MTH 9831 Assignment 7 (10/25/2018 - 10/31/2018).

Read Lecture 7. Additional references for this material are:

1. S. Shreve, Stochastic Calculus for Finance II, Sections 5.4, 9.2, 9.4.1, 9.4.2.

(1) (Self-financing condition for model with 2 stocks) Exercise 1 from lecture 7.

(2) (Replication guarantees that the price is unique) Exercise 3 from lecture 7.

(3) (Alternate proof of Theorem 3.3) Exercise 9.1.

(4) (Portfolios under change of numéraire) Exercise 9.2.

(5) (Change in volatility caused by change of numéraire) Exercise 9.3.

(6) (Explicit solution to a general linear SDE) Solve Exercise 6.1 following these steps:

   (a) Solve $dZ(u) = b(u)Z(u)du + \sigma(u)Z(u)dW(u)$ for $u \geq t$ with the initial condition $Z(t) = 1$.[1]

   (b) Find the SDE for the process $Y(u) := X(u)\left(\frac{1}{Z(u)}\right)$.

   (c) Solve the equation for $Y(u)$ obtained in (b) for $u \geq t$ with the initial condition $Y(t) = x$ and find the solution $X(u)$ to the original equation which satisfies the initial condition $X(t) = x$.

   Which of the processes and models that you already know satisfy a linear SDE?

[1] This is the analog of the so-called integrating factor for linear ODEs.