Topic 5: Confidence Interval Estimation Solutions

Q1

Since n = 36 > 30 from unknown population distribution, we may use the Central Limit Theorem to conclude that the sampling distribution of $\overline{X}$ is approximately normal. σ is known, so we use the Z-distribution.

$\overline{X} = 120, \quad \sigma = 24, \quad n = 36,$

$\alpha = 1 - 0.99 = 0.01, \quad Z_{0.01/2} = 2.575$

99% confidence interval for μ

$$= \overline{X} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 120 \pm 2.575\left(\frac{24}{\sqrt{36}}\right)$$

$$= [109.7, 130.3]$$

We are 99% confident that the population mean is between 109.7 and 130.33.

Q2

Since population of the retail value of greeting cards is normally distributed, we can conclude that the sampling distribution of $\overline{X}$ is also normally distributed.

σ is unknown, so we use the t-distribution.

$\overline{X} = 2.65, \quad s = 0.44, \quad n = 100,$

$\alpha = 1 - 0.95 = 0.05, \quad t_{0.05/2\,,\,100-1} = 1.9842$

95% confidence interval for μ

$$= \overline{X} \pm t_{0.05/2\,,\,100-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 2.65 \pm 1.9842\left(\frac{0.44}{\sqrt{100}}\right)$$

= [2.5627, 2.7373] in dollar

We are 95% confident that the true population mean value of all greeting cards in the store's inventory is between \$2.5627 and \$2.7373.

Q3

a) Since population of tread wear indexes is normally distributed, we can conclude that the sampling distribution of $\overline{X}$ is also normally distributed.

σ is unknown, so we use the <u>t-distribution</u>.

$\overline{X} = 195.3, \quad s = 21.4, \quad n = 18,$

$\alpha = 1 - 0.95 = 0.05, \quad t_{0.05/2,18-1} = 2.1098$

95% confidence interval for μ

$$= \overline{X} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 195.3 \pm 2.1098\left(\frac{21.4}{\sqrt{18}}\right)$$

$$= [184.6581, 205.9419]$$

We are 95% confident that the population mean tread wear index for tires produced by this manufacturer under this brand name is between 184.6581 and 205.9419.

b) No, a grade of 200 is in the interval

c) By comparing this value to the sample mean and sample standard deviation: It is not unusual. A tread-wear index of 210 for a particular tire is only (210-195.3)/21.4 = 0.69 standard deviation above the sample mean of 195.3

Q4

a) mean = $\frac{23.5 + 19.8 + 21.3 + 22.6 + 19.4 + 18.2 + 24.7 + 21.9 + 20.0 + 21.1}{10} = 21.25$

s.d.= $\sqrt{\frac{(23.5 - 21.25)^2 + ... + (21.1 - 21.25)^2}{10 - 1}}$ =1.9896

b) Assume the population distribution is normal

$\because$ σ is unknown, set $\sigma = s$ and n = 10 <30

$\therefore$ use t-distribution

$\alpha = 0.05$

Sampling error, E = $t_{\alpha/2,10-1}\frac{s}{\sqrt{n}} = t_{0.05/2,10-1}\frac{1.9896}{\sqrt{10}}$ =2.2622($\frac{1.9896}{\sqrt{10}}$) =1.4233

Q5

$\because$The population distribution is normal and σ is unknown, use t-distribution.

$\alpha = 0.05$, $t_{0.025,11} = 2.2010$

$\because$ CI = [20,30] = $\overline{X} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}}$

$\therefore$ $\overline{X}$ - (2.2010)$\frac{s}{\sqrt{12}}$ = 20 and $\overline{X}$ + (2.2010)$\frac{s}{\sqrt{12}}$ = 30

Thus, sample mean, $\overline{X}$ = 25 and sample s.d., s = 7.8694

Q6

a) $\sigma = 45, \quad \alpha = 1 - 0.9 = 0.1, \quad Z_{0.1/2} = 1.645$

"Correct to within $\pm$ 10 minutes" means sampling error E = 10.

$$n = \frac{(1.645)^2 (45)^2}{(10)^2} = 54.797 \approx 55\ (\textit{round up})$$

$\therefore$ The agency needs at least 55 observations in order to be 90% confident that the error of estimation is within $\pm$ 10 minutes.

b) $$n = \frac{(2.575)^2 (45)^2}{(10)^2} = 134.2702 \approx 135\ (\textit{round up})$$

$\therefore$ The agency needs at least 135 observations in order to be 99% confident that the error of estimation is within $\pm$ 10 minutes.

Q7

a) Assume that population is normal,

$\because$ n = 25 < 30, σ is unknown, use t-distribution

95% CI = $\overline{X} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}} = 9.7 \pm (2.0639)\frac{4}{\sqrt{25}}$ = [8.0489, 11.3511]

We are 95% confident that the population average number of absences is between 8.049 and 11.351 days.

b) $\because$ n = 25 < 30

$\therefore$ Assume that the number of absences is normally distributed, so that the sampling distribution of $\overline{X}$ is normal

c) $\alpha = 0.05$,

$$n = \frac{Z_{\frac{\alpha}{2}}{}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (4.5)^2}{(1.5)^2} = 34.57 \approx 35$$

Q8

a) Since the population is a normal distribution with known standard deviation (=3), normal distribution is used to construct the confidence interval.

For 95% confidence, $\alpha=0.05$, $Z_{\alpha/2}=1.96$

$\overline{X} = 22.8182$ $\sigma = \sqrt{9} = 3$

95%C.I. = [22.8182 – 1.96*3/sqrt(11) , 22.8182 + 1.96*3/sqrt(11)]

= [22.8182 – 1.7729, 22.8182+1.7729]

= [21.0453, 24.5911]

We are 95% confident that the population average number of vitamin supplements sold per day in the store is between 21.0453 and 24.5911

b) No, although the sample size is small, the sampling distribution follows normal distribution as the population distribution is normal. In addition, the population variance (standard deviation) is given, normal distribution, instead of t-distribution, should be used to construct the confidence interval.

c) The new observation is smaller, the new sample mean will be smaller as well. The confidence interval will shift to left with center at the new sample mean. However, the width of the interval will not be changed as the width of the confidence interval is related to level of confidence (α), population standard deviation (σ) and sample size (n). While these three values keep unchanged, the width of interval will not change.

d) 90% C.I. $Z_{\alpha/2}=1.645$

$E = 0.5$, $\sigma=3$

Sample size, $n \geq (Z_{\alpha/2} * \sigma / E)^2 = (1.645*3/0.5)^2 = (9.87)^2 = 97.4169 \cong 98$ (round up)