2. Solution:

* Because are continuous, is a linear combination of them. Thus, is also continuous.
* for each and

……

Because are independent random variables;, are independent random variables; and are independent.

Thus, are independent random variables.

* For all s>0, and t >= 0, and are independent, their covariance = 0

Because ],

the increment also has a normal distribution with mean 0 and variance s

So we can conclude that X(t) is a Brownian motion.

=

=

=

=

3.

(a) X(t) = -B(t)

* The negative multiplication maintains the continuity and independence of increments
* X(0) = -B(0) = 0

Because has normal distribution, X(t) still maintains normal distribution with mean 0 and variance s

Thus, is a Brownian motion.

(b) X(t) = where c > 0 is a constant

* Continuity and independence of increments still maintains.
* X(0) = = 0

= s

Thus, is a Brownian motion.

(c) X(t) =

The variance of increments is still related to t, so it’s not a Brownian motion.

(d) X(t) =

* for all s > 0, t >= 0

Thus, the variance of increments is still related to t for s . It’s not a Brownian motion.

(e) X(t) = , where s is fixed

* Because X(t) is a linear combination of Brownian motions, it maintains the independence of increments and is almost surely continuous. The increments are also normally distributed.
* For all m > 0,

=

= 0

=

= m

Thus, it’s a Brownian motion.