MTH 9831. Midterm preparation

- *Recommended materials*: Shreve II, lecture notes, handout on Ito's formula, solutions to quizzes, your homeworks. Stick to the basics, but learn them really well, with a good understanding of how they work, so that you can apply them with confidence in various situations.
- *Types of problems (different types can be combined in one problem):*

(1) Checking whether a given process is a martingale from the definition and correctly applying optional stopping theorems (as well as Bounded and monotone convergence theorems) for computing probabilities and expectations.

(2) Checking whether a given process is a Brownian motion from the definition. Computations involving Brownian motion (various distributions, expectations, etc. using a reflection principle, normality or martingale techniques).

(3) Checking whether a given process is a Markov process. Applying Markov and strong Markov properties in computations related to Brownian motion and random walks.

(4) Basic use of the stochastic representation formula of solutions to the heat equation.[1]

(5) Computation probabilities and expectations involving geometric BM.

(6) Computation of quadratic variation.

(7) Computation of cross variation.

(8) Finding out whether a given process is a (local) martingale using Itô's formula.

(9) Finding out whether a given process is a standard BM using Lévy's characterization of BM (dimensions 1 and 2).

(10) Computation of the distribution of a process that can be expressed as a stochastic integral of a deterministic integrand.

(11) Change from correlated to uncorrelated BMs and the other way around.

(12) Finding a closed form solution of simple SDEs (for example: generalized GBM, Ornstein-Uhlenbeck process, Vasicek, Black-Karasinski short rate models).

(13) Computation of the expected value of a function of an Itô process (i.e. finding the expectation, variance, MGF, etc.). In particular, computation of the mean and variance for a variety of short rate models (see http://en.wikipedia.org/wiki/Short_rate_model)).

[1] Example: use the stochastic representation formula to find the solution of the heat equation with $u(0,x) = x^2$ (ignoring the fact that it is not a bounded function with bounded derivatives).

(14) Finding a risk-neutral measure, pricing, and hedging for specific examples of a single stock model. This includes, in particular: BSM model and equation, portfolio value and discounted portfolio value dynamics and a self-financing condition.

(15) Writing market price of risk and hedging equations for a specific market model with many stocks (2 for the purposes of the midterm) and making conclusions about the existence of arbitrage and completeness of the market model.

The above list is not meant to be comprehensive but rather to help you in organizing your preparation for the exam.

- *You have to know rigorous definitions of:*

  (1) Brownian motion;
  (2) Quadratic and cross variation;
  (3) Stochastic integrals with respect to the BM or an Ito process (and their basic properties);
  (4) Risk-neutral measure;
  (5) Arbitrage;
  (6) Complete market.

- *You have to be able to state and apply the following theorems:*

  (1) Optional stopping theorems;
  (2) Ito-Doeblin formula (for BM and Ito processes, all types and dimensions). Particular case: integration by parts (i.e. Ito's product rule).
  (3) Levy's characterization of BM (dimensions 1 and 2)
  (4) Girsanov theorem (all dimensions).
  (5) Martingale representation theorem (all dimensions).
  (6) Two fundamental theorems of asset pricing.