7.4 静电场环路定理

电势能 电势

7.4 静电场环路定理　电势能　电势

※ 静电场力做功

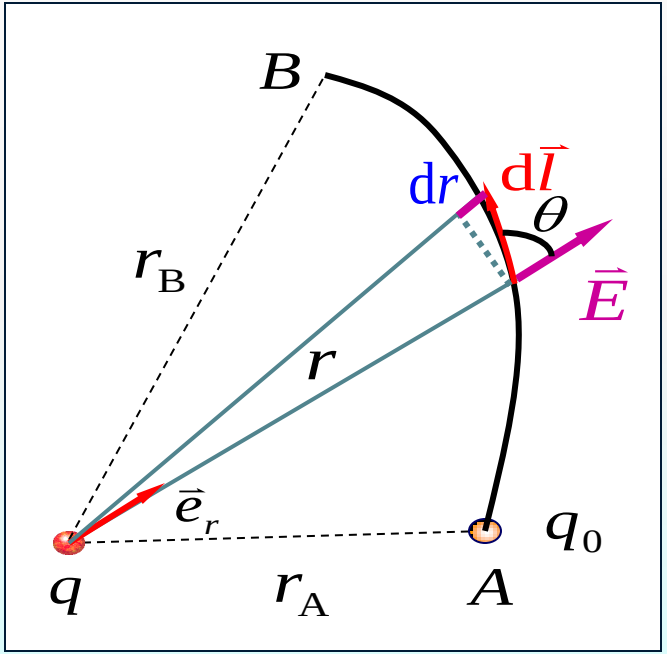
- 点电荷的电场力做功

$$\mathrm{d}W = q_0 \vec{E} \cdot \mathrm{d}\vec{l}$$

$$= \frac{q q_0}{4\pi\varepsilon_0 r^2} \vec{e}_r \cdot \mathrm{d}\vec{l}$$

$$\vec{e}_r \cdot \mathrm{d}\vec{l} = \mathrm{d}l \cos\theta = \mathrm{d}r$$

$$\mathrm{d}W = \frac{q q_0}{4\pi\varepsilon_0 r^2} \mathrm{d}r$$

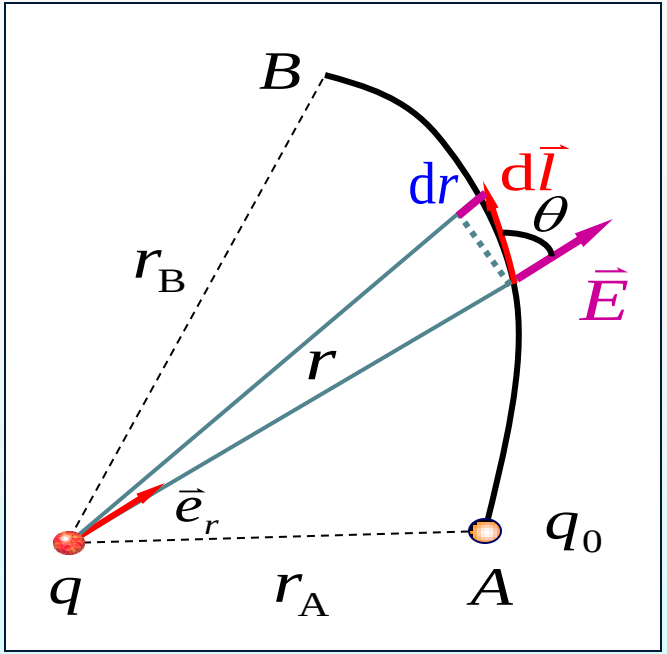


$$\mathrm{d}W = \frac{qq_0}{4\pi\varepsilon_0 r^2}\mathrm{d}r$$

$$W = \frac{qq_0}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{\mathrm{d}r}{r^2}$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

结论：W仅与q_0的始末位置有关，与路径无关.



- 任意带电体的电场　　（点电荷的组合）

$$\vec{E}=\sum_i \vec{E}_i$$

$$E_i=\frac{q_i}{4\pi\varepsilon_0 r_i^{\,2}}$$

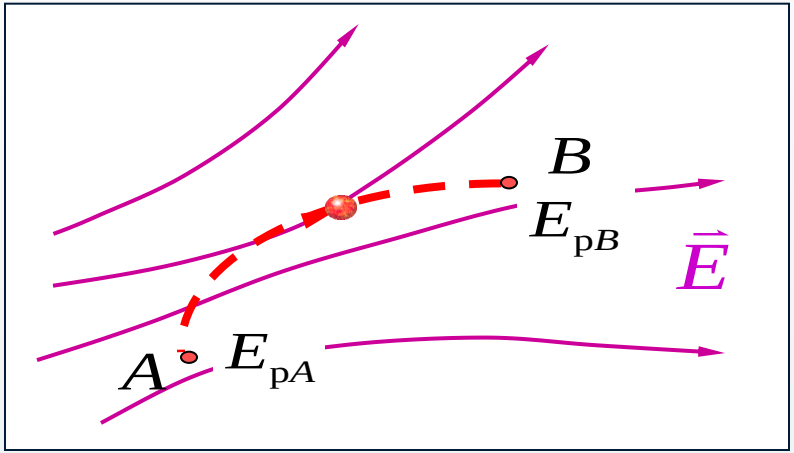
$$W=q_0\int_l \vec{E}\cdot \mathrm{d}\vec{l}\ =\sum_i q_0\int_l \vec{E}_i\cdot \mathrm{d}\vec{l}$$

$$=\sum_i \frac{q_i q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_{iA}}-\frac{1}{r_{iB}}\right)$$

结论：静电场力做功，与路径无关.

※ 电势能

静电场是保守场，静电场力是保守力. 静电场力所做的功就等于电荷电势能增量的负值(电势能的减少).



$$W_{AB} = E_{pA} - E_{pB} = -(E_{pB} - E_{pA})$$

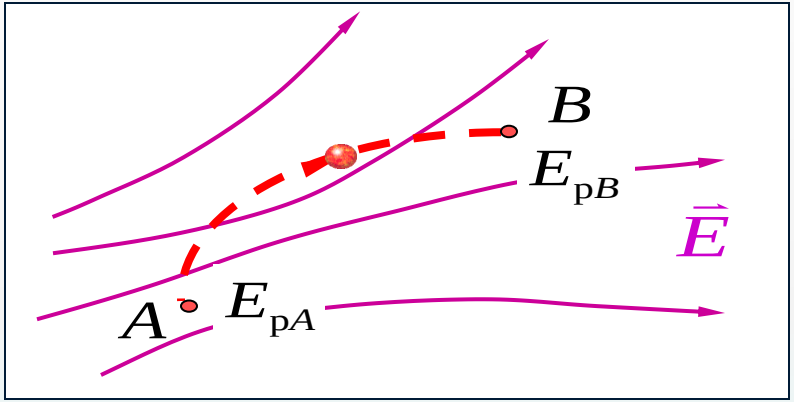
电场力做正功，电势能减少.

$$\int_{AB} q_0 \vec{E} \cdot \mathrm{d}\vec{l} = E_{\mathrm{p}A} - E_{\mathrm{p}B} = -(E_{\mathrm{p}B} - E_{\mathrm{p}A})$$

令 $E_{\mathrm{p}B} = 0$

$$E_{\mathrm{p}A} = \int_{AB} q_0 \vec{E} \cdot \mathrm{d}\vec{l}$$

电势能是系统的！



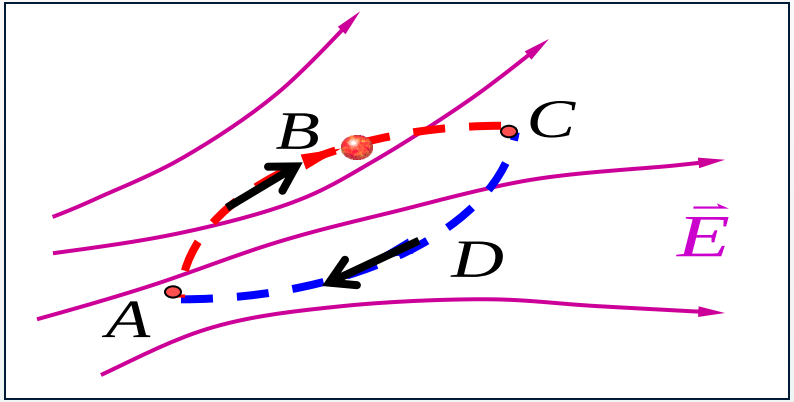
试验电荷q_0在电场中某点的电势能，在数值上等于把它从该点移到零电势能处静电场力所作的功.

※ 静电场的环路定理

$$q_0 \int_{ABC} \vec{E} \cdot \mathrm{d}\vec{l} = q_0 \int_{ADC} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$q_0 \left(\int_{ABC} \vec{E} \cdot \mathrm{d}\vec{l} + \int_{CDA} \vec{E} \cdot \mathrm{d}\vec{l} \right) = 0$$

$$\oint_l \vec{E} \cdot \mathrm{d}\vec{l} = 0$$



结论： 沿闭合路径一周，电场力作功为零.

静电力是保守力

静电场是保守场

※ 电势　　(电势能不能反映电场本身的性质)

$$W_{AB}=q_0\int_{AB}\vec{E}\cdot\mathrm{d}\vec{l}=-(E_{pB}-E_{pA})$$

注意区别：$\vec{E}$与E_P

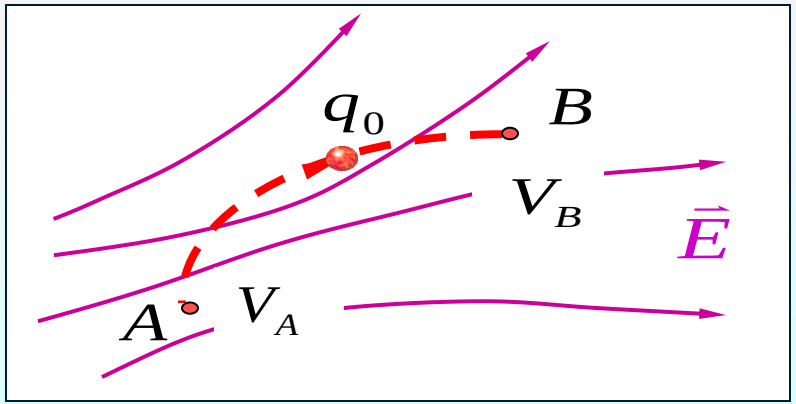
令 $V_A=E_{pA}/q_0$ A点电势，　$V_B=E_{pB}/q_0$ B点电势

$$\int_{AB}\vec{E}\cdot\mathrm{d}\vec{l}=-(V_B-V_A)$$

$$V_A=\int_{AB}\vec{E}\cdot\mathrm{d}\vec{l}+V_B$$

令 $V_B=0$

$$V_A=\int_{AB}\vec{E}\cdot\mathrm{d}\vec{l}$$



- **电势零点的选取：**

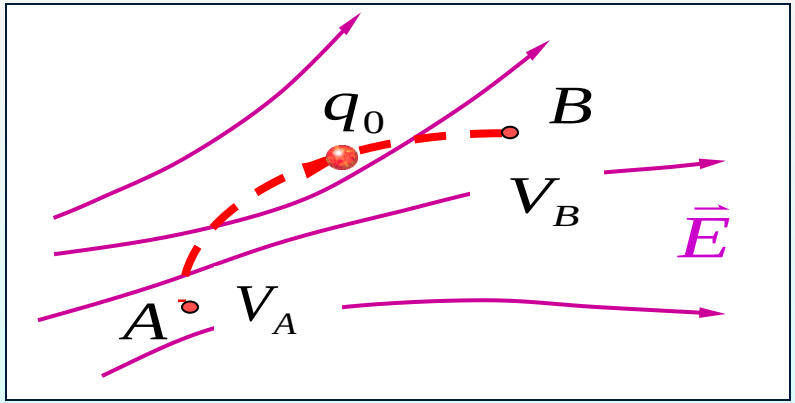
有限带电体以无穷远为电势零点，

实际问题中常选择地球电势为零.

$$V_A = \int_{A\infty} \vec{E} \cdot \mathrm{d}\vec{l}$$

- **电势的物理意义：**

在数值上等于，把单位正电荷从点A移到无限远处，静电场力作的功.



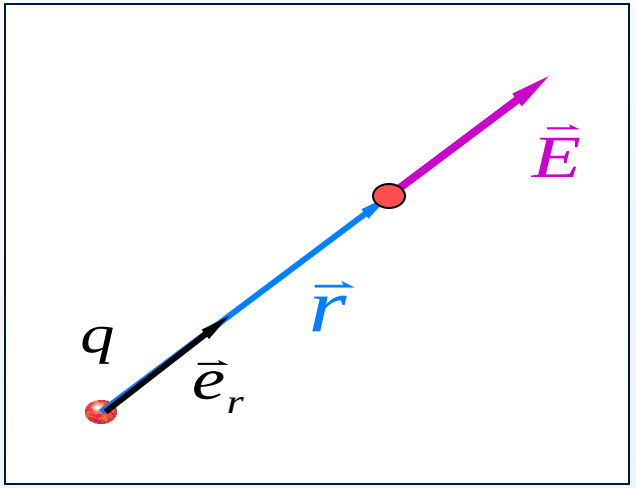
※ 点电荷的电势

$$\vec{E}=\frac{q}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

令 $V_\infty=0$

$$V=\int_r^\infty \vec{E}\cdot \mathrm{d}\vec{l}=\int_r^\infty \frac{q\mathrm{d}r}{4\pi\varepsilon_0 r^2}$$

$$V=\frac{q}{4\pi\varepsilon_0 r}$$



例 求电偶极子电场中任意一点*A*的电势。

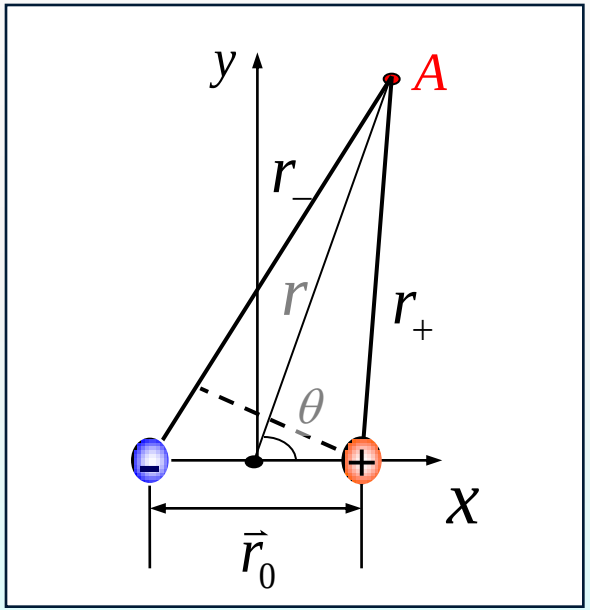
解

$$\begin{cases} V_+ = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_+} \\ V_- = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_-} \end{cases}$$

$$V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-}$$

$\because r_0 << r \qquad \therefore r_- - r_+ \approx r_0\cos\theta$

$$r_- r_+ \approx r^2 \qquad V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-}$$

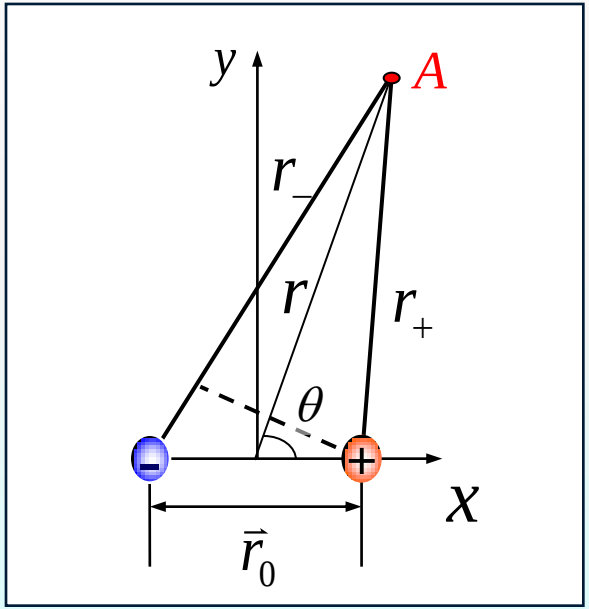


$$V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_+ r_-} \approx \frac{q}{4\pi\varepsilon_0}\frac{r_0\cos\theta}{r^2}$$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$r^2 = x^2 + y^2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{p}{4\pi\varepsilon_0}\frac{x}{(x^2+y^2)^{3/2}}$$



$$\begin{cases} \theta = 0 & V \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2} \\ \theta = \pi & V \approx -\dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2} \\ \theta = \dfrac{\pi}{2} & V = 0 \end{cases}$$

※ 电势差

注意区别:“电势差”与“电势增量”！

$$U_{AB}=V_A-V_B=\int_{AB}\vec{E}\cdot\mathrm{d}\vec{l}$$

$$\Delta V=V_B-V_A=-U_{AB}$$

将单位正电荷从A移到B时电场力作的功

几种常见的电势差（V）

生物电	10^{-3}	家用电器	110或220
普通干电池	1.5	高压输电线	已达1.0×10^{6}
汽车电源	12	闪电	10^{8}–10^{9}

※ 静电场力的功

$$W_{AB}=\int_{AB}q\vec{E}\cdot\mathrm{d}\vec{l}=q\int_{AB}\vec{E}\cdot d\vec{l}=qU_{AB}=q(V_A-V_B)$$

原子物理中能量单位：电子伏特（eV）

$$1\,\mathrm{eV}=1.602\times10^{-19}\,\mathrm{J}$$