6.
solution1:
$S(t)$ satisfies
$$dS(t) = \sigma S(t) dB(t), S(0) = A$$
apply Ito's formula to $S^2(t)$
$$dS^2(t) = 2S(t)dS(t) + d[S,S](t) = 2\sigma S(t)^2 dB(t) + \sigma^2 S^2(t)dt$$
$$E(S^2(t)) = S^2(0) + \int_0^t \sigma^2 E(S^2(t))dt$$
Let $m(t)$ be $E(S^2(t))$
$$m'(t) = \sigma^2 m(t)$$
then
$$m(t) = Ce^{\sigma^2 t}$$
$$E(S^2(t)) = Ce^{\sigma^2 t}$$
$S(0) = A$,then $E(S^2(0)) = Ce^{\sigma^2 0} = A^2$, $C = A^2$
$$E(S^2(t)) = A^2 e^{\sigma^2 t}$$
$$D(S(t)) = E(S^2(t)) - E(S(t))^2$$
and from $dS(t) = \sigma S(t)dB(t)$
$$S(t) - S(0) = \int_0^t \sigma S(t) dB(t)$$
$$E(S(t)) - A = 0$$
$S(t)$ has expectation A,then the variance of S(t) is $A^2(e^{\sigma^2 t} - 1)$
solution2:
verify that $S(t) = Ae^{\sigma B(t) - \sigma^2 t/2}$
$$dS(t) = A\sigma e^{\sigma B(t)-\sigma^2 t/2} dB(t) + A(-\frac{\sigma^2}{2} e^{\sigma B(t)-\sigma^2 t/2} + \frac{\sigma^2}{2} e^{\sigma B(t)-\sigma^2 t/2})dt = \sigma S(t)dB(t)$$
then use the moment generating function of normal distribution
$$E(S^2(t)) = A^2 e^{-\sigma^2 t} E(e^{2\sigma\sqrt{t}Z(t)}) = A^2 e^{-\sigma^2 t} e^{2\sigma^2 t} = A^2 e^{\sigma^2 t}$$
then the process is the same as above
7.
$$\int_0^t S(u)du = uS(u)|_0^t - \int_0^t udS(u) = tS(t) - \int_0^t u\sigma S(u)dB(u)$$
$$E(\int_0^t S(u)du) = E(tS(t)) = tA$$
$$Var(\int_0^t S(u)du) = \int_0^t E((u\sigma S(u))^2)du = \int_0^t u^2 \sigma^2 A^2 e^{\sigma^2 u} du = \sigma^{-4} A^2 (x^2 - 2x + 2)e^x|_0^{\sigma^2 t}$$
$$Var = A^2 t^2 - 2tA^2/\sigma^2$$