

January 14, 2014

Keng Low

Short Assignment: The Substitution Model

Trace the evaluation of the expression involving the `mystery` procedure:

`(mystery 7 2)`

- Evaluate `(mystery 7 2)` where `mystery` is defined below.

```
(define mystery
  (lambda ((a <number>) (b <integer>))
    (cond ((zero? b) 0)
          ((odd? b)
           (+ a (mystery (+ a a)
                         (quotient b 2))))
          (else
           (mystery (+ a a)
                     (quotient b 2))))))
```

- Evaluate `lambda` subexpression

`(lambda ((a <number>) (b <integer>)))`

Since this is the evaluation of a compound expression, the EVAL rule says we must first evaluate the subexpressions. The value obtained by evaluating the `lambda` subexpression is a procedure object, denoted:

`[PROC ((x <number>) (b <number>))]`

- Evaluate `cond` expression

`(cond ((zero? b) 0))`

`cond` is a special form, so we handle it specially (has its own rule in the model).

- Evaluate `((zero? b) 0)`

```
(PROC ((b <integer>)) ((zero?) (b)) 0)
```

- Evaluate 2

```
(PROC ((n <number>)) ((zero?) 2) 0)
```

We determine the value of the expression b is 2.

- Apply `zero?` procedure to 2

```
[PROC ((n <number>)) ((zero?) 2) 0]
```

We take the body of the procedure, `zero?` and substitute the corresponding arguments where the parameters appear.

- Evaluate `zero?` procedure

```
(zero? 2)
```

`zero? x` is a primitive procedure that returns `#t` if its numeric argument x is equal to zero, `#f` otherwise. The above expression returns `#f`. The consequent is not evaluated.

- Evaluate `cond` expression

```
(cond ((odd? b)
```

We evaluate `cond`, since the first test yields a false value, we evaluate the next test.

- Evaluate `((odd? b)`

```
(PROC ((b <integer>)) ((odd?) b)))
```

- Evaluate 2

```
(PROC ((b <integer>)) ((odd?) 2)))
```

We determine the value of the expression b is 2.

- Apply `odd?` procedure to 2

```
[PROC ((n <number>)) ((odd?) 2) 0]
```

- Evaluate `odd?` procedure

`odd? x` returns `#t` if its integer argument `x` is odd, `#f` otherwise. The consequent is not evaluated.

- Evaluate the `cond` expression

We evaluate `cond`, since the first test yields a false value, we evaluate the next test.

- Evaluate the `else` expression

The `else` clause is optional, but if it is present, and all the test expressions `test1` through `testn` have evaluated to `#f`, then `expr-e` is evaluated, and its value becomes the value of the `cond` expression.

- Evaluate `(mystery (+ a a) (quotient b 2))`

Evaluate `mystery`

- Evaluate `(+ a a)`

`(PROC (a <number>) (+ a a))`

- Evaluate 7

`(PROC (a <number>) (+ 7 7))`

- Apply procedure

`[PROC (a <number>) (+ 7 7)]`

- Evaluate `+`

`(+ 7 7)`

- Apply primitive `add`

`[add 7 7]`

Returns 14.

- Evaluate `(quotient b 2)`

`(PROC (b <integer>) (quotient b 2))`

- Evaluate 2

`(PROC (b <integer>) (quotient 2 2))`

- Apply `quotient` procedure

```
[PROC (b <integer>) (quotient 2 2)]
```

- Evaluate `(quotient 2 2)`

```
(quotient 2 2)
```

- Apply primitive `quotient`

```
[quotient 2 2]
```

`quotient` returns the quotient of a / b . In other words, if $a = qb + r$, with $0 \leq r < b$, then $(\text{quotient } a \ b)$ returns q . The above expression returns 1.0.

- Substitute the body of the procedure with values {14} and {1.0}.

```
(mystery 14 1.0)
```

- The process repeats, and so on and so forth.

`(cond ((zero? b) 0))` returns #f. We evaluate the next test.

`((odd? b))` returns #t. We evaluate the consequent.