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2.1
$$\begin{cases} Px^* + A^T v^* = -q \\ Ax^* = b \end{cases} \quad \therefore \quad \cancel{x^* = A^{-1}b} \quad \cancel{v^* = (A^T)^{-1}(-q - PA^{-1}b)}$$

$$\begin{cases} x^* = P^{-1}A^T(AP^{-1}A^T)^{-1} q \\ v^* = -(AP^{-1}A^T)^{-1} b - (AP^{-1}A^T)^{-1}AP^{-1} q \end{cases}$$

we need to assume P and $AP^{-1}A^T$ are invertible

if P has a non-trivial null space, then it is non-invertible

then there are more than one solutions to the original problem.

2.2
$$p^*(u,v) = \inf_{\text{constrained}}(f_0(x)) = L(x^{**}, \lambda^{**}, v^{**})$$
where $x^{**}, \lambda^{**}, v^{**}$ is optimal value for perturbed problem

$$L(x,\lambda,v) = f_0(x) + \sum_i \lambda_i (f_i - u_i) + \sum_j v_j (h_j - v_j)$$

$$\therefore \quad -\frac{\partial p^*(u,v)}{\partial u_i} = \lambda_i^{**} \qquad -\frac{\partial p^*(u,v)}{\partial v_j} = v_j^{**}$$

when $u=0$, $v=0$, perturbed problem back to original problem and $(x^{**},\lambda^{**},v^{**}) = (x^*,\lambda^*,v^*)$ where x^*,λ^*,v^* is optimal value for original problem

$$\therefore \quad -\frac{\partial p^*(0,0)}{\partial u_i} = \lambda_i^* \qquad -\frac{\partial p^*(0,0)}{\partial v_j} = v_j^*$$