

MTH 9831 Assignment 3 (09/27 - 10/3).

Read Lecture 3. Some additional references for this material are:

1. S. Shreve, Stochastic Calculus for Finance II, Sections 4.2, 4.3.
 2. A. Etheridge, A Course in Financial Calculus, Section 4.2.
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- (1) (Imitation of the technique which precedes the exercise) Exercise 1 from Lecture 3.
 - (2) (The difference between Ito and Stratonovich integrals) Exercise 3 from Lecture 3.
 - (3) (Stochastic integral of an elementary process is a martingale) Exercise 4 from Lecture 3.
 - (4) (Itô's isometry examples)
 - (a) Using only the material of Example 2.1 verify that for all $t \geq 0$

$$\mathbb{E} \left[\left(\int_0^t B(s) dB(s) \right)^2 \right] = \int_0^t \mathbb{E} [B(s)^2] ds.$$

- (b) Using Theorem 4.1 compute the variance of

$$\int_0^t (B(s))^n dB(s), \quad n \in \mathbb{N}.$$

The solution should include a derivation of the moment formula for the Brownian motion (if you want to use it).

- (5) Exercise 4.2 from the textbook.
- (6) Exercise 4.3 from the textbook.
- (7) (Review of Brownian motion and basic calculus) Use properties of Brownian motion to find the expectation and variance of $\int_0^t B^2(s) ds$. Hint: for the variance part, write the square of the integral as a double integral and only then take the expectation.