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Determination of multiple direction of arrival in antennas arrays with radial basis functions

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Abstract

This work presents a new proposal for the direction of arrival detection for more than one signal arriving simultaneously on an antennas array of linear or planar geometry using intelligent algorithms.

The *direction of arrival* (DOA) estimator is developed using the techniques of digital conventional beamforming, blind source separation (BSS) and the neural estimator *modular structure of radial basis functions* (MRBF). The developed MRBF has its capabilities extended due to the interaction with BSS technique, which makes an estimation of the steering vectors of the multiple plane waves that reach the array at the same frequency, that is, it separates mixed signals without a priori information.

Keywords: Blind source separation (BSS); Independent component analysis (ICA); Neural networks; Array processing; Direction of arrival (DOA)

1. Introduction

In way to the increasing development of the signal processing systems in the most diverse areas as personal communications, armaments and defense, geophysical explorations, data transmission and others, are the research in adaptive antennas, with new techniques, equipment and softwares. Despite the diversity, the research has a common objective: to increase the array directivity and to cancel interferences.

In this work a technique for direction of arrival identification for multiple incident signals on an antenna array and their separation was developed, being capable to operate in an unknown environment from a statistical point of view. This information can be useful to the adaptive beamformers, which respond to the signals in order to differentiate between interference signals and desired signals, eliminating the influence of the first one and increasing the directivity in the direction of the desired signal. The signals, carrying waves, impinge in such a way on linear arrays and in the planar arrays, which have been sufficiently employed, due to its versatility, in the beam modeling and beam directivity, as well as in other project factors (weight, aerodynamics, dimensions).

The most traditional methods for DOA determination are based on spectral techniques [14], maximum-likelihood criterion [9] and sub-space-based procedures [8,1], amongst which are the methods multiple signal classification (MUSIC) and minimum-norm [9,15,16]. The estimation of signal parameters via rotational invariance techniques (ESPRIT) method [12] also belong to this procedures class.

This work involves diverse techniques of adaptive processing and esteem of DOA in arrays of sensors, such as conventional beamforming in antennas arrays, blind separation of sources and neural networks. The result was the development of a multidisciplinar system for the final objective to detect DOAs of multiple sources with an intelligent technique based in neural networks of radial basis functions in a modular structure.

2. Conventional digital beamforming

Antennas with highly directive characteristics are a basic requirement in a large variety of applications. The antennas array are the solution for the problem of increasing the antenna directivity of only one element. By placing elementary radiators in one determined electric or geometric

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configuration, a highly directive beam can be got (high gain), that is, for the case of a receiver, signal is received by the array with a little interference from the environment.

In this work the conventional digital beamforming is considered in linear and planar arrays, whose descriptions are discussed in the following.

2.1. Beamforming in a linear array

In Fig. 1, a uniformly spaced linear array is shown with **M** identical isotropic elements, and the distance between the elements is represented by d. The plane wave impinge on the array in a θ angle in relation to the normal to the array and the difference of distance along one of the two ways is $d \sin\theta$.

A conventional beamformer with linear arrays of **M** elements introduces delays in the outputs of each sensor to compensate the delays of propagation of N wave fronts that reach the array. The pth sample ($p = 1, \ldots, P$) of the output of mth element of the beamformer is given by Eq. (1). Where $s_n(p)$ represents the signal emitted by the nth source, for $n = 1 \ldots N$. So $\mathbf{s}(p)$ (12) is the vector formed by the incident signals on the antenna

$$\mathbf{x}(p) = \sum_{n=1}^{N} \mathbf{a}(k_n) s_n(p) + \mathbf{v}(p). \tag{1}$$

The **M**-dimensional vectors $\mathbf{a}(k_n)$ are defined as the spatial transfer function between the nth emitting source and the array, and they receive the denomination of steering vectors. Each steering vector (2) is composed by the phased signal of output [11] from each element of the array a_{mn}. The normalized wavenumber k_n (3) relates the operation frequency to the angle θ_n of the nth incident signal. Therefore, $\mathbf{a}(k_n)$ is the vector that carry the DOA information θ_n

$$\mathbf{a}(k_n) = [a_{0n} \; a_{1n} \; \cdots \; a_{(\mathbf{M}-1)n}]^{\mathrm{T}} = [1 \; \mathrm{e}^{\mathrm{j}k_n} \; \cdots \; \mathrm{e}^{\mathrm{j}(\mathbf{M}-1)k_n}]^{\mathrm{T}}, \tag{2}$$

$$k_n = \frac{\omega}{c} d \sin\theta_n = \frac{2\pi}{\lambda} d \sin\theta_n. \tag{3}$$

2.2. Beamforming in a rectangular array

The planar rectangular arrays are obtained by placing the elements in a rectangular grid of $\mathbf{M} = M_x \times M_y$ elementary radiators, as shown in Fig. 2. The planar arrays supply radiation patterns more symmetrical with lower levels of lateral lobes. The distance between elements in both directions, d_x and d_y, is a half wavelength. The DOA of the incident signal $s_n(p)$ on the rectangular planar array is defined by the angles of azimuth (ϕ_n) and elevation (θ_n).

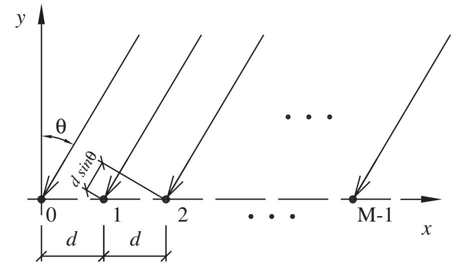

Fig. 1. Linear antennas array uniformly spaced.

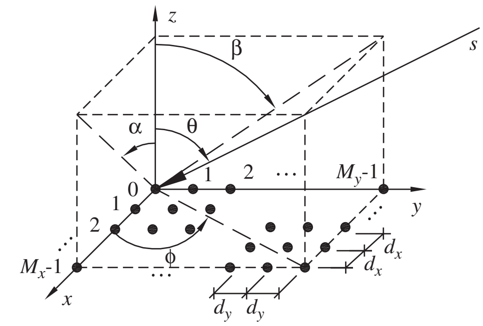

Fig. 2. Geometry of a rectangular planar antennas array.

The concepts of digital beamforming, techniques and algorithms for a linear antenna array are extended to a planar array. Then the pth sample of the output signal of the beamformer for a rectangular planar array [11] is formulated as in Eq. (4). The steering vector related to the nth signal is given by Eq. (5)

$$\mathbf{x}(p) = \sum_{n=1}^{N} \mathbf{a}(k_{\theta n}, k_{\phi n}) s_n(p) + \mathbf{v}(p), \tag{4}$$

$$\begin{aligned}\mathbf{a}(k_{\theta n}, k_{\phi n}) &= \left[a_{(0,0)n} \; a_{(1,0)n} \; \cdots \; a_{(M_x-1,0)n} \; a_{(0,1)n} \; \cdots \; a_{(M_x-1,M_y-1)n}\right]^{\mathrm{T}} \\ &= \left[1 \; \mathrm{e}^{\mathrm{j}k_{\theta n}} \; \cdots \; \mathrm{e}^{\mathrm{j}(M_x-1)k_{\theta n}} \; \mathrm{e}^{\mathrm{j}k_{\phi n}} \; \cdots \mathrm{e}^{\mathrm{j}[(M_x-1)k_{\theta n}+(M_y-1)k_{\phi n}]}\right]^{\mathrm{T}}.\end{aligned} \tag{5}$$

And the normalized wavenumbers for the nth angles of azimuth and elevation, are, respectively, given by Eqs. (6) and (7),

$$k_{\theta n} = \frac{2\pi}{\lambda} d_x \sin\alpha_n, \tag{6}$$

$$k_{\phi n} = \frac{2\pi}{\lambda} d_y \sin\beta_n, \tag{7}$$

where

$$\sin\alpha_n = \sin\theta_n \cos\phi_n, \tag{8}$$

$$\sin\beta_n = \sin\theta_n \sin\phi_n. \tag{9}$$

3. Blind source separation

The objective of the BSS method [7,2,17,13] is to get estimates of the vectors $\mathbf{a}(k_n)$ (2) or $\mathbf{a}(k_{\theta n}, k_{\phi n})$ (5)—from now on they will be assigned generically by $\mathbf{a}_n$—of the mixtures matrix $\mathbf{A}$, given by Eq. (10), through blind sources identification. That is, without any previous knowledge about the propagation model or the array calibration.

$$\mathbf{A} = [\mathbf{a}_1 \; \mathbf{a}_2 \; \ldots \; \mathbf{a}_N]. \tag{10}$$

The blind identification is based on statistical independence of the source signals $s_n(p)$ (12), the signal emitted by the nth source, which are exploited by using fourth order cumulant statistics [7]. And the blind identification of the steering vectors is based on the joint diagonalization of fourth-order cumulant matrices [2]. Basically the BSS calculation involves the steps of pre-processing of the input data, non-gaussianity measurement and optimization of some objective function. The operation on cumulants represents an advantage, since the algorithms do not require gradient descendent, preventing convergence problems and without the necessity of parameter adjustment to improve the performance. The disadvantage of this approach are the estimation and storage of a great number of cumulant matrices.

JADE—*joint approximate diagonalization of eigen-matrices*—is the chosen algorithm of blind identification [2] for this work. JADE uses second-order information through whitening of the output signal of the beamformer $\mathbf{x}(p)$ in Eq. (11), where $\mathbf{s}(p)$ (12) is the vector formed by the incident signals on the antenna. This ensures that the resulting signal is spatially white, that is, a signal whose components are uncorrelated and of unitary variances. And finally, $\mathbf{v}(p)$ is a vector of gaussian additive noise of dimension $\mathbf{M}$

$$\mathbf{x}(p) = \mathbf{A}\mathbf{s}^{\mathrm{T}}(p) + \mathbf{v}(p), \tag{11}$$

$$\mathbf{s}(p) = [s_1(p) \quad \ldots \quad s_N(p)]. \tag{12}$$

4. Modular structure of radial basis functions

The steering vectors $\mathbf{a}_n$ obtained from the operation of BSS with JADE algorithm has its respective DOA esteemed by the nth MRBF network [4,5], that assumes different configurations. For linear arrays, the MRBF network is illustrated as in Fig. 3, while for planar arrays the configuration is shown in Fig. 4. From these figures, a MRBF network is constituted by J experts modules, represented by *Exp.* and supervisor modules—one supervisor module for beamformers with linear array and two supervisor modules for beamformers with planar array, being one for the azimuth direction and the other one for the elevation direction—represented by *Supervisor*.

The input space is defined as the interval in which the data of the problem—angles—are contained. The input space then is partitioned into J distinct sub-spaces, and one expert is attributed to each sub-space. The supervisor module dedicates to all space input. The function of a supervisor is to evaluate which sub-region DOA of a given signal belongs to it, thus indicating the expert most prepared to supply the desired response, the closest one to the real DOA.

Each module is a radial basis functions (RBF) neural network [3,10], and the choice for this type of network in the modular structure was motivated by its architecture of direct propagation, with: one input layer, only one layer of hidden neurons that accomplish nonlinear processing and output layer of linear processing. This network allows a fast training and a high resolution response. In the hidden layer the nonlinear processing is accomplished by gaussian activation functions φ_{jk} (13), the kth radial basis function of the jth RBF module, with $k = 0, 1, \ldots, K-1$. Where υ_{jl} is the lth input vector of training, $\underline{c}_{jk}$ is the center and σ^2_{jk} is the variance. After the presentation of the training inputs,

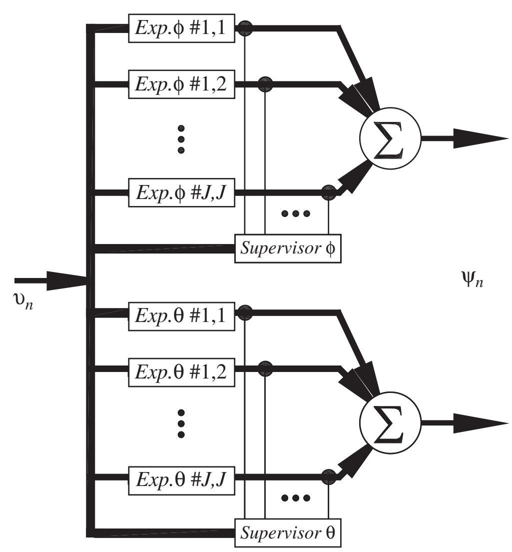

Fig. 4. MRBF System for planar arrays.

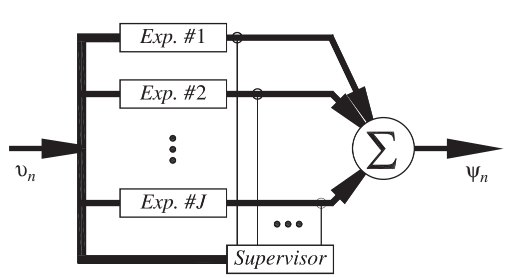

Fig. 3. MRBF System for linear arrays.

the radial basis functions (13) form the interpolation matrix Φ_j (14)

$$\varphi_{jk} = \exp\left(-\frac{1}{\sigma_{jk}^2}\|\upsilon_{jl} - \underline{c}_{jk}\|^2\right), \tag{13}$$

$$\Phi_j = \begin{pmatrix} \varphi(\upsilon_{j1}, \underline{c}_{j1}) & \varphi(\upsilon_{j1}, \underline{c}_{j2}) & \cdots & \varphi(\upsilon_{j1}, \underline{c}_{jK}) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi(\upsilon_{jL}, \underline{c}_{j1}) & \varphi(\upsilon_{jL}, \underline{c}_{j2}) & \cdots & \varphi(\upsilon_{jL}, \underline{c}_{jK}) & 1 \end{pmatrix}. \tag{14}$$

The output calculation of the module ψ_j is executed at the output layer of the RBF (15), that is, the linear combination of the interpolation matrix and the vector of synaptic weights $\underline{\varpi}_j$ (16):

$$\psi_j = \Phi_j \underline{\varpi}_j, \tag{15}$$

$$\underline{\varpi}_j = [\varpi_{j1}\ \ \varpi_{j2}\ \ \cdots\ \ \varpi_{jK}]^{\mathrm{T}}. \tag{16}$$

The training method by randomly selected fixed centers [6] was adopted for the modules because it favors the requirements of quickness and resolution. However, for input vectors of great dimensions, the training process becomes slow and the module response could not supply satisfactory resolution, due to the dimensions of the corresponding interpolation matrix. To solve this, the MRBF system was developed, which follows the principle of the modularity [6].

5. DOA estimation with MRBF system

The processing of the MRBF in both configurations (Figs. 3 and 4) is initiated by the *training* of the expert and supervisor modules, where the adjustment of the weights is processed. After that, each module will have reached high specialization in its respective sub-space, because the training examples of the same sub-space have lower variance. The posterior stage is the DOA determination, called *interpolation*, in which the MRBF receives a normalized steering vector and the modules supply its respective responses.

5.1. Training for a linear antennas array

The described training process in the following is refereed to a j expert for a linear antennas array. However, the supervisor network follows rigorously the same procedure, since the only difference is that the supervisor dedicates to all space input. A desired response vector is presented to the expert as in Eq. (17) and for each jlth angle from $\mathbf{d}_j$ a steering vector $\mathbf{a}_{jl}$ as in Eq. (2) is formed, then it must be normalized to form the lth input of the jth expert, the vector υ_{jl} (18)

$$\mathbf{d}_j = [\theta_{j1}\ \ \theta_{j2}\ \ \cdots\ \ \theta_{jL}], \tag{17}$$

$$\upsilon_{jl} = \frac{\mathbf{a}_{jl}}{\|\mathbf{a}_{jl}\|}. \tag{18}$$

The nonlinear mapping of the input space formed of each training vector υ_{jl} is executed in the hidden layer of the jth expert, composed of the gaussian radial basis functions φ_{jk} (13) and the interpolation matrix Φ_j is then generated (14).

The adaptation of synaptic weights is done by imposing that the output of jth expert ψ_j be equal to the vector of known angles $\mathbf{d}_j$, given by Eq. (19). Finally, $\underline{\varpi}_j$ (16) is adapted through the pseudo-inversion of Φ_j as in Eq. (20).

$$\psi_j = \mathbf{d}_j \Rightarrow \mathbf{d}_j = \Phi_j \underline{\varpi}_j, \tag{19}$$

$$\underline{\varpi}_j = \Phi_j^{+} \mathbf{d}_j. \tag{20}$$

5.2. Training for a planar array of antennas

There are J_ϕ sub-regions in the direction of azimuth and J_θ sub-regions in the direction of elevation, having a total of $J = J_\phi \times J_\theta$ experts. The desired responses of one expert $\mathbf{d}_{j\phi,j\theta}$ is given by Eq. (21)

$$\mathbf{d}_{j\phi,j\theta} = \begin{bmatrix} d_{j\phi} \\ d_{j\theta} \end{bmatrix} = \begin{bmatrix} \phi_{j\phi1} & \phi_{j\phi2} & \cdots & \phi_{j\phi L} \\ \theta_{j\theta1} & \theta_{j\theta2} & \cdots & \theta_{j\theta L} \end{bmatrix}. \tag{21}$$

Each column of $\mathbf{d}_{j\phi,j\theta}$ corresponds to a steering vector $\mathbf{a}(k_{j\theta l}, k_{j\phi l})$, that it will be normalized as in (18). The resultant input vector of training $\upsilon_{j\theta l,j\phi l}$ will be mapped in the nonlinear space $\Phi_{j\phi,j\theta}$ as in Eq. (14), executed in the hidden layer of the $(j\phi, j\theta)$th expert.

The synaptic weight adaptation is carried by imposing the corresponding outputs to the azimuth and elevation angles, respectively, $\psi_{j\phi}$ and $\psi_{j\theta}$, should be equal to the vectors of known angles $d_{j\phi}$ and $d_{j\theta}$, consequently, the equations of the expert output is given by (22). Finally, the weights $\underline{\varpi}_{\phi j}$ and $\underline{\varpi}_{j\theta}$ are adapted through the pseudo-inversion of $\Phi_{j\phi,j\theta}$ with respect to the respective desired response vectors (23). Where the weights of the $(j\phi, j\theta)$th expert form the matrix of Eq. (24)

$$\psi_{j\phi,j\theta} = \begin{bmatrix} \psi_{j\phi} \\ \psi_{j\theta} \end{bmatrix} = \begin{bmatrix} d_{j\phi} \\ d_{j\theta} \end{bmatrix} \Rightarrow \begin{bmatrix} d_{j\phi} \\ d_{j\theta} \end{bmatrix} = \begin{bmatrix} \Phi_{j\phi,j\theta}\underline{\varpi}_{\phi j} \\ \Phi_{j\phi,j\theta}\underline{\varpi}_{j\theta} \end{bmatrix}, \tag{22}$$

$$\begin{aligned} \underline{\varpi}_{\phi j} &= \Phi_{j\phi,j\theta}^{+} d_{j\phi}, \\ \underline{\varpi}_{j\theta} &= \Phi_{j\phi,j\theta}^{+} d_{j\theta}, \end{aligned} \tag{23}$$

$$\begin{bmatrix} \underline{\varpi}_{j\phi} \\ \underline{\varpi}_{j\theta} \end{bmatrix} = \begin{bmatrix} \varpi_{\phi j1} & \varpi_{\phi j2} & \cdots & \varpi_{\phi jK} \\ \varpi_{\theta j1} & \varpi_{\theta j2} & \cdots & \varpi_{\theta jK} \end{bmatrix}^{\mathrm{T}}. \tag{24}$$

5.3. Interpolation

From the weight vector $\underline{\varpi}_j$ of the experts and $\underline{\varpi}_{\mathrm{sup}}$ of the supervisor, the outputs ψ_j and ψ_{sup} will be obtained in response to the normalized steering vector of the nth incident signal υ_n

$$\upsilon_n = \frac{\mathbf{a}_n}{\|\mathbf{a}_n\|}. \tag{25}$$

Through the orientation of the supervisor network output ψ_{sup}, it will be determined which of the outputs ψ_j will be the output of the MRBF ψ_n corresponding to the nth incident wave.

5.3.1. Interpolation for a linear antennas array

When it is presented υ_n of Eq. (25) to the MRBF, as Fig. 3, are calculated the outputs of the supervisor $\psi_{\text{sup}\,n}$ (26) and each expert ψ_{jn} (27), forming a response vector of the experts $\psi_{\exp n}$ (28) for the nth signal.

$$\psi_{\text{sup}\,n} = \Phi_{\text{sup}\,n}\varpi_{\text{sup}}, \tag{26}$$

$$\psi_{jn} = \Phi_{jn}\varpi_j, \tag{27}$$

$$\psi_{\exp n} = [\psi_{1n} \;\; \psi_{2n} \;\; \cdots \;\; \psi_{Jn}]. \tag{28}$$

The output $\psi_{\text{sup}\,n}$ is transformed into a binary number, of J *bit*s. As the response $\psi_{\text{sup}\,n}$ belongs to only one subspace j, the corresponding binary number ψ_{bn} to the output of the supervisor for the nth signal will have only one *bit* 1 of order j, and all of the other *bit*s being equal to 0. The supervisor network receives this denomination because it selects, through the scalar product of Eq. (29) only the response of the expert j, that is DOA of the nth incident signal Ψ_n

$$\Psi_n = \psi_{jn} = \psi_{\exp n} \cdot \psi_{bn}^{\mathrm{T}}. \tag{29}$$

5.3.2. Interpolation for a planar antennas array

When υ_n is presented to the MRBF in accordance with Fig. 4, are calculated the outputs of the supervisor $\psi_{\text{sup}\,n}$ (30) and the outputs of each expert $\psi_{j\phi,j\theta n}$ (31) are calculated, forming a response matrix of the experts $\psi_{\exp n}$ (32) for the nth signal.

$$\psi_{\text{sup}\,n} = \begin{bmatrix} \psi_{\text{sup}\,\phi n} \\ \psi_{\text{sup}\,\theta n} \end{bmatrix} = \begin{bmatrix} \Phi_{\text{sup}\,n} \cdot \varpi_{\text{sup}\,\phi n} \\ \Phi_{\text{sup}\,n} \cdot \varpi_{\text{sup}\,\theta n} \end{bmatrix} \Rightarrow \begin{bmatrix} \psi_{b\phi n} \\ \psi_{b\theta n} \end{bmatrix}, \tag{30}$$

$$\psi_{j\phi,j\theta n} = \begin{bmatrix} \phi_{j\phi,j\theta n} \\ \theta_{j\phi,j\theta n} \end{bmatrix} = \begin{bmatrix} \Phi_{j\phi,j\theta n} \cdot \varpi_{j\phi n} \\ \Phi_{j\phi,j\theta n} \cdot \varpi_{j\theta n} \end{bmatrix}, \tag{31}$$

$$\psi_{\exp n} = \begin{bmatrix} \psi_{1\phi,1\theta n} & \psi_{1\phi,2\theta n} & \cdots & \psi_{1\phi,J\theta n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{J\phi,1\theta n} & \psi_{J\phi,2\theta n} & \cdots & \psi_{J\phi,J\theta n} \end{bmatrix}. \tag{32}$$

The output $\psi_{\text{sup}\,n}$ is transformed in binary numbers, of $J\phi$ and $J\theta$ *bit*s, respectively. Let the response $\psi_{\text{sup}\,\phi n}$, that it belongs to only one subspace $j\phi$. Then the binary number $\psi_{b\phi n}$ corresponding to the output of the supervisor for the nth signal in the azimuth direction will have only one *bit* 1 of order $j\phi$ and all of the other *bit*s being equal to 0. The same reasoning is used for the elevation direction, resulting in the binary number $\psi_{b\theta n}$.

The supervisor network guides through the scalar products of Eqs. (33) and (34) the selection of the expert network $(j\phi, j\theta)$ whose response is the DOA from the nth incident signal Ψ_n (35).

$$\phi_{j\phi,j\theta n} = \psi_{b\phi n}^{\mathrm{T}} \cdot \psi_{\exp \phi n} \cdot \psi_{b\theta n}, \tag{33}$$

$$\theta_{j\phi,j\theta n} = \psi_{b\phi n}^{\mathrm{T}} \cdot \psi_{\exp \theta n} \cdot \psi_{b\theta n}, \tag{34}$$

$$\Psi_n = \psi_{j\phi,j\theta n} = \begin{bmatrix} \phi_{j\phi,j\theta n} \\ \theta_{j\phi,j\theta n} \end{bmatrix}. \tag{35}$$

6. Simulations

The intelligent system developed for DOA determination, shown in Fig. 5, is composed by the following techniques: conventional beamforming, blind sources separation, and esteem of DOA with MRBF.

The processing is initiated with the reception of the plane waves by the antennas array of the conventional beamformer, where N independent and spatially separated carriers $s_n(p)$ reach the array in the same discrete instant time p. For each $s_n(p)$ is associated a corresponding DOA.

The output signal of the beamformer $\mathbf{x}(p)$ in Eq. (11), is a linear combination of incident signals and steering vectors, which is repassed to BSS block. Then, executing the JADE algorithm the matrix of steering vectors of the incident signals $\hat{\mathbf{A}}$ is esteemed.

The MRBF structure executes the DOA detection, and is trained for the following space inputs, in accordance with the type of array:

- Linear: $\theta : [0°; 80°]$;
- Planar: $\theta : [0°; 50°]$; $\phi : [0°; 50°]$.

Each expert have an angular range of 10°. Therefore, one will have eight experts for the system of beamformer with linear array and five experts for the system of beamformer with planar array.

The response of a supervisor for the system of beamformer with linear array and with planar array must

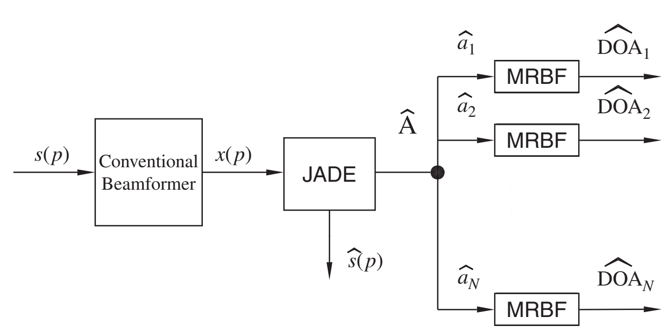

Fig. 5. DOA detection system.

be in accordance with Table 1. All of DOA pertaining to one of these intervals have the same supervisor response.

As approached in Section 2, a steering vector $\mathbf{a}_n$ brings the DOA information of a given signal. The estimates of the steering vectors $\hat{\mathbf{A}}$ are then repassed to a bank of N MRBF blocks, where DOAs of the signals are esteemed.

6.1. Beamformer with linear antennas array

The DOAs used in this simulation are exhibited in Table 2. The following parameters are considered: $\mathbf{M} = 5$ antenna elements, 3000 samples of the output signal of the beamformer, 40 training examples, 36 centers for the experts, 38 centers for the supervisor, SNR $= 20\,\text{dB}$ and SIR $= -10\,\text{dB}$.

The $P = 3000$ observed samples of the beamformer output (1) were processed by the JADE algorithm, generating the estimates of the steering vectors shown in Table 3.

The output ψ_n of the MRBF for the nth plane wave can be obtained through the scalar product between the response vector of the experts $\psi_{\exp n}$ and response vector of the supervisor $\psi_{\sup n}$, given by Eq. (29). Being thus, the DOAs were esteemed and shown with the percentual errors related to the real DOAs of Table 2.

The responses shown in Table 4 prove the capacity of the system to achieve the DOA detection of multiple incident signals on a linear antenna array in the presence of interferences and noises with a satisfactory resolution, due to the correct interaction among supervisor and experts modules.

6.2. Planar arrays

The DOA used in this simulation are shown in Table 5. The following parameters are considered in this simulation: $\mathbf{M} = 4 \times 4$ antenna elements, 10,000 samples of the signal of output of the array, 80 examples of training, 36 centers for the experts, 36 centers for the supervisor, SNR $= 2\,\text{dB}$ and SIR $= -5\,\text{dB}$.

After $P = 10,000$ samples of the beamformer output (4), the processing of the algorithm JADE generated the estimates of the steering vectors.

In accordance with Eq. (35), the output ψ_n of the MRBF for the nth plane wave can be obtained through the scalar products of Eqs. (33) and (34) with the binary responses of the supervisors $\psi^{\mathrm{T}}_{b\phi n}$ and $\psi^{\mathrm{T}}_{b\theta n}$.

The responses shown in Table 6 prove the capability of the system to accomplish the detection of DOA of multiple incident signals on a planar array of antennas in the

Table 1
Supervisors responses

Sub-Region	Interval	Linear array	Planar array
1	[0;10]	1 0 0 0 0 0 0 0	1 0 0 0 0
2	[10;20]	0 1 0 0 0 0 0 0	0 1 0 0 0
3	[20;30]	0 0 1 0 0 0 0 0	0 0 1 0 0
4	[30;40]	0 0 0 1 0 0 0 0	0 0 0 1 0
5	[40;50]	0 0 0 0 1 0 0 0	0 0 0 0 1
6	[50;60]	0 0 0 0 0 1 0 0	
7	[60;70]	0 0 0 0 0 0 1 0	
8	[70;80]	0 0 0 0 0 0 0 1	

Table 2
DOAs of the signals

n	1	2	3
DOA	60.2	20.4	45

Table 3
Steering vectors separated by BSS

$\hat{a}_1$	$[1.0000\ \ 0.4559-0.8970i\ \ -0.5891-0.8087i\ \ -0.9860+0.1465i\ \ -0.3268+0.9499i]^{\mathrm{T}}$
$\hat{a}_2$	$[1.0000\ \ -0.9156-0.4142i\ \ 0.6603+0.7468i\ \ -0.3083-0.9395i\ \ -0.0924+0.9908i]^{\mathrm{T}}$
$\hat{a}_3$	$[1.0000\ \ -0.6125-0.7773i\ \ -0.2326+0.9515i\ \ 0.8930-0.3884i\ \ -0.8339-0.4791i]^{\mathrm{T}}$

Table 4
DOAs esteemed by MRBF

n	1	2	3
Esteemed DOA	60.2253	20.4062	45.0884
Percentual error	−0.0420266	−0.0303922	−0.1964444

Table 5
DOAs for simulation with planar array

n	1	2	3
Azimuth DOA	23.3	4.6	46.9
Elevation DOA	36.0	22.3	13.1

Table 6
DOAs esteemed by MRBF

n	1	2	3
Azimuth DOA	23.2956	4.5241	46.9352
Azimuth percentual error	0.0188841	1.65	−0.0750533
Elevation DOA	36.1823	22.2892	13.0967
Azimuth percentual error	−0.5063889	0.0484305	0.0251908

presence of interferences and noises. These results were possible because the modules esteemed DOAs correctly in relation to the sub-regions that they belong and in relation to the choice of the expert networks responses.

7. Conclusions

A detector of directions of arrival for multiple signals incident on an antenna array was presented. The techniques that operated jointly to form the intelligent detector of DOA for multiple signals were described: conventional beamforming and the blind sources separation, in addition to the main contribution of this work, the direction of arrival detector for one signal on antennas arrays of linear or planar geometries based on neural networks of radial basis functions in modular structure. This technique presents advantages that were pointed by the versatility to operate with arrays of distinct geometries and high resolution of response. Simulations demonstrated that the three techniques work efficiently, receiving carrying waves in an environment with noise and the consequent beamforming, separating the steering vectors and, finally, getting DOA of the incident signals from relevant informations obtained from the blind separation algorithm, JADE, through system MRBF blocks. Future works will apply this detection system of multiple directions of arrival to acoustics waves, or any another application of signal processing where the separation and posterior DOA detection of multiple mixed signals in the output of sensor arrays is required.

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