Network Evolution

CS224W: Machine Learning with Graphs

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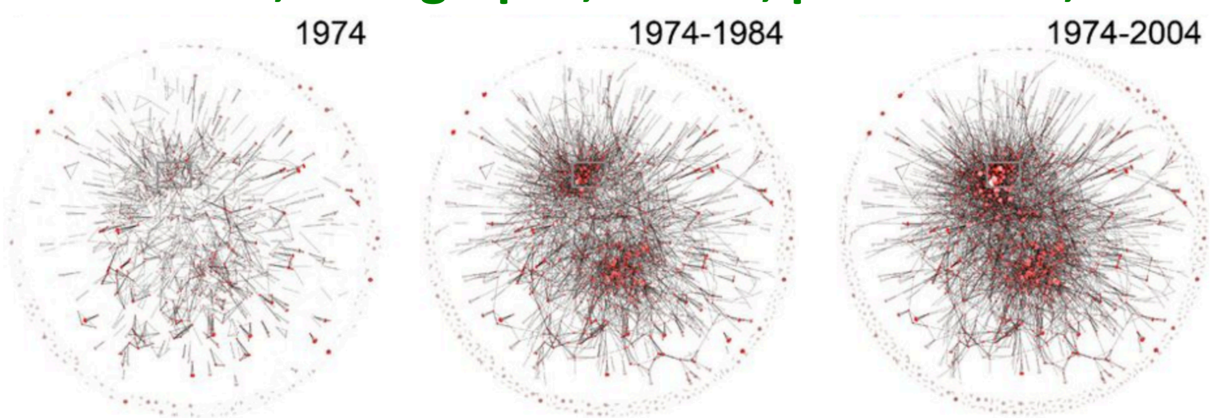
Evolving Networks

- **Evolving Networks** are networks that change as a function of time
- Almost all real world networks evolve either by **adding or removing nodes or links** over time
- **Examples**:
  - **Social networks**: people make and lose friends and join or leave the network
  - **Internet, web graphs, E-mail, phone calls, P2P networks, etc.**

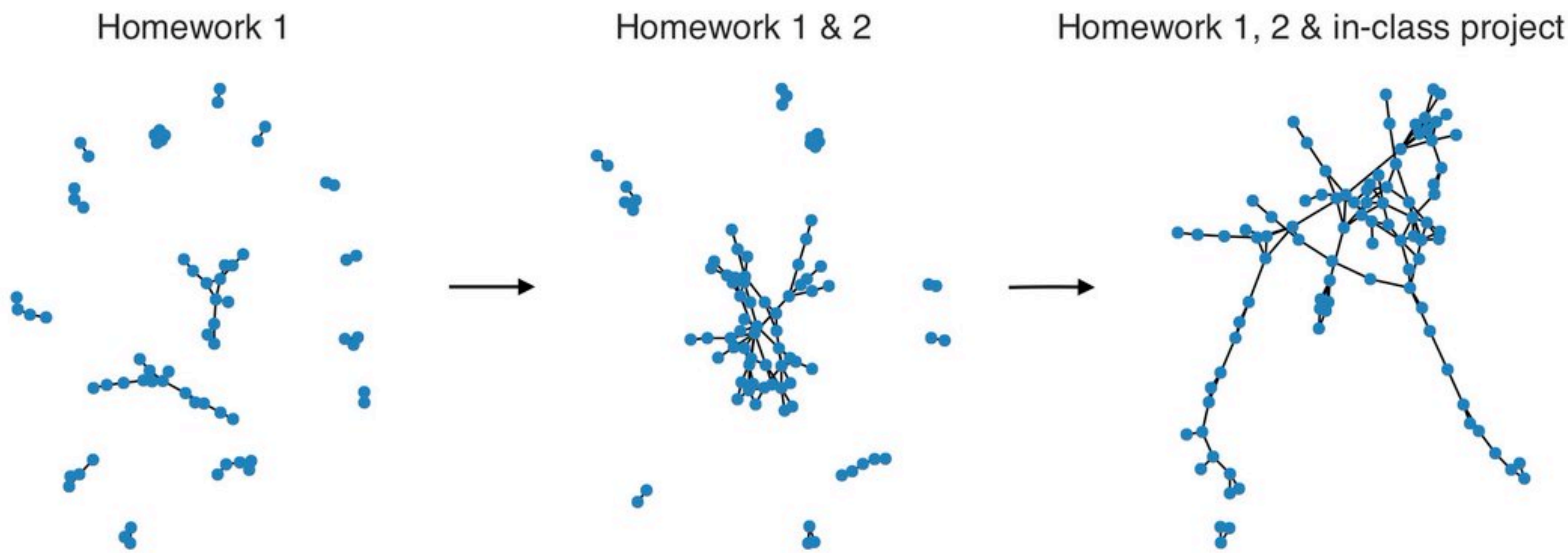


Collaborations in the journal Physical Review Letters (PRL)

[Perra et al. 2012]

Evolving Network Example

- Visualization of the student collaboration network

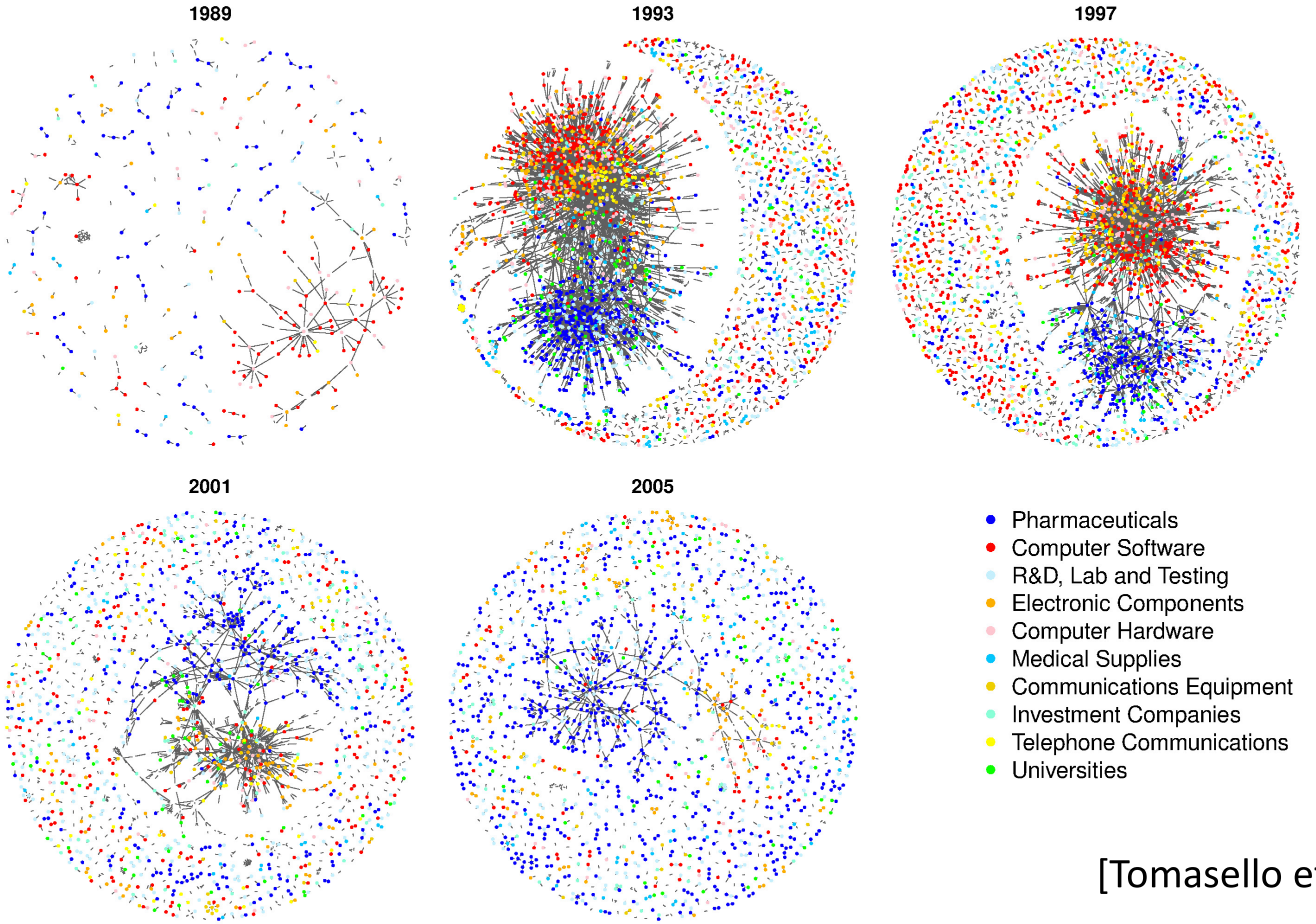


Nodes represent the students. An edge exists between two nodes if any of the two ever reported collaboration with the other in any of the assignments used to construct the network

[Burstein et al. 2018]

Evolving Network Example

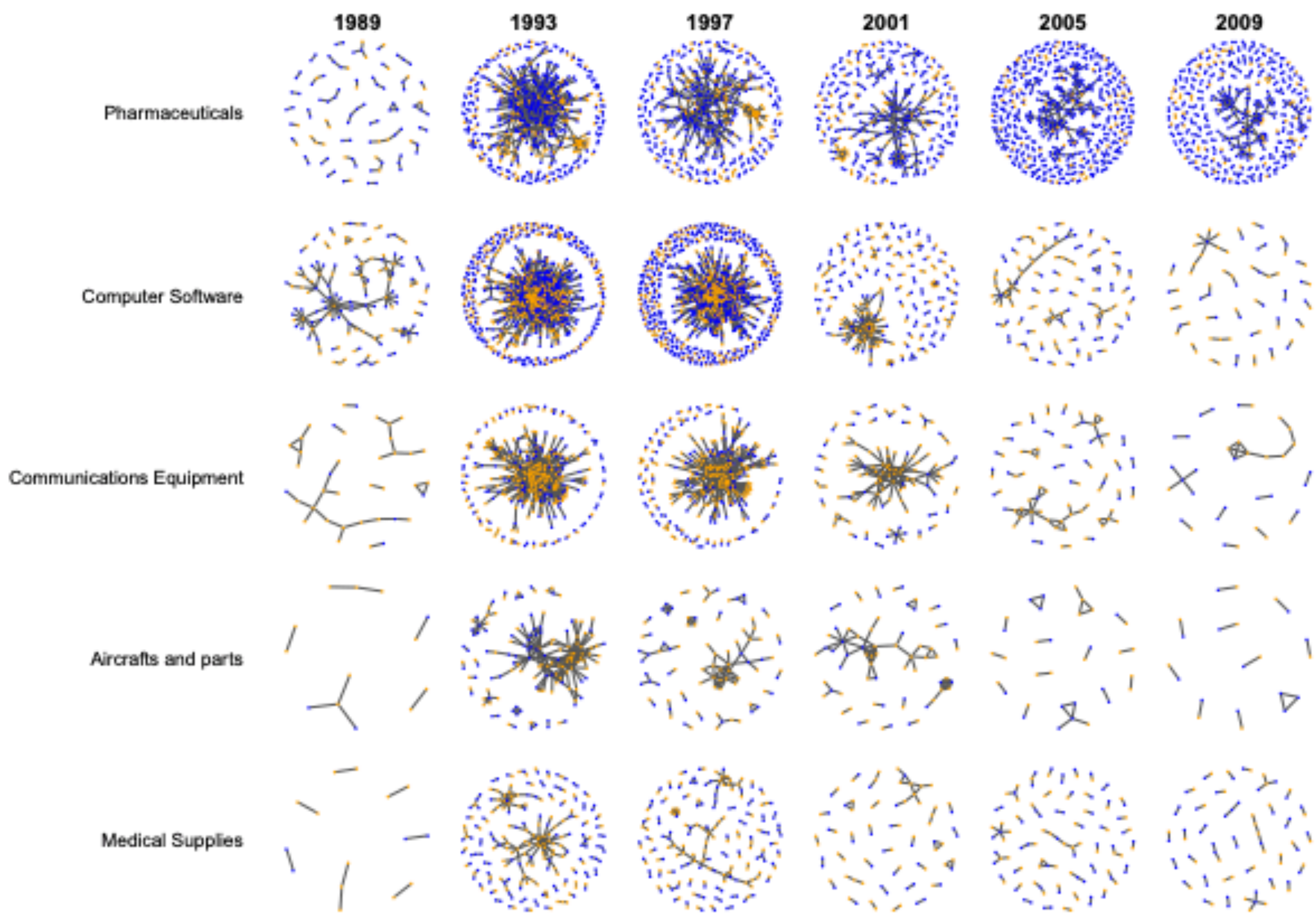
- Evolution of the pooled R&D network for the nodes belonging to the ten largest sectors



[Tomasello et al. 2017]

Evolving Network Example

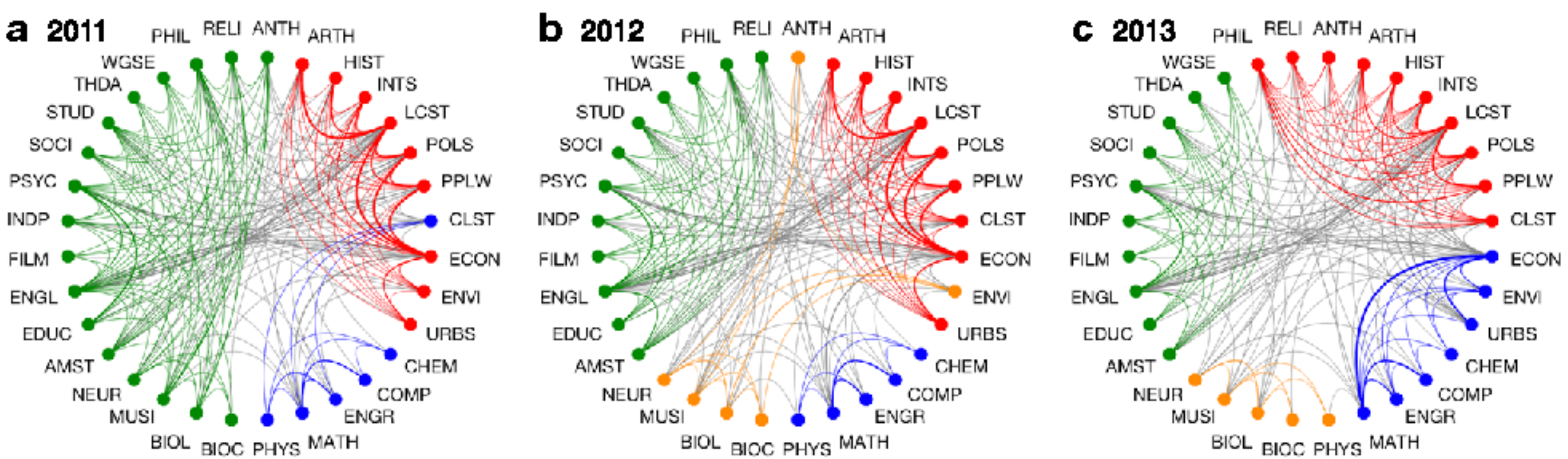
- Evolution of five selected sectoral R&D networks



Blue nodes represent the firms strictly belonging to the examined sector, while orange nodes represent their alliance partners belonging to different sectors [Tomasello et al. 2017]

Evolving Network Example

- Evolving network structure of academic institutions

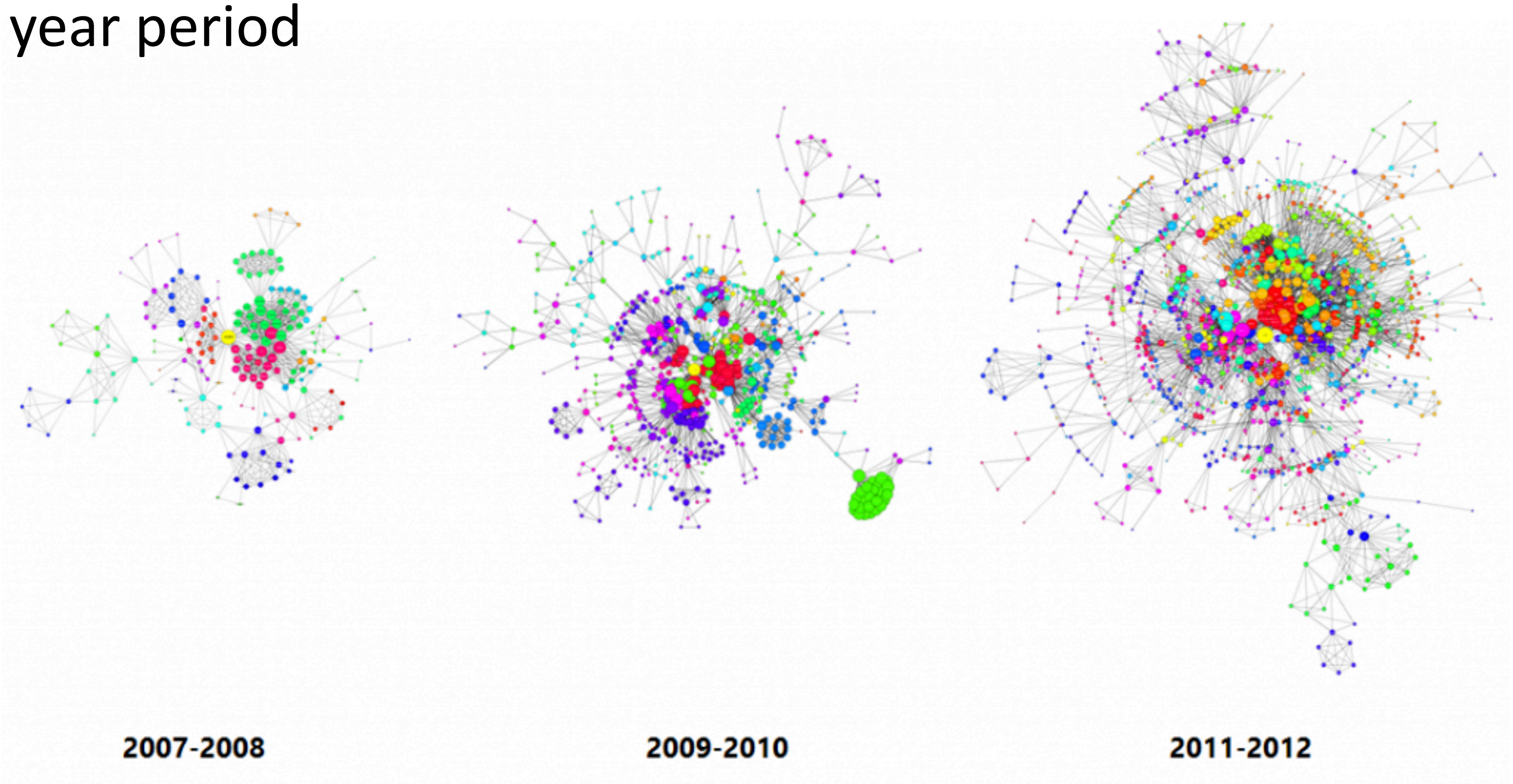


Community structure, indicated by color, for the networks from the three years 2011 to 2013. Different communities are indicated by different colors.

[Wang et al. 2017]

Evolving Network Example

- The largest components in Apple's inventor network over a 6-year period

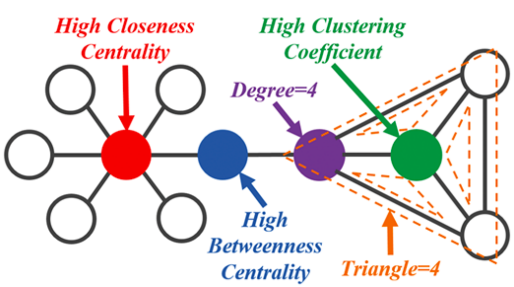


Each node reflects an inventor, each tie reflects a patent collaboration. Node colors reflect technology classes, while node sizes show the overall connectedness of an inventor by measuring their total number of ties/collaborations (the node's so-called *degree centrality*).

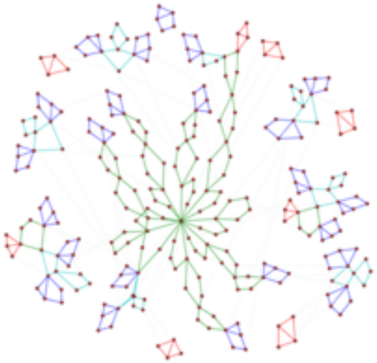
[kenedict.com]

Studying Evolving Networks

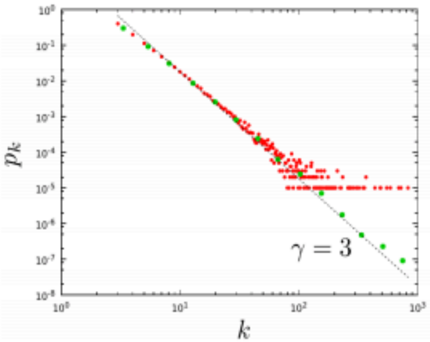
- **How do networks evolve?**
  - **How do networks evolve at the macro level?**
    - **Evolving network models, densification**
  - **How do networks evolve at the meso level?**
    - **Network motifs, communities**
  - **How do networks evolve at the micro level?**
    - **Node, link properties (degree, network centrality)**



Microscopic: Degree, centralities



Mesoscopic: Motifs, communities



Macroscopic: statistics

Macroscopic Evolution of Networks

Macroscopic Evolution

- **How do networks evolve at the macro level?**
  - What are global phenomena of network growth?
- **Questions:**
  - What is the relation between the number of nodes $n(t)$ and number of edges $e(t)$ over time t?
  - How does diameter change as the network grows?
  - How does degree distribution evolve as the network grows?

Network Evolution

- $N(t)$ ... nodes at time t
- $E(t)$ ... edges at time t
- Suppose that

$$N(t+1) = 2 \cdot N(t)$$

- **Q:** what is:

$$E(t+1) = ? \qquad \textbf{Is it } 2 \cdot E(t)?$$

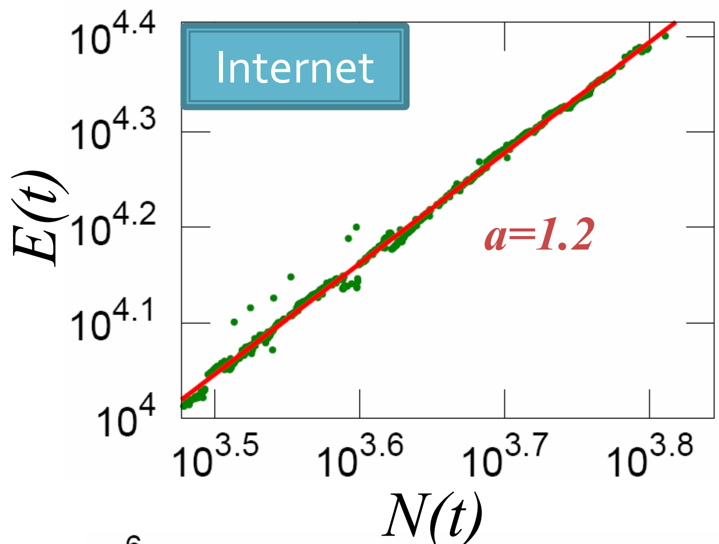
- **A: More than doubled!**
  - But obeying the **Densification Power Law**

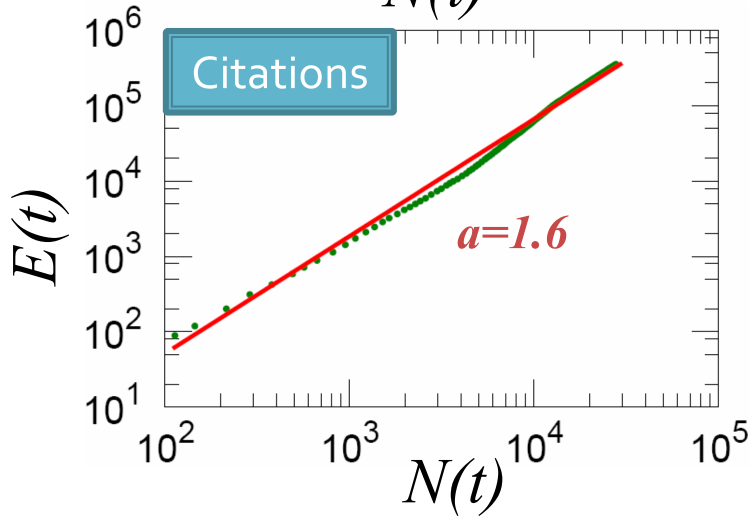
Q1) Network Evolution

- **What is the relation between the number of nodes and the edges over time?**
- ~~First guess: constant average degree over time~~
- Networks become **denser** over time
- **Densification Power Law:**

$$E(t) \propto N(t)^a$$

a … densification exponent (1 ≤ **a** ≤ 2)





Densification Power Law

- **Densification Power Law**
  - the number of edges grows faster than the number of nodes – **average degree is increasing**

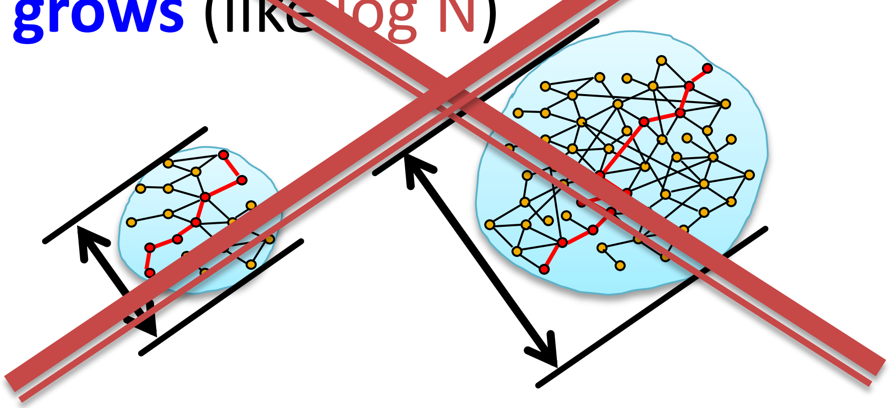
$$\boxed{E(t) \propto N(t)^a}$$ or equivalently $$\frac{\log(E(t))}{\log(N(t))} = const$$

a … densification exponent: 1 ≤ **a** ≤ 2:

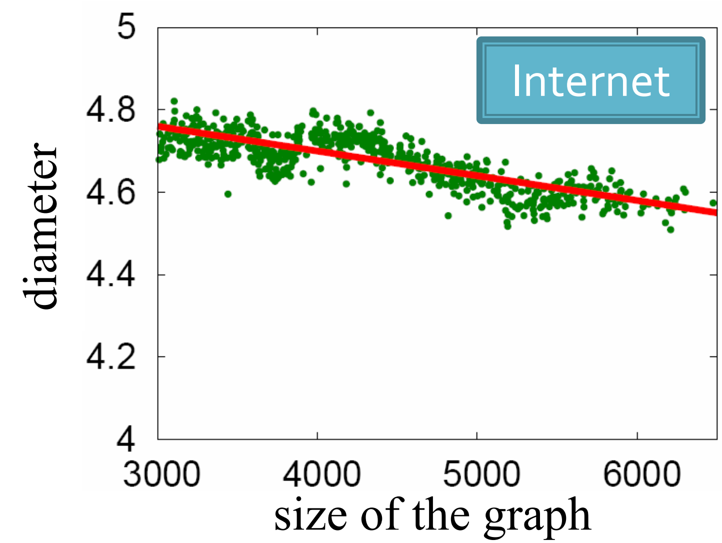
- **a=1: linear growth** – constant out-degree (traditionally assumed)
- **a=2: quadratic growth** – fully connected graph

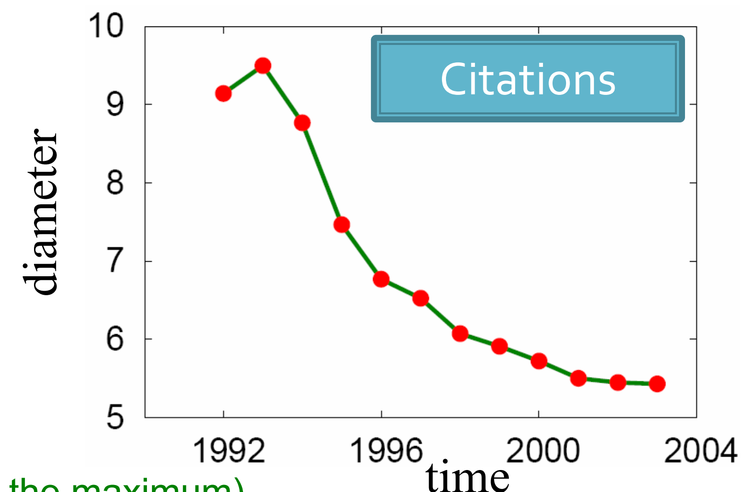
Q1) Network Evolution

- ~~Prior models and intuition say that the network **diameter slowly grows** (like log N)~~



- **Diameter shrinks over time**
  - As the <u>network grows</u> the distances between the nodes slowly **decrease**





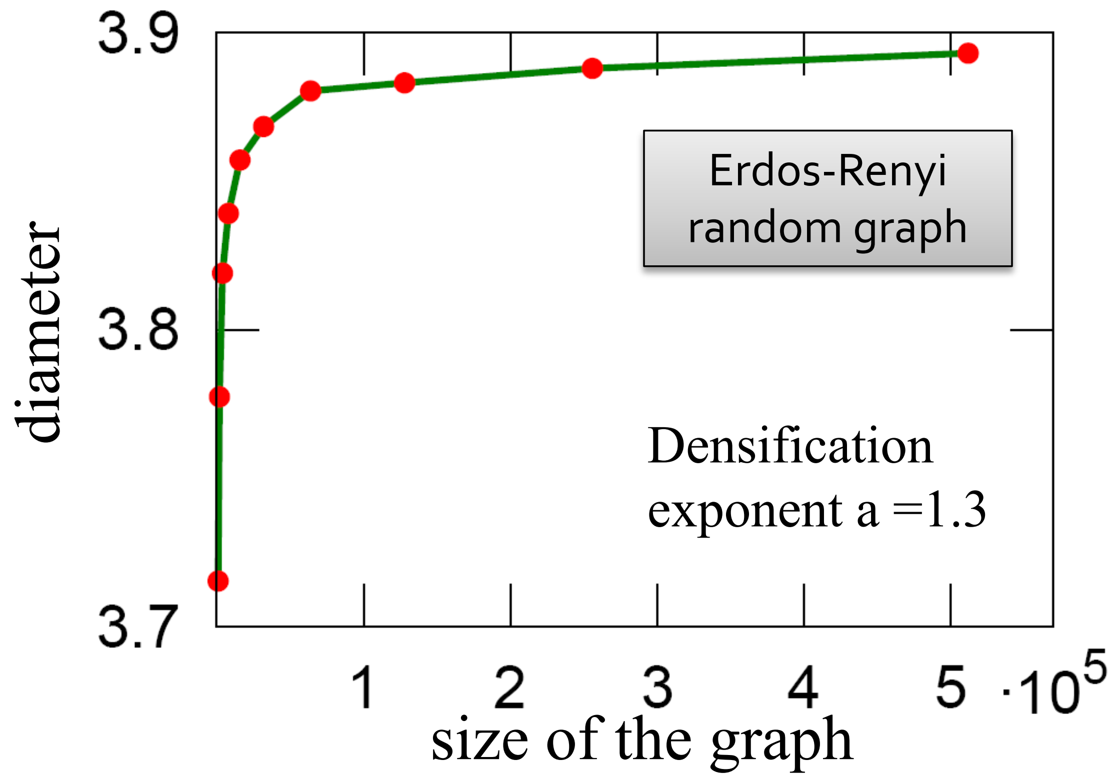
How do we compute diameter in practice?

-- Long paths: Take 90^{th}-percentile or average path length (not the maximum)

-- Disconnected components: Take only largest component or average only over connected pairs of nodes

Diameter of a Densifying G_{np}

Is shrinking diameter just a consequence of densification?
(answer by simulation)



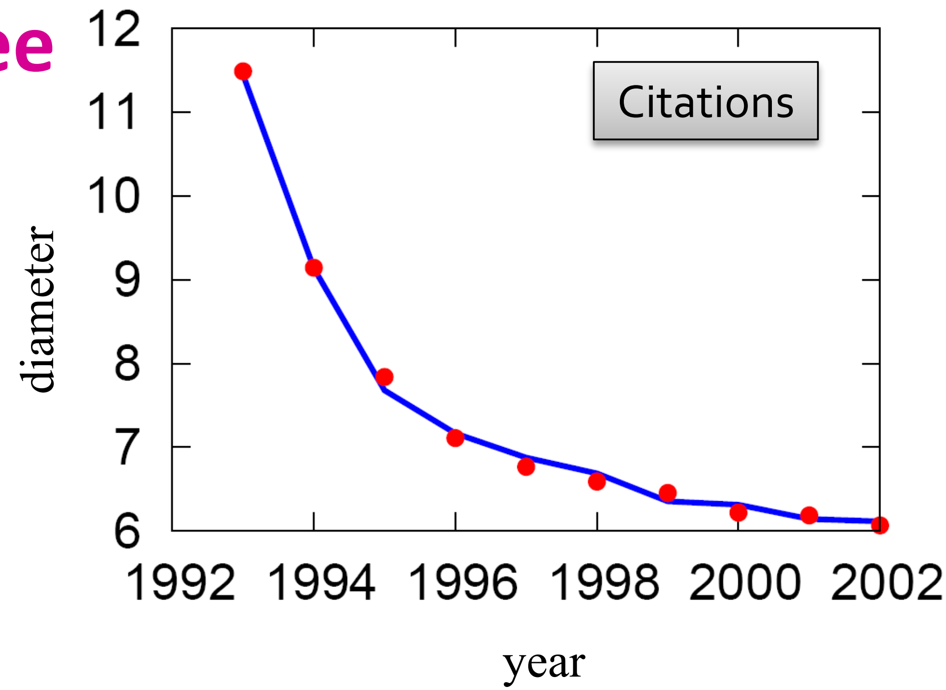
Densifying random graph has increasing diameter
$\Rightarrow$ There is more to shrinking diameter than just densification!

Diameter of a Rewired Network

Does the changing degree sequence explain the shrinking diameter?

Compare diameter of a:

- Real network (**red**)
- Random network with the same degree distribution (**blue**)

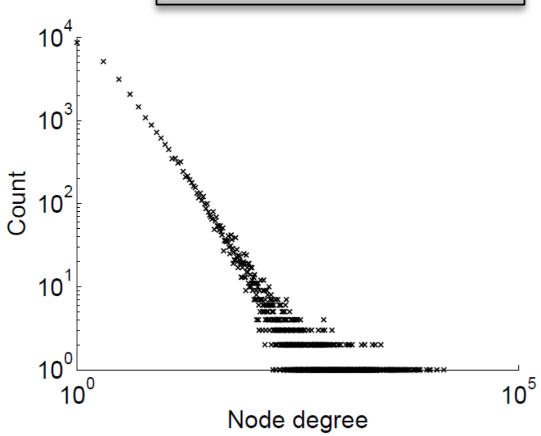


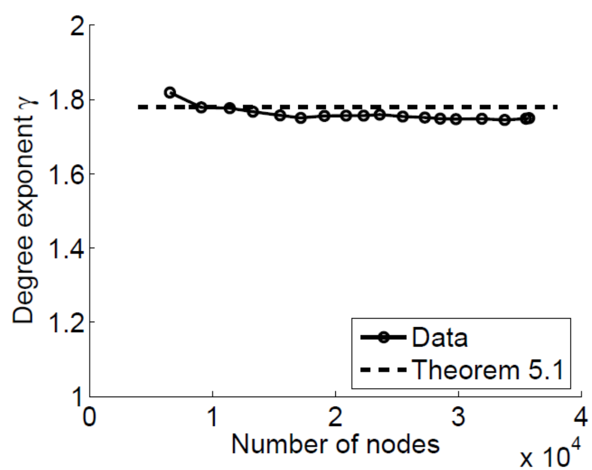
Densification + degree sequence gives shrinking diameter

Connecting Degrees & Densification

- **How does degree distribution evolve to allow for densification?**
- **Option 1)** **Degree exponent γ_t is constant:**
  - **Fact 1:** If $\gamma_t = \gamma \in [1, 2]$, then: $a = 2/\gamma$

Email network





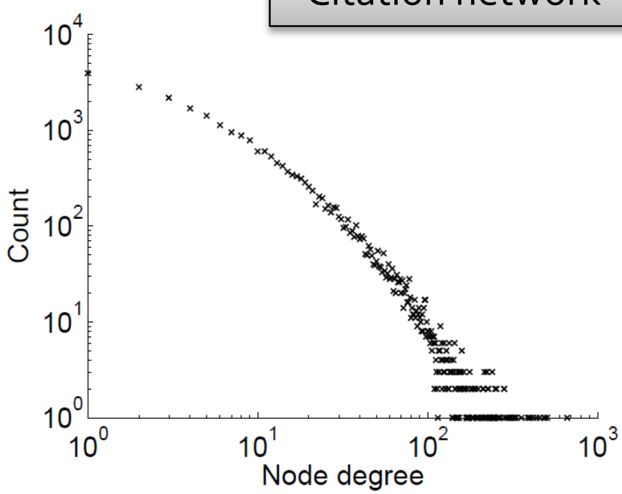
- Power-laws with exponents <2 have infinite expectations.
- So, by maintaining constant degree exponent α the average degree grows.

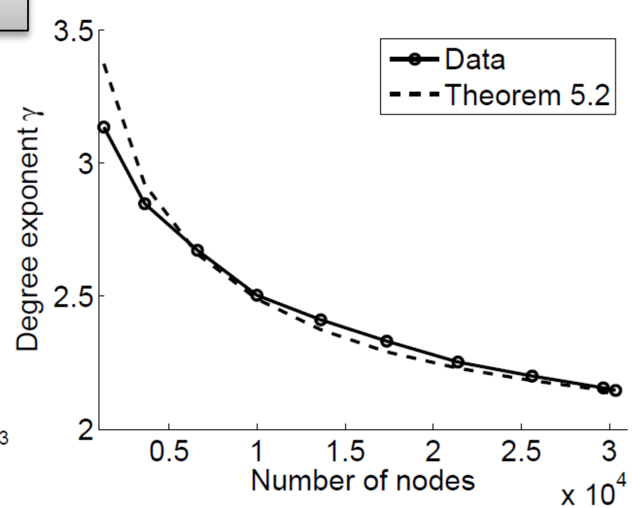
Connecting Degrees & Densification

- **How does degree distribution evolve to allow for densification?**
- **Option 2) γ_t evolves with graph size n:**
  - **Fact 2:** If $\gamma_t = \frac{4n_t^{x-1}-1}{2n_t^{x-1}-1}$, then: $a = x$

Notice: $\gamma_t \to 2$ as $n_t \to \infty$

Citation network





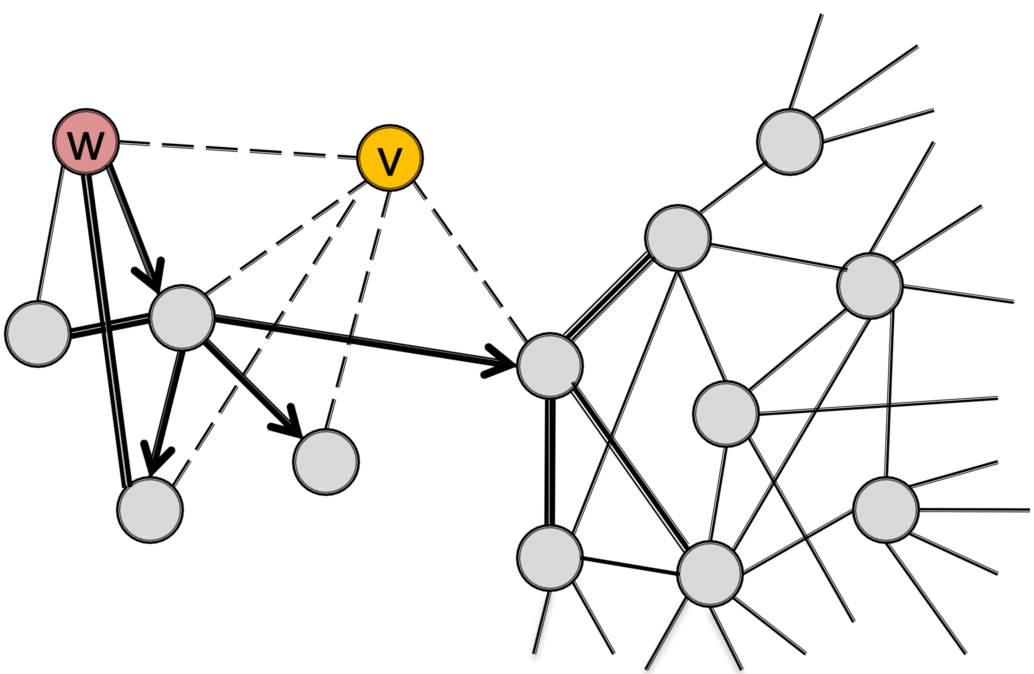
Remember, the expected degree in a power law is:

$$E[X] = \frac{\gamma_t - 1}{\gamma_t - 2} x_m$$

So γ_t has to decay as a function of graph size n_t for the avg. degree to go up.

Forest Fire Model

- **Want to model graphs that densify and have shrinking diameters**
- **Intuition:**
  - How do we meet friends at a party?
  - How do we identify references when writing papers?



Forest Fire Model

- **The Forest Fire model has 2 parameters:**
  - ***p*** … forward burning probability
  - ***r*** … backward burning probability
- **The model: Directed Graph**
  - Each turn a new node *v* arrives
  - Uniformly at random choose an "ambassador" node *w*
  - Flip 2 coins sampled from a geometric distribution (based on *p* and *r*) to determine the number of **in-** and **out-links** of *w* to follow, i.e., to "spread the fire" along
  - "Fire" spreads recursively until it dies
  - New node *v* links to all burned nodes

Geometric distribution:

$$\Pr(X = k) = (1 - p)^{k-1} p$$

Forest Fire Model

- **The Forest Fire model**
  - (1) v chooses an ambassador node w uniformly at random, and forms a link to w
  - (2) generate two random numbers x and y from geometric distributions with means $p/(1-p)$ and $rp/(1-rp)$
  - (3) v selects x out-links and y in-links of w incident to nodes that were not yet visited and form out-links to them
  - (4) v applies step (2) to the nodes found in step (3)

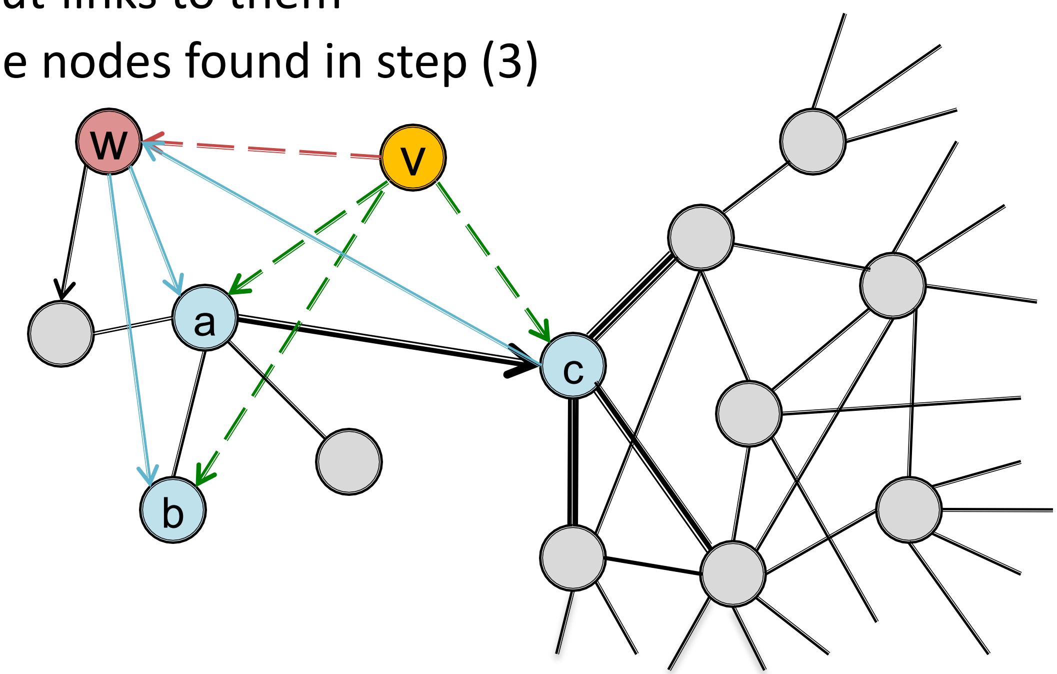
Example:

(1) Connect to a random node w

(2) Sample x=2, y=1

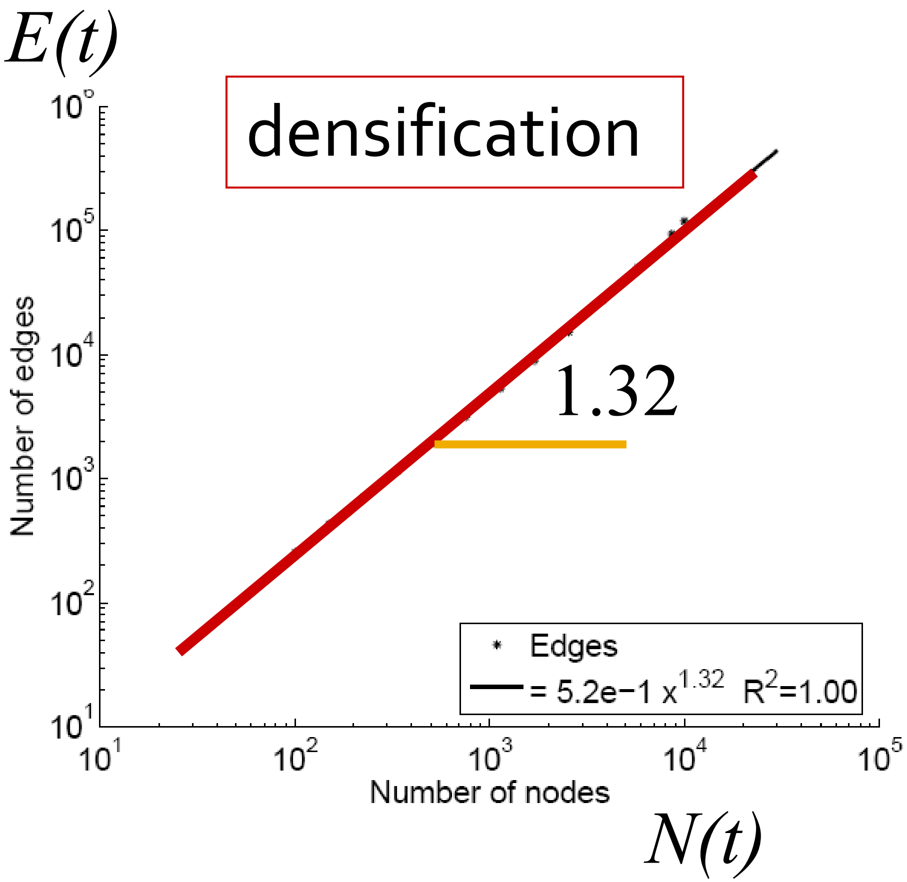
(3) Connect to 2 out- and 1 in-links of w, namely a, b, c

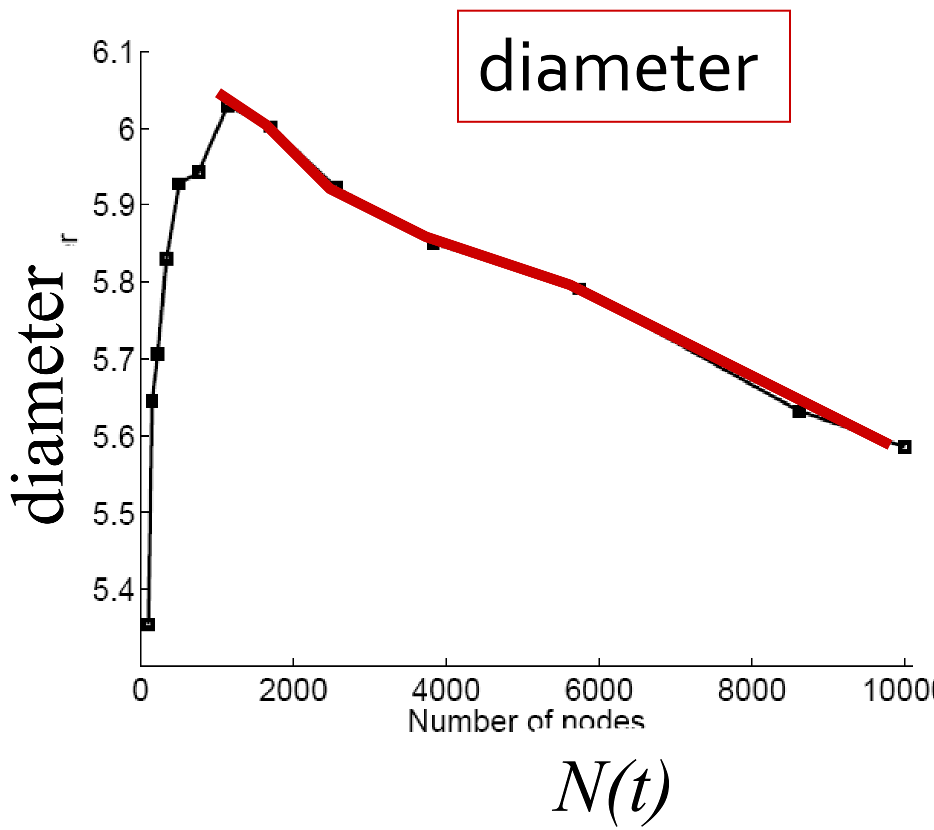
(4) Repeat the process for a, b, c



Forest Fire Model

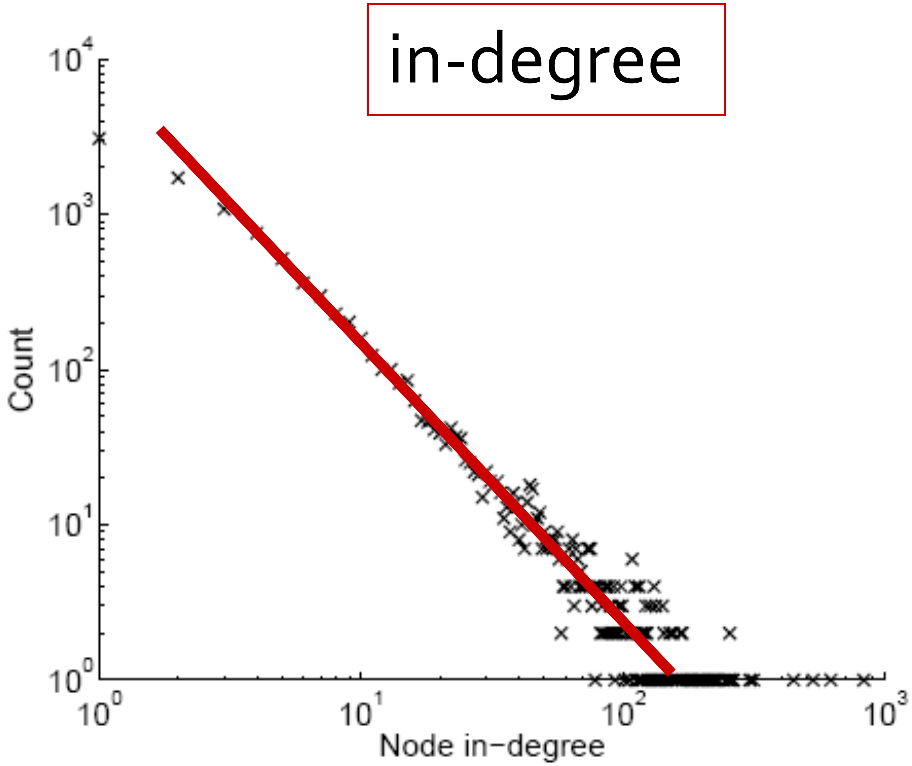
- **Forest Fire generates graphs that densify and have shrinking diameter**



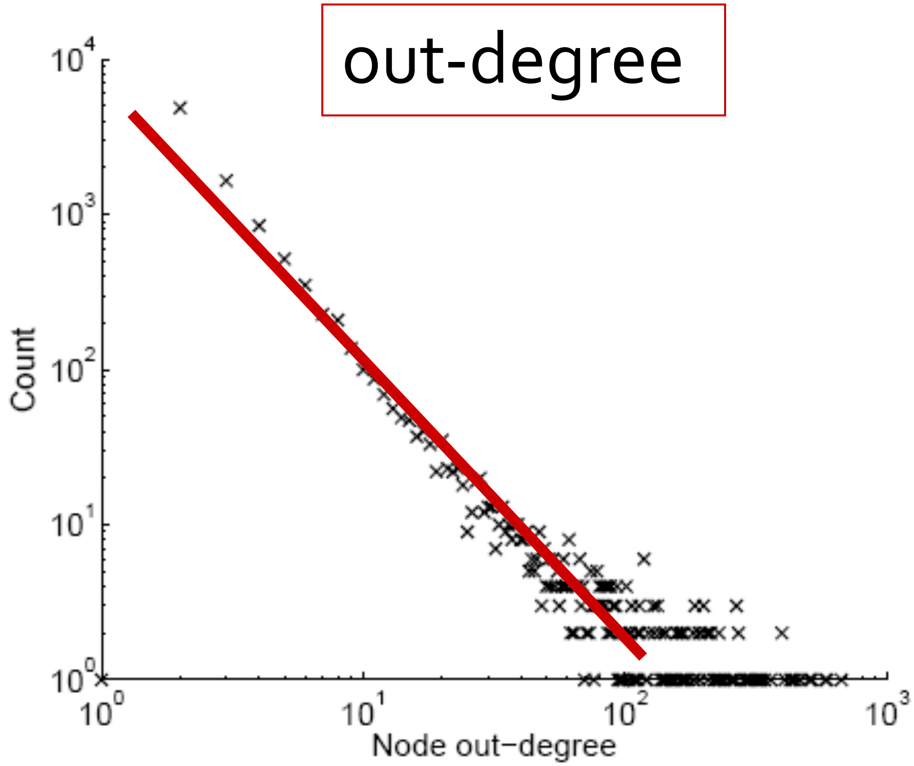


Forest Fire Model

- **Forest Fire also generates graphs with** **power-law degree distribution**



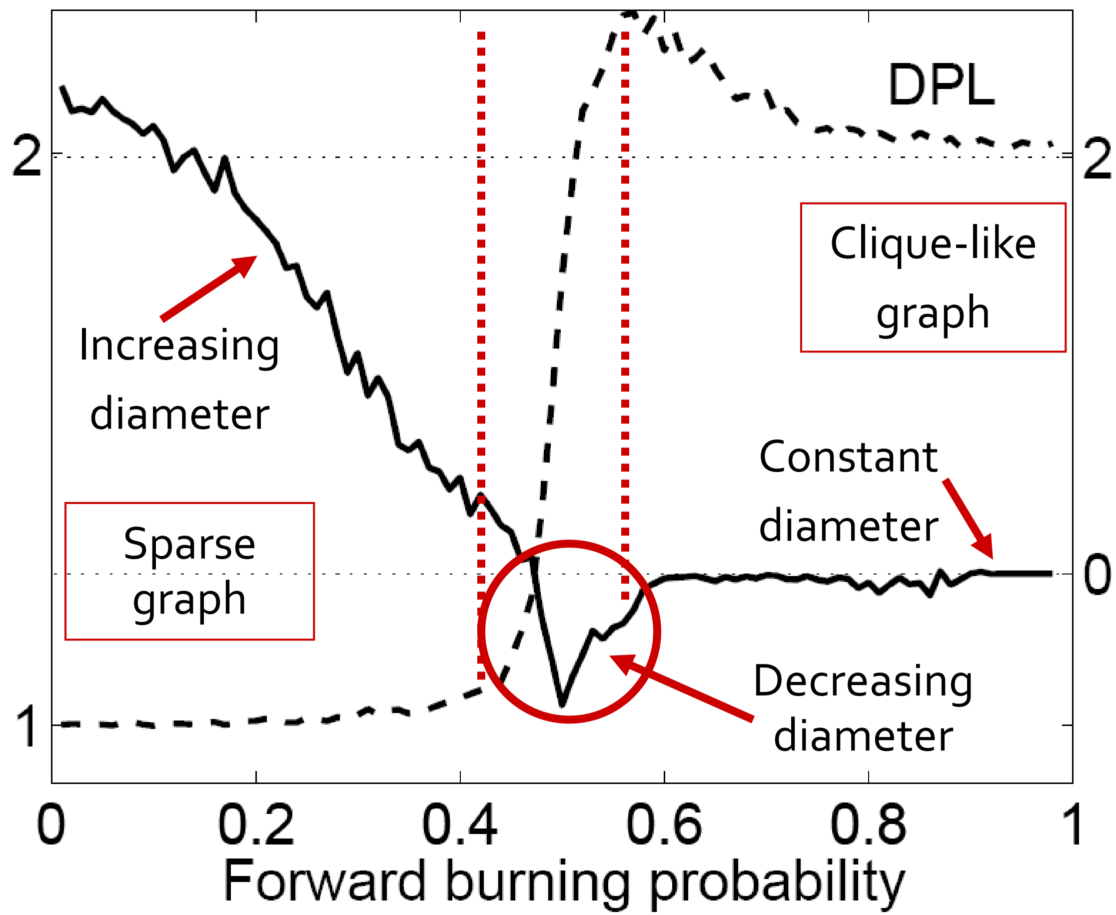
log count vs. log in-degree



log count vs. log out-degree

Forest Fire: Phase Transition

- Fix backward probability *r* and vary forward burning prob. *p*
- Notice a sharp transition between sparse and clique-like graphs
- **The "sweet spot" is very narrow**

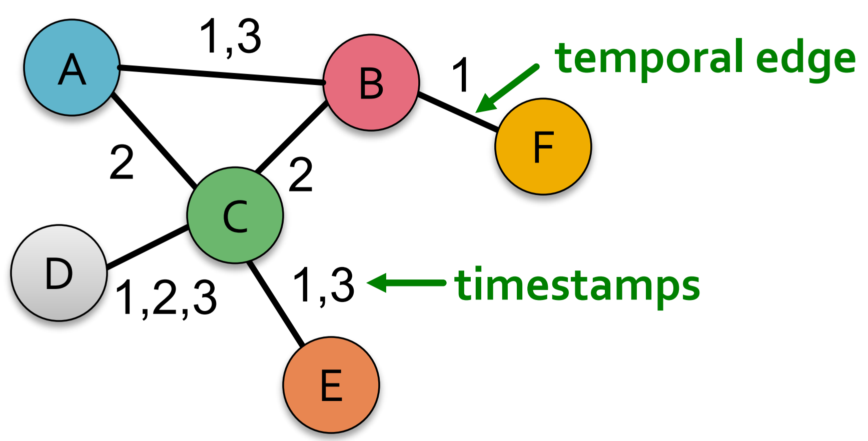


Temporal Networks

Temporal Networks

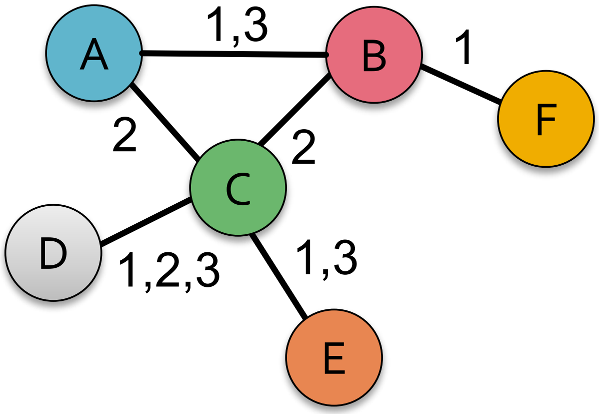
- **Temporal network**: A sequence of static directed graphs over the same (static) set of nodes V

- Each **temporal edge** is a timestamped ordered pair of nodes $(e_i = (u, v), t_i)$, where $u, v \in V$ and t_i is the timestamp at which the edge exists

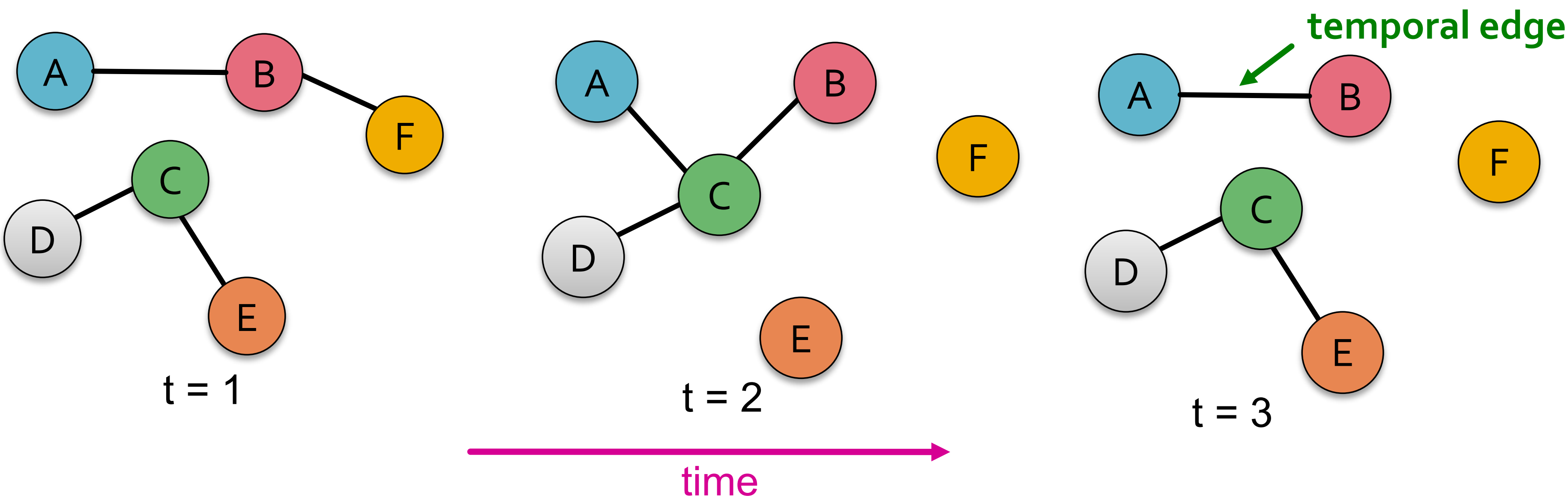


Temporal Networks

- Edges of a **temporal network** is are active only at certain points in time

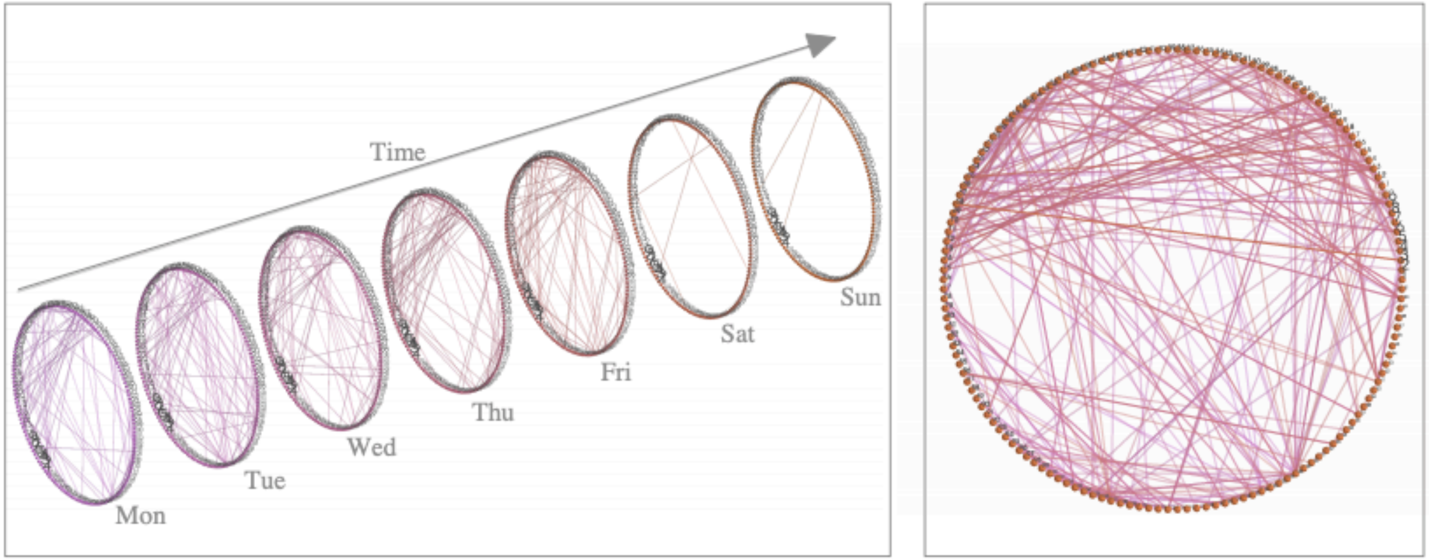


- A **temporal network** is a sequence of static directed graphs over the same (static) set of nodes V



Temporal Network Examples

- **Communication:** Email, phone call, face-to-face
- **Proximity networks:** Same hospital room, meet at conference, animals hanging out
- **Transportation:** train, flights…
- **Cell biology:** protein-protein, gene regulation



Email communication. A typical week of activity during Nov 2001 using 24-hour windows (left) and aggregated static graph (right). Nodes represent employees; a link between two employees exists if an email was sent by one of them to the other in that 24-hour window

[Tang et al, 2010]

Temporal Network Examples

- **Communication:** Email, phone call, face-to-face
- **Proximity networks:** Same hospital room, meet at conference, animals hanging out
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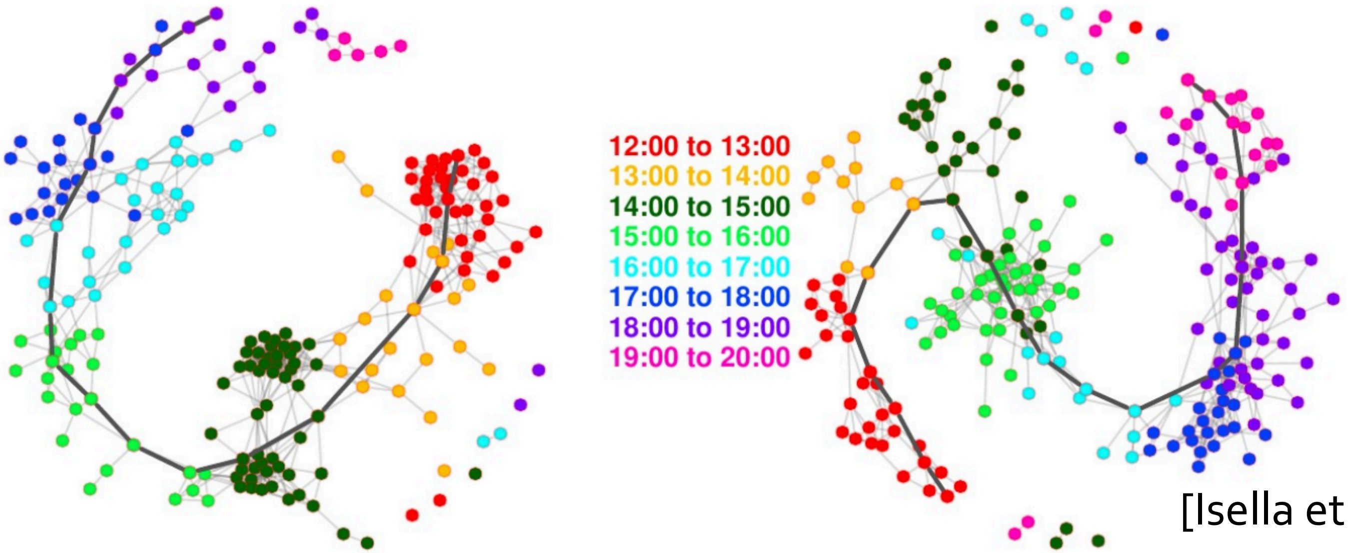


[Isella et al, 2011]

Aggregated networks for two different days of the Science Gallery museum deployment. Nodes are colored according to the corresponding visitor's entry time slot. The network diameter is highlighted in each case.

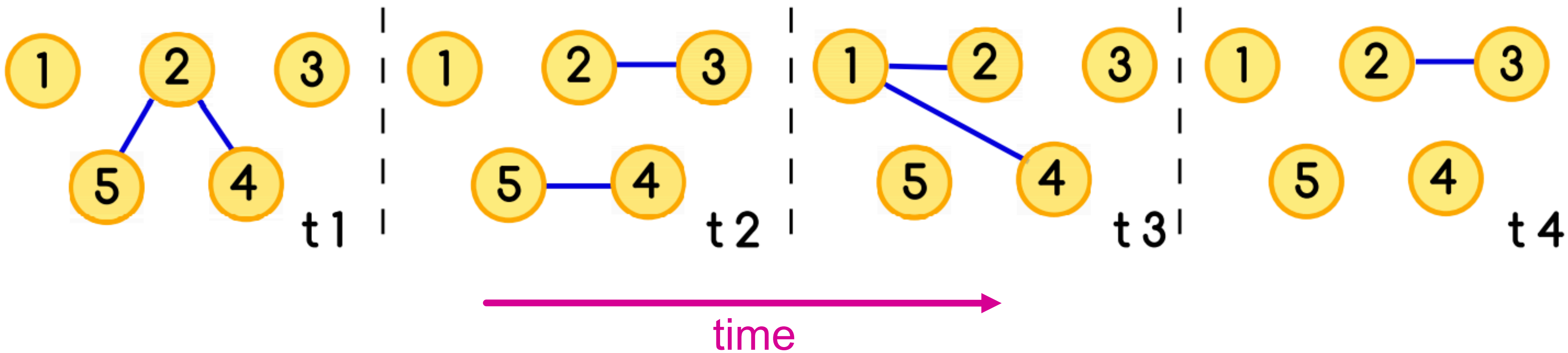
Microscopic Evolution of Networks

Microscopic Evolution

- **How do networks evolve at the micro level?**
  - What are local phenomena of network growth?
- **Questions:**
  - How do we define paths and walks in temporal networks?
  - How can we extend network centrality measures to temporal networks?

Temporal Path

- A **temporal path** is a sequence of edges (u_1, u_2, t_1), $(u_2, u_3, t_2), \dots, (u_j, u_{j+1}, t_j)$, for which $t_1 \leq t_2 \leq \cdots \leq t_j$ and each node is visited at most once



- **Example**:
  - The sequence of edges [(5,2),(2,1)] together with the sequence of times t_1, t_3 is a temporal path

How to Find Temporal Shortest Paths?

TPSP-Dijkstra algorithm: An adaptation of Dijkstra using a priority queue

Notation:

- n_s: source node
- n_t: target node
- t_q: time of the query (we calculate the distance from n_s to n_t between time t_s and time t_q)
- t_s: time that a node/edge joins the network
- t_e: time that a node/edge leaves the network
- w: edge weights
- $d[v]$: distance of n_s to v
- PQ: priority queue

Algorithm 2.1: TPSP-Dijkstra(TEG, n_s, n_t, t_q)

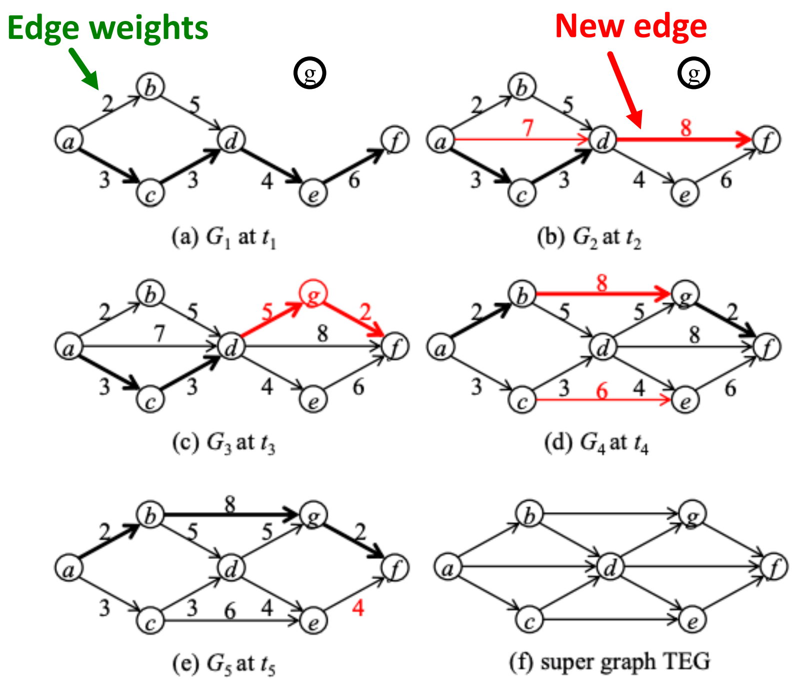
Input: Temporal evolving graph $TEG = (V, E, w, t_s, t_e)$, $n_s, n_t \in sub\text{-}V(t_q)$, and query time t_q

Output: Distance of the shortest path $p \subseteq$ sub-TEG(TEG, t_q)

$PQ \leftarrow \emptyset$
for all $v \in V \wedge t_s(v) \le t_q \wedge t_q < t_e(v)$ **do**
  $d[v] \leftarrow \infty$ ▷ Set distance to ∞ for all nodes
end for
$d[n_s] \leftarrow 0$ ▷ Set distance to 0 for n_s
$PQ.Insert(n_s, d[n_s])$ ▷ Insert (nodes, distances) to PQ
while $!PQ.empty()$ **do**
  $u \leftarrow PQ.ExtractMin()$ ▷ Extract the closest node from PQ
  if $u = n_t$ **then**
    return $d[n_t]$
  end if
  for all $e = (u, v) \in E$ **do** ▷ Verify if edge e is valid at t_q
    if $t_s(e) \le t_q \wedge t_q < t_e(e) \wedge d[u] + w(e) < d[v]$ **then**
      $d[v] \leftarrow d[u] + w(e)$ ▷ If so, update v's distance from n_s
      if $v \notin PQ$ **then**
        $PQ.Insert(v, d[v])$ ▷ insert $(v, d[v])$ to PQ
      else
        $PQ.DecreaseKey(v, d[v])$ ▷ or update $d[v]$ in PQ
      end if
    end if
  end for
end while
return ∞

How to Find Temporal Shortest Paths?

TPSP-Dijkstra algorithm: An adaptation of Dijkstra using a priority queue



Example of a temporally evolving graph.
Shortest path from *a* to *f* are marked in thick lines.

Algorithm 2.1: TPSP-Dijkstra(TEG, n_s, n_t, t_q)

Input: Temporal evolving graph $TEG = (V, E, w, t_s, t_e)$, $n_s, n_t \in sub\text{-}V(t_q)$, and query time t_q

Output: Distance of the shortest path $p \subseteq$ sub-TEG(TEG, t_q)

$PQ \leftarrow \emptyset$
for all $v \in V \wedge t_s(v) \leq t_q \wedge t_q < t_e(v)$ **do**
  $d[v] \leftarrow \infty$
end for
$d[n_s] \leftarrow 0$
$PQ.Insert(n_s, d[n_s])$
while !$PQ.empty()$ **do**
  $u \leftarrow PQ.ExtractMin()$
  if $u = n_t$ **then**
    return $d[n_t]$
  end if
  for all $e = (u, v) \in E$ **do**
    if $t_s(e) \leq t_q \wedge t_q < t_e(e) \wedge d[u] + w(e) < d[v]$ **then**
      $d[v] \leftarrow d[u] + w(e)$
      if $v \notin PQ$ **then**
        $PQ.Insert(v, d[v])$
      else
        $PQ.DecreaseKey(v, d[v])$
      end if
    end if
  end for
end while
return ∞

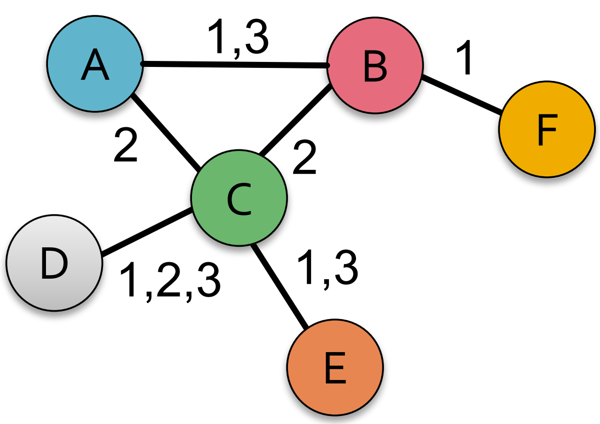
Temporal Centrality

- **Temporal Closeness**: Measure of how close a node is to any other node in the network at time interval $[0, t]$
  - Sum of **shortest (fastest) temporal** path lengths to all other nodes

$$c_{\text{clos}}(x,t) = \frac{1}{\sum_y \boxed{d(y,x|t)}}$$

length of the temporal shortest path from y to x from time 0 to time t

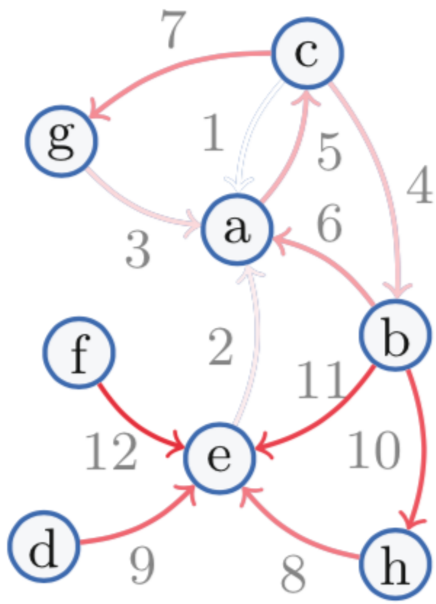
- **Example:**



$$c_{\text{clos}}(A,2) = \frac{1}{1+1+2+2+2} = 0.1$$

$d(A, C|t = 2)$

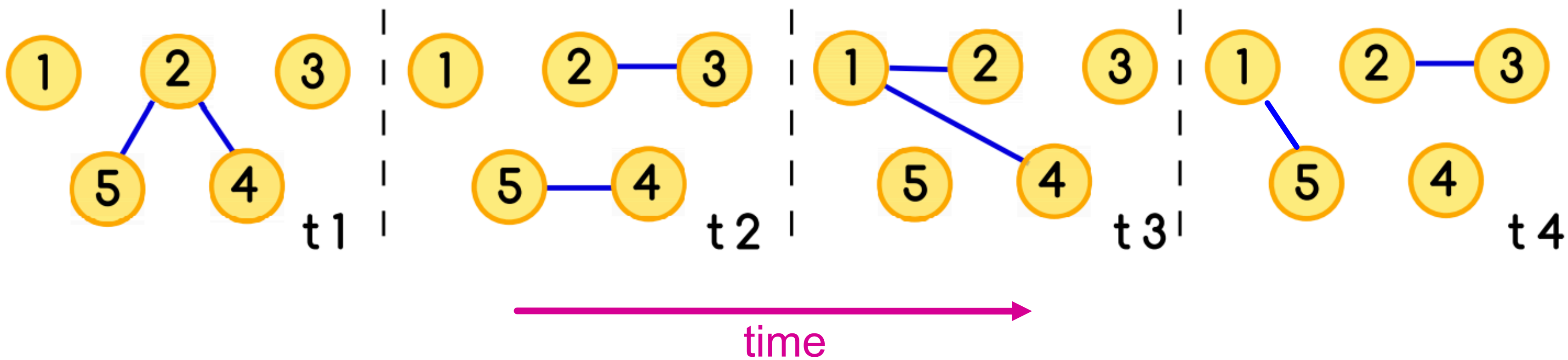
Temporal PageRank



- **Intuition:**
  - Node a initially receives many in-links and it should be considered important
  - After time $t = 8$, it does not receive any more in-links and thus its importance should diminish

Temporal Walk

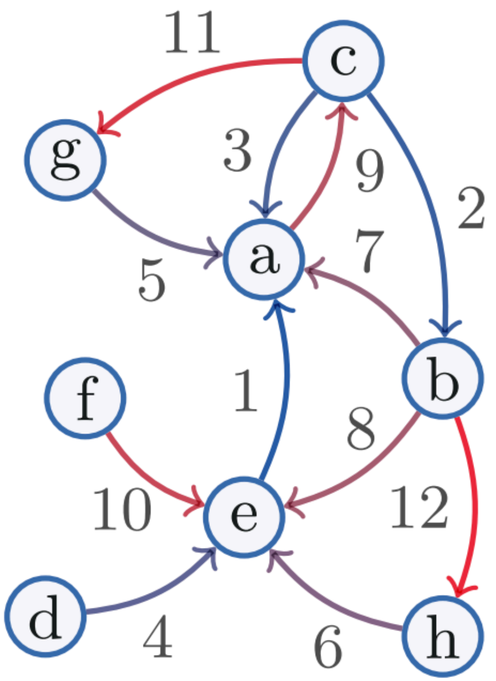
- A **temporal or time-respecting walk** is a sequence of edges $(u_1, u_2, t_1), (u_2, u_3, t_2), \ldots, (u_j, u_{j+1}, t_j)$, for which $t_1 \le t_2 \le \cdots \le t_j$



- **Example:**
  - The sequence of edges [(5,2),(2,1),(1,5)] together with the sequence of times t_1, t_3, t_4 is a temporal walk

Temporal PageRank: Idea

- **Idea**: Make a random walk only on temporal or time-respecting paths
  - Time-stamps increase along the path



- **c → b → a → c**: time respecting
- **a → c → b → a**: not time respecting

Temporal PageRank & Paths

How can we calculate the probability of a temporal path?

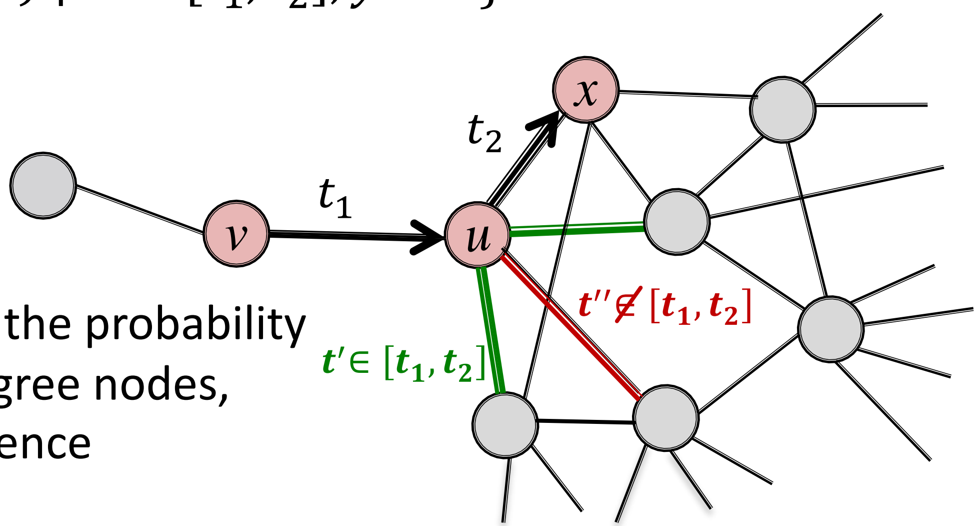
- The probability $P[(u,x,t_2)|(v,u,t_1)]$ of taking (u,x,t_2) given that we arrived via (u,v,t_1) decreases as the time difference (t_2-t_1) increases
- This can be modeled by an exponential distribution

$$P[(u,x,t_2)|(v,u,t_1)] = \beta^{|\Gamma_u|}$$

transition probability ($\boldsymbol{\beta} \in (\mathbf{0},\mathbf{1}]$)

  - Γ_u is the set of all temporal edges from u during the time $t' \in [t_1, t_2]$

$$\Gamma_u = \{(u,y,t') \mid t' \in [t_1,t_2], y \in V\}$$



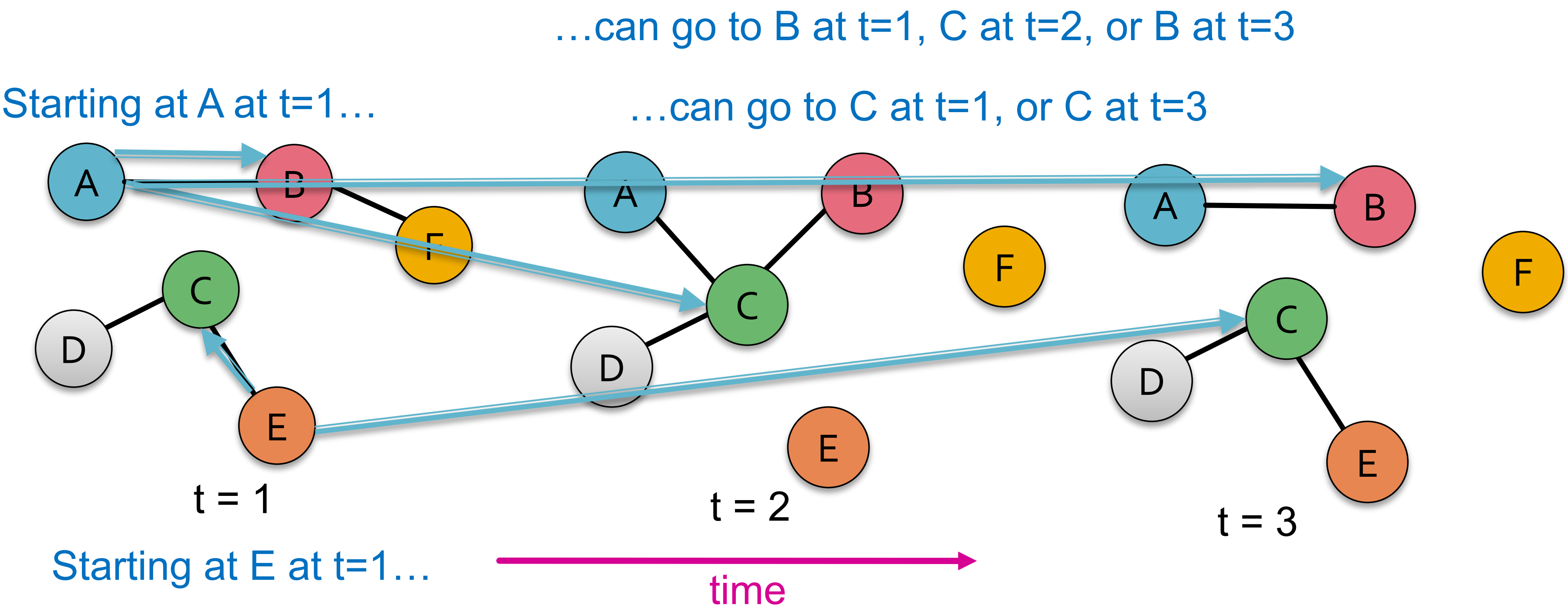
- Smaller values of β increase the probability of walks stopping at high degree nodes, but we get a slower convergence

Temporal PageRank: Intuition

- As $t \rightarrow \infty$, the temporal PageRank converges to the static PageRank: why?
- **Temporal PageRank is running regular PageRank on a time-augmented graph**:
  - Connect graphs at different time steps via time hops, and run PageRank on this time-extended graph
    - Node u at t_1 becomes a node (u, t_1) in this new graph
  - Transition probabilities given by $P[((u,t_1),(x,t_2))|((v,t_0),(u,t_1))] = \beta^{|\Gamma_u|}$
- As $t \rightarrow \infty$, $\beta^{|\Gamma_u|}$ becomes the uniform distribution
  - Graph looks as if we superimposed the original graphs from each time step → back to regular PageRank

Temporal PageRank: Example

Constructing the time-augmented graph:



Repeat for all nodes across all time steps!

Temporal PageRank: Definition

- **Temporal PageRank**:

$$r(u,t) = \sum_{v \in V} \sum_{k=0}^{t} (1-\alpha)\alpha^k \sum_{\substack{z \in Z(v,u|t) \\ |z|=k}} P[z|t]$$

  - $Z(v,u|t)$ is a set of all possible temporal walks from v to u **until time t**
  - α is the probability of starting a new walk

- As $t \to \infty$, the temporal PageRank converges to the static PageRank

- **Temporal Personalized PageRank**:

$$r(u,t) = \sum_{v \in V} \sum_{k=0}^{t} (1-\alpha)\alpha^k \frac{h^*(v)}{h'(v)} \sum_{\substack{z \in Z(v,u|t) \\ |z|=k}} P[z|t]$$

  - h^*: personalization vector
  - h': walk starting probability vector, $h'(u) = \frac{|(u,v,t) \in E: \ \forall v \in V|}{|E|}$

Temporal PageRank: Definition

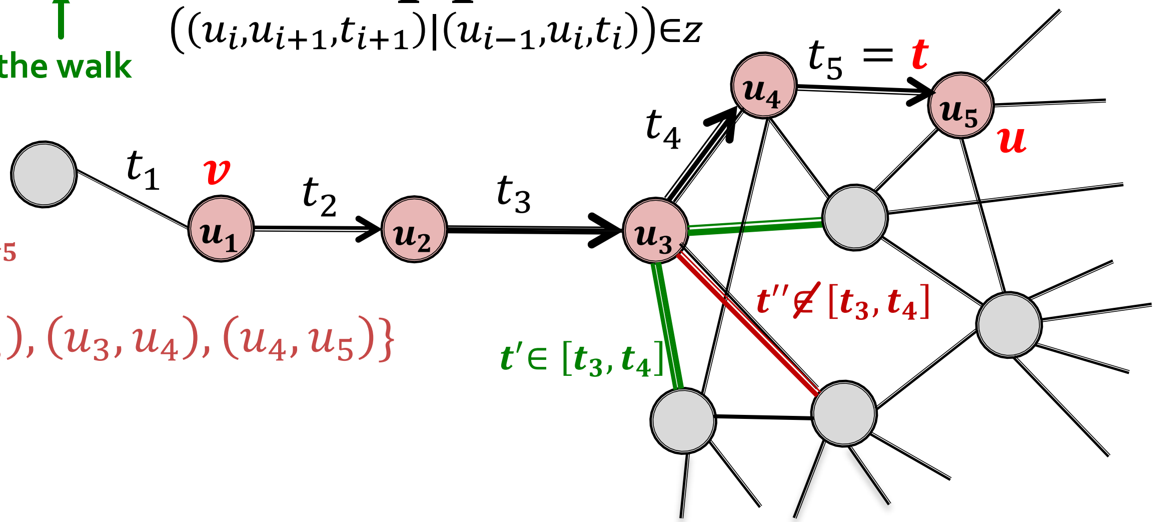
- **The weighted number of temporal walks z is then defined as**

$$c(z|t) = (1-\beta) \prod_{((u_i,u_{i+1},t_{i+1})|(u_{i-1},u_i,t_i))\in z} \beta^{|\Gamma_{u_i}|}$$

Probability of stopping the walk

Temporal walk from $v = u_1$ to $u = u_5$

$z = \{(u_1,u_2),(u_2,u_3),(u_3,u_4),(u_4,u_5)\}$



- $Z(u, v|t)$ is a set of all possible temporal walks from v to u **until time t**
- $P[z|t]$ is the probability of a particular walk $z \in Z(v, u|t)$

$$P[z \in Z(v,u|t)] = \frac{c(z|t)}{\boxed{\sum_{\substack{z' \in Z(v,x|t) \\ x \in V, |z'|=|z|}} c(z'|t)}}$$

number of all temporal walks that start at node v and have the same length as z

Temporal PageRank: how to compute?

- $r(u)$: Temporal PageRank estimate of u
- $s(u)$: Count of active walks visiting u

Algorithm 1. Temporal PageRank

input : E, transition probability $\beta \in (0, 1]$, probability of initiating a new walk α

$\boldsymbol{r} = \boldsymbol{0}$, $\boldsymbol{s} = \boldsymbol{0}$;

foreach $(u, v, t) \in E$ **do**

  $\boldsymbol{r}(u) = \boldsymbol{r}(u) + (1 - \alpha)$;

  $\boldsymbol{s}(u) = \boldsymbol{s}(u) + (1 - \alpha)$; ⊳ Initiating a new walk with probability 1 – α

  $\boldsymbol{r}(v) = \boldsymbol{r}(v) + \boldsymbol{s}(u)\alpha$; ⊳ With probability α we continue active walks that wait in u

  if $\beta \in (0, 1)$ **then**

    $\boldsymbol{s}(v) = \boldsymbol{s}(v) + \boldsymbol{s}(u)(1 - \beta)\alpha$; ⊳ Increment the active walks (active mass) count in the node v with appropriate normalization 1−β

    $\boldsymbol{s}(u) = \boldsymbol{s}(u)\beta$; ⊳ decrements the active mass count in node u

  else if $\beta = 1$ **then**

    $\boldsymbol{s}(v) = \boldsymbol{s}(v) + \boldsymbol{s}(u)\alpha$; ⊳ Increments the active walks (active mass) count in the node v with appropriate normalization 1−β

    $\boldsymbol{s}(u) = 0$; ⊳ decrements the active mass count in node u

normalize $\boldsymbol{r}$;

return $\boldsymbol{r}$;

Case Study — Temporal PageRank

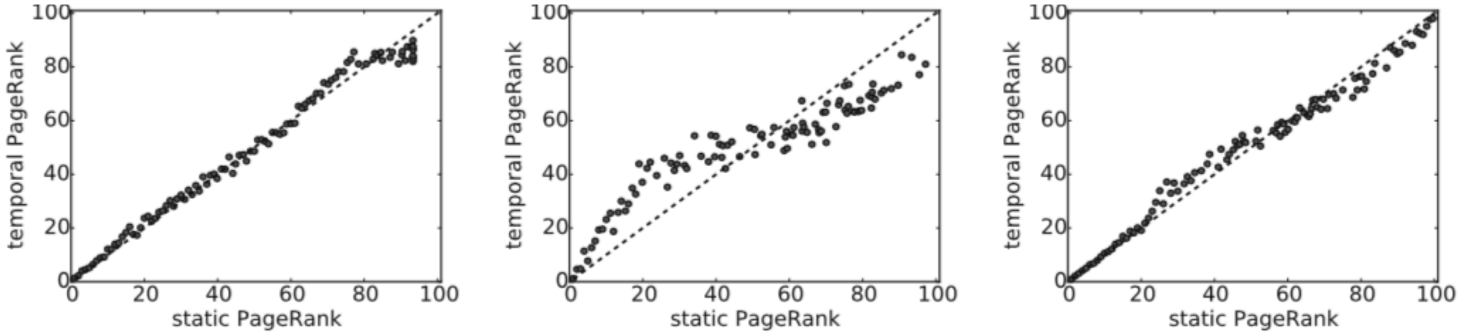
- **Datasets**:
  - **Facebook**: A 3-month subset of Facebook activity in a New Orleans regional community. The dataset contains an anonymized list of wall posts (interactions)
  - **Twitter**: Users' activity in Helsinki during 08.2010–10.2010. As interactions we consider tweets that contain mentions of other users
  - **Students**: An activity log of a student online community at the University of California, Irvine. Nodes represent students and edges represent messages

Case Study — Temporal PageRank

- **Experimental setup:**
  - For each network, static subgraph of n = 100 nodes is obtained by BFS from a random node
  - Edge weights are equal to the frequency of corresponding interactions and are normalized to sum to 1
  - Then a sequence of 100K temporal edges are sampled, such that each edge is sampled with probability proportional to its weight
  - In this setting, temporal PageRank is expected to converge to the static PageRank of a corresponding graph
  - Probability of starting a new walk is set to α = 0.85, and transition probability β for temporal PageRank is set to 0 unless specified otherwise.

Case Study — Temporal PageRank

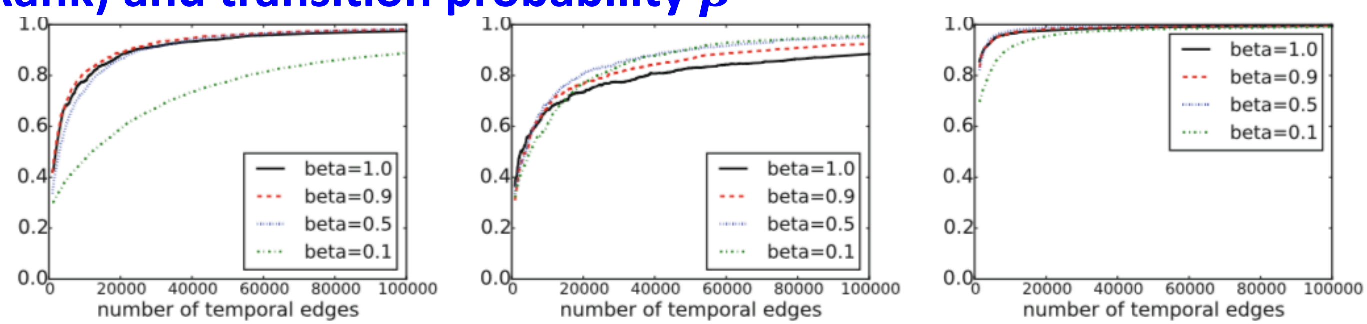
- **Comparison of temporal PageRank ranking with static PageRank ranking**



(a) *Facebook* (b) *Twitter* (c) *Students*

Rank correlation between static and temporal PageRank is high for top-ranked nodes and decreases towards the tail of ranking

- **Rank quality (Pearson corr. coeff. Between static and temporal PageRank) and transition probability β**

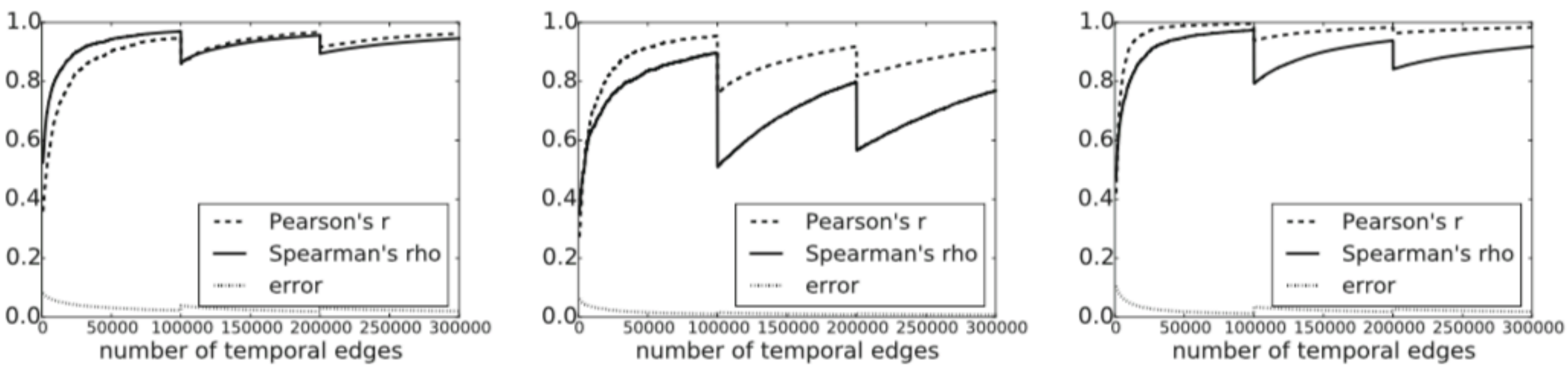


(a) *Facebook* (b) *Twitter* (c) *Students*

Smaller β corresponds to slower convergence rate, but better correlated rankings

Case Study — Temporal PageRank

- **Adaptation to concept drift (β=0.5)**
  - We start with a temporal network sampled from some static network
  - After sampling 10K temporal edges E_1, we change the weights of the static graph and sample another 10K temporal edges E_2
  - Similarly, a final set of edges E_3 is sampled after changing the weights
  - The algorithm on the concatenated sequence $E = < E_1, E_2, E_3 >$



(a) *Facebook* (b) *Twitter* (c) *Students*

Temporal PageRank is able to adapt to the changing distribution quite fast.
Error is the Euclidean distance on the PageRank vectors.

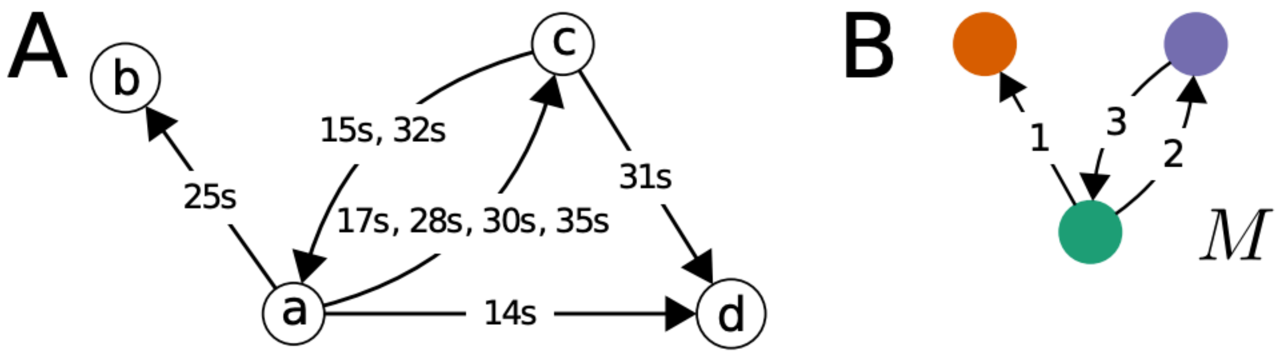
Mesoscopic Evolution of Networks

Mesoscopic Evolution

- **How do networks evolve at the mezo level?**
  - What are mesoscopic impact of network growth?
- **Questions:**
  - How does patterns of interaction change over time?
  - What can we infer about the network from the changes in temporal patterns?

Temporal Motifs

- $\boldsymbol{k}$ **−node** $\boldsymbol{l}$ **− edge** $\boldsymbol{\delta}$**-temporal motif**: is a sequence of l edges $(u_1, v_1, t_1), (u_2, v_2, t_2), \ldots (u_l, v_l, t_l)$ such that
  - $t_1 < t_2 < \ldots < t_l$ and t_l- $t_1 \leq \delta$,
  - The induced static graph from the edges is connected and has k nodes



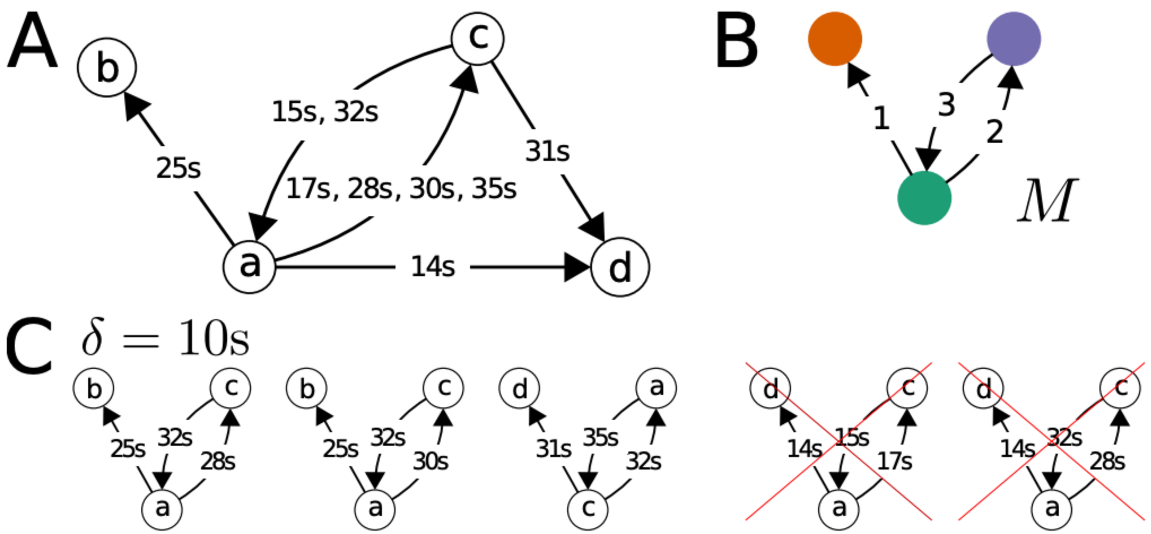
- Temporal motifs offer valuable information about the networks' evolution
  - For example to discover trends and anomalies in temporal networks

Temporal Motif Instances

- **Temporal Motif Instance**: A collection of edges in a temporal graph is an instance of a δ-temporal motif M if
  - It matches the same edge pattern, and
  - All of the edges occur in the right order specified by the motif, within a δ time window

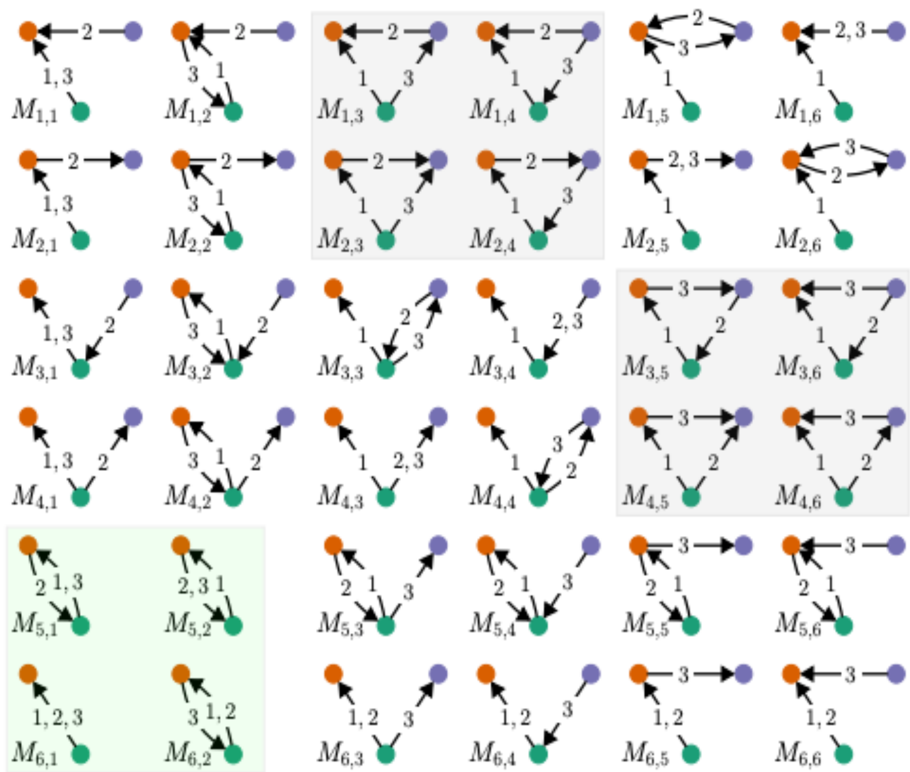
Temporal graph

δ-temporal motif instances



Case Study — Identifying Trends and Anomalies

- We study all 2- and 3- node motifs with 3 edges
  - We do not discuss how to count temporal motifs here



The green background highlights the four 2-node motifs (bottom left) and the grey background highlights the eight triangles.

Case Study — Temporal Motifs

- Real-world temporal datasets

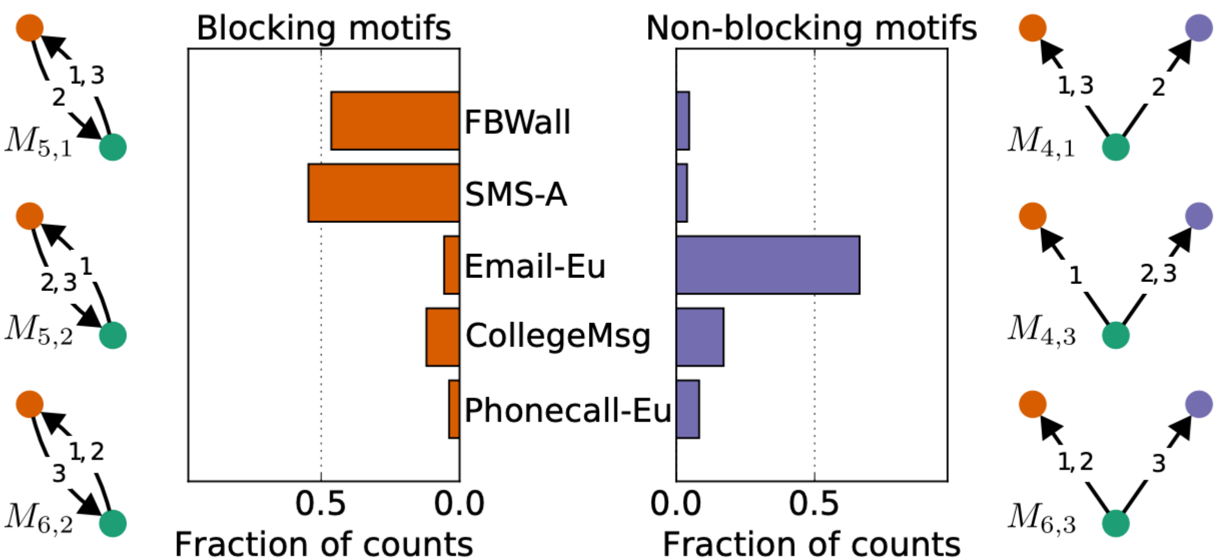
Table 1: Summary statistics of datasets.

dataset	# nodes	# static edges	# edges	time span (days)
EMAIL-EU	986	2.49K	332K	803
PHONECALL-EU	1.05M	2.74M	8.55M	7
SMS-A	44.1K	67.2K	545K	338
COLLEGEMSG	1.90K	20.3K	59.8K	193
STACKOVERFLOW	2.58M	34.9M	47.9M	2774
BITCOIN	24.6M	88.9M	123M	1811
FBWALL	45.8K	264K	856K	1560
WIKITALK	1.09M	3.13M	6.10M	2277
PHONECALL-ME	18.7M	360M	2.04B	364
SMS-ME	6.94M	51.5M	800M	89

[Paranjape et al. 2017]

Case Study — Temporal Motifs

- **Blocking communication**
  - If an individual typically waits for a reply from one individual before proceeding to communicate with another individual

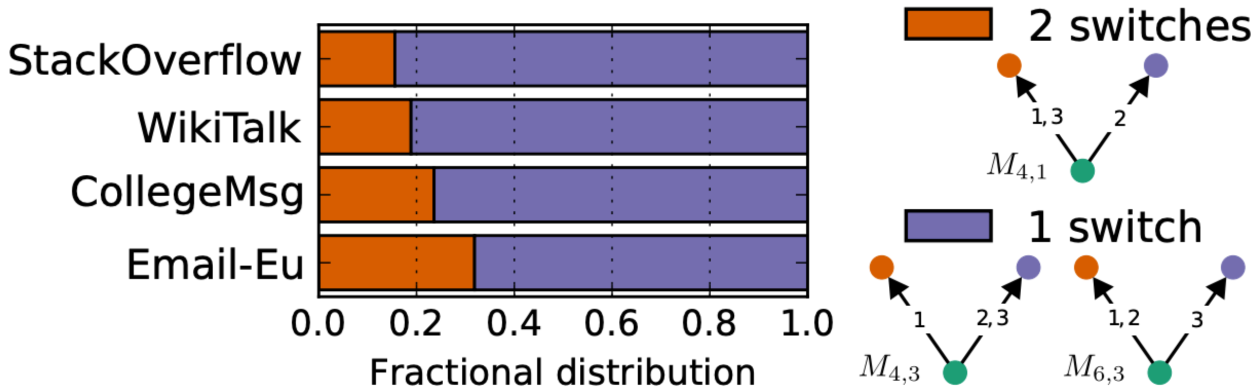


Fraction of all 2 and 3-node, 3-edge δ-temporal motif counts that correspond to two groups of motifs (δ = 1 hour). Motifs on the left capture "blocking" behavior, common in SMS messaging and Facebook wall posting, and motifs on the right exhibit "non-blocking" behavior, common in email.

[Paranjape et al. 2017]

Case Study — Temporal Motifs

- **Cost of Switching**
  - On Stack Overflow and Wikipedia talk pages, there is a high cost to switch targets because of peer engagement and depth of discussion
  - In the COLLEGEMSG dataset there is a lesser cost to switch because it lacks depth of discussion within the time frame of δ = 1 hour
  - In EMAIL-EU, there is almost no peer engagement and cost of switching is negligible

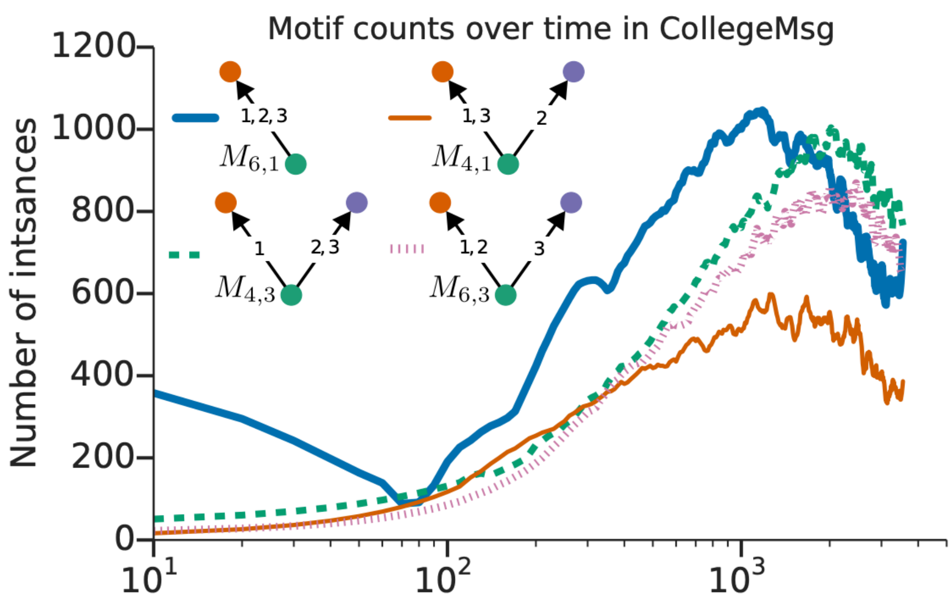


Distribution of switching behavior amongst the nonblocking motifs (δ = 1 hour). Switching is least common on Stack Overflow and most common in email.

[Paranjape et al. 2017]

Case Study — Temporal Motifs

- **Motif counts at varying time scales**
  - At small time scales, the motif consisting of three edges to a single neighbor occurs frequently
  - After 5 minutes, counts for the three motifs with one switch in the target grow at a faster rate than the counts for the motif with two switches

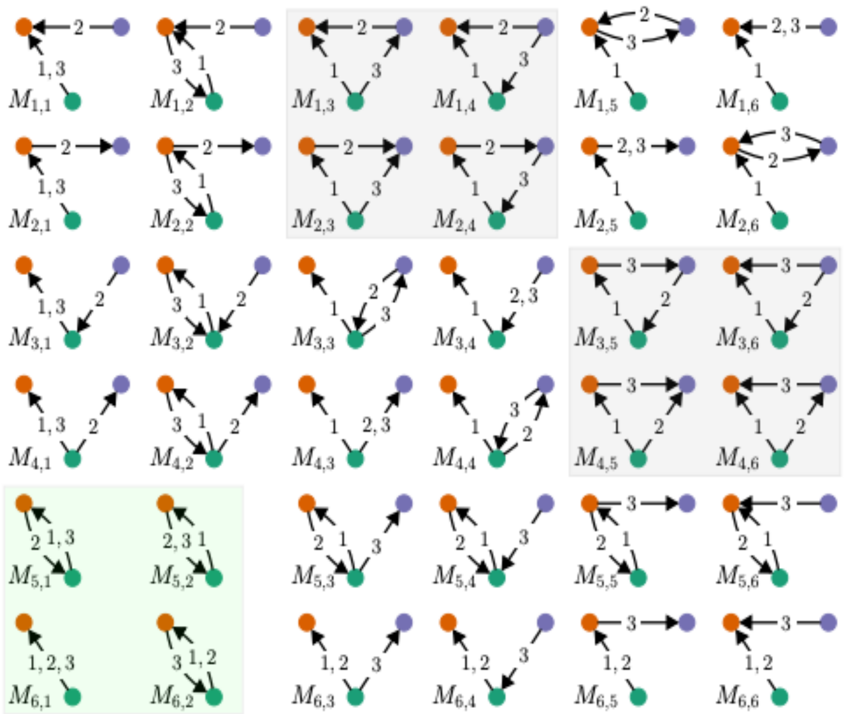


Counts over various time scales for the motifs representing a node sending 3 outgoing messages to 1 or 2 neighbors in the COLLEGEMSG dataset

[Paranjape et al. 2017]

Case Study — Identifying Trends and Anomalies

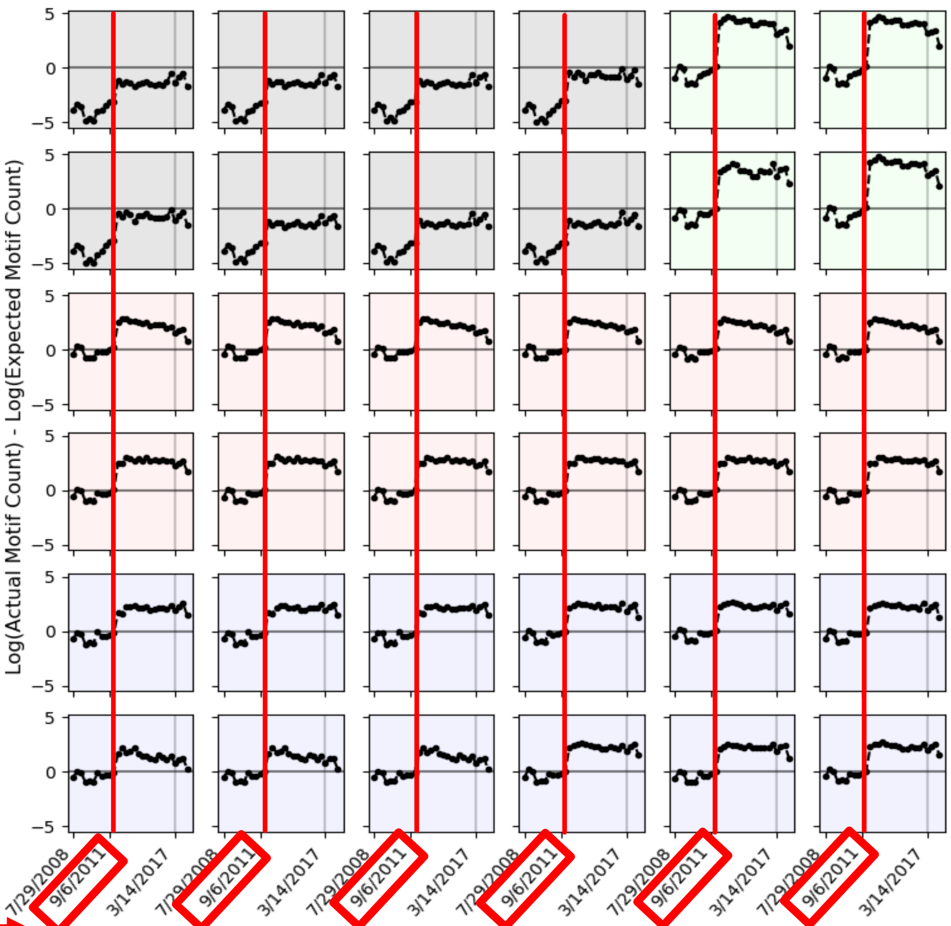
- To spot **trends and anomalies**, we have to spot **statistically significant** temporal motifs
  - To do so, we must compute the **expected number of occurrences of each motif**
  - We study all 2- and 3- node motifs with 3 edges



Case Study — Financial Network

- A European country's transaction log for all transactions larger than 50K Euros over 10 years from 2008 to 2018, with 118,739 nodes and 2,982,049 temporal edges (δ=90 days)

Anomalies: We can localize the time the financial crisis hits the country around September 2011 from the difference in the actual vs. expected motif frequencies



Financial crisis starts