

GE2262 Business Statistics

Topic 3: Discrete and Continuous Probability Distributions Exercises

Q1

In a recent survey concerning the age (to the nearest year) and weight (to the nearest 10 lb) of first-year university students, the following probability distribution was obtained:

Age	Weight				
	100	110	120	130	140
19	0.02	0.09	0.09	0.01	0.02
20	0.06	0.15	α	0.05	0.03
21	0.02	0.06	0.11	0.04	0.05

- Find the value of α
- Construct the probability distribution of the weights of these students.
- A student is selected at random. What do you expect his/her weight to be?
- What is the standard deviation of the distribution in (b)?
- Are “age” and “weight” independent? Why or why not?

Q2

An airline wants to overbook flights in order to reduce the numbers of vacant seats. For a certain flight, it is known that the probabilities of 0, 1, 2 and 3 vacant seats are 0.70, 0.15, 0.10 and 0.05 respectively.

- Find the mean and standard deviation for the number of vacant seats.
- What is the expected total number of vacant seats on 100 such flights?

Q3

Given the following probability distributions:

Distribution A		Distribution B	
X	$P(X)$	X	$P(X)$
0	0.50	0	0.05
1	0.20	1	0.10
2	0.15	2	0.15
3	0.10	3	0.20
4	0.05	4	0.50

- Compute the expected value for each distribution.
- Compute the standard deviation for each distribution.
- Compare the results of distributions A and B.

Q4

You are trying to develop a strategy for investing in two different stocks. The anticipated annual return for a \$1,000 investment in each stock has the following probability distribution:

Returns		
Stock X	Stock Y	Probability
-\$50	-\$100	0.1
20	50	0.3
100	130	0.4
150	200	0.2

- For each stock, compute the expected return and the standard deviation of return.
- Do you think that you will invest in stock X or stock Y? Explain.

Q5

When a customer places an order with Rudy's On-Line Office Supplies, a computerized accounting information system (AIS) automatically checks to see if the customer has exceeded his or her credit limit. Past records indicate that the probability of customers exceeding their credit limit is 0.05. Suppose that, on a given day, 20 customers place orders. Assume that the number of customers that the AIS detects as having exceeded their credit limit is distributed as a binomial random variable.

- What are the mean and standard deviation of the number of customers exceeding their credit limits?
- What is the probability that 0 customers will exceed their limits?
- What is the probability that 1 customer will exceed his or her limit?
- What is the probability that 2 or more customers will exceed their limits?

Q6

For e-commerce merchants, getting a customer to visit a Web site isn't enough. Merchants must also persuade online shoppers to spend money by completing a purchase. Experts at Consumer Consulting estimate that 88% of Web shoppers abandon their virtual shopping carts before completing their transaction. Consider a sample of 20 customers who visit an e-commerce Web site, and assume that the probability that a customer will leave the site before completing the transaction is 0.88. What is the probability that all 20 of the customers will leave the site without completing a transaction?

Q7

A task force of CityU sampled 200 students after the mid-term test to ask them whether they went shopping the weekend before the mid-term test or spent the weekend studying, and whether they did well or poorly on the mid-term test. The following result was obtained.

	Did Well on Mid-Term Test	Did Poorly on Mid-Term Test
Studied for Mid-Term Test	90	10
Went Shopping	30	70

- a) What is the probability that a randomly selected student did well on the mid-term test or went shopping the weekend before the mid-term test?
- b) A random sample of 10 students is selected. What is the probability that 2 of them did well on mid-term test and studied for mid-term test the weekend before the mid-term test? What distribution are you using? Why can you use such distribution?

Q8

Suppose that 1,000 patrons of a restaurant were asked whether they preferred beer or wine. 70% said that they preferred beer. 60% of patrons were male. 80% of the males preferred beer.

- a) What is the probability a randomly selected patron prefers wine?
- b) What is the probability a randomly selected patron is female and prefers wine?
- c) Suppose a randomly selected patron prefers wine, what is the probability that the patron is a male?
- d) Suppose 5 patrons were selected, what is the probability that at least four of them prefer beer?

Q9

MTR Corporation has to conduct surveys regularly to evaluate its service quality. According to previous studies, 87% of the passengers refuse to take part in such surveys.

If 15 passengers are selected randomly, what is the probability that at least 2 of them will respond to the survey?

Q10

According to Dental Association, 60% of all dentists use nitrous oxide (“laughing gas”) in their practice. Let x be the number of dentists who use laughing gas in practice in a random sample of five dentists. The probability distribution of x is as follows:

x	0	1	2	3	4	5
$P(x)$	0.0102	0.0768	0.2304	0.3456	0.2592	0.0778

- Find the probability that less than 2 dentists use laughing gas in a sample of five.
- Find $E(x)$. Interpret the result.
- Find the standard deviation of x .
- Based on the results of (b) and (c), show that the distribution of x is binomial with $n = 5$ and $\pi = 0.6$.

Q11

Given a normal distribution with $\mu = 100$ and $\sigma = 10$, what is the probability that

- $X > 85$?
- $X < 80$?
- $X < 80$ or $X > 110$?
- 80% of the values are between what two X values (symmetrically distributed around the mean)?

Q12

The breaking strength of plastic bags used for packaging produce is normally distributed, with a mean of 5 pounds per square inch and a standard deviation of 1.5 pounds per square inch. What proportion of the bags have a breaking strength of

- Less than 3.11 pounds per square inch?
- At least 3.8 pounds per square inch?
- Between 5 and 5.5 pounds per square inch?
- 95% of the breaking strength will be contained between what two values symmetrically distributed around the mean?

Q13

A statistical analysis of 1,000 long-distance telephone calls made from the headquarters of the Bricks and Clicks Computer Corporation indicates that the length of these calls is normally distributed with $\mu = 220$ seconds and $\sigma = 30$ seconds.

- What is the probability that a call lasted less than 175 seconds?
- What is the probability that a call lasted between 175 and 265 seconds?
- What is the probability that a calls lasted between 115 and 175 seconds?
- What is the length of a call if only 1% of all calls are shorter?

Q14

The exam marks of a large class of students follow a normal distribution with mean μ and standard deviation σ . 1% of the students got 90 or above. 10% of the students got 40 or below. The passing mark is 50.

- a) Find the values of μ and σ .
- b) Find the chance that a randomly selected student passes the exam.

Q15

The fill amount of bottles of soft drink has been found to be normally distributed with a mean amount of 2.0 liters and a standard deviation of 0.05 liter. Bottles that contain less than 95% of the listed net content (1.90 liters in this case) can make the manufacturer subject to penalty by the Consumer Council, whereas bottles that have a net content above 2.12 liters may cause excess spillage upon opening.

- a) What proportion of the bottles is subject to penalty by the Consumer Council?
- b) What proportion of the bottles is risking to excess spillage upon opening?
- c) In an effort to reduce the possible penalty due to insufficient net content in the bottles, the manufacturer has set out the following quality control requirement: 99% of bottles should comply with the Consumer Council's standard. To achieve this, the bottler decides to set the filling machine to a new mean amount. Determine the mean amount to be set for the bottle filling machine such that the above requirement can be met.

Q16

At the CityU Computer Service Centre, the loading time for e-Portal page on Internet Explorer is normally distributed with mean 3 seconds.

- a) Without doing the calculations, for a randomly selected student, which of the following intervals of loading time (in second) is the most likely to be: 2.9-3.1, 3.1-3.3, 3.3-3.5, 3.5-3.7? Which interval of loading time is the least likely to be? Explain.
- b) What is the chance that the loading time is exactly 2 seconds?

Q17

The volume of a randomly selected bottle of a new type of mineral water is known to have a normal distribution with a mean of 995ml and a standard deviation of 5ml. What is the volume that should be stamped on the bottle so that only 3% of bottles are underweight?