \le -2.37) + P(t \ge 2.37)$$
$$= 2 \times P(t \ge 2.37)$$
$$= 2 \times (0.01, 0.025)$$
$$= (0.02, 0.05)$$

Reject / Do not reject H_0:

- Reject H_0 if $t < C.V.$ or $t > C.V.$ or p-value < α
- There is evidence that the true mean is not xxx

- Do not reject H_0 as p-value > α
- There is no evidence that the true mean is not xxx

Z or t Test change

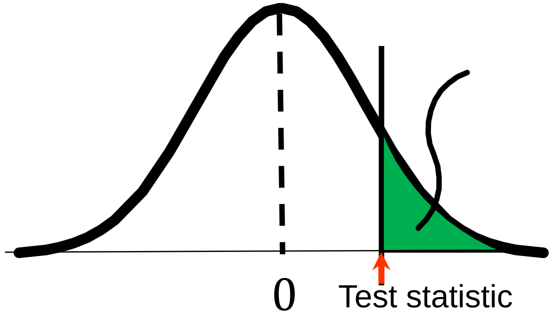
Z test

Critical value(s): obtained from the Z-table, Z_α or $Z_{\alpha/2}$

$$\alpha \propto \frac{1}{|\text{C.V.}|}$$

Test statistic, $\boldsymbol{Z}$:

$$n \propto \boldsymbol{Z}$$



t test

Critical value(s): obtained from the t-table, $t_{\alpha, df}$ or $t_{\alpha/2, df}$

$$\alpha \propto \frac{1}{|\text{C.V.}|}$$

$$df \propto \frac{1}{|\text{C.V.}|}$$

Test statistic, $\boldsymbol{t}$:

$$n \propto \boldsymbol{t}$$