Indian Institute of Technology, Bombay

Digital Signal Processing

EE - 338

Filter Design Assignment

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Contents

1 Student Details

Name : *Chandan Kumar*
Roll Number : *16D070061*
Filter Number(M) : 126

2 Filter-1(Bandpass) Details

1. Un-normalized discrete time filter specifications
Ideally sampled
filter number (M)-126
Since filter number is >75 , m = 126 - 75 = 51 and passband will be equiripple
q(m) = greatest integer strictly less than 0.1m
q(51) = 5
r(m) = m – 10q(m) = r(51) = 1
BL(m) = 5 + 1.4 q(m) + 4 r(m) = BL(51) = 16
BH(m) = BL(m) + 10 = BH(51) = 26
BH(51) = 26 and BL(51) = 16
The first filter is given to be a **Band-Pass** filter with passband from B_L(m) kHz to B_H(m) kHz.
Therefore the specifications are :-
• **Passband** : 16 kHz to 26 kHz
•**Transition Band** : 2 kHz on either side of passband
• **Stopband** : 14 kHz and 28-140 kHz (∵ Sampling rate is 320 kHz)
• **Tolerance** : 0.15 in magnitude for both Passband and Stopband
• **Passband Nature** : Equiripple
• **Stopband Nature** : Monotonic

2.2 Normalized Digital Filter Specifications

Sampling Rate = 320 kHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 160 kHz(*SamplingRate* /2) can be represented on the
normalized axis(ω) as :-

$$\omega = \frac{\Omega * 2\pi}{\Omega s (SamplingRate}$$

Therefore the corresponding normalized discrete filter specifications are :-
• **Passband** : 0.10000π to 0.1625π

- **Transition Band** : 0.0125π on either side of passband
- **Stopband** : 0-0.0875π and 0.1750π-0.8750π
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.3 Analog filter specifications for Band-pass analog filter using Bilinear Transformation

The bilinear transformation is given as :-

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
0.10000π	0.15838
0.1625π	0.2609
0.0875π	0.1383
0.1750π	0.2820
0	0
π	∞
0.8750π	5.0273

Therefore the corresponding analog filter specifications for the same type of analog filter using the
bilinear transformation are :-
- **Passband** : 0.15838($\Omega P1$) to 0.2609($\Omega P2$)
- **Transition Band** : 0.1383 to 0.15838 & 0.2609 to 0.2820
- **Stopband** : 0 to 0.1383($\Omega S1$) and 0.2820($\Omega S2$) to ∞
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.4 Frequency Transformation & Relevant Parameters

We need to transform a Band-Pass analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandpass transformation which is given as :-

$$\Omega_L = \frac{(\Omega 2 - \Omega 02)}{B\Omega}$$

The two parameters in the above equation are B and Ω0. They can be determined using the specifications of the bandpass analog filter using the following relations :-

$$\Omega_0 = \sqrt{\Omega P1 * \Omega P2} = \sqrt{0.15838 * 0.2609} = 0.2030$$

$$B = \Omega_{P2} - \Omega_{P1} = 0.2609 - 0.15838 = 0.10252$$

Ω	Ω_L
0+	$-\infty$
0.1383(Ω_{S1})	-1.55748
0.15838 (Ω_{P1})	-1 (-0.993)
0.2030Ω_0)	0
0.2609 (Ω_{P2})	1.00042
0.2820Ω_{S2})	1.32546
∞	∞

2.5 Frequency Transformed Low Pass Analog Filter Specifications
• **Passband Edge** : 1 (ΩLP)
• **Stopband Edge** : min(-ΩLS, , $\Omega LS2$) = min(1.55748, 1.32546) = 1.32546(ΩLS)
• **Tolerance** : 0.15 in magnitude for both Passband and Stopband
• **Passband Nature** : Equiripple
• **Stopband Nature** : Monotonic

2.6 Analog Lowpass Transfer Function

We need an Analog Filter which has an *equiripple passband* and a *monotonic stopband*. Therefore we need to design using the **Chebyshev** approximation. Since the

olerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way :-

$$D_1 = 1/(1 - \delta)^2 - 1 = 1/0.85^2 - 1 = 0.3841$$
$$D_2 = 1/\delta^2 - 1 = 1/0.15^2 - 1 = 43.44$$

Now choosing the parameter of the Chebyshev filter to be $\sqrt{D1}$, we get the min value of N as :-

$$N_{min} = \left\lceil \frac{cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_{LS}}{\Omega_{LP}})} \right\rceil$$

Nmin = [3.8853] = 4

Now, the poles of the transfer function can be obtained by solving the equation :-

$$1 + D_1 cosh^2(N_{min} cosh^{-1}(\frac{s}{j})) = 1 + 0.3841 cosh^2(4 cosh^{-1}(\frac{s}{j})) = 0$$

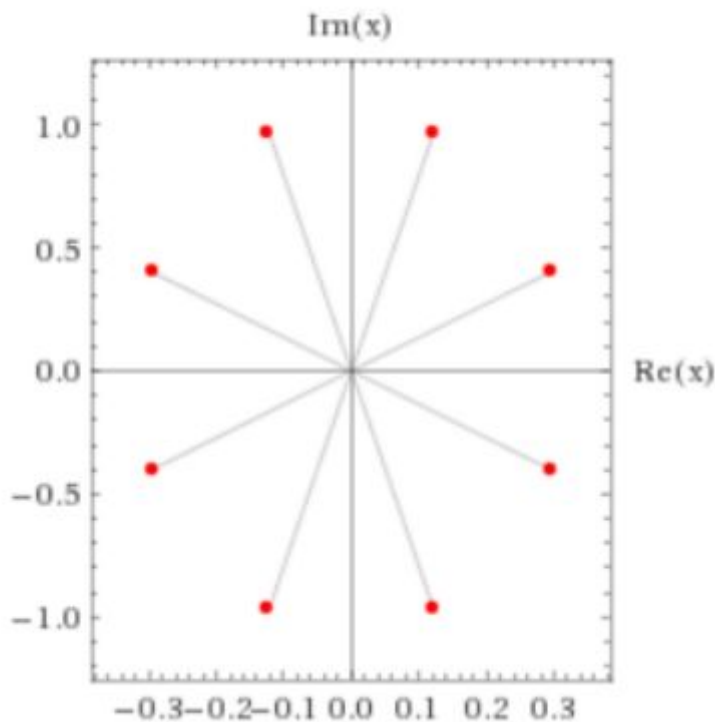Solving for the roots(using Wolfram) we get :-

Figure 1: Poles of Magnitude Plot of Analog LPF

Note that the above figure shows the poles of the Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

$$p1 = -0.1222 + 0.9698i$$
$$p2 = -0.1222 - 0.9698i$$
$$p3 = -0.2949 + 0.4017i$$
$$p4 = -0.2949 - 0.4017i$$

Using the above poles which are in the left half plane and the fact that N is even we can write the Analog Lowpass Transfer Function as :-

$$H_{analog}, \text{LP F (sL)} = \frac{(-1)4 \ p1p2p3p4}{\sqrt{(1+D1)}(sL -p\ 1)(sL-p2)(sL - p3)(sL - p4)}$$

$$Hanalog, LPF(sL) = \frac{0.198}{(s2\ L + 0.24432sL + 0.95545)(s2\ L + 0.58984sL + 0.24835)}$$

Note that since it is even order we take the DC Gain to be $\frac{1}{\sqrt{1+D1}}$ (since, epsilon is sqrt(D1))

2.7 Analog Bandpass Transfer Function
The transformation equation is given by :-

$$sL = \frac{s2 + \Omega02}{Bs}$$

Substituting the values of the parameters B(0.5151) and Ω_0(0.7650), we get :-

$$sL = \frac{(s2 + 0.041209)}{0.10252s} \qquad \left(\text{S2 is s\^2} \right)$$

Substituting this value into *Hanalog,LP F (sL)* we get *Hanalog,BP F (s)* as :-

Hanalog,BPF (s) =

0.2232 s^4

----

s^8 + 0.0856 s^7 + 0.1795 s^6 + 0.0113 s^5 + 0.0114 s^4 + 0.0005 s^3 + 0.0003 s^2

2.8 Discrete Time Filter Transfer Function
To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as :-

$$s = \frac{(1 - z-1)}{(1 + z-1)}$$

Using above equation we get *Hdiscrete,BPF (z)* from *Hanalog,BPF (s)* as :-

Hdiscrete,BPF (z) =

$$\frac{0.0173 - 0.0693\ z^{\wedge}{-2} + 0.1039\ z^{\wedge}{-4} - 0.0693\ z^{\wedge}{-6} + 0.0173\ z^{\wedge}{-8}}{(1 - 7.18\ z^{\wedge}{-1} + 23.16\ z^{\wedge}{-2} - 43.78\ z^{\wedge}{-3} + 52.98\ z^{\wedge}{-4} - 42.02\ z^{\wedge}{-5} + 21.34\ z^{\wedge}{-6} - 6.35\ z^{\wedge}{-7} + 0.849\ z^{\wedge}{-8})}$$

2.9 Realization using Direct Form II

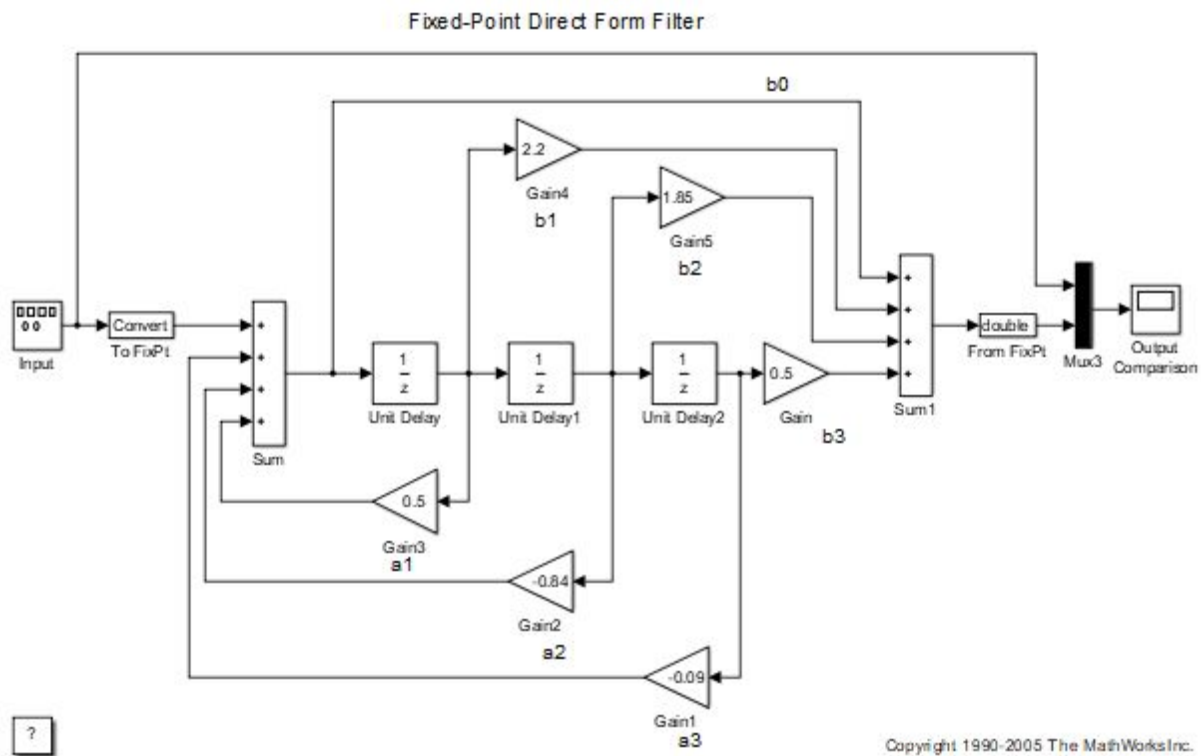(only screenshot since i don't know how to take the matlab image here)

Figure 2: Direct Form II Block Diagram for *Hdiscrete,BP F (z)*

Direct Form II is obtained by treating the transfer function H(z) = N(z)/D(z) as a cascade of 1/D(z) followed by N(z). The intermediate signal formed is the one whose samples get stored in the buffer.Thus it is advantageous in comparison to Direct Form I since it saves memory space.

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

2.10 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore δ = 0.15 and we get the minimum stopband attenuation to be :-

$$A = -20 \log(0.15) = 16.4782 dB$$

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega T}$$

Here $\Delta\omega T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega T = \frac{2kHz * 2\pi}{320kHz} = 0.0125\pi$$
$$\therefore N \geq 95.038$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **48** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters as done in class.

The z-transform can simply be read off from the sequence values since its finite sequence.

3 Filter-2(Bandstop) Details

3.1 Un-normalized Discrete Time Filter Specifications

Filter Number = 126
Since filter number is >75 , m = 126 - 75 = 51 and passband will be monotonic
q(m) = greatest integer strictly less than 0.1m.
q(m) = 5
r(m) = m - 10*q(m) = 51 - 10*5 = 1
BL(m) = 4 + 0.9*q(m) + 2*r(m) = 4 + 0.9*5 + 2*1 = 10.5
BH(m) = *BL*(m) + 10 = 10.5 + 10 = 20.5
The second filter is given to be a *Band-Stop* filter with stopband from *BL*(m) kHz to *BH*(m) kHz. Therefore the specifications are :-

      • **Stopband** : 10.5 kHz to 20.5 kHz

      • **Transition Band** : 2 kHz on either side of stopband

      • **Passband** : 8.5 kHz and 22.5-110 kHz (∵ Sampling rate is 250 kHz)

      • **Tolerance** : 0.15 in magnitude for both Passband and Stopband

      • **Passband Nature** : Monotonic

      • **Stopband Nature** : Monotonic

3.2 Normalized Digital Filter Specifications

Sampling Rate = 250 kHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 125 kHz(*SamplingRate* 2) can be represented on the normalized axis(ω) as :-

$$\omega = \frac{\Omega * 2\pi}{\Omega s (SamplingRate)}$$

Therefore the corresponding normalized discrete filter specifications are :-

      • **Stopband** : 0.084π to 0.164π

      • **Transition Band** : 0.016π on either side of stopband

      • **Passband** : 0-0.068π and 0.18π-0.88π

      • **Tolerance** : 0.15 in magnitude for both Passband and Stopband

      • **Passband Nature** : Monotonic

      • **Stopband Nature** : Monotonic

3.3 Analog filter specifications for Band-stop analog filter using Bilinear Transformation

The bilinear transformation is given as :-

$$\Omega = tan(\omega/2)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
0.084π	0.1327
0.164π	0.2634
0.068π	0.1077
0.18π	0.2905
0	0
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are :-

- **Stopband** : 0.1327($\Omega S1$) to 0.2634($\Omega S2$)

- **Transition Band** : 0.1077 to 0.1327 & 0.2634 to 0.2905

- **Passband** : 0 to 0.1077($\Omega P1$) and 0.2905($\Omega P2$) to ∞

- **Tolerance** : 0.15 in magnitude for both Passband and Stopband

- **Passband Nature** : Monotonic

- **Stopband Nature** : Monotonic

3.4 Frequency Transformation & Relevant Parameters

We need to transform a Band-Stop analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandstop transformation which is given as :-

$$\Omega_L = \frac{B\Omega}{\Omega 02 - \Omega 2}$$

The two parameters in the above equation are B and $\Omega 0$. They can be determined using the specifications of the bandpass analog filter using the following relations :-

$$\Omega 0 = \sqrt{\Omega P1 \Omega P2} = \sqrt{0.2905 * 0.1077} = 0.17688$$

$$B = \Omega p_1 - \Omega p_2 = 0.2905 - 0.1077 = 0.1828$$

Ω	ΩL
0+	0+
0.1077(ΩP_1)	1.0007
0.1327(ΩS_1)	1.7736
0.17688(Ω_{-0})	$+\infty$
0.17688(Ω_{+0})	$_{-}\infty$
0.2634(ΩS_2)	-1.264
0.2905ΩP_2)	-1.001
∞	0^{-}

3.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge** : 1 (ΩLP)
- **Stopband Edge** : min($\Omega LS1, -\Omega LS2$) = min(1.2637 1.7736) = 1.2637 (ΩLS)
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.6 Analog Lowpass Transfer Function

We need an Analog Filter which has a *monotonic passband* and a *monotonic stopband*. Therefore we need to design using the **Butterworth** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way :-

$$D1 = \frac{1}{(1-\delta)2} - 1 = \frac{1}{(0.85)2} - 1 = 0.3841$$

$$D_2 = \frac{1}{(\delta 2)^2} - 1 = \frac{1}{(0.15)^2} - 1 = 43.44$$

Now using the inequality on the order N of the filter for the Butterworth Approximation we get :-

$$N_{min} = \lceil \frac{\log \sqrt{\frac{D_2}{D_1}}}{\log \frac{\Omega_S}{\Omega_P}} \rceil$$

Nmin = [10.064] = 11

The cut-off frequency(Ωc) of the Analog LPF should satisfy the following constraint :-

$$\frac{\Omega_P}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_S}{D_2^{\frac{1}{2N}}}$$

1.04445 ≤ Ωc ≤ 1.064619

Thus we can choose the value of Ωc to be 1.054535
Now, the poles of the transfer function can be obtained by solving the equation :-

$$1 + \left(\frac{s}{j\Omega} \right) = 1 + \left(\frac{s}{j1.0545} \right)^{22}$$

Solving for the roots(using Wolfram) we get :-

Note that the above figure shows the poles of the
Figure 4: Poles of Magnitude Plot of Analog LPF
Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and
we can pick one from each pair to be a part of our Transfer Function).
 Butterworth_Filter_2
p1 = -0.1494 + 1.0393i
p2 = -0.4362 + 0.9551i
p3 = -0.6876 + 0.7935i
p4 = -0.8833 + 0.5677i
p5 = -1.0075 + 0.2958i
p6 = -1.0500 + 0.0000i
p7 = -1.0075 - 0.2958i
p8 = -0.8833 - 0.5677i
p9 = -0.6876 - 0.7935i
p10 = -0.4362 - 0.9551i
p11 = -0.1494 - 1.0393i

Using the above poles which are in the left half plane we can write the Analog Lowpass Transfer Function as :-
Hanalog,LP F (sL) =

$$\frac{(\Omega c)N}{(sL - p1)(sL - p2)(sL - p3)(sL - p4)(sL - p5)(sL - p6)(sL - p7)(sL - p8)L - p9)(sL - p10)(sL - p11)}$$

Note that the scaling of the numerator is done in order to obtain a DC gain of 1

3.7 Analog Bandstop Transfer Function

The transformation equation is given by :-

$$sL = \frac{Bs}{\Omega 20 + s2}$$

Substituting the values of the parameters B(0.4763) and Ω_0(0.4987), we get :-

$$sL = \frac{0.4763s}{0.2487 + s2}$$

Substituting this value into *Hanalog,LP F* (*sL*) we get *Hanalog,BSF* (*s*). It can be written in the form N(s)/D(s) where the coefficients of the polynomials N(s) and D(s) are given as :-
Hanalog,BSF (*s*). =

(s^22 + 0.3427 s^20 + 0.0534 s^18 + 0.005 s^16 + 0.0003 s^14)

(s^22 + 1.221 s^21 + 1.089 s^20 + 0.68 s^19 + 0.3501 s^18 + 0.1475 s^17 + 0.0535 s^16 + 0.0167 s^15 + 0.0046 s^14 + 0.0011 s^13 + 0.0002 s^12)

The coefficients of odd powers of s in N(s) are all 0.
3.8 Discrete Time Filter Transfer Function
To transform the analog domain transfer function into the discrete domain, we need to make use of
the Bilinear Transformation which is given as :-

$$s = \frac{(1 - z-1)}{(1 + z-1)}$$

Using above equation we get *Hdiscrete,BSF* (*z*) from *Hanalog,BSF* (*s*).It can be written in the form N(z)/D(z) which are given as :-

Hdiscrete,BSF (*z*) =

(0.0001 - 0.0006 z^-1 + 0.004 z^-2 - 0.0182 z^-3 + 0.063 z^-4 - 0.1729 z^-5 + 0.3851 z^-6 - 0.7076 z^-7 + 1.085 z^-8 - 1.398 z^-9 + 1.521 z^-10 - 1.398 z^-11 + 1.085 z^-12 - 0.7076 z^-13 + 0.3851 z^-14 - 0.1729 z^-15 + 0.063 z^-16 - 0.0182 z^-17 + 0.004 z^-18 - 0.0006 z^-19 + 0.0001 z^-20)

(0.0002 - 0.0016 z^-1 + 0.0093 z^-2 - 0.0378 z^-3 + 0.1172 z^-4 - 0.288 z^-5 + 0.5745 z^-6 - 0.9461 z^-7 + 1.301 z^-8 - 1.505 z^-9 + 1.47 z^-10 - 1.214 z^-11 + 0.8464 z^-12 - 0.4965 z^-13 + 0.2432 z^-14 - 0.0983 z^-15 + 0.0323 z^-16 - 0.0084 z^-17 + 0.0017 z^-18 - 0.0002 z^-19)
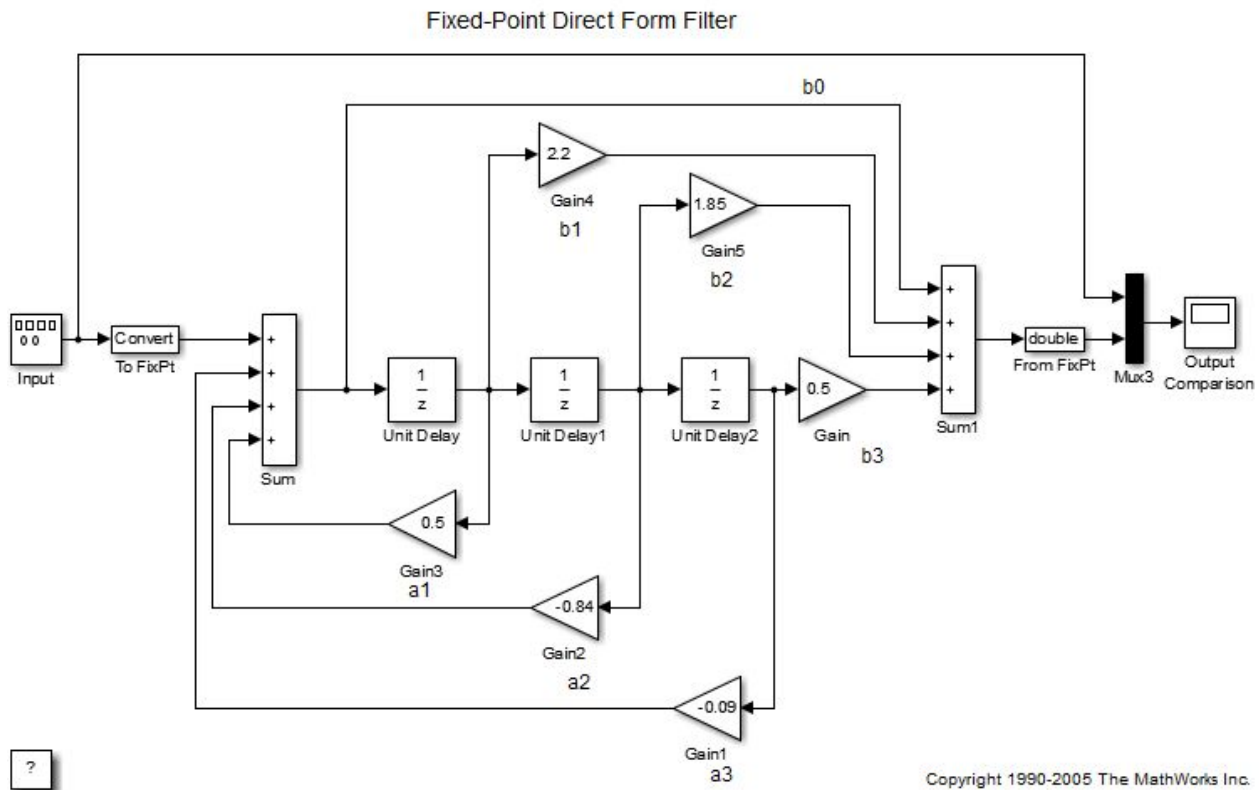
3.9 Realization using Direct Form II

Figure 5: Direct Form II Block Diagram for *Hdiscrete,BSF* (*z*)

Direct Form II is obtained by treating the transfer function H(z) = N(z)/D(z) as a cascade of 1/D(z) followed by N(z). The intermediate signal formed is the one whose samples get stored in the buffer.Thus it is advantageous in comparison to Direct Form I since it saves memory space.

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

3.10 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore δ = 0.15 and we get the minimum stopband attenuation to be :-

$$A = -20 \log(0.15) = 16.4782 dB$$

Since A < 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega T} = 74.25$$

Here $\Delta\omega T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega T = \frac{2kHz * 2\pi}{250kHz} = 0.016\pi$$

$$\therefore N \geq 75$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **53** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-stop impulse response samples were generated as the difference between three low-pass filters(all-pass - bandpass) as done in class.
The z-transform can simply be read off from the sequence values since its finite sequence.

4 MATLAB Plots
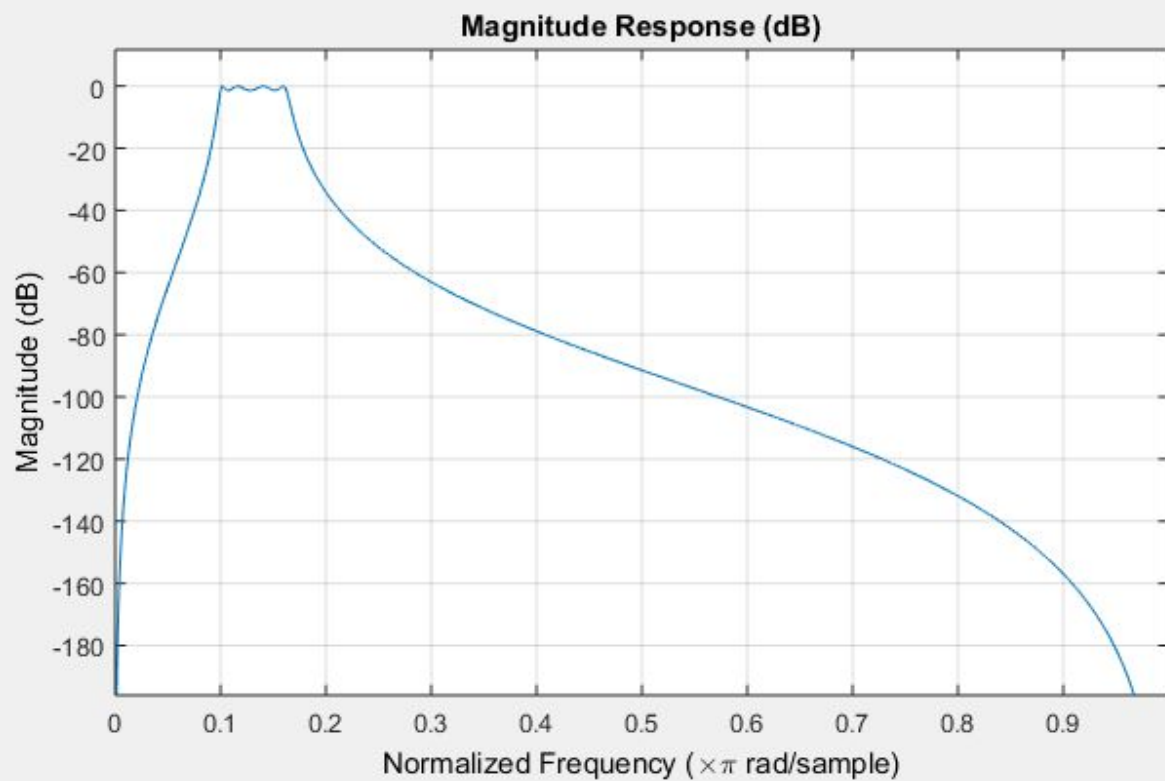
4.1 Filter 1 - Bandpass

4.1.1 IIR Filter

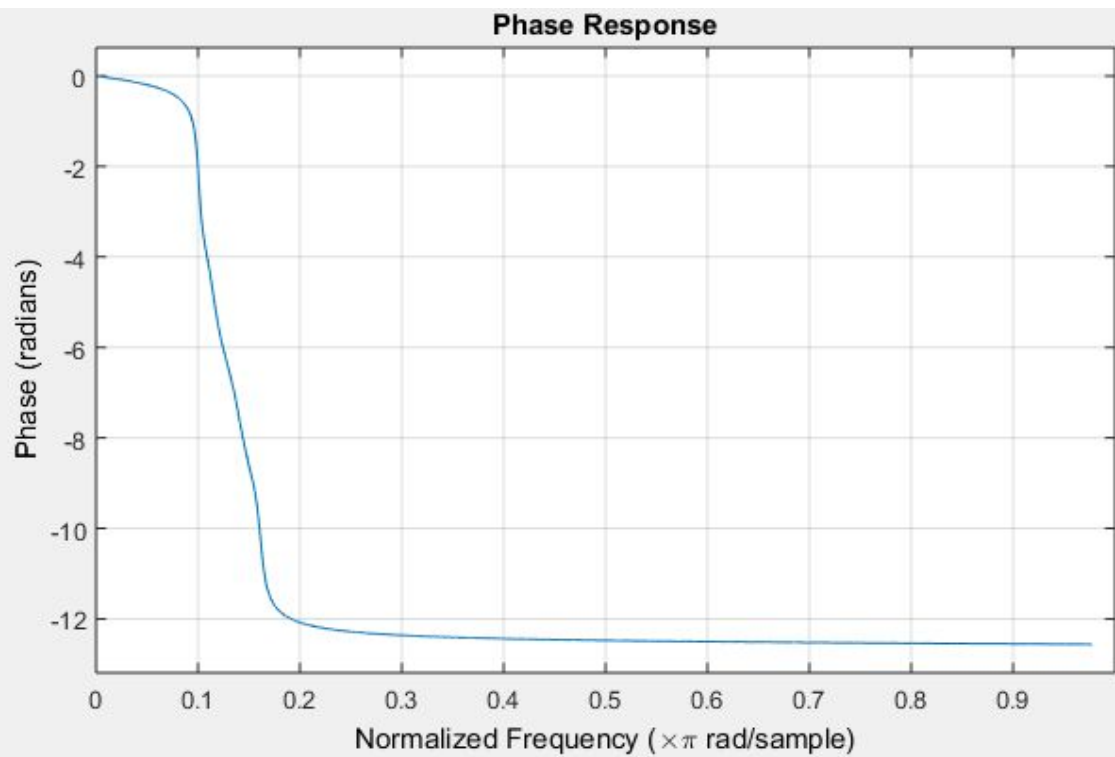Figure 7.a: Frequency Response with magnitude

Figure 7.a: Frequency Response with phase

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the **phase response** is **not linear**.
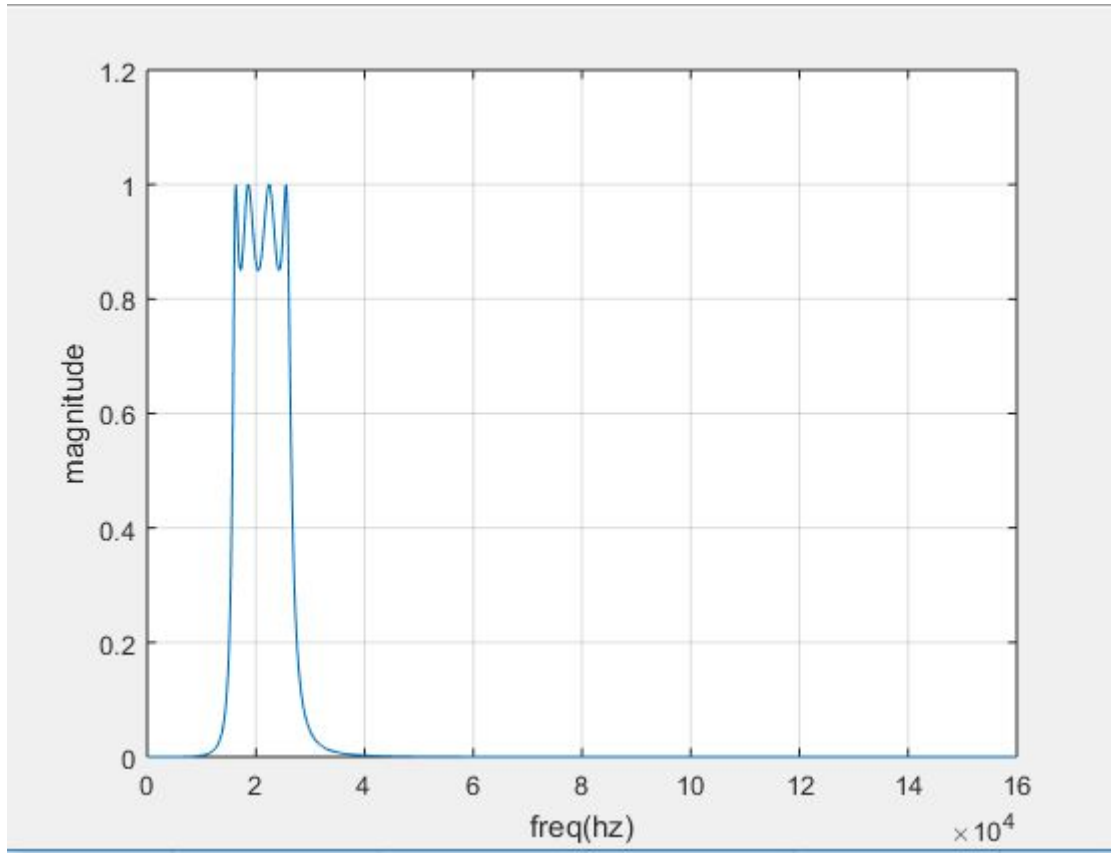Since all poles are inside the unit circle so it is stable.

Figure 9: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

4.1.2 FIR Filter

Figure 10: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter is indeed giving us a **Linear Phase** response which is desired.

Figure 11: Time Domain Sequence

Figure 12: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

4.2 Filter 2 - Bandstop
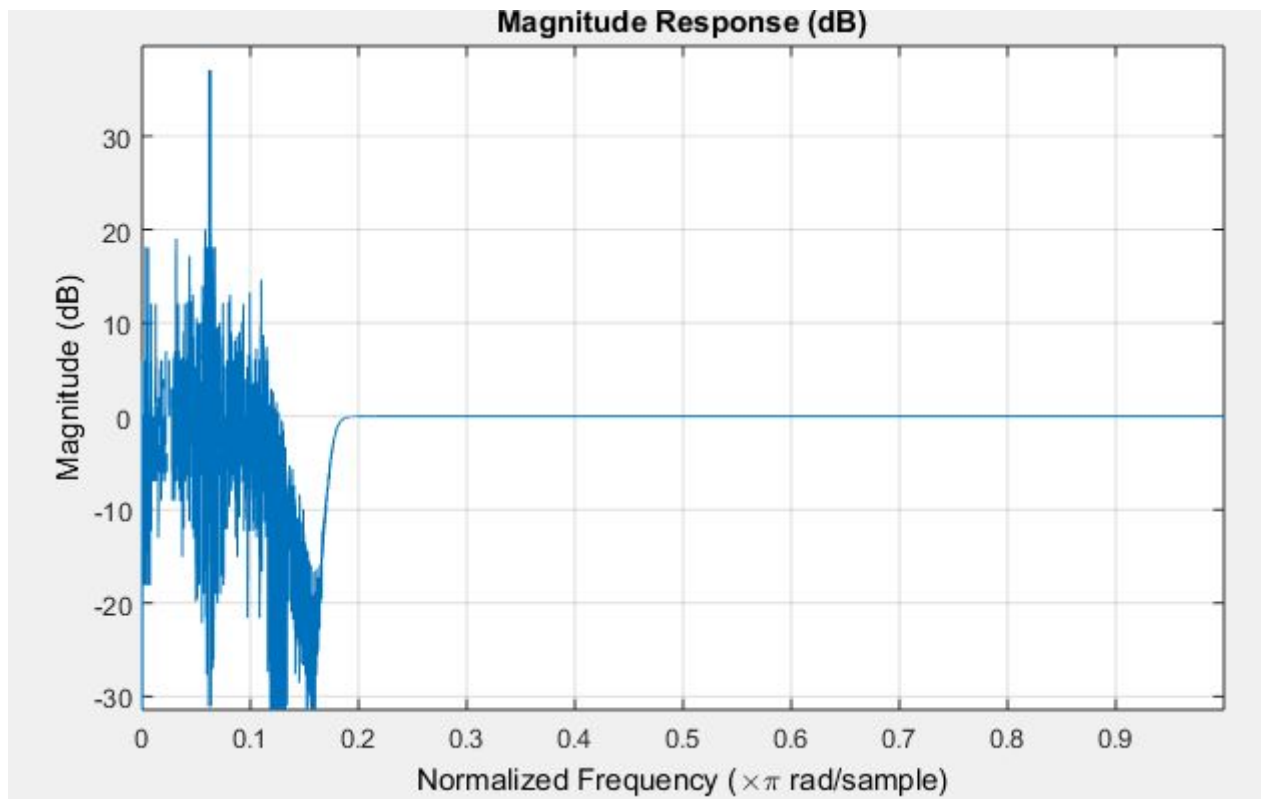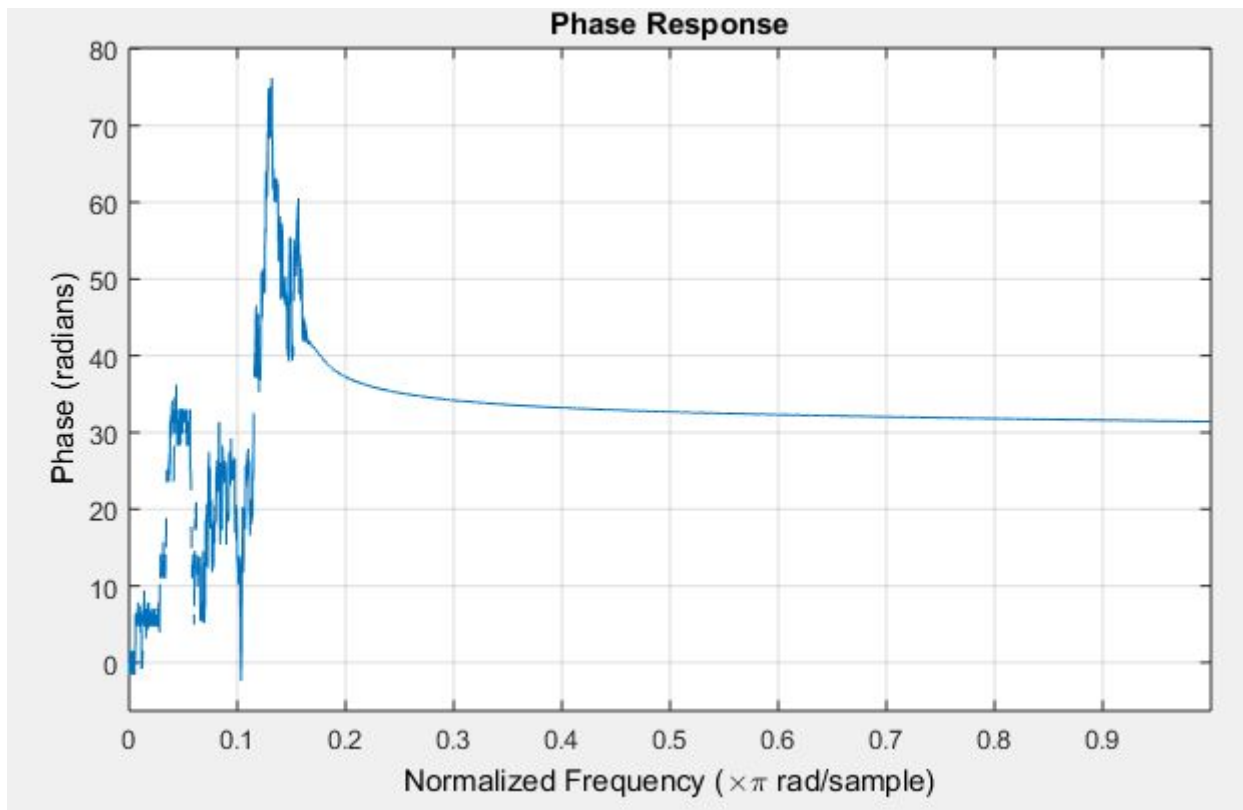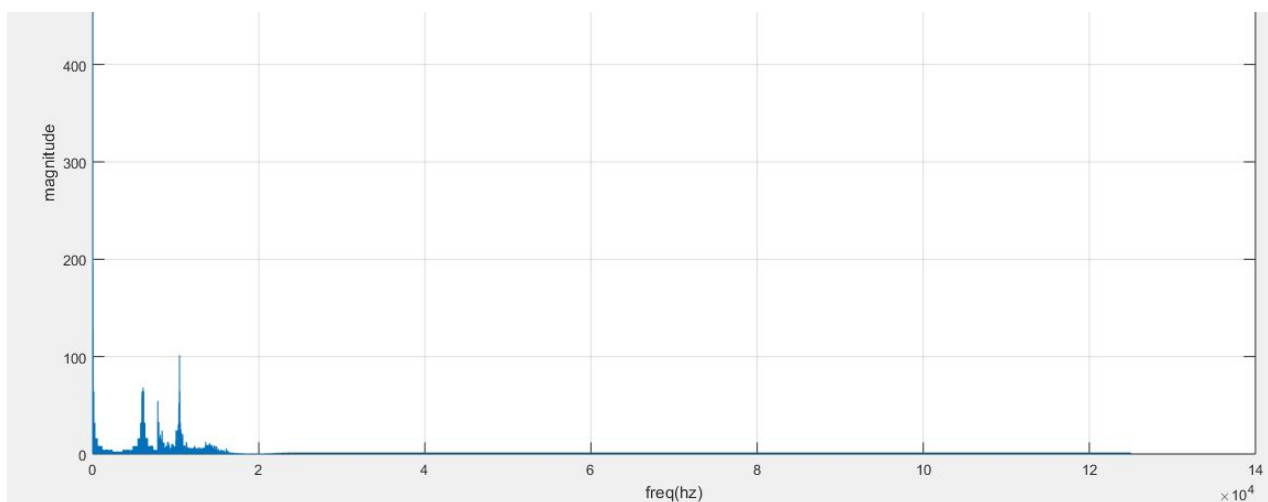
4.2.1 IIR Filter

Figure 13: Frequency Response with magnitude

Frequency Response with phase

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the **phase response** is **not linear**.

Figure 15: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.
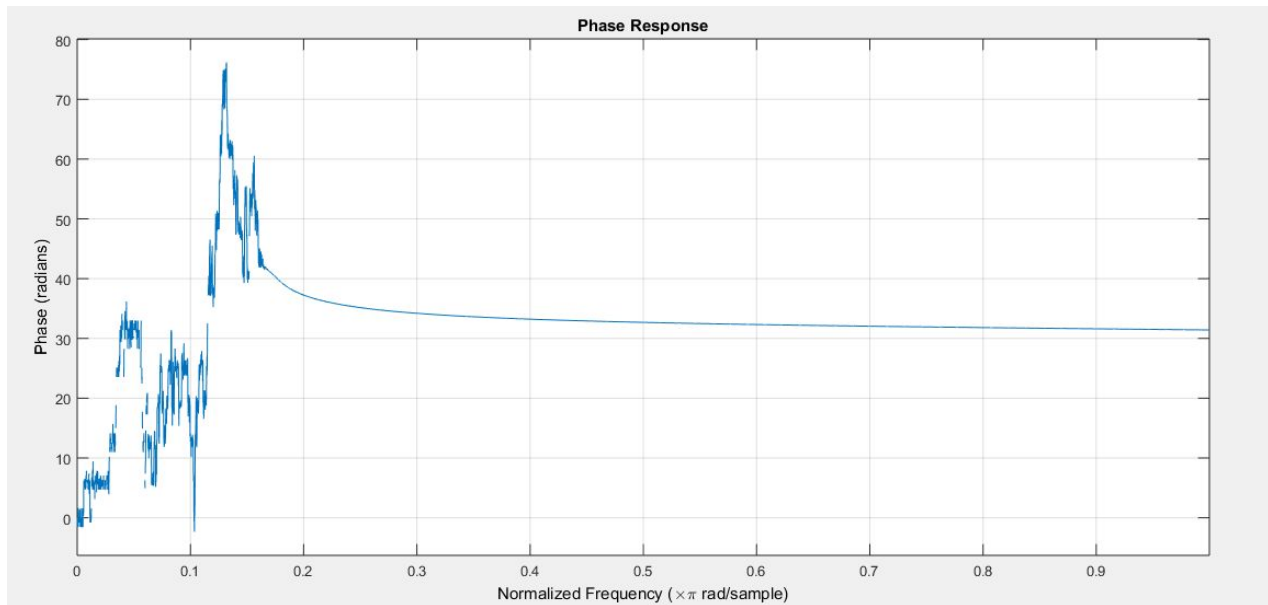
4.2.2 FIR Filter

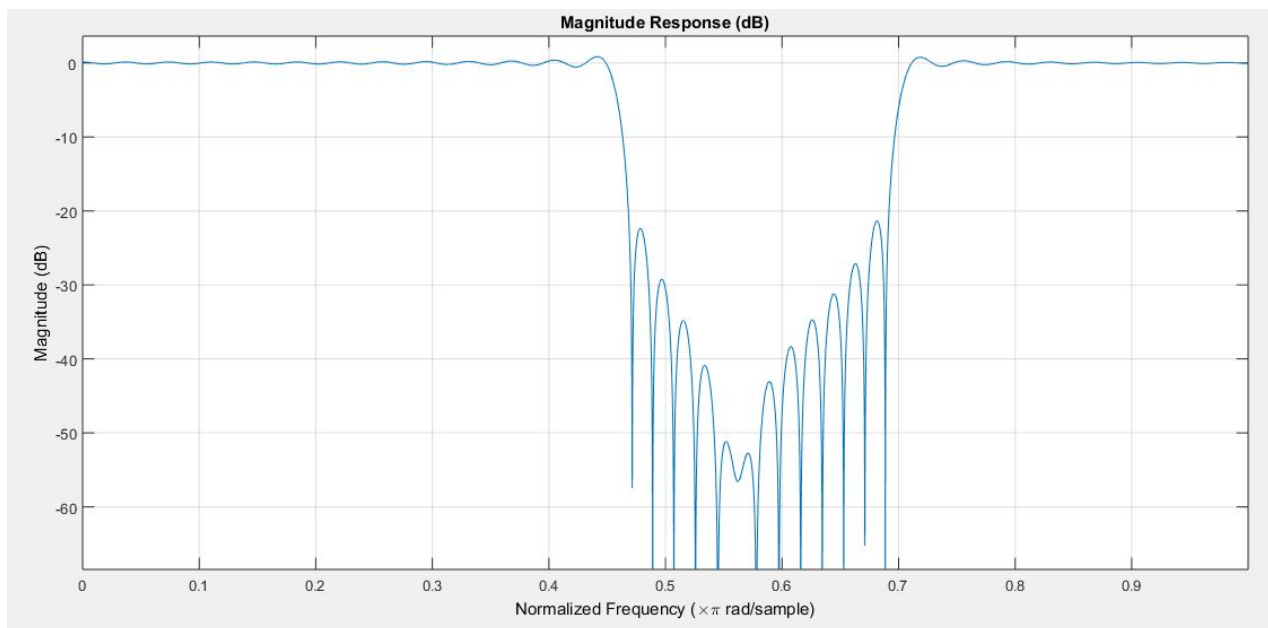Figure 16: Frequency Response with phase

Figure 16: Frequency Response with magnitude

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter indeed gives us a **Linear Phase** response which is desired.
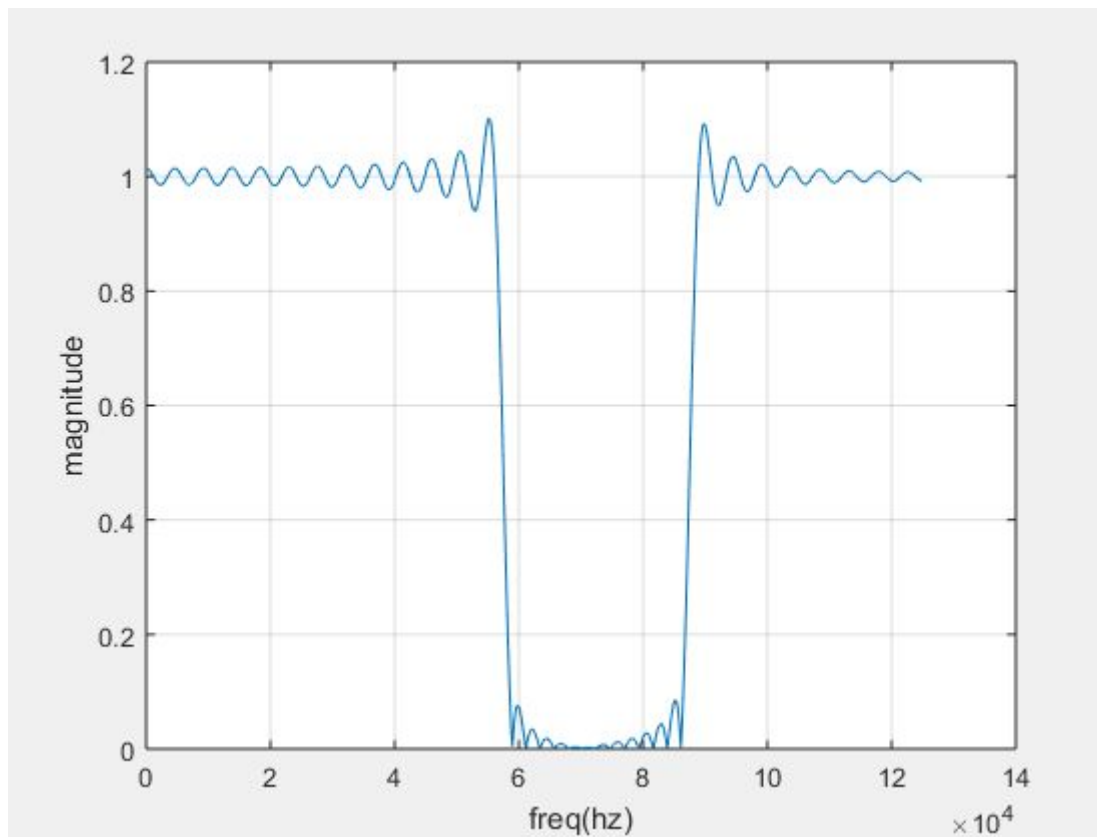
Figure 18: Magnitude Plot