Interest Rate and Credit Models

Homework Assignment #4

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Problems

1. Consider Andersen's method for pricing a single period CMDS as discussed in class. Let α be the participation rate (known). Assume the underlying spread is lognormally distributed, i.e. in the notation of pages 27 - 30 of Lecture Notes #7,

$$\varphi(x) = \frac{1}{\sqrt{2\pi\sigma^2 T_{i-1}}\, x} \exp\Big(- \frac{(\log(x/C_0) + \sigma^2 T_{i-1}/2)^2}{2\sigma^2 T_{i-1}} \Big)$$

   (i) Calculate $V_{cpn}(0)$ assuming no cap on the spread. Express the result in terms of the cumulative normal distribution.

   (ii) Calculate $V_{cpn}(0)$ assuming a cap of U on the spread. Express the result in terms of the cumulative normal distribution.

   (iii) What is the value of the cap option?

2. Consider the Marshall-Olkin copula $C_{MO}(u_1, u_2)$.

   (i) Calculate the Spearman rho of $C_{MO}(u_1, u_2)$.

   (ii) Calculate the limits λ_U and λ_L and determine whether the Marshall-Olkin copula has upper and / or lower tail dependencies.

(iii) Propose a simulation algorithm for $C_{MO}(u_1, u_2)$. Choose $\lambda_1 = 1.7, \lambda_2 = 0.7, \lambda_{12} = 0.3$, and generate 2000 samples from $C_{MO}(u_1, u_2)$. Plot the results in a 1×1-square.

3. Consider the following two-factor generalization of the one-factor Gaussian copula model:

$$Z_i = \beta_{i1} S_1 + \beta_{i2} S_2 + \sqrt{1 - \beta_{i1}^2 - \beta_{i2}^2}\, \varepsilon_i,$$

where $S_1, S_2 \sim N(0,1)$ are independent, $\beta_{i1}^2 + \beta_{i2}^2 < 1$, for each $i = 1, \ldots, N$, and $\varepsilon_i \sim N(0,1)$ are independent of S_1 and S_2.

(i) Determine the correlation structure in this model.

(ii) Derive, along the lines of the calculations done in class for the one-factor model, the LHP limit of this model:

(a) Find the conditional loss distribution $\mathsf{P}(L_N(T) = kl | S_1 = s_1, S_2 = s_2)$.

(b) Find the total loss distribution $\mathsf{P}(L_N(T) = kl)$.

(c) Calculate the $N \to \infty$ limit of $\mathsf{P}(l_N \leq xl)$, where $l_N \triangleq \frac{L_N(T)}{N}$.

4. Assume that the CCP calculates the margin requirements by means of VaR at the $p = 99\%$ confidence level. Regulatory authorities require that the CCP back test the performance of its risk model by calculating the number of breaches of the VaR model over the test period of the most recent 250 days (one year). Specifically, on each day of the test period, the daily portfolio loss[1] is compared to the calculated VaR level. A VaR breach means that the loss on a given day exceeds the VaR level.

(i) Assuming that the daily portfolio returns are independent of each other, find the probability distribution of the number of VaR breaches. What is the expected number of VaR breaches?

(ii) Determine numerically the probability of $k = 0, 1, \ldots, 10$ VaR breaches.

(iii) Portfolio back test showed 4 VaR breaches. Can one reject, at the confidence level of 95%, the hypothesis that the VaR model is working properly?

This assignment is due on April 8

[1] In reality, it is the n-day portfolio loss, where n is called the market period of risk (MPOR).