Interest Rate and Credit Models

Homework Assignment #1

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Problems

1. Assume an interest rate model with a constant instantaneous OIS rate r, and a constant instantaneous LIBOR rate l. Consider a spot starting swap maturing T years from now (T is an integer). For simplicity, assume that the coupon dates on both fixed and floating legs are equally spaced with all day count fractions equal to 0.5 and 0.25, respectively. Also, for simplicity, neglect the 2 business day lag between LIBOR fixing and start of an accrual period.

   (i) Find an explicit expression for the values of both legs of of the swap.

   (ii) Determine the par coupon on the swap.

2. Consider a credit model with a constant intensity λ, and a constant discounting (OIS) rate r. Consider a new CDS starting on a roll date and maturing T years from now (T is an integer) . For simplicity, assume that the roll dates are equally spaced with all day count fractions equal 0.25. The recovery rate is assumed to be R.

   (i) Find an explicit expression for the value of the protection leg of the swap.

   (ii) Find an explicit expression for the risky annuity.

(iii) Determine the par spread on the swap.

3. Repeat Problem 2 assuming a constant discounting rate r and the stochastic intensity $\lambda(t)$ introduced in Problem 4 of Homework Assignment #1.

4. (**Programming project**.) The enclosed spreadsheet contains a snapshot of actual par spreads (as of October 6, 2015) for several names: General Electric, JPMorgan Chase, Axis Capital, and MBIA. The values of the spreads are expressed in basis points (i.e. the value of 45 means 0.0045). In the calculations below, assume that the recovery rate is 40%, and the riskless rate r is constant and equal 1.5%.

   (i) Use the valuation formulas and the bootstrap method explained in class to build the survival curves for each of these names.

   (ii) Based on these curves, calculate the default probabilities of each of these names in $T = 1, 2, \ldots, 10$ years.

   (iii) For each of the names, compute the par spread for the 2Y into 5Y forward CDS.

   Note that Bloomberg has a number of screens containing the data and analytic for CDS, such as WCDS, CDSV, and others. I recommend that you spend some time familiarizing yourself with these screens.

5. Consider a frictionless market $\big(S_1(t), \ldots, S_n(t)\big)$ and a self-financing portfolio with weights $\big(w_1(t), \ldots, w_n(t)\big)$. Given a numeraire $\mathcal{N}(t)$, show that the portfolio expressed in terms of the relative prices $\big(S_1^{\mathcal{N}}(t), \ldots, S_n^{\mathcal{N}}(t)\big)$ is self-financing.

This assignment is due on March 11