**MTH 9831 Assignment 2**

**(1)**

According to Theorem 1.8 from lecture notes 2, we know that

where and all .

Thus

To find the joint density of , let take derivative w.r.t and successively, we have

since , the region where this joint density is non-zero is

**(2)**

**Part 1:**

Since , it follows that

Let , and take limit on both sides of inequality (1) and use equation (2), we have

which implies

**Part 2:**

Since means,

thus, we can rewrite as

Denote as , then, if and only if .

Note that , thus, according to the left continuity of probability, we have

Next, we will prove that

For any fixed value of m, denote as , then

since is a sequence of identically distributed integrable random variables,

Also

In conclusion,

So

Thus, according to the Borel-Cantelli Lemma (Part 1), we have

Plugging (4) into (3), it follows that