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1.1 i> to prove $\mathbb{E}(u(w+z)) \le u(w) \Rightarrow u$ is concave

let z is r.v., s.t. $\mathbb{P}(z+w=x)=\lambda$, $P(z+w=y)=1-\lambda$ $(0\le\lambda\le1)$

and $w=\lambda x+(1-\lambda)y$ to satisfy $\mathbb{E}(z)=0$

$\therefore \mathbb{E}(u(w+z)) = \lambda u(x)+(1-\lambda)u(y) \le u(w) = u(\lambda x+(1-\lambda)y)$

$\therefore$ u is concave

ii> to prove $\mathbb{E}(u(w+z)) \le u(w) \Leftarrow u$ is concave

according to Jensen inequality, $\mathbb{E}(u(w+z)) \le u(\mathbb{E}(w+z)) = u(w)$

1.2 according to A-P approximation

$u'(w)=e^{-kw}$ $\quad u''(w)=-ke^{-kw}$ $\quad \therefore A(w)=k$ $\quad \therefore \pi_{A-P}=\frac{1}{2}k\sigma^2$

$u(w-\pi) = -\frac{1}{k}e^{-k(w-\pi)}$

$$\mathbb{E}\,u(w+z) = \int_{-\infty}^{\infty} -\frac{1}{k}e^{-k(w+x)}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = -\frac{1}{k}e^{-kw+\frac{1}{2}k^2\sigma^2}$$

let $u(w-\pi)=\mathbb{E}\,u(w+z)$

$\therefore \pi = \frac{1}{2}k\sigma^2$

$\pi=\pi_{A-P}$

$\therefore$ A-P Approximation is exact

1.3. (a) $\Leftrightarrow$ (b)

(a) $\Leftrightarrow \frac{1}{2}\sigma^2 A_v(w) \geq \frac{1}{2}\sigma^2 A_u(w) \Leftrightarrow A_v(w) \geq A_u(w) \Leftrightarrow$ (b)

(b) $\Leftarrow$ (c)

$$A_v(w) \geq A_u(w) \Leftrightarrow -\frac{v''(w)}{v'(w)} \geq -\frac{u''(w)}{u'(w)} \Leftarrow \frac{\phi''(u)u'^2(w) + \phi'(u)u''(w)}{\phi'(u)u'(w)} \leq \frac{u''(w)}{u'(w)}$$

$\because u'>0,\ u''<0 \Leftarrow \phi'>0,\ \phi''<0$

(b) $\Rightarrow$ (c)

let $\phi(u) = v$

$$A_v(w) \geq A_u(w) \Rightarrow \frac{\phi''(u)u'^2(w) + \phi'(u)u''(w)}{\phi'(u)} \leq u''(w)$$

should hold for any u where $u'<0$ $u''>0$

$\therefore \phi'>0 \quad \phi''<0$

1.4 $\mathbb{E}u(w+z) = u(w+e)$

$\mathbb{E}v(w+z) = v(w+e') = a+bu(w+e')$

$\quad \parallel$

$a+b\mathbb{E}u(w+z) = a+bu(w+e)$

$\because u'>0 \quad \therefore u(w+e) = u(w+e') \Rightarrow e=e'$

$\therefore$ they have same certainty-equivalents