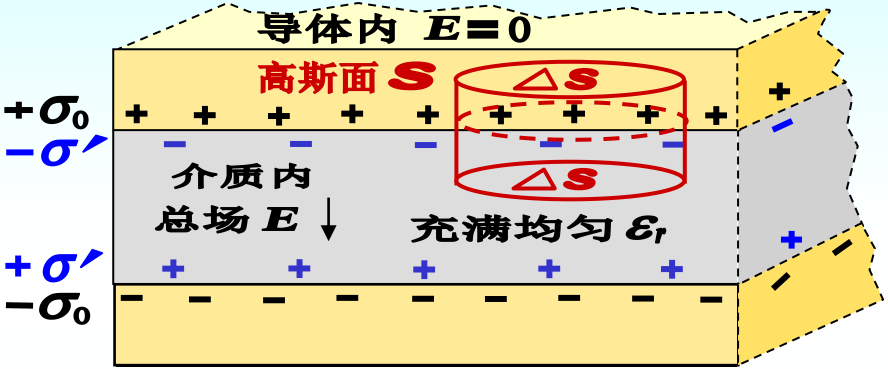
8.4 电位移 有介质时的高斯定理

※ 有介质时的高斯定理

加入电介质(ε_r)

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\sum_i q_i$$

$$= \frac{1}{\varepsilon_0}(\sigma_0 - \sigma')\Delta S$$



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{\sigma_0 \Delta S}{\varepsilon_0 \varepsilon_r} = \frac{1}{\varepsilon_0\varepsilon_r}\sum_i Q_{0i}$$

$$\because \sigma' = (1-\frac{1}{\varepsilon_r})\sigma_0$$

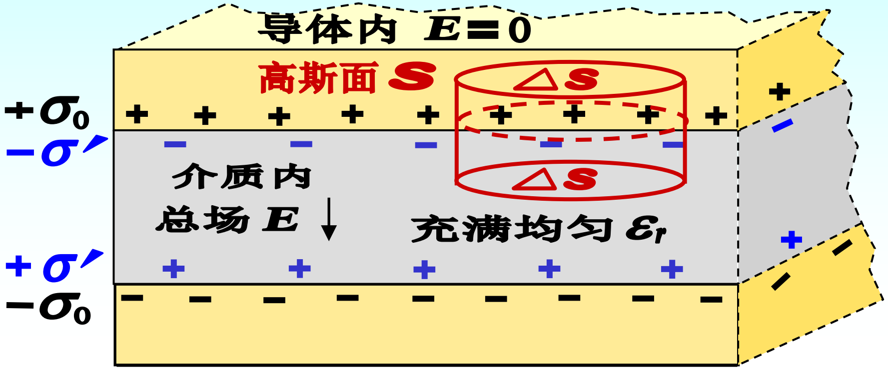
$$\boxed{\oint_S \varepsilon_0\varepsilon_r \vec{E}\cdot d\vec{S} = \sum_i Q_{0i}}$$

自由电荷的代数和

定义：电位移矢量$\vec{D}$

令：$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$

（电场中充满均匀各向同性电介质的情况下）



ε 电容率（介电常量）——决定于电介质种类的常数

$$\oint_S \underline{\varepsilon_0 \varepsilon_r \vec{E}} \cdot d\vec{S} = \sum_i Q_{0i}$$

自由电荷的代数和

有介质时的高斯定理

$$\boxed{\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = \sum_i Q_{0i}}$$

电介质中通过任一闭合曲面的电位移通量等于该曲面所包围的自由电荷的代数和。

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

在真空中： $\varepsilon_r = 1 \quad \varepsilon = \varepsilon_0$ $\quad \vec{D} = \varepsilon_0 \vec{E}$

$$\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = \oint_S \varepsilon_0 \vec{E}\cdot d\vec{S} = \sum_i Q_{0i}$$

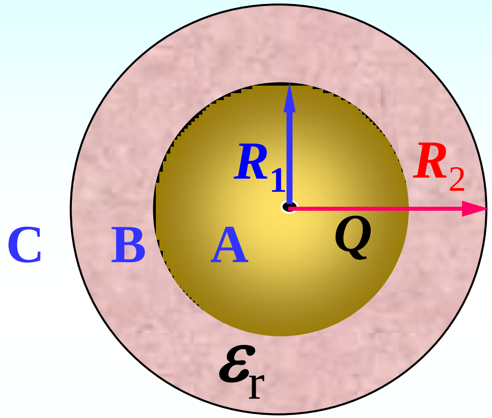
$$\oint_S \vec{E}\cdot \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0}\sum_i Q_{0i}$$

$$\oint_S \underline{\varepsilon_0 \varepsilon_r \vec{E}} \cdot \mathrm{d}\vec{S} = \sum_i Q_{0i}$$

自由电荷的代数和

例1 如图金属球半径为R_1、带电量$+Q$；均匀、各向同性电介质层外半径R_2、相对介电常数 ε_r ；

求： $\vec{D}$、$\vec{E}$ 的分布



解：对称性分析确定：

$\vec{E}$、$\vec{D}$ 沿矢径方向

由高斯定理： $\oint_S \vec{D}\cdot d\vec{S}=\sum_i q_{0i}$， $\vec{D}=\varepsilon_0\varepsilon_r\vec{E}=\varepsilon\vec{E}$

$$\oint \vec{D}\cdot d\vec{S}=\oint DdS=D\oint dS \qquad D\cdot 4\pi r^2=\sum_i q_{0i},$$

1) $r < R_1$ $\qquad D \cdot 4\pi r^2 = 0$

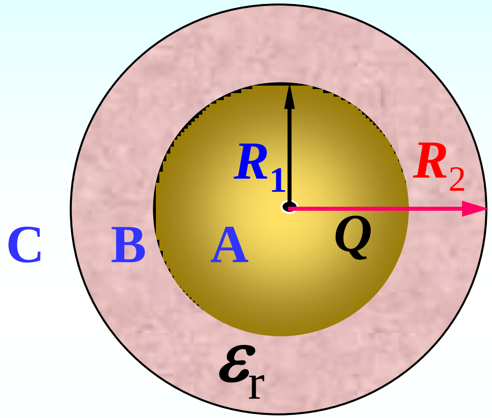
$D = 0, \quad E = 0$

2) $R_1 < r < R_2$ $\qquad D \cdot 4\pi r^2 = \sum_i q_{0i} = Q$

$D = \dfrac{Q}{4\pi r^2}, \quad E = \dfrac{Q}{4\pi\varepsilon\ r^2}$

3) $r > R_2$ $\qquad D \cdot 4\pi r^2 = \sum_i q_{0i} = Q$

$D = \dfrac{Q}{4\pi r^2}, \quad E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

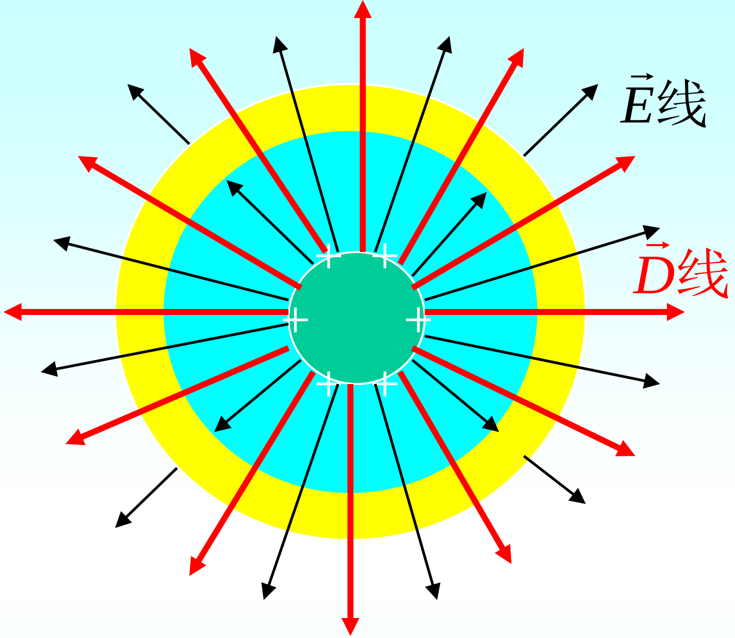


$$\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}}$$

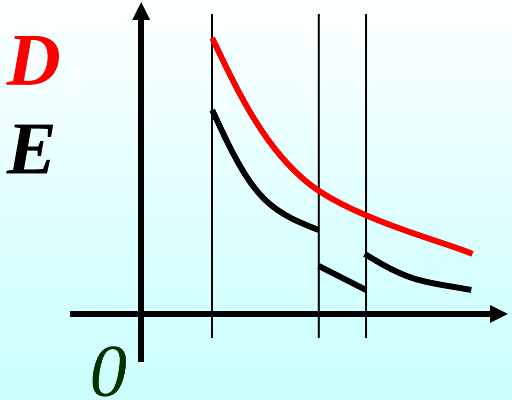
$$D \cdot 4\pi r^2 = \sum_i q_{0i}$$

※ 电位移线

电位移**D**线从正自由电荷发出，终止于负的自由电荷，与**E**线不同，**D**线在两介质界面上连续，而**E**线不连续。

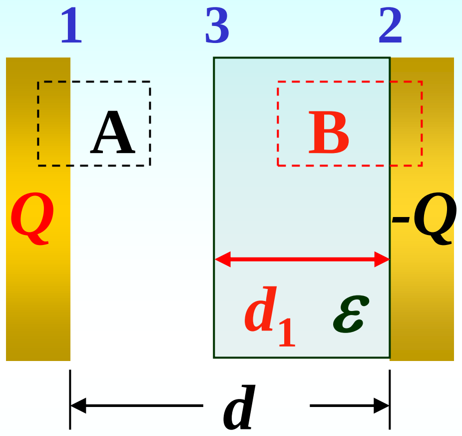


$$D=\frac{Q}{4\pi r^2},\qquad E=\frac{Q}{4\pi\varepsilon\ r^2}$$



例2 平板电容器极板间距d、面积S，带电量$\pm Q$，中间充一层厚度为d_1、介电常数为 ε 的均匀介质，

求：电场分布、极间电势差。



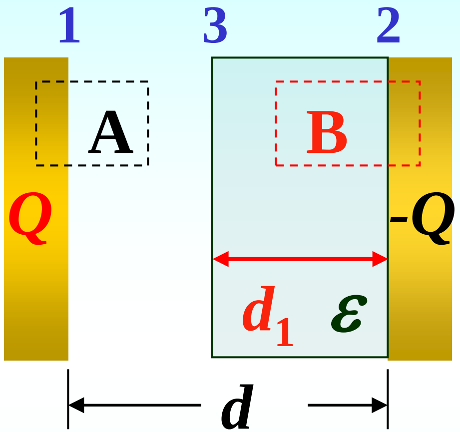
解：

$$\oint_S \vec{D}\cdot d\vec{S}=\sum_i q_{i0},\qquad \oint_S \vec{D}\cdot d\vec{S}=D\Delta S$$

$$D\Delta S=\sigma\Delta S,\quad \Rightarrow\quad D=\sigma\qquad \because D=\varepsilon E$$

$$E_{\mathrm{A}}=\frac{\sigma}{\varepsilon_0}\qquad E_{\mathrm{B}}=\frac{\sigma}{\varepsilon}$$

$$E_{\mathrm{A}} = \frac{\sigma}{\varepsilon_0} \qquad E_{\mathrm{B}} = \frac{\sigma}{\varepsilon}$$



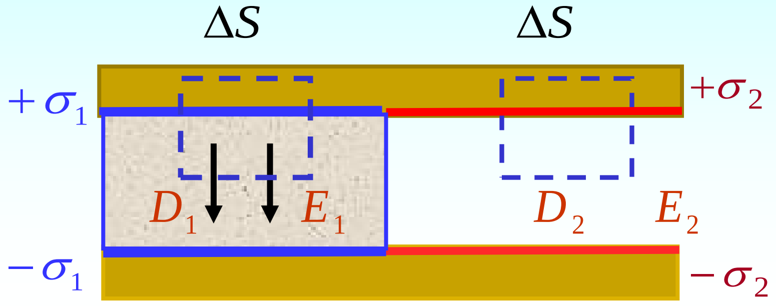
$$U_{12} = \int_1^2 \vec{E} \cdot d\vec{l}$$

$$= \int_1^3 \vec{E} \cdot d\vec{l} + \int_3^2 \vec{E} \cdot d\vec{l}$$

$$= \int_1^3 E_A dl + \int_3^2 E_B dl$$

$$= E_A(d - d_1) + E_B d_1 \quad = \frac{\sigma}{\varepsilon_0}(d - d_1) + \frac{\sigma}{\varepsilon} d_1$$

例3 两金属板间为真空，电荷面密度为$\pm\sigma_0$，电压 $U_0=300V$ 保持电量不变，一半空间充以的电介质 $\varepsilon_r=5$，板间电压变为多少？



解: 设金属板面积为**S** 间距为 **d**

$$U=Ed,\qquad U_0=E_0d=\frac{\sigma_0}{\varepsilon_0}d$$

$$U_1=E_1d,$$
$$\parallel\qquad\Rightarrow E_1=E_2$$
$$U_2=E_2d$$

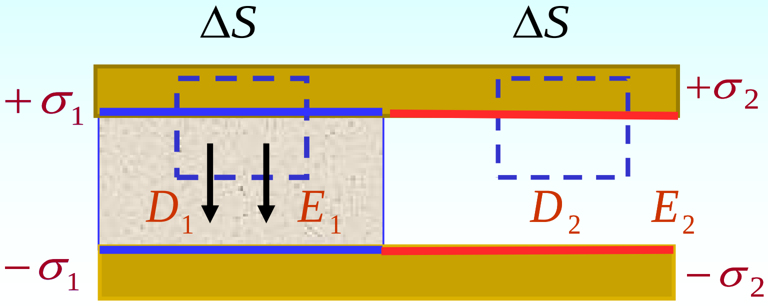
$$\oint_s\vec{D}_1\cdot\mathrm{d}\vec{S}=D_1\Delta S=\sigma_1\Delta S,\qquad D_1=\sigma_1,\quad E_1=\frac{D_1}{\varepsilon_0\varepsilon_r}=\frac{\sigma_1}{\varepsilon_0\varepsilon_r}$$

$$\parallel$$

同理可得：

$$D_2=\sigma_2,\quad E_2=\frac{D_2}{\varepsilon_0}=\frac{\sigma_2}{\varepsilon_0}\qquad\therefore\quad\sigma_2=\frac{\sigma_1}{\varepsilon_r}$$

例3 两金属板间为真空，电荷面密度为$\pm\sigma_0$，电压 $U_0 = 300V$ 保持电量不变，一半空间充以的电介质 $\varepsilon_r = 5$，板间电压变为多少？



解： 设金属板面积为**S** 间距为 **d**

由电荷守恒定律：

$$\sigma_1 \frac{S}{2} + \sigma_2 \frac{S}{2} = \sigma_0 S \qquad \sigma_1 + \sigma_2 = 2\sigma_0$$

$$\sigma_1 = \frac{2\varepsilon_r}{1+\varepsilon_r}\sigma_0 = \frac{5}{3}\sigma_0$$

$$E = E_1 = E_2 = \frac{\sigma_2}{\varepsilon_0} = \frac{2\sigma_0}{\varepsilon_0(1+\varepsilon_r)} = \frac{1}{3}E_0$$

$$\sigma_2 = \frac{2}{1+\varepsilon_r}\sigma_0 = \frac{1}{3}\sigma_0$$

$$U = Ed = \frac{1}{3}E_0 d = 100V$$

$$\therefore \quad \sigma_2 = \frac{\sigma_1}{\varepsilon_r}$$

END