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On the global robust asymptotic stability of BAM neural networks with time-varying delays

Xuyang Lou, Baotong Cui*

Research Center of Control Science and Engineering, Southern Yangtze University, 1800 Lihu Road, Wuxi, Jiangsu 214122, PR China

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Abstract

In this paper, the global robust asymptotic stability of a class of delayed bi-directional associative memory (BAM) neural networks, which contain variable uncertain parameters whose values are unknown but bounded, is studied. Some new sufficient conditions are presented for the global stability of BAM neural networks with time-varying delays by constructing Lyapunov functional and using linear matrix inequality (LMI), Halanay's inequality. A numerical example is presented to illustrate the effectiveness of our theoretical results.

Keywords: Stability; Bi-directional associative memory neural network; Time-varying delays; Lyapunov functional; Linear matrix inequality; Halanay's inequality

1. Introduction

A series of neural networks related to bi-directional associative memory (BAM) models have been proposed by Kosko in Refs. [10,14–16]. These models generalized the single-layer auto-associative Hebbian correlator to a two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks has good application perspective in pattern recognition. Cao et al. have considered the stability of this class of networks and presented criteria for the global stability and periodic oscillatory solution of delayed BAM neural networks (see, for example, [3,4,6,9]). Recently, we studied the global asymptotic stability of BAM neural networks with reaction–diffusion terms or impulses, see [11,23]. As Kosko considers the global stability for BAM neural networks, his approach requires severe constraint conditions of having symmetric connection weight matrix. To design this kind neural networks, vital data, such as the neurons fire rate, the synaptic interconnection weight and the signal transmission delays, etc., usually need to be measured, acquired and processed by means of statistical estimation which definitely leads to estimation errors. However, parameter fluctuation in neural network implementation on very large-scale integration (VLSI) chips is also unavoidable. It is important to ensure that system be stable or periodic stable with respect to these uncertainties in the design and applications of neural networks. In recent years, the robust stability of delayed neural networks have been investigated by many researchers (e.g. [1,5,8,21,22,24]).

Recently, linear matrix inequality (LMI)-based techniques have been successfully used to tackle various stability problems for neural networks with time delays (see, for example, [20,24,25]). The main advantage of the LMI-based approaches is that the LMI stability conditions can be solved numerically using the effective interior-point algorithm [2]. In [17], the global robust asymptotical stability is considered for multi-delayed interval neural networks based on LMI approach.

However, to the best of our knowledge, few authors have considered the global robust asymptotic stability of BAM neural networks with time-varying delays and uncertainties. In this paper, by constructing suitable Lyapunov functional based on LMI or Halanay's inequality, we derive some sufficient conditions for the global robust

*Corresponding author. Tel.: +86 10 5910639; fax: +86 10 5910633.
E-mail address: btcui@sohu.com (B. Cui).

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asymptotic stability of BAM neural networks with time-varying delays.

The rest of this paper is organized as follows. In Section 2, the problem to be investigated is stated and some definitions and lemmas are listed. Based on the Lyapunov–Krasovskii stability theory, LMI approach and Halanay's inequality, some robust asymptotic stability criteria for BAM neural networks with time-varying delays are obtained in Section 3. In addition, a numerical example is provided in Section 4, to support the results of the analysis. Finally, some conclusions are drawn in Section 5.

2. System description

We consider the following BAM model:

$$\begin{cases} x'(t) = -(C+\Delta C(t))x(t) + (W+\Delta W(t)) \\ \qquad \times f(y(t-\tau(t))) + I, \\ y'(t) = -(D+\Delta D(t))y(t) + (H+\Delta H(t)) \\ \qquad \times g(x(t-\sigma(t))) + J, \end{cases} \tag{1}$$

where $x=(x_1,x_2,\ldots,x_m)^{\mathrm{T}}$, $y=(y_1,y_2,\ldots,y_n)^{\mathrm{T}}$ are the neuron state vectors, $C=diag(c_1,c_2,\ldots,c_m)$, $D=diag(d_1, d_2,\ldots,d_n)$ are positive diagonal matrices, W, H are interconnection weight matrices, $\tau(t)=\tau_j(t)_{n\times 1}, \sigma(t)=\sigma_i(t)_{m\times 1}$ and $0\leqslant\tau(t)\leqslant\tau_0$, $0\leqslant\sigma(t)\leqslant\sigma_0$ are the time delays, and they are assumed that

$$0\leqslant\dot{\tau}(t)\leqslant\tau^*<1, \quad 0\leqslant\dot{\sigma}(t)\leqslant\sigma^*<1.$$

The $\Delta C(t), \Delta D(t), \Delta W(t), \Delta H(t)$ are parametric uncertainties, nonlinear active functions

$$f(y(t-\tau(t))) = (f_j(y_j(t-\tau_j(t))))_{n\times 1},$$
$$g(x(t-\sigma(t))) = g_i(x_i(t-\sigma_i(t)))_{m\times 1},$$

I, J denote the external inputs on the neurons.

The initial conditions associated with (1) are assumed to be of the form

$$\begin{cases} x(t)=\psi_x(t), & t\in[-\sigma_0,0), \\ y(t)=\psi_y(t), & t\in[-\tau_0,0). \end{cases} \tag{2}$$

In this paper, we do not need the assumption of monotonicity and smoothness of the activation functions $f_j(\cdot)$ and $g_i(\cdot)$, $i=1,2,\ldots,m$, $j=1,2,\ldots,n$. Instead, throughout this paper, we assume that

(H_1) The nonlinear activation function $f_j(\cdot)$ $g_i(\cdot)$ are globally Lipschitz, i.e.

$$|f_j(\xi_1)-f_j(\xi_2)|\leqslant a_j|\xi_1-\xi_2|,$$
$$|g_i(\xi_1)-g_i(\xi_2)|\leqslant b_i|\xi_1-\xi_2|, \tag{3}$$

for $i=1,2,\ldots,m$, $j=1,2,\ldots,n$, and $\xi_1,\xi_2\in R$, where a_j, b_i are positive constant numbers.

(H_2) For any $\xi\in R$ and $L_{1j}, L_{2i}>0$, $i=1,2,\ldots,m$, $j=1,2,\ldots,n$,

$$|f_j(\xi)|\leqslant L_{1j}, \quad |g_i(\xi)|\leqslant L_{2i}.$$

From the above assumption, we know they will ensure that system (1) has an equilibrium point $(x^*,y^*)=(x_1^*,x_2^*, \ldots,x_m^*,y_1^*,y_2^*,\ldots,y_n^*)^{\mathrm{T}}$.

For notational convenience, we will always shift an intended equilibrium point $(x^*,y^*)^{\mathrm{T}}$ of system (1) to the origin by letting

$$u(t)=x(t)-x^*, \quad v(t)=y(t)-y^*. \tag{4}$$

It is easy to transform system (1) into the following form:

$$\begin{cases} u'(t) = -(C+\Delta C(t))u(t) + (W+\Delta W(t)) \\ \qquad \times F(v(t-\tau(t))), \\ v'(t) = -(D+\Delta D(t))v(t) + (H+\Delta H(t)) \\ \qquad \times G(u(t-\sigma(t))), \end{cases} \tag{5}$$

with initial values

$$\begin{cases} u(t)=\phi_u(t), & t\in[-\sigma_0,0), \\ v(t)=\phi_v(t), & t\in[-\tau_0,0), \end{cases} \tag{6}$$

where

$$F_j(v_j(t-\tau_j(t))) = f_j(v_j(t-\tau_j(t))+y_j^*) - f_j(y_j^*),$$

$$G_i(u_i(t-\sigma_i(t))) = g_i(u_i(t-\sigma_i(t))+x_i^*) - g_i(x_i^*),$$

for $i=1,2,\ldots,m, j=1,2,\ldots,n$.

It follows from (H_1) and (H_2) that

$$\|F(X)\|\leqslant\|A\|\cdot\|X\|, \quad \|G(Y)\|\leqslant\|B\|\cdot\|Y\| \tag{7}$$

for all $t\geqslant 0$, where $\|\cdot\|$ denotes the Euclidean norm and

$$A=diag(a_1,a_2,\ldots,a_n), \quad B=diag(b_1,b_2,\ldots,b_m).$$

The time-varying uncertainties $\Delta C(t), \Delta D(t), \Delta W(t), \Delta H(t)$ are defined by

$$\Delta C(t)=L_0F_0(t)E_0, \quad \Delta D(t)=L_1F_1(t)E_1, \tag{8}$$

$$\Delta W(t)=L_2F_2(t)E_2, \quad \Delta H(t)=L_3F_3(t)E_3, \tag{9}$$

where $L_0,L_1,L_2,L_3,E_0,E_1,E_2,E_3$ are known constant matrices of appropriate dimensions, and $F_0(t), F_1(t), F_2(t), F_3(t)$ are unknown time-varying matrices with Lebesgue measurable elements bounded by

$$F_0^{\mathrm{T}}(t)F_0(t)\leqslant E, \quad F_1^{\mathrm{T}}(t)F_1(t)\leqslant E,$$
$$F_2^{\mathrm{T}}(t)F_2(t)\leqslant E, \quad F_3^{\mathrm{T}}(t)F_3(t)\leqslant E, \tag{10}$$

where E is the identity matrix of appropriate dimension.

In order to obtain our results, we need the following definitions and lemmas:

Definition 2.1 (*Liang and Cao [19]*). The equilibrium point $(x^*,y^*)=(x_1^*,x_2^*,\ldots,x_m^*,y_1^*,\ y_2^*,\ldots,y_n^*)^{\mathrm{T}}$ is said to be globally asymptotically stable if it is locally stable in the sense of Lyapunov and global attractive, where global attractiveness means that every trajectory tends to (x^*,y^*) as $t\to+\infty$, i.e. all solutions of (1) satisfy

$$\lim_{t\to\infty}x_i(t)=x_i^*, \quad \lim_{t\to\infty}y_j(t)=y_j^*,$$
$$i=1,2,\ldots,m, \quad j=1,2,\ldots,n. \tag{11}$$

Definition 2.2. For $(t,x) \in (t_{k-1}, t_k] \times R^n$, we define

$$D^+V(t,x) = \lim_{h\to 0^+} \sup \frac{1}{h}[V(t+h, x+h(f(t,x) + g(t,x))) - V(t,x)]. \quad (12)$$

Lemma 2.1 (*Hale and Lunel [12], Lyapunov–Krasovskii stability theorem*). *Consider the following functional differential equation*:

$$\dot{x}(t) = f(t, x_t), \quad t \geqslant t_0, \ x_{t_0} = \phi(\theta) \ \forall \theta \in [-\tau, 0], \quad (13)$$

where $x_t(\cdot)$, for given $t \geqslant t_0$, denotes the restriction of $x(\cdot)$ to the interval $[t-\tau, t]$ translated to $[-\tau, 0]$, namely $x_t(\theta) = x(t+\theta)$, $\forall \theta \in [-\tau, 0]$.

Assume that there exists a continuous functional $V(t,\phi)$ such that

(i) $V_1(\|\phi(0)\|) \leqslant V(t,\phi) \leqslant V_2(\|\phi(\theta)\|)$,
(ii) $\dot{V}(t, x_t) \leqslant -V_3(\|x(t)\|)$,

where V_1, V_2, V_3 are K-type function. Then the trivial solution of the upper functional differential equation is uniformly asymptotically stable.

Notice that condition (i) means that the function $V(t,\phi)$ is positive definite and has an infinitesimal upper limit.

Lemma 2.2 (*Li et al. [18]*). *Given any real matrices A, B, C of appropriate dimensions and a scalar $\varepsilon > 0$ such that $0 < C = C^{\mathrm{T}}$. Then, the following inequality holds*:

$$A^{\mathrm{T}}B + B^{\mathrm{T}}A \leqslant \varepsilon A^{\mathrm{T}}CA + \varepsilon^{-1}B^{\mathrm{T}}C^{-1}B, \quad (14)$$

where the superscript T *means the transpose of a matrix.*

Lemma 2.3 (*Cao et al. [5], Schur complement*). *Linear matrix inequality*:

$$\begin{pmatrix} Q(x) & S(x) \\ S^{\mathrm{T}}(x) & R(x) \end{pmatrix} > 0, \quad (15)$$

with $Q(x) = Q^{\mathrm{T}}(x), R(x) = R^{\mathrm{T}}(x)$ is the same as

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^{\mathrm{T}}(x) > 0.$$

Lemma 2.4 (*Halanay's inequality, Cao and Wang [7]*). *Let $a > b > 0$ and $v(t)$ be a non-negative continuous function on $[t_0 - \tau, t_0]$, and satisfy the following inequality*:

$$D^+v(t) \leqslant -av(t) + b \sup_{t-\tau \leqslant s \leqslant t} v(s), \quad t \geqslant t_0,$$

where τ is a non-negative constant, then there exists constants k, $\lambda > 0$ that satisfy

$$v(t) \leqslant k\mathrm{e}^{-\lambda(t-t_0)}, \quad t \geqslant t_0,$$

where $k = \sup_{t-\tau \leqslant s \leqslant t} v(s)$, λ is unique positive solution of the following equation:

$$\lambda = a - b\mathrm{e}^{\lambda\tau}.$$

3. Main results

In this section, we will discuss the global robust asymptotic stability of system (5) with the initial conditions (6) and give our main results.

Theorem 3.1. *If there exist symmetric positive matrices P_1, P_2 such that the following LMIs hold*:

$$\begin{bmatrix} \Upsilon_1 & P_1L_0 & \sqrt{\dfrac{1}{1-\tau^*}}P_1L_2 \\ L_0^{\mathrm{T}}P_1 & E & 0 \\ \sqrt{\dfrac{1}{1-\tau^*}}L_2^{\mathrm{T}}P_1 & 0 & E \end{bmatrix} < 0 \quad (16)$$

and

$$\begin{bmatrix} \Upsilon_2 & P_2L_1 & \sqrt{\dfrac{1}{1-\sigma^*}}P_2L_3 \\ L_1^{\mathrm{T}}P_2 & E & 0 \\ \sqrt{\dfrac{1}{1-\sigma^*}}L_3^{\mathrm{T}}P_2 & 0 & E \end{bmatrix} < 0, \quad (17)$$

where

$$\Upsilon_1 = -(P_1C + C^{\mathrm{T}}P_1) + E_0^{\mathrm{T}}E_0 + \frac{1}{1-\sigma^*}BQ_2B + \frac{1}{1-\tau^*}P_1^2,$$

$$\Upsilon_2 = -(P_2D + D^{\mathrm{T}}P_2) + E_1^{\mathrm{T}}E_1 + \frac{1}{1-\tau^*}AQ_1A + \frac{1}{1-\sigma^*}P_2^2,$$

$$Q_1 = W^{\mathrm{T}}W + E_2^{\mathrm{T}}E_2, \quad Q_2 = H^{\mathrm{T}}H + E_3^{\mathrm{T}}E_3.$$

Then the equilibrium (x^*, y^*) of (1) *is globally asymptotically stable dependent of delays in the sense that all solutions of* (1) *corresponding to initial values* (2) *satisfy*

$$\lim_{t\to\infty} x_i(t) = x_i^*, \quad \lim_{t\to\infty} y_j(t) = y_j^*, \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n. \quad (18)$$

Remark 3.1. Uncertain neural networks with multiple time delays can also be studied similarly.

Remark 3.2. The stability criterion (16)–(17) can be easily solved by using some existing software packages, for example, the MATLAB LMI toolbox.

Proof. We construct a Lyapunov–Krasovskii functional

$$V(t) = u^{\mathrm{T}}(t)P_1u(t) + \int_{t-\tau(t)}^{t} F^{\mathrm{T}}(v(s))Q_1F(v(s))\,\mathrm{d}s + v^{\mathrm{T}}(t)P_2v(t) + \int_{t-\sigma(t)}^{t} G^{\mathrm{T}}(u(s))Q_2G(u(s))\,\mathrm{d}s, \tag{19}$$

where $Q_1 = W^{\mathrm{T}}W + E_2^{\mathrm{T}}E_2, Q_2 = H^{\mathrm{T}}H + E_3^{\mathrm{T}}E_3$.

Calculating the upper right derivative D^+V of the Lyapunov functional V along the solution of (5), it follows that

$$\begin{aligned}
D^+V \leqslant & -u^{\mathrm{T}}(t)(P_1C + C^{\mathrm{T}}P_1)u(t) \\
& - v^{\mathrm{T}}(t)(P_2D + D^{\mathrm{T}}P_2)v(t) \\
& - u^{\mathrm{T}}(t)P_1L_0F_0(t)E_0u(t) \\
& - u^{\mathrm{T}}(t)E_0^{\mathrm{T}}F_0^{\mathrm{T}}(t)L_0^{\mathrm{T}}P_1u(t) \\
& + u^{\mathrm{T}}(t)P_1WF(v(t-\tau(t))) \\
& + u^{\mathrm{T}}(t)P_1L_2F_2E_2F(v(t-\tau(t))) \\
& + F^{\mathrm{T}}(v(t-\tau(t)))W^{\mathrm{T}}P_1u(t) \\
& + F^{\mathrm{T}}(v(t-\tau(t)))E_2^{\mathrm{T}}F_2^{\mathrm{T}}L_2^{\mathrm{T}}P_1u(t) \\
& + F^{\mathrm{T}}(v(t))Q_1F(v(t)) - (1-\tau^*) \\
& \times F^{\mathrm{T}}(v(t-\tau(t)))Q_1F(v(t-\tau(t))) \\
& - v^{\mathrm{T}}(t)P_2L_1F_1(t)E_1v(t) \\
& - v^{\mathrm{T}}(t)E_1^{\mathrm{T}}F_1^{\mathrm{T}}(t)L_1^{\mathrm{T}}P_2v(t) \\
& + v^{\mathrm{T}}(t)P_2HG(u(t-\sigma(t))) \\
& + v^{\mathrm{T}}(t)P_2L_3F_3E_3G(u(t-\sigma(t))) \\
& + G^{\mathrm{T}}(u(t-\sigma(t)))H^{\mathrm{T}}P_2v(t) \\
& + G^{\mathrm{T}}(u(t-\sigma(t)))E_3^{\mathrm{T}}F_3^{\mathrm{T}}L_3^{\mathrm{T}}P_2v(t) \\
& + G^{\mathrm{T}}(u(t))Q_2G(u(t)) - (1-\sigma^*) \\
& \times G^{\mathrm{T}}(u(t-\sigma(t)))Q_2G(u(t-\sigma(t))).
\end{aligned} \tag{20}$$

From Lemma 2.2 and (20), we have that

$$\begin{aligned}
D^+V \leqslant & -u^{\mathrm{T}}(t)(P_1C + C^{\mathrm{T}}P_1)u(t) - v^{\mathrm{T}}(t)(P_2D + D^{\mathrm{T}}P_2)v(t) \\
& + u^{\mathrm{T}}(t)P_1L_0L_0^{\mathrm{T}}P_1u(t) + u^{\mathrm{T}}(t)E_0^{\mathrm{T}}E_0u(t) \\
& + \frac{1}{1-\tau^*}u^{\mathrm{T}}(t)P_1^2u(t) \\
& + \frac{1}{1-\tau^*}u^{\mathrm{T}}(t)P_1L_2L_2^{\mathrm{T}}P_1u(t) \\
& + (1-\tau^*)F^{\mathrm{T}}(v(t-\tau(t)))W^{\mathrm{T}}WF(v(t-\tau(t))) \\
& + (1-\tau^*)F^{\mathrm{T}}(v(t-\tau(t)))E_2^{\mathrm{T}}E_2F(v(t-\tau(t))) \\
& + F^{\mathrm{T}}(v(t))Q_1F(v(t)) - (1-\tau^*) \\
& \times F^{\mathrm{T}}(v(t-\tau(t)))Q_1F(v(t-\tau(t))) \\
& + v^{\mathrm{T}}(t)P_2L_1L_1^{\mathrm{T}}P_2v(t) + v^{\mathrm{T}}(t)E_1^{\mathrm{T}}E_1v(t) \\
& + \frac{1}{1-\sigma^*}v^{\mathrm{T}}(t)P_2^2v(t) \\
& + \frac{1}{1-\sigma^*}v^{\mathrm{T}}(t)P_2L_3L_3^{\mathrm{T}}P_2v(t) \\
& + (1-\sigma^*)G^{\mathrm{T}}(u(t-\sigma(t)))H^{\mathrm{T}}HG(u(t-\sigma(t))) \\
& + (1-\sigma^*)G^{\mathrm{T}}(u(t-\sigma(t)))E_3^{\mathrm{T}}E_3G(u(t-\sigma(t))) \\
& + G^{\mathrm{T}}(u(t))Q_2G(u(t)) - (1-\sigma^*) \\
& \times G^{\mathrm{T}}(u(t-\sigma(t)))Q_2G(u(t-\sigma(t))) \\
= & \, u^{\mathrm{T}}(t)\Big[-(P_1C + C^{\mathrm{T}}P_1) + P_1L_0L_0^{\mathrm{T}}P_1 + E_0^{\mathrm{T}}E_0 \\
& + \frac{1}{1-\tau^*}P_1^2 + \frac{1}{1-\tau^*}P_1L_2L_2^{\mathrm{T}}P_1\Big]u(t) \\
& + \frac{1}{1-\tau^*}F^{\mathrm{T}}(v(t))Q_1F(v(t)) \\
& + v^{\mathrm{T}}(t)\Big[-(P_2D + D^{\mathrm{T}}P_2) + P_2L_1L_1^{\mathrm{T}}P_2 + E_1^{\mathrm{T}}E_1 \\
& + \frac{1}{1-\sigma^*}P_2^2 + \frac{1}{1-\sigma^*}P_2L_3L_3^{\mathrm{T}}P_2\Big]v(t) \\
& + \frac{1}{1-\sigma^*}G^{\mathrm{T}}(u(t))Q_2G(u(t)) \\
\leqslant & \, u^{\mathrm{T}}(t)\Big[-(P_1C + C^{\mathrm{T}}P_1) + P_1L_0L_0^{\mathrm{T}}P_1 + E_0^{\mathrm{T}}E_0 \\
& + \frac{1}{1-\tau^*}P_1^2 + \frac{1}{1-\tau^*}P_1L_2L_2^{\mathrm{T}}P_1 + \frac{1}{1-\sigma^*}BQ_2B\Big]u(t) \\
& + v^{\mathrm{T}}(t)\Big[-(P_2D + D^{\mathrm{T}}P_2) + P_2L_1L_1^{\mathrm{T}}P_2 + E_1^{\mathrm{T}}E_1 \\
& + \frac{1}{1-\sigma^*}P_2^2 + \frac{1}{1-\sigma^*}P_2L_3L_3^{\mathrm{T}}P_2 \\
& + \frac{1}{1-\tau^*}AQ_1A\Big]v(t) < 0.
\end{aligned} \tag{21}$$

Since V is bounded and has a limit $\widetilde{V}$ as $t \to \infty$ obviously $\widetilde{V} \geqslant 0$ and also $D^+V < 0$. This means that $\widetilde{V} = 0$, i.e. $u(t) \to 0$ and $v(t) \to 0$, as $t \to \infty$. This means that

$$\lim_{t\to\infty} x_i(t) = x_i^*, \quad \lim_{t\to\infty} y_j(t) = y_j^*, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n. \tag{22}$$

This completes the proof. □

In the following section, based on Halanay's inequality, we will study system (1) with $\Delta C(t) \equiv 0$, $\Delta D(t) \equiv 0$, $\Delta W(t) \equiv 0$, $\Delta H(t) \equiv 0$, i.e.

$$\begin{cases} x'(t) = -Cx(t) + Wf(y(t-\tau(t))) + I, \\ y'(t) = -Dy(t) + Hg(x(t-\sigma(t))) + J. \end{cases} \tag{23}$$

Similar to (5), we have

$$\begin{cases} u'(t) = -Cu(t) + WF(v(t-\tau(t))), \\ v'(t) = -Dv(t) + HG(u(t-\sigma(t))). \end{cases} \tag{24}$$

Theorem 3.2. *Assume that $f_j(\cdot)$, $g_i(\cdot)$ satisfy the hypotheses* (H_1) *and* (H_2), *and if there exist real constants α_{ij}, α_{ij}^*,*

β_j, β_j^*, ρ_{ij}, ρ_{ij}^*, ϱ_j, ϱ_j^* *and positive constants* $p\geqslant 1$ *such that*

$$\min\left\{\min_{1\leqslant i\leqslant m}\left(pc_i-(p-1)\sum_{j=1}^{n}|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}\right),\ \min_{1\leqslant j\leqslant n}\left(pd_j-(p-1)\sum_{i=1}^{m}|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}\right)\right\}$$
$$>\max\left\{\max_{1\leqslant i\leqslant m}\left(\sum_{j=1}^{n}|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}\right),\ \max_{1\leqslant j\leqslant n}\left(\sum_{i=1}^{m}|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}\right)\right\},\tag{25}$$

where $(p-1)\alpha_{ij}+\alpha_{ij}^*=1$, $(p-1)\beta_j+\beta_j^*=1$, $(p-1)\rho_{ij}+\rho_{ij}^*=1$, $(p-1)\varrho_j+\varrho_j^*=1$, *then system* (23) *has a unique equilibrium point* (x^*,y^*) *which is global asymptotically stable.*

Proof. Set

$$V(t)=\frac{1}{p}\left(\sum_{i=1}^{m}|u_i(t)|^p+\sum_{j=1}^{n}|v_j(t)|^p\right).\tag{26}$$

Calculate the upper right Dini derivative D^+V of V along the solution of system (24) as follows:

$$\begin{aligned}D^+V(t)\leqslant&\sum_{i=1}^{m}|u_i(t)|^{p-1}\left(-c_i|u_i(t)|+\sum_{j=1}^{n}|w_{ij}|a_j|v_j(t-\tau_j(t))|\right)\\&+\sum_{j=1}^{n}|v_j(t)|^{p-1}\left(-d_j|v_j(t)|+\sum_{i=1}^{m}|h_{ji}|b_i|u_i(t-\sigma_i(t))|\right)\\=&-\sum_{i=1}^{m}c_i|u_i(t)|^p+\sum_{i=1}^{m}\sum_{j=1}^{n}|w_{ij}|\cdot a_j\cdot|u_i(t)|^{p-1}||v_j(t-\tau_j(t))|\\&-\sum_{j=1}^{n}d_j|v_j(t)|^p+\sum_{j=1}^{n}\sum_{i=1}^{m}|h_{ji}|\cdot b_i\cdot|v_j(t)|^{p-1}||u_i(t-\sigma_i(t))|\\=&-\sum_{i=1}^{m}c_i|u_i(t)|^p-\sum_{j=1}^{n}d_j|v_j(t)|^p\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}(|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}|u_i(t)|^p)^{(p-1)/p}\times\left(|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}|v_j(t-\tau_j(t))|^p\right)^{1/p}\\&+\sum_{j=1}^{n}\sum_{i=1}^{m}(|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}|v_j(t)|^p)^{(p-1)/p}\times(|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}|u_i(t-\sigma_i(t))|^p)^{1/p}.\end{aligned}\tag{27}$$

Recall that the Young inequality $a^{1/p}b^{1/q}\leqslant(1/p)a+(1/q)b$ holds for any $a>0,b>0$ and $0\leqslant 1/p\leqslant 1$ with $1/p+1/q=1$, see, for instance, [13].

Let $a=(|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}|u_i(t)|^p)^{(p-1)/p}$, $b=(|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}|v_j(t-\tau_j(t))|^p)^{1/p}$, then we have

$$\begin{aligned}&(|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}|u_i(t)|^p)^{(p-1)/p}(|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}|v_j(t-\tau_j(t))|^p)^{1/p}\\&\quad\leqslant\frac{p-1}{p}|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}|u_i(t)|^p+\frac{1}{p}|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}|v_j(t-\tau_j(t))|^p.\end{aligned}\tag{28}$$

Similarly, let $a=(|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}|v_j(t)|^p)^{(p-1)/p}$, $b=(|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}|u_i(t-\sigma_i(t))|^p)^{1/p}$, then we can get

$$\begin{aligned}&(|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}|v_j(t)|^p)^{(p-1)/p}(|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}|u_i(t-\sigma_i(t))|^p)^{1/p}\\&\quad\leqslant\frac{p-1}{p}|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}|v_j(t)|^p+\frac{1}{p}|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}|u_i(t-\sigma_i(t))|^p.\end{aligned}\tag{29}$$

Substituting inequalities (28) and (29) into (27), we obtain

$$\begin{aligned}D^+V(t)\leqslant&-\sum_{i=1}^{m}c_i|u_i(t)|^p+\frac{p-1}{p}\sum_{i=1}^{m}\times\sum_{j=1}^{n}|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}|u_i(t)|^p\\&-\sum_{j=1}^{n}d_j|v_j(t)|^p+\frac{p-1}{p}\sum_{j=1}^{n}\times\sum_{i=1}^{m}|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}|v_j(t)|^p\\&+\frac{1}{p}\sum_{i=1}^{m}\sum_{j=1}^{n}|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}|v_j(t-\tau_j(t))|^p\\&+\frac{1}{p}\sum_{j=1}^{n}\sum_{i=1}^{m}|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}|u_i(t-\sigma_i(t))|^p\\\leqslant&-\mu V(t)+v\sup_{t-\max\{\tau_0,\sigma_0\}\leqslant s\leqslant t}V(t),\end{aligned}\tag{30}$$

where

$$\mu=\min\left\{\min_{1\leqslant i\leqslant m}\left(pc_i-(p-1)\sum_{j=1}^{n}|w_{ij}|^{p\alpha_{ij}}d_j^{p\beta_j}\right),\ \min_{1\leqslant j\leqslant n}\left(pd_j-(p-1)\sum_{i=1}^{m}|h_{ji}|^{p\rho_{ji}}b_i^{p\varrho_i}\right)\right\},$$

$$v=\max\left\{\max_{1\leqslant i\leqslant m}\left(\sum_{j=1}^{n}|w_{ij}|^{p\alpha_{ij}^*}d_j^{p\beta_j^*}\right),\ \max_{1\leqslant j\leqslant n}\left(\sum_{i=1}^{m}|h_{ji}|^{p\rho_{ji}^*}b_i^{p\varrho_i^*}\right)\right\}.$$

In view of (25) it implies that $\mu>v>0$. By Lemma 2.4, there exist k, $\lambda>0$ such that

$$V(t)\leqslant k\mathrm{e}^{-\lambda t},\quad t\geqslant 0.$$

That is,

$$\left(\sum_{i=1}^{m}|u_i(t)|^p+\sum_{j=1}^{n}|v_j|^p\right)\leqslant pk\mathrm{e}^{-\lambda t}.$$

Thus, system (23) is global exponentially stable, which implies system (23) is global asymptotically stable. □

Corollary 3.1. *Assume that $f_j(\cdot)$, $g_i(\cdot)$ satisfy the hypotheses* (H_1) *and* (H_2), *and suppose further that*

$$\min\left\{\min_{1\leqslant i\leqslant m} c_i,\ \min_{1\leqslant j\leqslant n} d_j\right\} > \max\left\{\max_{1\leqslant i\leqslant m}\left(\sum_{j=1}^{n}|w_{ij}|a_j\right),\ \max_{1\leqslant j\leqslant n}\left(\sum_{i=1}^{m}|h_{ji}|b_i\right)\right\}, \quad (31)$$

then system (23) *has a unique equilibrium point* (x^*, y^*) *which is global asymptotically stable.*

4. A numerical example

In this section, we give an illustrative example for our main result.

Example 4.1. Consider system (5) with time-varying delays: $\tau(t)=\frac{1}{2}\sin^2(t)$, $\sigma(t)=\frac{1}{3}\cos^2(t)$ with the initial values of the system as follows:

$$\begin{cases}\phi_{p1}(t)=\phi_{p2}(t)=0, & t\in[-\frac{1}{3},0),\\ \phi_{q1}(t)=\phi_{q1}(t)=0, & t\in[-\frac{1}{2},0),\end{cases}$$

and take

$$C=\begin{bmatrix}2.6 & 0\\ 0 & 2.1\end{bmatrix},\quad W=\begin{bmatrix}1.1 & 1\\ -0.2 & 0.1\end{bmatrix},$$

$$D=\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix},\quad H=\begin{bmatrix}0.9 & 0.1\\ -0.1 & 0.1\end{bmatrix},$$

$$L_0=\begin{bmatrix}-0.2 & 0.2\\ 0.2 & 0.2\end{bmatrix},\quad L_1=\begin{bmatrix}-0.4 & 0.3\\ 0.3 & 0.4\end{bmatrix},$$

$$L_2=\begin{bmatrix}0.2 & 0.5\\ 0.1 & -0.3\end{bmatrix},\quad L_3=\begin{bmatrix}-0.2 & 0.4\\ 0.4 & 0.2\end{bmatrix},$$

$$F_0(t)=\begin{bmatrix}\sin(t) & 0\\ 0 & \cos(t)\end{bmatrix},\quad F_1(t)=\begin{bmatrix}\sin^2(t) & 0\\ 0 & 0.2\cos^2(t)\end{bmatrix},$$

$$F_2(t)=\begin{bmatrix}0.5\sin^3(t) & 0\\ 0 & \cos^3(t)\end{bmatrix},$$

$$F_3(t)=\begin{bmatrix}1-2\sin^2(t) & 0\\ 0 & 1-2\cos^2(t)\end{bmatrix},$$

$$E_0=L_0,\quad E_1=L_1,\quad E_2=L_2,\quad E_3=L_3,$$

$$P_1=P_2=E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

and $f_1(\xi)=f_2(\xi)=g_1(\xi)=g_2(\xi)=[|\xi+1|-|\xi-1|]/2$. Then we can easily obtain that

$$-(P_1C+C^{\mathrm{T}}P_1)+P_1L_0L_0^{\mathrm{T}}P_1+E_0^{\mathrm{T}}E_0+\frac{1}{1-\tau^*}P_1^2+\frac{1}{1-\tau^*}P_1L_2L_2^{\mathrm{T}}P_1+\frac{1}{1-\sigma^*}BQ_2B=\begin{bmatrix}-2.0775 & -0.2300\\ -0.2300 & -1.7575\end{bmatrix}<0,$$

$$-(P_2D+D^{\mathrm{T}}P_2)+P_2L_1L_1^{\mathrm{T}}P_2+E_1^{\mathrm{T}}E_1+\frac{1}{1-\sigma^*}P_2^2+\frac{1}{1-\sigma^*}P_2L_3L_3^{\mathrm{T}}P_2+\frac{1}{1-\tau^*}AQ_1A=\begin{bmatrix}-3.0500 & 0.5750\\ 0.5750 & -1.0250\end{bmatrix}<0.$$

Using Theorem 3.1, we can find that the system is robust asymptotically stable.

Remark 4.1. There are few references which aim at global robust asymptotic stability of BAM neural networks via LMI. As we can see, when $\Delta C(t)\equiv 0, \Delta W(t)\equiv 0, \Delta D(t)\equiv 0, \Delta H(t)\equiv 0$, system (1) is BAM neural networks, which has been studied in Refs. [10,4,9].

Remark 4.2. Let $\lambda_i=1$, $\mu_i=1$, $i=1,2,\ldots,n$ in [3] and we choose $\Delta C(t)\equiv 0, \Delta W(t)\equiv 0, \Delta D(t)\equiv 0, \Delta H(t)\equiv 0$ in the above example, i.e. Example 4.1. Then, apply the example to Theorems 1 and 2 in [3], it follows that all the conditions are satisfied in Theorems 1 and 2 (see [3]). Therefore, it is clear to derive that the conditions of Theorem 3.1 are near the conditions of Theorem 1 in [3] for global asymptotic stability of BAM neural networks with constant delay.

5. Conclusion

A robust stability criterion for general delayed BAM neural networks with parametric uncertainties and time-varying delay has been presented. The stability criterion is given in terms of linear matrix inequality (LMI) which can be easily solved by some existing software packages. An example has been provided to illustrate the effectiveness of our theoretical results.

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Xuyang Lou was born in 1982. He is presently a Ph.D. candidate in Research Center of Control Science and Engineering, Southern Yangtze University, China and received the B.S. degree from the Zhejiang Ocean University, China in 2004. His current research interests include nonlinear systems, neural networks and stability theory.

Baotong Cui was born in 1960. He received the M.S. degree in applied mathematics from the Department of Mathematics, Anhui University in 1989 and the Ph.D. degree in control theory and control engineering from the College of Automation Science and Engineering, South China University of Technology, China in 2003. He was a post-doctoral fellow at Shanghai Jiao Tong University, China from 2003 to 2005. He joined the Department of Mathematics, Binzhou University, Shangdong, China in 1982, where he became an associate professor in 1993, and a full professor in 1995. In 2003, he moved to the Southern Yangtze University, China where he is a full professor and vice-director for Research Center of Control Science and Engineering and vice-dean for College of Communication and Control Engineering. His current research interests include systems analysis, stability theory, artificial neural networks impulsive and chaos synchronization.