restart;

$with(PolynomialTools):$
$with(LinearAlgebra):$

Section 2.1

Main equations in Lemma 4 :

$P1(x1, x2) := l1 \cdot x1^2 + 2 \cdot x1 \cdot x2 + m1 \cdot x2^2 - (l1 + m1 + 2)$

$$P1 := (x1, x2) \mapsto l1 \cdot x1^2 + 2 \cdot x2 \cdot x1 + m1 \cdot x2^2 - l1 - m1 - 2 \tag{1}$$

$P2(x2, x3) := l2 \cdot x2^2 + 2 \cdot x2 \cdot x3 + m2 \cdot x3^2 - (l2 + m2 + 2)$

$$P2 := (x2, x3) \mapsto l2 \cdot x2^2 + 2 \cdot x3 \cdot x2 + m2 \cdot x3^2 - l2 - m2 - 2 \tag{2}$$

$P3(x3, x4) := l3 \cdot x3^2 + 2 \cdot x3 \cdot x4 + m3 \cdot x4^2 - (l3 + m3 + 2)$

$$P3 := (x3, x4) \mapsto l3 \cdot x3^2 + 2 \cdot x4 \cdot x3 + m3 \cdot x4^2 - l3 - m3 - 2 \tag{3}$$

$P4(x4, x1) := l4 \cdot x4^2 + 2 \cdot x4 \cdot x1 + m4 \cdot x1^2 - (l4 + m4 + 2)$

$$P4 := (x4, x1) \mapsto l4 \cdot x4^2 + 2 \cdot x1 \cdot x4 + m4 \cdot x1^2 - l4 - m4 - 2 \tag{4}$$

Equations for *li* and *mi* in Lemma 6, where i = 1, 2, 3, 4 :

$eq1 := \frac{1 - l1 \cdot m1}{(1 + l1)^2}; eq2 := \frac{1 - l2 \cdot m2}{(1 + m2)^2}; eq3 := \frac{1 - l2 \cdot m2}{(1 + l2)^2}; eq4 := \frac{1 - l3 \cdot m3}{(1 + m3)^2}; eq5 := \frac{1 - l3 \cdot m3}{(1 + l3)^2}; eq6 := \frac{1 - l4 \cdot m4}{(1 + m4)^2}; eq7 := \frac{1 - l4 \cdot m4}{(1 + l4)^2}; eq8 := \frac{1 - l1 \cdot m1}{(1 + m1)^2};$

=

$$\frac{-l1\, m1 + 1}{(1 + l1)^2}$$

$$\frac{-l2\, m2 + 1}{(1 + m2)^2}$$

$$\frac{-l2\, m2 + 1}{(1 + l2)^2}$$

$$\frac{-l3\, m3 + 1}{(1 + m3)^2}$$

$$\frac{-l3\ m3+1}{(1+l3)^2}$$

$$\frac{-l4\ m4+1}{(1+m4)^2}$$

$$\frac{-l4\ m4+1}{(1+l4)^2}$$

$$\frac{-l1\ m1+1}{(1+m1)^2} \qquad \textbf{(5)}$$

Solving system of equations in Lemma 6 for *li* and *mi* in terms of *ki*, where i = 1, 2, 3, 4 :

$$solve\left(\left\{eq1=\frac{1}{k2^2}, eq2=\frac{1}{k2^2}, eq3=\frac{1}{k3^2}, eq4=\frac{1}{k3^2}, eq5=\frac{1}{k4^2}, eq6=\frac{1}{k4^2}, eq7=\frac{1}{k1^2}, eq8=\frac{1}{k1^2}\right\}, \{l1, m1, l2, m2, l3, m3, l4, m4\}\right)$$

$$\left\{l1=-\frac{k2^2-1}{k1\ k2-1}, l2=-\frac{k3^2-1}{k2\ k3-1}, l3=\frac{k4^2-1}{k3\ k4+1}, l4=-\frac{k1^2-1}{k4\ k1-1}, m1=-\frac{k1^2-1}{k1\ k2-1}, m2=-\frac{k2^2-1}{k2\ k3-1}, m3=\frac{k3^2-1}{k3\ k4+1}, m4=-\frac{k4^2-1}{k4\ k1-1}\right\}, \left\{l1=-\frac{k2^2-1}{k1\ k2-1}, l2=-\frac{k3^2-1}{k2\ k3-1}, l3=\frac{k4^2-1}{k3\ k4+1}, l4=\frac{k1^2-1}{k4\ k1+1}, m1=-\frac{k1^2-1}{k1\ k2-1}, m2=-\frac{k2^2-1}{k2\ k3-1}, m3=\frac{k3^2-1}{k3\ k4+1}, m4=\frac{k4^2-1}{k4\ k1+1}\right\}, \left\{l1=-\frac{k2^2-1}{k1\ k2-1}, l2=-\frac{k3^2-1}{k2\ k3-1}, l3=-\frac{k4^2-1}{k3\ k4-1}, l4=-\frac{k1^2-1}{k4\ k1-1}, m1=-\frac{k1^2-1}{k1\ k2-1}, m2=-\frac{k2^2-1}{k2\ k3-1}, m3=-\frac{k3^2-1}{k3\ k4-1}, m4=-\frac{k4^2-1}{k4\ k1-1}\right\}, \left\{l1=-\frac{k2^2-1}{k1\ k2-1}, l2=-\frac{k3^2-1}{k2\ k3-1}, l3=-\frac{k4^2-1}{k3\ k4-1}, l4=\frac{k1^2-1}{k4\ k1+1}, m1=-\frac{k1^2-1}{k1\ k2-1}, m2=-\frac{k2^2-1}{k2\ k3-1}, m3=-\frac{k3^2-1}{k3\ k4-1}, m4=\frac{k4^2-1}{k4\ k1+1}\right\}, \left\{l1=-\frac{k2^2-1}{k1\ k2-1}, l2=\frac{k3^2-1}{k2\ k3+1}, l3=\frac{k4^2-1}{k3\ k4+1}, l4=-\frac{k1^2-1}{k4\ k1-1}, m1=-\frac{k1^2-1}{k1\ k2-1}, m2=\frac{k2^2-1}{k2\ k3+1}, m3=\frac{k3^2-1}{k3\ k4+1}, m4=-\frac{k4^2-1}{k4\ k1-1}\right\}, \left\{l1=-\frac{k2^2-1}{k1\ k2-1}, l2=\frac{k3^2-1}{k2\ k3+1}, l3=\frac{k4^2-1}{k3\ k4+1}, l4=\frac{k1^2-1}{k4\ k1+1}, m1=-\frac{k1^2-1}{k1\ k2-1}, m2=\frac{k2^2-1}{k2\ k3+1}, m3=\frac{k3^2-1}{k3\ k4+1}, m4 \qquad \textbf{(6)}$$

$$= \frac{k4^2-1}{k4\,k1+1}\Big\}, \Big\{l1 = -\frac{k2^2-1}{k1\,k2-1}, l2 = \frac{k3^2-1}{k2\,k3+1}, l3 = -\frac{k4^2-1}{k3\,k4-1}, l4 = -\frac{k1^2-1}{k4\,k1-1},$$

$$m1 = -\frac{k1^2-1}{k1\,k2-1}, m2 = \frac{k2^2-1}{k2\,k3+1}, m3 = -\frac{k3^2-1}{k3\,k4-1}, m4 = -\frac{k4^2-1}{k4\,k1-1}\Big\}, \Big\{l1 =$$

$$-\frac{k2^2-1}{k1\,k2-1}, l2 = \frac{k3^2-1}{k2\,k3+1}, l3 = -\frac{k4^2-1}{k3\,k4-1}, l4 = \frac{k1^2-1}{k4\,k1+1}, m1 = -\frac{k1^2-1}{k1\,k2-1}, m2$$

$$= \frac{k2^2-1}{k2\,k3+1}, m3 = -\frac{k3^2-1}{k3\,k4-1}, m4 = \frac{k4^2-1}{k4\,k1+1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2 = -\frac{k3^2-1}{k2\,k3-1}, l3$$

$$= \frac{k4^2-1}{k3\,k4+1}, l4 = -\frac{k1^2-1}{k4\,k1-1}, m1 = \frac{k1^2-1}{k1\,k2+1}, m2 = -\frac{k2^2-1}{k2\,k3-1}, m3 = \frac{k3^2-1}{k3\,k4+1}, m4$$

$$= -\frac{k4^2-1}{k4\,k1-1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2 = -\frac{k3^2-1}{k2\,k3-1}, l3 = \frac{k4^2-1}{k3\,k4+1}, l4 = \frac{k1^2-1}{k4\,k1+1}, m1$$

$$= \frac{k1^2-1}{k1\,k2+1}, m2 = -\frac{k2^2-1}{k2\,k3-1}, m3 = \frac{k3^2-1}{k3\,k4+1}, m4 = \frac{k4^2-1}{k4\,k1+1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2$$

$$= -\frac{k3^2-1}{k2\,k3-1}, l3 = -\frac{k4^2-1}{k3\,k4-1}, l4 = -\frac{k1^2-1}{k4\,k1-1}, m1 = \frac{k1^2-1}{k1\,k2+1}, m2 = -\frac{k2^2-1}{k2\,k3-1},$$

$$m3 = -\frac{k3^2-1}{k3\,k4-1}, m4 = -\frac{k4^2-1}{k4\,k1-1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2 = -\frac{k3^2-1}{k2\,k3-1}, l3 =$$

$$-\frac{k4^2-1}{k3\,k4-1}, l4 = \frac{k1^2-1}{k4\,k1+1}, m1 = \frac{k1^2-1}{k1\,k2+1}, m2 = -\frac{k2^2-1}{k2\,k3-1}, m3 = -\frac{k3^2-1}{k3\,k4-1}, m4$$

$$= \frac{k4^2-1}{k4\,k1+1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2 = \frac{k3^2-1}{k2\,k3+1}, l3 = \frac{k4^2-1}{k3\,k4+1}, l4 = -\frac{k1^2-1}{k4\,k1-1}, m1$$

$$= \frac{k1^2-1}{k1\,k2+1}, m2 = \frac{k2^2-1}{k2\,k3+1}, m3 = \frac{k3^2-1}{k3\,k4+1}, m4 = -\frac{k4^2-1}{k4\,k1-1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2$$

$$= \frac{k3^2-1}{k2\,k3+1}, l3 = \frac{k4^2-1}{k3\,k4+1}, l4 = \frac{k1^2-1}{k4\,k1+1}, m1 = \frac{k1^2-1}{k1\,k2+1}, m2 = \frac{k2^2-1}{k2\,k3+1}, m3$$

$$= \frac{k3^2-1}{k3\,k4+1}, m4 = \frac{k4^2-1}{k4\,k1+1}\Big\}, \Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2 = \frac{k3^2-1}{k2\,k3+1}, l3 = -\frac{k4^2-1}{k3\,k4-1}, l4 =$$

$$-\frac{k1^2-1}{k4\,k1-1}, m1 = \frac{k1^2-1}{k1\,k2+1}, m2 = \frac{k2^2-1}{k2\,k3+1}, m3 = -\frac{k3^2-1}{k3\,k4-1}, m4 = -\frac{k4^2-1}{k4\,k1-1}\Big\},$$

$$\Big\{l1 = \frac{k2^2-1}{k1\,k2+1}, l2 = \frac{k3^2-1}{k2\,k3+1}, l3 = -\frac{k4^2-1}{k3\,k4-1}, l4 = \frac{k1^2-1}{k4\,k1+1}, m1 = \frac{k1^2-1}{k1\,k2+1}, m2$$

$$= \frac{k2^2-1}{k2\,k3+1}, m3 = -\frac{k3^2-1}{k3\,k4-1}, m4 = \frac{k4^2-1}{k4\,k1+1}\Big\}$$

Using $li + 1 > 0$, $mi + 1 > 0$, where i = 1, 2, 3, 4, the values of li and mi in terms ki are :

$$l1 := \frac{k2^2 - 1}{k1\,k2 + 1};\ l2 := \frac{k3^2 - 1}{k2\,k3 + 1};\ l3 := \frac{k4^2 - 1}{k3\,k4 + 1};\ l4 := \frac{k1^2 - 1}{k4\,k1 + 1};\ m1 := \frac{k1^2 - 1}{k1\,k2 + 1};\ m2 := \frac{k2^2 - 1}{k2\,k3 + 1};\ m3 := \frac{k3^2 - 1}{k3\,k4 + 1};\ m4 := \frac{k4^2 - 1}{k4\,k1 + 1}$$

$$l1 := \frac{k2^2 - 1}{k1\,k2 + 1}$$

$$l2 := \frac{k3^2 - 1}{k2\,k3 + 1}$$

$$l3 := \frac{k4^2 - 1}{k3\,k4 + 1}$$

$$l4 := \frac{k1^2 - 1}{k4\,k1 + 1}$$

$$m1 := \frac{k1^2 - 1}{k1\,k2 + 1}$$

$$m2 := \frac{k2^2 - 1}{k2\,k3 + 1}$$

$$m3 := \frac{k3^2 - 1}{k3\,k4 + 1}$$

$$m4 := \frac{k4^2 - 1}{k4\,k1 + 1} \tag{7}$$

:= :=
:= :=

Section 2.2

Functions x1, x2, x3, x4 :

$$x1 := \frac{(1+t)^2\cdot(1-k1)+1+k1}{2\cdot(1+t)};\ x2 := \frac{(1+t)^2\cdot(1+k2)+1-k2}{2\cdot(1+t)};\ x3 := \frac{(1+t)^2\cdot(1-k3)+1+k3}{2\cdot(1+t)};\ x4 := \frac{(1+t)^2\cdot(1+k4)+1-k4}{2\cdot(1+t)}$$

$$x1 := \frac{(1+t)^2 (1-k1) + 1 + k1}{2+2t}$$

$$x2 := \frac{(1+t)^2 (1+k2) + 1 - k2}{2+2t}$$

$$x3 := \frac{(1+t)^2 (1-k3) + 1 + k3}{2+2t}$$

$$x4 := \frac{(1+t)^2 (1+k4) + 1 - k4}{2+2t} \tag{8}$$

Verification of equations in Lemma 4 for *xi, li, mi,* where i = 1, 2, 3, 4 :

simplify(*P1*(*x1*, *x2*)); *simplify*(*P2*(*x2*, *x3*)); *simplify*(*P3*(*x3*, *x4*)); *simplify*(*P4*(*x4*, *x1*))

$$0$$

$$0$$

$$0$$

$$0 \tag{9}$$

:= :=
:= :=

Section 2.3

Quadratic form in the left side of equation (4) :

eq := *collect*(*simplify*((*l2* + *m2* + 2) · *P1*(*x1*, *x2*) − (*l1* + *m1* + 2) · *P2*(*x2*, *x3*)), { *x1*, *x2*, *x3* }, *distributed*)

$$eq := (l2 + m2 + 2)\, l1\, x1^2 + (2\, m2 + 2\, l2 + 4)\, x2\, x1 + (-l1\, l2 + m1\, m2 - 2\, l2 + 2\, m1)\, x2^2 + (-l1\, m2 - m2\, (m1 + 2))\, x3^2 + (-2\, l1 - 2\, m1 - 4)\, x3\, x2 \tag{10}$$

Matrix of coefficients of quadratic form :

$Quad\_form\_Matrix := Matrix\left(\left[\left[(l2+m2+2)\,l1, \frac{(2\,l2+2\,m2+4)}{2}, 0\right], \left[\frac{(2\,l2+2\,m2+4)}{2}, -l1\,l2+m1\,m2-2\,l2+2\,m1, \frac{(-2\,l1-2\,m1-4)}{2}\right], \left[0, \frac{(-2\,l1-2\,m1-4)}{2}, -l1\,m2 - m2\,(m1+2)\right]\right]\right)$

$$Quad\_form\_Matrix := \begin{bmatrix} (l2+m2+2)\,l1 & l2+m2+2 & 0 \\ l2+m2+2 & -l1\,l2+m1\,m2-2\,l2+2\,m1 & -l1-m1-2 \\ 0 & -l1-m1-2 & -l1\,m2-m2\,(m1+2) \end{bmatrix} \tag{11}$$

Verifying that the determinant of *Quad_form_Matrix* is equal to the left side of equation (5) :

$simplify\left(Determinant(Quad\_form\_Matrix) - (l1+m1+2)\cdot(l2+m2+2)\cdot\left((m2+1)^2\cdot(1-m1\cdot l1) - (l1+1)^2\cdot(1-l2\cdot m2)\right)\right)$

$$0 \tag{12}$$

:= :=
:= :=

Section 2.4

Equations (6) :

$Eq6 := simplify\left(Determinant\left(Matrix\left(\left[[m1, 0, l2, 0], [2\cdot x1, m1, 2\cdot x3, l2], \left[l1\cdot x1^2 - (l1+m1+2), 2\cdot x1, m2\cdot x3^2 - (l2+m2+2), 2\cdot x3\right], \left[0, l1\cdot x1^2 - (l1+m1+2), 0, m2\cdot x3^2 - (l2+m2+2)\right]\right]\right)\right)\right)$

$$Eq6 := \left(l1^2\,x1^4 + \left(-2\,l1^2-4\,l1-4\right)x1^2 + (2+l1)^2\right)l2^2 + \left(\left(-2\,l1\,\left(m2\,x3^2-m2-2\right)x1^2 + 8\,x1\,x3 + 2\,\left(m2\,x3^2-m2-2\right)(2+l1)\right)m1 - 4\,x1\,\left(l1\,x1^2\,x3 + \left(-m2\,x3^2+m2+2\right)x1 - x3\,(2+l1)\right)\right)l2 + 4\left(\frac{(x3-1)\,(x3+1)\,(m2\,x3+m2+2)\,(m2\,x3-m2-2)\,m1}{4} + \left(l1\,x1^2\,x3 + \left(-m2\,x3^2+m2+2\right)x1 - x3\,(2+l1)\right)x3\right)m1 \tag{13}$$

Equations (7) :

$$Eq7 := l1 \cdot x1^2 \cdot x3 - m2 \cdot x3^2 \cdot x1 - (l1 + 2) \cdot x3 + (m2 + 2) \cdot x1$$

$$Eq7 := l1\, x1^2\, x3 - m2\, x3^2\, x1 - x3\, (2 + l1) + (m2 + 2)\, x1 \tag{14}$$

Equations$\cdot$ (6 *prime*) :

$$Eq6prime := simplify\Big(Determinant\Big(Matrix\big(\big[[l4, 0, m3, 0], [2 \cdot x1, l4, 2 \cdot x3, m3], [m4 \cdot x1^2 - (m4 + l4 + 2), 2 \cdot x1, l3 \cdot x3^2 - (m3 + l3 + 2), 2 \cdot x3], [0, m4 \cdot x1^2 - (m4 + l4 + 2), 0, l3 \cdot x3^2 - (m3 + l3 + 2)]\big]\big)\Big)\Big)$$

$$Eq6prime := \big(l3^2\, x3^4 + (-2\, l3^2 - 4\, l3 - 4)\, x3^2 + (2 + l3)^2\big)\, l4^2 + \Big(\big((-2\, m4\, (l3\, x3^2 - l3 - 2)\, x1^2 + 8\, x1\, x3 + 2\, (m4 + 2)\, (l3\, x3^2 - l3 - 2)\big)\, m3 - 4\, \big(-m4\, x3\, x1^2 + (l3\, x3^2 - l3 - 2)\, x1 + x3\, (m4 + 2)\big)\, x3\Big)\, l4 + 4\, m3 \left(\frac{(x1 - 1)\, (x1 + 1)\, (m4\, x1 + m4 + 2)\, (m4\, x1 - m4 - 2)\, m3}{4} + \big(-m4\, x3\, x1^2 + (l3\, x3^2 - l3 - 2)\, x1 + x3\, (m4 + 2)\big)\, x1\right) \tag{15}$$

Equations$\cdot$ (7 *prime*) :

$$Eq7prime := m4 \cdot x1^2 \cdot x3 - l3 \cdot x3^2 \cdot x1 - (m4 + 2) \cdot x3 + (l3 + 2) \cdot x1$$

$$Eq7prime := m4\, x3\, x1^2 - l3\, x3^2\, x1 - x3\, (m4 + 2) + (2 + l3)\, x1 \tag{16}$$

Important equation for comperison of coefficients :

$$Eq := collect\big(simplify(\alpha \cdot Eq6 - Eq6prime), \{x1, x2, x3\}, distributed\big)$$

$$Eq := (\alpha\, l1^2\, l2^2 - m3^2\, m4^2)\, x1^4 + (-4\, \alpha\, l1\, l2 + 4\, m3\, m4)\, x3\, x1^3 + \big(((-2\, l1\, m1\, m2 + 4\, m2)\, l2 + 4\, l1\, m1)\, \alpha + (2\, l3\, m3\, m4 - 4\, m4)\, l4 - 4\, l3\, m3\big)\, x1^2\, x3^2 + \Big(-2\, l2\, \big((l1^2 + 2\, l1 + 2)\, l2 - (m2 + 2)\, (l1\, m1 - 2)\big)\, \alpha - 2\, \big(m4\, (2 + l3)\, l4 + (-m4^2 - 2\, m4 - 2)\, m3 - 2\, l3 - 4\big)\, m3\Big)\, x1^2 + (-4\, m1\, m2\, \alpha + 4\, l4\, l3)\, x1\, x3^3 + \big(4\, ((l1 + 2\, m1 + 2)\, l2 + m1\, (m2 + 2))\, \alpha + 4\, (-l3 - 2\, m3 - 2)\, l4 - 4\, m3\, (m4 + 2)\big)\, x1\, x3 + (\alpha\, m1^2\, m2^2 - l3^2\, l4^2)\, x3^4 + \Big(2\, \big(m2\, (2 + l1)\, l2 + (-m2^2 - 2\, m2 - 2)\, m1 - 2\, l1 - 4\big)\, m1\, \alpha + 2\, l4\, \big((l3^2 + 2\, l3 + 2)\, l4 - (m4 + 2)\, (l3\, m3 - 2)\big)\Big)\, x3^2 + ((2 + l1)\, l2 - m1\, (m2 \tag{17}$$

$+ 2))^2 \alpha - ((2 + l3)\, l4 - m3\, (m4 + 2))^2$

This function returns the coefficient t of polynomial P with variable list T:

```
coefff := proc(P, T, t)
local L, H, i, k :
L := [coeffs(P, T, 'h')] : H := [h] : k := 0 :
for i from 1 to nops(H) do
if H[i] = t then k := L[i] fi:
od:
k;
end proc:
```

Comparing coefficients of $x1x3^3$, $x3^4$, $x1^3x3$, $x1^4$, $x1^2x3^2$, $x1x3$, 1, $x3^2$, $x1^2$ in (6) and (6') :

$Vector([coefff(Eq, [x1, x2, x3], x1 \cdot x3^3) = 0, coefff(Eq, [x1, x2, x3], x3^4) = 0, coefff(Eq, [x1, x2, x3], x1^3 \cdot x3) = 0, coefff(Eq, [x1, x2, x3], x1^4) = 0, coefff(Eq, [x1, x2, x3], x1^2 \cdot x3^2) = 0, coefff(Eq, [x1, x2, x3], x1 \cdot x3) = 0, coefff(Eq, [x1, x2, x3], 1) = 0, coefff(Eq, [x1, x2, x3], x3^2) = 0, coefff(Eq, [x1, x2, x3], x1^2) = 0])$

$$\begin{aligned}
&[[-4\,\alpha\, m1\, m2 + 4\, l3\, l4 = 0],\\
&[\alpha\, m1^2\, m2^2 - l3^2\, l4^2 = 0],\\
&[-4\,\alpha\, l1\, l2 + 4\, m3\, m4 = 0],\\
&[\alpha\, l1^2\, l2^2 - m3^2\, m4^2 = 0],\\
&[((-2\, l1\, m1\, m2 + 4\, m2)\, l2 + 4\, l1\, m1)\,\alpha + (2\, l3\, m3\, m4 - 4\, m4)\, l4 - 4\, l3\, m3 = 0],\\
&[4\,((l1 + 2\, m1 + 2)\, l2 + m1\,(m2 + 2))\,\alpha + 4\,(-l3 - 2\, m3 - 2)\, l4 - 4\, m3\,(m4 + 2) = 0\\
&],\\
&[((2 + l1)\, l2 - m1\,(m2 + 2))^2\,\alpha - ((2 + l3)\, l4 - m3\,(m4 + 2))^2 = 0],\\
&[2\,\big(m2\,(2 + l1)\, l2 + \big(-m2^2 - 2\, m2 - 2\big)\, m1 - 2\, l1 - 4\big)\, m1\,\alpha + 2\, l4\,\big(\big(l3^2 + 2\, l3\\
&+ 2\big)\, l4 - (m4 + 2)\,(l3\, m3 - 2)\big) = 0],\\
&[-2\, l2\,\big(\big(l1^2 + 2\, l1 + 2\big)\, l2 - (m2 + 2)\,(l1\, m1 - 2)\big)\,\alpha - 2\,\big(m4\,(2 + l3)\, l4 + \big(-m4^2\\
&- 2\, m4 - 2\big)\, m3 - 2\, l3 - 4\big)\, m3 = 0]]
\end{aligned} \tag{18}$$

:= :=

:= :=

Section 2.5

Equation (20) and condition $\alpha := 1$ in Case 2 :

$l2 := \frac{(m3+1)\cdot(l4+1)}{m1+1} - 1;\ l3 := \frac{m1\cdot m2}{l4};\ m4 := \frac{l1\cdot(m3\cdot l4 + m3 + l4 - m1)}{m3\cdot(m1+1)};\ \alpha := 1$

$$l2 := \frac{(m3+1)\ (l4+1)}{m1+1} - 1$$

$$l3 := \frac{m1\ m2}{l4}$$

$$m4 := \frac{l1\ (m3\ l4 + l4 - m1 + m3)}{m3\ (m1+1)}$$

$$\alpha := 1 \tag{19}$$

Substitution (20) in (17) :

$factor(numer(factor(coefff(\ Eq, [x1, x2, x3], 1))))$

$$4\ (m1 + m3 + 2)\ (-m1 + l4)\ \left(l1\ l4\ m3 - m1^2\ m2 + l1\ l4 - l1\ m1 + l1\ m3 - l4\ m1 + m3\ l4 - m1^2 - m1\ m2 + m3\ m1 - 2\ m1 + 2\ m3\right) \tag{20}$$

Value of A :

$A := expand((m3 + 1)\cdot(l1 + 1)\cdot(l4 + 1) - (m1 + 1)\cdot(m1\cdot m2 + l1 + l4 + m1 - m3 + 1))$

$$A := l1\ l4\ m3 - m1^2\ m2 + l1\ l4 - l1\ m1 + l1\ m3 - l4\ m1 + m3\ l4 - m1^2 - m1\ m2 + m3\ m1 - 2\ m1 + 2\ m3 \tag{21}$$

Substitution (20) in (18) :

$factor\left(numer\left(factor\left(coefff\left(Eq, [x1, x2, x3], x3^2\right)\right)\right)\right)$

$$4\ (-m1 + l4)\ \left(m1^2\ m2\ m3 + m1\ m2\ m3^2 + l1\ l4\ m3 + l1\ m1\ m3 + l4\ m1\ m3 + m1^2\ m3 + 2\ m1\ m2\ m3 + l1\ l4 + l1\ m3 + m3\ l4 + 3\ m3\ m1 + 2\ m3\right) \tag{22}$$

Value of B :

$B := m1 \cdot m2 + m2 \cdot m3 + l1 + m2 + l4 + m1 + 2$

$$B := m1\, m2 + m2\, m3 + l1 + l4 + m1 + m2 + 2 \tag{23}$$

The form of $A + (1 + m3) \cdot m1 \cdot B$ looks like long factor of (22) :

$expand(A + (1 + m3) \cdot m1 \cdot B)$

$$m1^2\, m2\, m3 + m1\, m2\, m3^2 + l1\, l4\, m3 + l1\, m1\, m3 + l4\, m1\, m3 + m1^2\, m3 + 2\, m1\, m2\, m3 + l1\, l4 + l1\, m3 + m3\, l4 + 3\, m3\, m1 + 2\, m3 \tag{24}$$

:= :=
:= :=

Section 2.6

Checking $A + (m1 - m3) \cdot B$:

$factor(A + (m1 - m3) \cdot B)$

$$(m3 + 1)\,(l1\, l4 - m2\, m3) \tag{25}$$