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Neural network control of flexible-link manipulators using sliding mode

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Abstract

This paper focuses on tracking control problem of flexible-link manipulators. In order to alleviate the effects of nonlinearities and uncertainties, a combined control strategy based on neural network (NN) and the concept of sliding mode control (SMC) is proposed systematically. The chattering phenomenon in conventional SMC is eliminated by incorporated a saturation function in the proposed controller, and the computation burden caused by model dynamics is reduced by applying a two-layer NN with an analytical approximated upper bound, which is used to implement a certain functional estimate. In addition, the Lyapunov analysis can guarantee the signals of closed-loop system bounded and the online NN adaptive laws can make the system states converge to the sliding surface. Furthermore, the boundary layer thickness as well as the gain of corrective control term is also discussed in detail. At last, the theoretic results are validated on the flexible-link manipulator experimental system in Tsinghua University.

Keywords: Flexible-link manipulators; Sliding mode control; Neural network; Lyapunov theory

1. Introduction

Flexible-link manipulators are being intensively studied due to the challenging requirements of fast and precise tip-position control in various industry and space applications. Their potential advantages over rigid ones include less mass, faster operation, lower energy consumption, higher load-carrying capability, wider operation range and so on. On the contrary, structural flexibility also gives rise to elastic deflections, distributed parameters, nonlinearity, strong coupling, nonminimum phase etc. Therefore, how to improve control performance under disturbances becomes a major issue. As a powerful method of tackling uncertain nonlinear systems, sliding mode control (SMC) [8] has been widely used in the control of flexible-link manipulators, such as [15,3,7,12]. In conventional SMC, two inherent difficulties must be confronted: (1) the chattering phenomenon resulted from delay of switching and imprecision of sliding surface in practice [8,16]; (2) the complete knowledge of the plant dynamics [10,17] employed to design SMC controller.

To produce chattering-free sliding mode, Bartolini et al. [2] utilized higher sliding mode in the SMC controller, but this method often demands that the higher-order derivatives of the state variables are available for measurement. Otherwise, observers are needed consequently. In [11], Khalil gave an alternative method to reduce chattering by decreasing the amplitude of the corrective term, however, this algorithm is unable to eliminate chattering completely. Therefore, in this paper we employ the boundary layer method [9,16] to avoid chattering, i.e. a saturation function is incorporated in the proposed controller to smooth the control output.

Since the traditional SMC design is a model-based control approach, the partial knowledge of model dynamics will deteriorate the control performance. In an effort to avoid the complete knowledge of the plant dynamics, some researchers used NNs to estimate plant dynamics. Sundareshan and Askew [17] developed an NN-based scheme for adaptively accomplishing a variable structure controller to drive a flexible manipulator arm, but the NN is only used for payload identification and also needs the off-line training with the back-propagation

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algorithm. With an adaptive bound estimation algorithm Wai [18] presented a sliding-mode neural network (NN) controller for rigid-link manipulators, however, two facts should not be ignored: (1) there is a sign function in the robust term, which may cause chattering of controller output; (2) the upper bound of the uncertain term is only depended on the sliding surface, in fact, the estimated error bounds may be affected by network architecture. Ertugrul and Kaynak [6] utilized two parallel NNs to realize a neuro-SMC, which can be regarded as an inverse dynamics implementation using NN based on gradient descent method, and yet the stability analysis of closed-loop system is not carried out. Moreover, the neuro-SMC cannot be directly extended to flexible-link manipulators, where the number of control variables is strictly less than the number of mechanical degrees of freedom. In this paper, a two-layer NN with an analytical approximated upper bound is applied to estimating a certain function existed in the proposed controller, and the online NN adaptive laws can guarantee the stability of the closed-loop system in the sense of Lyapunov. Furthermore, the whole design process does not involve the prior knowledge of the flexible-link manipulators, such as the positive definite symmetry

property of inertia matrix and the skew-symmetry property of coriolis/centripetal matrix and inertia matrix. As a result, the composite controller cannot only eliminate the chattering phenomenon, but also make the system dynamics converge to the sliding surface.

This paper is organized as follows. Section 2 gives a brief description of the dynamic model and output redefinition of flexible-link manipulators. In Section 3 the proposed controller is presented in detail and the closed-loop stability analysis is also carried out based on Lyapunov theory. Section 4 implements the controller on the flexible-link manipulator experimental system in Tsinghua University. Finally, conclusions are drawn in Section 5.

2. Dynamic model

The closed-form dynamic equations of a flexible-link manipulator obtained from Lagrangian approach have the following form [4,5]:

$$\begin{bmatrix} M_{\theta\theta} & M_{\theta\delta} \\ M_{\theta\delta}^T & M_{\delta\delta} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} C_{\theta\theta} & C_{\theta\delta} \\ C_{\delta\theta} & C_{\delta\delta} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} F_\theta \\ F_\delta \end{bmatrix} + \begin{bmatrix} G_\theta \\ G_\delta \end{bmatrix} + \begin{bmatrix} \tau_{d\theta} \\ \tau_{d\delta} \end{bmatrix} = \begin{bmatrix} \tau \\ \mathbf{0} \end{bmatrix}, \tag{1}$$

where $\theta \in R^m$ is the vector of joint position variables, $\delta \in R^q$ is the vector of flexible modes, $M_{\theta\theta}$, $M_{\theta\delta}$, $M_{\delta\delta}$ are the blocks of inertia matrix M, $C_{\theta\theta}$, $C_{\theta\delta}$, $C_{\delta\theta}$ and $C_{\delta\delta}$ are the blocks of Coriolis and centrifugal matrix C, F_θ and F_δ are the components of the friction vector F, G_θ and G_δ are the components of the gravity vector G, $\tau_{d\theta}$ and $\tau_{d\delta}$ are the bounded un-modeled dynamics, τ is the vector of control torques.

The dynamic equations (1) can also be rewritten as

$$M_{\theta\theta}\ddot{\theta} + M_{\theta\delta}\ddot{\delta} + C_{\theta\theta}\dot{\theta} + C_{\theta\delta}\dot{\delta} + F_\theta + G_\theta + \tau_{d\theta} = \tau, \tag{2}$$

$$M_{\theta\delta}^{T}\ddot{\theta} + M_{\delta\delta}\ddot{\delta} + C_{\delta\theta}\dot{\theta} + C_{\delta\delta}\dot{\delta} + F_\delta + G_\delta + \tau_{d\delta} = \mathbf{0}. \tag{3}$$

From (3) we can obtain

$$\ddot{\delta} = -M_{\delta\delta}^{-1}\left(M_{\theta\delta}^{T}\ddot{\theta} + C_{\delta\theta}\dot{\theta} + C_{\delta\delta}\dot{\delta} + F_\delta + G_\delta\right) - M_{\delta\delta}^{-1}\tau_{d\delta}. \tag{4}$$

Substituting (4) into (2) yields

$$\begin{aligned} &M_{\theta\theta}\ddot{\theta} + C_{\theta\theta}\dot{\theta} - M_{\theta\delta}M_{\delta\delta}^{-1}\left(M_{\theta\delta}^{T}\ddot{\theta} + C_{\delta\theta}\dot{\theta} + C_{\delta\delta}\dot{\delta} + F_\delta + G_\delta\right) \\ &\quad + C_{\theta\delta}\dot{\delta} + F_\theta + G_\theta - M_{\theta\delta}M_{\delta\delta}^{-1}\tau_{d\delta} + \tau_{d\theta} = \tau. \end{aligned} \tag{5}$$

The tip-position of the link i can be expressed as $y_i = \theta_i + \eta_i d_i/l_i$ $(i = 1, \ldots, m)$, where $\eta_i = 0$ corresponds to the joint position and $\eta_i = 1$ corresponds to the tip position, l_i is the length of flexible link i, d_i is the tip deflection of link i, $d_i = \sum_{j=1}^{N_i} \phi_{ij}(x_i)\delta_{ij}(t)$, $\phi_{ij}(x_i)$ is the spatial mode shape function [4], $\delta_{ij}(t)$ is the jth flexible mode of link i, N_i is the number of flexible modes of link i. Thereby, the dimension of flexible modes $q = m(N_1 + \cdots + N_m)$.

3. NN control using sliding mode

Generally speaking, NN can approximate any complicated nonlinear function and SMC is robust to parametric uncertainties. The flowing parts will show how these two control methods are combined.

3.1. Basic idea of NN approximation

Here, we employ a simple two-layer NN to approximate a general smooth nonlinear function on a compact set $S \in R^p$. According to the NN approximation property, we have

$$f(x) = W^{T}\sigma\left(V^{T}x\right) + \varepsilon(x), \tag{6}$$

where $x = [1 \;\; x_1 \cdots x_p]^T$ is the input to NN, $\sigma(\cdot)$ is an active function, W and V are defined as the collection of NN weights for output and hidden layer, respectively, and $\varepsilon(x)$ is the NN approximation error. Including "1" in x allows one to incorporate the thresholds as the first column of V^T, and then, any tuning of NN weights also involves tuning of thresholds as well. For convenience of narration, let us define NN weight error $\tilde{W} = W - \hat{W}$, $\tilde{V} = V - \hat{V}$ ("$\wedge$" represents estimation value), denote the norm of a vector x is defined by $\|x\| = \sqrt{x^T x}$, and the norm of a vector function $f(x)$ is $\|f(x)\| = \sup\{|f(x)| : x \in R^{p+1}\}$.

Assuming that $f(x)$ is absolutely integrable, i.e.

$$\int_{-\infty}^{\infty} |f(x)|\,dx \leqslant c, \tag{7}$$

where c is a sufficiently large positive constant. We can establish the following approximation theorem.

Theorem 1. *For every function* $f(x)$ *satisfying* (7) *and every sigmoidal function* $\sigma(z) = 1/(1+\exp(-z))$, *if* $\xi \geqslant \sqrt{n}\ln n$ (*n is the number of the hidden-layer neurons*), *there exists an NN functional estimate* $f_n(x) = W^{\mathrm{T}}\sigma(V^{\mathrm{T}}x) \in G_{\sigma,\xi}$ *such that*

$$\|\varepsilon(x)\| \leqslant O(n^{-1/2}) + \|f(0)\|, \tag{8}$$

where $G_{\sigma,\xi} = \{\gamma\sigma(\xi(ax+b)) : |\gamma| \leqslant 2c, |a|_B \leqslant 1, |b| \leqslant 1\}$, $|a|_B = \sup_{x\in B}|a\cdot x|$, B *is a bounded set in* R^{p+1}, $O(n^{-1/2})$ *denotes the order* $n^{-1/2}$ *approximation.*

Proof. Consider (6) and let

$$\bar{f}(x) = f(x) - f(0)$$

the proof of Theorem 1 can be completed based on Theorem 3 in [1]. □

Remark 1. Theorem 1 indicates that, for a special two-layer NN, the upper bound of NN functional reconstruction error is affected by NN structure (i.e. the number of hidden-layer neurons) and also related to the initial value of the estimated function.

3.2. Conventional SMC design

As a nonlinear robust control approach, SMC should finish the following two subtasks. One is determining the sliding surface which can be regarded as the desired system dynamics, the other is designing the equivalent control [1] which can force the system dynamics to converge to the sliding surface.

Usually, the sliding surface function can be selected as [16]

$$s = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)^r (x_\mathrm{d} - x), \tag{9}$$

where λ is a strictly positive constant, x_d is the desired system state and x is the actual system state. The order of sliding surface r is often less 1 than the one of x, so as to derive the equivalent controller according to system dynamics. Although higher-order sliding mode may counter the chattering phenomenon [9], complicated computation based on exact knowledge of system dynamics is also required. To avoid this, a simple selection of the sliding surface for the flexible-link manipulator is considered.

$$s = x_\mathrm{d} - x, \tag{10}$$

where $x = \theta + \Phi\delta$ are the tip positions of flexible links, $\Phi = diag\{\Phi_1, \Phi_2, \ldots, \Phi_m\}$, $\Phi_i = [\phi_{i1}, \phi_{i2}, \ldots, \phi_{iN_i}]$, $\delta = [\delta_{11}, \delta_{12}, \ldots, \delta_{1N_1}, \ldots, \delta_{m1}, \delta_{m2}, \ldots, \delta_{mN_m}]^{\mathrm{T}}$.

The equivalent control term can be obtained by setting the derivative of (10) to zero based on the standard SMC design technique. Utilizing (5) gives

$$\dot{s} = \dot{x}_\mathrm{d} - \dot{\theta} - \Phi\dot{\delta} = \dot{x}_\mathrm{d} - \dot{\theta} - \Phi\dot{\delta} - \tau + f_1 = f - \tau, \tag{11}$$

where $f_1 = M_{\theta\theta}\ddot{\theta} + C_{\theta\theta}\dot{\theta} + M_{\theta\delta}S + C_{\theta\delta}\dot{\delta} + F_\theta + G_\theta - M_{\theta\delta} M_{\delta\delta}^{-1}\tau_{\mathrm{d}\delta} + \tau_{\mathrm{d}\theta}$, $f = \dot{x}_\mathrm{d} + f_1 - \dot{\theta} - \Phi\dot{\delta}$. In order to satisfy the reaching condition $s\dot{s}^{\mathrm{T}} \leqslant 0$ [8], a corrective control term $K_1\,\mathrm{sgn}(s)$ (K_1 is positive-definite diagonal matrix) is also needed. Thus, the standard SMC controller can be obtained.

$$\tau = f + K_1\,\mathrm{sgn}(s). \tag{12}$$

For purpose of design integrity, a simple stability analysis based on Lyapunov theory is carried out. Define the Lyapunov function candidate

$$V_1 = ss^{\mathrm{T}}/2. \tag{13}$$

Differentiating (13), using the dynamics of (11) and the controller (12) produces

$$\dot{V}_1 = s\dot{s}^{\mathrm{T}} = -sK_1\,\mathrm{sgn}(s) = -K_1|s| \leqslant 0$$

which means that the SMC controller (12) can maintain the asymptotical stability of the closed-loop system. Unfortunately, the nonlinear function f is often not known exactly due to the uncertain system dynamics and the corrective term in (12) may result in chattering phenomenon. Even with known f, the computation burden of the equivalent control cannot be avoided. To deal with the chattering phenomenon and complicated computation in conventional SMC design, we use a saturation function and an NN to reconstruct the SMC controller.

3.3. NN design with SMC

The saturation function is chosen as

$$sat(s) = \begin{cases} s/L, & \|s\| \leqslant L, \\ \mathrm{sgn}(s/L), & \|s\| > L \end{cases} \tag{14}$$

which is shown in Fig. 1. Fig. 2 illustrates the boundary of system state, where $s = 0$ means that state variable x equals to the desired trajectory x_d. With the similar idea of controller (12), the proposed sliding-mode NN controller is of the following form:

$$\tau = \hat{f} + K_2\,sat(s). \tag{15}$$

where $\hat{f} = \hat{W}^{\mathrm{T}}\sigma(\hat{V}^{\mathrm{T}}x)$ is a certain two-layer NN estimate, $K_2 = diag\{k_{21}, k_{22}, \ldots, k_{2m}\}$ is a positive-definite diagonal

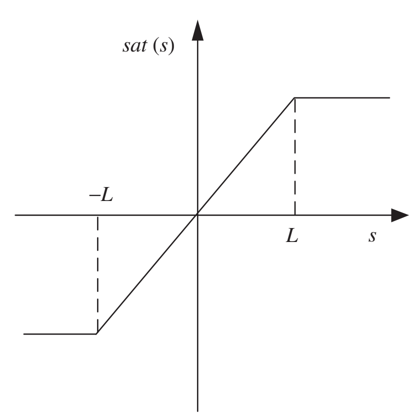

Fig. 1. The saturation function.

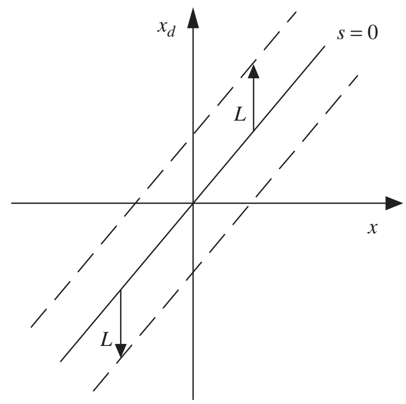

Fig. 2. The boundary layer of system state.

gain matrix which should guarantee that the sliding surface is attractable in a wide range.

Based on the norm bound properties of flexible-link manipulators [14] and the linearization transformation [13] on $\hat{V}^{\mathrm{T}}x$ of the sigmoid function (i.e. the active function)

$$\sigma\left(V^{\mathrm{T}}x\right) = \sigma\left(\hat{V}^{\mathrm{T}}x\right) + \sigma'\left(\hat{V}^{\mathrm{T}}x\right)\tilde{V}^{\mathrm{T}}x + O\left(\hat{V}^{\mathrm{T}}x\right)^2 \tag{16}$$

with $\sigma'\left(\hat{V}^{\mathrm{T}}x\right) = \partial\sigma(z)/\partial z|_{z=\hat{V}^{\mathrm{T}}x}$, the whole controller design and the stability analysis of closed-loop system can be summarized in Theorem 2.

Theorem 2. Consider the dynamic system described by (1) and the sliding surface (10), for the bounded, continuous desired tip trajectory with bounded velocity, controller (15) can guarantee the boundedness of the close-loop system signals. And the NN adaptive laws are given by

$$\dot{\hat{W}} = \alpha\sigma\left(\hat{V}^{\mathrm{T}}x\right)s^{\mathrm{T}} - \alpha\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\hat{V}^{\mathrm{T}}xs^{\mathrm{T}}, \tag{17}$$

$$\dot{\hat{V}} = \beta xs^{\mathrm{T}}\hat{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right), \tag{18}$$

where α, β are positive constants. Moreover, the system states converge to the sliding surface in a wide range.

Proof. Without using the special properties of flexible-link manipulators, we choose the following Lyapunov function candidate

$$V_L = s^{\mathrm{T}}s/2 + \mathrm{tr}\left(\tilde{W}^{\mathrm{T}}\tilde{W}\right)/2\alpha + \mathrm{tr}\left(\tilde{V}^{\mathrm{T}}\tilde{V}\right)/2\beta. \tag{19}$$

Computing the derivative of (19) along the sliding surface (10) yields

$$\dot{V}_L = s^{\mathrm{T}}\dot{s} + \mathrm{tr}\left(\tilde{W}^{\mathrm{T}}\dot{\tilde{W}}\right)/\alpha + \mathrm{tr}\left(\tilde{V}^{\mathrm{T}}\dot{\tilde{V}}\right)/\beta.$$

Considering (6), (11) and (15) we have

$$\begin{aligned}\dot{V}_L &= s^{\mathrm{T}}\left(W^{\mathrm{T}}\sigma\left(V^{\mathrm{T}}x\right) + \varepsilon(x) - \hat{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right) - K_2\,sat(s)\right)\\ &\quad + \mathrm{tr}\left(\tilde{W}^{\mathrm{T}}\dot{\tilde{W}}\right)/\alpha + \mathrm{tr}\left(\tilde{V}^{\mathrm{T}}\dot{\tilde{V}}\right)/\beta.\end{aligned}$$

By taking (16) into account, we obtain that

$$\begin{aligned}&W^{\mathrm{T}}\sigma\left(V^{\mathrm{T}}x\right) + \varepsilon(x) - \hat{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right) - K_2\,sat(s)\\ &= W^{\mathrm{T}}\sigma\left(V^{\mathrm{T}}x\right) - \hat{W}^{\mathrm{T}}\sigma\left(V^{\mathrm{T}}x\right) + \hat{W}^{T}\sigma\left(V^{\mathrm{T}}x\right)\\ &\quad - \hat{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right) + \varepsilon(x) - K_2\,sat(s)\\ &= \tilde{W}^{\mathrm{T}}\sigma\left(V^{\mathrm{T}}x\right) + \hat{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right) + \varepsilon(x) - K_2\,sat(s)\\ &= \tilde{W}^{\mathrm{T}}\left(\sigma\left(\hat{V}^{\mathrm{T}}x\right) + \sigma'\left(\hat{V}^{\mathrm{T}}x\right)\tilde{V}^{T}x + O\left(\hat{V}^{\mathrm{T}}x\right)^2\right)\\ &\quad + \hat{W}^{\mathrm{T}}\left(\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\tilde{V}^{\mathrm{T}}x + O\left(\tilde{V}^{\mathrm{T}}x\right)^2\right) + \varepsilon - K_2\,sat(s)\\ &= \tilde{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right) - \tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\hat{V}^{\mathrm{T}}x + \hat{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\tilde{V}^{\mathrm{T}}x\\ &\quad + \tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)V^{\mathrm{T}}x + WO\left(\tilde{V}^{\mathrm{T}}x\right)^2 + \varepsilon(x) - K_2\,sat(s)\end{aligned}$$

Since

$$s^{\mathrm{T}}\tilde{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right) = \mathrm{tr}\left(\tilde{W}^{\mathrm{T}}\sigma\left(\hat{V}^{\mathrm{T}}x\right)s^{\mathrm{T}}\right),$$

$$s^{\mathrm{T}}\tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\hat{V}^{\mathrm{T}}x = \mathrm{tr}\left(\tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\hat{V}^{\mathrm{T}}xs^{\mathrm{T}}\right),$$

$$s^{\mathrm{T}}\hat{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\tilde{V}^{\mathrm{T}}x = \mathrm{tr}\left(\tilde{V}^{\mathrm{T}}xs^{\mathrm{T}}\tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\right)$$

we have

$$\begin{aligned}\dot{V}_L &= \mathrm{tr}\left(\tilde{W}^{\mathrm{T}}\left(\alpha^{-1}\dot{\tilde{W}} + \sigma\left(\hat{V}^{\mathrm{T}}x\right)s^{\mathrm{T}} - \sigma'\left(\hat{V}^{\mathrm{T}}x\right)\hat{V}^{\mathrm{T}}xs^{\mathrm{T}}\right)\right)\\ &\quad + \mathrm{tr}\left(\tilde{V}^{\mathrm{T}}\left(\beta^{-1}\dot{\tilde{V}} + xs^{\mathrm{T}}\hat{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)\right)\right)\\ &\quad + s^{\mathrm{T}}\left(\tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)V^{\mathrm{T}}x + WO\left(\tilde{V}^{\mathrm{T}}x\right)^2 + \varepsilon(x) - K_2\,sat(s)\right)\end{aligned}$$

If the adaptive laws (17) and (18) are adopted

$$\dot{V}_L = s^{\mathrm{T}}\left(\tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)V^{\mathrm{T}}x + WO\left(\tilde{V}^{\mathrm{T}}x\right)^2 + \varepsilon(x) - K_2\,sat(s)\right)$$

Consider (8) and assume

$$\|\tilde{W}^{\mathrm{T}}\sigma'\left(\hat{V}^{\mathrm{T}}x\right)V^{\mathrm{T}}x\| \leqslant \varepsilon_1, \quad \|WO\left(\tilde{V}^{\mathrm{T}}x\right)^2\| \leqslant \varepsilon_2$$

and then we can discuss the value of K_2.

In the region $\|s\| > L$, we have

$$\begin{aligned}\dot{V}_L &\leqslant \|s\|(\|\varepsilon_1 + \varepsilon_2\| + \|O\left(n^{-1/2}\right)\| + \|f(0)\|)\\ &\quad - \sum_{i=1}^{m} k_{2i}\|s_i\| \leqslant (\|\varepsilon_1 + \varepsilon_2\| + \|O\left(n^{-1/2}\right)\|\\ &\quad + \|f(0)\|)(\|s_1\| + \cdots + \|s_m\|) - \sum_{i=1}^{m} k_{2i}\|s_i\|.\end{aligned}$$

Thus, for $i = 1, \ldots, m$

$$k_{2i} > \|O(n^{-1/2})\| + \|f(0)\| + \|\varepsilon_1 + \varepsilon_2\| \tag{20}$$

can guarantee $\dot{V}_L \leqslant 0$.

On the other hand, in the region $\|s\| \leqslant L$, we obtain

$$\dot{V}_L \leqslant \|s\| \left(\|\varepsilon_1 + \varepsilon_2\| + \|f(0)\| + \left\| O\left(n^{-1/2}\right) \right\| \right) - \sum_{i=1}^{m} k_{2i} \|s_i\|^2 / L \leqslant (\|\varepsilon_1 + \varepsilon_2\| + \|f(0)\| + \|O\left(n^{-1/2}\right)\|)(\|s_1\| + \cdots + \|s_m\|) - \sum_{i=1}^{m} k_{2i}\|s_i\|^2/L.$$

For

$$L\left(\|\varepsilon_1 + \varepsilon_2\| + \|f(0)\| + \left\|O\left(n^{-1/2}\right)\right\| \right) \Big/ \min_i (k_{2i}) < \|s\| \leqslant L$$

$$\dot{V}_L \leqslant 0. \quad \square \tag{21}$$

Remark 2. If (20) and (21) are satisfied, then $\dot{V}_L < 0$ for $\|s\| \neq 0$. This also implies that $\|s\|$ is bounded and can be kept as small as desired by increasing the gains of K_2, i.e., the proposed controller (15) can make the system states converge to the sliding surface .

Remark 3. K_2 is an important parameter matrix, which can be effected by $f(0)$, n, W and V. If K_2 is properly selected, Theorem 2 shows that $\varepsilon_1 \to 0$ and $\varepsilon_2 \to 0$. Therefore, the value of K_2 should not be less than $(\|O(n^{-1/2})()\| + \|f(0)\|)$, which depends on the property of NN approximation and the initial value of system dynamics.

Remark 4. If $\Delta = \|O(n^{-1/2})\| + \|f(0)\| + \|\varepsilon_1 + \varepsilon_2\|$ is known, from (21) we can obtain the analytical lower bound of $\|s\|$. Unfortunately, the boundedness of uncertainty is not known as a priori, so the boundary layer thickness in our proposed method still needs some trial and error in order to obtain the desired performance.

4. Experimental results

In this section the proposed control strategy will be implemented on the two-link flexible manipulator test bed of Tsinghua University shown in Fig. 3. Both flexible links are driven by dc motors mounted at hubs. To counteract the gravity of the two flexible links, an adjustable counterweight is installed at the other end.

A list of the main characteristics of the practical implementation is given as below. The first flexible link is an antirust aluminum beam (LF2 M) with the following parameters: dimensions 73 cm × 0.8 cm × 0.23 cm, link uniform density $\rho_1 = 2.68 \times 10^3\,\text{kg/m}^3$, flexural link rigidity $E_1 I_1 = 4.662\,\text{N/m}^2$. The second link is a duralumin beam (6012-T4) with the following parameters: dimensions 43 cm × 0.4 cm × 0.015 cm, link uniform density $\rho_2 = 2.70 \times 10^3\,\text{kg/m}^3$, flexural link rigidity $E_1 I_1 = 0.28575\,\text{N/m}^2$. The payload mass at the second link tip is $m_p = 0.06\,\text{kg}$. In order to detect the robot tip vibrations, two JN-06E accelerometers are attached to the tips of the links for tip-acceleration measurement and these signals are further converted into position signals by a low-pass filter, a high-pass filter and a double integration circuit because the operating range of the tip sensor is limited in the experimental test bed. Additionally, two separate analog input and output channels are included. Each analog input contains a low-pass filter to limit noise and provide anti-aliasing protection and a 12-b analog/digital (A/D) converter. Each analog output consists of a low-pass filter to smooth output signal and an 8-b digital/analog (D/A) converter. The sampling period is 5 ms.

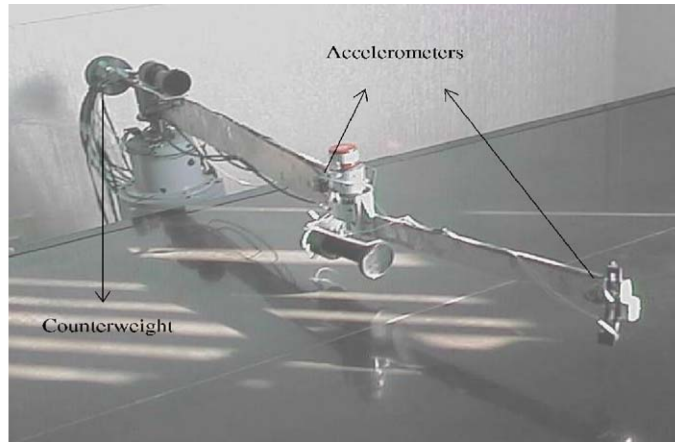

Fig. 3. Flexible-link manipulator experimental system.

As for the control goal, the final desired tip position of each flexible link

$$x_{1d} = 40\sin(0.5t), \quad x_{2d} = 70\sin(0.8t) \tag{22}$$

should be reached. In this experiment, NN has five hidden-layer neurons and two output neurons. The inputs to the NN are given by $[1, s, x, x_d, y]^T$, where $y = [y_1, y_2]^T$ are the tip deflections of the two links, and the initial NN weights are all zeros. The other parameters $\alpha = 5$, $\beta = 5$, $K_2 = diag\{10, 20\}$.

To investigate the change of control output caused by boundary layer thickness L, two cases are considered: (1) $L = 1^\circ$ and (2) $L = 10^\circ$. It should be also noted that $L = 1^\circ$ is equivalent to $\pi \times 0.73 \times 1/180$ rad tip deflection of the first link or $\pi \times 0.43 \times 1/180$ rad tip deflection of the second link.

Experimental results for $L = 1^\circ$ are shown in Figs. 4–7. Tip tracking errors of both links are depicted in Figs. 4 and 5, respectively. Figs. 6 and 7 illustrate the control outputs of two links. As for $L = 10^\circ$, Figs. 8 and 9 are the experimental results of tip tracking errors and the corresponding control outputs can be seen from Figs. 10 and 11.

From the above experimental results we can see that:

(1) the proposed SMC controller (15) based on NN can stabilize the close-loop system of the flexible-link manipulator and can give satisfied performances even without known system dynamics, while the conventional SMC controller (9) cannot be implemented because f is unknown exactly;

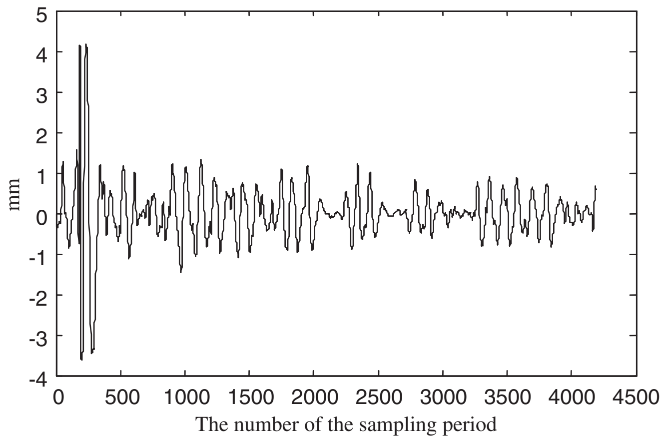

Fig. 4. Tip tracking error of the first link.

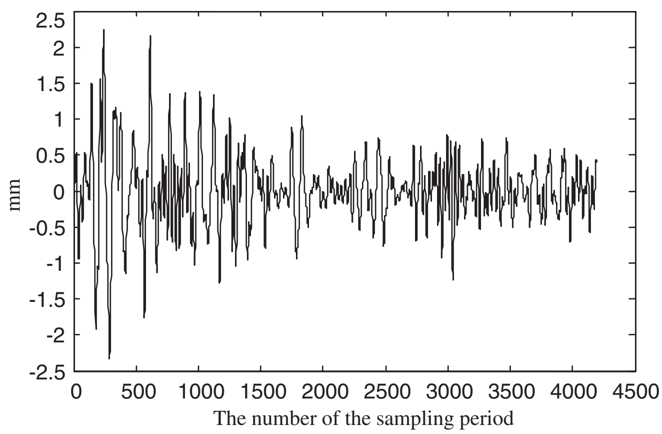

Fig. 5. Tip tracking error of the second link.

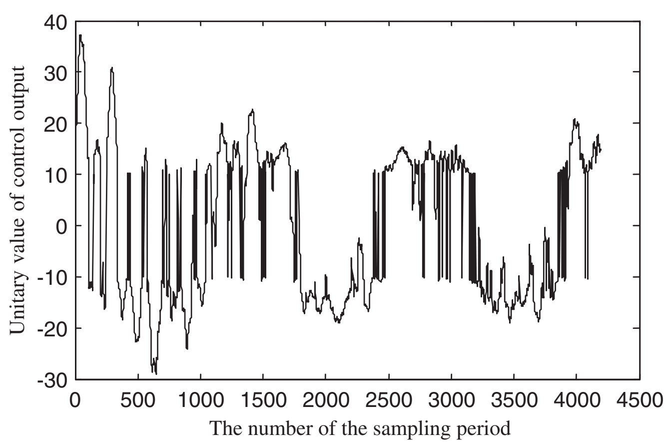

Fig. 6. Control output of the first link.

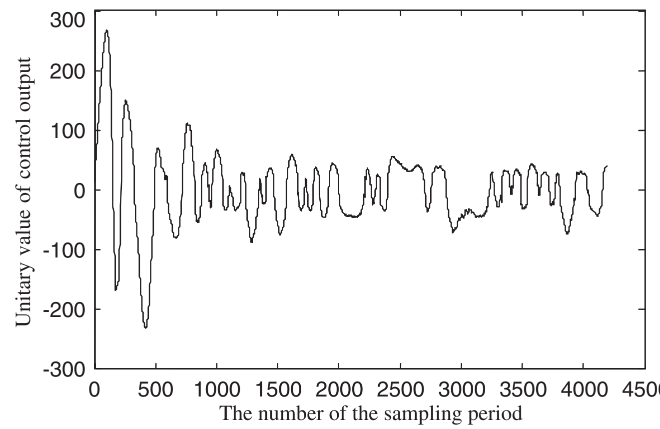

Fig. 7. Control output of the second link.

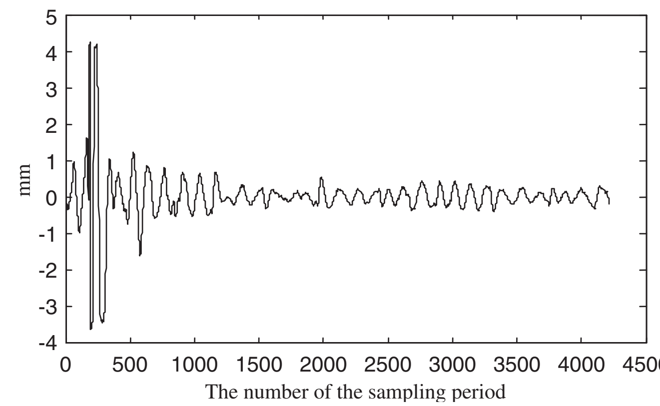

Fig. 8. Tip tracking error of the first link

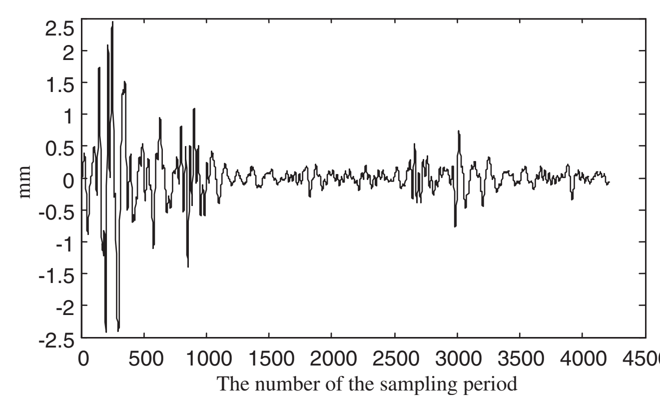

Fig. 9. Tip tracking error of the second link.

(2) good tip tracking performances can be guaranteed by NN adaptive laws (17) (18) designed according to Lyapunov theory;

On the other hand, some practical issues should not be ignored:

(1) although a lower bound of K_2 is given in an analytical form of (20) or (21), in practice, the value of it still depends on the physical system and needs some trial and error. The specified boundary layer thickness L can directly affect the behavior of control output. As is shown in Figs. 6 and 7 and Figs. 10 and 11, different values of L may produce different control outputs, which may result in various tracking performances. If L is very small, chattering phenomenon also appears in control, such as in Figs. 6 and 7. Therefore, a proper L may lie on the actual system and performance requirements.

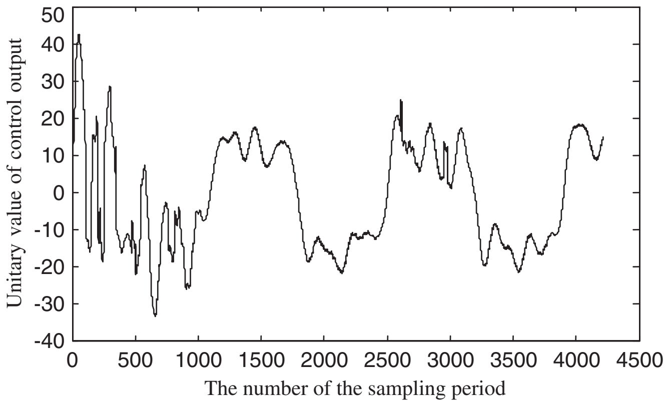

Fig. 10. Control output of the first link.

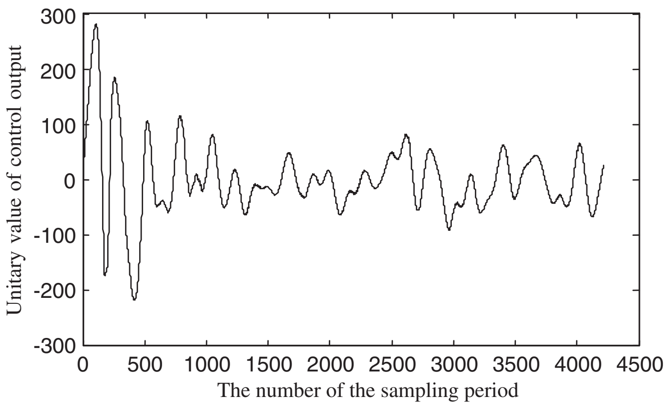

Fig. 11. Control output of the second link.

5. Conclusion

Due to nonlinearities and uncertainties the exact dynamic characteristics of flexible-link manipulators are very difficult to obtain, therefore, a controller combining the robustness of SMC and the approximation of NN has been proposed in this paper. The Lyapunov theory can guarantee the signals of closed-loop system bounded as well as the sliding surface attractable in a wide range with the online NN adaptive laws. The experimental results also show that the proposed control strategy is feasible in the practical flexible-link manipulator and the controller (15) has potential in dealing with the nonlinearities and uncertainties of the system. However, the nonlinear function to be estimated is often unknown, so the gain of the corrective term still requires some trial and error in practice. Specially, how to decide the boundary layer thickness of the proposed method needs future studies.

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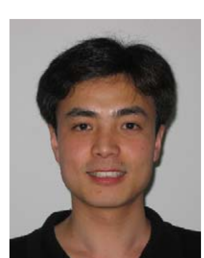

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