MTH 9878 - INTEREST RATES

BARUCH COLLEGE

HOMEWORK #5

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1.

(i) Model (1) is inadequate for prepayment modeling because the density which is represented by

$$f(t) = e^{\lambda(t)} = e^{\lambda\gamma(\lambda t)^{\gamma-1}}$$

has undesirable properties for prepayment modelling. For $\gamma > 1$, the density increases in t and decreases in t for $\gamma < 1$. Prepayments in reality are more likely near the middle of the period between the settlement date and the maturity date. More importantly, the intensity is only a function of time and thus ignores most of the factors that affect prepayment.

(ii) The survival probability is represented by

$$S(t,T) = \exp\left\{-\int_t^T \lambda(s)ds\right\}$$
$$= \exp\left\{-\int_t^T \lambda\gamma(\lambda s)^{\gamma-1}ds\right\} = e^{-\lambda^\gamma(T^\gamma - t^\gamma)}$$

(iii) The expression for the valuation of a TBA is

$$P(T) = \mathbb{E}\left[\sum_j Z^\pi(T,T_j)(C_j + \bar{\lambda}(T_{j-1},T_j)B_j\right]$$
$$= \mathbb{E}\left[\sum_j e^{-\lambda^\gamma(T^\gamma - T_j{}^\gamma) - r(T-T_j)}\left(C_j + \left(1 - e^{\lambda^\gamma\left(T_j^\gamma - T_{j-1}^\gamma\right)}\right)B_j\right)\right]$$
$$= \mathbb{E}\left[\sum_j e^{-\lambda^\gamma(T^\gamma - T_j{}^\gamma) - r(T-T_j)}\left(C_j + \left(1 - e^{\lambda^\gamma\left(T_j^\gamma - T_{j-1}^\gamma\right)}\right)B_j\right)\right]$$

the expression is deterministic so

$$= \sum_j e^{-\lambda^\gamma(T^\gamma - T_j{}^\gamma) - r(T-T_j)}\left(C_j + \left(1 - e^{\lambda^\gamma\left(T^\gamma - T_{j-1}^\gamma\right)}\right)B_j\right)$$

2.

(i) Model (2) is inadequate for prepayment modeling because it does not depend on the interest rate at time t. A better way of modelling the probability of prepayment would have the hazard rate depend on the factors that affect the likelihood of prepayment.

(ii) The survival probability is represented by

$$S(t,T) = \exp\left\{-\int_t^T \lambda(s)ds\right\}$$
$$= \exp\left\{-\int_t^T \frac{\gamma\lambda(\lambda s)^{\gamma-1}}{1+(\lambda s)^\gamma}ds\right\} = \exp\left\{\log(1+(\lambda s)^\gamma)\ \Big|_{\mathrm{t}}^{\mathrm{T}}\right\}$$
$$= \exp\left\{\log\left(\frac{1+(\lambda T)^\gamma}{1+(\lambda t)^\gamma}\right)\right\} = \frac{1+(\lambda T)^\gamma}{1+(\lambda t)^\gamma}$$

(iii) The expression for the valuation of a TBA is

$$P(T) = \mathbb{E}\left[\sum_j Z^{\pi}(T,T_j)(C_j + \bar{\lambda}(T_{j-1},T_j)B_j\right]$$
$$= \mathbb{E}\left[\sum_j e^{-r(T-T_j)}\frac{1+(\lambda T)^\gamma}{1+(\lambda T_{j-1})^\gamma}\left(C_j + \left(1 - \frac{1+(\lambda T_{j-1})^\gamma}{1+(\lambda T_j)^\gamma}\right)B_j\right)\right]$$

the expression is deterministic so

$$= \sum_j e^{-r(T-T_j)}\frac{1+(\lambda T)^\gamma}{1+(\lambda T_{j-1})^\gamma}\left(C_j + \left(1 - \frac{1+(\lambda T_{j-1})^\gamma}{1+(\lambda T_j)^\gamma}\right)B_j\right)$$

(iv) I would choose model (2) because the hazard rate has the ability to adjust to create a density such that prepayment is more likely near the middle between the settlement date and the maturity date.