HW4

April 3, 2019

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2 Problem 4.1

```
In [1]: import pandas as pd
        import numpy as np
        from sklearn.linear_model import LinearRegression
        import statsmodels.api as sm
        from scipy.stats import pearsonr
```

```
In [2]: data = pd.read_csv('BetaExample.txt',sep="|")
        data.DATE=pd.to_datetime(data.DATE)
        data=data.dropna()
```

```
In [3]: data1=data.loc[data.TICKER=="IBM"]
        data2=data.loc[data.TICKER=="AAPL"]
        data3=data.loc[data.TICKER=="TSLA"]
```

2.1 (a)

2.1.1 IBM

```
In [4]: d=data1[data1.DATE<pd.to_datetime("2015-01-01")]
        X=np.array(d[["RM"]])
        y=np.array(d["R"])
```

```
In [5]: est = sm.OLS(y, X)
        est2 = est.fit()
        print(est2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.332
Model:                            OLS   Adj. R-squared:                  0.332
Method:                 Least Squares   F-statistic:                     3133.
Date:                Wed, 03 Apr 2019   Prob (F-statistic):               0.00
Time:                        10:09:54   Log-Likelihood:                 17657.
No. Observations:                6301   AIC:                        -3.531e+04
```

```
Df Residuals:                    6300   BIC:                        -3.531e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
x1             0.9067      0.016     55.970      0.000       0.875       0.938
==============================================================================
Omnibus:                     1227.530   Durbin-Watson:                   1.964
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            34867.449
Skew:                           0.179   Prob(JB):                         0.00
Kurtosis:                      14.519   Cond. No.                         1.00
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

$\beta_{IBM} = 0.91$**, P is low, we reject the null hypothesis.**

2.1.2 AAPL

```
In [6]: d=data2[data2.DATE<pd.to_datetime("2015-01-01")]
        X=np.array(d[["RM"]])
        y=np.array(d["R"])
```

```
In [7]: est = sm.OLS(y, X)
        est2 = est.fit()
        print(est2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.192
Model:                            OLS   Adj. R-squared:                  0.192
Method:                 Least Squares   F-statistic:                     1500.
Date:                Wed, 03 Apr 2019   Prob (F-statistic):          1.61e-294
Time:                        10:09:54   Log-Likelihood:                 13915.
No. Observations:                6301   AIC:                        -2.783e+04
Df Residuals:                    6300   BIC:                        -2.782e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
x1             1.1363      0.029     38.728      0.000       1.079       1.194
==============================================================================
Omnibus:                     1964.903   Durbin-Watson:                   1.980
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           232261.428
Skew:                          -0.417   Prob(JB):                         0.00
```

```
Kurtosis:                      32.732   Cond. No.                         1.00
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

$\beta_{AAPL} = 1.14$, **P is low, we reject the null hypothesis.**

2.1.3 TSLA

```
In [8]: d=data3[data3.DATE<pd.to_datetime("2015-01-01")]
        X=np.array(d[["RM"]])
        y=np.array(d["R"])
```

```
In [9]: est = sm.OLS(y, X)
        est2 = est.fit()
        print(est2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.122
Model:                            OLS   Adj. R-squared:                  0.121
Method:                 Least Squares   F-statistic:                     157.4
Date:                Wed, 03 Apr 2019   Prob (F-statistic):           6.51e-34
Time:                        10:09:54   Log-Likelihood:                 2219.2
No. Observations:                1135   AIC:                            -4436.
Df Residuals:                    1134   BIC:                            -4431.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
x1             1.3101      0.104     12.548      0.000       1.105       1.515
==============================================================================
Omnibus:                      262.175   Durbin-Watson:                   2.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2947.095
Skew:                           0.726   Prob(JB):                         0.00
Kurtosis:                      10.760   Cond. No.                         1.00
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

$\beta_{TSLA} = 1.14$, **P is low, we reject the null hypothesis.**

2.2 (b)

let $h = (a, b, c)^T$

from self-financing condition, we get $a + b + c = 0$

from unit exposure to AAPL, we get $b = 1$

from zero exposure to beta, we get $a\beta_{IBM} + b\beta_{AAPL} + c * \beta_{TSLA} = 0$

we use following code to solve this linear problem

```
In [33]: a = np.array([[1,1,1], [0,1,0],[0.91,1.14,1.31]])
         b = np.array([0.0,1.0,0.0])
         x = np.linalg.solve(a, b)
         x

Out[33]: array([-0.425,  1.   , -0.575])
```

$\therefore h = (-0.425, 1, -0.575)^T$

2.3 (C)

```
In [11]: d1=data1[data1.DATE>=pd.to_datetime("2015-01-01")]
         d2=data2[data2.DATE>=pd.to_datetime("2015-01-01")]
         d3=data3[data3.DATE>=pd.to_datetime("2015-01-01")]

In [17]: sum(d1.R*(-0.425))+sum(d2.R*1)+sum(d3.R*(-0.575))

Out[17]: -0.021396825000000078
```

cumulative sum of return in 2015 is -0.0214

2.4 (d)

```
In [13]: ret=np.array(d1.R*(-0.425))+np.array(d2.R*1)+np.array(d3.R*(-0.575))
         ret_m=np.array(data1[data1.DATE>=pd.to_datetime("2015-01-01")].RM)

In [14]: pearsonr(ret,ret_m)

Out[14]: (0.02866801914913187, 0.6512598504889181)
```

P-value is 0.65, not greater than 0.95. So it is not significantly different from 0.

3 Problem 4.2

3.1 (a)

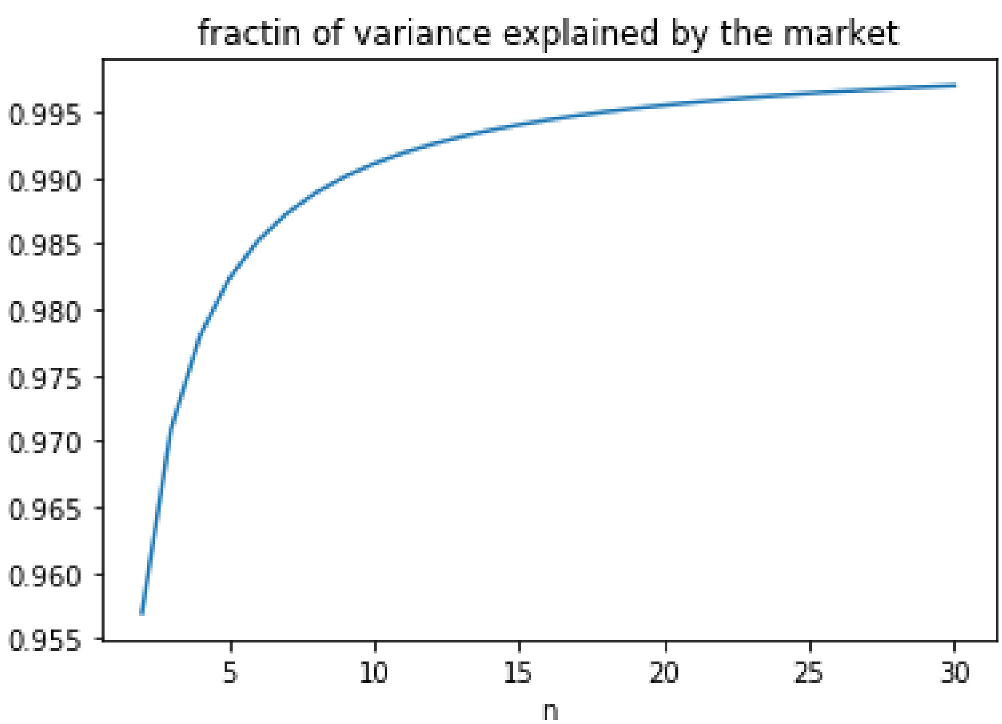
$$E[h'r] = \frac{1}{n}\sum_i E[r_i] = \beta E[r_m] + \frac{1}{n}\sum_i E[\epsilon_i]$$

$$V[h'r] = \frac{1}{n^2}\sum_i(\beta^2\sigma_m^2 + \sigma_i^2) + \frac{1}{n^2}\sum_{i,j,i\neq j}\beta^2\sigma_m^2 = \beta^2\sigma_m^2 + \sum_i \frac{1}{n^2}\sigma_i^2$$

$$\therefore f(\beta, \sigma_m^2) = \beta^2\sigma_m^2 \qquad g(\sigma_1^2, \ldots, \sigma_n^2) = \frac{1}{n^2}\sum_i \sigma_i^2$$

3.2 (b)

In [2]:
```
beta=0.5
sigmaM=0.2
sigmaI=0.03

def f(n):
    return beta**2*sigmaM**2

def g(n):
    return sigmaI**2/n

def fraction(n):
    return f(n)/(f(n)+g(n))

x=[]
y=[]
for i in range(2,31):
    x.append(i)
    y.append(fraction(i))

import matplotlib.pyplot as plt
plt.plot(x,y)
plt.title("fractin of variance explained by the market")
plt.xlabel('n')
plt.show()
```

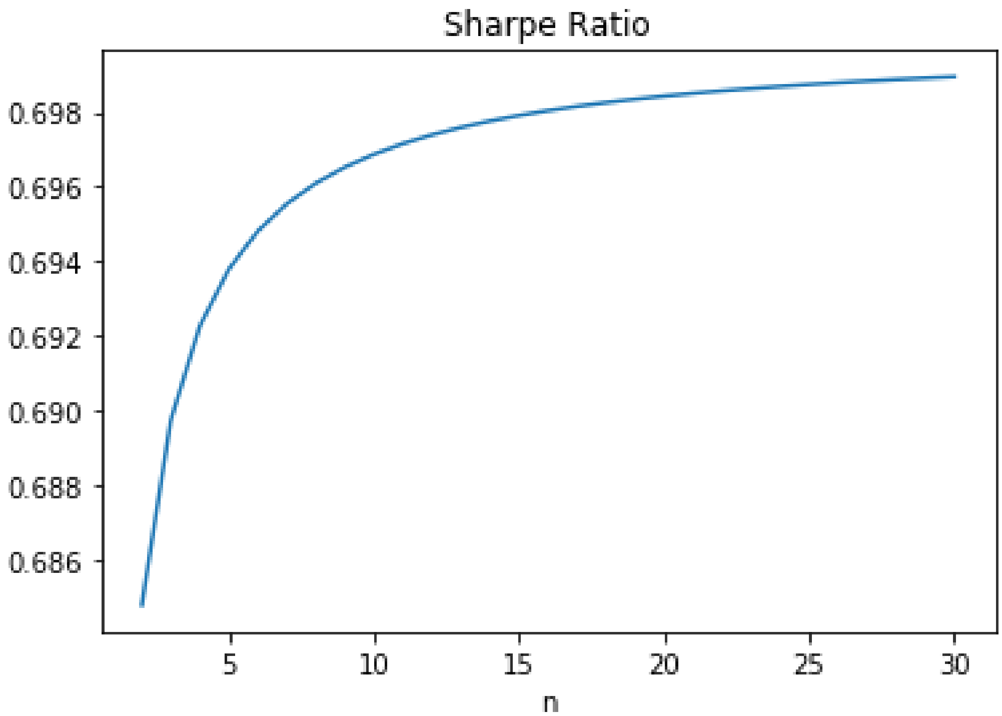


3.3 (c)

In this case,

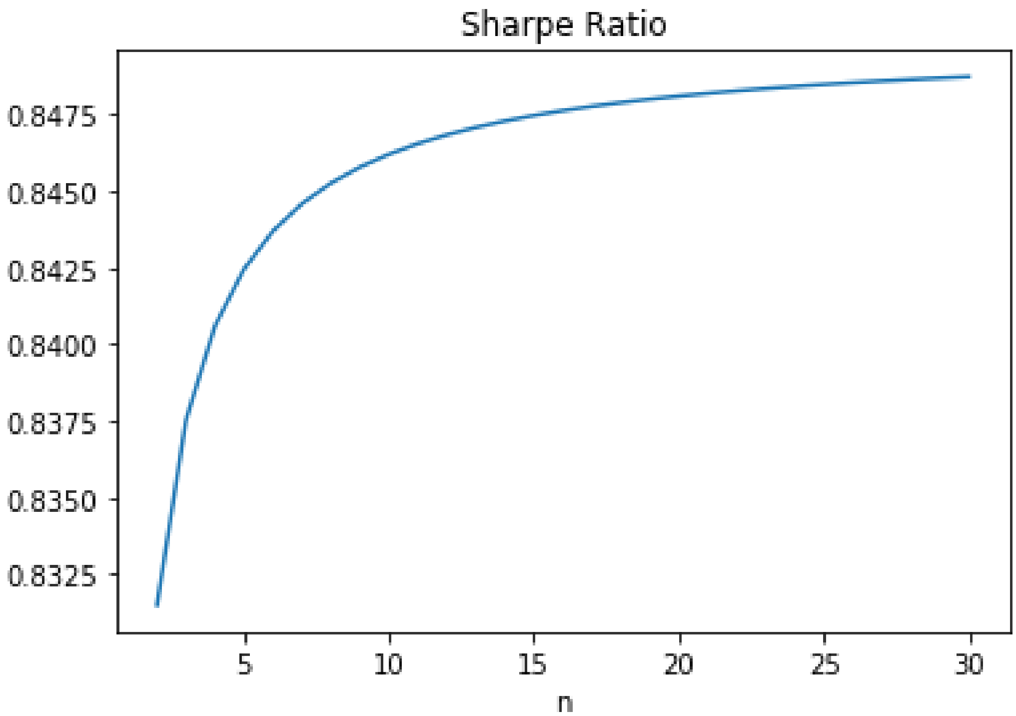
$$E[h'r] = \frac{1}{n}\sum_i \beta E[r_M] + E[\epsilon_i]$$

```
In [3]: Eepsilon=1.5*sigmaI
        ErM=0.07

        def E(n):
            return beta*ErM+Eepsilon

        def V(n):
            return f(n)+g(n)

        def Sharpe(n):
            return (E(n)-0.01)/np.sqrt(V(n))

        x=[]
        y=[]
        for i in range(2,31):
            x.append(i)
            y.append(Sharpe(i))

        plt.plot(x,y)
        plt.title("Sharpe Ratio")
        plt.xlabel('n')
        plt.show()
```



If we change Sharpe of ϵ_i from 1.5 to 2.0

```
In [4]: Eepsilon=2.0*sigmaI

        x=[]
        y=[]
        for i in range(2,31):
            x.append(i)
            y.append(Sharpe(i))

        plt.plot(x,y)
        plt.title("Sharpe Ratio")
        plt.xlabel('n')
        plt.show()
```



3.4 (d)

Suppose Sharpe of ϵ_i is 1.5

$$E[h'r] = E[\epsilon_i] = 1.5\sigma_i$$

$$V[h'r] = \sigma_i^2$$

$$\therefore Sharpe = \frac{1.5\sigma_i - 0.01}{\sigma_i} = 1.4$$

Compare to above scenario, this is much better.