MTH 9831 Assignment 11 (11/29/2015 - 12/05/2015).

Let $\{N(t)\}_{t\geq 0}$ be a Poisson process with intensity λ and $M(t) = N(t) - \lambda t$ be a compensated Poisson process.

(1) Prove Theorem 3.2 from Lecture 11. Most proofs are written in the textbook, so please do not submit this problem as a part of your homework. This is a "quizzable exercise".

(2) Exercise 11.1.

(3) Exercise 11.7.

(4) Evaluate the following stochastic integrals:

(a) $\displaystyle\int_0^t N(s)\,dN(s)$;

(b) $\displaystyle\int_0^t N(s-)\,dN(s)$;

(c) $\displaystyle\int_0^t M(s-)\,dM(s)$; (hint: use Itô's formula for $M^2(t)$);

(d) $\displaystyle\int_0^t M(s)\,dM(s)$;

(5) Let λ, $\tilde{\lambda} \in (0, \infty)$. Apply Itô-Doeblin formula to show that $Z(t) := Z(0)e^{(\lambda-\tilde{\lambda})t}\left(\dfrac{\tilde{\lambda}}{\lambda}\right)^{N(t)}$, $t \geq 0$, satisfies the equation

$$dZ(t) = \frac{\tilde{\lambda} - \lambda}{\lambda}\, Z(t-)\, dM(t).$$

Conclude that $\{Z(t)\}_{t\geq 0}$ is a martingale.

(6) Suppose that $\sigma > -1$ and a jump process $\{S(t)\}_{t\geq 0}$ satisfies

$$S(t) = S(0) + \sigma \int_0^t S(u-)\, dM(u).$$

Show that $S(t) = S(0)e^{-\lambda\sigma t}(1+\sigma)^{N(t)}$. Hint: use the same approach as in the Example 3 about Doléans-Dade exponential in Lecture 11.