Topic 6: Hypothesis Testing Solutions

Q1

a) H_0: $\mu = 14.6$ hour
H_1: $\mu \neq 14.6$ hour

b) A Type I error is the mistake of **concluding** that the mean number of hours studied at your school is **different** from the 14.6 hour benchmark reported by Business Week when **in fact** it is not any different.
(Type I Error occurs if reject the H_0 when it is true)

c) A Type II error is the mistake of **not concluding** that the mean number of hours studied at your school is **different** from the 14.6 hour benchmark reported by Business Week when it is **in fact** different.
(Type II Error occurs if do not reject the H_0 when it is false)

Q2

a) $H_0 : \mu \geq 160$
$H_1 : \mu < 160$
Let μ be the population mean of withdrawal

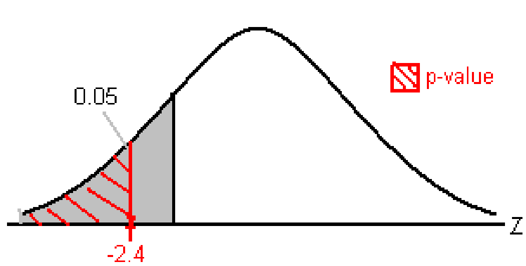
Population distribution unknown, $\because$ n = 36> 30, by Central Limit Theorem, the sampling distribution of $\overline{X}$ is approximately normal
σ known (σ=30) $\therefore$ Z test can be used (Lower-tail test)

At $\alpha = 0.05$, Critical value = -Z = -1.645
Reject H_0 if Z < -1.645

$$Z = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{148 - 160}{30 / \sqrt{36}} = -2.4$$

Since Z = -2.4 < -1.645, Reject H_0 at $\alpha = 0.05$
There is sufficient evidence that the population mean amount of money withdrawn from ATMs per customer transaction is less than $160.

b) p -value = P(Z≤-2.4) = 0.0082
Probability of obtaining a test statistic -2.4 or less is 0.0082, given H_0 is true.



Q3

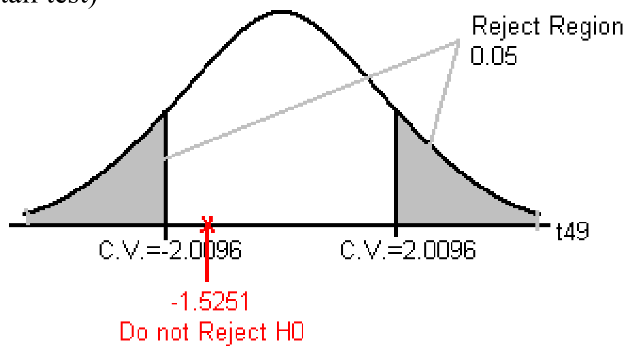
a) H0: $\mu = 8.17$ ounces

H1: $\mu \neq 8.17$ ounces

∵ n = 50 > 30 from unknown population distribution, by Central Limit Theorem, the sampling distribution of $\overline{X}$ is approximately normal

∵ σ unknown ∴ t test should be used (two-tail test)

$\alpha = 0.05$, Critical value $= \pm t_{\frac{0.05}{2},50-1} = \pm 2.0096$



Reject H0 if t < -2.0096 or t > 2.0096

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{8.159 - 8.17}{0.051/\sqrt{50}} = -1.5251$$

Since t = -1.5251 >-2.0096 and <2.0096

Do not Reject H0 at $\alpha = 0.05$

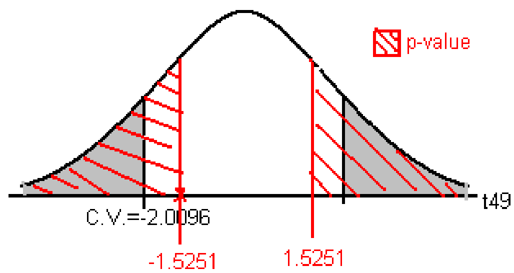
There is insufficient evidence that population mean amount is different from 8.17 ounces.

b) p -value $= P(t \leq -1.5251) + P(t \geq 1.5251) = 2 \times P(t \geq 1.5251)$

$= 2 \times (0.05, 0.1) = (0.1, 0.2)$

Interpretation:

Probability of obtaining a test statistics 1.5251 or more or -1.5251 or less is between 0.1 and 0.2 exclusively, given H0 is true.



Q4

a) H0: $\mu \geq 2.8$ feet

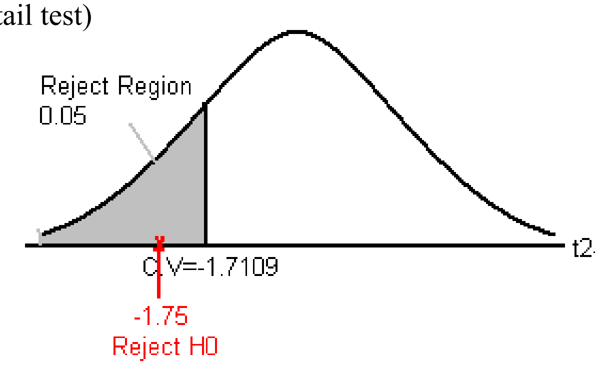
H1: $\mu < 2.8$ feet

∵ the population is normal distribution, the sampling distribution of $\overline{X}$ is also normal distribution

∵ σ unknown ∴ t test should be used (lower-tail test)

$\alpha = 0.05$, Critical value $= -t_{0.05,25-1} = -1.7109$

Reject H0 if t < -1.7109



$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{2.73 - 2.8}{0.2/\sqrt{25}} = -1.75$$

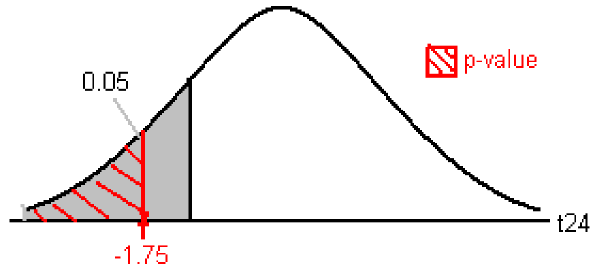
Since t = -1.75 < -1.7109

Reject H0 at $\alpha = 0.05$

There is sufficient evidence that the production equipment needs adjustment.

b) H_0: $\mu \geq 2.8$ feet
H_1: $\mu < 2.8$feet

$\because$ the population is normal distribution, the sampling distribution of $\overline{X}$ is also normal distribution
$\because$ σ unknown $\therefore$ t test should be used (lower-tail test)
$\alpha = 0.05$
Reject H_0 if p-value < 0.05



$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{2.73 - 2.8}{0.2/\sqrt{25}} = -1.75$$

p -value = P(t ≤ -1.75) = (0.025, 0.05)

Since p-value = (0.025, 0.05) < 0.05
Reject H_0 at $\alpha = 0.05$
There is sufficient evidence that the production equipment needs adjustment.

c) Probability of obtaining a test statistics -1.75 or less is between 0.025 and 0.05 exclusively, given H_0 is true.

d) The conclusions are the same.

Q5
Assume population distribution is normal
Let μ be the true average waiting time at back in commercial district.
H_0: $\mu \geq 5$
H_1: $\mu < 5$
$\because$ n < 30 and σ unknown
$\therefore$ Use *t*-test
$\alpha = 0.05$, $t_{critical} = -t_{0.05,15-1} = -1.7613$; Reject H_0 if t < -1.7613
$\bar{X} = 4.286667$, s = 1.637985
$t = \frac{4.286667-5}{\frac{1.637985}{\sqrt{15}}} = -1.6867 > -1.7613$
$\therefore$ We do not reject H_0 at $\alpha = 0.05$. There is insufficient evidence that the population average waiting time is less than 5 mins.

Q6

a) As $n = 18 < 30$, we need to assume the population distribution is normal.

b) Let μ be the population mean of overweight

$H_0 : \mu = 10$

$H_1 : \mu \neq 10$

$\because n = 18 < 30$, σ is unknown, assume population distribution is normal

$\therefore$ Use the t test, $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ (two-tail test)

$\alpha = 0.01$; Critical Value $= -t_{\frac{\alpha}{2}, n-1} = -t_{0.005,17} = -2.8982$

$= t_{\frac{\alpha}{2}, n-1} = t_{0.005,17} = 2.8982$

Reject H_0 if $t < -2.8982$ or $t > 2.8982$

$\bar{x} = 12.4 \quad \mu_0 = 10 \quad s = 2.7 \quad n = 18$

$$t = \frac{12.4 - 10}{2.7/\sqrt{18}} = 3.7712$$

$\because t = 3.7712 > 2.8982$

$\therefore$ We reject H_0.

There is sufficient evidence that the population mean overweight is not 10 pounds

c) Type I error (α)

= P (do not agree the claim of 10-pound overweight when in fact the claim is true)

Type II error (β)

= P (Agree the claim of 10-pound overweight when in fact the claim is false)

Q7

a) n = 40 > 30 and σ is unknown, use t-distribution

H_0: $\mu \leq 10$

H_1: $\mu > 10$

$\alpha = 0.01$; $t_{critical} = t_{0.01,\,39} = 2.4258$

$$t = \frac{\overline{X} - 10}{\frac{s}{\sqrt{40}}} > 2.4528, \text{ as } H_0 \text{ is rejected}$$

Now sample size, n$\uparrow$, n' = 41, hence

$t'_{critical} = t_{0.01,\,40} = 2.4233$, and

$\frac{\sigma}{\sqrt{n}} \downarrow$, as a result, t' will be increased

i.e. t' > t > 2.4258 > 2.4233

$\therefore$ H_0 is still rejected

b) Now α'' = 0.05

$t''_{critical} = 1.6839$, with 40 degrees of freedom

$\because$ t' > 2.4233 from above

$\therefore$ t' > 2.4233 > 1.6839

Therefore, H_0 is still rejected