Interest Rate and Credit Models

Homework Assignment #5

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1. Using the market data sets provided for the previous homework assignments, calculate approximately the following CMS rates:

   - 10 year CMS settling in 1 year and paying 3 months later, and
   - 10 year CMS settling in 5 years and paying 3 months later.

   For simplicity, use the Black model approach (the first method on page 30 of Lecture Notes #6). It produces less accurate results but is much simpler than the replication method. What fraction of the CMS convexity correction comes from the payment delay?

2. In this problem, all formula numbers refer to Lecture Notes #10. We consider an affine term structure model, i.e. a short rate model, with Q_0-dynamics given by equation (1), in which the zero coupon price is given by equation (42).

   (i) Let, as usual, $P(t, T)$ denote the time t price of a zero coupon bond for maturity T. Argue that $P(t, T)$ satisfies the following partial differential equation:

$$\frac{\partial P}{\partial t} + \frac{1}{2}\,\sigma(t, r)^2\,\frac{\partial^2 P}{\partial r^2} + \mu(t, r)\frac{\partial P}{\partial r} = rP,$$
$$P(T, T) = 1.$$

This fact does not, of course, require that the model be affine.

(ii) Conclude that in an affine term structure model with coefficients $A(t,T)$ and $B(t,T)$,

$$\frac{\partial \log A}{\partial t} - r\,\frac{\partial B}{\partial t} + \frac{1}{2}\,\sigma(t,r)^2 B^2 - \mu(t,r)B = r,$$

with

$$\begin{aligned} A(T,T) &= 1, \\ B(T,T) &= 0. \end{aligned}$$

(iii) Show that if the coefficients in the SDE (1) are of the form (43), then the partial differential equation above reduces to the system (44) of ordinary differential equations.

The purpose of the following two problems is to implement a simple 1-factor LIBOR market model as in Lecture Notes #11. We consider the following 1-factor normal LMM:

$$\begin{aligned} dL_j(t) &= \Delta_j(t, L(t))dt + \sigma_j(t)dZ(t), \\ L_j(0) &= L_{j,0}. \end{aligned} \tag{1}$$

where $\sigma_j(t)$ is a deterministic instantaneous volatility function which should be calibrated to the market. For simplicity, we make the following assumptions:

(i) We assume that $\sigma_j(t) = 0.0085$, for all t and j (thus circumventing the issue of calibrating the model...).

(ii) The LIBOR / OIS basis is zero, and so the model is a single curve model.

(iii) The dynamics is written under the terminal forward measure, and thus the drift terms Δ_j are given by the appropriate formulas stated in Lecture Notes #11.

For the initial value of the SDEs (1) you could use the corresponding forward calculated from the curve that you have built in Homework Assignment #1.
I also suggest that you take into account the following points:

(i) Ideally the implementation should be done in Python (or C++). Make sure that the Gaussian random numbers are generated by means of a quality algorithm (such as described in Lecture Notes #11).

(ii) Use the spectral decomposition algorithm to simulate a Brownian motion.

3. Implement the model using Euler's scheme (note that for the normal LMM, Euler's and Milstein's schemes are identical). For drift term calculations, implement the ability to do both:

   (i) the exact calculation, and

   (ii) the frozen curve approximation.

4. Apply your model to a 1Y into 10Y European receiver swaption struck at 3.872%. Use 2,000 simulated paths to carry out the calculation. If you wish to implement a variance reducing method, you may consider using antithetic variables.

   (i) How accurate is your calculation? Compare against a run with 5,000 simulated paths.

   (ii) Compare the performance and accuracy of both drift term calculation methods.

This assignment is due on April 29