5.1 试证
$$W(x) = (\mathbf{I}_1' \mathbf{x} + c_1) - (\mathbf{I}_2' \mathbf{x} + c_2)$$

若
$$\Sigma_1 = \Sigma_2 = \cdots = \Sigma_k = \Sigma$$

$$egin{aligned} d^2(oldsymbol{x},\pi_i) &= (oldsymbol{x} - oldsymbol{\mu}_i)^{'} oldsymbol{\Sigma}_i^{-1}(oldsymbol{x} - oldsymbol{\mu}_i) \ &= oldsymbol{x}^{'} oldsymbol{\Sigma}_i^{-1} oldsymbol{x} - 2 oldsymbol{\mu}_i^{'} oldsymbol{\Sigma}_i^{-1} oldsymbol{x} + oldsymbol{\mu}_i^{'} oldsymbol{\Sigma}_i^{-1} oldsymbol{\mu}_i \ &= oldsymbol{x}^{'} oldsymbol{\Sigma}^{-1} oldsymbol{x} - 2 (oldsymbol{I}_i^{'} oldsymbol{x} - c_i) \end{aligned}$$

其中
$$oldsymbol{I}_{i}^{'}=oldsymbol{\Sigma}_{i}^{-1}oldsymbol{\mu}_{i},c_{i}=-rac{1}{2}oldsymbol{\mu}_{i}^{'}oldsymbol{\Sigma}^{-1}oldsymbol{\mu}_{i},i=1,2,\cdots,k$$

当组数k=2时,

$$egin{aligned} \left\{ m{x} \in \pi_1, \ if \ m{I_1'}m{x} + c_1 \geq m{I_2'}m{x} + c_2 \ m{x} \in \pi_2, \ if \ m{I_1'}m{x} + c_1 < m{I_2'}m{x} + c_2 \end{aligned}
ight. \ \left\{ m{x} \in \pi_2, \ if \ m{I_1'}m{x} + c_1 < m{I_2'}m{x} + c_2 \end{aligned}
ight. \ \left\{ m{d}^2(m{x}, \pi_1) - m{d}^2(m{x}, \pi_2) = m{x'} m{\Sigma}^{-1}m{x} - 2(m{I_1'}m{x} - c_1) - [m{x'} m{\Sigma}^{-1}m{x} - 2(m{I_2'}m{x} - c_2)] \\ &= 2[(m{I_2'}m{x} - c_2) - (m{I_1'}m{x} - c_1)] \end{aligned} \ W(x) = (m{I_1'}m{x} + c_1) - (m{I_2'}m{x} + c_2) \\ \left\{ m{x} \in \pi_1, \ if \ W(m{x}) \geq 0 \ (m{d}^2(m{x}, \pi_1) \leq m{d}^2(m{x}, \pi_2) \\ m{x} \in \pi_2, \ if \ W(m{x}) < 0 \ (m{d}^2(m{x}, \pi_1) > m{d}^2(m{x}, \pi_2) \end{aligned}
ight.$$

5. 2

假定 $\Sigma_1 = \Sigma_2$

$$egin{aligned} ar{m{x}} &= rac{1}{2} (ar{m{x}}_1 + ar{m{x}}_2) = rac{1}{2} \cdot igg[inom{4}{2} + inom{3}{-1}igg] = inom{3.5}{0.5} \ \hat{m{a}} &= m{S}_p^{-1} (ar{m{x}}_1 - ar{m{x}}_2) = inom{6.5}{1.1}{1.1} \cdot inom{4}{8.4} \cdot igg[inom{4}{2} - inom{3}{-1}igg] = inom{0.0955}{0.3446} \ \hat{m{W}}(m{x}) = \hat{m{a}}'(m{x} - ar{m{x}}) \ &= igg\{m{x} \in \pi_1, \ 0.0955m{x}_1 + 0.3446m{x}_2 - 0.5066 \geq 0 \\ m{x} \in \pi_2, \ 0.0955m{x}_1 + 0.3446m{x}_2 - 0.5066 < 0 \ \end{pmatrix}$$

$$W(\boldsymbol{x}_0) = 0.0955 \times 2 + 0.3446 \times 1 - 0.5066 = 0.0290 \ge 0$$
, ∴ \boldsymbol{x} 判为组 π_1 。

5.3

考虑误判代价时

$$egin{aligned} l &= 1: p_2 c(1|2) f_2(oldsymbol{x}_0) + p_3 c(1|3) f_3(oldsymbol{x}_0) = 111 \ l &= 2: p_1 c(2|1) f_1(oldsymbol{x}_0) + p_3 c(2|3) f_3(oldsymbol{x}_0) = 15.56 \ l &= 3: p_1 c(3|1) f_1(oldsymbol{x}_0) + p_2 c(3|2) f_2(oldsymbol{x}_0) = 65.24 \end{aligned}$$

由于l=2时的值15.56达到最小,所以将 x_0 判归为 π_2 。

不考虑误判代价时

$$p_1 f_1({m x}_0) = 0.253 \ p_2 f_2({m x}_0) = 0.225 \ p_3 f_3({m x}_0) = 0.210$$

由于0.253达到最大,所以将 x_0 判归为 π_1 。

5.4

$$\because \pi_i \sim N_p(oldsymbol{\mu_i}, oldsymbol{\Sigma_i}), i = 1, 2, \therefore$$

$$egin{split} rac{f_1(oldsymbol{x})}{f_2(oldsymbol{x})} &= rac{(2\pi)^{-p/2}|oldsymbol{\Sigma}_1|^{-1/2}\exp\left[-rac{1}{2}d^2(oldsymbol{x},\pi_1)
ight]}{(2\pi)^{-p/2}|oldsymbol{\Sigma}_2|^{-1/2}\exp\left[-rac{1}{2}d^2(oldsymbol{x},\pi_2)
ight]} &= rac{|oldsymbol{\Sigma}_1|^{-1/2}}{|oldsymbol{\Sigma}_2|^{-1/2}}\exp\left\{-rac{1}{2}[d^2(oldsymbol{x},\pi_1)-d^2(oldsymbol{x},\pi_2)]
ight\}}{|oldsymbol{\Sigma}_2|^{-1/2}}\exp\left\{-rac{1}{2}[d^2(oldsymbol{x},\pi_1)-d^2(oldsymbol{x},\pi_2)]
ight\}}{c(2|1)p_1} &\geq rac{c(1|2)p_2}{c(2|1)p_1} &\\ oldsymbol{x}\in\pi_2, rac{|oldsymbol{\Sigma}_1|^{-1/2}}{|oldsymbol{\Sigma}_2|^{-1/2}}\exp\left\{-rac{1}{2}[d^2(oldsymbol{x},\pi_1)-d^2(oldsymbol{x},\pi_2)]
ight\}}{c(2|1)p_1} &\leq rac{c(1|2)p_2}{c(2|1)p_1} &\\ &\frac{1}{2}[a^2(oldsymbol{x},\pi_1)-a^2(oldsymbol{x},\pi_2)] &\frac{1}{2}[a^2(oldsymbol{x},\pi_2)] &\frac{1}{2}[a^2(oldsymbol$$

(1) $\Sigma_1 = \Sigma_2 = \Sigma$ 的判别

$$\left\{egin{aligned} m{x} \in \pi_1, \; if \; m{a}^{'}(m{x} - ar{m{\mu}}) \geq rac{c(1|2)p_2}{c(2|1)p_1} \ m{x} \in \pi_2, \; if \; m{a}^{'}(m{x} - ar{m{\mu}}) < rac{c(1|2)p_2}{c(2|1)p_1} \end{array}
ight.$$

$$W(m{x}) = -rac{1}{2}[d^{2}(m{x},\pi_{1}),d^{2}(m{x},\pi_{2})] = m{a}^{'}(m{x}-ar{m{\mu}})$$

(2) $\Sigma_1 \neq \Sigma_2$ 的判别

$$\left\{egin{aligned} m{x} \in \pi_1, \; if \; d^2(m{x}, \pi_1) - d^2(m{x}, \pi_2) & \leq 2 \ln \left[rac{c(2|1)p_1|m{\Sigma}_2|^{1/2}}{c(1|2)p_1|m{\Sigma}_1|^{1/2}}
ight] \ m{x} \in \pi_2, \; if \; d^2(m{x}, \pi_1) - d^2(m{x}, \pi_2) & > 2 \ln \left[rac{c(2|1)p_1|m{\Sigma}_2|^{1/2}}{c(1|2)p_1|m{\Sigma}_1|^{1/2}}
ight] \end{array}
ight.$$