**Exercises on Asymptotic Analysis of Algorithms**

1. What is the smallest value of n such that an algorithm whose running time is 100n2 runs faster than an algorithm whose running time is 2n on the same machine?”

2. “Consider the searching problem:

**Input:** A sequence of n numbers A = <a1, a2, . . . , an> and a value v.

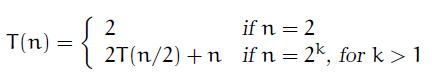
**Output:** An index i such that v = A[i] or the special value nil if v does not appear in A.

Write pseudo-code for linear search, which scans through the sequence, looking for v.”

3. Consider linear search again. How many elements of the input sequence need to be checked on the average, assuming that the element being searched for is equally likely to be any element in the array? How about in the worst case? What is the average-case and worst-case running times of linear search in ɵ-notation? Justify your

answers.

4. Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence



is T(n)= n log n

5. Describe a ɵ (n lg n)-time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x.

6. Is 2n+1 = O(2n)? Is 22n = O(2n)?”

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7. Find a simple formula for Also verify with induction.



8. Show that the solution of

9. Show that the solution of this recurrence is also Ω (n lg n). Conclude that the solution is ɵ (n lg n).

10. Draw the recursion tree for T(n) = 4T(n/2) + cn, where c is a constant, and provide a tight asymptotic bound on its solution. Verify your bound by the substitution method.

11. Use the master method to give tight asymptotic bounds for the following recurrences.

a. T(n) = 4T(n/2) + n.

b. T(n) = 4T(n/2) + n2.

c. T(n) = 4T(n/2) + n3.

12. Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Assume that T(n) is constant for n≤2. Make your bounds as tight as possible, and justify your answers.

a. T(n) = 2T(n/2) + n3.

b. T(n) = T(9n/10) + n.

c. T(n) = 16T(n/4) + n2.

d. T(n) = 7T(n/3) + n2.