Compressed Suffix Memory Algorithm for Reinforcement Learning

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***Abstract*—Instance-based approaches are effective ways to solve reinforcement learning problems. Utile Suffix Memory (USM) algorithm has shown decent results for distinguishing different states from instance chains and generating Q-value of actions of each state, but involving exponentially expanded state space and a number of redundant states. In this paper we propose a new state space compressed algorithm, called Compressed Suffix Memory (CSM) algorithm. CSM algorithm obtains heuristic information of the environment by a blind exploration, for example, the maximum L1 distance between instances in instance chains and goal frequencies, to improve efficiency and resist overfitting. Boltzmann sampling is adopted to balance between exploration and exploitation. Experiments show that both the efficiency and the effect have been improved a lot by CSM algorithm compared with USM algorithm.**

***Index Terms*—keyword1, keyword2, keyword3**

I. Introduction

Reinforcement learning is learning what to do—how to map situations to actions—so as to maximize a numerical reward signal in a provided environment [1]. In many reinforcement learning scenarios such as robotic exploration [2] and autonomous driving [3], the agent is only able to gain partial and noisy observations form environment, so POMDP (partially observable Markov decision process) model is wildly adopted. According to the observations, reward and the historical information, POMDPs provide a rich mathematical approach to solving sequential problems by calculating the Q-value of actions.

As the agent does not directly observe the underlying state, the generation of the state space is the key to reinforcement learning algorithms. Many instance-based methods have been put forward, including Nearest Sequence Memory (NSM) algorithm [4] and Utile Suffix Memory (USM) algorithm [5]. USM algorithm presents the state space by tree-nodes in a suffix tree building from the instance chains, and is proved effective maximizing the Q-value of actions. However, the state space of USM algorithm exponentially expands during iteration and comprises many redundant states, which reduces the efficiency. Furthermore, because the ϵ-greedy policy of USM lacks of the exploitation of overall information, it may lead to overfitting.

In this paper, we propose a new algorithm, called Compressed Suffix Memory (CSM) algorithm, which optimizes the generation of a utile tree and decision process. First, the heuristic information is obtained by the blind exploration of the environment, e.g., l as the effective path length from start to goal and p as the probability of goal during the exploration. Second, the maximum depth of the suffix tree is limited to $2l$ and the minimum instances required to trigger state splitting, the threshold b, is deduced from p. Finally, after initializing the agent with a random policy, Boltzmann sampling approach will be applied. Experiment has shown that both the efficiency and the effect have been greatly improved by CSM algorithm compared with USM algorithm.

The paper outline follows. We will briefly review reinforcement in partially observable environment and Utile Suffix Memory (USM) algorithm, in Section 2. Next, we propose Compressed Suffix Memory (CSM) algorithm, which takes advantage of heuristic information and optimizes the generation of the suffix tree, in Section 3. We measure the performance of CSM algorithm and see improvements comparing with USM algorithm, in Section 4. We close in Section 5, with a brief summary and possible means of improvement of CSM algorithm.

II. Background

A. Reinforcement Learning in Partially Observable Environment

Reinforcement learning is about how an agent learning to map states to actions and produce a maximized reward in a provided environment. In general, a reinforcement learning agent interacts with the environment over time. At each time step t, it determines its state s_t from a state space S, and chooses a best action a_t from an action space A according to the policy π. The agent gets an instant reward r_t according to the reward function $R(s_t, a_t)$ and transfer to the next state s_{t+1} according to the transition probability $T(s_{t+1}|s_t, a_t)$. The reward is normally discounted with factor $\gamma \in (0, 1]$, and the accumulated reward at t_n is defined as $R_{t_n} = \sum_{t=0}^{t_n} \gamma^t r_t$. When a problem satisfies the Markov property, the problem can be formulated as a Markov decision process (MDP), which is defined by the 5-tuple (S, A, T, R, γ).

However, in most cases, an agent cannot directly observe the states of the underlying MDP model in the provided environment, but can only deduce a state by an observation. It

is necessary to bring in partially observable Markov decision process (POMDP). It defines Ω as a set of observations and O as a set of conditional observation probabilities-mapping current state s_t and previous action a_{t-1} to the probability of current observation o_t. In that way, a POMDP can be defined by the 7-tuple $(S, A, T, R, \Omega, O, \gamma)$.

When the model is successfully learned, i.e., all elements of the 7-tuple are generated to fit the problem, there are plenty of algorithms to calculate a best policy [6]. However, in model-free methods, an agent can learn with trail-and-error from experience directly and a policy can generated before grasping all information of a model [7], which is normally faster and more suitable real time problem. An important way to implement model-free methods is to make an agent comprise some sort of internal memory [5], [8], [9] (see Figure 1). For example, the states of a model can be expressed by some nearest observations and internal states of an agent. After learning the state space, the agent can make decisions using model-free methods, such as HQ-learning [10].

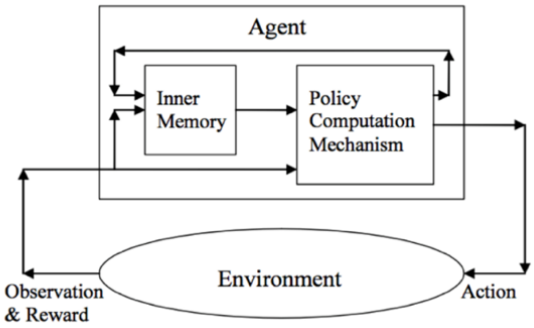


Fig. 1: The agent model that has internal memory

The internal memory of an agent can comprise instances that record what it has encountered at each time step. Nearest Sequence Memory (NSM) and Utile Suffix Memory (USM) algorithms are based on the instances.

B. Utile Suffix Memory Algorithm

The interaction between the agent implementing USM algorithm and the environment is described by A, O, R, which are finite set of actions, finite set of observations and a reward function. Like other instance-based algorithms, USM algorithm records each of its raw instances [5]. At each time step t, the agent executes action $a_t \in A$ to get a new instance, gets observation $o_{t+1} \in O$, and get a instant reward r_{t+1} according to R, which is determined by the environment. A new instance is formulated as

$$T_{t+1} = (T_t, a_t, o_{t+1}, r_{t+1}). \tag{1}$$

T_{t+1} is T_t's successor. It is obvious that instance chains are built from during learning process.

In order to deduce state space from those instance chains, USM algorithm creates a suffix tree, whose leaves present the state space and store clustered instances. Each node of the tree can thus be uniquely identified by the string of labels on the path from node to the root, and the string is called the node's suffix. An instance is always deposited into the nodes whose suffix matches its observation and action context, or suffix. That is, for an instance T_i, if its suffix $[..., o_{t-3}, a_{t-3}, o_{t-2}, a_{t-2}, o_{t-1}, a_{t-1}]$ matches the suffix of a certain node, it would be put into that node (see Figure 2). The set of instances that a node contains is written as $I(s)$. The suffix tree leaf which instance T belongs to is written as $L(T)$. It is inevitable that the action layer and the observation layer appears alternately when tree grows (see Figure 2).

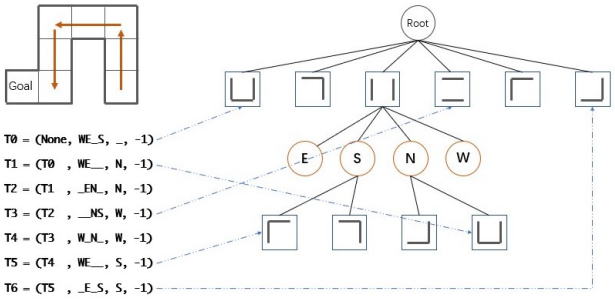


Fig. 2: The agent navigates itself in a maze (probably not the first time), generates a sequence of instances, and builds a suffix tree. The action layer and the observation layer appear alternately.

Besides the general nodes, there is another type of node called "fringe" node [5]. The fringe nodes are actually the deepest layers of the suffix tree, however, we treated them the inner instance buckets of the leaves previously mentioned. Fringe nodes also contain instances according to the same suffix criterion used by non-fringe nodes. That is, if the suffix of a leaf is $[a_{t-2}, o_{t-1}, a_{t-1}]$, an 1-layer deeper fringe nodes of it will stores the instance that matches suffix $[o_{t-2}, a_{t-2}, o_{t-1}, a_{t-1}]$. The agent decides whether it should promote fringe nodes to leaves by Kolmogorov-Smirnov test, which determine the instances in the fringes and that of their parent are drawn the same distribution.

The steps of USM algorithm is as below:

1) The agent begins with a suffix tree that comprises a root node and a layer of observation nodes as leaves, i.e., the agent only acts according to its observation.
2) The agent chooses an action according to ϵ-greedy policy, executes it and generates an instance (see Equation 1).
3) The agent inserts the instance into the suffix tree. The instance is classified to leaves and fringes with the same suffix with that instance.
4) The agent triggers K-S test every n additions of instance. If the Q-value of fringes and their parent are from different distributions, the fringes will be promoted to leaves. The instance will always be added to fringes and leaves that have the same suffix with the instance. Q-value table will be updated after every addition of

instance. Time step increases and algorithm jumps to step 2.

There are two obvious deficiencies of USM algorithm. Firstly, in USM algorithm, the number of states may exponentially increase as the number of steps grows, but many states are redundant, which correspond to the same state in the real environment. Thus, it will reduce the efficiency of the algorithm. Secondly, since the initial Q value of all leaves is initialized with 0, the ϵ-greedy policy may lead to overfitting. Because the ϵ-greedy policy of USM treats the second best and the worst the same, it drastically decrease the probability to explore other actions which may get higher reward. For example, we found that in the first few steps, the agent may get negative default return after choosing actions at some states, which decreases the probability of exploring those actions when at those states again.

III. Compressed Suffix Memory Algorithm

Exploration and exploitation is a key issue in reinforcement learning. Heuristic information of environment can help agent understand state space better. Agent could first make a blind policy exploration of the environment, obtaining some interaction experience, and apply it to following reinforcement learning. The effective path length to goal and the frequency of goals decently represents the scale of the environment and can be used to improve the algorithm. In addition, more overall information should be considered.

1) The depth of the suffix tree should be constrained. The 3rd Quartile of path lengths to goal and the maximum tree depth are denoted as l_3 and d. Leaves with suffix longer than $2l_3$ are considered redundant. However, the exploitation of immature policy can cause overfitting, which makes d grows greater than $2l_3$. So, an alterable guardian depth l is needed. On the one hand, l starts with a relatively small value and will gradually get closer to l_3 through learning. On the other hand, $2l$ should always greater than d. The initial l is set as:
$$l = \max L_T/2, \tag{2}$$
and will be updated as:
$$l \leftarrow l + \alpha(l_3 - l) \ s.t. \ l + \alpha(l_3 - l) > d/2. \tag{3}$$
L_T is a set of L1 distances, which comprises the maximum L1 distance between each instance and the initial instance in each instance chain during the blind exploration; α is the learning rate.
2) Instances in tree node should be denser. K-S test performs better with more statistical sample. Agent should only trigger K-S test when leaves and their fringe nodes are holding enough instances, namely, more than a threshold b. Because the ideal b varies with environment itself. In practice, as the frequency of goal in blind exploration, p, well describes the problem scale, b is strongly correlated with p. Agent is encouraged to match longer suffix when problem scale is bigger. Therefore, the smaller the p is, the smaller the b is set.
3) More overall information should be taken into account. The main problem of ϵ-greedy approach is that it lacks the exploitation of overall information, such as those Q-value which is not the maximum. We use Boltzmann sampling to balance between exploration and exploitation, which assigns actions with similar Q-value similar executing probabilities to encourage agent to exploit them all, rather than only the best. The probability to choose an action a_i at leaf s with temperature t is
$$p(s, a_i) = e(s, a_i) / \sum_a e(s, a), \tag{4}$$
which e is
$$e(s, a) = \exp((Q(s, a) - \max Q(s))/t) \tag{5}$$

The details and the pseudo code (see Algorithm 1) form of CSM algorithm is as below.

1) The agent begins with a suffix tree that comprises a root node and an layer of observation nodes as leaves, i.e., the agent only acts according to its observation.
2) The agent is initialized with a randomly policy. The guardian length l and minimum number of instances to do K-S test, b, are set during initialization. The maximum depth of the suffix tree is set to $2l$.
3) The agent executes action and interacts with the environment at time step t. It records its learning history as an instance T_t (see Equation 1). For each treenode s, if its suffix matches the suffix of T_t. Its instance set are updated as:
$$I(s) \leftarrow I(s) \cup T_t. \tag{6}$$
Let $L(T)$ be the leaf which instance T belongs to. $L(T_t)$ is cached to agent memory. l is updated (see Equation 3). If the agent reaches the goal, it repositions itself randomly to one of the provided start positions, and updates the value of l_3.
4) For each instance added, the agent does one Bellman iteration with the leaves of of the states:
$$Q(s, a) \leftarrow R(s, a) + \gamma Pr(s'|s, a)U(s'). \tag{7}$$
Let $I(s, a)$ be the subset of $I(s)$ that contains all the instances that executed action a. $U(s')$ is the utility of the state s', calculated as $U(s) = \max_{a \in A} Q(s, a)$. $R(s, a)$ and $Pr(s'|s, a)$ are the estimated immediate reward and the transition probability, respectively, that drawn from the instance chains:
$$R(s, a) = \frac{\sum_{T_i \in I(s,a)} r_i}{|I(s, a)|}, \tag{8}$$
$$Pr(s'|s, a) = \frac{|\forall T_i \in I(s, a) \ s.t. \ L(T_{i+1} = s')|}{|I(s, a)|}. \tag{9}$$
5) Kolmogorov-Smirnov is performed when $L(T_t)$ is not the same as $L(T_{t-1})$ and $L(T_t)$ holds enough instances

(more than b). The expected discounted reward of instance T_i is written as $H(T_i)$, and is defined as:

$$H(T_i) = r_i + \gamma U(L(T_{i+1})) \tag{10}$$

Every $Q(T_i)$ of instances stored in leaf and those stored in immediate fringe nodes is calculated. Therefore, $|A|$ pairs of smaple are prepared for Kolmogorov-Smirnov test. If there existing a certain pair that is believed not from the same distribution, those fringe nodes would be promoted as leaves.

6) Agent does Boltzmann sampling to choose the next action, i.e., with probability $p(s, a_i)$ the agent chooses a_i as a_{t+1} (see Equation 4). In practice, agent is encouraged to do random exploration for n_r times, and then perform Boltzmann sampling method with descendant temperature. Time step increase to $t + 1$ and algorithm jumps to step 3.

IV. Experiment

A. Experiment Setup

We run algorithms with three well-known benchmarks: Tiger-Grid, Hallway and McCallum as well as a large Prim Maze generated by Prim algorithm randomly (see Figure 3). The characteristics of benchmarks are described below (see Table I), where $|S|$, $|A|$, $|\Omega|$, r_d, r_p, r_g, denotes the number of states, the number of actions, the number of observations, default reward, penalty and goal, respectively. The $|A|$ is 4 because the action of the agent normally includes: moving south, west, east and north. However, problems differ in some details.

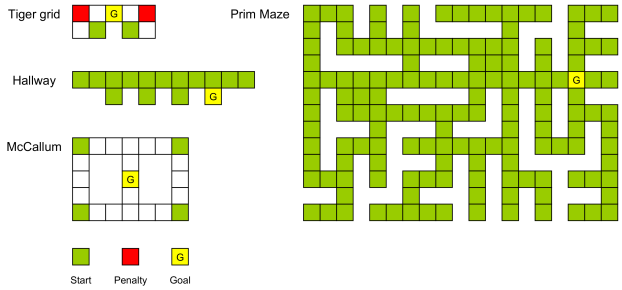


Fig. 3: Benchmark mazes

TABLE I: Benchmarks

Problem	$\|S\|$	$\|A\|$	$\|\Omega\|$	r_d	r_p	r_g
Tiger-Grid	9	4	7	-0.1	-4	16
Hallway	15	4	4	-0.1		32
McCallum	23	4	9	-0.1		18
Prim Maze	154	4	16	-0.1		96

In Tiger-Grid, the agents are initially assigned to one of two definite starting positions. The probability of each starting position is 0.5. The goal of the model is to make the agent reach the goal position as soon as possible. Once the agent reaches the goal position, it will be reset to two starting positions. Tiger-Grid's maze comprises two penalty positions where agent will receive a negative reward.

In Hallway, similar to Tiger-Grid problem but still different, agent will be reset randomly to a non-goal state. The Hallway does not include any penalty position. Furthermore, the hallway adds 4 landmarks as hint for agent to discover hidden states.

In McCallum, after reached the goal, the agent will be randomly reset at one of the 4 corners. There are 3 south-north hallways, which are deliberately designed for agent to

Algorithm 1 Compressed Suffix Algorithm

Input: iteration steps n, radom steps in iteration n_r
Output: average discounted return ADR

Initialize guardian length l, and minimum number b of instance required to do K-S test by blind exploration
Initialize temperature t as positive infinity
Initialize a suffix tree with depth limitation $2l$
Initialize agent with random start position
for $i = 0$ to n **do**
  if $i > n_r$ **then**
    t decreases from 0.5 to nearly 0 as i increases
  end if
  Choose action a by Boltzmann sampling and execute it (see Equation 4)
  while not moved after executed a **do**
    Choose action a by Boltzmann sampling with higher t and execute it
    if the agent reaches goal **then**
      Update l_3
    end if
  end while
  Generate instance T according to the execution of a just now
  Insert T into the suffix tree, update l, and find the leaf node s the instance belongs
  $I(s) = I(s) \cup T_t$
  $Q(s, a) = R(s, a) + \gamma Pr(s'|s, a)U(s')$
  $R(s, a) = \sum_{T_i \in I(s,a)} r_i / |I(s, a)|$
  $Pr(s'|s, a) = |\forall T_i \in I(s, a)\ s.t.\ L(T_{i+1} = s')| / |I(s, a)|$
  if $|I(s)| > b$ **then**
    do K-S test between $H(T)$s of s and its fringe nodes, get p_value p
    if $p < 0.1$ **then**
      promote fringe nodes to leaves
    end if
  end if
  if agent at goal **then**
    Initialize agent with random start position
  end if
end for
Use Boltzmann sampling at low temperature on $Q(s, a)$ to produce average discounted reward ADR (see Equation 11)
return ADR

distinguish, since they are ambiguous.

In Prim Maze, which is randomly generated by Prim algorithm and mean to be big. $|S|$ of Prim Maze is several times the S of others. When the agent arrives at the goal, Prim Maze will reset the agent to a random state in the maze.

We evaluate average discounted reward, ADR, in the experiment:

$$ADR = \frac{\sum_{i=0}^{n_{trails}} \sum_{j=0}^{n_{steps}} \gamma^j r^j}{n_{trails}}, \tag{11}$$

n_{trails} is the number of trails we test our learning result; n_{steps} is the maximum steps to try to reach goal. ADR is widely considered to be a good evaluation of the quality of a value function. The higher the ADR is, the more intelligent the agent is considered to be. We track the changing of ADR through time to determine whether CSM algorithm helps the agent to learn faster and better.

Our experiment uses a discount factor $\gamma = 0.9$, an exploration probability $\epsilon = 0.1$ for USM algorithm. The boundary value of the Kolmogorov-Smirnov test is $p = 0.1$. We iterate the USM, CSM and random algorithm 2048 times, and set check points per 48 iteration. In each check point we test our model 256 times within 96 steps, i.e., n_{trial} is 256 (see Equation 11) in ADR calculation.

B. Experiment Results

The experiment results of CSM, USM and random algorithms are shown below (see Figure 4). It should be noted that even if n_trail-set to 256-seems large enough, the results of ADR may still fluctuate. Therefore, we record (ADR, time) tuples in the table below (see Table **??**), when ADR starts to become relatively stationary, rather than when ADR gets maximum.

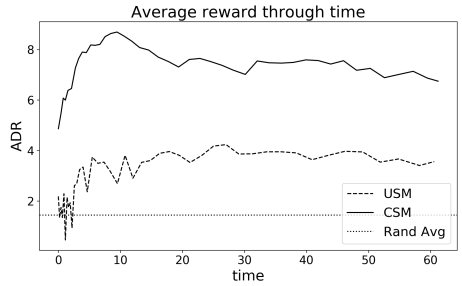
TABLE II: Table Type Styles

	ADR		**Time**	
Problem	USM	CSM	USM	CSM
Tiger-Grid	3.81	8.70	10.75	9.45
Hallway	8.43	10.13	13.52	6.67
McCallum	2.82	5.90	34.39	14.15
Prim Maze	7.13	8.30	49.30	18.24

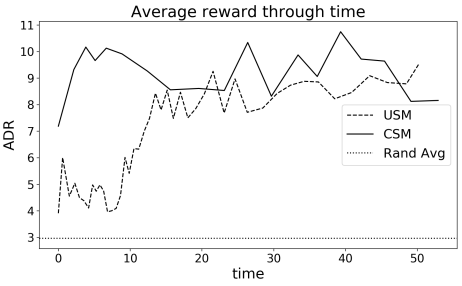
In Tiger-Grid, CSM algorithm quickly reached a very high ADR in a short time after the start of the experiment. As the experiment progressed, the ADR of CSM algorithm converged to 7.5. In contrast, the ADR of USM algorithm fluctuated drastically in the early stage of the experiment, and became stable at around 4, which was only half of the CSM algorithm.

In Hallway, the ADR of both algorithms fluctuated violently, and converged to a similar value. However, when comparing time taken to reach those, CSM algorithm reached plateau ADR taken the half runtime of USM algorithm, i.e., the agent adopted CSM algorithm have learned twice as fast as agent adopted USM algorithm in Hallway.

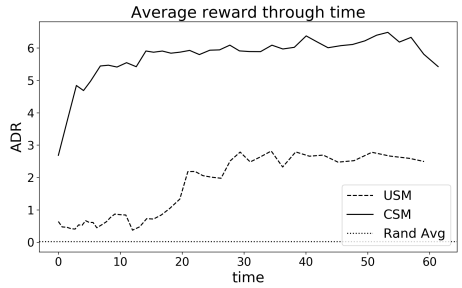
In McCallum, similar to the performance in Tiger-Grid, CSM algorithm quickly reached a very high ADR in short time after the start of the experiment. As the experiment



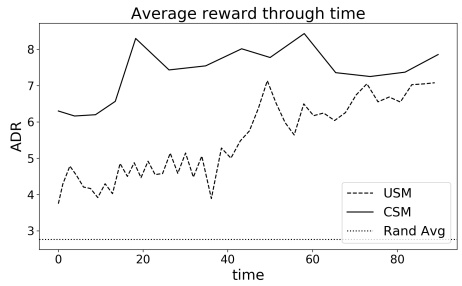
(a) Tiger Grid



(b) Hallway



(c) McCallum



(d) Prim Maze

Fig. 4: Performance of different agent on different benchmarks; y-axis is ADR, x-axis is runtime in seconds.

progressed, the ADR of CSM algorithm was basically stable at 6. On the other hand, the ADR of USM algorithm have not significantly increased in the early stage. What's more, USM algorithm got a far less max ADR than CSM algorithm, which is only about 3. The figure indicated that in McCallum, CSM algorithm could find a better policy more quickly than USM algorithm.

In larger scale problems, such as the Prim Maze, CSM algorithm also performed faster and better than USM. From the experiment results, CSM algorithm achieved an ADR about 8 in about 20 seconds. USM algorithm, meanwhile, only reached around 5. Even if in the mid-stage of the experiment, where the ADR of USM has experienced arapid growth, its time cost was almost three times that of CSM, and had a still lower ADR.

In addition, it can be seen from the experiment result that at the beginning of the experiment, although both algorithms have not gained experience through learning, CSM has obtained a higher ADR than USM. This can be explained in two ways:

1) CSM algorithm uses the Boltzmann sampling method to choose action, the agent does not only perform the best action at the stage of exploitation, but other non-optimal actions, which resist overfitting at early stage.
2) CSM rises the $temperature$ of Boltzmann sampling when agent continuously bumps into walls, which helps agent get out of that position. That ensures the efficiency of exploration.

It is for these two reasons that CSM algorithm is able to understand the environment better and discover the goal faster than USM algorithm, hence obtaining a higher ADR.

Overall, It can be seen that CSM algorithm helps agent to learn faster and better than USM algorithm

V. Conclusion

The paper presents an reinforcement learning algorithm to apply heuristic information into instance-based decision making. We utilized heuristic information in three aspects: constraining the depth of the suffix tree using L1 distance, densifying instances in tree node according to environment scale and utilizing Boltzmann sampling to balance exploration and exploitation. The experiment results confirmed that CSM algorithm effectively generates good policies, and outperforms USM algorithm in every benchmark.

In the future work, the computational resources cost by repeatedly suffix matching is needed to be reduced; time cost of Boltzmann sampling is needed to be reduced. Furthermore, more effective heuristic standards based on exploration experience have to be considered.

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