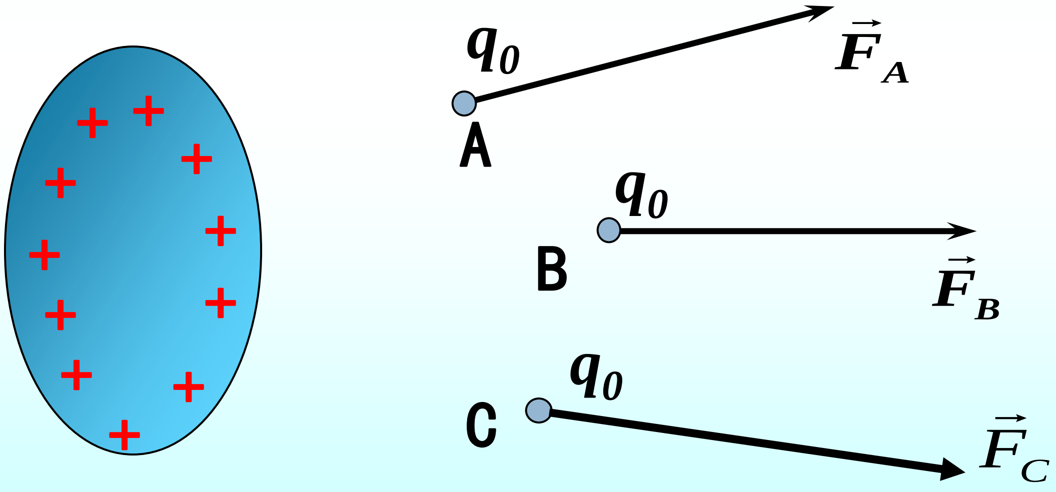
7.2 电场强度 场强叠加原理

※ 电场强度 描述电场中各点电场的强弱和方向的物理量

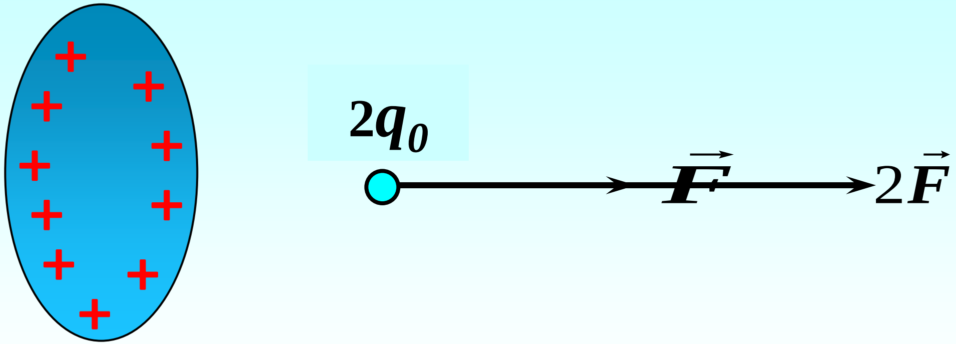
1. 试探(验)电荷q_0

静电场的最基本特征：

对引入电场中的其他电荷产生作用力.



试探电荷： 电量充分地小， 线度足够地小， 带正电。



2. 电场强度矢量

试验表明：对于给定的场点，比值 $\dfrac{\vec{F}}{q_0}$ 与试探电荷无关

电场强度定义： $\boxed{\vec{E}=\dfrac{\vec{F}}{q_0}}$

描述场中各点电场的强弱和方向的物理量

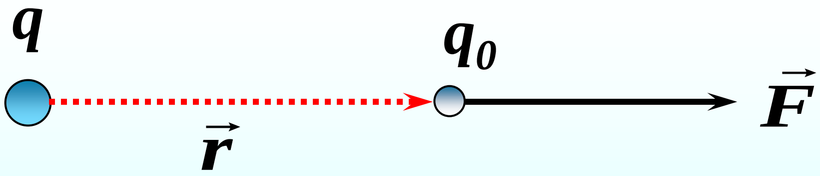
电场中某点的电场强度:大小等于单位电荷在该点受力的大小,
方向为正电荷在该点受力的方向

※ 电场强度的计算

$$\boxed{\vec{E}=\frac{\vec{F}}{q_0}} \qquad \vec{E}=\vec{E}(x,y,z)$$

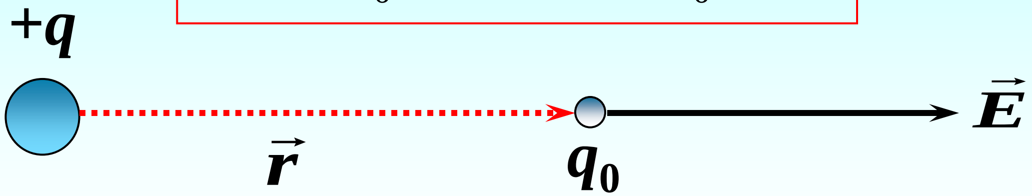
$$\vec{r}=r\cdot\vec{e}_r$$

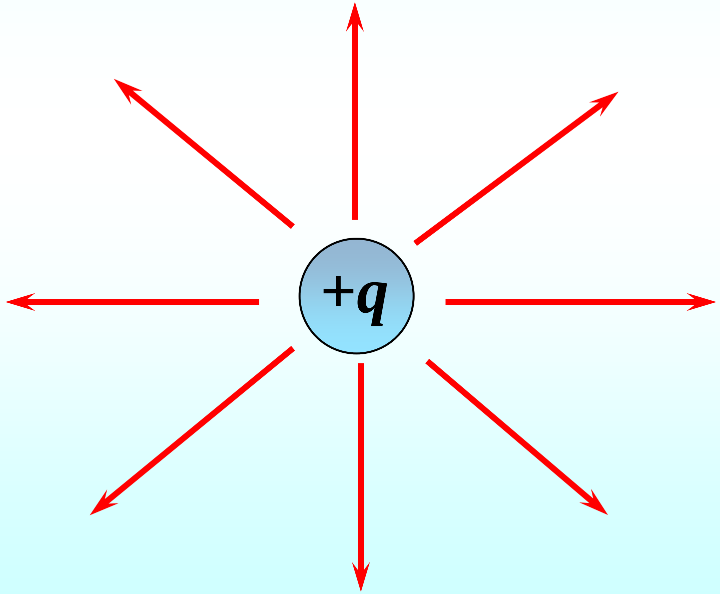
(1) 点电荷 q 的场强



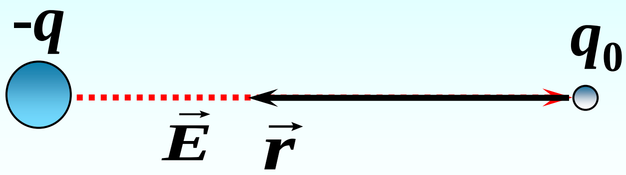
$$\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^3}\vec{r} \qquad \vec{E}=\frac{\vec{F}}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{e}_r$$

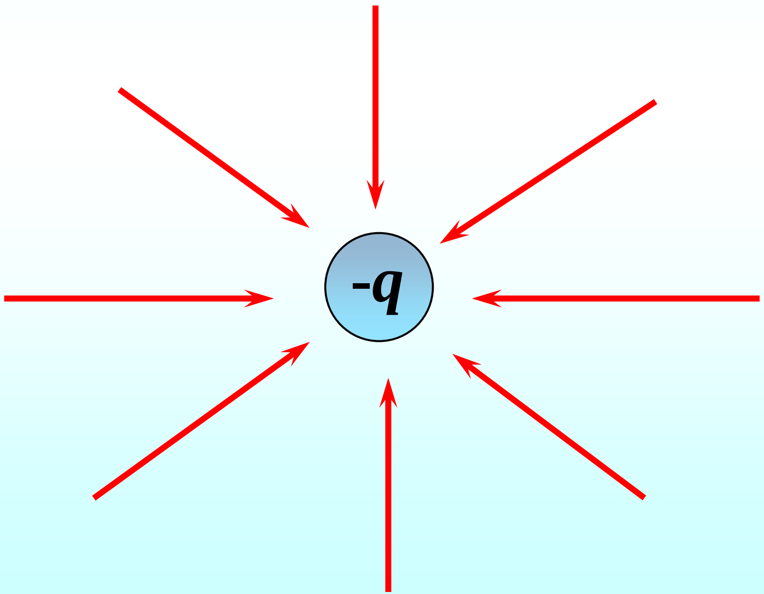
$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{e}_r$$





$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{e}_r$$





(2) 点电荷系的场强

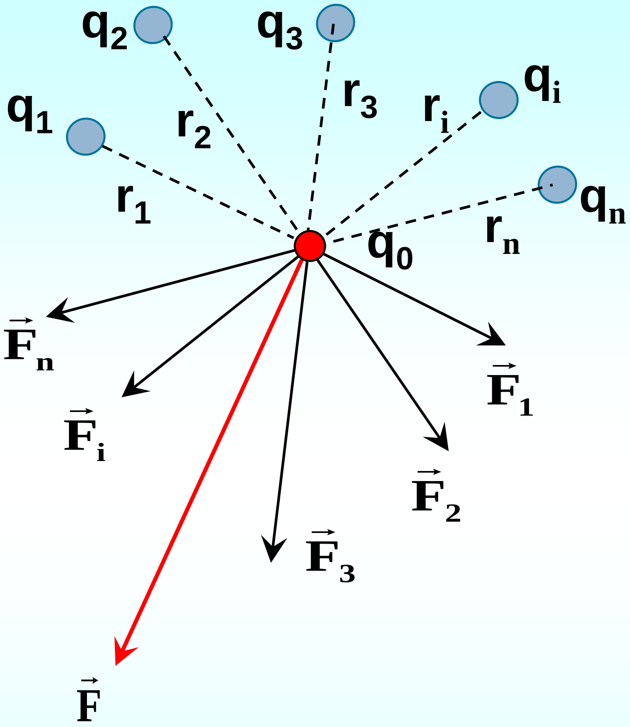
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\vec{e}_{r1}$$

$$\vec{E}_2 = \frac{\vec{F}_2}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\vec{e}_{r2}$$

$$\vec{E}_i = \frac{\vec{F}_i}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\vec{e}_{ri}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\vec{F}_1}{q_0} + \frac{\vec{F}_2}{q_0} + \cdots + \frac{\vec{F}_n}{q_0} = \sum_{i=1}^{n}\frac{\vec{F}_i}{q_0}$$



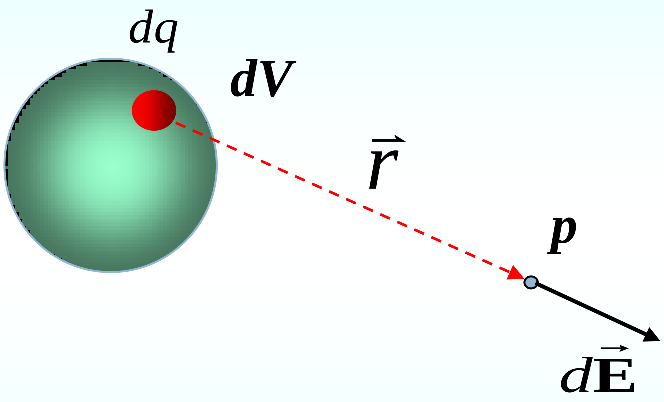
$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n = \sum_{i=1}^{n}\vec{E}_i = \sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\vec{e}_{ri}}$$

（场强叠加原理）

(3) 连续分布电荷的场强(电场强度)

微积分思想

$$d\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{e}_r$$



$$\boxed{\vec{E}_p=\int d\vec{E}=\frac{1}{4\pi\varepsilon_0}\int_{(V)}\frac{dq}{r^2}\vec{e}_r}$$

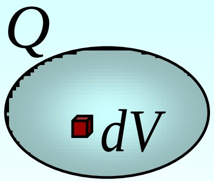
※ 场强叠加原理

任意带电体的场强

$$\boxed{\vec{E}=\sum_i\vec{E}_i=\int d\vec{E}=\int\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\vec{e}_r}$$

电荷密度

体分布



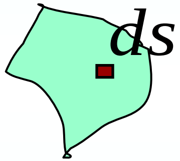
$$dq = \rho dV$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iiint_{(V)} \frac{\rho dV}{r^2} \vec{e}_r$$

ρ：电荷体密度

$$\boxed{\rho = \frac{dq}{dV}}$$

面分布



$$dq = \sigma dS$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint_{(S)} \frac{\sigma dS}{r^2} \vec{e}_r$$

σ：电荷面密度

$$\boxed{\sigma = \frac{dq}{ds}}$$

线分布

$$dq = \lambda dl$$

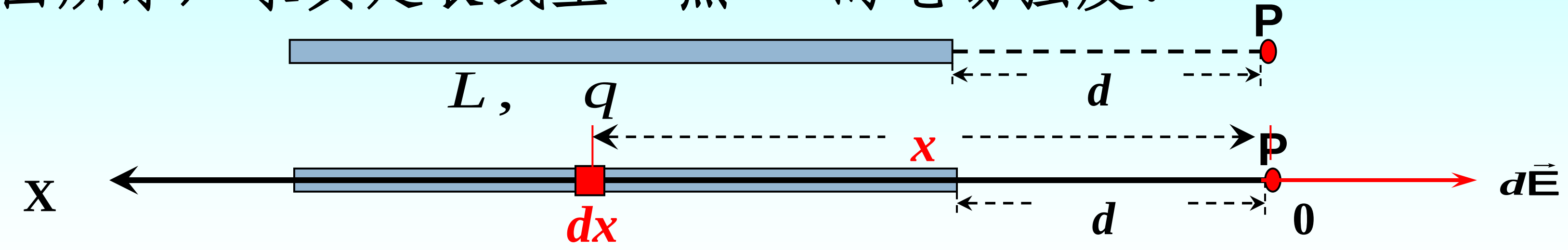
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{(L)} \frac{\lambda dl}{r^2} \vec{e}_r$$

λ：电荷线密度

$$\boxed{\lambda = \frac{dq}{dl}}$$

例1：设有一均匀带电直线段，长度为L，总电荷量为q，（如图所示）求其延长线上一点 P 的电场强度.

解



建坐标系如图所示, 在坐标为 x 处取一线元 dx, 视为点电荷,电量为:

微积分思想

$$dq = \lambda dx, \qquad \lambda = \frac{q}{L} \qquad d\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}\vec{i}$$

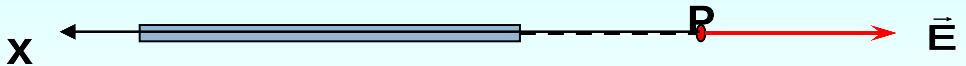
$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\int_d^{d+L}\frac{\lambda dx}{x^2}\vec{i} = -\frac{1}{4\pi\varepsilon_0}\left(-\frac{\lambda}{x}\right)\vec{i}\,\Bigg|_d^{d+L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{d+L} - \frac{1}{d}\right)\vec{i} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{d(d+L)}\vec{i}$$

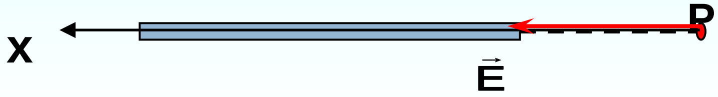
讨论

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{d(d+L)}\vec{i}$$

1) $q > 0$　$\vec{E}$沿x负方向



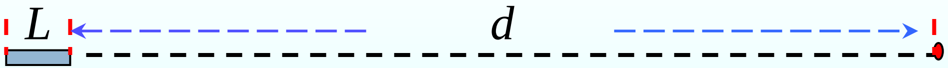
$q < 0$　$\vec{E}$沿x正方向



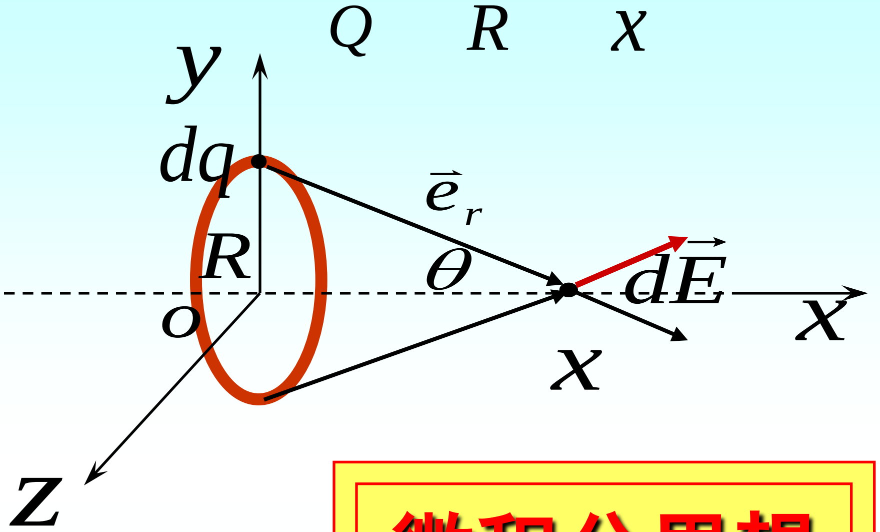
2）可以大致检查此题结果是否正确

当$d >> L$时，

$$\vec{E} \approx -\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\vec{i}$$



例2. 求均匀带电圆环轴线上的场强



解：

在圆环上任取电荷元 dq

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

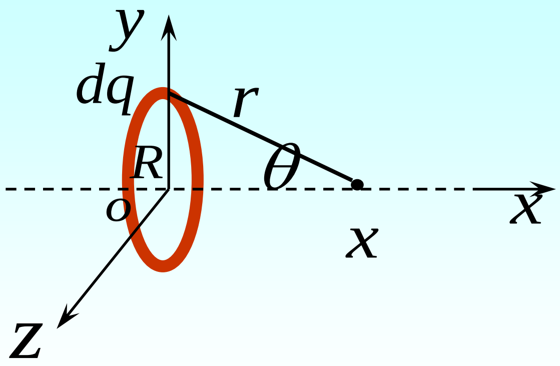
微积分思想

$$dE_x = dE\cos\theta$$
$$dE_{\perp x} = dE\sin\theta$$

由对称性分析知垂直x轴的场强为0

$$\Rightarrow \quad \vec{E} = E_x\vec{i}$$

$$\vec{E} = E_x \vec{i} = \vec{i} \int dE_x$$



$$E = E_x = \int_{(Q)} \frac{dq}{4\pi\,\varepsilon_0 r^2} \cos\theta$$

$$= \frac{\cos\theta}{4\pi\varepsilon_0 r^2} \int_{(Q)} dq \longrightarrow \boxed{E = \frac{xQ}{4\pi\,\varepsilon_0 \left(x^2 + R^2\right)^{3/2}}}$$

$$\cos\theta = \frac{x}{r}$$

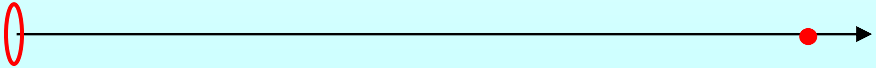
$$r = \sqrt{R^2 + x^2}$$

若 $x >> R$

$$\boxed{E = \frac{Q}{4\pi\,\varepsilon_0 x^2} = \frac{Q}{4\pi\,\varepsilon_0 r^2}}$$

点电荷

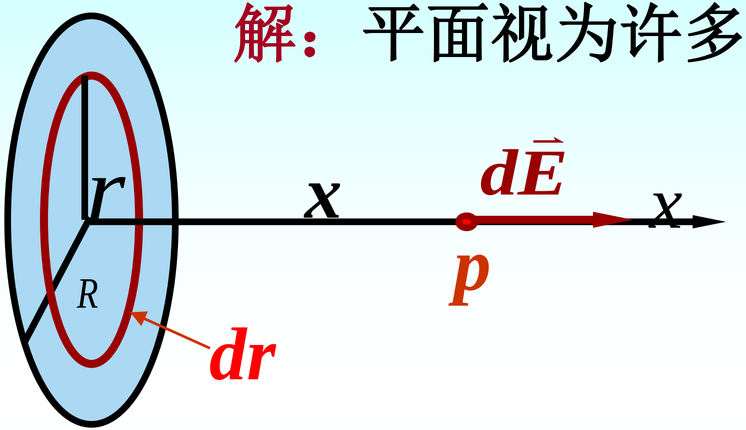
若 $x = 0, \quad E = ?$



例 3. 求总电量Q ,半径R 的均匀带电圆盘轴线上的场强。

解：平面视为许多同心圆环组成

微积分思想



$$\mathrm{d}\vec{E}=\frac{x\mathrm{d}Q\vec{i}}{4\pi\varepsilon_o\left(x^2+r^2\right)^{\frac{3}{2}}}$$

$$\mathrm{d}Q=\frac{Q}{\pi R^2}\cdot 2\pi r\mathrm{d}r=\frac{2Qr\mathrm{d}r}{R^2}$$

$$E=\frac{xQ}{2\pi\varepsilon_o R^2}\int_0^R\frac{r\mathrm{d}r}{\left(x^2+r^2\right)^{\frac{3}{2}}}$$

$$\boxed{\vec{E}=\frac{Q}{2\pi\varepsilon_o R^2}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)\vec{i}}$$

当 $R>>x$
$$\vec{E}=\frac{Q\qquad\vec{i}}{2\pi\varepsilon_0 R^2}=\frac{\sigma}{2\varepsilon_o}\vec{i}$$

无限大带电平面场强

$(\sigma=Q/\pi R^2)$

例4.　长为L的均匀带电直杆，电荷线密度为λ

求 它在空间一点 P 产生的电场强度。

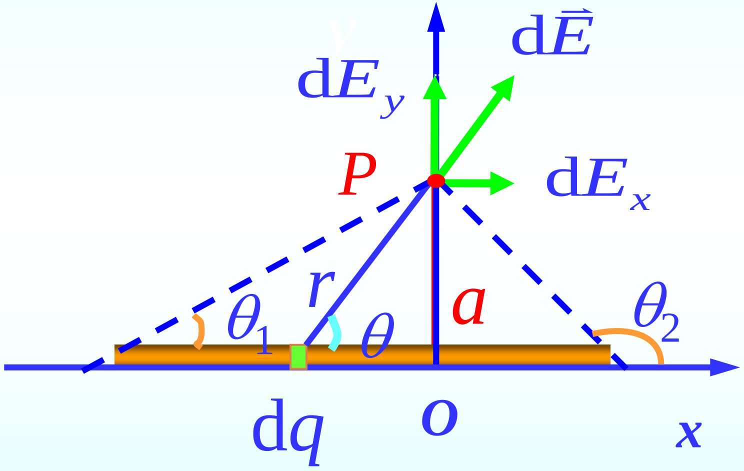
（P点到杆的垂直距离为 a ）

解 $dq = \lambda dx$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r^2}$$

$$dE_x = dE\cos\theta$$

$$dE_y = dE\sin\theta$$



由图上的几何关系

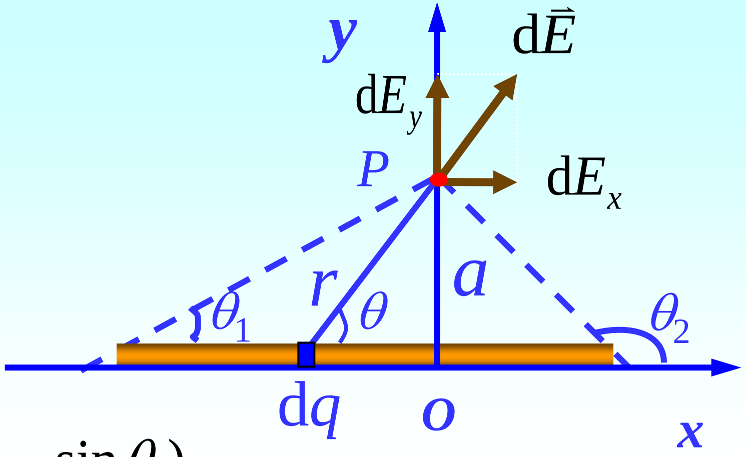
$$x = -a\tan\theta$$

$$dx = a\csc^2\theta d\theta$$

$$r^2 = a^2 + x^2 = a^2\csc^2\theta$$

微积分思想

$$\left.\begin{aligned} dE_x &= \frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta d\theta \\ dE_y &= \frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta d\theta \end{aligned}\right\}$$



$$E_x = \int dE_x = \int_{\theta_1}^{\theta_2}\frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$

$$E_y = \int dE_y = \int_{\theta_1}^{\theta_2}\frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$$

讨论： 无限长直导线

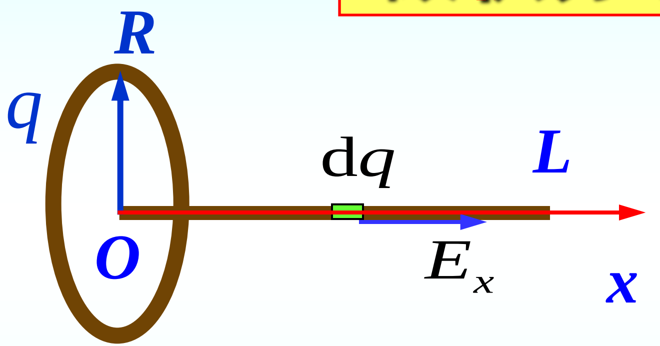
$\theta_1 = 0$ $\qquad \theta_2 = \pi$ $\qquad E_y = \dfrac{\lambda}{2\pi\varepsilon_0 a}$ $\qquad E_x = 0$

例5 已知圆环带电量为q，杆的电荷线密度为λ，长为L

求：杆对圆环的作用力

微积分思想

解 $\mathrm{d}q = \lambda \mathrm{d}x$



$$E_x = \frac{1}{4\pi\varepsilon_0} \frac{qx}{(R^2 + x^2)^{3/2}}$$

$$\mathrm{d}F = E_x \mathrm{d}q = E_x \lambda \mathrm{d}x$$

$$F = \int_0^L \frac{q\lambda x \mathrm{d}x}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}} = \frac{q\lambda}{4\pi\varepsilon_0} \int_0^L \frac{x \mathrm{d}x}{(R^2 + x^2)^{3/2}}$$

END