TS. HW2. Group H.

Q3: i we first calculate $p(x_1, x_2 \cdots x_T | \theta, x_0)$

$$p(x_1 \cdots x_T | x_0, \theta) = \frac{1}{(2\pi\sigma^2)^{T/2}} \cdot \exp\left(-\frac{1}{2\sigma^2}\sum_{t=1}^{T} \varepsilon_t^2\right).$$

(Note that $\hat{\varepsilon}_t = x_t - \alpha - \beta x_{t-1} - \delta t \sim N(0, \sigma^2)$).

Thus. $p(x_1 \cdots x_T | x_0, \theta) = \frac{1}{(2\pi\sigma^2)^{T/2}} \cdot \exp\left(-\frac{1}{2\sigma^2}\sum_{t=1}^{T} (x_t - \alpha - \beta x_{t-1} - \delta t)^2\right)$.

ii draw log-likelihood estimation.

$$-\log L(\theta | y) = \frac{1}{2} T \log \sigma^2 + \frac{1}{2\sigma^2}\sum_{t=1}^{T}(x_t - \alpha - \beta x_{t-1} - \delta t)^2 + \text{constant}$$

Minimizing this function yields:

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\delta} \end{pmatrix} = \begin{pmatrix} T & \sum_{t=1}^{T} x_{t-1} & \sum_{t=1}^{T} t \\ \sum_{t=1}^{T} x_{t-1} & \sum_{t=1}^{T} x_{t-1}^2 & \sum_{t=1}^{T} t x_{t-1} \\ \sum_{t=1}^{T} t & \sum_{t=1}^{T} t x_{t-1} & \sum_{t=1}^{T} t^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{t=1}^{T} x_t \\ \sum_{t=1}^{T} x_t x_{t-1} \\ \sum_{t=1}^{T} t \cdot x_t \end{pmatrix}.$$

$$\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^{T} (x_t - \hat{\alpha} - \hat{\beta} x_{t-1} - \hat{\delta} t)^2.$$