Topic 3: Discrete and Continuous Probability Distributions Solutions

Q11

$X \sim N(100, 10^2)$

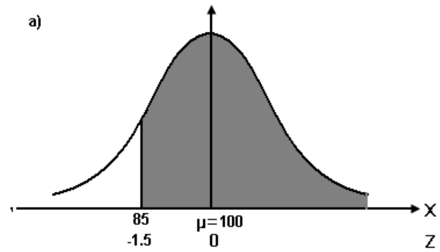
a) $P(X > 85)$ $= P\left(Z > \frac{85-100}{10}\right)$

$= P(Z > -1.5)$

$= 1 - P(Z <= -1.5)$

$= 1 - 0.0668$

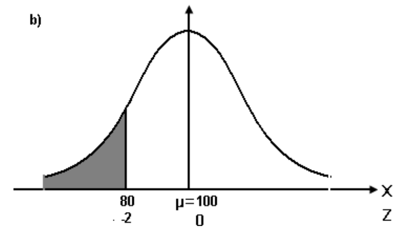
$= 0.9332$



b) $P(X < 80)$ $= P\left(Z < \frac{80-100}{10}\right)$

$= P(Z < -2)$

$= 0.0228$



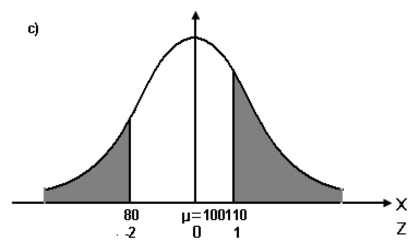
c) $P(X < 80 \text{ or } X > 110) = P\left(Z < \frac{80-100}{10}\right) + P\left(Z > \frac{110-100}{10}\right)$

$= P(Z < -2) + P(Z > 1)$

$= 0.0228 + 1 - P(Z <= 1)$

$= 0.0228 + 1 - 0.8413$

$= 0.1815$



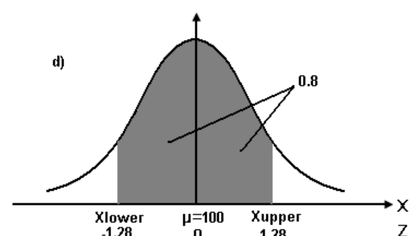
d) $P(X_{lower} \le X \le X_{upper}) = 0.8$

$$P\left(\frac{X_{lower}-100}{10} \le Z \le \frac{X_{upper}-100}{10}\right) = 0.8$$

$P(Z \le -1.28) = 0.10$ and $P(Z \le 1.28) = 0.90$

$Z = \frac{X_{lower}-100}{10} = -1.28$ and $Z = \frac{X_{upper}-100}{10} = 1.28$

$X_{lower} = -1.28(10) + 100 = 87.2$ and $X_{upper} = 1.28(10) + 100 = 112.8$



Q12

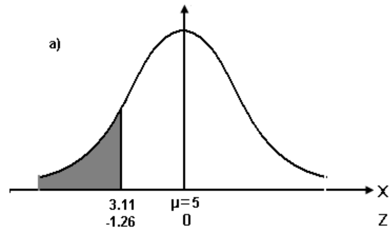
Let X be the breaking strength of plastic bags.

$X \sim N(5, 1.5^2)$

a) $P(X < 3.11) = P\left(Z < \frac{3.11-5}{1.5}\right)$

$= P(Z < -1.26)$

$= 0.1038$



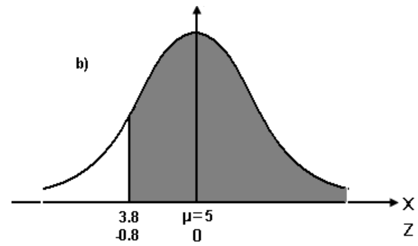
b) $P(X >= 3.8) = P\left(Z >= \frac{3.8-5}{1.5}\right)$

$= P(Z >= -0.8)$

$= 1 - P(Z < -0.8)$

$= 1 - 0.2119$

$= 0.7881$

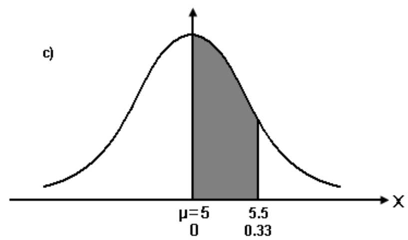


c) $P(5 \leq X \leq 5.5) = P\left(\frac{5-5}{1.5} \leq Z \leq \frac{5.5-5}{1.5}\right)$

$= P(0 \leq Z \leq 0.33)$

$= 0.6293 - 0.5$

$= 0.1293$



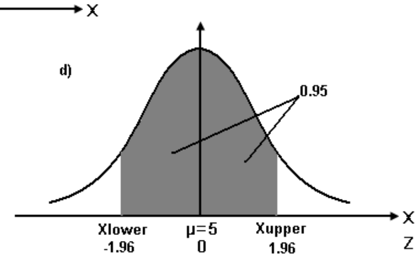
d) $P(X_{lower} \leq X \leq X_{upper}) = 0.95$

$$P\left(\frac{X_{lower}-5}{1.5} \leq Z \leq \frac{X_{upper}-5}{1.5}\right) = 0.95$$

$P(Z \leq -1.96) = 0.025$ and $P(Z \leq 1.96) = 0.975$

$Z = \frac{X_{lower}-5}{1.5} = -1.96$ $\quad$ $Z = \frac{X_{upper}-5}{1.5} = 1.96$

$X_{lower} = -1.96(1.5) + 5 = 2.06$ $\quad$ $X_{upper} = 1.96(1.5) + 5 = 7.94$

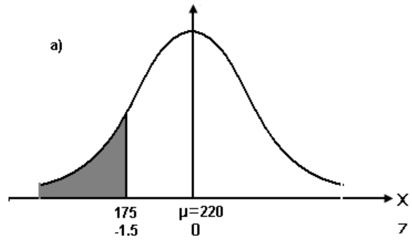


Q13

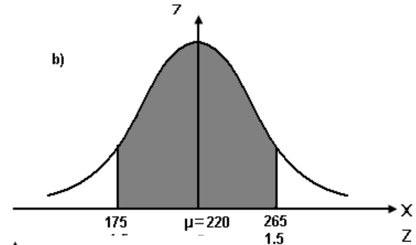
Let X be the length of long-distance telephone call.

$X \sim N(220, 30^2)$

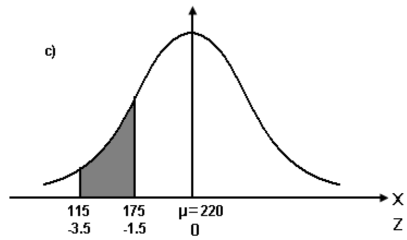
a) P (X < 175) $= P\left(Z < \frac{175-220}{30}\right)$
$= P(Z < -1.5)$
$= 0.0668$



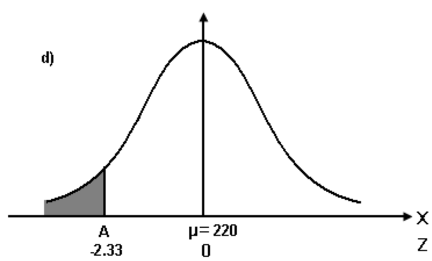
b) $P(175 \le X \le 265) = P\left(\frac{175-220}{30} \le Z \le \frac{265-220}{30}\right)$
$= P(-1.5 \le Z \le 1.5)$
$= 0.9332 - 0.0668$
$= 0.8664$



c) $P(115 \le X \le 175) = P\left(\frac{115-220}{30} \le Z \le \frac{175-220}{30}\right)$
$= P(-3.5 \le Z \le -1.5)$
$= 0.0668 - 0.00023$
$= 0.06657$



d) Let A be the length of a call if only 1 % of all calls are shorted
P (X < A) = 0.01
Since P (Z<-2.33) = 0.01
$\frac{A-220}{30} = -2.33$
A = -2.33 (30) + 220 = 150.1



Q14

a) Let X be the exam marks of the student
$X \sim N(\mu, \sigma^2)$

$P(X \ge 90) = 0.01$
$P\left(Z \ge \frac{90-\mu}{\sigma}\right) = 0.01$
$\frac{90-\mu}{\sigma} = 2.33$
$90 - \mu = 2.33\sigma$......(1)

$P(X \le 40) = 0.1$
$P\left(Z \le \frac{40-\mu}{\sigma}\right) = 0.1$
$\frac{40-\mu}{\sigma} = -1.28$
$40 - \mu = -1.28\sigma$......(2)

By solving equation (1) & (2)
$(1)-(2): 50 = 3.61\sigma$
$\sigma = 13.85$
Sub $\sigma = 13.85$ into (2)
$\mu = 57.73$

b) $P(X \ge 50) = P\left(Z \ge \frac{50-57.73}{13.85}\right) = P(Z \ge -0.56) = 0.7123$

Q15

a) $X \sim N(2, 0.05^2)$

$P(x < 1.9) = P(Z < \frac{1.9-2}{0.05}) = P(Z < -2) = 0.0228$ (2.28%)

The proportion of the bottles is subject to penalty by the Customer Council is 0.0228.

b) $P(x > 2.12) = P(Z > \frac{2.12-2}{0.05}) = P(Z > 2.4) = 0.0082$ (0.82%)

The proportion of the bottles is risking to excess spilling upon opening is 0.0082.

c) P(x < 1.9) = 1 - 0.99

$\Pr(Z > \frac{1.9-\mu}{0.05}) = 0.01$

$\frac{1.9-\mu}{0.05} = -2.33$

$\mu = 2.0165$

Q16

a) ∵ The loading time is normally distributed with mean of 3 seconds

Most likely: 2.9-3.1, since it lies in the central part of the normal distribution model, which has the largest area, thus the largest probability to occur.

Less likely: 3.5-3.7, since it is the farthest interval from the mean, thus has the least probability to occur under the normal distribution model.

b) P(exactly 2) = 0, since it is a line, not an area, this probability = 0

Q17

Let x be the volume that should be stamped on the bottle:

P(X < x) = 0.03

$P(Z < \frac{x-995}{5}) = 0.03$

$\frac{x-995}{5} = -1.88$

x = 985.6