Time Series Analysis- GroupH - Assignment 2

Group H: Shenyi Mao, Yueting Jiang, Chenyu Zhao, Jose Ferreira

Question 1

In [1]:
```
%matplotlib inline

import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.ar_model import AR
from statsmodels.tsa.arima_model import ARMA
import statsmodels as sm
import pandas as pd
```

In [2]:
```
#Simulate AR(1)
 alpha=0.1
 beta = 0.3
 sigma=0.005
 T_list=[100,250,1250]
 N = 2000


 for T in T_list:
     alpha_sum = 0
     beta_sum = 0
     sigma_sum = 0
     for i in range(0,N):
         x0=alpha/(1-beta)
         x=np.zeros(T+1)
         x[0]=x0
         eps=np.random.normal(0.0,sigma,T)
         for i in range(1,T+1):
             x[i]=alpha+beta*x[i-1]+eps[i-1]

         y=x[0:T]
         yp=x[1:(T+1)]
         m=np.sum(y)/T
         mp=np.sum(yp)/T
         betaCMLE=np.inner(y-m,yp-mp)/np.inner(y-m,y-m)
         alphaCMLE=mp-betaCMLE*m
         sigmaCMLE=np.sqrt(np.inner(yp-betaCMLE*y-alphaCMLE,
                                    yp-betaCMLE*y-alphaCMLE)/T)

         alpha_sum += alphaCMLE
         beta_sum += betaCMLE
         sigma_sum += sigmaCMLE
```

```
        alpha_hat = alpha_sum / N
        beta_hat = beta_sum / N
        sigma_hat = sigma_sum / N
        print("T = ", T, ",(alpha,beta,sigma) = " ,[alpha_hat,beta_hat,sigma_hat])
```

```
T =  100 ,(alpha,beta,sigma) =  [0.1024153805831, 0.2831247621365, 0.004951490497068]
T =  250 ,(alpha,beta,sigma) =  [0.1009745294900, 0.2932032633954, 0.004971390214618]
T =  1250 ,(alpha,beta,sigma) =  [0.1002329906964, 0.2983269719304, 0.004993956810083]
```

Comment

- We run simulations N(=2000) times to give T(=100,250,1250) observations from the given AR(1) model and then fit the observartions into AR(1) to give the average MLE estimates.
- The output is:
$$T = 100, [\hat{\alpha}, \hat{\beta}, \hat{\sigma}] = [0.102415, 0.283124, 0.00495149]$$
$$T = 250, [\hat{\alpha}, \hat{\beta}, \hat{\sigma}] = [0.100974, 0.293203, 0.00497139]$$
$$T = 1250, [\hat{\alpha}, \hat{\beta}, \hat{\sigma}] = [0.100232, 0.298326, 0.00499396]$$
we can see that the estimates are biased compare with the given parameters
$$[\alpha, \beta, \sigma] = [0.1, 0.3, 0.005]$$
- for these biased estimates,we can see that when T goes larger, the bias get smaller because the error is O($\frac{1}{N}$),which means that they are consistant.

Question 2

```
In [7]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        from statsmodels.tsa.stattools import adfuller
```

```
In [37]: def perform_adf_test(xt):
             """
             input:  a timeseries
             output: outputs the statistic values from the ADF test
             """
             test = adfuller(xt)
             print('ADF Statistic: %f' % test[0])
             print('p-value: %f' % test[1])
             for key, value in test[4].items():
                 print('\t%s: %.3f' % (key, value))

         def plot_series(eur, diff_rates):
             plt.figure()
```

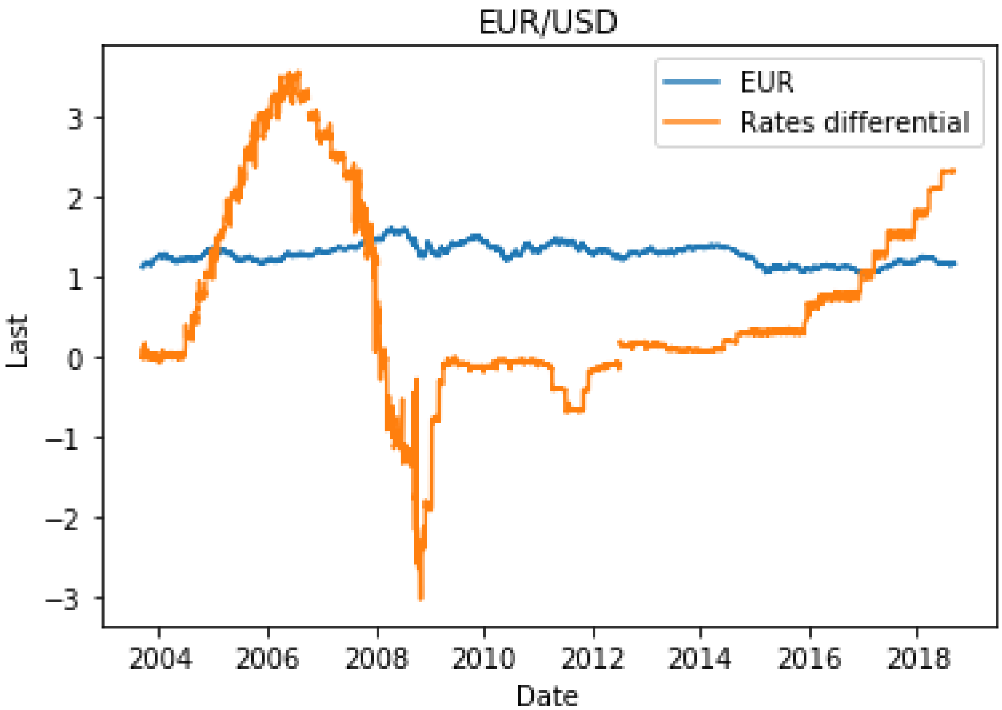
```
    plt.plot(eur,label='EUR')
    plt.plot(diff_rates,label='Rates differential')
    plt.title('EUR/USD')
    plt.xlabel('Date')
    plt.ylabel('Last')
    plt.legend()
    plt.show()

def plot_coint_residual(u):
    plt.figure()
    plt.plot(u,label='Cointegrating residual')
    plt.title('EUR/USD - (FED-ECB)')
    plt.xlabel('Date')
    plt.ylabel('Last')
    plt.legend()
    plt.show()
```

In [54]:
```
def perform_engle_granger_test(eur, diff_rates, coint_residual, confidence):
    """
    input:a cointegrating residual vector and the level of confidence
    output: True if the series are cointegrated
    """
    # run Dickey-Fuller on each series separately to check if they are I(d).
    print("EURUSD")
    eur_p = perform_adf_test(eur)
    print("Diff")
    diff_p = perform_adf_test(diff_rates)
    # this should be stationary
    print("delta EURUSD")
    lagged_eur = eur.diff().dropna()
    lag_eur_p = perform_adf_test(lagged_eur)

    # this too should be stationary
    print("delta rates")
    lagged_rates = diff_rates.diff().dropna()
    lag_rates_p = perform_adf_test(lagged_rates)



    # test unit root on the cointegration residual process
    # this would need to be stationary to say there is cointegration

    print("u")
    perform_adf_test(coint_residual)
    test = adfuller(coint_residual)
    p_value=test[1]
    if (p_value>confidence):
        print("The cointegrating residual is not stationary")
```

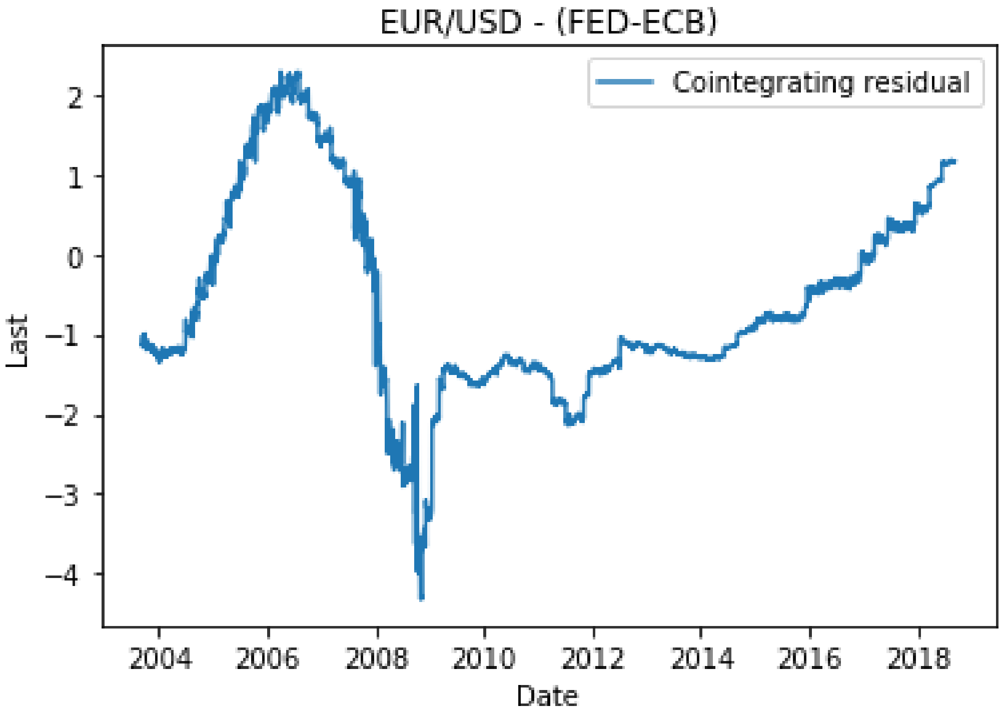
```
            return False

        print ("The series are cointegrated")
        return True
```

```
In [57]: def problem_2():
             # Section 2.1: Perform a cointegration test
             # Read data and generate time series
             eur = pd.read_excel('HW2_Data.xlsx', sheet_name = 'EUR',
                                 index_col=0, parse_dates=True, infer_datetime_format=True)
             fed = pd.read_excel('HW2_Data.xlsx', sheet_name = 'FEDL01',
                                 index_col=0, parse_dates=True, infer_datetime_format=True)
             ecb = pd.read_excel('HW2_Data.xlsx', sheet_name = 'EUORDEPOT',
                                 index_col=0, parse_dates=True, infer_datetime_format=True)

             diff = fed - ecb

             plot_series(eur, diff)
             eur_rates = eur['Last Price'].dropna()
             diff_interest_rates = diff['Last Price'].dropna()
             merged_data = pd.concat([eur_rates.rename('eur'), diff_interest_rates.rename
             ('rates')], axis=1)
             print(merged_data.head())
             clean_data = merged_data.dropna()
             u = (clean_data['rates'] - clean_data['eur'])

             plot_coint_residual(u)

             perform_engle_granger_test(eur_rates, diff_interest_rates, u, confidence=0.05)
```

```
In [58]: problem_2()
```




```
              eur  rates
Date
2003-09-11  1.1210   0.00
2003-09-12  1.1291   0.02
2003-09-15  1.1284   0.11
2003-09-16  1.1177  -0.03
2003-09-17  1.1285  -0.03
```




```
EURUSD
ADF Statistic: -1.966646
p-value: 0.301386
        5%: -2.862
        1%: -3.432
        10%: -2.567
Diff
ADF Statistic: -1.016293
p-value: 0.747276
        5%: -2.862
        1%: -3.432
        10%: -2.567
delta EURUSD
ADF Statistic: -62.279418
p-value: 0.000000
        5%: -2.862
        1%: -3.432
        10%: -2.567
delta rates
ADF Statistic: -9.600190
p-value: 0.000000
        5%: -2.862
        1%: -3.432
        10%: -2.567
```

```
u
ADF Statistic: -0.930077
p-value: 0.777906
        5%: -2.862
        1%: -3.432
        10%: -2.567
The cointegrating residual is not stationary
```

Comment

First, we run ADF on EURUUSD, the short interest difference between Fed and ECB. The test result shows that both series are not stationary at 5% significance level. And the first difference of both series are stationary at 5% significance level. Thus we can conclude that both series are I(1).

Second, we use the given cointegration vector $\alpha = (1, -1)$ to calculate the residual of the cointegration process u_t. Then we test the stationarity of the reriduals. The test shows that the residual is not stationary at 5% significance level. Thus the vector $\alpha = (1, -1)$ is not the cointegration vector of EURUSD exchange rate and the differential between Fed and ECB short interest rates in these currencies.

TS. HW2. Group H.

Q3: i we first calculate $p(x_1, x_2 \cdots x_T | \theta, x_0)$

$$p(x_1 \cdots x_T | x_0, \theta) = \frac{1}{(2\pi\sigma^2)^{T/2}} \cdot \exp\left(-\frac{1}{2\sigma^2} \sum_{t=1}^{T} \varepsilon_t^2\right).$$

(Note that $\hat{\varepsilon}_t = x_t - \alpha - \beta x_{t-1} - \delta t \sim N(0, \sigma^2)$).

Thus. $$p(x_1 \cdots x_T | x_0, \theta) = \frac{1}{(2\pi\sigma^2)^{T/2}} \cdot \exp\left(-\frac{1}{2\sigma^2} \sum_{t=1}^{T} (x_t - \alpha - \beta x_{t-1} - \delta t)^2\right).$$

ii draw log-likelihood estimation.

$$-\log L(\theta | y) = \frac{1}{2} T \log \sigma^2 + \frac{1}{2\sigma^2} \sum_{t=1}^{T} (x_t - \alpha - \beta x_{t-1} - \delta t)^2 + \text{constant}$$

Minimizing this function yields:

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\delta} \end{pmatrix} = \begin{pmatrix} T & \sum_{t=1}^{T} x_{t-1} & \sum_{t=1}^{T} t \\ \sum_{t=1}^{T} x_{t-1} & \sum_{t=1}^{T} x_{t-1}^2 & \sum_{t=1}^{T} t x_{t-1} \\ \sum_{t=1}^{T} t & \sum_{t=1}^{T} t x_{t-1} & \sum_{t=1}^{T} t^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{t=1}^{T} x_t \\ \sum_{t=1}^{T} x_t x_{t-1} \\ \sum_{t=1}^{T} t \cdot x_t \end{pmatrix}.$$

$$\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^{T} (x_t - \hat{\alpha} - \hat{\beta} x_{t-1} - \hat{\delta} t)^2.$$