**GE2262 Business Statistics**
**Topic 4: Sampling Distributions Exercises**

**Q1**
Given a normal distribution with $\mu = 100$ and $\sigma = 12$, if you select a sample of $n = 36$, what is the probability that $\overline{X}$ is

a) Less than 95?
b) Between 95 and 97.5?
c) Above 102.2?
d) There is a 65% chance that $\overline{X}$ is above what value?

**Q2**
The diameter of a brand of Ping-Pong balls is normally distributed, with a mean of 1.30 inches and a standard deviation of 0.05 inch. If you select a random sample of 25 Ping-Pong balls,

a) What is the sampling distribution of the mean?
b) What is the probability that the sample mean is less than 1.28 inches?
c) What is the probability that the sample mean is between 1.31 and 1.33 inches?
d) The probability is 60% that the sample mean will be between what two values, symmetrically distributed around the population mean?

**Q3**
Time spent using e-mail per session is normally distributed with $\mu$ = 8 minutes and $\sigma$ = 2 minutes. If you select a random sample of 16 sessions,

a) What is the probability that the sample mean is between 7.8 and 8.2 minutes?
b) What is the probability that the sample mean is between 7.5 and 8 minutes?
c) If you select a random sample of 100 sessions, what is the probability that the sample means is between 7.8 and 8.2 minutes?
d) Explain the difference in the results of (a) and (c).

**Q4**
In a recent survey concerning the age (to the nearest year) and weight (to the nearest 10 lb) of first-year university students, the following probability distribution was obtained:

| Age | Weight | | | | |
|---|---|---|---|---|---|
| | 100 | 110 | 120 | 130 | 140 |
| 19 | 0.02 | 0.09 | 0.09 | 0.01 | 0.02 |
| 20 | 0.06 | 0.15 | α | 0.05 | 0.03 |
| 21 | 0.02 | 0.06 | 0.11 | 0.04 | 0.05 |

A sample of 36 first-year students is taken. Find the approximate chance that their total weight is at most 4350 lb.

**Q5**
At the CityU Computer Service Centre, the loading time for e-Portal page on Internet Explorer is normally distributed with mean 3 seconds.

A random sample of 5 computers is drawn. What is the chance that their total loading time is at least 15 seconds?

**Q6**
Suppose there is a population with population size N = 3. The variable of interest is the salary (X) of individuals. The values of X are 18, 20 and 22 (in thousand dollars).

a) Find the mean (μ) and standard deviation (σ) for the population distribution.

In the process of developing sampling distribution, all possible samples (taken with replacement) of size n = 2 are obtained. The sample mean ( $\overline{X}$ ) is considered as the sample statistic.

b) What are the possible values of this sample mean random variable? Develop the probability distribution of the sample mean.
c) Show that the sample statistic $\overline{X}$ is an unbiased estimator of μ.
d) Denote $\sigma_{\overline{X}}$ the standard deviation of $\overline{X}$, verify the following relationship: $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$.
e) Does the sampling distribution of $\overline{X}$ follows a Normal Distribution? Explain.

**Q7**
To investigate the length of time working for an employer, researchers at the CityU sampled 344 business students and asked them a question: Over the course of your lifetime, what is the maximum number of years you expect to work for any one employer? The resulting sample had sample mean $\overline{X}$ = 19.1 years and sample standard deviation s=6 years. Assume the sample of students was randomly selected from the 5800 undergraduate students in CityU.

a) What are reasonable estimators of population mean and population standard deviation?
b) What is the sampling distribution of $\overline{X}$ ? Why?
c) If the population mean was 18.5 years, what is P( $\overline{X}$ ≥19.1 years)?
d) If the population mean was 19.5, what is P( $\overline{X}$ =19.1 years)?
e) If P( $\overline{X}$ ≥19.1 years) = 0.5, what is the population mean?
f) If P( $\overline{X}$ ≥19.1 years) = 0.2, without calculation, can you tell that the population mean is greater or less than 19.1 years? Explain.

**Q8**
The following data represent the responses (Y for yes and N for no) from a sample of 40 college students to the question "Do you currently own shares in any stocks?"

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N | N | Y | N | N | Y | N | N | N | Y | N | N | Y | N | N | Y | N | N | N | Y |
| N | N | N | N | N | N | N | N | N | N | Y | N | N | N | Y | N | N | N | N | N |

a) Find the sample proportion of college students who own shares.
b) Find the standard error of the sample proportion of college students who own shares.

**Q9**

You plan to conduct a marketing experiment in which students are to taste one of two different brands of soft drink. Their task is to correctly identify the brand tasted. You select a random sample of 200 students and assume that the students have no ability to distinguish between the two brands. (Hint: If an individual has no ability to distinguish between the two soft drinks, then each brand is equally likely to be selected.)

a) What is the probability that the sample will have between 50% and 60% of the identifications correct?
b) The probability is 90% that the sample percentage is contained within what symmetrical limits of the population percentage?
c) What is the probability that the sample percentage of correct identifications is greater than 65%?
d) Which is more likely to occur – more than 60% correct identifications in the sample of 200 or more than 55% correct identifications in a sample of 1,000? Explain.