Final Exam preparation

- Recommended materials:

  Shreve II, lecture notes, handout on Ito's formula, solutions to quizzes and midterm, and your solutions to homeworks.

  Try to really understand the basics and how they work.

- You should be proficient in:

  (1) Computation of quadratic variation, cross variation for Itô and jump processes.

  (2) Computation of the distribution of a process that can be expressed as an integral of a deterministic integrand with respect to a BM.

  (3) Finding out whether a given process is a martingale, super- or sub-martingale (using definitions or Itô's formula).

  (4) Applications of optional stopping theorems.

  (5) Using reflection principle and calculating probability distributions and related quantities involving hitting times.

  (6) Finding out whether a given process is a standard BM (dimensions 1 and 2).

  (7) Computations involving a change of measure for BM with a drift and jump processes.

  (8) Finding a closed form solution of simple SDEs (for example: linear SDEs as in Exercise 6.1, various short rate models, geometric Poisson process, Doléans-Dade exponential).

  (9) Computation of the expected value of a function of an Ito or jump process (i.e. finding the expectation, variance, covariance, MGF, etc.).

  (10) Evaluation of simple stochastic integrals.

  (11) Writing equations for the dynamics of a portfolio value process and a discounted portfolio value process. Self-financing condition.

  (12) Pricing and hedging within Black-Scholes framework (includes simple market models with several stocks).

  (13) Computations related to changing a numéraire.

  (14) Using Feynman-Kac formula (dimensions 1 and 2).

  (15) Computing generators and their adjoints for diffusion processes. Writing Kolmogorov's forward and backward equations.

  (16) Identifying Markov processes and using Markov property.

- You have to know rigorous definitions and basic properties of:

(1) Brownian Motion (BM), Geometric Brownian Motion (GBM), generalized GBM, Poisson process, compound Poisson process.
(2) Martingales.
(3) Stopping times.
(4) Quadratic and cross variation (for Itô and jump processes).
(5) Stochastic integral with respect to BM, Ito process, and jump process (among their basic properties, Itô isometry).
(6) Risk-neutral measures.
(7) Arbitrage.
(8) Complete market.
(9) Markov process.
(10) Convex/concave functions.
(11) Stochastic differential equation, its components (drift and diffusion); diffusion process.
(12) Jump process.
(13) Black-Scholes-Merton (BSM) model and Black-Scholes-Merton PDE.

- You have to be able to state and apply the following theorems:

(1) Ito-Doeblin formula (for BM, Itô, and jump processes, all types and dimensions). Particular case: integration by parts (i.e. Ito's product rule).
(2) Levy's characterization of BM (dimensions 1 and 2)
(3) Girsanov theorem (all dimensions).
(4) Martingale representation theorem (all dimensions).
(5) Two fundamental theorems of asset pricing.
(6) Feynman-Kac representation (dimentions 1 and 2).
(7) Optional stopping (=sampling) theorems (Shreve II, p. 342, 373, see also refresher notes).
(8) Analytic and probabilistic characterizations of the price of American put options (perpetual and finite expiration).
(9) Jensen's inequality and conditional Jensen's inequality.

These lists are meant to help you to organize your preparation for the final exam.