MA3650 - Numerical Methods for Differential Equations

Luke Dando

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satisfy $L_i(x_i) = 1$ and $L_i(x_j) = 0$ for $j \neq i$. Now, given $n+1$ points $(x_i, y_i) \in \mathbb{R}^2$, $0 \leq i \leq n$, with $x_i \neq x_j$ when $i \neq j$. The Lagrange polynomial p is a polynomial of degree up to n equal to

$$p(x) = \sum_{i=0}^{n} y_i L_i(x). \tag{2}$$

This is *linear* if $n = 1$ and *quadratic* if $n = 2$.

18.2 Vandermonde Method

Given $p(x) = ax^2 + bx + c$, with a, b and c to be determined from the conditions $p(x_i) = y_i$, $i = 0, 1, 2$, we can input this into a matrix know as the Vandermonde matrix and solve it for values a, b and c.

$$\begin{pmatrix} x_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix}. \tag{3}$$

This can be extended for larger polynomials trivially.