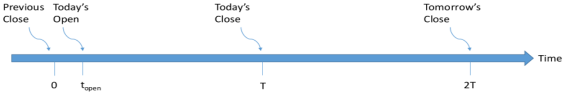
Optimal Trading with Alpha Predictors

Filippo Passerini, Samuel E. Vazquez

Lecturer: Chenyu Zhao (Part I), Haocheng Gu (Part II)

April 30, 2019

Set up, Market order only



- Price dynamic $dP_t = \alpha_t dt + \sqrt{\nu} dW_t$
  Decompose alpha $\alpha_t := \bar{\alpha} + x_t, \quad \mathbb{E}[x_t] = 0$
  Intra-day signal general dynamic $dx_t = \mu(t,x)dt + \sqrt{\eta(t,x)}dZ_t$
  Evolution of our position $dq_t = u_t dt$
- Objective function, only market order
  $\Omega(t,x,q) = \min_{\{u_s|s\in(t,T)\}} \mathbb{E}\left[\int_t^T C|u_S|ds + K\int_t^T u_s^2 ds - \int_t^{2T} \alpha_s q_s ds + \frac{1}{2}\lambda\nu \int_t^{2T} q_s^2 ds \,|q_t = q, \; x_t = x\right]$
  cost of market order + temporary market impact + overnight gain and risk

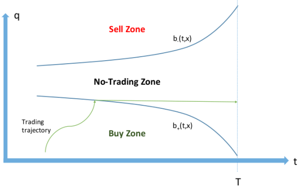
Market order only

- Tracking Markowitz portfolio $\bar{q} = \frac{\bar{\alpha}}{\lambda\nu}$
- New objective function
  $V(t, x, q) := \Omega(t, x, q) + g(t, x)q + \frac{1}{2}\lambda\nu\bar{q}^2(2T - t)$
  where $g(t, x) := \int_t^{2T} \mathbb{E}[x_s | x_t = x] ds$
- New HJB

$$\hat{D}_{t,x}V + \frac{1}{2}\lambda\nu(q - \bar{q})^2 + \min_u \left[C|u| + Ku^2 + \left(\frac{\partial V}{\partial q} - g\right)u \right] = 0$$

Terminal Condition $V(T, x, q) = \frac{1}{2}\lambda\nu T(q - \bar{q})^2$

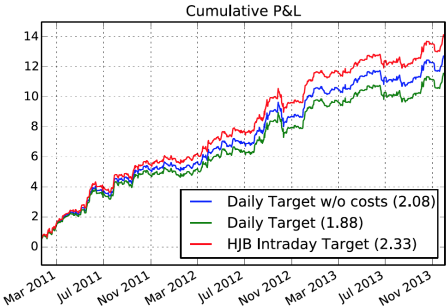
$$u = \begin{cases} \frac{1}{2K}\left(g - c - \frac{\partial V}{\partial q}\right), & \text{if } g > c + \frac{\partial V}{\partial q}, \textit{ buy}. \\ 0, & \text{otherwise, } \textit{no trading}. \\ -\frac{1}{2K}\left(-g - c + \frac{\partial V}{\partial q}\right), & \text{if } g < -c + \frac{\partial V}{\partial q}, \textit{ sell}. \end{cases}$$



- Now the function is

$$\hat{D}_{t,x}V + \frac{1}{2}\lambda\nu(q-\bar{q})^2 - Ku^2 = 0$$

- Ignoring $-Ku^2$ and we have $V \approx \frac{1}{2}\lambda\nu(2T-t)(q-\bar{q})^2$
- The boundaries $b_{\pm}(t,x) = \bar{q} + \frac{1}{\lambda\nu(2T-t)}(g(t,x) \mp C)$ where $g(t,x) := \int_t^{2T} \mathbb{E}[x_s|x_t = x]ds$
- Simulate using mean reverting alpha process and get



Adding limit order

- With limit order and market order, the position can be written as

$$dq_t = (m_t^+ - m_t^- + 1t^+ lt^+ - 1t^- lt^-)dt$$

- New HJB equation is

$$\begin{aligned} 0 \;=\; & \hat{D}_{t,x,y} \cdot V + \frac{1}{2}\lambda\nu(q-\bar{q})^2 \\ & + \min_{m^\pm, l^\pm} \left[m^+ \left(C + \frac{\partial V}{\partial q} - g \right) - P^+ l^+ \left(C - \frac{\partial V}{\partial q} + g \right) \right. \\ & \left. + m^- \left(C - \frac{\partial V}{\partial q} + g \right) - P^- l^- \left(C + \frac{\partial V}{\partial q} - g \right) \right] \end{aligned}$$

- New zones and simulation results are