应用多元统计作业

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应用统计学1701

5.1 试证$W(x)=(\boldsymbol{I}_1^{'}\boldsymbol{x}+c_1)-(\boldsymbol{I}_2^{'}\boldsymbol{x}+c_2)$

若$\boldsymbol{\Sigma_1}=\boldsymbol{\Sigma_2}=\cdots=\boldsymbol{\Sigma_k}=\boldsymbol{\Sigma}$

$$\begin{aligned}d^2(\boldsymbol{x},\pi_i)&=(\boldsymbol{x}-\boldsymbol{\mu}_i)^{'}\boldsymbol{\Sigma_i^{-1}}(\boldsymbol{x}-\boldsymbol{\mu_i})\\&=\boldsymbol{x}^{'}\boldsymbol{\Sigma}_i^{-1}\boldsymbol{x}-2\boldsymbol{\mu}_i^{'}\boldsymbol{\Sigma}_i^{-1}\boldsymbol{x}+\boldsymbol{\mu}_i^{'}\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i\\&=\boldsymbol{x}^{'}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}-2(\boldsymbol{I}_i^{'}\boldsymbol{x}-c_i)\end{aligned}$$

其中$\boldsymbol{I}_i^{'}=\boldsymbol{\Sigma_i^{-1}}\boldsymbol{\mu}_i,c_i=-\dfrac{1}{2}\boldsymbol{\mu}_i^{'}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_i,i=1,2,\cdots,k$

当组数$k=2$时，

$$\begin{cases}\boldsymbol{x}\in\pi_1,\ if\ \boldsymbol{I}_1^{'}\boldsymbol{x}+c_1\geq\boldsymbol{I}_2^{'}\boldsymbol{x}+c_2\\\boldsymbol{x}\in\pi_2,\ if\ \boldsymbol{I}_1^{'}\boldsymbol{x}+c_1<\boldsymbol{I}_2^{'}\boldsymbol{x}+c_2\end{cases}$$

$$\begin{aligned}d^2(\boldsymbol{x},\pi_1)-d^2(\boldsymbol{x},\pi_2)&=\boldsymbol{x}^{'}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}-2(\boldsymbol{I}_1^{'}\boldsymbol{x}-c_1)-[\boldsymbol{x}^{'}\boldsymbol{\Sigma}^{-1}\boldsymbol{x}-2(\boldsymbol{I}_2^{'}\boldsymbol{x}-c_2)]\\&=2[(\boldsymbol{I}_2^{'}\boldsymbol{x}-c_2)-(\boldsymbol{I}_1^{'}\boldsymbol{x}-c_1)]\end{aligned}$$

$$\begin{aligned}&W(x)=(\boldsymbol{I}_1^{'}\boldsymbol{x}+c_1)-(\boldsymbol{I}_2^{'}\boldsymbol{x}+c_2)\\&\begin{cases}\boldsymbol{x}\in\pi_1,\ if\ W(\boldsymbol{x})\geq0\ (d^2(\boldsymbol{x},\pi_1)\leq d^2(\boldsymbol{x},\pi_2)\\\boldsymbol{x}\in\pi_2,\ if\ W(\boldsymbol{x})<0\ (d^2(\boldsymbol{x},\pi_1)>d^2(\boldsymbol{x},\pi_2)\end{cases}\end{aligned}$$

5.2

假定$\boldsymbol{\Sigma_1}=\boldsymbol{\Sigma_2}$

$$\begin{aligned}\bar{\boldsymbol{x}}&=\frac{1}{2}(\bar{\boldsymbol{x}}_1+\bar{\boldsymbol{x}}_2)=\frac{1}{2}\cdot\left[\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}3\\-1\end{pmatrix}\right]=\begin{pmatrix}3.5\\0.5\end{pmatrix}\\\hat{\boldsymbol{a}}&=\boldsymbol{S}_p^{-1}(\bar{\boldsymbol{x}}_1-\bar{\boldsymbol{x}}_2)=\begin{pmatrix}6.5&1.1\\1.1&8.4\end{pmatrix}\cdot\left[\begin{pmatrix}4\\2\end{pmatrix}-\begin{pmatrix}3\\-1\end{pmatrix}\right]=\begin{pmatrix}0.0955\\0.3446\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\hat{W}(\boldsymbol{x})&=\hat{\boldsymbol{a}}^{'}(\boldsymbol{x}-\bar{\boldsymbol{x}})\\&=\begin{cases}\boldsymbol{x}\in\pi_1,\ 0.0955\boldsymbol{x}_1+0.3446\boldsymbol{x}_2-0.5066\geq0\\\boldsymbol{x}\in\pi_2,\ 0.0955\boldsymbol{x}_1+0.3446\boldsymbol{x}_2-0.5066<0\end{cases}\end{aligned}$$

$W(\boldsymbol{x}_0)=0.0955\times2+0.3446\times1-0.5066=0.0290\geq0,\therefore\boldsymbol{x}$判为组$\pi_1$。

5.3

考虑误判代价时

$$\begin{aligned}&l=1:p_2c(1|2)f_2(\boldsymbol{x}_0)+p_3c(1|3)f_3(\boldsymbol{x}_0)=111\\&l=2:p_1c(2|1)f_1(\boldsymbol{x}_0)+p_3c(2|3)f_3(\boldsymbol{x}_0)=15.56\\&l=3:p_1c(3|1)f_1(\boldsymbol{x}_0)+p_2c(3|2)f_2(\boldsymbol{x}_0)=65.24\end{aligned}$$

由于$l=2$时的值15.56达到最小，所以将$\boldsymbol{x}_0$判归为π_2。

不考虑误判代价时

$$\begin{aligned} p_1 f_1(\boldsymbol{x}_0) &= 0.253 \\ p_2 f_2(\boldsymbol{x}_0) &= 0.225 \\ p_3 f_3(\boldsymbol{x}_0) &= 0.210 \end{aligned}$$

由于0.253达到最大，所以将$\boldsymbol{x}_0$判归为π_1。

5.4

$\because \pi_i \sim N_p(\boldsymbol{\mu_i}, \boldsymbol{\Sigma_i}), i=1,2, \therefore$

$$\frac{f_1(\boldsymbol{x})}{f_2(\boldsymbol{x})} = \frac{(2\pi)^{-p/2}|\boldsymbol{\Sigma}_1|^{-1/2}\exp\left[-\frac{1}{2}d^2(\boldsymbol{x},\pi_1)\right]}{(2\pi)^{-p/2}|\boldsymbol{\Sigma}_2|^{-1/2}\exp\left[-\frac{1}{2}d^2(\boldsymbol{x},\pi_2)\right]} = \frac{|\boldsymbol{\Sigma}_1|^{-1/2}}{|\boldsymbol{\Sigma}_2|^{-1/2}}\exp\left\{-\frac{1}{2}[d^2(\boldsymbol{x},\pi_1)-d^2(\boldsymbol{x},\pi_2)]\right\}$$

$$\begin{cases} \boldsymbol{x}\in\pi_1,\ \frac{|\boldsymbol{\Sigma}_1|^{-1/2}}{|\boldsymbol{\Sigma}_2|^{-1/2}}\exp\{-\frac{1}{2}[d^2(\boldsymbol{x},\pi_1)-d^2(\boldsymbol{x},\pi_2)]\} \geq \frac{c(1|2)p_2}{c(2|1)p_1} \\ \boldsymbol{x}\in\pi_2,\ \frac{|\boldsymbol{\Sigma}_1|^{-1/2}}{|\boldsymbol{\Sigma}_2|^{-1/2}}\exp\{-\frac{1}{2}[d^2(\boldsymbol{x},\pi_1)-d^2(\boldsymbol{x},\pi_2)]\} < \frac{c(1|2)p_2}{c(2|1)p_1} \end{cases}$$

(1) $\boldsymbol{\Sigma}_1=\boldsymbol{\Sigma}_2=\boldsymbol{\Sigma}$的判别

$$\begin{cases} \boldsymbol{x}\in\pi_1,\ if\ \boldsymbol{a}'(\boldsymbol{x}-\bar{\boldsymbol{\mu}}) \geq \dfrac{c(1|2)p_2}{c(2|1)p_1} \\ \boldsymbol{x}\in\pi_2,\ if\ \boldsymbol{a}'(\boldsymbol{x}-\bar{\boldsymbol{\mu}}) < \dfrac{c(1|2)p_2}{c(2|1)p_1} \end{cases}$$

$W(\boldsymbol{x}) = -\dfrac{1}{2}[d^2(\boldsymbol{x},\pi_1), d^2(\boldsymbol{x},\pi_2)] = \boldsymbol{a}'(\boldsymbol{x}-\bar{\boldsymbol{\mu}})$

(2) $\boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2$的判别

$$\begin{cases} \boldsymbol{x}\in\pi_1,\ if\ d^2(\boldsymbol{x},\pi_1)-d^2(\boldsymbol{x},\pi_2) \leq 2\ln\left[\dfrac{c(2|1)p_1|\boldsymbol{\Sigma}_2|^{1/2}}{c(1|2)p_1|\boldsymbol{\Sigma}_1|^{1/2}}\right] \\ \boldsymbol{x}\in\pi_2,\ if\ d^2(\boldsymbol{x},\pi_1)-d^2(\boldsymbol{x},\pi_2) > 2\ln\left[\dfrac{c(2|1)p_1|\boldsymbol{\Sigma}_2|^{1/2}}{c(1|2)p_1|\boldsymbol{\Sigma}_1|^{1/2}}\right] \end{cases}$$