7.5 电势叠加原理

电场强度与电势梯度

※ 电势叠加原理

点电荷 $\boxed{V = \dfrac{q}{4\pi\varepsilon_0 r}}$

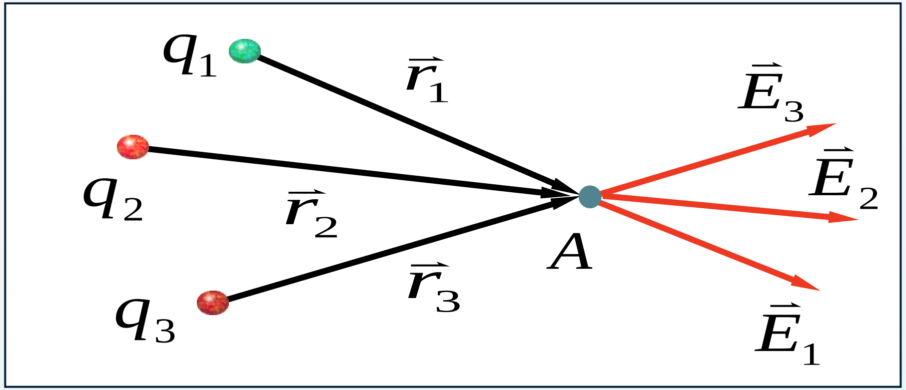
- 点电荷系

$$\vec{E} = \sum_i \vec{E}_i$$

$$V_A = \int_{A\infty} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$= \sum_{i=1}^{n} \int_{A\infty} \vec{E}_i \cdot \mathrm{d}\vec{l}$$

$$= \sum_{i=1}^{n} V_i$$

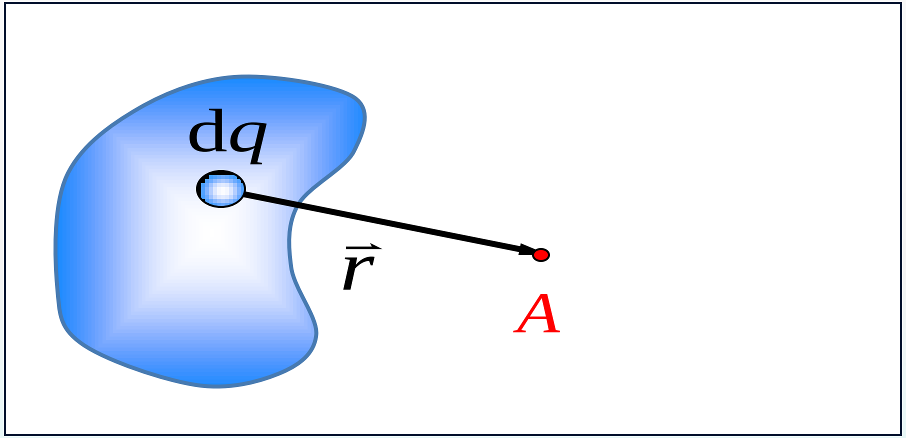


$$\boxed{V_A = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{q_i}{4\pi\varepsilon_0 r_i}}$$

◆ 电荷连续分布时

点电荷 $\boxed{V=\dfrac{q}{4\pi\varepsilon_0 r}}$

$$\mathrm{d}q=\rho\mathrm{d}V_{体}$$



$$\mathrm{d}V=\frac{\mathrm{d}q}{4\pi\varepsilon_0 r}$$

$$\boxed{V_A=\frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{r}}$$

微积分思想

计算电势的方法

（1）利用 $$\boxed{V=\int_{r,\infty}\vec{E}\cdot\mathrm{d}\vec{l}}$$

已知在积分路径上 $\vec{E}$ 的分布函数

有限大带电体，选无限远处电势为零.

（2）利用点电荷电势和电势叠加原理

$$\boxed{V=\frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{r}}$$

例1 正电荷q均匀分布在半径为R的细圆环上， 求圆环轴线上距环心为x处的点P的电势.

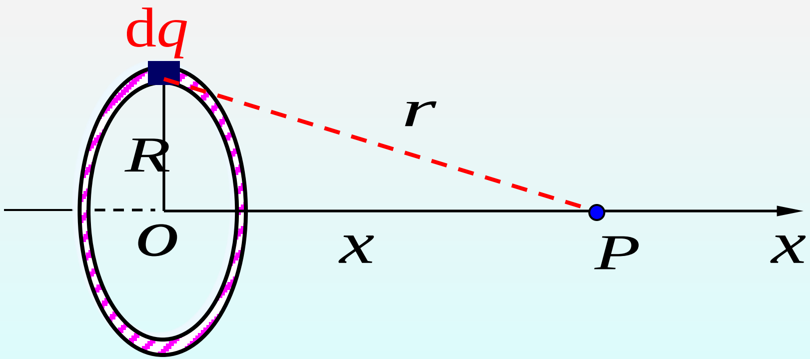
解 $$\mathrm{d}V_P = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}q}{r}$$

微积分思想

$$V_P = \frac{1}{4\pi\varepsilon_0 r}\int \mathrm{d}q$$

$$= \frac{q}{4\pi\varepsilon_0 r}$$

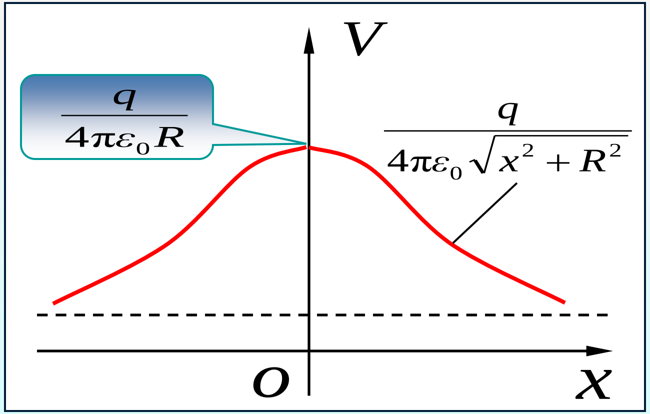
$$= \frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

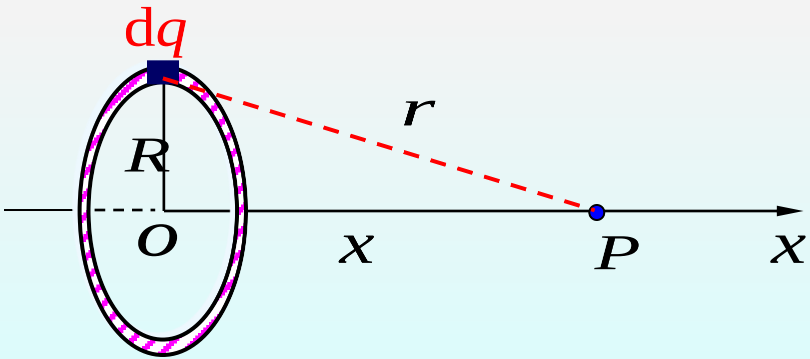


讨 论

$$V_P = \frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

$$x=0,\quad V_0 = \frac{q}{4\pi\varepsilon_0 R} \qquad\qquad x \gg R,\quad V_P = \frac{q}{4\pi\varepsilon_0 x}$$





带电圆环的电势：

$$dV = \frac{dq}{4\pi\varepsilon_0\sqrt{x^2+r^2}}$$

◆ 例2：求通过一均匀带电圆平面中心且垂直平面的轴线上任意点的电势.

$$dq = \sigma 2\pi r dr$$

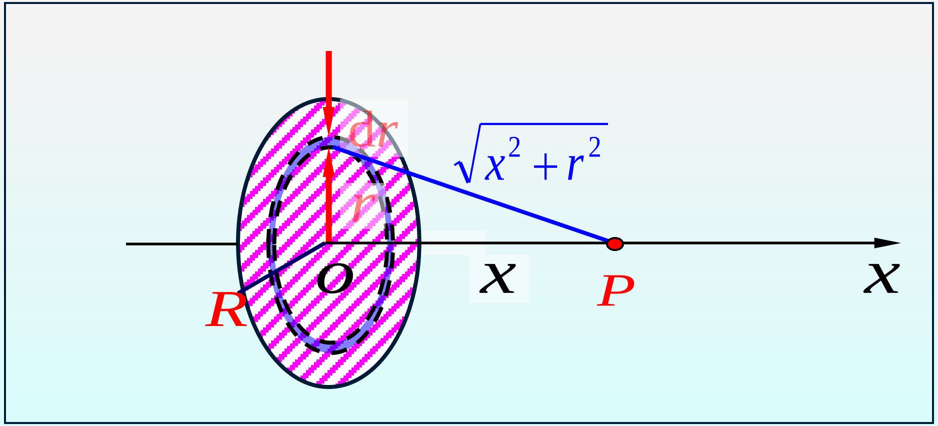
$$V = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma 2\pi r dr}{\sqrt{x^2+r^2}} = \frac{\sigma}{2\varepsilon_0}(\sqrt{x^2+R^2}-x)$$

微积分思想

$$x >> R$$

$$\sqrt{x^2+R^2} \approx x + \frac{R^2}{2x}$$

$$V \approx \frac{Q}{4\pi\varepsilon_0 x}$$



例3 真空中有一电荷为Q，半径为R的均匀带电球面.

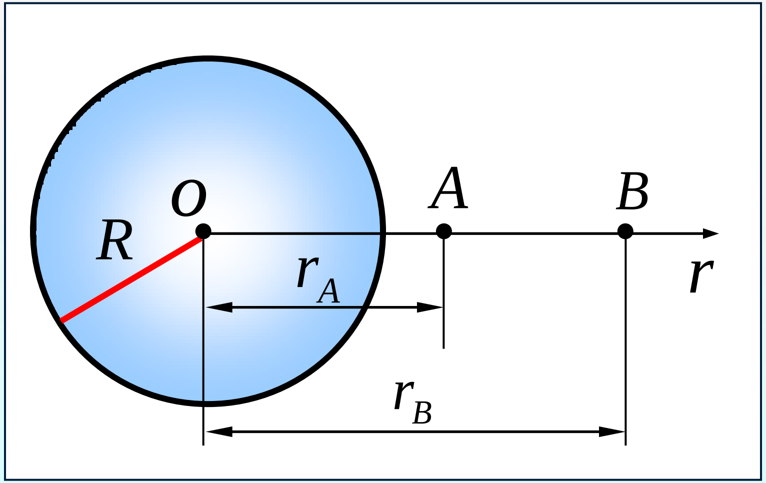
试求

（1）球面外两点间的电势差；

（2）球面内两点间的电势差；

（3）球面外任意点的电势；

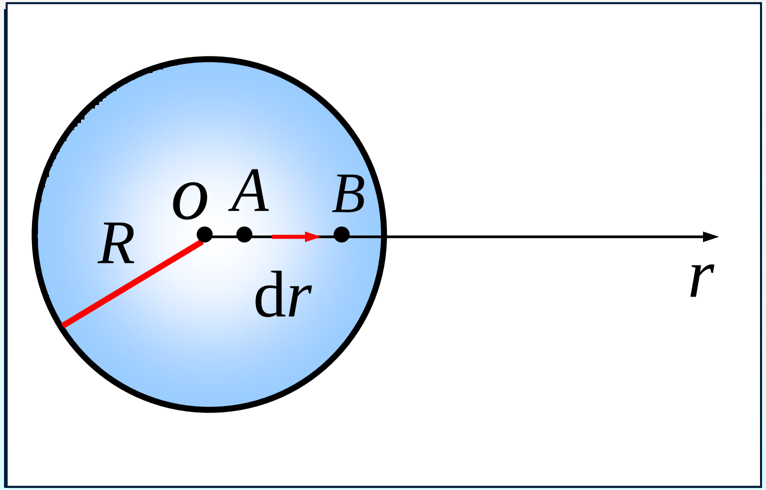
（4）球面内任意点的电势.



解 $$E=\begin{cases}0 & r<R\\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r>R\end{cases}$$

（1） $r>R$ $$V_A-V_B=\int_{r_A}^{r_B}\vec{E}\cdot\mathrm{d}\vec{r}=\frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{\mathrm{d}r}{r^2}$$

$$=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A}-\frac{1}{r_B}\right)$$



（2） $r<R$

$$V_A-V_B=\int_{r_A}^{r_B}\vec{E}\cdot\mathrm{d}\vec{r}=0$$

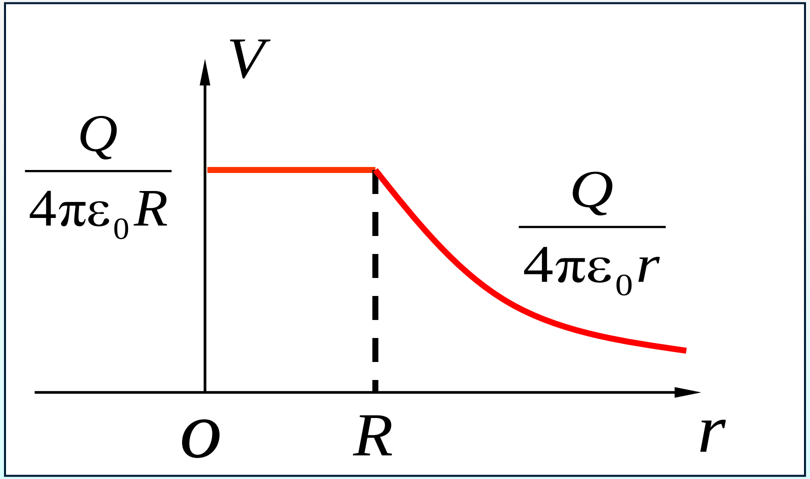
（3）$r > R$ 令 $r_B \approx \infty$ $V_\infty = 0$

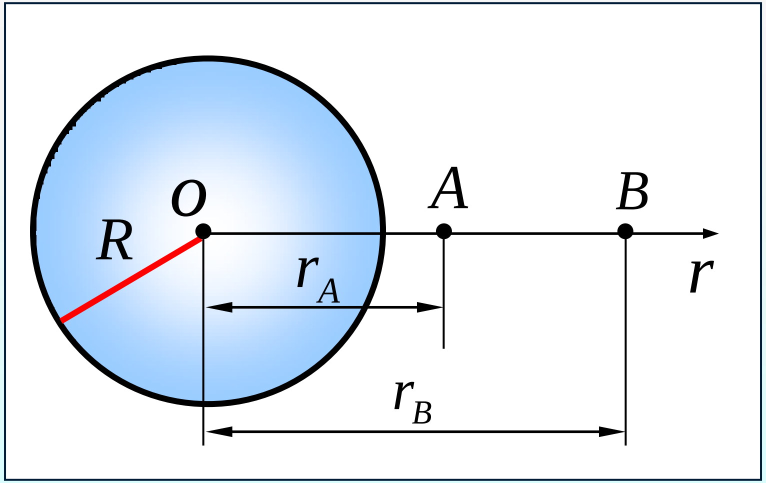
$$V_A - V_B = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

或：$V_A = \int_{r_A}^{\infty} \vec{E}\cdot \mathrm{d}\vec{r}$

$$= \frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{\infty}\frac{\mathrm{d}r}{r^2}$$

（4）$r < R$ $$V(r) = \int_r^R \vec{E}\cdot\mathrm{d}\vec{r} + \int_R^\infty \vec{E}\cdot\mathrm{d}\vec{r} = \frac{Q}{4\pi\varepsilon_0 R}$$





例4 求“无限长”带电直导线的电势附近的电势.

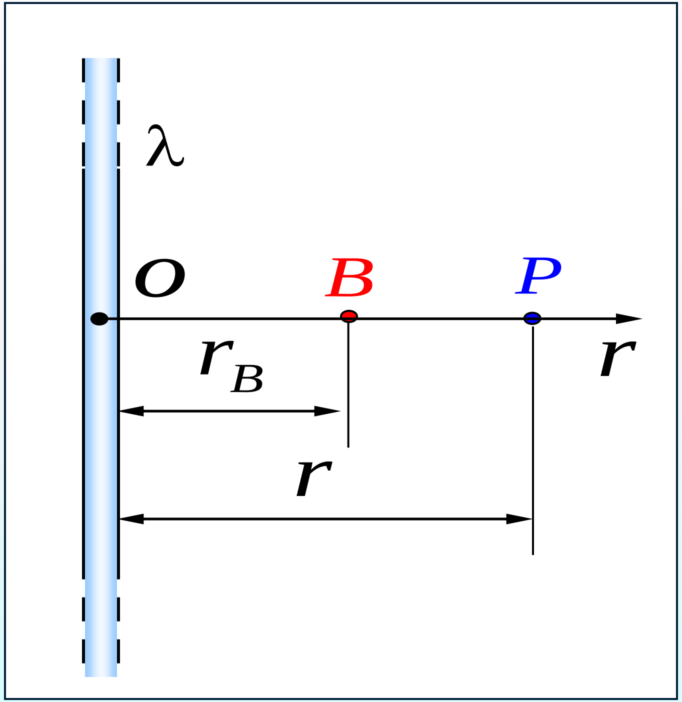
解 令 $V_B = 0$

$$V_P = \int_r^{r_B} \vec{E} \cdot d\vec{r}$$

$$= \int_r^{r_B} \frac{\lambda}{2\pi\varepsilon_0 r} dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r_B}{r}$$

讨论：能否选 $V_\infty = 0$?



※ 等势面

电场中电势相等的点所构成的面.

◆ 电荷沿等势面移动时，电场力做功为零.

$$W_{AB}=q(V_A-V_B)=\int_a^b q\vec{E}\cdot \mathrm{d}\vec{l}=0$$

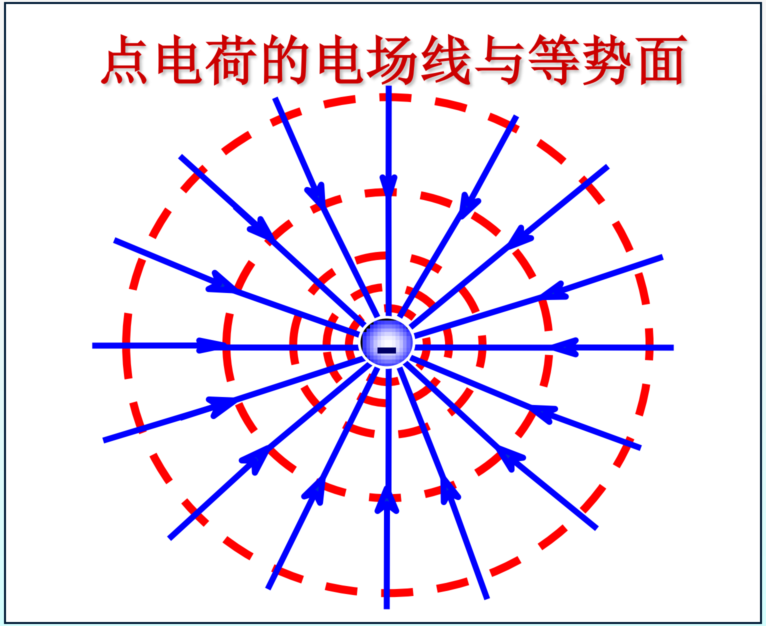
$$\boxed{\therefore \quad \vec{E}\perp \mathrm{d}\vec{l}}$$

◆ 某点的电场强度与通过该点的等势面垂直.

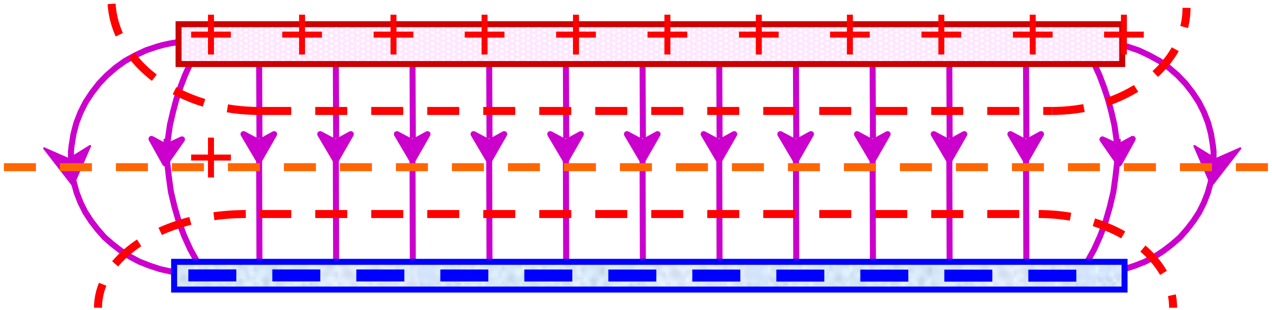
- 用等势面的疏密表示电场的强弱.

任意两相邻等势面间的电势差相等.

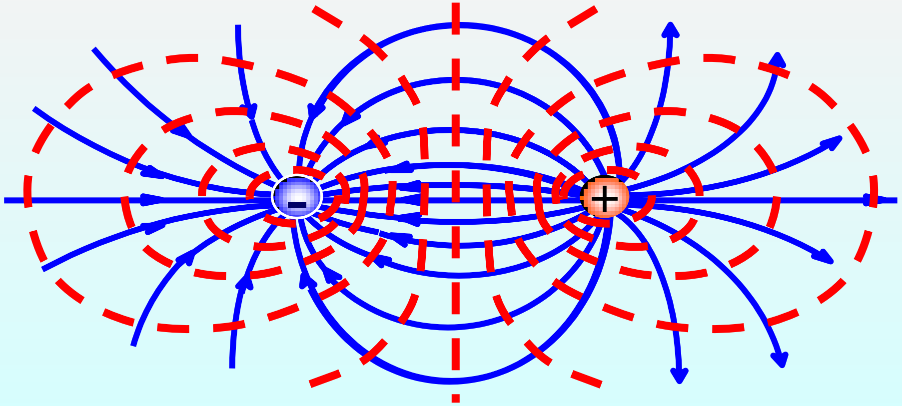
等势面越密的地方，电场强度越大.



两平行带电平板的电场线和等势面



一对等量异号点电荷的电场线和等势面



※ 电场强度与电势梯度

$$\boxed{-\Delta V = U_{AB} = V_A - V_B = \int_{AB} \vec{E} \cdot \mathrm{d}\vec{l}}$$

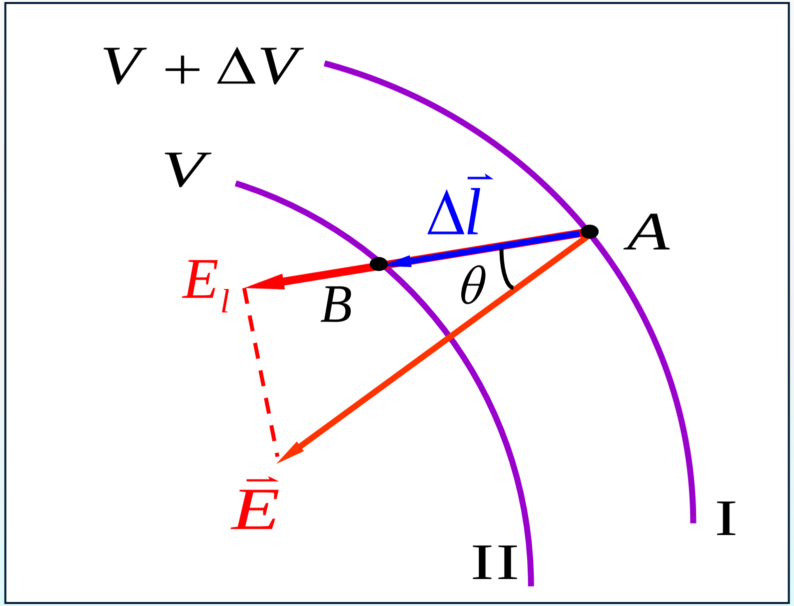
$$-\Delta V = \vec{E} \cdot \Delta\vec{l}$$

$$= E\Delta l \cos\theta$$

$$E\cos\theta = E_l$$

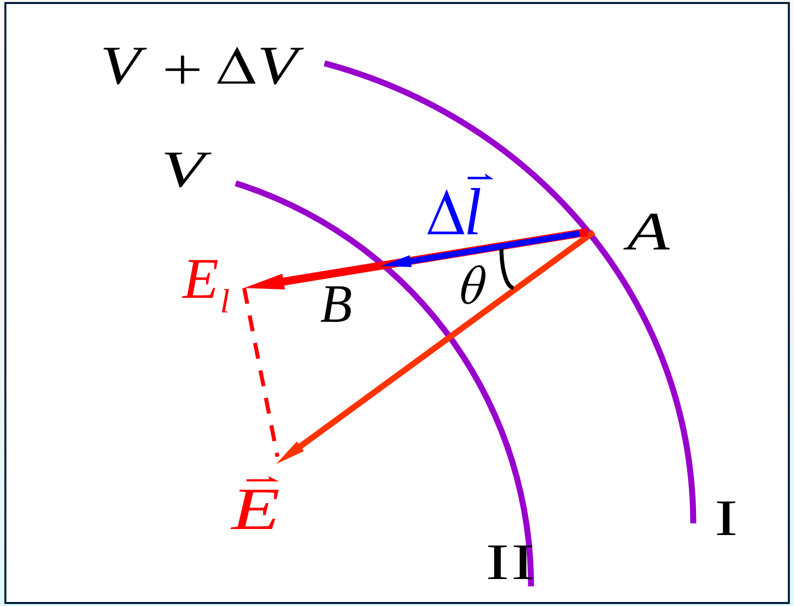
$$E_l = -\frac{\Delta V}{\Delta l}$$

$$E_l = -\lim_{\Delta l \to 0} \frac{\Delta V}{\Delta l} = -\frac{\mathrm{d}V}{\mathrm{d}l}$$



$$E_l = -\frac{\mathrm{d}V}{\mathrm{d}l}$$

电场中某一点的电场强度沿任一方向的分量，等于这一点的电势沿该方向上电势(空间)变化率(方向微商)的负值.



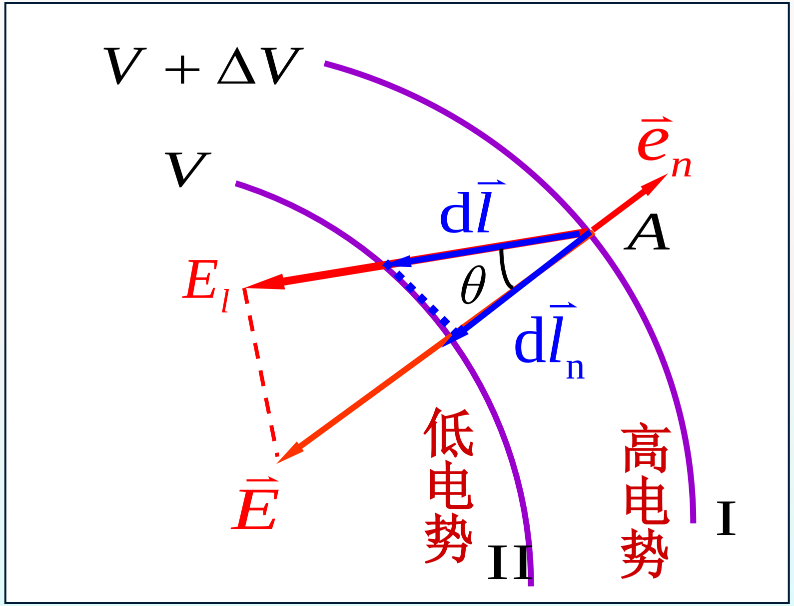
$$E_l = -\frac{\mathrm{d}V}{\mathrm{d}l} \qquad E_\mathrm{n} = -\frac{\mathrm{d}V}{\mathrm{d}l_\mathrm{n}} \qquad \because \mathrm{d}l > \mathrm{d}l_\mathrm{n} \qquad \therefore E_\mathrm{n} > E_l$$

$$\boxed{\vec{E} = -\frac{\mathrm{d}V}{\mathrm{d}l_\mathrm{n}}\vec{e}_\mathrm{n} = -\nabla V}$$

电势梯度

$$\nabla V = \mathrm{grad}\, V = \frac{dV}{dl_n}\vec{e}_n$$

$$= \left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$



电场强度 $\begin{cases} \text{大小} & |\vec{E}| = \left|\dfrac{\mathrm{d}V}{\mathrm{d}l_\mathrm{n}}\right| \\ \text{方向} & \text{由高电势处指向低电势处} \end{cases}$

电场强度等于电势梯度的负值

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right) = -\mathrm{grad}V = -\nabla V$$

求电场强度的三种方法

- 利用电场强度叠加原理
- 利用高斯定理
- 利用电势与电场强度的关系

例5 用电场强度与电势的关系，求均匀带电细圆环轴线上一点的电场强度的大小.

解 $$V=\frac{q}{4\pi\varepsilon_0 r}=\frac{q}{4\pi\varepsilon_0(x^2+R^2)^{1/2}}$$

$$E=E_x=-\frac{\partial V}{\partial x}$$

$$=-\frac{\partial}{\partial x}\left[\frac{q}{4\pi\varepsilon_0(x^2+R^2)^{1/2}}\right]$$

$$=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$