Area Preserving Combescure Transformations
(Auxiliary computations)

Olimjoni Pirahmad, Helmut Pottmann, and Mikhail Skopenkov

Here, we check the auxiliary computations of the proofs.

restart;

with(*PolynomialTools*) :
with(*LinearAlgebra*) :

Section 1

Changing variables l_i and m_i for $i = 1, 2, 3, 4$ by:

$l_1 := \frac{\mu_1 - 1}{\lambda_1}$; $m_1 := \frac{\lambda_1 - 1}{\mu_1}$; $l_2 := \frac{\mu_2 - 1}{\lambda_2}$; $m_2 := \frac{\lambda_2 - 1}{\mu_2}$;
$l_3 := \frac{\mu_3 - 1}{\lambda_3}$; $m_3 := \frac{\lambda_3 - 1}{\mu_3}$; $l_4 := \frac{\mu_4 - 1}{\lambda_4}$; $m_4 := \frac{\lambda_4 - 1}{\mu_4}$;

$$l_1 := \frac{\mu_1 - 1}{\lambda_1}$$

$$m_1 := \frac{\lambda_1 - 1}{\mu_1}$$

$$l_2 := \frac{\mu_2 - 1}{\lambda_2}$$

$$m_2 := \frac{\lambda_2 - 1}{\mu_2}$$

$$l_3 := \frac{\mu_3 - 1}{\lambda_3}$$

$$m_3 := \frac{\lambda_3 - 1}{\mu_3}$$

$$l_4 := \frac{\mu_4 - 1}{\lambda_4}$$

$$m_4 := \frac{\lambda_4 - 1}{\mu_4} \tag{1}$$

Equating equations in Lemma 2.14 (b) to $1 / k_i^2$ for $i = 1, 2, 3, 4$:

$simplify\left(\frac{1 - l_1 \cdot m_1}{(1 + m_1)^2}\right) = \frac{1}{k_1^2}$; $simplify\left(\frac{1 - l_1 \cdot m_1}{(1 + l_1)^2}\right) = \frac{1}{k_2^2}$; $simplify\left(\frac{1 - l_2 \cdot m_2}{(1 + m_2)^2}\right) = \frac{1}{k_2^2}$;
$simplify\left(\frac{1 - l_2 \cdot m_2}{(1 + l_2)^2}\right) = \frac{1}{k_3^2}$; $simplify\left(\frac{1 - l_3 \cdot m_3}{(1 + m_3)^2}\right) = \frac{1}{k_3^2}$; $simplify\left(\frac{1 - l_3 \cdot m_3}{(1 + l_3)^2}\right) = \frac{1}{k_4^2}$;
$simplify\left(\frac{1 - l_4 \cdot m_4}{(1 + m_4)^2}\right) = \frac{1}{k_4^2}$; $simplify\left(\frac{1 - l_4 \cdot m_4}{(1 + l_4)^2}\right) = \frac{1}{k_1^2}$;

$$\frac{\mu_1}{(\lambda_1 + \mu_1 - 1)\, \lambda_1} = \frac{1}{k_1^2}$$

$$\frac{\lambda_1}{(\lambda_1 + \mu_1 - 1)\, \mu_1} = \frac{1}{k_2^2}$$

$$\frac{\mu_2}{(\lambda_2 + \mu_2 - 1)\, \lambda_2} = \frac{1}{k_2^2}$$

$$\frac{\lambda_2}{(\lambda_2 + \mu_2 - 1)\, \mu_2} = \frac{1}{k_3^2}$$

$$\frac{\mu_3}{(\lambda_3 + \mu_3 - 1)\, \lambda_3} = \frac{1}{k_3^2}$$

$$\frac{\lambda_3}{(\lambda_3 + \mu_3 - 1)\, \mu_3} = \frac{1}{k_4^2}$$

$$\frac{\mu_4}{(\lambda_4 + \mu_4 - 1)\, \lambda_4} = \frac{1}{k_4^2}$$

$$\frac{\lambda_4}{(\lambda_4 + \mu_4 - 1)\, \mu_4} = \frac{1}{k_1^2} \tag{2}$$

Note that $\mu_i/\lambda_i > 0$ because of $l_i + 1 > 0,\ m_i + 1 > 0,$ where $i = 1, 2, 3, 4.$

In the system (2) we devide equation $1-2, 3-4, 5-6, 7-8$
and taking square root, we obtain :

$$\frac{\mu_1}{\lambda_1} = \frac{k_2}{k_1};\ \frac{\mu_2}{\lambda_2} = \frac{k_3}{k_2};\ \frac{\mu_3}{\lambda_3} = \frac{k_4}{k_3};\ \frac{\mu_4}{\lambda_4} = \frac{k_1}{k_4};$$

$$\frac{k_2}{k_1} = \frac{k_2}{k_1}$$

$$\frac{k_3}{k_2} = \frac{k_3}{k_2}$$

$$\frac{k_4}{k_3} = \frac{k_4}{k_3}$$

$$\frac{k_1}{k_4} = \frac{k_1}{k_4} \tag{3}$$

Then equations of the system (2) becomes linear :

$$\mu_1 = \frac{k_2}{k_1}\cdot\lambda_1;\ \mu_1 + \lambda_1 - 1 = k_1\cdot k_2;\ \mu_2 = \frac{k_3}{k_2}\cdot\lambda_2;\ \mu_2 + \lambda_2 - 1 = k_2\cdot k_3;$$

$$\mu_3 = \frac{k_4}{k_3}\cdot\lambda_3;\ \mu_3 + \lambda_3 - 1 = k_3\cdot k_4;\ \mu_4 = \frac{k_1}{k_4}\cdot\lambda_4;\ \mu_4 + \lambda_4 - 1 = k_4\cdot k_1;$$

$$\mu_1 = \frac{k_2\,\lambda_1}{k_1}$$

$$\lambda_1 + \mu_1 - 1 = k_1\,k_2$$

$$\mu_2 = \frac{k_3\,\lambda_2}{k_2}$$

$$\lambda_2 + \mu_2 - 1 = k_2\,k_3$$

$$\mu_3 = \frac{k_4\,\lambda_3}{k_3}$$

$$\lambda_3 + \mu_3 - 1 = k_3\,k_4$$

$$\mu_4 = \frac{k_1 \lambda_4}{k_4}$$

$$\lambda_4 + \mu_4 - 1 = k_4 k_1 \tag{4}$$

It has a solution :

$$solve\left(\left\{\mu_1 = \frac{k_2}{k_1} \cdot \lambda_1, \mu_1 + \lambda_1 - 1 = k_1 \cdot k_2, \mu_2 = \frac{k_3}{k_2} \cdot \lambda_2, \mu_2 + \lambda_2 - 1 = k_2 \cdot k_3, \mu_3 = \frac{k_4}{k_3} \cdot \lambda_3, \mu_3 + \lambda_3 - 1 = k_3 \cdot k_4, \mu_4 = \frac{k_1}{k_4} \cdot \lambda_4, \mu_4 + \lambda_4 - 1 = k_4 \cdot k_1\right\}, \{\mu_1, \lambda_1, \mu_2, \lambda_2, \mu_3, \lambda_3, \mu_4, \lambda_4\}\right)$$

$$\left\{\lambda_1 = \frac{k_1 (k_1 k_2 + 1)}{k_1 + k_2}, \lambda_2 = \frac{k_2 (k_2 k_3 + 1)}{k_2 + k_3}, \lambda_3 = \frac{k_3 (k_3 k_4 + 1)}{k_3 + k_4}, \lambda_4 = \frac{k_4 (k_4 k_1 + 1)}{k_1 + k_4}, \mu_1 = \frac{k_2 (k_1 k_2 + 1)}{k_1 + k_2}, \mu_2 = \frac{k_3 (k_2 k_3 + 1)}{k_2 + k_3}, \mu_3 = \frac{k_4 (k_3 k_4 + 1)}{k_3 + k_4}, \mu_4 = \frac{k_1 (k_4 k_1 + 1)}{k_1 + k_4}\right\} \tag{5}$$

Setting :

$$\lambda_1 := \frac{k_1 (k_1 k_2 + 1)}{k_1 + k_2}; \lambda_2 := \frac{k_2 (k_2 k_3 + 1)}{k_2 + k_3}; \lambda_3 := \frac{k_3 (k_3 k_4 + 1)}{k_3 + k_4}; \lambda_4 := \frac{k_4 (k_4 k_1 + 1)}{k_1 + k_4};$$
$$\mu_1 := \frac{k_2 (k_1 k_2 + 1)}{k_1 + k_2}; \mu_2 := \frac{k_3 (k_2 k_3 + 1)}{k_2 + k_3}; \mu_3 := \frac{k_4 (k_3 k_4 + 1)}{k_3 + k_4}; \mu_4 := \frac{k_1 (k_4 k_1 + 1)}{k_1 + k_4}$$

$$\lambda_1 := \frac{k_1 (k_1 k_2 + 1)}{k_1 + k_2}$$

$$\lambda_2 := \frac{k_2 (k_2 k_3 + 1)}{k_2 + k_3}$$

$$\lambda_3 := \frac{k_3 (k_3 k_4 + 1)}{k_3 + k_4}$$

$$\lambda_4 := \frac{k_4 (k_4 k_1 + 1)}{k_1 + k_4}$$

$$\mu_1 := \frac{k_2 (k_1 k_2 + 1)}{k_1 + k_2}$$

$$\mu_2 := \frac{k_3 (k_2 k_3 + 1)}{k_2 + k_3}$$

$$\mu_3 := \frac{k_4\left(k_3 k_4 + 1\right)}{k_3 + k_4}$$

$$\mu_4 := \frac{k_1\left(k_4 k_1 + 1\right)}{k_1 + k_4} \tag{6}$$

Then the value of l_i, m_i where i = 1, 2, 3, 4 is :

$l_1 := simplify\left(\frac{\mu_1 - 1}{\lambda_1}\right)$; $m_1 := simplify\left(\frac{\lambda_1 - 1}{\mu_1}\right)$; $l_2 := simplify\left(\frac{\mu_2 - 1}{\lambda_2}\right)$;
$m_2 := simplify\left(\frac{\lambda_2 - 1}{\mu_2}\right)$; $l_3 := simplify\left(\frac{\mu_3 - 1}{\lambda_3}\right)$; $m_3 := simplify\left(\frac{\lambda_3 - 1}{\mu_3}\right)$;
$l_4 := simplify\left(\frac{\mu_4 - 1}{\lambda_4}\right)$; $m_4 := simplify\left(\frac{\lambda_4 - 1}{\mu_4}\right)$;

$$l_1 := \frac{k_2^2 - 1}{k_1 k_2 + 1}$$

$$m_1 := \frac{k_1^2 - 1}{k_1 k_2 + 1}$$

$$l_2 := \frac{k_3^2 - 1}{k_2 k_3 + 1}$$

$$m_2 := \frac{k_2^2 - 1}{k_2 k_3 + 1}$$

$$l_3 := \frac{k_4^2 - 1}{k_3 k_4 + 1}$$

$$m_3 := \frac{k_3^2 - 1}{k_3 k_4 + 1}$$

$$l_4 := \frac{k_1^2 - 1}{k_1 k_4 + 1}$$

$$m_4 := \frac{k_4^2 - 1}{k_1 k_4 + 1} \tag{7}$$

:= :=

:= :=

restart;

$with(PolynomialTools):$
$with(LinearAlgebra):$

Section 2

Equations in Lemma 2.12 are :

$P_1 := l_1 \cdot x_1^2 + 2 \cdot x_1 \cdot x_2 + m_1 \cdot x_2^2 - (l_1 + m_1 + 2)$

$$P_1 := l_1 x_1^2 + m_1 x_2^2 + 2 x_1 x_2 - l_1 - m_1 - 2 \tag{8}$$

$P_2 := l_2 \cdot x_2^2 + 2 \cdot x_2 \cdot x_3 + m_2 \cdot x_3^2 - (l_2 + m_2 + 2)$

$$P_2 := l_2 x_2^2 + m_2 x_3^2 + 2 x_2 x_3 - l_2 - m_2 - 2 \tag{9}$$

$P_3 := l_3 \cdot x_3^2 + 2 \cdot x_3 \cdot x_4 + m_3 \cdot x_4^2 - (l_3 + m_3 + 2)$

$$P_3 := l_3 x_3^2 + m_3 x_4^2 + 2 x_3 x_4 - l_3 - m_3 - 2 \tag{10}$$

$P_4 := l_4 \cdot x_4^2 + 2 \cdot x_4 \cdot x_1 + m_4 \cdot x_1^2 - (l_4 + m_4 + 2)$

$$P_4 := l_4 x_4^2 + m_4 x_1^2 + 2 x_4 x_1 - l_4 - m_4 - 2 \tag{11}$$

Setting the functions :

$x_1 := \frac{(1+t)^2 \cdot (1-k_1) + 1 + k_1}{2 \cdot (1+t)}; x_2 := \frac{(1+t)^2 \cdot (1+k_2) + 1 - k_2}{2 \cdot (1+t)};$
$x_3 := \frac{(1+t)^2 \cdot (1-k_3) + 1 + k_3}{2 \cdot (1+t)}; x_4 := \frac{(1+t)^2 \cdot (1+k_4) + 1 - k_4}{2 \cdot (1+t)}$

$$x_1 := \frac{(1+t)^2 (1-k_1) + 1 + k_1}{2 + 2t}$$

$$x_2 := \frac{(1+t)^2 (1+k_2) + 1 - k_2}{2 + 2t}$$

$$x_3 := \frac{(1+t)^2 (1-k_3) + 1 + k_3}{2 + 2t}$$

$$x_4 := \frac{(1+t)^2\left(1+k_4\right)+1-k_4}{2+2\,t} \tag{12}$$

Verification of the equations in Lemma 2.12 :

$simplify\left(P_1\right); simplify\left(P_2\right); simplify\left(P_3\right); simplify\left(P_4\right)$

$$0$$
$$0$$
$$0$$
$$0 \tag{13}$$

:= :=
:= :=

Section 3

Left side of equation (2.9) :

$eq := collect\Big(\Big(\big(l_2+m_2+2\big)\cdot\big(l_1\cdot x_1^{\,2}+2\cdot x_1\cdot x_2+m_1\cdot x_2^{\,2}-\big(l_1+m_1+2\big)\big)-\big(l_1+m_1+2\big)\cdot\big(\big(l_2\cdot x_2^{\,2}+2\cdot x_2\cdot x_3+m_2\cdot x_3^{\,2}-\big(l_2+m_2+2\big)\big)\big)\Big), \{x_1, x_2, x_3\}, distributed\Big)$

$$eq := \big(l_2+m_2+2\big)\,l_1\,x_1^2+\big(2\,l_2+2\,m_2+4\big)\,x_2\,x_1+\big(\big(l_2+m_2+2\big)\,m_1-\big(l_1+m_1+2\big)\,l_2\big)\,x_2^2+\big(-2\,l_1-2\,m_1-4\big)\,x_3\,x_2-m_2\,\big(l_1+m_1+2\big)\,x_3^2+\big(l_2+m_2+2\big)\,\big(-l_1-m_1-2\big)-\big(l_1+m_1+2\big)\,\big(-l_2-m_2-2\big) \tag{14}$$

Matrix of coefficients of the left side of equation (2.9) :

$M := Matrix\left(\left[\left[\left(l_2+m_2+2\right)l_1, \frac{\left(2\cdot l_2+2\cdot m_2+4\right)}{2}, 0\right], \left[\frac{\left(2\cdot l_2+2\cdot m_2+4\right)}{2}, -l_1\,l_2+m_1\,m_2-2\,l_2+2\,m_1, \frac{\left(-2\,l_1-2\,m_1-4\right)}{2}\right], \left[0, \frac{\left(-2\,l_1-2\,m_1-4\right)}{2}, -l_1\,m_2-m_2\left(m_1+2\right)\right]\right]\right)$

$$M := \begin{bmatrix} (l_2+m_2+2)\,l_1 & l_2+m_2+2 & 0 \\ l_2+m_2+2 & -l_1 l_2 + m_1 m_2 - 2 l_2 + 2 m_1 & -l_1 - m_1 - 2 \\ 0 & -l_1 - m_1 - 2 & -l_1 m_2 - m_2 (m_1+2) \end{bmatrix} \quad \textbf{(15)}$$

Verifying that the det(M) and left side of (2.10) are the same :

$$simplify\Big(Determinant(M) - (l_1 + m_1 + 2)\cdot(l_2 + m_2 + 2)\cdot\big((m_2 + 1)^2\cdot(1 - m_1\cdot l_1) - (l_1 + 1)^2\cdot(1 - l_2\cdot m_2)\big)\Big)$$

$$0 \quad \textbf{(16)}$$

:= :=
:= :=

Section 4

Equation (2.11) :

$$Eq\_2\_11 := simplify\Big(Determinant\Big(Matrix\Big(\big[[m_1, 0, l_2, 0], [2\cdot x_1, m_1, 2\cdot x_3, l_2], [l_1\cdot x_1^2 - (l_1 + m_1 + 2), 2\cdot x_1, m_2\cdot x_3^2 - (l_2 + m_2 + 2), 2\cdot x_3], [0, l_1\cdot x_1^2 - (l_1 + m_1 + 2), 0, m_2\cdot x_3^2 - (l_2 + m_2 + 2)]\big]\Big)\Big)\Big)$$

$$Eq\_2\_11 := \left(l_1^2 x_1^4 + \left(-2 l_1^2 - 4 l_1 - 4\right) x_1^2 + \left(2 + l_1\right)^2\right) l_2^2 + \Big(\big(-2 l_1 \left(m_2 x_3^2 - m_2 - 2\right) x_1^2 + 8 x_1 x_3 + 2 \left(m_2 x_3^2 - m_2 - 2\right) \left(2 + l_1\right)\big) m_1 - 4 x_1 \big(l_1 x_1^2 x_3 + \left(-m_2 x_3^2 + m_2 + 2\right) x_1 - x_3 \left(2 + l_1\right)\big)\Big) l_2 + 4 m_1 \left(\frac{\left(x_3 - 1\right)\left(x_3 + 1\right)\left(m_2 x_3 + m_2 + 2\right)\left(m_2 x_3 - m_2 - 2\right) m_1}{4} + \left(l_1 x_1^2 x_3 + \left(-m_2 x_3^2 + m_2 + 2\right) x_1 - x_3 \left(2 + l_1\right)\right) x_3\right) \quad \textbf{(17)}$$

Equation (2.12) :

$$Eq\_2\_12 := l_1\cdot x1^2\cdot x_3 - m_2\cdot x_3^2\cdot x_1 - (l_1 + 2)\cdot x_3 + (m_2 + 2)\cdot x_1$$

$$Eq\_2\_12 := l_1 x1^2 x_3 - m_2 x_3^2 x_1 - x_3 \left(2 + l_1\right) + \left(m_2 + 2\right) x_1 \quad \textbf{(18)}$$

Equation (2.11') :

$Eq\_2\_11\_pr := simplify\Big(Determinant\Big(Matrix\Big(\big[\,[l_4, 0\,, m_3\,, 0\,], [2\cdot x_1, l_4\,, 2\cdot x_3\,, m_3], [m_4\cdot x_1{}^2 - (m_4 + l_4 + 2), 2\cdot x_1, l_3\cdot x_3{}^2 - (m_3 + l_3 + 2), 2\cdot x_3], [0, m_4\cdot x_1{}^2 - (m_4 + l_4 + 2), 0, l_3\cdot x_3{}^2 - (m_3 + l_3 + 2)]\big]\Big)\Big)\Big)$

$$Eq\_2\_11\_pr := \left(l_3^2 x_3^4 + \left(-2\, l_3^2 - 4\, l_3 - 4\right) x_3^2 + \left(2 + l_3\right)^2\right) l_4^2 + \left(\left(-2\, m_4 \left(l_3\, x_3^2 - l_3 - 2\right) x_1^2 + 8\, x_1\, x_3 + 2\left(m_4 + 2\right)\left(l_3\, x_3^2 - l_3 - 2\right)\right) m_3 - 4\left(-m_4\, x_3\, x_1^2 + \left(l_3\, x_3^2 - l_3 - 2\right) x_1 + x_3\left(m_4 + 2\right)\right) x_3\right) l_4 + 4\, m_3 \left(\frac{\left(x_1 - 1\right)\left(x_1 + 1\right)\left(m_4\, x_1 + m_4 + 2\right)\left(m_4\, x_1 - m_4 - 2\right) m_3}{4} + x_1\left(-m_4\, x_3\, x_1^2 + \left(l_3\, x_3^2 - l_3 - 2\right) x_1 + x_3\left(m_4 + 2\right)\right)\right) \quad \textbf{(19)}$$

Equation (2.12'):

$Eq\_2\_12\_pr := m_4\cdot x_1{}^2\cdot x_3 - l_3\cdot x_3{}^2\cdot x_1 - (m_4 + 2)\cdot x_3 + (l_3 + 2)\cdot x_1$

$$Eq\_2\_12\_pr := m_4\, x_1^2\, x_3 - l_3\, x_1\, x_3^2 - \left(m_4 + 2\right) x_3 + x_1\left(l_3 + 2\right) \quad \textbf{(20)}$$

Comperison of coefficients of the equation $\alpha\cdot Eq\_2\_11 - Eq\_2\_11\_pr$:

$Eq := collect\big(simplify(\alpha\cdot Eq\_2\_11 - Eq\_2\_11\_pr), \{x_1, x_2, x_3\}, distributed\big)$

$$Eq := \left(\alpha\, l_1^2\, l_2^2 - m_3^2\, m_4^2\right) x_1^4 + \left(-4\,\alpha\, l_1\, l_2 + 4\, m_3\, m_4\right) x_3\, x_1^3 + \left(-2\, l_2\left(\left(l_1^2 + 2\, l_1 + 2\right) l_2 - \left(2 + m_2\right)\left(m_1\, l_1 - 2\right)\right)\alpha - 2\, m_3\left(m_4\left(2 + l_3\right) l_4 + \left(-m_4^2 - 2\, m_4 - 2\right) m_3 - 2\, l_3 - 4\right)\right) x_1^2 + \left(\left(\left(-2\, l_1\, m_1\, m_2 + 4\, m_2\right) l_2 + 4\, m_1\, l_1\right)\alpha + \left(2\, l_3\, m_3\, m_4 - 4\, m_4\right) l_4 - 4\, l_3\, m_3\right) x_3^2\, x_1^2 + \left(-4\,\alpha\, m_1\, m_2 + 4\, l_3\, l_4\right) x_1\, x_3^3 + \left(-4\left(\left(-l_1 - 2\, m_1 - 2\right) l_2 - m_1\left(2 + m_2\right)\right)\alpha - 4\left(l_3 + 2\, m_3 + 2\right) l_4 - 4\, m_3\left(m_4 + 2\right)\right) x_1\, x_3 + \left(\alpha\, m_1^2\, m_2^2 - l_3^2\, l_4^2\right) x_3^4 + \left(2\, m_1\left(m_2\left(2 + l_1\right) l_2 + \left(-m_2^2 - 2\, m_2 - 2\right) m_1 - 2\, l_1 - 4\right)\alpha + 2\, l_4\left(\left(l_3^2 + 2\, l_3 + 2\right) l_4 - \left(m_4 + 2\right)\left(l_3\, m_3 - 2\right)\right)\right) x_3^2 + \left(\left(2 + l_1\right) l_2 - m_1\left(2 + m_2\right)\right)^2\alpha - \left(\left(2 + l_3\right) l_4 - m_3\left(m_4 + 2\right)\right)^2 \quad \textbf{(21)}$$

:= :=

:= :=

Section 5

Equation (2.25) and $\alpha = 1$:

$$l_2 := \frac{(m_3+1)\cdot(l_4+1)}{m_1+1} - 1;\ l_3 := \frac{m_1\cdot m_2}{l_4};\ m_4 := \frac{l_1\cdot(m_3\cdot l_4 + m_3 + l_4 - m_1)}{m_3\cdot(m_1+1)};\ \alpha := 1$$

$$l_2 := \frac{(m_3+1)\,(l_4+1)}{m_1+1} - 1$$

$$l_3 := \frac{m_1\,m_2}{l_4}$$

$$m_4 := \frac{l_1\,(l_4\,m_3 + l_4 - m_1 + m_3)}{m_3\,(m_1+1)}$$

$$\alpha := 1 \tag{22}$$

Expression A :

$$A := (m_3+1)\cdot(l_1+1)\cdot(l_4+1) - (m_1+1)\cdot(m_1\cdot m_2 + l_1 + l_4 + m_1 - m_3 + 1)$$

$$A := (m_3+1)\,(l_1+1)\,(l_4+1) - (m_1+1)\,(m_1\,m_2 + l_1 + l_4 + m_1 - m_3 + 1) \tag{23}$$

Expression B :

$$B := m_1\cdot m_2 + m_2\cdot m_3 + l_1 + m_2 + l_4 + m_1 + 2$$

$$B := m_1\,m_2 + m_2\,m_3 + l_1 + l_4 + m_1 + m_2 + 2 \tag{24}$$

Substitution (2.25) in (2.22) and comparing with required equation :

$$\mathit{simplify}\left(\frac{(m_1+1)^2}{4}\cdot\left(\left((2+l_1)\,l_2 - m_1\,(2+m_2)\right)^2\alpha - \left(\left((2+l_3)\,l_4 - m_3\,(m_4+2)\right)^2\right)\right) - (m_1 + m_3 + 2)\cdot(l_4 - m_1)\cdot A\right)$$

$$0 \tag{25}$$

Substitution (2.25) in (2.23) and comparing with required equation :

$$simplify\left(\frac{m_3\cdot(m_1+1)}{4}\cdot\left(2\,m_1\left(m_2\left(2+l_1\right)l_2+\left(-m_2^2-2\,m_2-2\right)m_1-2\,l_1-4\right)\alpha+2\,l_4\left(\left(l_3^2+2\,l_3+2\right)l_4-\left(m_4+2\right)\left(l_3\,m_3-2\right)\right)\right)-\left(l_4-m_1\right)\cdot\left(A+\left(m_3+1\right)\cdot m_1\cdot B\right)\right)$$

$$0 \tag{26}$$

:= :=
:= :=

Section 6

Expression $A+(m_1-m_3)\cdot B$:

$$factor\left(A+\left(m_1-m_3\right)\cdot B\right)$$

$$\left(m_3+1\right)\left(l_1\,l_4-m_2\,m_3\right) \tag{27}$$