提高练习八　参考解答

一、填空题

1.若$u=\arctan\dfrac{y}{x}$，则$\dfrac{\partial u}{\partial x}=$ $\underline{-\dfrac{y}{x^2+y^2}}$.

2.由$xy+yz+zx=1$确定隐函数$z=f(x,y)$，则

$\dfrac{\partial z}{\partial x}=$ $\underline{-\dfrac{y+z}{x+y}}$.

3.函数$z=\dfrac{\sqrt{x}}{\sqrt{2-x^2-y^2}}-\dfrac{1}{\sqrt{y-x}}$的定义域为

$D=\{(x,y)|$ $\underline{x^2+y^2<2,y>x\geq 0}$ $\}$.

4. 已知 $f(x+y, x-y)=x^2y+y^2$，则 $f(x,y)=$ $\underline{\dfrac{x-y}{2}[(\dfrac{x+y}{2})^2+\dfrac{x-y}{2}]}$.

5. $u=\sin(x^2+y^2+z^2)$，则 $grad\ u=$ $\underline{\qquad\qquad\qquad}$.

$$2\cos(x^2+y^2+z^2)\cdot(x\vec{i}+y\vec{j}+z\vec{k})$$

二、选择题

1. 下列极限存在的是（ D ）.

（A）$\lim\limits_{\substack{x\to0\\y\to0}}\dfrac{x}{x+y}$ （B）$\lim\limits_{\substack{x\to0\\y\to0}}\dfrac{1}{x+y}$ （C）$\lim\limits_{\substack{x\to0\\y\to0}}\dfrac{x^2}{x+y}$ （D）$\lim\limits_{\substack{x\to0\\y\to0}}x\sin\dfrac{1}{x+y}$

2. 使 $\dfrac{\partial^2 z}{\partial x\partial y}=2x-y$ 成立的函数是（ B ）.

(A) $z=x^2y-\dfrac{1}{2}xy^2+e^{x+y}$ (B) $z=x^2y-\dfrac{1}{2}xy^2+e^{x}$

(C) $z=x^2y-\dfrac{1}{2}xy^2+\sin xy$ (D) $z=x^2y-\dfrac{1}{2}xy^2+e^{xy}+3$

3. 曲线$\begin{cases} x = t, \\ y = 2t^2, \\ z = 3t^3 \end{cases}$在点$(1,2,3)$处的一个切向量为（ C ）.

(A)$\{1,2,3\}$　　(B)$\{2,4,6\}$　　(C)$\{1,4,9\}$　　(D)$\{1,4,8\}$

4. 函数$f(x,y) = 4(x-y) - x^2 - y^2$（ A ）.

(A)有极大值 8　　(B)有极小值 8

(C)无极值　　(D)有无极值不确定

5. $u = e^{-x}\sin\dfrac{x}{y}$，则$\dfrac{\partial^2 u}{\partial x \partial y}$在点$\left(2, \dfrac{1}{\pi}\right)$处的值为（ C ）.

(A)$\dfrac{\pi}{e}$　　(B)$\left(\dfrac{\pi}{e}\right)^3$　　(C)$\left(\dfrac{\pi}{e}\right)^2$　　(D)1

三、设$u = f(xy, x+2y)$，f有连续的二阶偏导，求 $\dfrac{\partial^2 u}{\partial x \partial y}$.

解：$\dfrac{\partial u}{\partial x} = f_1' \cdot y + f_2'$,

$$\frac{\partial^2 u}{\partial x \partial y} = (f_{11}'' x + 2f_{12}'')y + f_1' + xf_{21}'' + 2f_{22}''$$

$$= f_1' + xyf_{11}'' + (x+2y)f_{12}'' + 2f_{22}''$$

四、由$x + y + z = \sqrt{xyz}$确定z是x, y的函数，求 dz.

解：$dz = \dfrac{\partial z}{\partial x} dx + \dfrac{\partial z}{\partial y} dy$

$$= -\frac{1}{2\sqrt{xyz} - xy}[(2\sqrt{xyz} - yz)dx + (2\sqrt{xyz} - xz)dy]$$

五、设$M_0(x_0, y_0, z_0)$是$z = xf\left(\dfrac{y}{x}\right)$上一点，

求证M_0处的法线垂直于向径$\overrightarrow{OM_0}$.

解：令$F(x,y,z) = z - xf(\dfrac{y}{x})$

$$\because \overrightarrow{OM_0} = \{x_0, y_0, z_0\},$$

$$\vec{n} = \{\frac{\partial F}{\partial x}\bigg|_{M_0}, \frac{\partial F}{\partial y}\bigg|_{M_0}, \frac{\partial F}{\partial z}\bigg|_{M_0}\} = \{f - \frac{y_0}{x_0}f', f', -1\}$$

$$\therefore \overrightarrow{OM_0} \cdot \vec{n} = x_0(f - \frac{y_0}{x_0}f') + y_0 f' - z_0 = 0$$证毕

六、求曲线$\begin{cases} x^2+y^2+z^2=6 \\ x+y+z=0 \end{cases}$在点(1,–2,1)处的切线和法平面方程.

解: 切向量$T=\left\{\begin{vmatrix} F_y & F_z \\ G_y & G_z \end{vmatrix},\begin{vmatrix} F_z & F_x \\ G_z & G_x \end{vmatrix},\begin{vmatrix} F_x & F_y \\ G_x & G_y \end{vmatrix}\right\}$

$$=\left\{\begin{vmatrix} -4 & 2 \\ 1 & 1 \end{vmatrix},\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix},\begin{vmatrix} 2 & -4 \\ 1 & 1 \end{vmatrix}\right\}=\{-6,0,6\}$$

切线: $\dfrac{x-1}{-1}=\dfrac{y+2}{0}=\dfrac{z-1}{1}$

法平面: $-(x-1)+0(y+2)+(z-1)=0$

即: $x-z=0$

七、设$f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0,\\ 0, & x^2+y^2=0,\end{cases}$,

讨论$f(x,y)$在（0,0）处：

⑴ 偏导数是否存在； ⑵ 是否可微.

解：(1)偏导数存在

$$\lim_{\Delta x\to 0}\frac{f(0+\Delta x,0)-f(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\dfrac{\Delta x\cdot 0}{\sqrt{(\Delta x)^2+0^2}}}{\Delta x}=0$$

$$\lim_{\Delta y\to 0}\frac{f(0,0+\Delta y)-f(0,0)}{\Delta y}=0$$

(2)不可微.

$$\lim_{\rho\to 0}\frac{\Delta z - f'_x(0,0)\Delta x - f'_y(0,0)\Delta y}{\rho}$$

$$\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\frac{f(\Delta x,\Delta y) - f(0,0) - f'_x(0,0)\Delta x - f'_y(0,0)\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}}$$

$$= \lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\frac{\Delta x\Delta y}{\Delta x^2 + \Delta y^2},$$ 该极限不存在,

$$\Delta z - f'_x(0,0)\Delta x - f'_y(0,0)\Delta y \neq o(\rho)$$

所以在(0,0)点不可微.