Available online at www.sciencedirect.com
ScienceDirect
Neurocomputing 70 (2006) 164–172
NEUROCOMPUTING
www.elsevier.com/locate/neucom

Dynamics of periodic Cohen–Grossberg neural networks with varying delays ☆

Zhaohui Yuan[a,*], Lifen Yuan[b], Lihong Huang[a]

[a]*College of Mathematics and Econometrics, Hunan University, Changsha, Hunan 410082, China*
[b]*College of Information Science, Hunan Normal University, Changsha, Hunan 410081, China*

Received 23 August 2005; received in revised form 2 February 2006; accepted 17 April 2006
Communicated by S. Arik
Available online 14 June 2006

Abstract

In this paper, a periodic Cohen–Grossberg neural networks with varying delay is considered. Under the generalization of dropping the boundedness and differentiability hypotheses for activation functions as well as differentiability of the varying delays with derivative not greater than 1, some novel sufficient conditions are given to guarantee the existence and global exponential stability of periodic solutions. Our results obtained here improve and extend some previously related results. In addition, two examples are also to illustrate the theory.

Keywords: Global exponential stability; Periodic solution; Neural networks; Varying delay

1. Introduction

In recent years, considerable attention has been paid to study the dynamics of neural networks due to the application in many fields such as image and signal processing, pattern recognition, optimization and content-addressable memory. Such applications heavily depend on the dynamic behavior of the networks, therefore, the analysis of these dynamic behaviors is a necessary step for practical design of neural networks. Cohen and Grossberg [8] proposed a class of neural networks in 1983, which is described by a system of differential equations

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i(x_i)\left[b_i(x_i(t)) - \sum_{j=1}^{n} w_{ij} f_j(x_j(t))\right],\quad i = 1, 2, \ldots, n, \tag{1}$$

where n denotes the number of neurons in the network, x_i denotes the state variable associated to ith neuron, a_i represents an amplification function, and b_i is an appropriately behaved function. The $n \times n$ connection matrix $W = (w_{ij})$ tells how the neurons are connected in the network, and the activation function f_j shows the output of the jth neuron.

However, both in biological and artificial neural networks, the interconnections between neurons are generally asynchronous. As a result, time delays are inevitably encountered in neural networks. It is also important to incorporate time delay into the model equations of the network. Discrete delays were introduced into system (1) by considering the following system [25]:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i(x_i)\left[b_i(x_i(t)) - \sum_{k=0}^{K}\sum_{j=1}^{n} w_{ij}^{(k)} f_j(x_j(t-\tau_k)) + u_i\right],\quad i = 1, 2, \ldots, n, \tag{2}$$

where the delays τ_k ($k = 1, 2, \ldots, K$) are arranged such that $0 = \tau_0 < \tau_1 < \cdots < \tau_K$. Further studies were taken by [5,19,22–24]. In electronic implementation of neural networks, the delays are usually time variant and sometimes vary dramatically with time due to the finite switch speed of

☆This work was supported by the National Science Foundation of China (10371034).
*Corresponding author. Tel./fax: +86 7318823056.
E-mail address: yzhh312@tom.com (Z. Yuan).

0925-2312/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.neucom.2006.04.002

amplifiers and faults in the electrical circuits. Therefore, a more generalized Cohen–Grossberg neural networks with variable coefficients and time-varying delays model

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{n} w_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} m_{ij} f_j(x_j(t-\tau_{ij}(t))) + u_i\right], \quad t \geqslant 0, \quad i = 1, 2, \ldots, n, \tag{3}$$

were studied and global asymptotic and exponential stability results were obtained in Refs. [3,13,17,26]. System (3) is quite general and it includes several well-known neural network models as its special cases such as Hopfield neural networks [2,1,30,11,16,14,21,27,29], cellular neural networks [6,7,20], etc. However, the existing results for system (3) in [8,22–26,5,19,13] are all based on the assumption on boundedness of activation function $f_i(v)$ $(i = 1, 2, \ldots, n)$. We also notice that most results deal with varying delays $\tau_{ij}(t)$ were built on such an assumption: the delays $\tau_{ij}(t)$ are differential and their derivatives were simultaneously required to be not greater than 1 (see, for example, [3,17,2,21,29]). This assumption may impose a very strict constraint on model because time delays sometimes vary dramatically with time in real circuits. On the other hand, it is well known that studies on neural dynamical systems not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory, almost periodic oscillatory properties, chaos, and bifurcation. As a fundamental significance in the control of regular dynamical functions, the existence and global exponential stability of periodic solution for neural networks have been studied by some authors [4,9,12,15,18,28,31,32], but to the best of the author's knowledge, few authors have considered periodic oscillatory solutions for Cohen–Grossberg neural network with varying delays and this fact motivates our work.

The main objective of this paper is to give some sufficient conditions for the existence and global exponential stability of periodic solutions for a class of generalized periodic Cohen–Grossberg neural network with varying delays under some weaker conditions. The main results extend and develop the existent outcome in earlier literature. Furthermore, it is worthwhile to mention that our approach is more applied because it is less restrictive and independent of delay. The rest of this paper is organized as follows. In Section 2, a model description and some preliminaries are given. In Section 3, our main results on the existence as well as the global exponential stability of periodic solution are presented by some analytical techniques. These criteria are all independent of the magnitudes and derivatives of the varying delays, and so the varying delays under these conditions are harmless. Some comparisons and example are given in Section 4 to demonstrate our results. Finally, conclusions are drawn in Section 5.

2. Model description and preliminaries

In this paper, we investigate the following generalized periodic Cohen–Grossberg neural network with varying delays described by

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i(x_i(t))\left[b_i(t, x_i(t)) - \sum_{j=1}^{n} w_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^{n} m_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) + u_i(t)\right], \quad t \geqslant 0, \quad i = 1, 2, \ldots, n, \tag{4}$$

where the continuous functions $w_{ij}(t), m_{ij}(t), u_i(t)$ and $\tau_{ij}(t)$ are ω-periodic: $w_{ij}(t)$ weights the strength of the jth neuron on the ith neuron at time t; $m_{ij}(t)$ is the strength of the jth neuron on the ith neuron at time $t - \tau_{ij}(t)$; $u_i(t)$ denotes the input to ith neuron; $\tau_{ij}(t)$ corresponds to the transmission delay along the axon of jth neuron and is a nonnegative function, the positive continuous function $a_i : R \to R^+$ represents an amplification function and there exist two positive constant numbers $\underline{\alpha}_i$ and $\overline{\alpha}_i$ such that

$$0 < \underline{\alpha}_i \leqslant a_i(v) \leqslant \overline{\alpha}_i \quad \forall v \in R, \quad i = 1, 2, \ldots, n, \tag{5}$$

the continuous function $b_i : R^+ \times R \to R$ denotes an appropriately behaved function which is ω-periodic with respect to its first argument and there exists some continuous positive ω-periodic function $c_i(t)$ such that

$$\frac{b_i(t, u) - b_i(t, v)}{u - v} \geqslant c_i(t) > 0 \quad \forall u \neq v, \quad u, v \in R, \quad i = 1, 2, \ldots, n. \tag{6}$$

The initial conditions associated with (4) are given in form

$$x_i(s) = \phi_i(s) \in C([-\tau, 0], R), \quad i = 1, 2, \ldots, n, \tag{7}$$

and

$$\|\phi\| = \max_{1 \leqslant i \leqslant n} \sup_{s \in [-\tau, 0]} \{|\phi_i(s)|\} \quad \text{for } \phi = (\phi_1, \phi_2, \ldots, \phi_n)^{\mathrm{T}} \in C([-\tau, 0], R^n),$$

where $\tau = \max_{1 \leqslant i,j \leqslant n}\{\tau_{ij}(t) : t \in [0, \infty)\}$.

Assume that each function $f_i(u)$ $(i = 1, 2, \ldots, n)$ is continuous on R, possesses the following properties:

(A_1) There exist real numbers $d_i \geqslant 0$ and $k_i \geqslant 0$ such that for $u \in R$ holds,

$$|f_i(u)| \leqslant k_i|u| + d_i, \quad i = 1, 2, \ldots, n.$$

(A_2) There exist real numbers $k_i \geqslant 0$ such that for $u, v \in R$ holds,

$$|f_i(u) - f_i(v)| \leqslant k_i|u - v|, \quad i = 1, 2, \ldots, n.$$

Note that Assumption (A_1) is implied by (A_2), whereas (A_1) does not imply that the function f_i is globally Lipschitz continuous.

Definition 1. The periodic solution $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^{\mathrm{T}}$ of system (4) is called globally exponentially stable if there exist $M > 0$ and positive constant μ such that

$$|x_i(t) - x_i^*(t)| \leqslant M\|\phi - \phi^*\|\mathrm{e}^{-\mu t}, \quad t \geqslant 0, \ \ i = 1, 2, \ldots,$$

holds for any solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}}$ of system (4), where ϕ and ϕ^* are the initial functions of solutions $x(t)$ and $x^*(t)$, respectively.

In order to use Mawhin's continuation theorem to study the existence of solution for system (4), now, we should make some preparations.

Let X, Z be real Banach spaces and I be the identity mapping; $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim(\operatorname{Ker} L) = \operatorname{codim}(\operatorname{Im} L) < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm qmapping of index zero, there must exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Ker} L = \operatorname{Im} P$, $\operatorname{Ker} Q = \operatorname{Im} L$, and $X = \operatorname{Ker} L \oplus \operatorname{Ker} P$, $Z = \operatorname{Im} L \oplus \operatorname{Im} Q$. It follows that $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : \operatorname{Dom} L \cap \operatorname{Ker} P \to \operatorname{Im} L$ is invertible, and the inverse of this map is denoted by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN : \overline{\Omega} \to Z$ and $K_P(I-Q)N : \overline{\Omega} \to X$ are compact on $\overline{\Omega}$. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 1 (*Continuation theorem, Gaines and Mawhin [10]*). *Let $\Omega \subset X$ be an open bounded set and L be a Fredholm mapping of index zero. Assume that $N : X \to Z$ is a continuous operator and is L-compact on $\overline{\Omega}$. Furthermore, suppose that*

(a) *for each $\lambda \in (0,1)$ and $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;*
(b) *for each $x \in \partial\Omega \cap \operatorname{Ker} L$, $QNx \neq 0$;*
(c) $\deg\{JQNx, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$.

Then, $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

For the sake of simplicity, the following notations will be used throughout this paper:

$$\hat{c}_i = \frac{1}{\omega}\int_0^{\omega} c_i(t)\,\mathrm{d}t, \quad [f(t)]^+ = \max_{t \in [0,\omega]}\{|f(t)|\},$$
$$[f(t)]^l = \min_{t \in [0,\omega]}\{|f(t)|\},$$

where f is a continuous ω-periodic function.

3. Main results

In this section, we will give the sufficient conditions on existence of solution of system (4) and analyze the exponential stability of solution.

Theorem 1. *Assume that (A_1) holds and there exist n positive numbers $\xi_1, \xi_2, \ldots, \xi_n$ such that*

$$\delta \overset{\text{def}}{=} \max_{1 \leqslant i \leqslant n}\left\{\sum_{j=1}^{n} k_j \xi_i^{-1} \xi_j [c_i^{-1}(t)(|w_{ij}(t)| + |m_{ij}(t)|)]^+\right\} < 1, \qquad (8)$$

then system (4) has at least one ω-periodic solution.

Proof. Define $X = Z = \{x = x(t) \in C(R, R^n) : x(t+\omega) = x(t)\}$, endowed with norm

$$\|x(t)\| = \max_{1 \leqslant i \leqslant n} \sup_{[0,\omega]}\{\xi_i^{-1}|x_i(t)|\} \quad \forall x \in X (\text{or } Z).$$

Then, X and Z are both real Banach spaces.

Let

$$L : \operatorname{Dom} L \cap X \to Z, \quad Lx = x' \quad \forall x \in X$$

where $\operatorname{Dom} L = C^1(R, R^n) \cap X$

and

$$N : X \to Z,$$

$$N\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} -a_1(x_1(t))\left[b_1(t, x_1(t)) - \sum_{j=1}^{n} w_{1j}(t) f_j(x_j(t)) - \sum_{j=1}^{n} m_{1j}(t) f_j(x_j(t - \tau_{1j}(t))) + u_1(t)\right] \\ -a_2(x_2(t))\left[b_2(t, x_2(t)) - \sum_{j=1}^{n} w_{2j}(t) f_j(x_j(t)) - \sum_{j=1}^{n} m_{2j}(t) f_j(x_j(t - \tau_{2j}(t))) + u_2(t)\right] \\ \vdots \\ -a_n(x_n(t))\left[b_n(t, x_n(t)) - \sum_{j=1}^{n} w_{nj}(t) f_j(x_j(t)) - \sum_{j=1}^{n} m_{nj}(t) f_j(x_j(t - \tau_{nj}(t))) + u_n(t)\right] \end{bmatrix}.$$

Define two projectors P and Q as

$$P(x) = \frac{1}{\omega}\int_0^{\omega} x\,\mathrm{d}t, \quad x = x(t) \in X$$

and

$$Q(z) = \frac{1}{\omega}\int_0^{\omega} z\,\mathrm{d}t, \quad z = z(t) \in Z.$$

Obviously, $\operatorname{Ker} L = R^n$, $\operatorname{Im} L = \{(x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in X : \int_0^\omega x_i(t)\,\mathrm{d}t = 0, i = 1, 2, \ldots, n\}$ is closed in X and $\dim(\operatorname{Ker} L) = \operatorname{codim}(\operatorname{Im} L) = n$. Hence, L is a Fredholm mapping of index zero. Moreover, we have $\operatorname{Ker} L = \operatorname{Im} P$, $\operatorname{Ker} Q = \operatorname{Im} L$. Let $L_P = L|_{\operatorname{Dom} L \cap \operatorname{Ker} P}$, then the generalized inverse (to L) $K_P = L_P^{-1} : \operatorname{Im} L \to \operatorname{Dom} L \cap \operatorname{Ker} P$ is given by

$$K_P(z) = \int_0^t z(s)\,\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t z(s)\,\mathrm{d}s\,\mathrm{d}t.$$

Hence, $K_P(I-Q)g : X \to X$,

$$x \to \int_0^t g(x(s))\,\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^\eta g(x(s))\,\mathrm{d}s\,\mathrm{d}\eta + \left(\frac{1}{2} - \frac{t}{\omega}\right)\int_0^\omega g(x(t))\,\mathrm{d}t,$$

where $g = (g_1, g_2, \ldots, g_n)^{\mathrm{T}}$, g_i $(i = 1, 2, \ldots, n)$ are continuous and ω-periodic.

Thus, $QN : X \to R^n$ and $K_P(I-Q)N : X \to R^n$ are both continuous by Lebesgue convergence theorem, and moreover, using the Arzela–Ascoli theorem, $QN(\overline{\Omega})$ and $K_P(I-Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$.

Corresponding to the operator equation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$\begin{aligned} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = & -\lambda a_i(x_i(t))\left[b_i(t, x_i(t)) - \sum_{j=1}^n w_{ij}(t) f_j(x_j(t)) \right. \\ & \left. - \sum_{j=1}^n m_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) + u_i(t)\right], \quad t \geqslant 0, \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{9}$$

Suppose that $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in X$ is a solution of (9) for $\lambda \in (0, 1)$. Then $x_i(t)$ is continuously differentiable. Thus, there exist $t_i \in [0, \omega]$ such that $|x_i(t_i)| = [x_i(t)]^+$. Hence, $\mathrm{d}x_i(t)/\mathrm{d}t|_{t=t_i} = 0$. This implies

$$\begin{aligned} b_i(t_i, x_i(t_i)) = & \sum_{j=1}^n w_{ij}(t_i) f_j(x_j(t_i)) \\ & + \sum_{j=1}^n m_{ij}(t_i) f_j(x_j(t_i - \tau_{ij}(t_i))) - u_i(t_i). \end{aligned} \tag{10}$$

Noting that the assumption on $b_i(t, x_i)$ in (6) implies that

$|b_i(t, x_i)| \geqslant c_i(t)|x_i| - |b_i(t, 0)|$,

it follows from Assumption (A_1) and (10) that

$$\begin{aligned} c_i(t_i)|x_i(t_i)| \leqslant & \sum_{j=1}^n |w_{ij}(t_i)|(k_j|x_j(t_i)| + d_j) \\ & + \sum_{j=1}^n |m_{ij}(t_i)|(k_j|x_j(t_i - \tau_{ij}(t_i))| + d_j) + |u_i(t)| \\ & + |b_i(t_i, 0)| \\ \leqslant & \sum_{j=1}^n (|w_{ij}(t_i)| + |m_{ij}(t_i)|)k_j|x_j(t_j)| \\ & + \sum_{j=1}^n d_j(|w_{ij}(t_i)| + |m_{ij}(t_i)|) + |u_i(t_i)| + |b_i(t_i, 0)|, \end{aligned}$$

it follows that

$$\begin{aligned} \xi_i^{-1}|x_i(t_i)| \leqslant & \sum_{j=1}^n k_j \xi_i^{-1} \xi_j c_i^{-1}(t_i)(|w_{ij}(t_i)| + |m_{ij}(t_i)|)\xi_j^{-1}|x_j(t_j)| \\ & + \sum_{j=1}^n d_j \xi_i^{-1} c_i^{-1}(t_i)(|w_{ij}(t_i)| \\ & + |m_{ij}(t_i)|) + \xi_i^{-1} c_i^{-1}(t_i)(|u_i(t_i)| + |b_i(t_i, 0)|). \end{aligned} \tag{11}$$

Without loss of generality, we assume that there must exist some $k \in \{1, 2, \ldots, n\}$ such that $\|x(t)\| = \xi_k^{-1}|x_k(t_k)|$. Thus, from (11), we have

$$\begin{aligned} \|x(t)\| \leqslant & \sum_{j=1}^n k_j \xi_k^{-1} \xi_j c_k^{-1}(t_k)(|w_{kj}(t_k)| + |m_{kj}(t_k)|)\|x(t)\| \\ & + \sum_{j=1}^n d_j \xi_k^{-1} c_k^{-1}(t_k)(|w_{kj}(t_k)| + |m_{kj}(t_k)|) \\ & + \xi_k^{-1} c_k^{-1}(t_k)(|u_k(t_k)| + |b_k(t_k, 0)|) \\ \leqslant & \delta\|x(t)\| + D, \end{aligned}$$

where

$$D = \max_{1 \leqslant i \leqslant n}\left\{\sum_{j=1}^n d_j \xi_i^{-1}[c_i^{-1}(t)(|w_{ij}(t)| + |m_{ij}(t)|)]^+ + \xi_i^{-1}[c_i^{-1}(t)(|u_i(t)| + |b_i(t, 0)|)]^+\right\}.$$

Therefore,

$$\|x(t)\| \leqslant \frac{D}{1-\delta} \stackrel{\text{def}}{=} H, \quad t \in [0, \omega]. \tag{12}$$

Now, we take $\Omega = \{x \in X : \|x\| < H + 1\}$. From (12), we see that $Lx \neq \lambda Nx$ for $x \in \partial\Omega \cap \operatorname{Dom} L$. This Ω satisfies condition (a) in Lemma 1. When $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^n$, x is a constant vector in R^n with $\|x\| = \max_{1 \leqslant i \leqslant n}\{\xi_i^{-1}|x_i|\} = H + 1$. It follows that

$$\begin{aligned} (QNx)_i = \frac{1}{\omega}\int_0^\omega \left\{ -a_i(x_i)\left[b_i(t, x_i) - \sum_{j=1}^n (w_{ij}(t) \right.\right. \\ \left.\left. + m_{ij}(t)) f_j(x_j) + u_i(t)\right]\right\} \mathrm{d}t. \end{aligned} \tag{13}$$

Without loss of generality, we assume that there exists some $k \in \{1, 2, \ldots, n\}$ such that $\|x\| = \xi_k^{-1}|x_k| = H + 1$.

We claim that

$$|QNx|_k > 0. \tag{14}$$

Obviously, it is suffice to prove

$$\frac{\xi_k^{-1}\operatorname{sgn} x_k}{a_k(x_k)}(QNx)_k<0. \tag{15}$$

In fact, it is easy to verify that

$$\begin{aligned}
&\frac{\xi_k^{-1}\operatorname{sgn} x_k}{a_k(x_k)}(QNx)_k\\
&\quad=\frac{\xi_k^{-1}\operatorname{sgn} x_k}{a_k(x_k)}\frac{1}{\omega}\int_0^{\omega}\Bigg\{-a_k(x_k)\Bigg[b_k(t,x_k)\\
&\qquad-\sum_{j=1}^{n}(w_{kj}(t)+m_{kj}(t))f_j(x_j)+u_k(t)\Bigg]\Bigg\}\,\mathrm{d}t\\
&\quad\leqslant\frac{1}{\omega}\int_0^{\omega}\Bigg[-\xi_k^{-1}b_k(t,x_k)\operatorname{sgn} x_k\\
&\qquad+\sum_{j=1}^{n}\xi_k^{-1}(|w_{kj}(t)|+|m_{kj}(t)|)|f_j(x_j)|+\xi_k^{-1}|u_k(t)|\Bigg]\,\mathrm{d}t
\end{aligned} \tag{16}$$

holds. Note that condition (6) implies

$$b_k(t,x_k)\operatorname{sgn} x_k\geqslant c_k(t)|x_k|-|b_k(t,0)|. \tag{17}$$

Applying Assumption (A_1) and inequality (17) to (16), we obtain

$$\begin{aligned}
&\frac{\xi_k^{-1}\operatorname{sgn} x_k}{a_k(x_k)}(QNx)_k\\
&\quad\leqslant\frac{1}{\omega}\int_0^{\omega}\Bigg\{-\xi_k^{-1}[c_k(t)|x_k|-|b_k(t,0)|]\\
&\qquad+\sum_{j=1}^{n}\xi_k^{-1}(|w_{kj}(t)|+|m_{kj}(t)|)|f_j(x_j)|+\xi_k^{-1}|u_k(t)|\Bigg\}\mathrm{d}t\\
&\quad\leqslant\frac{1}{\omega}\int_0^{\omega}c_k(t)\Bigg\{-\xi_k^{-1}|x_k|\\
&\qquad+\sum_{j=1}^{n}k_j\xi_k^{-1}\xi_j c_k(t)^{-1}(|w_{kj}(t)|+|m_{kj}|)\xi_j^{-1}|x_j|\\
&\qquad+\sum_{j=1}^{n}d_j\xi_k^{-1}c_k^{-1}(t)(|w_{kj}(t)|+|m_{kj}(t)|)\\
&\qquad+\xi_k^{-1}c_k(t)^{-1}(|u_k(t)|+|b_k(t),0|)\Bigg\}\mathrm{d}t\\
&\quad\leqslant\frac{1}{\omega}\int_0^{\omega}c_k(t)[-(H+1)+\delta(H+1)+D]\,\mathrm{d}t\\
&\quad=\hat{c}_k[-(H+1)+\delta(H+1)+D]\\
&\quad=\hat{c}_k[-(H+1)+\delta(H+1)+(1-\delta)H]\\
&\quad=-\hat{c}_k(1-\delta)\\
&\quad<0.
\end{aligned} \tag{18}$$

Therefore, (15) holds, and hence

$$QNx\neq 0 \quad \text{for } x\in\partial\Omega\cap\operatorname{Ker}L,$$

thus condition (b) of Lemma 1 is satisfied.

Furthermore, let

$$\Psi(\rho,x)=(\Psi_1(\rho,x),\ldots,\Psi_n(\rho,x))^{\mathrm{T}}\overset{\text{def}}{=}-\rho x+(1-\rho)JQNx \tag{19}$$

for all $x=(x_1,x_2,\ldots,x_n)^{\mathrm{T}}\in\Omega\cap\operatorname{Ker}L=\Omega\cap R^n$ and $\rho\in[0,1]$, where the isomorphism J is from $\operatorname{Im}\Omega$ onto $\operatorname{Ker}L$ and we take it as the identity mapping, since $\operatorname{Ker}L=\operatorname{Im}Q$. When $x\in\partial\Omega\cap\operatorname{Ker}L$ and $\rho\in[0,1]$, $x=(x_1,x_2,\ldots,x_n)^{\mathrm{T}}$ is a constant vector in R^n with $\|x\|=H+1$. We assume that there exists some $k\in\{1,2,\ldots,n\}$ such that $\|x\|=\xi_k^{-1}|x_k|=H+1$.

We claim that

$$\Psi_k(\rho,x)\neq 0 \quad \text{for any } \rho\in[0,1], \tag{20}$$

that is,

$$\begin{aligned}
&-\rho x_k+\frac{1-\rho}{\omega}\int_0^{\omega}\Bigg\{-a_k(x_k)\Bigg[b_k(t,x_k)-\sum_{j=1}^{n}(w_{kj}(t)\\
&\quad+m_{kj}(t))f_j(x_k)+u_k(t)\Bigg]\Bigg\}\,\mathrm{d}t\neq 0.
\end{aligned} \tag{21}$$

In fact, using (18) and (19), we have

$$\begin{aligned}
&\frac{\xi_k^{-1}\operatorname{sgn} x_k}{a_k(x_k)}\Psi_k(\rho,x)\\
&\quad=-\rho\frac{\xi_k^{-1}|x_k|}{a_k(x_k)}+(1-\rho)\frac{\xi_k^{-1}\operatorname{sgn} x_k}{a_k(x_k)}(QNx)_k<0\\
&\qquad\text{for any } \rho\in[0,1].
\end{aligned}$$

Thus, (20) holds. Therefore,

$$\Psi(\rho,x)\neq 0 \quad \text{for any } x\in\partial\Omega\cap\operatorname{Ker}L,$$

and hence

$$\begin{aligned}
\deg\{JQNx,\Omega\cap\operatorname{Ker}L,0\}&=\deg\{-x,\Omega\cap\operatorname{Ker}L,0\}\\
&=(-1)^n\neq 0.
\end{aligned}$$

Condition (c) of Lemma 1 is also satisfied. Thus, by Lemma 1, we conclude that $Lx=Nx$ has at least one solution in X, that is, system (4) has at least one ω-periodic solution. This completes the proof of Theorem 1. □

Theorem 2. *Assume that (A_2) holds and there exist n positive numbers $\xi_1,\xi_2,\ldots,\xi_n$ such that*

$$\sum_{j=1}^{n}k_j\overline{\alpha}_j\underline{\alpha}_i^{-1}\xi_i^{-1}\xi_j[c_i^{-1}(t)(|w_{ij}(t)|+|m_{ij}(t)|)]^{+}<1 \quad \textit{for } i=1,2,\ldots,n, \tag{22}$$

then system (4) has at least one ω-periodic solution $x^*(t)=(x_1^*(t),x_2^*(t),\ldots,x_n^*(t))^{\mathrm{T}}$, which is globally exponentially stable.

Proof. Obviously, the inequality (22) implies (8) holds and the Assumption (A_2) implies (A_1) holds. Hence, from Theorem 1 we conclude that system (4) has at least one ω-periodic solution $x^*(t)=(x_1^*(t),x_2^*(t),\ldots,x_n^*(t))^{\mathrm{T}}$. Assume that $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^{\mathrm{T}}$ be an arbitrary solution of system (4). From (4), it is easy to verify that there exist some $\mu \in (0, \underline{\alpha}_i[c_i(t)]^l)$ such that

$$\gamma \overset{\text{def}}{=} \max_{1\leqslant i\leqslant n}\left\{\sum_{j=1}^{n} k_j\overline{\alpha}_j\xi_i^{-1}\xi_j[(\underline{\alpha}_i c_i(t)-\mu)^{-1}(|w_{ij}(t)| + e^{\mu\tau}|m_{ij}(t)|)]^{+}\right\} < 1. \tag{23}$$

Let

$$y_i(t) = \left|\int_{x_i^*(t)}^{x_i(t)} \frac{1}{a_i(s)}\,ds\right|.$$

Obviously

$$\frac{1}{\overline{\alpha}_i}|x_i(t)-x_i^*(t)| \leqslant y_i(t) \leqslant \frac{1}{\underline{\alpha}_i}|x_i(t)-x_i^*(t)|. \tag{24}$$

From system (4) and inequality (24), we have

$$\begin{aligned}
\frac{dy_i(t)}{dt} &= \left(\frac{x_i'(t)}{a_i(x_i(t))} - \frac{x_i^{*\prime}(t)}{a_i(x_i^*(t))}\right)\operatorname{sgn}(x_i(t)-x_i^*(t))\\
&= -(b_i(t,x_i(t)) - b_i(t,x_i^*(t))\operatorname{sgn}(x_i(t)-x_i^*(t))\\
&\quad + \left[\sum_{j=1}^{n} w_{ij}(t)(f_j(x_j(t)) - f_j(x_j^*(t)))\right.\\
&\quad \left. + \sum_{j=1}^{n} m_{ij}(t)(f_j(x_j(t-\tau_{ij}(t))) - f_j(x_j^*(t-\tau_{ij})))\right]\\
&\quad \times \operatorname{sgn}(x_i(t)-x_i^*(t))\\
&\leqslant -c_i(t)|x_i(t)-x_i^*(t)| + \sum_{j=1}^{n}|w_{ij}(t)|k_j|x_j(t)-x_j^*(t)|\\
&\quad + \sum_{j=1}^{n}|m_{ij}(t)|k_j|x_j(t-\tau_{ij}(t)) - x_j^*(t-\tau_{ij}(t))|\\
&\leqslant -\underline{\alpha}_i c_i(t)y_i(t) + \sum_{j=1}^{n} k_j\overline{\alpha}_j|w_{ij}(t)|y_j(t)\\
&\quad + \sum_{j=1}^{n} k_j\overline{\alpha}_j|m_{ij}(t)|y_j(t-\tau_{ij}(t)).
\end{aligned} \tag{25}$$

Set $V_i(t) = \xi_i^{-1}y_i(t)e^{\mu t}$ if $t\geqslant 0$ and $V_i(t) = \xi_i^{-1}y_i(t)$ otherwise. Then, for $t\geqslant 0$ and $i = 1,2,\ldots,n$, it follows from (25) that

$$\begin{aligned}
V_i(t) &\leqslant \exp\left[\int_0^t(\mu-\underline{\alpha}_i c_i(s))\,ds\right]V_i(0)\\
&\quad + \sum_{j=1}^{n} k_j\overline{\alpha}_j\xi_i^{-1}\xi_j\exp\left[\int_0^t(\mu-\underline{\alpha}_i c_i(s))\,ds\right]\\
&\quad \cdot \int_0^t \exp\left[\int_0^s(\underline{\alpha}_i c_i(\theta)-\mu)\,d\theta\right](|w_{ij}(s)|V_j(s)\\
&\quad + |m_{ij}(s)|e^{\mu\tau}V_j(s-\tau_{ij}(s)))\,ds\\
&\leqslant \exp\left[\int_0^t(\mu-\underline{\alpha}_i c_i(s))\,ds\right]V_i(0)\\
&\quad + \sum_{j=1}^{n} k_j\overline{\alpha}_j\xi_i^{-1}\xi_j\exp\left[\int_0^t(\mu-\underline{\alpha}_i c_i(s))\,ds\right]\\
&\quad \cdot \int_0^t \exp\left[\int_0^s(\underline{\alpha}_i c_i(\theta)-\mu)\,d\theta\right](|w_{ij}(s)|\|V_j(s)\|_\tau\\
&\quad + e^{\mu\tau}|m_{ij}(s)|\|V_j(s)\|_\tau)\,ds,
\end{aligned} \tag{26}$$

where $\|V_i(t)\|_\tau = \sup_{s\in[t-\tau,t]}\{|V_i(s)|\}$. Set

$$B = \max_{1\leqslant i\leqslant n}\{\xi_i^{-1}\|y_i(0)\|_\tau\}.$$

We will prove that, for any sufficiently small constant $\varepsilon > 0$,

$$V_i(t) < B+\varepsilon \quad \text{for all } t\geqslant 0 \text{ and } i\in\{1,2,\ldots,n\}. \tag{27}$$

By a way of contrary, without loss of generality, we assume that there must exist some t^* and $k\in\{1,2,\ldots,n\}$ such that

$$V_k(t^*) = B+\varepsilon, \quad V_i(t)\leqslant B+\varepsilon \quad \text{for } t\in[0,t^*),\ i\in\{1,2,\ldots,n\}.$$

It follows from (23) and (26) that

$$\begin{aligned}
B+\varepsilon &= V_k(t^*)\\
&\leqslant \exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,ds\right]V_k(0)\\
&\quad + \sum_{j=1}^{n} k_j\overline{\alpha}_j\xi_k^{-1}\xi_j\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,ds\right]\\
&\quad \cdot \int_0^{t^*}\exp\left[\int_0^s(\underline{\alpha}_k c_k(\theta)-\mu)\,d\theta\right](|w_{kj}(s)|\|V_j(s)\|_\tau\\
&\quad + e^{\mu\tau}|m_{kj}(s)|\|V_j(s)\|_\tau)\,ds\\
&\leqslant (B+\varepsilon)\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,ds\right]\\
&\quad + (B+\varepsilon)\sum_{j=1}^{n} k_j\overline{\alpha}_j\xi_k^{-1}\xi_j\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,ds\right]\\
&\quad \cdot \int_0^{t^*}\exp\left[\int_0^s(\underline{\alpha}_k c_k(\theta)-\mu)\,d\theta\right](|w_{kj}(s)|\\
&\quad + e^{\mu\tau}|m_{kj}(s)|)\,ds\\
&\leqslant (B+\varepsilon)\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,ds\right] + (B+\varepsilon)\\
&\quad \cdot \sum_{j=1}^{n} k_j\overline{\alpha}_j\xi_k^{-1}\xi_j[(\underline{\alpha}_k c_k(s)-\mu)^{-1}(|w_{kj}(s)|\\
&\quad + e^{\mu\tau}|m_{kj}(s)|)]^{+}\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,ds\right]\\
&\quad \cdot \int_0^{t^*}\exp\left[\int_0^s(\underline{\alpha}_k c_k(\theta)-\mu)\,d\theta\right](\underline{\alpha}_k c_k(s)-\mu)\,ds
\end{aligned}$$

$$
\begin{aligned}
&\leqslant (B+\varepsilon)\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,\mathrm{d}s\right]\\
&\quad+(B+\varepsilon)\cdot\gamma\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\mathrm{d}s\right]\\
&\quad\cdot\left(\exp\left[\int_0^{t^*}(\underline{\alpha}_k c_k(s)-\mu)\,\mathrm{d}s\right]-1\right)\\
&<(B+\varepsilon)\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,\mathrm{d}s\right]\\
&\quad+(B+\varepsilon)\cdot\left(1-\exp\left[\int_0^{t^*}(\mu-\underline{\alpha}_k c_k(s))\,\mathrm{d}s\right]\right)\\
&=B+\varepsilon,
\end{aligned}
$$

which is a contradiction. Therefore, (27) holds. Let $\varepsilon\to 0$. Then we have

$$V_i(t)\leqslant B \quad \text{for all } t\geqslant 0 \text{ and } i\in\{1,2,\dots,n\}.$$

Thus, for all $t\geqslant 0$ and $i\in\{1,2,\dots,n\}$, from (24) and (27), we obtain

$$
\begin{aligned}
|x_i(t)-x_i^*(t)|&\leqslant\overline{\alpha}_i y_i(t)=\overline{\alpha}_i\xi_i V_i(t)\mathrm{e}^{-\mu t}\\
&\leqslant\overline{\alpha}_i\xi_i\max_{1\leqslant j\leqslant n}\left\{\xi_j^{-1}\sup_{s\in[-\tau,0]}|y_j(s)|\right\}\mathrm{e}^{-\mu t}\\
&\leqslant\overline{\alpha}_i\xi_i\max_{1\leqslant j\leqslant n}\left\{\xi_j^{-1}\underline{\alpha}_j^{-1}\sup_{s\in[-\tau,0]}|\phi_j(s)-\phi_j^*(s)|\right\}\mathrm{e}^{-\mu t}\\
&\leqslant\overline{\alpha}_i\xi_i\max_{1\leqslant j\leqslant n}\{\xi_j^{-1}\underline{\alpha}_j^{-1}\}\|\phi-\phi^*\|\mathrm{e}^{-\mu t}\\
&\leqslant M\|\phi-\phi^*\|\mathrm{e}^{-\mu t},
\end{aligned}
$$

where $M=\max_{1\leqslant i,j\leqslant n}\{\overline{\alpha}_i\underline{\alpha}_j^{-1}\xi_i\xi_j^{-1}\}$, $\phi=(\phi_1,\phi_2,\dots,\phi_n)^{\mathrm{T}}$ and $\phi^*=(\phi_1^*,\phi_2^*,\dots,\phi_n^*)^{\mathrm{T}}$ are the initial functions of solutions $x(t)$ and $x^*(t)$, respectively. Obviously, $M>0$. This implies that the periodic solution $x^*(t)=(x_1^*(t),x_2^*(t),\dots,x_n^*(t))^{\mathrm{T}}$ of system (4) is globally exponentially stable and we complete the proof of Theorem 2. □

Remark 1. If system (4) becomes an autonomous system (3) with varying delays, then it is easy to verify that the periodic solution in Theorem 1 is an equilibrium, and we can conclude this equilibrium is globally exponentially stable under the same conditions in Theorem 2. However, if we use the approaches in [19–27,5–7,1–3,13,17,30,11,16,14,29], then we have to assume either the activation $f_i(v)$ is bounded or the time delays $\tau_{ij}(t)$ are differentiable and their derivative $\tau_{ij}'(t)$ are not greater than 1 (see, e.g., [3,17,26]). Thus our method is more general and the conditions in our theorems (the varying delays $\tau_{ij}(t)$ $(i=1,2,\dots,n)$ are only required to be nonnegative and bounded) presented here are milder and less restrictive even though system (4) is an autonomous system. Compared with the results in some existing work, our results improve and extend some previously related results in [22–26,5,19,3,13,17].

Remark 2. If we assume that $a_i(x_i)\equiv 1$, $b_i(t,x_i)=c_i(t)x_i$, then our results here are available to Hopfield neural networks. Obviously, some existing results in [3,17,21,29,9] are improved and generalized because our results are independent of delays $\tau_{ij}(t)$.

4. Numerical example

In this section, we give two examples to demonstrate our criteria.

Example 1. Consider the following the autonomous system:

$$
\begin{cases}
x_1'(t)=-(7+\cos(x_1(t)))[8x_1(t)-3f(x_1(t-\tau_1(t)))\\
\qquad -8f(x_2(t-\tau_2(t)))+1],\\
x_2'(t)=-(4+\sin(x_2(t)))[8x_2(t)-f(x_1(t-\tau_1(t)))\\
\qquad -2f(x_2(t-\tau_2(t)))+2],
\end{cases}
\tag{28}
$$

where $\tau_1(t)=2\sin^2 t$, $\tau_2(t)=2|\cos t|$ and $f(v)=0.5(v+\sin v)$. Obviously, the function $f(v)$ satisfies Assumption (A_2) with $k_1=k_2=1$. The inequality (22) reduces to

$$\tfrac{8}{6}\cdot\tfrac{1}{8}\cdot 3+\tfrac{5}{6}\cdot\tfrac{1}{8}\cdot 8\xi_1^{-1}\xi_2<1,$$

$$\tfrac{8}{3}\cdot\tfrac{1}{8}\cdot 1\xi_2^{-1}\xi_1+\tfrac{5}{3}\cdot\tfrac{1}{8}\cdot 2<1.$$

The solution is $\frac{5}{3}<\xi_1/\xi_2<\frac{7}{4}$. Thus, from Remark 1, we see that system (28) has a unique equilibrium which is globally exponentially stable. However, all results in [22–26,5,19,3,13,17] cannot be applied to system (28).

Example 2. Consider the following the periodic system:

$$
\begin{cases}
x_1'(t)=-(7+\cos(x_1(t)))[(9-\sin t)x_1(t)\\
\qquad -(2+\sin t)f(x_1(t-\tau_1(t)))\\
\qquad -(4\sqrt{5}\cos t)f(x_2(t-\tau_2(t)))+\sin t],\\
x_2'(t)=-(4+\cos(x_2(t)))[(9+\cos t)x_2(t)\\
\qquad -(\cos t)f(x_1(t-\tau_1(t)))\\
\qquad -(\sqrt{5}\sin t)f(x_2(t-\tau_2(t)))+2\cos t],
\end{cases}
\tag{29}
$$

where $\tau_1(t)=2\sin^2 t, \tau_2(t)=2|\cos t|$ and $f(v)=0.5(v+\sin v)$. It is straightforward to check that

$$[c_1^{-1}(t)|m_{11}(t)|]^+=\left[\frac{2+\sin t}{9-\sin t}\right]^+=\frac{3}{8},$$

$$
\begin{aligned}
[c_1^{-1}(t)|m_{12}(t)|]^+&=\left[\frac{4\sqrt{5}\cos t}{9-\sin t}\right]^+\\
&=\frac{4\sqrt{5}|\cos t|}{9-\sin t}\Big|_{t=\pi/2-\arctan 4\sqrt{5}}=1,
\end{aligned}
$$

$$[c_2^{-1}(t)|m_{21}(t)|]^+=\left[\frac{\cos t}{9+\cos t}\right]^+=\frac{1}{8},$$

$$\left[c_2^{-1}(t)|m_{22}(t)|\right]^+ = \left[\frac{\sqrt{5}\sin t}{9+\cos t}\right]^+ = \frac{\sqrt{5}|\sin t|}{9+\cos t}\Big|_{t=\pi/2+\arctan(4\sqrt{5})^{-1}} = \frac{1}{4}.$$

From Example 1, we know that inequality (22) holds. Hence, we conclude that system (29) has a 2π-periodic solution which is globally exponentially stable. However, all results in [4,15,28] cannot be applied to system (29).

5. Conclusions

Some new results for existence and global exponential stability of solution of periodic Cohen–Grossberg neural networks with varying delays have been presented. The results obtained here improved and generalized some existing results. It is worthwhile to mention that our method is more practical because we drop the boundedness of activation function and differentiability of delay $\tau_{ij}(t)$ with derivative $\tau'_{ij}(t) < 1$.

References

[1] S. Arik, Stability analysis of delayed neural networks, IEEE Trans. Circuits Syst. 47 (2000) 1089–1092.
[2] S. Arik, An analysis of exponential stability of delayed neural networks with time varying delays, Neural Networks 17 (2004) 1027–1031.
[3] S. Arik, Z. Orman, Global stability analysis of Cohen–Grossberg neural networks with time varying delays, Phys. Lett. A 341 (2005) 410–421.
[4] J. Cao, J. Liang, Boundedness and stability for Cohen–Grossberg neural network with time-varying delays, J. Math. Anal. Appl. 296 (2004) 665–685.
[5] T. Chen, L. Rong, Delay-independent stability analysis of Cohen–Grossberg neural networks, Phys. Lett. A 317 (2003) 436–449.
[6] L.O. Chua, L. Yang, Cellular neural networks: theory, IEEE Trans. Circuits Syst. 35 (1988) 1257–1272.
[7] L.O. Chua, L. Yang, Cellular neural networks: applications, IEEE Trans. Circuits Syst. 35 (1988) 1273–1290.
[8] M.A. Cohen, S. Grossberg, Absolute stability and global pattern formation and parallel memory storage by competitive neural networks, IEEE Trans. Syst. Man Cybern. 13 (1983) 815–826.
[9] Q. Dong, K. Matsui, X. Huang, Existence and stability of periodic solutions for Hopfield neural network equations with periodic input, Nonlinear Anal. 49 (2002) 471–479.
[10] R.E. Gaines, J.L. Mawhin, Coincidence Degree and Nonlinear Differential Equations, Springer, Berlin, 1977.
[11] K. Gopalsamy, X.Z. He, Stability in asymmetric Hopfield nets with transmission delays, Physica D 76 (1994) 344–358.
[12] S. Guo, L. Huang, B. Dai, Z. Zhong, Global existence of periodic solutions of BAM neural networks with variable coefficients, Phys. Lett. A 317 (2003) 97–106.
[13] C.C. Hwang, C.J. Chen, T.L. Liao, Globally exponential stability of generalized Cohen–Grossberg neural networks with delays, Phys. Lett. A 319 (2003) 157–166.
[14] X. Li, L. Huang, H. Zhu, Global stability of cellular neural networks with constant and variable delays, Nonlinear Anal. 53 (2003) 319–333.
[15] Y. Li, Existence and stability of periodic solutions for Cohen–Grossberg neural networks with multiple delays, Chaos Solitons Fractals 20 (2004) 459–466.
[16] X. Liao, C. Li, LMI approach to asymptotical stability of multi-delayed neural networks, Physica D 200 (2005) 139–155.
[17] J. Liu, Global exponential stability of Cohen–Grossberg neural networks with time-varying delays, Chaos Solitons Fractals 26 (2005) 935–945.
[18] Z. Liu, L. Liao, Existence and global exponential stability of periodic solution of cellular neural networks with time-varying delays, J. Math. Anal. Appl. 290 (2004) 247–262.
[19] W. Lu, T. Chen, New conditions on global stability of Cohen–Grossberg neural networks, Neural Comput. 15 (2003) 1173–1189.
[20] T. Roska, C.W. Wu, L.O. Chua, Stability of cellular neural networks with dominant nonlinear and delay-type templates, IEEE Trans. Circuits Syst. 40 (1993) 270–272.
[21] Q. Song, Z. Zhao, Global dissipativity of neural networks with both variable and unbounded delays, Chaos Solitons Fractals 25 (2005) 393–401.
[22] F. Tu, X. Liao, Harmless delays for global asymptotic stability of Cohen–Grossberg neural network, Chaos Solitons Fractals 26 (2005) 927–933.
[23] L. Wang, X. Zou, Exponential stability of Cohen–Grossberg neural networks, Neural Networks 15 (2002) 415–422.
[24] L. Wang, X. Zou, Harmless delays in Cohen–Grossberg neural networks, Physica D 170 (2002) 162–173.
[25] H. Ye, A.N. Michel, K. Wang, Qualitative analysis of Cohen–Grossberg neural networks with multiple delays, Phys. Rev. E 51 (1995) 2611–2618.
[26] K. Yuan , J. Cao, Global exponential stability of Cohen–Grossberg neural networks with multiple time-varying delays, Advances in Neural Networks—ISNN 2004, International Symposium on Neural Networks, August 2004 Proceedings, Dalian, China, 2004, pp. 78–83.
[27] Z. Yuan, D. Hu, L. Huang, G. Dong, On the global asymptotic stability analysis of delayed neural networks, Int. J. Bifurcation Chaos 15 (2005) 4019–4025.
[28] Z. Yuan, L. Huang, D. Hu, G. Dong, Existence and global exponential stability of periodic solution for Cohen–Grossberg neural networks with delays, Nonlinear Anal. RWA 7 (2006) 572–590.
[29] E. Yucel, S. Arik, New exponential stability results for delayed neural networks with time varying delays, Physica D 191 (2004) 314–322.
[30] H. Zhao, Global exponential stability and periodicity of cellular neural networks with delays, Phys. Lett. A 336 (2005) 331–341.
[31] Y. Zheng, T. Chen, Global exponential stability of delayed periodic dynamical systems, Phys. Lett. A 322 (2004) 344–355.
[32] J. Zhou, Z. Liu, G. Chen, Dynamics of periodic delayed neural networks, Neural Networks 17 (2004) 87–101.

Zhaohui Yuan received M.S. and Ph.D. in applied mathematics from Hunan University, China, in 2000 and 2003, respectively.

He is an Associate Professor in the Department of Mathematics in Hunan University. From October 2003 to April 2006, he was with the College of Mechatronics and Automation, National University of Defense Technology, China, where he was a Post-Doctoral Fellow. His current interests include theory of functional differential equations and difference equations, and their applications to dynamics of Neural Networks.

Lifen Yuan received a B.Sc. in communication engineering from Changsha Railway University, China, in 1999, and an M.S. in circuit and system from Hunan University, China, in 2003.

Since 2003, she is a lecturer of Communication in Hunan Normal University. Her current interests include the research of signal processing, neural networks and its application, pattern recognition.

Lihong Huang was born in Hunan, China, in 1963. He received the B.S. degree in mathematics in 1984 from Hunan Normal University, Changsha, China, and the M.S. and the Ph.D. degrees in applied mathematics from Hunan University, Changsha, China, in 1988 and 1994, respectively.

From July 1988 to June 2000 he was with the Department of Applied Mathematics at Hunan University, where he was an associate professor of applied mathematics from July 1994 to May 1997. In June 1997 he became a professor and doctoral advisor of applied mathematics and Chair of the Department of Applied Mathematics. Since July 2000 he has been Dean of the College of Mathematics and Econometrics at Hunan University, Changsha, China.

He is the author or co-author of more than 100 journal papers, six edited books. His research interests are in the areas of dynamics of neural networks, and qualitative theory of differential equations and difference equations.