Induction Recipe

① what variable?

② what property?

③ Base case

④ induction.
  — use I.H. (induction Hypotheses).

b

lead up

I. Postage Stamp problem

P.O. is trying to save $.

Claim: Using 3 and 5 cent stamps suffice for any postage (> 7 ¢).

pf: By induction on n, amt of postage ①

we SHOW:

$$P(n): \quad n = 3a + 5b \quad \text{for some} \quad ②$$
$$a, b \in \mathbb{N}.$$

3. **Base case:**

$$8 = 3 + 5$$
$$9 = 3 + 3$$
$$10 = 5 + 5$$

4. **Induction step:** | Use strong induction |

I.H. $\Big\{$ Assume $k = 3a_k + 5b_k$ for all $8 \le k < n$.

$a_k, b_k \in \mathbb{N}.$ ↑

NOS $\quad n = 3\alpha + 5\beta \quad (\alpha, \beta \in \mathbb{N}).$

well.... $n - 3 = 3a + 5b \quad$ by I.H.

so.... $n = (n-3) + 3$

$$= 3a + 5b + 3$$
$$= (3)(a+1) + 5b$$
$$= 3(a+1) + 5b \quad \underline{\qquad}$$

II a $\mathbb{N}, \mathbb{Q}, \mathbb{R}, \& \mathbb{Z}$

a proof that $\sqrt{2}$ is irrational.

Thm: $\sqrt{2}$ is irrational.

Pf: By induction.

we show $\left(\dfrac{p}{q}\right)^2 \neq 2$ For all $p, q \in \mathbb{N}$

by induction on q.

① what variable? $q \in \mathbb{N}$

② what is $\mathbb{N}$ property?

$$\left(\dfrac{p}{q}\right)^2 \neq 2 \qquad \forall p \in \mathbb{N}.$$

③ Base case, $q = 1$

$$\dfrac{1}{q^2} = 1 \neq 2$$

$$\dfrac{2^2}{q^2} = 4 > 2$$

$$\forall p > 2, \quad \dfrac{p^2}{q^2} > 4.$$

④ INDUCTION

Assume $\left(\frac{P}{b}\right)^2 \neq 2$ $\forall p \in \mathbb{N}$ and $\forall q \in \mathbb{N}$, $q < n$. $\Big]$ $\underline{\underline{IH}}$

Nos $\left(\frac{P}{n}\right)^2 \neq 2$. $\forall P$, BY contradiction.

Claim 1: if $\left(\frac{P}{n}\right)^2 = 2$ Then $0 < p - n < n$.

claim 2: if $\left(\frac{P}{n}\right)^2 = 2$ Then $\left(\frac{2n - p}{p - n}\right)^2 = 2$

which contradicts the I.H.