

# MA3650 - Numerical Methods for Differential Equations

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March 10, 2021

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satisfy  $L_i(x_i) = 1$  and  $L_i(x_j) = 0$  for  $j \neq i$ . Now, given  $n + 1$  points  $(x_i, y_i) \in \mathbb{R}^2$ ,  $0 \leq i \leq n$ , with  $x_i \neq x_j$  when  $i \neq j$ . The Lagrange polynomial  $p$  is a polynomial of degree up to  $n$  equal to

$$p(x) = \sum_{i=0}^n y_i L_i(x). \quad (2)$$

This is *linear* if  $n = 1$  and *quadratic* if  $n = 2$ .

## 18.2 Vandermonde Method

Given  $p(x) = ax^2 + bx + c$ , with  $a$ ,  $b$  and  $c$  to be determined from the conditions  $p(x_i) = y_i$ ,  $i = 0, 1, 2$ , we can input this into a matrix known as the Vandermonde matrix and solve it for values  $a$ ,  $b$  and  $c$ .

$$\begin{pmatrix} x_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix}. \quad (3)$$

This can be extended for larger polynomials trivially.