

Emerging Markets and Inflation

Fall 2019
Yury Blyakhman



1. Personal intro

2. What do Desk Quant supporting Market Making business do?

- Salespeople help clients to determine the best strategies for hedging or investment
- Traders work on price quotes for the products, and manage portfolio risks they take through trading with our customers
- Quants work with trading on pricing models, risk analysis, hedging strategy as well as marketing on new products

Emerging Markets and Inflation

Lecture 1. Introduction to Emerging Markets

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Agenda for Today

1. Class introduction: syllabus and structure
2. Lecture 1. Introduction to Emerging Markets

Agenda for Today. Class Introduction

1. Class introduction: syllabus and structure

- a) Summary
- b) Detailed Course Outline
- c) Class Structure

2. Lecture 1. Introduction to Emerging Markets

Summary:

- Seven three-hour classes
- Two main topics:
 - Emerging Markets (EM)
 - Inflation
- Particulars:
 - Always start from Economics
 - Turn to derivatives, discuss practical pricing and risk management
 - Follow the details of Linear (Flow) products in Rates and FX
 - Use a lot of EM examples, spend the whole day in Brazil
 - Inflation and special Latin America (LatAm) cases. Real Rates
 - Classic Inflation modelling: 3-Factor, Libor Models

Detailed Course Outline:

1. Introduction to Emerging Markets

- a. Definitions and market' drivers
- b. Local Markets overview: economics and derivatives' trading
- c. Emerging Markets as an Asset Class: compare to classic Rates, FX or Credit markets
- d. EM tradable Indices and their derivatives. Pricing and hedging practices
- e. Practical overview of Debt Exchange Warrants

2. Linear Rates and FX Introduction. Part 1: FX

- a. Introduction to Interest Rates and FX
- b. Basic Interest Rates Concepts
- c. Linear FX Instruments
 - i. FX Spot and Forward
 - ii. Non-Deliverable or Cash Settled FX Forward
 - iii. Forward Starting FX Forward and Convexity Adjustment(s)
 - iv. FX Future and Convexity Adjustment(s)

Detailed Course Outline (continued):

3. Linear Rates and FX Introduction. Part 2: Rates

a. Fixed Income Instruments and Curves

- i. Bonds and Asset Swaps
- ii. Evolution of a Bond market into a Swap market in Emerging Markets
- iii. Single Currency and Cross Currency Swaps

b. Interest Rate (Yield) Curve Bootstrapping

- i. Calibration
- ii. Markets and Patterns across Developed and Emerging Markets
- iii. New Instruments evolution in Emerging Markets

c. Differential Discounting

- i. Multiple CSA Discounting Introduction
- ii. Special cases of Local Collateral in Emerging Markets

Detailed Course Outline (continued):

4. Brazil

- a. Brazil benchmarks and Day Count Conventions
- b. FX Products. Onshore and Offshore market. FX Convertibility
- c. Linear Interest Rate products
 - i. IR Futures. IR/FX Futures
 - ii. Onshore USD rates and extension to other Currencies
 - iii. CDI Swap and Percentage CDI Swap
- d. Interest Rate options
 - i. CDI Swaption and DI Future Option
 - ii. CDI Cap and IDI Options

Detailed Course Outline (continued):

5. Introduction to Inflation

- a. Economics of Inflation. Inflation measures
- b. Inflation-Linked products and markets
 - i. Securities
 - ii. Breakeven Inflation
 - iii. Futures and Inflation swaps
 - iv. Inflation-linked derivatives market
- c. Bootstrapping Inflation curve
 - i. Breakeven and Zero Coupon Swap Inflation curve
 - ii. Mean-reverting process in building the inflation curve and forward inflation pricing
- d. Inflation seasonality

Detailed Course Outline (continued):

6 - 7. Inflation in Latin America and Classic Inflation Modelling

- a. The Real Rate Economy: Inflation in Latin America
 - i. Real Rate Currency
 - ii. Latin America Inflation-Indexed bonds overview
 - iii. A Little Bit of History
- b. Inflation in Chile: Forwards
- c. Inflation in Brazil: Inflation-linked Bonds and Swaps
- d. Inflation in Colombia: Term-on-Term Inflation in Real Rates Economy
- e. The 3-Factor Jarrow - Yildirim model of Inflation
 - i. HJM and Short Rate refresher
 - ii. Detailed JY derivation
- f. Application of the 3-F inflation model in Colombia
- g. Pricing of Zero Coupon Inflation Indexed Swap (ZCIS)
- h. Pricing of Year-on-Year Inflation Indexed Swap (YYIS)
 - i. YYIS pricing with JY model
 - ii. YYIS pricing with First Market Model
 - iii. YYIS pricing with Better Market Model

Course Structure

■ Lectures

- PowerPoint slides available on the class' forum site before the class
- Slides will stay available throughout the semester
- There will be no hard copies

■ Home works

- All 100% practical covering structures virtually identical to real trades
- Strictly due before the next class submitted to your TA
- Submission format is free
- Each lecture will start with the homework review
- E-copy of solutions will not be available
- Cumulative Home works grade will be worth 20% of the final grade

■ In-class final exam

Agenda for Today. Introduction to Emerging Markets

1. Class introduction: syllabus and structure

2. Lecture 1. Introduction to Emerging Markets

- a) Emerging Markets definition, drivers, map
- b) EM Local Markets. Latest Overview
- c) Emerging Markets as an Asset Class. Regional differences
- d) Emerging Markets Indices and Derivatives on them
 - EMBI+ and the EM Contagion effect
 - GBI-EM and Derivatives pricing
- e) Debt Exchange Warrants
 - Pricing model and risk management
 - Parameters estimation

Definitions

Collection of countries with economic similarities

■ Collection of BRICs:

- BRIC = Brazil, Russia, India, China as in [[O'N 2001](#)]
- BRICS = BRIC + South Africa
- BRICET = BRIC + Eastern Europe, Turkey
- BRICM = BRIC + Mexico
- BRICK = BRIC + South Korea

■ And mortars:

- MIST: Mexico, Indonesia, South Korea, Turkey
- MINT: Mexico, Indonesia, Nigeria, Turkey
- CIVETS: Colombia, Indonesia, Vietnam, Egypt, Turkey, South Africa

■ Come up with your acronym?

Definitions (continued)

- Development over last 20 years, forming G20 with 11 EM countries
- *Less Economically Developed Countries* term used in 1970s
- “*Emerging Markets*” coined in early 1980s by A. van Agtmael from the World Bank’s International Financial Corporation
- No agreed upon definition, but common characteristics are →
- [JPMorgan 2006]:
 - Markets and economies in transition, hence not stable. Countries have begun to open up their markets and “emerge”. They have stock and capital markets that foreign investors can participate in;
 - Lack of historical economic and political stability, regulatory / legal framework and transparency;
 - High growth and liberalization / privatizations, but heavily regulated with barriers and capital controls;
 - Markets susceptible to financial and currency crisis, often a reflection of the domestic banking system strength

Drivers [JPMorgan 2006]

1. Systemic:

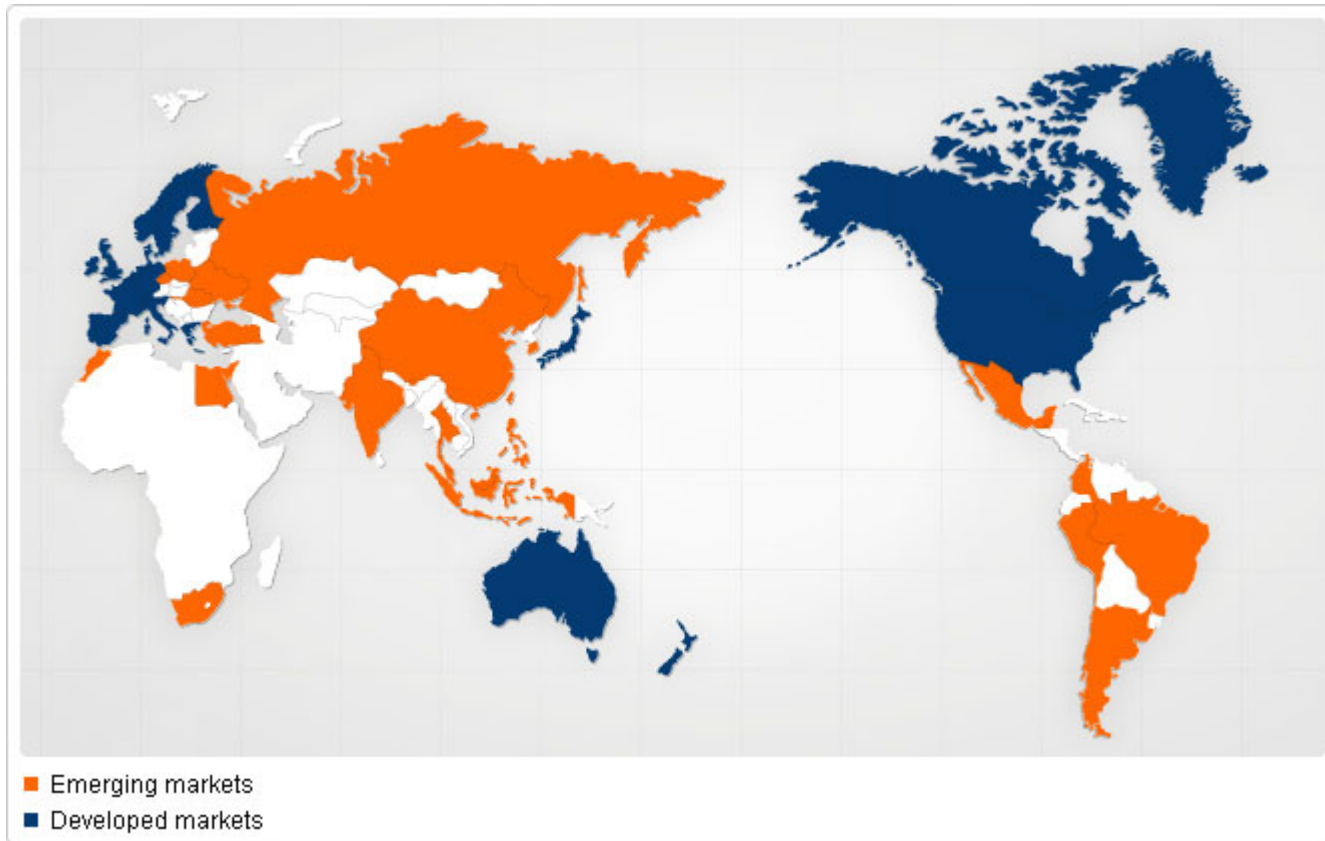
- General risk appetite and liquidity constraints
- State of economy, monetary policy, levels of inflation and interest rates
- Scare of contagion effect (will be shown later)

2. Idiosyncratic:

- Political process, fiscal discipline
- History of payments, defaults (at least 7 for Argentina so far)
- Economic growth, reforms, transparency, etc.

Emerging Markets Definitions

Map [Mirae 2013]



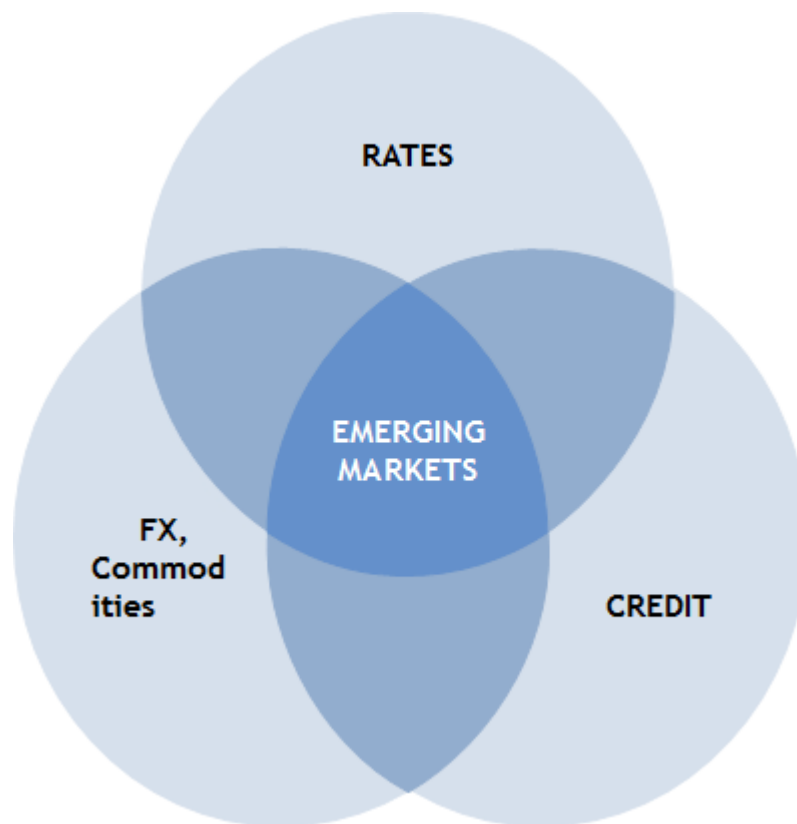
Local Markets Overview [JPMorgan 2014]

- Continue to depend on US monetary policy and even more so now than before
- Local yields recovery was very moderate connected to low money inflow into dedicated EM strategies and capital flights from them
- Regionally Emerging Eastern Europe Middle East and Africa (EMEA EM) sees less of the inflow compared to Asia and Latin America (LatAm) due to the ever increased geopolitical risks
- EM FX liquidity continues to improve since 2012, but is still low

Emerging Markets as an Asset Class

What makes it special?

- [[Fabozzi 2002](#)] suggests Brady debt restructuring in 1990's as first step towards Asset Class recognition
- Look at XCcy IR Swap in EM [[JPMorgan 2006](#)]:
 1. IR Fixed - Float Single Ccy Swap
 2. Deliverable XCcy Swap
 3. Non-Deliverable XCcy Swap
 4. Local Ccy Collateral in off-market mode
 5. Rates and FX correlated to Credit
 6. Commodities dependency



Emerging Markets as an Asset Class

Regional differences. Latin America

■ Mexico:

- Deep capital markets, high products sophistication
- Economy highly correlated to US
- Only free floating and convertible currency in LatAm
- Heavy regulatory environment limits development in the derivatives space

■ Chile:

- Strong and stable capital markets with high products sophistication
- Freely convertible, but non-deliverable FX (will be defined later)
- Heavy attention to the inflation market due to historical reasons

■ Brazil (to be covered in more details later):

- Large scope of exchange traded products and deep capital control
- Non Convertible FX
- Regulatory framework is becoming more fluid
- Largest local bonds market in EM
- High inflation and wide range of inflation linked products
- Special conventions historical going back to high interest rates

Regional differences. Europe, Africa, Middle East

■ Poland, Czech, Hungary:

- Free floating and convertible FX
- Not in Eurozone, but market is following EUR both in depth and products sophistication

■ Turkey:

- Low product sophistication with visible growing and liberalization
- Free floating and convertible FX
- Liquid Fixed Income market

■ South Africa:

- One of the deepest FX and Rates markets in EM
- Fully convertible and free floating FX
- Fixed coupon bonds are among the most liquid in EM, but FX Options are less liquid
- Heavy regulatory requirement from SA Reserve Bank (SARB)

■ Israel:

- Deep capital markets
- FX is managed float and convertible
- Highly liquid local bonds, FX Options, Rates derivatives

Regional differences. Asia

■ South Korea:

- Most developed, dynamic and liquid markets in EM Asia
- Credit Default Swap (CDS) spread is close to Japan
- Large and active IR Options market

■ China:

- Complex variety of onshore (CNY) and Hong Kong traded deliverable CNB
- Gradual evolution of regulations
- Active and liquid government bonds market

Developed Markets Promotion

- What does it take to promote an EM country to DM?
- Who makes the promotion?
- FTSE Russel promotes Poland to DM in 2018:

As of September 2018, Poland will leave the FTSE Emerging All Cap Index (where its weight, as of March 2018, was 1.33%) and join the FTSE Developed All Cap Index, where its index weight is projected to be 0.154%

- FTSE promotion criteria:
 - Quality of Market - the quality of regulation, presence of a derivatives market
 - Materiality
 - Consistency and Predictability
 - Cost Limitation - the cost of implementing a change
 - Stability - promotion in response to permanent changes in market status
 - Market Access
- Do other Index owners have their own rules?

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[Overview \[J.P.Morgan Markets\]](#)

- Powerful tool providing investors with an access to multi-component markets without trading components directly
- Scalar collection of assets (benchmarks), or investment strategy replicating underlyings in a transparent manner
- In EM justified by lack of access to Local Markets directly and thus are among the most popular ones
- Think of your own Index as it will be needed for the Home work later

EM families

■ External Sovereigns

- EMBI (Emerging Markets Bond Index) family: USD EM debt benchmark covering securities issued by sovereign and quasi-sovereign entities
- NEXGEM (Next Generation Market Index) for less liquid population of EM Economies with frequent and large debt issuance

■ External Credit

- CEMBI (Corporate Emerging Markets Bond Index)
- JACI (JPMorgan Credit Asia Index)
- CACI (Central America and Caribbean Index)

■ FX

- ELMI (EM Local Market Index) good at tracking short term returns
- LACI (Latin America Currencies Index)

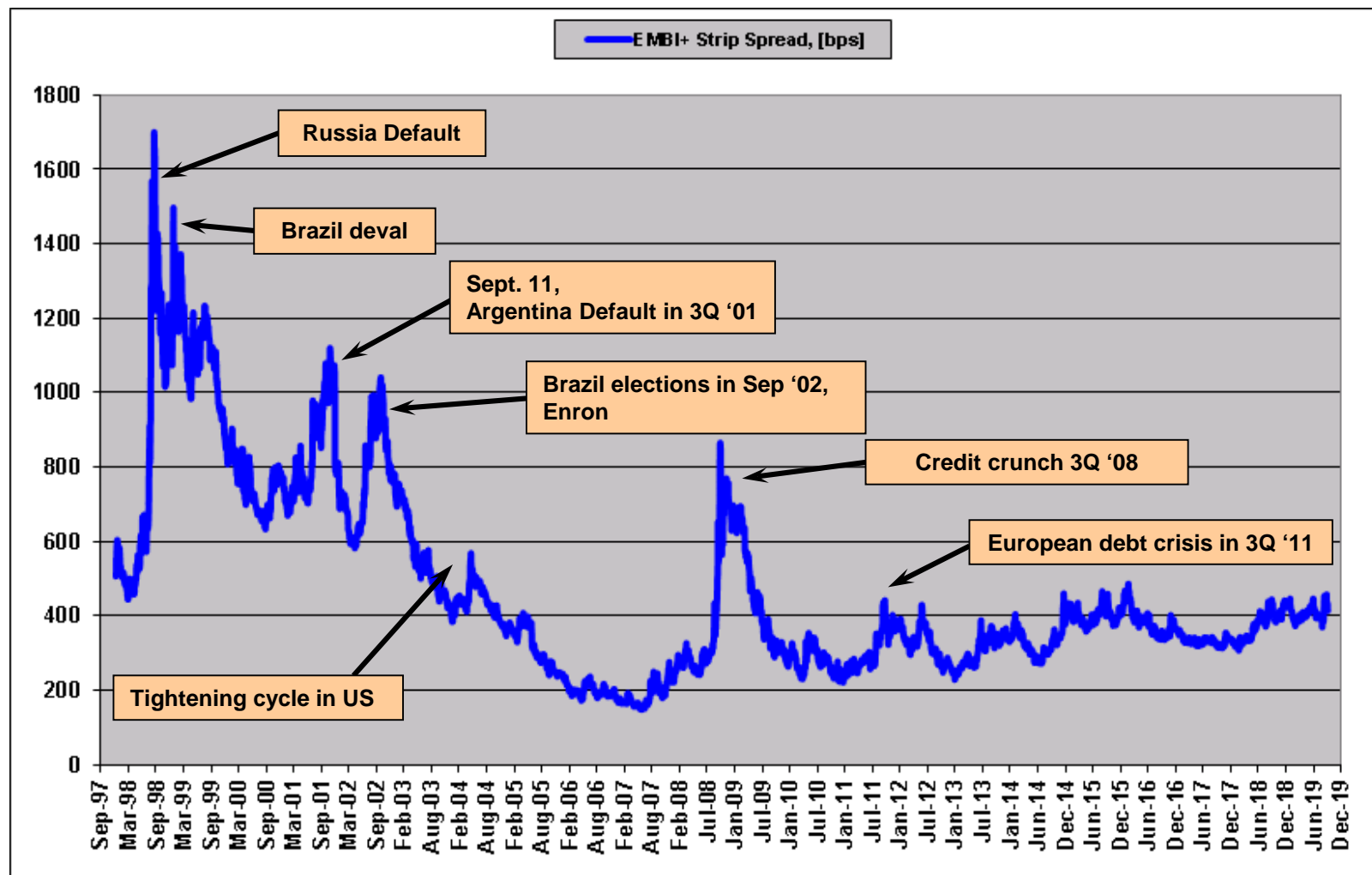
■ Local Government Bonds

- GBI-EM provides exposure to local currency denominated sovereign debt and FX rates

EMBI+ and EM Contagion effect

- Among the first offered and proved to be a great measure for EM investors
- Weighting of its components uses complex liquidity criteria with rebalance
- Stripped Spread measures the credit risk premium over US Treasury bonds:
 - A difference between Yield to Maturity of a bond to the Yield to Maturity of the corresponding point on the US Treasury curve
 - Measures constituents' sovereign risk since it also accounts for collateral

EMBI+ and EM Contagion effect (continued)



GBI-EM Index [JPMorgan GBI-EM]

- Tracks local currency bonds issued by EM governments
- As of June 2014 it included 16 countries split in 4 regions:
 - Asia: Indonesia, Malaysia, Philippines and Thailand
 - Europe: Hungary, Poland, Romania, Russia, Turkey
 - Latin America: Brazil, Chile, Colombia, Mexico, Peru
 - Africa: Nigeria, South Africa
- Excludes countries with explicit capital control
- Positioned as the investable benchmark

GBI-EM Index (continued)

- Monthly rebalanced with inclusion / removal of countries and issuances
- Includes fixed coupon bonds with very strict **liquidity** requirements:
 - Pricing: trading with enough frequency to prevent stale pricing
 - Availability: regularly traded at size at acceptable bid-offer
 - Replication cost: no additional cost to replicate index via weighted components

* Liquidity as ability to trade in sizable amounts without affecting the market. We will need this later for hedging

GBI-EM Index. Definition

Define some notations:

- $CP(t)$: Clean Price of a bond at time t
- $IA(t)$: Accrued Interest of a bond at time t
- $DP(t)$: Dirty Price of a bond at time t . $DP(t) = CP(t) + IA(t)$
- $MV(t)$: Market Value (outstanding amount) of a bond at time t
- $FX(t)$: FX rate in terms of foreign currency per US dollars at time t
- $r(t)$: One-day return at time t
- $w(t)$: Weight at time t
- $I(t)$: Index value at time t

GBI-EM Index. Definition (contd)

- Global Index consists of collection weighted Regional Indices
- Regional Index is a weighted collection of USD denominated Country Indices
- Country USD denominated index is FX converted from Local Ccy denominated country Index
- Ccy denominated country index is collection of weighted returns on individual bond prices

GBI-EM Index. One bond return

- One-day return of a bond in local currency:

$$r_B^{(L)}(t) = \frac{CP(t) + IA(t)}{CP(t-1) + IA(t-1)} - 1 = \frac{DP(t)}{DP(t-1)} - 1$$

- One-day return of a bond in USD:

$$r_B^{(U)}(t) = [1 + r_B^{(L)}(t)][1 + r_{FX}(t)] - 1$$

where

$$r_{FX}(t) = \frac{FX(t-1)}{FX(t)} - 1$$

GBI-EM Index. Country Index

- Country Index in Local currency for N_{CB} bonds:

$$\begin{aligned} I_C^{(L)}(t) &= I_C^{(L)}(t-1) \cdot \left[1 + r_C^{(L)}(t) \right] \\ &= I_C^{(L)}(t-1) \cdot \left[1 + \sum_{i=1}^{N_{CB}} w_{B_i}(t) \cdot r_{B_i}^{(L)}(t) \right] \end{aligned}$$

- Here $w_{B_i}(t)$ is weight of i -th bond for country C :

$$w_i(t) = \frac{MV_i^{(L)}(t-1)}{\sum_{k=1}^{NC} MV_k^{(L)}(t-1)}$$

- Country index in USD then:

$$I_C^{(U)}(t) = I_C^{(U)}(t-1) \cdot \left[\left(1 + r_C^{(L)}(t) \right) \cdot \left(1 + r_{FX}(t) \right) \right]$$

GBI-EM Index. Regional Index

- Weighted combo of Country Indices
- Defined as month-to-date return since last rebalance at s :

$$\hat{r}_C^{(L)}(t) = \frac{I_C^{(L)}(t)}{I_C^{(L)}(s)} - 1$$
$$\hat{r}_C^{(U)}(t) = [1 + \hat{r}_C^{(L)}(t)][1 + \hat{r}_{FX}(t)] - 1$$

- So for N_{RC} countries in a Region:

$$I_R^{(U)}(t) = I_R^{(U)}(s) \left[1 + \sum_{j=1}^{N_{RC}} w_{C_j}(s) \cdot \hat{r}_{C_j}^{(U)}(t) \right]$$

- With country weight $w_C(s)$ as of last rebalance defined as

$$w_C(s) = \frac{MV_C^{(U)}(s)}{\sum_{j=1}^{N_{RC}} MV_{C_j}^{(U)}(s)}$$

GBI-EM Index. Global Index

- Expressed similarly to Regional via weighted collection:

$$I(t) = I(s) \left[1 + \sum_{k=1}^{N_R} w_{R_k}(s) \cdot \hat{r}_{R_k}^{(U)}(t) \right]$$
$$w_R(s) = \frac{MV_R^{(U)}(s)}{\sum_{j=1}^{N_R} MV_{R_j}^{(U)}(s)}$$

where $w_R(s)$ is the weight of the Region in the global index at the last rebalancing date in terms of its total market value in USD

- So, why all the details?..

GBI-EM Index. Total Return Swap

- Total Return Swap (TRS): financial contract that fully transfers risk of an underlying asset from one party to another
- No asset ownership is required
- Risk transferred includes Market and Credit risk
- For position Q on an Index I till maturity T Future Value $FV(T)$ is

$$FV_T = Q \cdot \left\{ \frac{I_T}{I_o} - \Phi \right\},$$

$$\Phi = \begin{cases} 0, & \text{with final exchange} \\ 1, & \text{without final exchange} \end{cases}$$

(1)

- Now we need some dynamics assumptions!

GBI-EM Index. Total Return Swap. Pricing

Funding rate or general rate of growth:

- Forward price of an index via Forward price of a bond:

$$P_{t,T} = \frac{P_{t,t}}{Z^{Fund}_{t,T}}$$

- here Z^{Fund} is standard price of a Discount Zero Coupon bond paying \$1 at time T . Comes from Repo market for a bond
- Repo: a combination of a security's sale and an agreement of later purchase for a pre-agreed price on a pre-agreed day
- So let us start building Forward price of an Index via assumptions on Repo rates for 200+ bonds in it and market information on FX growth:

GBI-EM Index. Total Return Swap. Pricing

Use Country index as a starting point:

$$I_T^U = I_o^U \left\{ 1 + \left(\frac{I_T^L}{I_o^L} \frac{FX_o}{FX_T} - 1 \right) \right\} = I_o^U \left\{ 1 + \left(\frac{I_{T-1}^L (1 + R/365)}{I_o^L} \frac{FX_o}{FX_T} - 1 \right) \right\},$$

$$R = \sum_i w_i \left(\frac{DP_T^i}{DP_{T-1}^i} - 1 \right) = \sum_i w_i \left(\frac{DP_{T-1}^i (1 + \rho/365)}{DP_{T-1}^i} - 1 \right) = \sum_i w_i \rho/365$$

$$FX_T = FX_{T-1} \frac{Z_{T-1,T}^U}{Z_{T-1,T}^L}$$

here ρ is an overnight repo rate for individual bond.

■ Next is to extend overnight repo to a term one...

GBI-EM Index. Total Return Swap. Pricing

- Stop here as it already is getting too complicated...
- Instead use generic arbitrage free assumption in Eq. (1):

$$\begin{aligned}
 FV_T &= Q \cdot \left\{ \frac{I_T}{I_o} - 1 \right\} = Q \cdot \left\{ \frac{I_t}{I_o} \cdot \frac{I_T}{I_t} - 1 \right\} = Q \cdot \left\{ \frac{I_o(1+R_t)}{I_o} \cdot \frac{I_t(1+\rho_T)}{I_t} - 1 \right\} \\
 &= Q \{ (1+R_t) \cdot (1+\rho_T) - 1 \} = Q(R_t + \rho_T + R_t \cdot \rho_T)
 \end{aligned} \tag{2}$$

- Now TRS depends only on realized Index return and generic Funding rate ρ from time t to maturity T
- Could even dissect into regional (hedge-able) funding (using fv for Regional)

$$fv_T = q_i \cdot \left\{ \frac{I^R_T}{I^R_o} - 1 \right\} = q_i \cdot (R^R_t + \rho^R_T + R^R_t \cdot \rho^R_T)$$

Mexico Debt Exchange Warrants of 2005

- Switch foreign currency (debt) holding into locally denominated
- Enticing as allows to rely on domestic based financing
- And allows to exclude FX risk due to convertible nature of MXN Peso
- More details:
 - Compare local market MBono securities to offshore UMS
 - Option (warrant) involves cash-neutral switch basket ATMF option
 - Option to exchange any UMS bond from deliverable basket into pre-defined MBono with price quanto'd into USD

Quanto is a derivative where payoff measured in currency X is made in currency Y. Good introduction to quanto math could be found in Ch. 19.9 of (Hull, 2000)

General description

■ Actual pay-off at maturity:

$$\pi_T = \text{MAX} \left[f_B \hat{P}_B(T) - f_{U_1} P_{U_1}(T), f_B \hat{P}_B(T) - f_{U_2} P_{U_2}(T), \dots, 0 \right]$$

where:

f_B - face value of MBono;

$\hat{P}_B(T)$ - MXN spot price of MBono at option maturity, quanto'd in USD

f_{U_i} - face value of the i -th UMS bond from the deliverable basket

$P_{U_i}(T)$ - spot price of the i -th UMS bond at option maturity

Option valuation

- Start with 1 UMS bond in a basket
- Two assets exchange option valuation as in [[Margrabe'78](#)]:

$$\begin{aligned}\pi_o &= \mathbb{E}\left[Z_{0,T} \cdot \max\left(f_B \hat{P}_B(T) - f_U P_U(T), 0\right)\right] \\ &= \mathbb{E}\left[Z_{0,T} \cdot \max\left(f_B P_B(T) e^{\rho_{B,FX} \cdot \sigma_B \cdot \sigma_{FX} \cdot \tau} - f_U P_U(T), 0\right)\right] \\ &\stackrel{def}{=} \mathbb{E}\left[Z_{0,T} \cdot \max(B - U, 0)\right] \\ &= Z_{0,T} \cdot B \cdot \{B \cdot N(d_1) - U \cdot N(d_2)\}\end{aligned}\tag{3}$$

$Z_{0,T}$ - USD discount factor from today to expiry T

$P_U(T)$ - forward price of UMS bond at expiry

$P_B(T)$ - forward price of MBono bond at expiry in MXN

$\hat{P}_B(T)$ - MXN forward price of MBono bond at expiry quanto'd into USD

Option valuation

- Continue with two assets case as in [[Margrabe'78](#)]:

$e^{\rho_{B,FX} \cdot \sigma_B \cdot \sigma_{FX} \cdot \tau}$ - standard quanto adjustment from numeraire change

$\rho_{B,FX}$ - MBono price and FX rate correlation

σ_B - MBono price volatility

σ_{FX} - MXN forward price of MBono bond at expiry quanto'd into USD

τ - option's tenor

And standard notations for cumulative normal density $N(^*)$, d_1 and d_2

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{B}{U}\right) + \frac{\hat{\sigma}^2 \tau}{2}}{\hat{\sigma} \sqrt{\tau}} \\ d_2 &= d_1 - \hat{\sigma} \sqrt{\tau} \end{aligned}$$

Option valuation

- Two assets variance $\hat{\sigma}^2$ could be easily derived as

$$\hat{\sigma}^2 = \sigma_B^2 - 2\rho_{B,U}\sigma_B\sigma_U + \sigma_U^2$$

σ_U - UMS price volatility

$\rho_{B,U}$ - MBono price and UMS price correlation

HW1: Derive variance of two assets exchange warrant

Option valuation for full deliverable basket

- Price of a basket at time 0:

$$\begin{aligned}\pi_o &= \mathbb{E}\left[Z_{0,T} \cdot \text{MAX}\left(f_B \hat{P}_B(T) - f_{U_1} P_{U_1}(T), f_B \hat{P}_B(T) - f_{U_2} P_{U_2}(T), \dots, 0\right)\right] \\ &\stackrel{\text{def}}{=} \mathbb{E}\left[Z_{0,T} \cdot \text{MAX}\left(B - U_1, B - U_2, \dots, 0\right)\right]\end{aligned}\tag{4}$$

- Margrabe doesn't help us, so what can we do?→
- Multi-dimensional Monte-Carlo

Parameters estimation

- What parameters do we need? Let's look at Eq. [4] again →
- Individual Volatilities and Correlations matrix for all assets
- Where and how can we get them? →
- Of course in the market →
- But no Warrants market to calibrate parameters directly. Next? →
- Bond Derivatives with vols. Bond Options, Bond Future Options →
- Bond Derivatives with correlations →
- So we have nothing...

Parameters estimation. Historical estimates

1. Assets distribution:

- Goodness of Fit (GoF) or Chi Square
- What measure will let us say “lognormal enough”? →
- Compare to other assets with traded options:
 - US treasury Options
 - Mexico IR (TIIE) Swaptions + Correlation of Swap rates to Bond yields

Parameters estimation. Historical estimates

2. Volatilities. Suggest two approaches:

A. Standard Deviation of bond prices returns in log space:

$$\sigma = n \sqrt{\frac{n \left(\sum_i^n x_i^2 \right) - \left(\sum_i^n x_i \right)^2}{n(n-1)}}$$
$$x_i = \ln \left(\frac{P_{i+1}}{P_i} \right)$$

Questions to consider:

- How big rolling window to take?
- What do we do with vol as a function of time or asset prices?

Parameters estimation. Historical estimates

2. Volatilities. Suggest two approaches:

B. Correlation to IR Options:

- Yield vs. IR Swap → Yield vol. How do we get Price vol? →
- Simple assumption for price P_t and yield to maturity Y_t :

- Assuming

$$\begin{cases} dP_t = \sigma_P P_t dW_t \\ dY_t = \mu dt + \sigma_Y Y_t dW_t \end{cases}$$

$$\sigma_P \equiv E \left[\frac{dP_t}{P_t} \right] \quad \text{and} \quad \sigma_Y \equiv E \left[\frac{dY_t}{Y_t} \right]$$

- Results in a simple

$$\begin{cases} dP_t = \frac{dP_t}{dY_t} dY_t \\ \sigma_P P_t \propto \frac{dP_t}{dY_t} \sigma_Y Y_t \end{cases} \Rightarrow \sigma_P = \sigma_Y \cdot \text{BondDuration} \cdot Y_t$$

Parameters estimation. Historical estimates

3. Correlations:

- Same as before, go for simple historical correlations:

$$\left\{ \begin{array}{l} A_i = \ln \left(\frac{(P_1)_{i+1}}{(P_1)_i} \right) \\ B_i = \ln \left(\frac{(P_2)_{i+1}}{(P_2)_i} \right) \end{array} \right. \Rightarrow \rho = \text{Corr}(A, B)$$

- Parameters are ready, but how stable is our Monte-Carlo?

HW2 discuss pricing vanilla CALL Option on a generic Index

Given:

- Index is newly created: not much of historical index prices is available
- Index components have been traded for a while and do have historical prices
- Feel free to use earlier discussed GBI-EM index as example to make question more detailed
- There is no option market for this index

Questions:

- Discuss what model we could use to price this option. Can we start with Black-Sholes?
- Discuss how could we extract or derive parameters for this model: Index' volatility?
- If we are to deviate from Black-Sholes a bit and to introduce some simple Local Vol as vol for strike, how could we mark this smile and where from?
- How would we hedge this option if ever traded?

Summary

- Justified and agreed on pricing model
- Estimated parameters via some simple assumptions
- Implemented stand-alone pricing engine
- Discussed stability and potential ways of improvement
- What is next? →
 - Hedging: what can we hedge and how? →
 - Credit considerations? →
 - Stochastic behavior and jumps of EM bond yields [Matovu'07]
 - In times of economic distress when needed the most

Practical Trading Lessons

- Market opens, prices flow in, volume picks up
- Where does market trade compared to where we estimated the warrant price to be?
- Spot on! Are we that good?
- Maybe, but not necessarily:
 - Lack of model parameters leads to very similar assumptions made by quants using exactly the same, and only available market data sets...

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