SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

II YEAR / IV SEMESTER

B.E - COMPUTER SCIENCE ENGINEERING
B.E – CYBER SECURITY

1918402 –PROBABILITY AND QUEUEING THEORY

Regulation – 2019

Academic Year – 2023 - 24

Prepared by

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<table>
<tr><th>S.No</th><th>QUESTIONS</th><th>BT Level</th><th>Competence</th><th>COs</th></tr>
<tr><td colspan="5">UNIT I RANDOM VARIABLES 9L+3T
Discrete and continuous random variables – Moments – Moment generating functions - Binomial, Poisson, Geometric, Uniform, Exponential and Normal distributions.</td></tr>
<tr><td colspan="4">Part - A (2 MARK QUESTIONS)</td><td></td></tr>
<tr><td>1.</td><td>The number of hardware failures of a computer system in a week of operations has the following p.d.f, Find the mean of the number of failures in a week.
<table><tr><td>No.of failures</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Probability</td><td>.18</td><td>.28</td><td>.25</td><td>.18</td><td>.06</td><td>.04</td><td>.01</td></tr></table></td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>2.</td><td>The number of hardware failures of a computer system in a week of operations has the following p.d.f, Calculate the value of K.
<table><tr><td>No.of failures</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Probability</td><td>K</td><td>2 K</td><td>2 K</td><td>K</td><td>3 K</td><td>K</td><td>4 K</td></tr></table></td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>3.</td><td>Check whether the function given by $f(x) = \frac{x+2}{25}$ for x=1,2,3,4,5 can serve as the probability distribution of a discrete random variable.</td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>4.</td><td>If the random variable X takes the values 1,2,3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, find the probability distribution of X</td><td>BTL-1</td><td>Remembering</td><td>CO1</td></tr>
<tr><td>5.</td><td>The RV X has the following probability distribution:
<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td></tr><tr><td>P(x)</td><td>0.4</td><td>k</td><td>0.2</td><td>0.3</td></tr></table>Find k and the mean value of X</td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>6.</td><td>If $f(x) = K(x + x^2)$ in $1 < x < 5$ is a pdf of a continuous random variables. Find the value of K.</td><td>BTL-1</td><td>Remembering</td><td>CO1</td></tr>
<tr><td>7.</td><td>The p.d.f of a continuous random variable X is $f(x) = k(1+x)$, $2 < x < 5$ Find k.</td><td>BTL-1</td><td>Remembering</td><td>CO1</td></tr>
<tr><td>8.</td><td>For a continuous distribution $f(x) = k(x - x^2), 0 \leq x \leq 1$, where k is a constant. Find k.</td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>9.</td><td>If $f(x) = kx^2, 0 < x < 3$, is to be a density function, find the value of k.</td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>10.</td><td>If the probability that a target is destroyed on any one shot is 0.5, Find the probability that it would be destroyed an 6^{th} attempt.</td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>11.</td><td>The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution.</td><td>BTL-1</td><td>Remembering</td><td>CO1</td></tr>
<tr><td>12.</td><td>The mean and variance of binomial distribution are 5 and 4. Determine the distribution.</td><td>BTL-2</td><td>Understanding</td><td>CO1</td></tr>
<tr><td>13.</td><td>If 3% of the electric bulbs manufactured by a company are defective, Find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.</td><td>BTL-1</td><td>Remembering</td><td>CO1</td></tr>
</table>

14.	Messages arrive at a switchboard in a poisson manner at an average rate of six per hour. Find the probability for exactly two messages arrive within one hour.	BTL-2	Understanding	CO1
15.	The number of monthly breakdowns of a computer is a random variable having Poisson distribution with mean 1.8. Find the probability that this computer will function for a month with only one breakdown.	BTL-1	Remembering	CO1
16.	If X is a Poisson variate such that $2P(X=0)+P(X=2)=2P(X=1)$, find E(X)	BTL-2	Understanding	CO1
17.	The probability that a candidate can pass in an examination is 0.6. What is the probability that he will pass in third trial?	BTL-1	Remembering	CO1
18.	If $f(x)=\frac{x^2}{3}, -1<x<2$ is the pdf of the random variable X ,then find p(0<x<1)	BTL-2	Understanding	CO1
19.	If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test on the fourth trial	BTL-1	Remembering	CO1
20.	If X has uniform distribution in (-3,3), find $P(\|x-2\|<2)$	BTL-2	Understanding	CO1
21.	Let X be a random variable with moment generating function $M_X(t)=\frac{(2e^t+1)^4}{81}$ .Find its mean and variance.	BTL-1	Remembering	CO1
22.	A Random variable X is uniformly distributed between 3 and 15. Find the variance of X.	BTL-2	Understanding	CO1
23.	A continuous RV X has the density function $ce^{-\frac{x}{5}}, x>0$. Find c. Create E(x) and Var(X)	BTL-1	Remembering	CO1
24.	If X is a normal random variable with mean 3 and variance 9, find the probability that X lies between 2 and 5.	BTL-2	Understanding	CO1
25.	A normal distribution has mean μ = 20 and standard deviation σ = 10. Evaluate $(15 \le X \le 40)$.	BTL-2	Understanding	CO1
	PART – B (13 MARK QUESTIONS)			
1.(a)	A random variable X has the following probability distribution: X: 0, 1, 2, 3, 4, 5, 6, 7 P(X): 0, k, $2k$, $2k$, $3k$, k^2, $2k^2$, $7k^2$+k Find (i) the value of k (ii) $P(1.5<X<4.5/X>2)$	BTL-2	Understanding	CO1
1.(b)	Find the MGF of Binomial distribution and hence find its mean and variance	BTL-1	Remembering	CO1
2.(a)	The probability mass function of a discrete R. V X is given in the following table: X: -2, -1, 0, 1, 2, 3 P(X=x): 0.1, K, 0.2, $2k$, 0.3, k Find (1) Find the value of k, (2) P(X<1),(3) $P(-1<X\le 2)$	BTL-2	Understanding	CO1
2.(b)	Obtain the MGF of Poisson distribution and hence find its mean and variance	BTL-1	Remembering	CO1
3.(a)	The probability mass function of a discrete random variable X is given in the following table X: 0, 1, 2, 3, 4, 5, 6, 7, 8 P(X): a, 3a, 5a, 7a, 9a, 11a, 13a, 15a, 17a Find (i) the value of a , (ii) $P(X<3)$, (iii) Mean of X, (iv) Variance of X.	BTL-2	Understanding	CO1

3.(b)	Deduce the MGF of a geometric distribution and hence find the mean and variance	BTL-1	Remembering	CO1
4.(a)	If the discrete random variable X has the probability function given by the table. x: 1, 2, 3, 4; $P(x)$: $k/3$, $k/6$, $k/3$, $k/6$ Find the value of k and Cumulative distribution of X.	BTL-2	Understanding	CO1
4.(b)	Derive the MGF of Uniform distribution and hence deduce the mean and variance	BTL-1	Remembering	CO1
5.(a)	If the probability mass function of a random variable X is given by P(X=x) = kx^3, x=1,2,3,4. Find the value of k, mean and variance of X.	BTL-3	Applying	CO1
5.(b)	Deduce the MGF of Exponential distribution and hence find its mean and variance	BTL-1	Remembering	CO1
6.(a)	Find the MGF , mean and variance of the random variable X has the pdf $f(x) = \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, otherwise \end{cases}$	BTL-2	Understanding	CO1
6.(b)	State and prove the memory less property of exponential distribution	BTL-3	Applying	CO1
7.(a)	In a large consignment of electric bulbs, 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that i) all are good bulbs ii) atmost there are 3 defective bulbs iii) exactly there are 3 defective bulbs.	BTL-2	Understanding	CO1
7.(b)	A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins are defective, what is the probability that a box fail to meet the guaranteed quality.	BTL3	Applying	CO1
8.	Obtain the MGF of a normal distribution and hence find its mean and variance	BTL-1	Remembering	CO1
9.(a)	If a random variable X has p.d.f $f(x) = \begin{cases} \frac{1}{4}, & \lvert X \rvert < 2 \\ 0, & Otherwise \end{cases}$ Find (a) $P(X < 1)$ (b) $P(\lvert X\rvert > 1)$ (c) $P(2X + 3 > 5)$.	BTL-2	Understanding	CO1
9.(b)	Out of 2000 families with 4 children each, Find how many family would you expect to have i) at least 1 boy ii) 2 boys.	BTL-3	Applying	CO1
10.(a)	Find the MGF of the random variable X having the probability density function f(x) = $\begin{cases} \frac{x}{4}e^{-\frac{x}{2}}, & x > 0 \\ 0 & ,\text{otherwise} \end{cases}$. Also find the mean and variance	BTL-2	Understanding	CO1
10.(b)	4 coins were tossed simultaneously. What is the probability of getting (i) 2 heads, (ii) at least 2 heads, (iii) at most 2 heads.	BTL-4	Analyzing	CO1
11.(a)	A random variable X has c.d.f $F(x) = \begin{cases} 0, if x < -1 \\ a(1 + x), if -1 < x < 1 \\ 1, if x \geq 1 \end{cases}$. Find the value of a. Also P(X>1/4) and $P(-0.5 \leq X \leq 0)$.	BTL-2	Understanding	CO1
11.(b)	The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, then what is the probability that during the next second the number of alpha particles emitted from 1 gram is (1) at most 6 (2) at least 2 and (3) at least and at most5	BTL-4	Analyzing	CO1

12.	If f(x) = $\begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & elsewhere \end{cases}$ is the p.d.f of X. Calculate (i) The value of a , (ii) The cumulative distribution function of X (iii) If X_1, X_2 and X_3 are 3 independent observations of X. Find the probability that exactly one of these 3 is greater than 1.5?	BTL-2	Understanding	CO1
13.(a)	The Probability distribution function of a R.V. X is given by $f(x) = \frac{4x(9-x^2)}{81}$, $0 \le x \le 3$. Find the mean, variance.	BTL-2	Understanding	CO1
13.(b)	The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without breakdown (2) with only one breakdown and (3) with at least one breakdown.	BTL-3	Applying	CO1
14.(a)	Messages arrive at a switch board in a Poisson manner at an average rate of 6 per hour. Find the probability that exactly 2 messages arrive within one hour, no messages arrives within one hour and at least 3 messages arrive within one hour	BTL-3	Applying	CO1
14.(b)	An electrical firm manufactures light bulbs that have a life, before burn out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.	BTL-3	Applying	CO1
15.(a)	The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. (a) What is the probability that the repair time exceeds 2 hours? (b) What is the conditional probability that a repair time exceeds at least 10 hours that its distribution exceeds 9 hours?	BTL-4	Analyzing	CO1
15.(b)	Let X be a Uniformly distributed R. V. over [-5, 5]. Evaluate (i) $P(X \le 2)$ (ii) $P(\|X\|>2)$ (iii) Cumulative distribution function of X (iv) Var(X)	BTL-4	Analyzing	CO1
16.(a)	Buses arrive at a specified stop at 15 minutes interval starting at 7am that is, 7,7:15,7:30,7:45, and so on, If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 am, evaluate the probability that he waits (a) Less than 5 minutes for a bus and (b) At least 12 minutes for a bus	BTL-4	Analyzing	CO1
16.(b)	The marks obtained by a number of students for a certain subject is assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are taken at random from this set Find the probability that exactly 2 of them will have marks over 70?	BTL-4	Analyzing	CO1
17.	In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and Standard Deviation of 60 hours. Find the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours and (iii) more than 1920 hours burs less than 2160 hours.	BTL-3	Applying	CO1
18.(a)	The length of time a person speaks over phone follows exponential distribution with mean 6 mins. What is the probability that the person will talk for (1) more than 8 mins (2) between 4 and 8 mins.	BTL-2	Understanding	CO1
18.(b)	A car hire firm has 2 cars. The number of demands for a car on each day	BTL-3	Applying	CO1

	is distributed as poisson variate with mean 0.5. Calculate the portion of days on which (1) Neither car is used (2) Some demand is refused.			
	PART C(15 Mark Questions)			
1.	Out of 2000 families with 4 children each, Create how many family would you expect to have i) at least 1 boy ii) 2 boys and 2 girls iii) at most 2 girls iv) children of both genders.	BTL-3	Applying	CO1
2.	In a certain factory manufacturing razor blades, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) No defective (ii) One defective (iii) Two defective blades Respectively in a consignment of 10,000 packet.	BTL-4	Analyzing	CO1
3.	Buses arrive at a specified stop at 15 minutes interval starting at 6 AM ie they arrive at 6 AM, 6.15AM, 6.30 AM and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 6 and 6.30 AM. Evaluate the probability that he waits (i) Less than 5 minutes for a bus. (ii) More than 10 minutes for a bus.	BTL-3	Applying	CO1
4.	The daily consumption of milk in excess of 20,000 liters in a town is approximately exponentially distributed with parameter 1/3000. The town has a daily stock of 35,000L. What is the probability that of 2 days selected at random, the stock is insufficient for both days?	BTL-4	Analyzing	CO1
5.	In an Engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60% between 60% and 75% and above 75%respectively. In a particular year 10% of the students failed in the examination and 5% of the students get distinction. Find the percentage of students who have got first class and second class. Assume normal distribution of marks.	BTL-4	Analyzing	CO1

UNIT II TWO – DIMENSIONAL RANDOM VARIABLES 9L+3T

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem

	PART-A(2 MARK QUESTIONS)			
1.	The joint probability distribution of X and Y is given by $p(x,y)=\frac{x+y}{21}$, $x = 1,2,3$; $y = 1, 2$. Find the marginal probability distributions of X	BTL-2	Understanding	CO2
2.	The joint probability function (X,Y) is given by $P(x,y) = k(2x+3y)$, $x = 0,1,2$ $y = 1,2,3$, Find the value of K.	BTL-2	Understanding	CO2
3.	Find the probability distribution of X + Y from the bivariate distribution of (X,Y) given below:	BTL-2	Understanding	CO2
4.	If the joint pdf of X and Y is given by f(x,y)=2,in 0≤x<y≤1, Find E(X)	BTL-1	Remembering	CO2
5.	Find the marginal distributions of X and Y from the bivariate distribution of (X,Y) given below:	BTL-2	Understanding	CO2
6.	Find the value of k,if the joint density function of (X,Y) as	BTL-1	Remembering	CO2

Table for question 3:

X \ Y	1	2
1	0.4	0.2
2	0.3	0.1

Table for question 5:

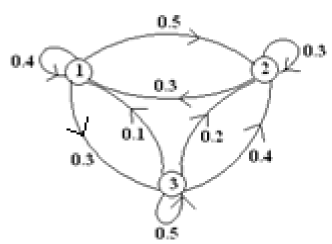
X \ Y	1	2
1	0.1	0.2
2	0.3	0.4

	$f(x,y)=\begin{cases} k(1-x)(1-y), 0<x<4, 1<y<5 \\ 0, otherwise \end{cases}$			
7.	If the joint probability density function of a random variable X and Y is given by $f(x,y)=\begin{cases} \frac{x^3y^3}{16}, 0<x<2, 0<y<2 \\ 0, \quad otherwise \end{cases}$. Obtain the marginal density function of X.	BTL-1	Remembering	CO2
8.	The joint pdf of the random variable (X,Y) is given by $f(x,y)=Kxye^{-(x^2+y^2)}, x>0, y>0$ Find the value of K.	BTL-1	Remembering	CO2
9.	The joint probability density function of a random variable (X,Y) is $f(x,y)=k\,e^{-(2x+3y)}, x\geq 0,\ y\geq 0$. Point out the value of k.	BTL-2	Understanding	CO2
10.	If the joint pdf of (X, Y) is $f(x,y)=\begin{cases} \frac{1}{4}, 0<x,y<2 \\ 0, \ otherwise \end{cases}$. Find $P(X+Y\leq 1)$	BTL-1	Remembering	CO2
11.	Let X and Y be random variables with joint density function f(x,y) = $\begin{cases} 4xy,\ 0<x<1,\ 0<y<1 \\ 0,\ otherwise \end{cases}$ formulate the value of E(XY)	BTL-2	Understanding	CO2
12.	Let the joint density function of a random variable X and Y be given by $f(x,y) = 8xy, \quad 0<y\leq x\leq 1$.Calculate the marginal probability function of X	BTL-1	Remembering	CO2
13.	What is the condition for two random variables are independent?	BTL-2	Understanding	CO2
14.	If the joint probability density function of X and Y is $f(x,y)=e^{-(x+y)}, x,y\geq 0$.Are X and Y independent	BTL-1	Remembering	CO2
15.	State any tow properties of correlation coefficient	BTL-2	Understanding	CO2
16.	Write the angle between the regression lines	BTL-1	Remembering	CO2
17.	The regression equations are x + 6y = 14 and 2x + 3y = 1. Evaluate the correlation coefficient between X & Y .	BTL-1	Remembering	CO2
18.	If $\bar{X}=970, \bar{Y}=18, \sigma_x=38, \sigma_y=2$ and r = 0.6 , Find the line of regression of X on Y.	BTL-2	Understanding	CO2
19.	In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible; Variance of X = 9; Regression equations are 8X - 10Y + 66 = 0 and 40X-18Y = 214. Find the mean values of X and Y?	BTL-1	Remembering	CO2
20.	The regression equations are 3x + 2y = 26 and 6x + y = 31. Find the correlation coefficient**.**	BTL-2	Understanding	CO2
21.	State central limit theorem	BTL-1	Remembering	CO2
22.	Prove that $-1\leq r_{xy}\leq 1$	BTL-2	Understanding	CO2
23.	The equations of two regression lines are 3x+2y=19 and 3y+9x=46. Obtain the mean of X and Y.	BTL-1	Remembering	CO2
24.	The equations of two regression lines are 3x+2y=19 and 3y+9x=46. Derive the correlation coefficient between X and Y.	BTL-1	Remembering	CO2
25.	State the equations of two regression lines.	BTL-2	Understanding	CO2
PART B (13 Mark Questions)				

<table>
<tr><td>1.</td><td>From the following table for bivariate distribution of (X, Y). Find
(i) $P(X \leq 1)$ (ii)$P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv)$P(X \leq 1 / Y \leq 3)$ (v) $P(Y \leq 3 / X \leq 1)$ (vi)$P(X + Y \leq 4)$
<table>
<tr><td>Y / X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>$\frac{1}{32}$</td><td>$\frac{2}{32}$</td><td>$\frac{2}{32}$</td><td>$\frac{3}{32}$</td></tr>
<tr><td>1</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td></tr>
<tr><td>2</td><td>$\frac{1}{32}$</td><td>$\frac{1}{32}$</td><td>$\frac{1}{64}$</td><td>$\frac{1}{64}$</td><td>0</td><td>$\frac{2}{64}$</td></tr>
</table></td><td>BTL-2</td><td>Understanding</td><td>CO2</td></tr>
<tr><td>2.(a)</td><td>The two dimensional random variable (X, Y) has the joint probability mass function $f(x, y) = \frac{x+2y}{27}, x = 0,1,2; y = 0,1,2$. Find the marginal distributions of X and Y. Also find the conditional distribution of Y given X = 1 also find the conditional distribution of X given Y = 1.</td><td>BTL-3</td><td>Applying</td><td>CO2</td></tr>
<tr><td>2.(b)</td><td>The joint pdf a bivariate R.V(X, Y) is given by
$$f(x, y) = \begin{cases} Kxy \ , & 0 < x < 1, 0 < y < 1 \\ 0 \ , & otherwise \end{cases}$$
(1) Find K. (2) Find P(X+Y<1). (3) Are X and Y independent R.V's.</td><td>BTL-3</td><td>Applying</td><td>CO2</td></tr>
<tr><td>3.(a)</td><td>If the joint pdf of (X, Y) is given by P(x, y) = K(2x+3y), x=0, 1, 2, 3, y = 1, 2, 3 Find all the marginal probability distribution. Also find the probability distribution of X+Y.</td><td>BTL-3</td><td>Applying</td><td>CO2</td></tr>
<tr><td>3.(b)</td><td>The joint pdf of the RV (X,Y) is given by $f(x, y) = kxye^{-(x^2+y^2)}$, $x > 0, y > 0$. Find the value of k. Also prove that X and Y are independent</td><td>BTL-4</td><td>Analyzing</td><td>CO2</td></tr>
<tr><td>4.</td><td>The following table represents the joint probability distribution of the discrete RV (X,Y). Find all the marginal and conditional distributions.
<table>
<tr><td>Y</td><td colspan="3">X</td></tr>
<tr><td></td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>1</td><td>1/2</td><td>1/6</td><td>0</td></tr>
<tr><td>2</td><td>0</td><td>1/9</td><td>1/5</td></tr>
<tr><td>3</td><td>1/18</td><td>1/4</td><td>2/15</td></tr>
</table></td><td>BTL-2</td><td>Understanding</td><td>CO2</td></tr>
<tr><td>5.</td><td>Find the marginal distribution of X and Y and also P($P(X \leq 1, Y \leq 1)$, $P(X \leq 1), P(Y \leq 1)$. Check whether X and Y are independent. The joint probability mass function of X and Y is
<table>
<tr><td>Y / X</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>0</td><td>0.10</td><td>0.04</td><td>0.02</td></tr>
<tr><td>1</td><td>0.08</td><td>0.20</td><td>0.06</td></tr>
<tr><td>2</td><td>0.06</td><td>0.14</td><td>.030</td></tr>
</table></td><td>BTL-2</td><td>Understanding</td><td>CO2</td></tr>
<tr><td>6.</td><td>The joint pdf of two dimensional random variables (X,Y) is given by
$f(x, y) = \begin{cases} 25e^{-5y}, 0 < x < 0.2, y > 0 \\ 0, otherwise \end{cases}$ Find the covariance of x and y.</td><td>BTL-4</td><td>Analyzing</td><td>CO2</td></tr>
</table>

7.	If the joint pdf of a two-dimensional RV(X,Y) is given by $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}; 0 < x < 1, 0 < y < 2 \\ 0,\ elsewhere \end{cases}$. Find (i) $P\left(X > \frac{1}{2}\right)$ (ii) $P(Y < \frac{1}{2}, X < \frac{1}{2})$ (iii) $P\left(Y < \frac{1}{2} / X < \frac{1}{2}\right)$	BTL-3	Applying	CO2
8.	The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$. Compute (i) $P\left(X > 1 / Y < \frac{1}{2}\right)$ (ii) $P\left(Y < \frac{1}{2} / X > 1\right)$ (iii) $P(X+Y) \le 1$.	BTL-3	Applying	CO2
9.	(b)The joint pdf of X and Y is given by f(x,y)=$\begin{cases} kx(x-y), 0 < x < 2, -x < y < x \\ 0,\quad otherwise \end{cases}$ (i)Find K (ii) Find $f_x(x)$ and $f_y(y)$	BTL-3	Applying	CO2
10.	Find the Coefficient of Correlation between industrial production and export using the following table <table><tr><td>Production (X)</td><td>14</td><td>17</td><td>23</td><td>21</td><td>25</td></tr><tr><td>Export (Y)</td><td>10</td><td>12</td><td>15</td><td>20</td><td>23</td></tr></table>	BTL-2	Understanding	CO2
11.	Find the correlation coefficient for the following heights of fathers X,their sons Y and also find the equations of regression lines. Hence find the height of son when the height of father is 71 <table><tr><td>X</td><td>65</td><td>66</td><td>67</td><td>67</td><td>68</td><td>69</td><td>70</td><td>72</td></tr><tr><td>Y</td><td>67</td><td>68</td><td>65</td><td>68</td><td>72</td><td>72</td><td>69</td><td>71</td></tr></table>	BTL-2	Understanding	CO2
12.	Obtain the lines of regression <table><tr><td>X</td><td>50</td><td>55</td><td>50</td><td>60</td><td>65</td><td>65</td><td>65</td><td>60</td><td>60</td></tr><tr><td>Y</td><td>11</td><td>14</td><td>13</td><td>16</td><td>16</td><td>15</td><td>15</td><td>14</td><td>13</td></tr></table>	BTL-2	Understanding	CO2
13.	If $f(x,y) = \frac{6-x-y}{8}$, $0 \le x \le 2$, , $2 \le y \le 4$ for a bivariate random variable (X,Y), Evaluate the correlation coefficient ρ.	BTL-3	Applying	CO2
14.	Two random variables X and Y have the joint density function $f(x,y) = x + y, 0 \le x \le 1, 0 \le y \le 1$. Evaluate the Correlation coefficient between X and Y.	BTL-4	Analyzing	CO2
15.(a)	20 dice are thrown. Find the approximate probability that the sum obtained is between 65 and 75 using central limit theorem	BTL-3	Applying	CO2
15.(b)	The two regression lines are 4x-5y+33=0 and 20x-9y=107. Find the mean of X and Y. Also find the correlation coefficient between them	BTL-3	Applying	CO2
16.(a)	If $X_1, X_2, X_3, \cdots X_n$ are Poisson variates with mean 2, use central limit theorem to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + X_3 + \cdots + X_n$ and n=75.	BTL-4	Analyzing	CO2
16.(b)	If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of U = X-Y.	BTL-4	Analyzing	CO2
17.(a)	If X and Y independent Random Variables with pdf $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$. Devise the density function of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are they independent?	BTL-4	Analyzing	CO2
17.(b)	Two random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} x + y; 0 \le x \le 1, 0 \le y \le 1 \\ 0,\ otherwise \end{cases}$. Find the probability density function of the random variable U = XY.	BTL-3	Applying	CO2

18.(a)	A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using central limit theorem find what probability that we can assert that the mean of the sample will not differ from μ more than 4?	BTL-4	Analyzing	CO2
18.(b)	If X and Y follows an exponential distribution with parameters 2 and 3 respectively and are independent, Create the probability density function of U =X+Y	BTL-3	Applying	CO2
	PART-C(15 Mark Questions)			
1.	Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 blue balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, Find the probability distribution of X and Y.	BTL-3	Applying	CO2
2.	Out of the two lines of regression given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, which one is the regression line of X on Y? Analyze the equations to find the means of X and Y. If the variance of X is 12, find the variance of Y.	BTL-4	Analyzing	CO2
3.	From the following data , Find (i)The two regression equations (ii) The coefficient of correlation between the marks in Mathematics and Statistics (iii) The most likely marks in Statistics when marks in Mathematics are 30 Marks in Maths : 25 28 35 32 31 36 29 38 34 32 Marks in Statistics: 43 46 49 41 36 32 31 30 33 39	BTL-2	Understanding	CO2
4.	For a particular brand of TV picture tube, it is known that the mean operating life of the tubes is 1000 hours with a standard deviation of 250 hours, Devise the probability that the mean for a random sample of size 25 will be between 950 and 1050 hours?	BTL-4	Analyzing	CO2
5.	The lifetime of a certain brand of an electric bulb may be considered a RV with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem, that the average life time of 60 bulbs exceeds 1250 h.	BTL-3	Applying	CO2
UNIT III : RANDOM PROCESSES **9L+3T** Classification – Stationary process – Markov process– Markov chain – Chapman Kolmogorov equations – Limiting distributions– Poisson process-Gaussian process.				
	PART-A(2 Mark Questions)			
1.	What are the four types of a stochastic process?	BTL-1	Remembering	CO3
2.	Define Discrete Random sequence with example.	BTL-1	Remembering	CO3
3.	Define Discrete Random Process with example.	BTL-1	Remembering	CO3
4.	Define Continuous Random sequence with example.	BTL-1	Remembering	CO3
5.	Define Continuous Random Process with example.	BTL-1	Remembering	CO3
6.	Define wide sense stationary process. .	BTL-1	Remembering	CO3
7.	Define Strict Sense Stationary Process.	BTL-1	Remembering	CO3
8.	Show that the random process $X(t) = A\cos(\omega_c t + \theta)$ is not stationary if it is assumed that A and ω_c are constants and θ is a uniformly distributed variable on the interval (0,π).	BTL-2	Understanding	CO3
9.	A random process X (t) = A sin t + B cos t where A and B are independent random variables with zero means and equal standard deviations. Find the mean of the process.	BTL-1	Remembering	CO3

10.	Consider the random process X $(t) = cos\ (t + \phi)$, where ϕ is uniform random variable in $\left(-\pi/2, \pi/2\right)$. Check whether the process is stationary.	BTL-2	Understanding	CO3
11.	Consider the random process X $(t) = cos\ (\omega_0 t + \theta)$, where θ is uniform random variable in $(-\pi, \pi)$. Check whether the process is stationary or not	BTL-1	Remembering	CO3
12.	Find the mean of a stationary random process whose auto correlation function is given by $R_{(Z)} = \dfrac{25Z^2 + 36}{6.25Z^2 + 4}$.	BTL-2	Understanding	CO3
13.	Find the mean of a stationary random process whose auto correlation function is given by $R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}$.	BTL-2	Understanding	CO3
14.	A random process has the autocorrelation function $R_{xx}(\tau) = \frac{4\tau^2+6}{\tau^2+1}$, find the mean square value of the problem.	BTL-2	Understanding	CO3
15.	Compute the mean value of the random process whose auto correlation function is given by $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$.	BTL-2	Understanding	CO3
16.	Define Poisson process.	BTL-1	Remembering	CO3
17.	State and two properties of Poisson process.	BTL-1	Remembering	CO3
18.	Check whether the Poisson process $X(t)$ given by the probability law $P\{X(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0,1,2,\cdots$is stationary or not.	BTL-1	Remembering	CO3
19.	A hospital receives on an average of 3 emergency calls in 10 minutes interval. What is the probability that there are 3 emergency calls in a 10 minute interval	BTL-2	Understanding	CO3
20.	Define Markov chain	BTL-1	Remembering	CO3
21.	State Chapman- Kolmogorov theorem	BTL-1	Remembering	CO3
22.	Consider the Markov chain with 2 states and transition probability matrix $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Find the stationary probabilities of the chain.	BTL-1	Remembering	CO3
23.	The one-step transition probability matrix of a Markov chain with states (0,1) is given by $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Evaluate whether it is irreducible Markov chain?	BTL-2	Understanding	CO3
24.	Obtain the transition matrix of the following transition diagram. 	BTL-1	Remembering	CO3
25.	Check whether the Markov chain with transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ is irreducible or not?	BTL-2	Understanding	CO3
	PART-B (13 Marks Questions)			

1.	The process {X(t)} whose probability distribution under certain conditions is given by $P\{X(t)=n\}=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2\\ \dfrac{at}{(1+at)}, & n=0\end{cases}$ Show that it is not stationary.	BTL-2	Understanding	CO3
2.(a)	A radioactive source emits particles at a rate of 5 per minute in accordance with poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 partcles are recorded in 4 minute period	BTL-2	Understanding	CO3
2.(b)	Find the mean and autocorrelation of the Poisson processes	BTL-1	Remembering	CO3
3.(a)	If the random process {X(t)} takes the value -1 with probability 1/3 and takes the value +1 with probability 2/3, find whether {X(t)} is a stationary process or not.	BTL-3	Applying	CO3
3.(b)	Prove that the sum of two independent Poisson process is a Poisson process.	BTL-1	Remembering	CO3
4.(a)	Consider a random process $X(t) = B\ cos\ (50\ t + \Phi)$ where B and Φ are independent random variables. B is a random variable with mean 0 and variance 1.Φ is uniformly distributed in the interval [-π,π]. Determine the mean and auto correlation of the process.	BTL-3	Applying	CO3
4.(b)	Prove that the difference of two independent Poisson process is not a Poisson process.	BTL-1	Remembering	CO3
5.(a)	Show that the random process $X(t) = A\cos(\omega t + \theta))$ is wide sense stationary, if A and ω are constant and θ is a uniformly distributed random variable in (0, 2π).	BTL-3	Applying	CO3
5.(b)	A fisherman catches a fish at a poisson rate of 2 per hour from a large lake with lots of fish. If he starts fishing at 10 am. What is the probability that he catches one fish by 10.30 am and three fishes by noon.	BTL-4	Analyzing	CO3
6.(a)	Suppose that customers arrive at a bank according to poisson process with mean rate of 3 per minute. Find the probability that during a time of two minutes(1)Exactly 4 customers arrive (2) Greater than 4 customers arrive (3)Fewer than 4 customers arrive	BTL-3	Applying	CO3
6.(b)	Prove that the inter arrival time of the Poisson process follows exponential distribution	BTL-1	Remembering	CO3
7.	Show that the random process $X(t) = Acos\omega t + Bsin\omega t$ is wide sense stationary process if A and B are random variables such that $E(A) = E(B) = 0, E(A^2) = E(B^2)$ and $E(AB) = 0$	BTL-2	Understanding	CO3
8.	A salesman's territory consists of three regions A, B, C. He never sells in the same region on successive days. If he sells in region A, then the next day he sells in B. However, if he sells either B or C, then the next day he is twice as likely to sell in A as in the other region. Explain How often does he sell in each of the regions in the steady state?	BTL-3	Applying	CO3
9.	There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. The state of the related markov chain is the number of red marbles in urn A after the interchange. What is the probability that there are 2 red marbles in urn A after the interchange? What is the probability that there are 2 red marbles in urn A after 3 steps?	BTL-3	Applying	CO3

	In the long run, What is the probability that there are 2 red marbles in urn A			
10.(a)	A hard disk fails in a computer system and it follows a poisson distribution with mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If we have extra hard disks and the next supply is not due in 10 weeks, Find the probability that the machine will not be out of order in next 10 weeks.	BTL-4	Analyzing	CO3
10.(b)	The probability of a dry day following a rainy day is 1/3 and that the probability of a rainy day following a dry day is ½. Given that May 1st is a dry day. Obtain the probability that May 3rd is a dry day also May 5th is a dry day	BTL-3	Applying	CO3
11.	Find the limiting state probabilities associated with the following transition probability matrix $\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$	BTL-4	Analyzing	CO3
12.	The transition probability matrix of a Markov chain $\{X_n\}$, n = 1,2,3, …. having 3 states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is P(0) = (0.7, 0.2, 0.1). Evaluate i) $P(X_2 = 3)$ ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$	BTL-2	Understanding	CO3
13.	Consider the Markov chain $\{X_n$, n= 0, 1, 2,3…..} having 3 states space S={1,2,3} and one step TPM $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ and initial probability distribution $P(X_0=i)=1/3$,i= 1,2,3. Compute (1) $P(X_3=2. X_2=1,X_1=2/X_0=1)$ (2) $P(X_3=2, X_2=1/X_1=2,X_0=1)$ (3) $P(X_2=2/X_0=2)$ (4) Invariant Probabilities of the Markov Chain.	BTL-2	Understanding	CO3
14.(a)	Let $\{X_n$: n = 1,2,3 …… } be a Markov chain on the space S = {1,2,3} with one step t.p.m $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$ 1. Sketch the transition diagram, 2. Is the chain irreducible? Explain. 3. Is the chain ergodic? Explain.	BTL-2	Understanding	CO3
14.(b)	If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, Evaluate the probability that the interval between 2 consecutive arrivals is (a) more than 1 minute, (b) between 1 minute and 2 minutes and (c) 4 minutes or less	BTL-4	Analyzing	CO3
15.	Three boys A, B and C are throwing a ball to each other. *A* always throws the ball to *B* and *B* always throws the ball to *C* but *C* is just as likely to throw the ball to *B* as to *A*. Show that the process is Markovian. Find the transition probability matrix and classify the states	BTL-4	Analyzing	CO3

16.	Consider a Markov chain chain $\{X_n, n= 0, 1, 2,\ldots..\}$ having states space S={ 1,2} and one step TPM $P=\begin{bmatrix} \frac{4}{10} & \frac{6}{10} \\ \frac{8}{10} & \frac{2}{10} \end{bmatrix}$. (1) Draw a transition diagram, (2) Is the chain irreducible? (3) Is the state -1 ergodic? Explain. (4) Is the chain ergodic? Explain	BTL-4	Analyzing	CO3
17.	Classify the states of the Markov chain for the one-step transition probability matrix $P=\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$ with state space S = {1,2,3}	BTL-4	Analyzing	CO3
18.	Using limiting behavior of homogeneous markov chain, Find the steady state probability of the chain given by the transaction probability matrix $P=\begin{pmatrix} 0.1 & 0.6 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$	BTL-3	Applying	CO3
	Part C: 15 - MARK QUESTIONS			
1.	On a given day, a retired English professor, Dr. Charles Fish amuses himself with only one of the following activities reading (i), gardening (ii) or working on his book about a river valley (iii) for $1 \le i \le 3$, let $X_n = i$, if Dr. Fish devotes day *n* to activity *i*. Suppose that $\{X_n : n=1,2\ldots\}$ is a Markov chain, and depending on which of these activities on the next day is given by the t. p. m $P=\begin{bmatrix} 0.30 & 0.25 & 0.45 \\ 0.40 & 0.10 & 0.50 \\ 0.25 & 0.40 & 0.35 \end{bmatrix}$ Find the proportion of days Dr. Fish devotes to each activity.	BTL-2	Understanding	CO3
2.	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives to work in the long run.	BTL-3	Applying	CO3
3.	A machine goes out of order whenever a component fails. The failure of this part follows a Poisson process with mean rate of 1 per week. Find the probability that 2 weeks have a elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks	BTL-4	Analyzing	CO3
4.	A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, Evaluate the transition probability matrix P of the Markov chain $\{X_n\}$. Find also $P\{X_2= 6\}$ and P^2.	BTL-3	Applying	CO3
5.	A student's study habits are as follows: If he studies one night, he is 70% sure not to study next night. On the other hand, if he does not study one night, he is 60% sure not to study the next nights as well. In the long run how often does he study?	BTL-4	Analyzing	CO3

UNIT IV : QUEUEING MODELS

Markovian queues – Birth and death processes – Single and multiple server queueing models – Little's formula –

Queues with finite waiting rooms – Queues with impatient customers: Balking and reneging.				
PART-A(2 Mark Questions)				
1.	For (M/M/1): (∞/FIFO) model, Write the Little's formula.	BTL2	Understanding	CO4
2.	Find the probability of at least 10 customers in the system (M/M/1): (∞/FIFO) queue system, if λ=6 *per hour* and μ= 8 *per hour?*			CO4
3.	Find the probability that a customer has to wait more than 15 *min* to get his service completed in a (M/M/1): (∞/FIFO) queue system, if λ=6 *per hour* and μ= 10 per hour?	BTL2	Understanding	CO4
4.	For a (M/M/1): (∞/FIFO) queue system, if λ=4 per hour and μ=6 per hour, find the average queue length.	BTL4	Analyzing	CO4
5.	If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket. What he expect to be seated for the start of the picture?	BTL3	Applying	CO4
6.	Write the effective arrival rate for M/M/1: K/FIFO queueing model	BTL1	Remembering	CO4
7.	Give the formula for average waiting time of a customer in the queue for (M/M/1): (K/FIFO).	BTL1	Remembering	CO4
8.	If λ= 3 per hour, μ= 4 per hour and maximum capacity K = 7 in a (M/M/1) (K/FIFO) system, Find the average number of customers in the system.	BTL4	Analyzing	CO4
9.	A drive in banking service is modeled as an M/M/1 queueing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99 percent of the time. How fast should the service rate be?	BTL4	Analyzing	CO4
10.	Describe the formula for W_s and W_q for the M/M/1/N queueing system.	BTL1	Remembering	CO4
11.	For (M/M/C): (N/FIFO) model, Write the formula for (*a*) average number of customers in the queue. (*b*) Average waiting time in the system.	BTL4	Analyzing	CO4
12.	State the characteristics of a Queueing model.	BTL1	Remembering	CO4
13.	Write Kendall's notation for Queueing Model.	BTL2	Understanding	CO4
14.	What are the service disciplines available in the queueing model?	BTL1	Remembering	CO4
15.	Write the formulae for P_0 and P_n in a Poisson queue system in the steady – state	BTL3	Applying	CO4
16.	Define steady state and transient state queueing systems			CO4
17.	Consider an M/M/C queueing system. Find the probability that an arriving customer is forced to join the queue.	BTL4	Analyzing	CO4
18.	Draw the transition diagram for M/M/1 queueing model.	BTL1	Remembering	CO4
19.	A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 *minutes* while the cashier can serve 10 customers in 5 *minutes*. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, Find the average time a customer spends in the system.	BTL1	Remembering	CO4
20.	In a 3 server infinite capacity Poisson queue model if $\frac{\lambda}{\mu C}=\frac{2}{3}$, Calculate P_0.	BTL1	Remembering	CO4
21.	State the characteristics of a Queueing model.	BTL1	Remembering	CO4

22.	If the inter arrival time and service time in a public telephone booth with a single phone follow exponential distribution with means of 10 and 8 minutes respectively. Find the average number of callers in the booth at any time.	BTL2	Understanding	CO4
23.	If the arrival and departure rates in a M/M/1 queue are ½ per minute and 2/3 per minute respectively, find the average waiting time of a customer in the queue.	BTL2	Understanding	CO4
24.	If there are 2 servers in an infinite capacity Poisson queue system with λ= 10 per hour and μ = 15 per hour, Examine the percentage of idle time for each server?	BTL4	Analyzing	CO4
25.	Find the traffic intensity for an (M/M/C): (∞ /FIFO) queue with λ= 10 per hour and μ = 15 per hour and 2 servers.	BTL2	Understanding	CO4
PART-B (13 Mark Questions)				
1.	Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every10 minutes, and the service time is exponential random variable with 8 minutes. Apply M/M/1 queueing model a) Find the average number of customers L_s in the shop. b) Find the average number of customers L_q in the queue. c) Find the average time a customer spends in the system in the shop W_s d) What is the probability that the server is idle?	BTL3	Applying	CO4
2.	A TV repairman finds that the time spend on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per8 – hour day. Point out the repairman's expected idle time in each day? How many jobs are ahead of the average set just brought in?	BTL3	Applying	CO4
3.	On average 96 patients per (24 hour) day require the service of an emergency clinic. Also an average a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs.100 per patient treated to obtain an average service time of 10 minutes, and that each minute of decrease in this average time would cost Rs .10 per patient treated .Analyze how much would have to be budgeted by the clinic to decrease the average size of the queue from 1 1/3 patients to ½ patient?	BTL4	Analyzing	CO4
4.	A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only .It has been found that the service time distributions for both deposits and withdrawals are exponential with mean time of 3 min per customer. Depositors are found to arrive in Poisson fashion throughout the day with mean arrival rate of 16 per hour Withdrawers also arrive in a Poisson fashion with mean arrival rate 14 per hour. What would be the effect on the average waiting time for the customers if each teller handles both withdrawals and deposits? What would be the effect, if this could only be accomplished by increasing the service time to 3.5 min?	BTL4	Analyzing	CO4
5.	There are three typists in an office. Each typist can type an average of 6 Letters per hour .If letters arrive for being typed at the rate of 15 letters per hour, Analyze the following a) What fraction of the time all the typists will be busy? b) What is the average number of letters waiting to be typed? c) What is the average time a letter has to spend for waiting and for being typed? d) What is the probability that a letter will take longer than 20 min waiting	BTL4	Analyzing	CO4

	to be typed?			
6.	A 2 – person barber shop has 5 chair to accommodate waiting customers.Potential customers ,who arrive when all 5 chairs are full,leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair .Compute P_0 , P_1,P_7 , E(N_q) and E(W)	BTL4	Analyzing	CO4
7.	A car servicing station has 2 bays where service can be offered simultaneously. Because of space limtation,only 4 cars are accepted for servicing . The arrival pattern is Poisson with 12 cars per day.The service time in both bays is exponentially distributed with μ = 8 cars per day per bay. Write the average number of cars in the service station , the average number of cars waiting for service time a car spends in the system	BTL4	Creating	CO4
8.	The railway marshalling yard is sufficient only for trains (there being 11 lines, one of which is earmarked for the shunting engine to reverse itself from the crest of the hump to the rear of the train). Trains arrive at the rate of 25 trains per day, inter – arrival time and service time follow exponential with an average of 30 minutes. Estimate the probability that the yard is empty and average queue length.	BTL4	Analyzing	CO4
9.	Derive the average number of customers in the system, in queue and average waiting time of a customer in the system and in queue for an $M/M/1:\infty/FIFO$ queueing system	BTL1	Remembering	CO4
10.	Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min. a) Find the average number of persons waiting in the system b) What is the probability that a person arriving at the booth will have to wait in the queue? c) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete his call? d) Estimate the fraction of the day when the phone will be in use e) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 min. for phone .By how much the flow of arrivals should increase in order to justify a second booth?	BTL2	Understanding	CO4
11.	Customers arrive at a one-man barber shop according to a Poisson with a mean inter arrival time of 20 min Customers spend an average of 15 min in the barber's chair 1) What is the expected number of customers in the barber shop ?In the Queue? 2) What is the probability that a customer will not have to wait for a hair cut? 3) How much can a customer expect to spend in the barbershop? 4) What are the average time customers spend in the queue? 5) What is the probability that the waiting time in the system is greater than 30 min? 6) What is the probability that there are more than 3 customers in the system?	BTL2	Understanding	CO4
12.	Customers arrive at a one-man barber shop according to a Poisson with a mean inter arrival time of 12 min Customers spend an average of 10min in the barber's chair 1) What is the expected number of customers in the barber shop?	BTL2	Understanding	CO4

	2) What is the expected number of customers In the Queue? 3) What is the probability that a customer will not have to wait for a hair cut? 4) How much can a customer expect to spend in the barbershop? 5) What are the average time customers spend in the queue? 6) What is the probability that the waiting time in the system is greater than 30 min?			
13.	In a given M / M / 1 queueing system, the average arrivals is 4 customers per minute, $\rho = 0.7$. Find the 1) mean number of customers L_s in the system 2) mean number of customers L_q in the queue 3) probability that the server is idle 4) mean waiting time W_s in the system 5) mean waiting time W_q in the queue	BTL2	Understanding	CO4
14.	A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour. Evaluate the following (a)What is the probability that an arrival would have to wait in line? (b)Find the average waiting time, average time spent in the system and the average number of cars in the system (c) For what percentage of time would a pump be idle on an average?	BTL4	Analyzing	CO4
15.	A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min and if people arrive in Poisson fashion at the rate of 10 per hour a) What is the probability that a customer has to wait for service? b) What is the expected percentage of idle time for each girl? c) If the customer has to wait in the queue, what is the expected length of the waiting time?	BTL4	Analyzing	CO4
16.	A tele phone exchange has two long distance operators. The telephone company finds that during peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service of these calls approximately exponentially distributed with mean length of 5 minutes. (1) What is the probability that a subscriber will have to wait for his long distance calls during the peack hours of the day? If the subcribers will wait and are serviced in turn, what is the expected waiting time?	BTL4	Analyzing	CO4
17.	Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains. 1.Find the probability that the yard is empty 2. Find the average number of trains in the system	BTL4	Analyzing	CO4
18.	Consider a single server queueing system with Poisson input exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units in the system is 2. Apply single server finite capacity to 1. Find the steady state probability distribution of the number of calling units in the system. 2. Find the expected number of calling units in the system and in queue.	BTL3	Applying	CO4

	3. Find the average waiting time in the system and in the queue.			
Part C (15 Mark Questions)				
1.	A repairman is to be hired to repair machines which breakdown at the average rate of 3 per hour the breakdown follow Poisson distribution. Non –productive time of machine is considered to cost Rs 16/hour. Two repair men have been interviewed. One is slow but cheap while the other is fast and expensive. The slow repairman charges Rs.8 per hour and he services at the rate of 4 per hour The fast repairman demands Rs .10 per hour and services at the average rate 6 per hour. Which repairman should be hired?	BTL3	Applying	CO4
2.	A departmental store has single cashier. During the rush hours customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Point out the following 1. What is the probability that the cashier is idle? 2. What is the average number of customers in the queueing system? 3. What is the average time a customer spends in the system? 4. What is the average number of customers in the queue? 5. What is the average time a customer spends in the queue?	BTL2	Understanding	CO4
3.	A petrol pump station has 2 pumps. The service times follow the exponential distribution with a mean of 4minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle?	BTL4	Analyzing	CO4
4.	At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms. down the river. Tankers arrive according to a Poisson process with a mean of 1 for every 2 hours. It takes for an unloading crew, on the average, 10 *hours* to unload a tanker, the unloading time follows an exponential distribution Develop and Determine (i) How many tankers are at the port on the average? (ii) How long does a tanker spend at the port on the average? (iii)What is the average arrival rate at the overflow facility?	BTL4	Analyzing	CO4
5.	Patients arrive at a clinic according to a Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour. 1. Derive the effective arrival rate at the clinic 2. What is the probability that an arriving patient does not have to wait? 3. What is the expected waiting time until a patient is discharged from the clinic?	BTL4	Analyzing	CO4

UNIT V: ADVANCED QUEUEING MODELS

Finite source models – M/G/1 queue – Pollaczek Khinchine formula – M/D/1 and $M/E_K/1$ as special cases – Series queues – Open Jackson networks.

PART-A (2 Mark Questions)				
1.	Express Pollaczek- Khintchine formula.	BTL1	Remembering	CO5
2.	Write P-K formula for the case when the service time is constant.	BTL1	Remembering	CO5
3.	Write down the formula for (M/D/1) : (∞/GD) model			CO5
4.	Define effective arrival rate with respect to an (M ∣ M ∣ 1): (GD / N/∞) queuing model.	BTL1	Remembering	CO5
5.	What do you mean by E_k in the $M/E_k/$ 1 queueing model?	BTL2	Understanding	CO5

6.	For an M/G/1 model if λ=5 and μ=6 min and σ =1/20, find the length of the queue.	BTL1	Remembering	CO5
7.	A one man barber shop taken 25 minutes to complete a haircut. If customers arrive in a Poisson fashion at an average rate of 1 per 40 minutes find the average length of the queue.	BTL1	Remembering	CO5
8.	In an M/D/1 queueing system, an arrival rate of customer is 1/6 per minute and the server takes exactly 4 minutes to serve a customer. Calculate the mean number of customers in the system.	BTL2	Understanding	CO5
9.	The arrival of trucks to a factory for unloading is Poisson with arrival rate of 3 trucks per hour. The unloading time is constant with exactly 4 customers per hour. What is the expected number of trucks in queue?	BTL2	Understanding	CO5
10.	Describe series queue.	BTL1	Remembering	CO5
11.	Define a two-stage series queue.	BTL3	Applying	CO5
12.	Define Series Queue with blocking.	BTL1	Remembering	CO5
13.	Define a tandem queue.	BTL3	Applying	CO5
14.	Draw the state transition diagram of a two stage sequential queue model with blocking for the stage2	BTL4	Analyzing	CO5
15.	State any two examples for series queues.	BTL4	Analyzing	CO5
16.	A transfer line has two machines M1 and M2 with unlimited buffer space in between. Parts arrives the transfer line at the rate of 1 part every 2 minutes. The processing rates of M1 and M2 are 1 per minutes and 2 per minutes respectively. Find the average number of parts in M1.	BTL1	Remembering	CO5
17.	Consider a series facility with two sequential stations with respective service rates 3/min and 4/min. The arrival rate is 2/min. What is the average service time of the system, if the system could be approximated by a two stage tandem queue?	BTL1	Remembering	CO5
18.	Define an open Jackson network	BTL4	Analyzing	CO5
19.	State Jackson's theorem for an open network.	BTL3	Applying	CO5
20.	Compose classification of queuing networks.	BTL4	Analyzing	CO5
21.	Distinguish between open and closed networks.	BTL1	Remembering	CO5
22.	What do you mean by bottleneck of a network?	BTL2	Understanding	CO5
23.	Write down the characteristics of an open Jackson network.	BTL1	Remembering	CO5
24.	Define a closed Jackson network.	BTL1	Remembering	CO5
25.	State Burke's theorem used in queueing theory.	BTL1	Remembering	CO5
PART-B (13 Mark Questions)				
1.	State and Derive Pollaczek - Khinchine formula.	BTL4	Analyzing	CO5
2.	In a college canteen, it was observed that there is only one waiter who takes exactly 4 minutes to serve a cup of coffee once the order has been placed with him. If students arrive in the canteen at an average rate of 10 per hour, how much time one is expected to spend waiting for his turn to place the order.	BTL1	Remembering	CO5
3.	A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, Find the average time a customer spends in the shop. Also, Find the average time a customer must wait for service?	BTL2	Understanding	CO5
4.	For$(M/E_2/\ 1)$: (FIFO/∞/∞) queueing model with $\lambda=\frac{6}{5}$ per hour and $\mu=\frac{3}{2}$per hour, find the average waiting time of a customer. Also find the average time he spends in the system	BTL4	Analyzing	CO5

5.	Consider a queuing system where arrivals according to a Poisson distribution with mean 5/hr. Find expected waiting time in the system if the service time distribution is Uniform from t = 5 min to t = 15 minutes	BTL4	Analyzing	CO5
6.	Find Ls, Lq, Ws andWq. Automatic car wash facility operates with only one Bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. (i)If the service time for all cars is constant and equal to 10 min (ii) Uniform distribution between 8 and 12 minutes (iii) Normal distribution with mean 12 minutes and SD 3minutes (iv) Follows discrete distribution 4,8 & 15 minutes with corresponding probability 0.2,0.6&0.2	BTL4	Analyzing	CO5
7.	If a patient who goes to a single doctor clinic for a general check up has to go through 4 phases. The doctor takes on the average 4 minutes for each phase of the check up and the time taken for each phase is exponentially distributed. If the arrivals of the patients at the clinic are approximately Poisson at the average rate of 3 per hour, what is the average time spent by a patient (i) in the examination (ii) waiting in the clinic?	BTL4	Analyzing	CO5
8.	In a computer programs for execution arrive according to Poisson law with a mean of 5 per minute. Assuming the system is busy, Find L_q, L_s,W_q , W_s if the service time is uniform between 8 and 12 sec.	BTL1	Remembering	CO5
9.	Find the average calling rate for the services of the crane and what is the average delay in getting service? In a heavy machine shop, the overhead crane is 75% utilized. Time study observations gave the average slinging time as 10.5 minutes with a standard deviation of 8.8 minutes. If the average service time is cut to 8.0 minutes with a standard deviation of 6.0 minutes, how much reduction will occur on average in the delay of getting served?	BTL3	Applying	CO5
10.	In a two-station service facility, queues are not allowed. Customers arrive at the facility at an average rate of 4 per hour; the server at each station serves at the rate of 5 customers per hour. If arrivals are Poisson and service times are exponential, find the probability that an arriving customer enter the system (a) The probability that an arriving customer enters the system. (b) effective arrival rate. (c) Average (expected) number of customers in the system. Expected time of a customer spends in the system.	BTL2	Understanding	CO5
11.	In a charity clinic there are two doctors, one assistant doctor D1 and his senior doctor D2. The Junior doctor tests and writes the case sheet and then sends to the senior for diagnosis and Prescription of medicine. Only one patient is allowed to enter the clinic at a time due to capacity of space. A patient who has finished with D1 has to wait till the patient with D2 has finished. If Patients arrive according to Poisson with rate 1 per hour and service times are independent and Follow exponential with parameters 3 and 2, Find (i) the probability of a customer entering the Clinic, (ii) the average number of customers in the clinic, (iii) the average time spent by a patient Who entered the clinic.	BTL2	Understanding	CO5
12.	A repair facility is shared by a large number of machines for repair. The facility has two sequential stations with respective rates of service 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1	BTL3	Applying	CO5

	per hour. Assuming that the system behavior may be approximated by a two-station tandem queue Find (i) the average number of customers in both stations, (ii) The average repair time. (iii) The probability that both service stations are idle.			
13.	There are two service stations S1 and S2 in a line with unlimited buffer space in between. Customers arrives S1 at a rate of 1 per every 2 min. The service time rates of S1 and S2 are 1 and 2 per min. respectively. Find (i) the average number of customers at S1 and S2 (ii) The average waiting times at S1and S2 (iii) the total waiting time in the system.	BTL2	Understanding	CO5
14.	In a book shop there are two sections, one for text books and the other for note books Customers from side arrived at the text book section at a Poisson rate of 4 per hour and at the notebook section at a Poison rate of 3per hour. The service rates of T.B and N.B sections respectively 8 and 10 per hour. customer upon completion of service at T.B section is equally likely to go to the N.B section or to leave the book shop, where as a customer upon completion of service at N.B section will go to the T.B section with probability 1/3 and will leave the book shop otherwise. Find the joint steady state probability that there are 4 customers in the T.B section and 2 customers In the N.B section. Find also the average number of customers in the book shop and the average waiting time of the customers in the shop. Assume that there is only one sales man in each section.	BTL2	Understanding	CO5
15.	In a departmental store, there are two sections namely grocery section and perishable section. Customers from outside arrive the G-section according to a Poisson process at a mean rate of 10 per hour and they reach the p-section at a mean rate of 2 per hour. The service times at both the sections are exponentially distributed with parameters 15 and 12 respectively. On finishing the job in G-section, a customer is equally likely to go to the P-section or leave the store, where as a customer on finishing his job in the P-section will go to the G- section with probability 0.25 and leave the store otherwise. Assuming that there is only one salesman in each section, find (i) the probability that there are 3 customers in the G-section and 2 customers in the P-section, (ii) the average waiting time of a customer in the store.	BTL4	Analyzing	CO5
16.	Consider a system with two servers where customers arrive from outside the system in a Poisson fashion at server 1 at the rate of 4 per hour and at server 2 at a rate of 5 per hour. The customers are served at station 1 and station2 at the rate of 8 hour and 10 hour respectively. A customer after completion of service at server 1 is equally likely will go to server 2 or to leave the system. A departing customer from server 2 will go to server 1, 25% of the time and will depart from the system otherwise. Find the (i) The total arrival rates at server1 and server 2. (ii) The limiting probability of n customers at server 1 and m customers at server2 (iii) Expected number of customers in the system Expected time a customer spends in the system	BTL2	Understanding	CO5

<table>
<tr><td>17.</td><td>In a network of 3 service station 1,2, 3 customer arrive at 1,2,3 from outside in accordance with Poisson process having rate 5, 10, 15 respectively. The service time at the stations are exponential with respect rate 10, 50, 100, A customer completing service at station -1 is equally likely to (i) go to station 2 (ii) go to station 3 or (iii) leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go station 2 or leave the system. (a) Find the average number customer in the system consisting of all the three stations? (b) Examine the average time a customer spend in the system?</td><td>BTL4</td><td>Analyzing</td><td>CO5</td></tr>
<tr><td>18.</td><td>The open Jackson network the following information are given:
<table>
<tr><td></td><td></td><td></td><td></td><td colspan="3">r_{ij}</td></tr>
<tr><td>Station</td><td>C_j</td><td>μ_j</td><td>r_j</td><td>i = 1</td><td>i =2</td><td>i =3</td></tr>
<tr><td>1</td><td>1</td><td>10</td><td>1</td><td>0</td><td>0.1</td><td>0.4</td></tr>
<tr><td>2</td><td>2</td><td>10</td><td>4</td><td>0.6</td><td>0</td><td>0.4</td></tr>
<tr><td>3</td><td>1</td><td>10</td><td>3</td><td>0.3</td><td>0.3</td><td>0</td></tr>
</table>
Find (i) the joint probability for the number of customers in 1st , 2nd and 3rd stations are 2,3,4 respectively.
(ii) the expected number of customer in each station.
(iii) the expected total number of customers in the system
(iv) the expected total waiting time in the system.</td><td>BTL2</td><td>Understanding</td><td>CO5</td></tr>
<tr><td colspan="4">Part C (15 Mark Questions)</td><td></td></tr>
<tr><td>1</td><td>Consider a single server. Poisson input queue with a mean arrival rate of 10/hr. Currently the server works according to an exponential distribution with a mean service time of 5 minutes. Management has a training course after which service time will follow non-exponential distribution and the mean service time will increase to 5.5 minutes, but the standard deviation will decrease from five minutes (exponential case) to 4 minutes. Should the server undergo training?</td><td>BTL2</td><td>Understanding</td><td>CO5</td></tr>
<tr><td>2</td><td>In a car manufacturing plant the loading crane takes exactly 10 minutes to load a car into a wagon and again come back position to load another car.If the arrivals of the car is a Poisson stream at an average of one every 20 minutes. Calculate the following
(1) Average number of cars in the system
(2) Average number of cars in the queue
(3) The Average waiting time of cars in the system
(4) The Average waiting time of cars in the queue</td><td>BTL-4</td><td>Analyzing</td><td>CO5</td></tr>
<tr><td>3</td><td>An average of 120 students arrives each hour (inter arrival times are exponential) at the controller's office to get their hall tickets. To complete the process a candidate must pass through counters. Each counter consists of a single server, service times at each counter are exponential with the following mean times: counter1, 20 seconds; conuter2, 15 seconds and counter3, 12 seconds. On the average evaluate how many students will be present in the controller's office?</td><td>BTL-4</td><td>Analyzing</td><td>CO5</td></tr>
<tr><td>4</td><td>Consider two servers. An average of 8 customers per hour from outside at server1 and an average of 17 customers arrive at server2. Inter arrival times are exponential server1 can serve at an exponential rate of 20 customers per hour and server2 can serve at an exponential rate of 30 customers per hour. After completing service at station1, half the customers leave the system and half go to server2. After completing service at station 2 ¾ of the customers complete the server and ¼ return</td><td>BTL-4</td><td>Analyzing</td><td>CO5</td></tr>
</table>

	to server1.Find the expected number of customers at each server. Find the average time a customer spends in the system.			
5.	There are two salesman in a ration shop one in charge of billing and receiving payment and the other in charge of weighing and delivering the items. Due to limited availability of space, only one customer is allowed to enter the shop that too when the billing clerk is free. The customer who has finished his billing job has to wait until the delivery section becomes free. If customers arrive in accordance with a Poisson process at rate 1 and the service times of two clerks are independent and have exponential rates of 3 and 2. Find (a) The proportion of customers who enter the ration shop (b) The average number of customers in the shop The average amount of time that an entering customer spends in the shop.	BTL-3	Applying	CO5

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