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MIT NULL

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adapted from KTH ACM Contest Template Library

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Contest (1)

template.cpp

15 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
#define all(x) x.begin(), x.end()
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.sync_with_stdio(0); cin.tie(0);
  cin.exceptions(cin.failbit);
}
```

hash.sh

1 lines

```
tr -d '[:space:]' | md5sum | cut -d ' ' -f 1
```

hash-cpp.sh

1 lines

```
cpp -P -fpreprocessed | tr -d '[:space:]' | md5sum | cut -d
     ' ' -f 1
```

Makefile

37 lines

```
CXX = g++
CXXFLAGS = -Wall -Wextra -pedantic -std=c++11 -O2 -Wshadow
    -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -
    Wshift-overflow=2 -Wduplicated-cond -Wcast-qual -Wcast
    -align
DEBUGFLAGS = -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -
    fsanitize=address -fsanitize=undefined -fno-sanitize-
    recover=all -fstack-protector -D_FORTIFY_SOURCE=2

CXXFLAGS += $(DEBUGFLAGS)

TARGET := $(notdir $(CURDIR))
EXECUTE := ./$(TARGET)

CASES := $(sort $(basename $(wildcard *.in)))
TESTS := $(sort $(basename $(wildcard *.out)))

all: $(TARGET)

clean:
  -rm -rf $(TARGET) *.res

%: %.cpp
  $(LINK.cpp) $< $(LOADLIBES) $(LDLIBS) -o $@

run: $(TARGET)
  time $(EXECUTE)

%.res: $(TARGET) %.in
  time $(EXECUTE) < $*.in > $*.res

%.out: %

test_%: %.res %.out
  diff $*.res $*.out

runs: $(patsubst %,%.res,$(CASES))
test: $(patsubst %,test_%,$(TESTS))

.PHONY: all clean run test test_% runs

.PRECIOUS: %.res
```

vimrc

5 lines

```
set nocp ai bs=2 cul hls ic is lbr ls=2 mouse=a nu ru sc
    scs smd so=3 sw=4 ts=4
filetype plugin indent on
syntax on

map gA m'ggVG"+y''
```

nanorc

3 lines

```
set tabsize 4
set const
set autoindent
```

Miscellaneous (for now) (2)

2.1 Misc

directed-MST.cpp

Description: Finds the minimum spanning arborescence from the root. (any more notes?)

72 lines

```
#define rep(i, n) for (int i = 0; i < n; i++)

#define N 110000
#define M 110000
#define inf 2000000000

struct edg {
    int u, v;
    int cost;
} E[M], E_copy[M];

int In[N], ID[N], vis[N], pre[N];

// edges pointed from root.
int Directed_MST(int root, int NV, int NE) {
  for (int i = 0; i < NE; i++)
    E_copy[i] = E[i];
    int ret = 0;
    int u, v;
    while (true) {
        rep(i, NV)   In[i] = inf;
        rep(i, NE) {
            u = E_copy[i].u;
            v = E_copy[i].v;
            if(E_copy[i].cost < In[v] && u != v) {
                In[v] = E_copy[i].cost;
                pre[v] = u;
            }
        }
        rep(i, NV) {
            if(i == root)   continue;
            if(In[i] == inf)    return -1; // no solution
        }

        int cnt = 0;
        rep(i, NV) {
          ID[i] = -1;
          vis[i] = -1;
        }
        In[root] = 0;

        rep(i, NV) {
            ret += In[i];
            int v = i;
            while(vis[v] != i && ID[v] == -1 && v != root)
                {
                vis[v] = i;
                v = pre[v];
            }
            if(v != root && ID[v] == -1) {
                for(u = pre[v]; u != v; u = pre[u]) {
                    ID[u] = cnt;
                }
                ID[v] = cnt++;
            }
        }
        if(cnt == 0)    break;
        rep(i, NV) {
            if(ID[i] == -1) ID[i] = cnt++;
        }
        rep(i, NE) {
            v = E_copy[i].v;
            E_copy[i].u = ID[E_copy[i].u];
            E_copy[i].v = ID[E_copy[i].v];
            if(E_copy[i].u != E_copy[i].v) {
                E_copy[i].cost -= In[v];
            }
        }
        NV = cnt;
        root = ID[root];
    }
    return ret;
}
```

graph-clique.cpp

Description: Max clique N<64. Bit trick for speed. clique solver calculates both size and consitution of maximum clique uses bit operation to accelerate searching graph size limit is 63, the graph should be undirected can optimize to calculate on each component, and sort on vertex degrees can be used to solve maximum independent set

80 lines

```
class clique {
  public:
  static const long long ONE = 1;
  static const long long MASK = (1 << 21) - 1;
  char* bits;
  int n, size, cmax[63];
  long long mask[63], cons;
  // initiate lookup table
  clique() {
    bits = new char[1 << 21];
    bits[0] = 0;
    for (int i = 1; i < (1<<21); ++i)
      bits[i] = bits[i >> 1] + (i & 1);
  }
  ~clique() {
    delete bits;
  }
  // search routine
  bool search(int step,int siz,LL mor,LL con);
  // solve maximum clique and return size
  int sizeClique(vector<vector<int> >& mat);
  // solve maximum clique and return set
  vector<int>getClq(vector<vector<int> >&mat);
};
// step is node id, size is current sol., more is available
     mask, cons is constitution mask
bool clique::search(int step, int size,
                    LL more, LL cons) {
  if (step >= n) {
    // a new solution reached
    this->size = size;
    this->cons = cons;
    return true;
  }
```

```
  long long now = ONE << step;
  if ((now & more) > 0) {
    long long next = more & mask[step];
    if (size + bits[next & MASK] +
        bits[(next >> 21) & MASK] +
        bits[next >> 42] >= this->size
     && size + cmax[step] > this->size) {
      // the current node is in the clique
      if (search(step+1,size+1,next,cons|now))
        return true;
    }
  }
  long long next = more & ~now;
  if (size + bits[next & MASK] +
      bits[(next >> 21) & MASK] +
      bits[next >> 42] > this->size) {
    // the current node is not in the clique
    if (search(step + 1, size, next, cons))
      return true;
  }
  return false;
}
// solve maximum clique and return size
int clique::sizeClique(vector<vector<int> >& mat) {
  n = mat.size();
  // generate mask vectors
  for (int i = 0; i < n; ++i) {
    mask[i] = 0;
    for (int j = 0; j < n; ++j)
      if (mat[i][j] > 0) mask[i] |= ONE << j;
  }
  size = 0;
  for (int i = n - 1; i >= 0; --i) {
    search(i + 1, 1, mask[i], ONE << i);
    cmax[i] = size;
  }
  return size;
}
// calls sizeClique and restore cons
vector<int> clique::getClq(
    vector<vector<int> >& mat) {
  sizeClique(mat);
  vector<int> ret;
  for (int i = 0; i < n; ++i)
    if ((cons&(ONE<<i)) > 0) ret.push_back(i);
  return ret;
}
```

graph-dominator-tree.cpp

Description: Dominator Tree.

107 lines

```
#define N 110000 //max number of vertices

vector<int> succ[N], prod[N], bucket[N], dom_t[N];
int semi[N], anc[N], idom[N], best[N], fa[N], tmp_idom[N];
int dfn[N], redfn[N];
int child[N], size[N];
int timestamp;

void dfs(int now) {
  dfn[now] = ++timestamp;
  redfn[timestamp] = now;
  anc[timestamp] = idom[timestamp] = child[timestamp] =
      size[timestamp] = 0;
  semi[timestamp] = best[timestamp] = timestamp;
  int sz = succ[now].size();
  for(int i = 0; i < sz; ++i) {
    if(dfn[succ[now][i]] == -1) {
      dfs(succ[now][i]);
      fa[dfn[succ[now][i]]] = dfn[now];
    }
    prod[dfn[succ[now][i]]].push_back(dfn[now]);
  }
}

void compress(int now) {
  if(anc[anc[now]] != 0) {
    compress(anc[now]);
    if(semi[best[now]] > semi[best[anc[now]]])
      best[now] = best[anc[now]];
    anc[now] = anc[anc[now]];
  }
}

inline int eval(int now) {
  if(anc[now] == 0)
    return now;
  else {
    compress(now);
    return semi[best[anc[now]]] >= semi[best[now]] ? best[
        now]
      : best[anc[now]];
  }
}

inline void link(int v, int w) {
  int s = w;
  while(semi[best[w]] < semi[best[child[w]]]) {
    if(size[s] + size[child[child[s]]] >= 2*size[child[s]])
        {
      anc[child[s]] = s;
      child[s] = child[child[s]];
    } else {
      size[child[s]] = size[s];
      s = anc[s] = child[s];
    }
  }
  best[s] = best[w];
  size[v] += size[w];
  if(size[v] < 2*size[w])
    swap(s, child[v]);
  while(s != 0) {
    anc[s] = v;
    s = child[s];
  }
}

// idom[n] and other vertices that cannot be reached from n
    will be 0
void lengauer_tarjan(int n) { // n is the root's number
  memset(dfn, -1, sizeof dfn);
  memset(fa, -1, sizeof fa);
  timestamp = 0;
  dfs(n);
  fa[1] = 0;
  for(int w = timestamp; w > 1; --w) {
    int sz = prod[w].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(prod[w][i]);
      if(semi[w] > semi[u])
        semi[w] = semi[u];
    }
    bucket[semi[w]].push_back(w);
    //anc[w] = fa[w]; link operation for o(mlogm) version
                link(fa[w], w);
    if(fa[w] == 0)
      continue;
    sz = bucket[fa[w]].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(bucket[fa[w]][i]);
      if(semi[u] < fa[w])
        idom[bucket[fa[w]][i]] = u;
      else
        idom[bucket[fa[w]][i]] = fa[w];
    }
    bucket[fa[w]].clear();
  }
  for(int w = 2; w <= timestamp; ++w) {
    if(idom[w] != semi[w])
      idom[w] = idom[idom[w]];
  }
  idom[1] = 0;
  for(int i = timestamp; i > 1; --i) {
    if(fa[i] == -1)
      continue;
    dom_t[idom[i]].push_back(i);
  }
  memset(tmp_idom, 0, sizeof tmp_idom);
  for (int i = 1; i <= timestamp; i++)
    tmp_idom[redfn[i]] = redfn[idom[i]];
  memcpy(idom, tmp_idom, sizeof idom);
}
```

string-SAM.cpp

Description: Suffix Automaton (SAM)

37 lines

```
int n,i,init,L,len,ll,q,h,ch,p,last[1700000],n1[1700000],du
    [1700000],s[1700000],fa[800001],l[1700000],son
    [1700000][3],par[1700000];
char S[8000001],k;
long long ans,sum[1600001];
void ins(int p,int ss,int k)
{
  int np=++len,q,nq;
  l[np]=l[p]+1;
  s[np]=1;
  while (p&&!son[p][k]) son[p][k]=np,p=par[p];
  if (!p) par[np]=1;
  else {
    q=son[p][k];
    if (l[p]+1==l[q]) par[np]=q;
    else {
      nq=++len;
      l[nq]=l[p]+1;
      s[nq]=0;
      memset(son[nq], son[q], sizeof son[q]);
      par[nq]=par[q];
      par[q]=nq;
      par[np]=nq;
      while (p&&son[p][k]==q) son[p][k]=nq,p=par[p];
    }
  }
  last[ss]=np;
}
int main()
{
  read(n);
  last[1]=init=len=1;
  for (i=2;i<=n;i++)
  {
    read(fa[i]);
    for (k=getchar();k<=32;k=getchar());
    ins(last[fa[i]],i,k-'a');
  }
}
```

math-simplex.cpp

Description: Simplex algorithm. WARNING- segfaults on empty (size 0) max cx st Ax<=b, x>=0 do 2 phases; 1st check feasibility; 2nd check boundedness and ans

40 lines

```
vector<double> simplex(vector<vector<double> > A, vector<
    double> b, vector<double> c) {
```

```
  int n = (int) A.size(), m = (int) A[0].size()+1, r = n, s
       = m-1;
  vector<vector<double> > D = vector<vector<double> > (n+2,
       vector<double>(m+1));
  vector<int> ix = vector<int> (n+m);
  for (int i=0; i<n+m; i++) ix[i] = i;
  for (int i=0; i<n; i++) {
    for (int j=0; j<m-1; j++)D[i][j]=-A[i][j];
    D[i][m-1] = 1;
    D[i][m] = b[i];
    if (D[r][m] > D[i][m]) r = i;
  }
  for (int j=0; j<m-1; j++) D[n][j]=c[j];
  D[n+1][m-1] = -1; int z = 0;
  for (double d;;) {
    if (r < n) {
      swap(ix[s], ix[r+m]);
      D[r][s] = 1.0/D[r][s];
      for (int j=0; j<=m; j++) if (j!=s) D[r][j] *= -D[r][s
           ];
      for(int i=0; i<=n+1; i++)if(i!=r) {
        for (int j=0; j<=m; j++) if(j!=s) D[i][j] += D[r][j
             ] * D[i][s];
        D[i][s] *= D[r][s];
      }
    }
    r = -1; s = -1;
    for (int j=0; j <m; j++) if (s<0 || ix[s]>ix[j]) {
      if (D[n+1][j]>eps || D[n+1][j]>-eps && D[n][j]>eps) s
           = j;
    }
    if (s < 0) break;
    for (int i=0; i<n; i++) if(D[i][s]<-eps) {
      if (r < 0 || (d = D[r][m]/D[r][s]-D[i][m]/D[i][s]) <
           -eps
        || d < eps && ix[r+m] > ix[i+m]) r=i;
    }
    if (r < 0) return vector<double>(); // unbounded
  }
  if (D[n+1][m] < -eps) return vector<double>(); //
       infeasible
  vector<double> x(m-1);
  for (int i = m; i < n+m; i ++) if (ix[i] < m-1) x[ix[i]]
       = D[i-m][m];
  printf("%.2lf\n", D[n][m]);
  return x; // ans: D[n][m]
}
```

graph-negative-cycle.cpp

Description: negative cycle

33 lines

```
double b[N][N];

double dis[N];
int vis[N], pc[N];

bool dfs(int k) {
  vis[k] += 1; pc[k] = true;
  if (vis[k] > N)
    return true;
  for (int i = 0; i < N; i++)
    if (dis[k] + b[k][i] < dis[i]) {
      dis[i] = dis[k] + b[k][i];
      if (!pc[i]) {
        if (dfs(i))
          return true;
      } else return true;
    }
  pc[k] = false;
  return false;
}

bool chk(double d) {
  for (int i = 0; i < N; i ++)
    for (int j = 0; j < N; j ++) {
      b[i][j] = -a[i][j] + d;
    }
  for (int i = 0; i < N; i++)
    vis[i] = false, dis[i] = 0, pc[i] = false;
  for (int i = 0; i < N; i++)
    if (!vis[i] && dfs(i))
      return true;
  return false;
}
```

string-sa+lcp.cpp

Description: SA + LCP

31 lines

```
int wa[maxn],wb[maxn],wv[maxn],ws[maxn];
int cmp(int *r,int a,int b,int l) {
  return r[a]==r[b]&&r[a+l]==r[b+l];
}
void da(int *r,int *sa,int n,int m) {
  int i,j,p,*x=wa,*y=wb,*t;
  for(i=0;i<m;i++) ws[i]=0;
  for(i=0;i<n;i++) ws[x[i]=r[i]]++;
  for(i=1;i<m;i++) ws[i]+=ws[i-1];
  for(i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
  for(j=1,p=1;p<n;j*=2,m=p){
    for(p=0,i=n-j;i<n;i++) y[p++]=i;
    for(i=0;i<n;i++)
      if(sa[i]>=j) y[p++]=sa[i]-j;
    for(i=0;i<n;i++) wv[i]=x[y[i]];
    for(i=0;i<m;i++) ws[i]=0;
    for(i=0;i<n;i++) ws[wv[i]]++;
    for(i=1;i<m;i++) ws[i]+=ws[i-1];
    for(i=n-1;i>=0;i--) sa[--ws[wv[i]]]=y[i];
      for(t=x,x=y,y=t,p=1,x[sa[0]]=0,i=1;i<n;i++)
        x[sa[i]]=cmp(y,sa[i-1],sa[i],j)?p-1:p++;
  }
}
//height[i]: lcp(sa[i],sa[i-1])
int rank[maxn],height[maxn];
void calheight(int *r,int *sa,int n) {
  int i,j,k=0;
  for(i=1;i<=n;i++) rank[sa[i]]=i;
  for(i=0;i<n;height[rank[i++]]=k)
    for(k?k--:0,j=sa[rank[i]-1]; r[i+k]==r[j+k]; k++);
}
```

poly-exp-log.cpp

Description: polynomial exp/log

78 lines

```
const int mo=998244353,g=3;
int qp(int a,int b){
  int ans=1;
  do{if(b&1)ans=1ll*ans*a%mo;a=1ll*a*a%mo;}while(b>>=1);
  return ans;
}
int w[2][N+1];
void dft(int *a,int n,bool v){//0 <= a[i] < mo
  int j=0;
  rep(i,0,n){
    if(i>j)swap(a[i],a[j]);
    for (int l=n>>1;(j^=l)<l;l>>=1);
  }
  for (int i=2;i<=n;i<<=1)
    for (int j=0,s=n/i;j<n;j+=i)
      rep(l,0,i>>1){
        int t=1ll*a[j+l+(i>>1)]*w[v][s*l]%mo;
        a[j+l+(i>>1)]=(a[j+l]+mo-t)%mo;
        a[j+l]=(a[j+l]+t)%mo;
} if(v){
    int y=qp(n,mo-2);
    rep(i,0,n)a[i]=1ll*a[i]*y%mo;
  }
}

void init(int n){
  int ww=qp(g,(mo-1)/n);
  w[0][0]=1;
  rep(i,1,n+1)w[0][i]=1ll*w[0][i-1]*ww%mo;
  rep(i,0,n+1)w[1][i]=w[0][n-i];
}
void mul(int *a,int *b,int n){
  static int x[N];
  rep(i,0,2*n)x[i]=b[i];
  init(2*n);
  dft(x,2*n,0);
  dft(a,2*n,0);
  rep(i,0,2*n)a[i]=1ll*x[i]*a[i]%mo;
  dft(a,2*n,1);
  rep(i,n,2*n)a[i]=0;
}

void inv(int *a,int n,int *b){
  static int x[N];
  b[0]=qp(a[0],mo-2);b[1]=0;
  for (int m=2;m<=n;m<<=1){
    rep(i,0,m) x[i]=a[i],x[i+m]=b[i+m]=0;
    init(2*m);
    dft(x,2*m,0);
    dft(b,2*m,0);
    rep(i,0,2*m) b[i]=1ll*b[i]*(2-1ll*x[i]*b[i]%mo+mo)%mo;
    dft(b,2*m,1);
    rep(i,m,2*m)b[i]=0;
  }
}

void Ln(int *a,int n){
  static int x[N];
  a[0]=1;inv(a,n,x);
  rep(i,0,n-1)a[i]=1ll*(i+1)*a[i+1]%mo;
  mul(a,x,n);
  per(i,1,n)a[i]=1ll*a[i-1]*qp(i,mo-2)%mo;
  a[0]=0;
}

void Exp(int *a,int n,int *r){
  static int x[N];
  r[0]=1;r[1]=0;
  for (int m=2;m<=n;m<<=1){
    rep(i,0,m)x[i]=r[i];
    Ln(x,m);
    rep(i,0,m)x[i]=(a[i]-x[i]+mo)%mo;
    x[0]=(x[0]+1)%mo;
    rep(i,m,2*m) r[i]=x[i]=0;
    mul(r,x,m);
    rep(i,m,2*m)r[i]=0;
  }
}
```

nim-product.cpp

Description: Nim Product.

17 lines

```
using ull = uint64_t;
ull _nimProd2[64][64];
ull nimProd2(int i, int j) {
  if (_nimProd2[i][j]) return _nimProd2[i][j];
  if ((i & j) == 0) return _nimProd2[i][j] = 1ull << (i|j);
  int a = (i&j) & -(i&j);
  return _nimProd2[i][j] = nimProd2(i ^ a, j) ^ nimProd2((i
       ^ a) | (a-1), (j ^ a) | (i & (a-1)));
```

```
}
ull nimProd(ull x, ull y) {
  ull res = 0;
  for (int i = 0; x >> i; i++)
    if ((x >> i) & 1)
      for (int j = 0; y >> j; j++)
        if ((y >> j) & 1)
          res ^= nimProd2(i, j);
  return res;
}
```

schreier-sims.cpp

Description: Check group membership of permutation groups 52 lines

```
struct Perm {
  int a[N];
  Perm() {
    for (int i = 1; i <= n; ++i) a[i] = i;
  }
  friend Perm operator* (const Perm &lhs, const Perm &rhs)
      {
    static Perm res;
    for (int i = 1; i <= n; ++i) res.a[i] = lhs.a[rhs.a[i
        ]];
    return res;
  }
  friend Perm inv(const Perm &cur) {
    static Perm res;
    for (int i = 1; i <= n; ++i) res.a[cur.a[i]] = i;
    return res;
  }
};
class Group {
  bool flag[N];
  Perm w[N];
  std::vector<Perm> x;
public:
  void clear(int p) {
    memset(flag, 0, sizeof flag);
    for (int i = 1; i <= n; ++i) w[i] = Perm();
    flag[p] = true;
    x.clear();
  }
  friend bool check(const Perm&, int);
  friend void insert(const Perm&, int);
  friend void updateX(const Perm&, int);
} g[N];
bool check(const Perm &cur, int k) {
  if (!k) return true;
  int t = cur.a[k];
  return g[k].flag[t] ? check(g[k].w[t] * cur, k - 1) :
      false;
}
void updateX(const Perm&, int);
void insert(const Perm &cur, int k) {
  if (check(cur, k)) return;
  g[k].x.push_back(cur);
  for (int i = 1; i <= n; ++i) if (g[k].flag[i]) updateX(
      cur * inv(g[k].w[i]), k);
}
void updateX(const Perm &cur, int k) {
  int t = cur.a[k];
  if (g[k].flag[t]) {
    insert(g[k].w[t] * cur, k - 1);
  } else {
    g[k].w[t] = inv(cur);
    g[k].flag[t] = true;
    for (int i = 0; i < g[k].x.size(); ++i) updateX(g[k].x[
        i] * cur, k);
  }
}
```

Data structures (3)

LCT.cpp

Description: Link-Cut Tree. 106 lines

```
struct T {
  bool rr;
  T *son[2], *pf, *fa;
} f1[N], *ff = f1, *f[N], *null;

void downdate(T *x) {
  if (x -> rr) {
    x -> son[0] -> rr = !x -> son[0] -> rr;
    x -> son[1] -> rr = !x -> son[1] -> rr;
    swap(x -> son[0], x -> son[1]);
    x -> rr = false;
  }
  // add stuff
}

void update(T *x) {
  // add stuff
}

void rotate(T *x, bool t) {
  T *y = x -> fa, *z = y -> fa;
  if (z != null)  z -> son[z -> son[1] == y] = x;
  x -> fa = z;
  y -> son[t] = x -> son[!t];
  x -> son[!t] -> fa = y;
  x -> son[!t] = y;
  y -> fa = x;
  update(y);
}

void xiao(T *x) {
  if (x -> fa != null)  xiao(x -> fa), x -> pf = x -> fa ->
      pf;
  downdate(x);
}

void splay(T *x) {
  xiao(x);
  T *y, *z;
  while (x -> fa != null) {
    y = x -> fa; z = y -> fa;
    bool t1 = (y -> son[1] == x), t2 = (z -> son[1] == y);
    if (z != null) {
      if (t1 == t2) rotate(y, t2), rotate(x, t1);
      else  rotate(x, t1), rotate(x, t2);
    }else rotate(x, t1);
  }
  update(x);
}

void access(T *x) {
  splay(x);
  x -> son[1] -> pf = x;
  x -> son[1] -> fa = null;
  x -> son[1] = null;
  update(x);
  while (x -> pf != null) {
    splay(x -> pf);
    x -> pf -> son[1] -> pf = x -> pf;
    x -> pf -> son[1] -> fa = null;
    x -> pf -> son[1] = x;
    x -> fa = x -> pf;
    splay(x);
  }
  x -> rr = true;
}

bool Cut(T *x, T *y) {
  access(x);
  access(y);
  downdate(y);
  downdate(x);
  if (y -> son[1] != x || x -> son[0] != null)
    return false;
  y -> son[1] = null;
  x -> fa = x -> pf = null;
  update(x);
  update(y);
  return true;
}

bool Connected(T *x, T *y) {
  access(x);
  access(y);
  return x == y || x -> fa != null;
}

bool Link(T *x, T *y) {
  if (Connected(x, y))
    return false;
  access(x);
  access(y);
  x -> pf = y;
  return true;
}

int main() {
  read(n); read(m); read(q);
  null = new T; null -> son[0] = null -> son[1] = null ->
      fa = null -> pf = null;
  for (int i = 1; i <= n; i++) {
    f[i] = ++ff;
    f[i] -> son[0] = f[i] -> son[1] = f[i] -> fa = f[i] ->
        pf = null;
    f[i] -> rr = false;
  }
  // init null and f[i]
}
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming.
Time: $\mathcal{O}(\log N)$ 32 lines

```
bool Q;
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const {
    return Q ? p < o.p : k < o.k;
  }
};

struct LineContainer : multiset<Line> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
```

```
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y
        ));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
    return l.k * x + l.m;
  }
};
```

Numerical (4)

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a,b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is *eps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}\left(\log((b-a)/\epsilon)\right)$

14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

Polynomial.h

17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
        b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax)
    {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0]*x^0 + ... + a[n-1]*x^{n-1}$. For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`

"../number-theory/ModPow.h" 20 lines

```
vector<ll> BerlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  trav(x, C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(S);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
          mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

HillClimbing.h

Description: Poor man's optimization for unimodal functions.

16 lines

```
typedef array<double, 2> P;

double func(P p);

pair<double, P> hillClimb(P start) {
  pair<double, P> cur(func(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
      P p = cur.second;
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(func(p), p));
    }
  }
  return cur;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

8 lines

```
double quad(double (*f)(double), double a, double b) {
  const int n = 1000;
  double h = (b - a) / 2 / n;
  double v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: `double z, y;`
`double h(double x) { return x*x + y*y + z*z <= 1; }`
`double g(double y) { ::y = y; return quad(h, -1, 1); }`
`double f(double z) { ::z = z; return quad(g, -1, 1); }`
`double sphereVol = quad(f, -1, 1), pi = sphereVol*3/4;`
16 lines

```
typedef double d;
d simpson(d (*f)(d), d a, d b) {
  d c = (a+b) / 2;
  return (f(a) + 4*f(c) + f(b)) * (b-a) / 6;
}
d rec(d (*f)(d), d a, d b, d eps, d S) {
  d c = (a+b) / 2;
  d S1 = simpson(f, a, c);
  d S2 = simpson(f, c, b), T = S1 + S2;
  if (abs (T - S) <= 15*eps || b-a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps/2, S1) + rec(f, c, b, eps/2, S2);
}
d quad(d (*f)(d), d a, d b, d eps = 1e-8) {
  return rec(f, a, b, eps, simpson(f, a, b));
}
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$
15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$
18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
Time: $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case.
68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$
38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:
"SolveLinear.h"
7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
```

```
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2 m\right)$

34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}\left(N\right)$

26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>&
    super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i]
        == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

4.1 Fourier transforms

FastFourierTransform.h

Description: Computes $\hat{f}(k) = \sum_x f(x) \exp(-2\pi i k x / N)$ for all k. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i] b[x - i]$. a and b should be of roughly equal size. For convolutions of integers, consider using a number-theoretic transform instead, to avoid rounding issues.
Time: $\mathcal{O}\left(N \log N\right)$

<valarray> 29 lines

```
typedef valarray<complex<double> > carray;
void fft(carray& x, carray& roots) {
  int N = sz(x);
  if (N <= 1) return;
  carray even = x[slice(0, N/2, 2)];
  carray odd = x[slice(1, N/2, 2)];
  carray rs = roots[slice(0, N/2, 2)];
  fft(even, rs);
  fft(odd, rs);
  rep(k,0,N/2) {
    auto t = roots[k] * odd[k];
    x[k    ] = even[k] + t;
    x[k+N/2] = even[k] - t;
  }
}

typedef vector<double> vd;
vd conv(const vd& a, const vd& b) {
  int s = sz(a) + sz(b) - 1, L = 32-__builtin_clz(s), n =
      1<<L;
  if (s <= 0) return {};
  carray av(n), bv(n), roots(n);
  rep(i,0,n) roots[i] = polar(1.0, -2 * M_PI * i / n);
  copy(all(a), begin(av)); fft(av, roots);
  copy(all(b), begin(bv)); fft(bv, roots);
  roots = roots.apply(conj);
  carray cv = av * bv; fft(cv, roots);
  vd c(s); rep(i,0,s) c[i] = cv[i].real() / n;
  return c;
}
```

NumberTheoreticTransform.h

Description: Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For other primes/integers, use two different primes and combine with CRT. May return negative values.
Time: $\mathcal{O}\left(N \log N\right)$

"ModPow.h" 38 lines

```
const ll mod = (119 << 23) + 1, root = 3; // = 998244353
// For p < 2^30 there is also e.g. (5 << 25, 3), (7 << 26,
    3),
// (479 << 21, 3) and (483 << 21, 5). The last two are >
    10^9.

typedef vector<ll> vl;
void ntt(ll* x, ll* temp, ll* roots, int N, int skip) {
  if (N == 1) return;
  int n2 = N/2;
  ntt(x     , temp, roots, n2, skip*2);
  ntt(x+skip, temp, roots, n2, skip*2);
  rep(i,0,N) temp[i] = x[i*skip];
  rep(i,0,n2) {
    ll s = temp[2*i], t = temp[2*i+1] * roots[skip*i];
    x[skip*i] = (s + t) % mod; x[skip*(i+n2)] = (s - t) %
        mod;
  }
}
void ntt(vl& x, bool inv = false) {
  ll e = modpow(root, (mod-1) / sz(x));
  if (inv) e = modpow(e, mod-2);
```

```
  vl roots(sz(x), 1), temp = roots;
  rep(i,1,sz(x)) roots[i] = roots[i-1] * e % mod;
  ntt(&x[0], &temp[0], &roots[0], sz(x), 1);
}
vl conv(vl a, vl b) {
  int s = sz(a) + sz(b) - 1; if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L
      ;
  if (s <= 200) { // (factor 10 optimization for |a|,|b| =
       10)
    vl c(s);
    rep(i,0,sz(a)) rep(j,0,sz(b))
      c[i + j] = (c[i + j] + a[i] * b[j]) % mod;
    return c;
  }
  a.resize(n); ntt(a);
  b.resize(n); ntt(b);
  vl c(n); ll d = modpow(n, mod-2);
  rep(i,0,n) c[i] = a[i] * b[i] % mod * d % mod;
  ntt(c, true); c.resize(s); return c;
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) trav(x, a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

"euclid.h" 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod);
      }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

6 lines

```
const ll mod = 1000000007; // faster if const
ll modpow(ll a, ll e) {
  if (e == 0) return 1;
  ll x = modpow(a * a % mod, e >> 1);
  return e & 1 ? x * a % mod : x;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
modsum(to, c, k, m) $= \sum_{i=0}^{to-1} (ki + c) \% m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.

19 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (k) {
    ull to2 = (to * k + c) / m;
    res += to * to2;
    res -= divsum(to2, m-1 - c, m, k) + to2;
  }
  return res;
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for large c.
Time: $\mathcal{O}(64/bits \cdot \log b)$, where $bits = 64 - k$, if we want to deal with k-bit numbers.

19 lines

```
typedef unsigned long long ull;
const int bits = 10;
// if all numbers are less than 2^k, set bits = 64-k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
  ull x = a * (b & (po - 1)) % c;
  while ((b >>= bits) > 0) {
    a = (a << bits) % c;
    x += (a * (b & (po - 1))) % c;
  }
  return x % c;
}
ull mod_pow(ull a, ull b, ull mod) {
  if (b == 0) return 1;
  ull res = mod_pow(a, b / 2, mod);
  res = mod_mul(res, res, mod);
  if (b & 1) return mod_mul(res, a, mod);
  return res;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots.
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, often $\mathcal{O}(\log p)$

"ModPow.h" 30 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1);
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1;
  int r = 0;
  while (s % 2 == 0)
    ++r, s /= 2;
  ll n = 2; // find a non-square mod p
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p);
  ll g = modpow(n, s, p);
  for (;;) {
    ll t = b;
    int m = 0;
    for (; m < r; ++m) {
      if (t == 1) break;
      t = t * t % p;
    }
    if (m == 0) return x;
    ll gs = modpow(g, 1 << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
    r = m;
  }
}
```

5.2 Primality

eratosthenes.h

Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 $\approx$ 0.8 s. Runs 30% faster if only odd indices are stored.

11 lines

```
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
  isprime.set(); isprime[0] = isprime[1] = 0;
  for (int i = 4; i < lim; i += 2) isprime[i] = 0;
  for (int i = 3; i*i < lim; i += 2) if (isprime[i])
    for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
  vi pr;
  rep(i,2,lim) if (isprime[i]) pr.push_back(i);
  return pr;
}
```

MillerRabin.h

Description: Miller-Rabin primality probabilistic test. Probability of failing one iteration is at most 1/4. 15 iterations should be enough for 50-bit numbers.
Time: 15 times the complexity of $a^b \mod c$.

"ModMulLL.h" 16 lines

```
bool prime(ull p) {
  if (p == 2) return true;
  if (p == 1 || p % 2 == 0) return false;
  ull s = p - 1;
  while (s % 2 == 0) s /= 2;
  rep(i,0,15) {
    ull a = rand() % (p - 1) + 1, tmp = s;
    ull mod = mod_pow(a, tmp, p);
    while (tmp != p - 1 && mod != 1 && mod != p - 1) {
      mod = mod_mul(mod, mod, p);
```

```
      tmp *= 2;
    }
    if (mod != p - 1 && tmp % 2 == 0) return false;
  }
  return true;
}
```

factor.h

Description: Pollard's rho algorithm. It is a probabilistic factorisation algorithm, whose expected time complexity is good. Before you start using it, run `init(bits)`, where bits is the length of the numbers you use. Returns factors of the input without duplicates.

Time: Expected running time should be good enough for 50-bit numbers.

"ModMulLL.h", "MillerRabin.h", "eratosthenes.h" — 35 lines

```
vector<ull> pr;
ull f(ull a, ull n, ull &has) {
  return (mod_mul(a, a, n) + has) % n;
}
vector<ull> factor(ull d) {
  vector<ull> res;
  for (int i = 0; i < sz(pr) && pr[i]*pr[i] <= d; i++)
    if (d % pr[i] == 0) {
      while (d % pr[i] == 0) d /= pr[i];
      res.push_back(pr[i]);
    }
  //d is now a product of at most 2 primes.
  if (d > 1) {
    if (prime(d))
      res.push_back(d);
    else while (true) {
      ull has = rand() % 2321 + 47;
      ull x = 2, y = 2, c = 1;
      for (; c==1; c = __gcd((y > x ? y - x : x - y), d)) {
        x = f(x, d, has);
        y = f(f(y, d, has), d, has);
      }
      if (c != d) {
        res.push_back(c); d /= c;
        if (d != c) res.push_back(d);
        break;
      }
    }
  }
  return res;
}
void init(int bits) {//how many bits do we use?
  vi p = eratosthenes_sieve(1 << ((bits + 2) / 3));
  pr.assign(all(p));
}
```

5.3 Divisibility

euclid.h

Description: Finds the Greatest Common Divisor to the integers a and b. Euclid also finds two integers x and y, such that $ax + by = \gcd(a, b)$. If a and b are coprime, then x is the inverse of $a \pmod b$.

7 lines

```
ll gcd(ll a, ll b) { return __gcd(a, b); }

ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) { ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d; }
  return x = 1, y = 0, a;
}
```

Euclid.java

Description: Finds {x, y, d} s.t. ax + by = d = gcd(a, b).

11 lines

```
static BigInteger[] euclid(BigInteger a, BigInteger b) {
  BigInteger x = BigInteger.ONE, yy = x;
  BigInteger y = BigInteger.ZERO, xx = y;
  while (b.signum() != 0) {
    BigInteger q = a.divide(b), t = b;
    b = a.mod(b); a = t;
    t = xx; xx = x.subtract(q.multiply(xx)); x = t;
    t = yy; yy = y.subtract(q.multiply(yy)); y = t;
  }
  return new BigInteger[]{x, y, a};
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's totient* or *Euler's phi* function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. The *cototient* is $n - \phi(n)$. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$. $\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$

Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.

Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p \ \forall a$.

10 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for(int i = 3; i < LIM; i += 2)
    if(phi[i] == i)
      for(int j = i; j < LIM; j += i)
        (phi[j] /= i) *= i-1;
}
```

5.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$. For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$

21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x
      ;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf
        ),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives
          us a
      // better approximation; if b = a/2, we *may* have
          one.
      // Return {P, Q} here for a more canonical
          approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)
          ) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`

Time: $\mathcal{O}(\log(N))$

24 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N
      ]
  assert(!f(lo)); assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

5.5 Chinese remainder theorem

chinese.h

Description: Chinese Remainder Theorem.

`chinese(a, m, b, n)` returns a number x, such that $x \equiv a \pmod m$ and $x \equiv b \pmod n$. For not coprime n, m, use `chinese_common`. Note that all numbers must be less than 2^{31} if you have Z = unsigned long long.

Time: $\log(m + n)$

"euclid.h" — 13 lines

```
template<class Z> Z chinese(Z a, Z m, Z b, Z n) {
  Z x, y; euclid(m, n, x, y);
  Z ret = a * (y + m) % m * n + b * (x + n) % n * m;
  if (ret >= m * n) ret -= m * n;
  return ret;
}

template<class Z> Z chinese_common(Z a, Z m, Z b, Z n) {
  Z d = gcd(m, n);
  if (((b -= a) %= n) < 0) b += n;
  if (b % d) return -1; // No solution
  return d * chinese(Z(0), m/d, b/d, n/d) + a;
```

```
}
```

5.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.8 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11		12		13		14		15	
$n!$	4.0e7		4.8e8		6.2e9		8.7e10		1.3e12	
n	16		17		20		25		30	
$n!$	2.1e13		3.6e14		2e18		2e25		3e32	
n	40		50		100		150		171	
$n!$	8e47		3e64		9e157		6e262		>DBL_MAX	

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.)

Time: $\mathcal{O}(n)$

6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  trav(x, v) r = r * ++i + __builtin_popcount(use & -(1 <<
      x)),
    use |= 1 << x;                    // (note: minus, not
        ~!)
  return r;
}
```

6.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

6.2.2 Binomials

binomialModPrime.h

Description: Lucas' thm: Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \ldots + n_1 p + n_0$ and $m = m_k p^k + \ldots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$. fact and invfact must hold precomputed factorials / inverse factorials, e.g. from ModInverse.h.

Time: $\mathcal{O}(\log_p n)$

10 lines

```
ll chooseModP(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] %
        p;
    n /= p; m /= p;
  }
  return c;
}
```

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$.

6 lines

```
multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$

$$\sum_{k=0}^{n} c(n,k) x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) =$
$8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) =$
$0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.2 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.3 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

6.3.4 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.5 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

bellmanFord.h

Description: Calculates shortest path in a graph that might have negative edge distances. Propagates negative infinity distances (sets dist = -inf), and returns true if there is some negative cycle. Unreachable nodes get dist = inf.

Time: $\mathcal{O}(EV)$ 27 lines

```
typedef ll T; // or whatever
struct Edge { int src, dest; T weight; };
struct Node { T dist; int prev; };
struct Graph { vector<Node> nodes; vector<Edge> edges; };

const T inf = numeric_limits<T>::max();
bool bellmanFord2(Graph& g, int start_node) {
  trav(n, g.nodes) { n.dist = inf; n.prev = -1; }
  g.nodes[start_node].dist = 0;

  rep(i,0,sz(g.nodes)) trav(e, g.edges) {
    Node& cur = g.nodes[e.src];
    Node& dest = g.nodes[e.dest];
    if (cur.dist == inf) continue;
    T ndist = cur.dist + (cur.dist == -inf ? 0 : e.weight);
    if (ndist < dest.dist) {
      dest.prev = e.src;
      dest.dist = (i >= sz(g.nodes)-1 ? -inf : ndist);
    }
  }
  bool ret = 0;
  rep(i,0,sz(g.nodes)) trav(e, g.edges) {
    if (g.nodes[e.src].dist == -inf)
      g.nodes[e.dest].dist = -inf, ret = 1;
  }
  return ret;
}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge distances. Input is an distance matrix m, where $m[i][j] =$ `inf` if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.

Time: $\mathcal{O}(N^3)$ 12 lines

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], {});
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);
    }
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices (array idx), such that there are edges only from left to right. The function returns false if there is a cycle in the graph.

Time: $\mathcal{O}(|V| + |E|)$ 18 lines

```
template<class E, class I>
bool topo_sort(const E &edges, I &idx, int n) {
  vi indeg(n);
  rep(i,0,n)
    trav(e, edges[i])
      indeg[e]++;
  queue<int> q; // use priority queue for lexic. smallest ans.
  rep(i,0,n) if (indeg[i] == 0) q.push(-i);
  int nr = 0;
  while (q.size() > 0) {
    int i = -q.front(); // top() for priority queue
    idx[i] = nr++;
    q.pop();
    trav(e, edges[i])
      if (--indeg[e] == 0) q.push(-e);
  }
  return nr == n;
}
```

7.2 Euler walk

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, also put it->second in s (and then ret).

Time: $\mathcal{O}(E)$ where E is the number of edges. 27 lines

```
struct V {
  vector<pii> outs; // (dest, edge index)
  int nins = 0;
};
vi euler_walk(vector<V>& nodes, int nedges, int src=0) {
  int c = 0;
  trav(n, nodes) c += abs(n.nins - sz(n.outs));
  if (c > 2) return {};
  vector<vector<pii>::iterator> its;
  trav(n, nodes)
    its.push_back(n.outs.begin());
  vector<bool> eu(nedges);
  vi ret, s = {src};
  while(!s.empty()) {
    int x = s.back();
    auto& it = its[x], end = nodes[x].outs.end();
    while(it != end && eu[it->second]) ++it;
    if(it == end) { ret.push_back(x); s.pop_back(); }
    else { s.push_back(it->first); eu[it->second] = true; }
  }
  if(sz(ret) != nedges+1)
    ret.clear(); // No Eulerian cycles/paths.
  // else, non-cycle if ret.front() != ret.back()
  reverse(all(ret));
  return ret;
}
```

7.3 Network flow

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ 51 lines

```
typedef ll Flow;
struct Edge {
  int dest, back;
  Flow f, c;
};

struct PushRelabel {
  vector<vector<Edge>> g;
  vector<Flow> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n)
      {}

  void add_edge(int s, int t, Flow cap, Flow rcap=0) {
    if (s == t) return;
    Edge a = {t, sz(g[t]), 0, cap};
    Edge b = {s, sz(g[s]), 0, rcap};
    g[s].push_back(a);
    g[t].push_back(b);
  }

  void add_flow(Edge& e, Flow f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  Flow maxflow(int s, int t) {
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;
    rep(i,0,v) cur[i] = g[i].data();
    trav(e, g[s]) add_flow(e, e.c);

    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
```

```
        trav(e, g[u]) if (e.c && H[u] > H[e.dest]+1)
          H[u] = H[e.dest]+1, cur[u] = &e;
        if (++co[H[u]], !--co[hi] && hi < v)
          rep(i,0,v) if (hi < H[i] && H[i] < v)
            --co[H[i]], H[i] = v + 1;
        hi = H[u];
      } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
        add_flow(*cur[u], min(ec[u], cur[u]->c));
      else ++cur[u];
    }
  }
};
```

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}\left(E^2\right)$

81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<VL> cap, flow, cost;
  vi seen;
  VL dist, pi;
  vector<pii> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost(cap
        ),
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});

    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i], i
            });
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      trav(i, ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
      trav(i, red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p
            ]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    rep(i,0,N) rep(j,0,N) totcost += cost[i][j] * flow[i][j
        ];
    return {totflow, totcost};
  }

  // If some costs can be negative, call this before
      maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        trav(to, ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.

35 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T
    >>& graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;

  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;

    rep(i,0,ptr) {
      int x = q[i];
      trav(e, graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
}
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

31 lines

```
pair<int, vi> GetMinCut(vector<vi>& weights) {
  int N = sz(weights);
  vi used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    vi w = weights[0], added = used;
    int prev, k = 0;
    rep(i,0,phase){
      prev = k;
      k = -1;
      rep(j,1,N)
        if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
      if (i == phase-1) {
        rep(j,0,N) weights[prev][j] += weights[k][j];
        rep(j,0,N) weights[j][prev] = weights[prev][j];
        used[k] = true;
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        rep(j,0,N)
          w[j] += weights[k][j];
        added[k] = true;
      }
    }
  }
  return {best_weight, best_cut};
}
```

7.4 Matching

hopcroftKarp.h

Description: Find a maximum matching in a bipartite graph.
Usage: vi ba(m, -1); hopcroftKarp(g, ba);
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

45 lines

```
bool dfs(int a, int layer, const vector<vi>& g, vi& btoa,
      vi& A, vi& B) {
  if (A[a] != layer) return 0;
  A[a] = -1;
  trav(b, g[a]) if (B[b] == layer + 1) {
    B[b] = -1;
    if (btoa[b] == -1 || dfs(btoa[b], layer+2, g, btoa, A,
        B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(const vector<vi>& g, vi& btoa) {
```

```
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), -1);
    cur.clear();
    trav(a, btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay += 2) {
      bool islast = 0;
      next.clear();
      trav(a, cur) trav(b, g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && B[b] == -1) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      trav(a, next) A[a] = lay+1;
      cur.swap(next);
    }
    rep(a,0,sz(g)) {
      if(dfs(a, 0, g, btoa, A, B))
        ++res;
    }
  }
}
```

DFSMatching.h

Description: This is a simple matching algorithm but should be just fine in most cases. Graph g should be a list of neighbours of the left partition. n is the size of the left partition and m is the size of the right partition. If you want to get the matched pairs, $match[i]$ contains match for vertex i on the right side or -1 if it's not matched.
Time: $\mathcal{O}(EV)$ where E is the number of edges and V is the number of vertices.

24 lines

```
vi match;
vector<bool> seen;
bool find(int j, const vector<vi>& g) {
  if (match[j] == -1) return 1;
  seen[j] = 1; int di = match[j];
  trav(e, g[di])
    if (!seen[e] && find(e, g)) {
      match[e] = di;
      return 1;
    }
  return 0;
}
int dfs_matching(const vector<vi>& g, int n, int m) {
  match.assign(m, -1);
  rep(i,0,n) {
    seen.assign(m, 0);
    trav(j,g[i])
      if (find(j, g)) {
        match[j] = i;
        break;
      }
  }
  return m - (int)count(all(match), -1);
}
```

WeightedMatching.h

Description: Min cost bipartite matching. Negate costs for max cost.
Time: $\mathcal{O}(N^3)$

75 lines

```
typedef vector<double> vd;
bool zero(double x) { return fabs(x) < 1e-10; }
double MinCostMatching(const vector<vd>& cost, vi& L, vi& R
    ) {
  int n = sz(cost), mated = 0;
  vd dist(n), u(n), v(n);
  vi dad(n), seen(n);

  rep(i,0,n) {
    u[i] = cost[i][0];
    rep(j,1,n) u[i] = min(u[i], cost[i][j]);
  }
  rep(j,0,n) {
    v[j] = cost[0][j] - u[0];
    rep(i,1,n) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  L = R = vi(n, -1);
  rep(i,0,n) rep(j,0,n) {
    if (R[j] != -1) continue;
    if (zero(cost[i][j] - u[i] - v[j])) {
      L[i] = j;
      R[j] = i;
      mated++;
      break;
    }
  }

  for (; mated < n; mated++) { // until solution is
      feasible
    int s = 0;
    while (L[s] != -1) s++;
    fill(all(dad), -1);
    fill(all(seen), 0);
    rep(k,0,n)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    for (;;) {
      j = -1;
      rep(k,0,n){
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;
      int i = R[j];
      if (i == -1) break;
      rep(k,0,n) {
        if (seen[k]) continue;
        auto new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    rep(k,0,n) {
      if (k == j || !seen[k]) continue;
      auto w = dist[k] - dist[j];
      v[k] += w, u[R[k]] -= w;
    }
    u[s] += dist[j];

    while (dad[j] >= 0) {
      int d = dad[j];
      R[j] = R[d];
      L[R[j]] = j;
      j = d;
    }
    R[j] = s;
    L[s] = j;
  }
  auto value = vd(1)[0];
  rep(i,0,n) value += cost[i][L[i]];
  return value;
}
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod.
Time: $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h" 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  trav(pa, ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
      } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod
            ;
      }
      swap(fi,fj);
    }
  }
  return ret;
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is an independent set.

"DFSMatching.h" 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  int res = dfs_matching(g, n, m);
  seen.assign(m, false);
  vector<bool> lfound(n, true);
  trav(it, match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    trav(e, g[i]) if (!seen[e] && match[e] != -1) {
```

```
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

7.5 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.
Time: $\mathcal{O}(E+V)$

24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  trav(e,g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));

  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;
  }
  return val[j] = low;
}
template<class G, class F> void scc(G& g, F f) {
  int n = sz(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
}
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: int eid = 0; ed.resize(N);
for each edge (a,b) {
ed[a].emplace_back(b, eid);
ed[b].emplace_back(a, eid++); }
bicomps([&](const vi& edgelist) {...});
Time: $\mathcal{O}(E+V)$

33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F f) {
  int me = num[at] = ++Time, e, y, top = me;
  trav(pa, ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).
Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.set_value(2); // Var 2 is true
ts.at_most_one({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

57 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int add_var() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f^1].push_back(j);
    gr[j^1].push_back(f);
  }
  void set_value(int x) { either(x, x); }

  void at_most_one(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = add_var();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    trav(e, gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    ++time;
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = time;
      if (values[x>>1] == -1)
        values[x>>1] = !(x&1);
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

7.6 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Possible optimization: on the top-most recursion level, ignore 'cands', and go through nodes in order of increasing degree, where degrees go down as nodes are removed.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R
    ={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

7.7 Trees

TreePower.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

25 lines

```
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
```

```
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
}
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected. Can also find the distance between two nodes.
Usage: `LCA lca(undirGraph);`
`lca.query(firstNode, secondNode);`
`lca.distance(firstNode, secondNode);`
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" 37 lines

```
typedef vector<pii> vpi;
typedef vector<vpi> graph;

struct LCA {
  vi time;
  vector<ll> dist;
  RMQ<pii> rmq;

  LCA(graph& C) : time(sz(C), -99), dist(sz(C)), rmq(dfs(C)
      ) {}

  vpi dfs(graph& C) {
    vector<tuple<int, int, int, ll>> q(1);
    vpi ret;
    int T = 0, v, p, d; ll di;
    while (!q.empty()) {
      tie(v, p, d, di) = q.back();
      q.pop_back();
      if (d) ret.emplace_back(d, p);
      time[v] = T++;
      dist[v] = di;
      trav(e, C[v]) if (e.first != p)
        q.emplace_back(e.first, v, d+1, di + e.second);
    }
    return ret;
  }

  int query(int a, int b) {
    if (a == b) return a;
    a = time[a], b = time[b];
    return rmq.query(min(a, b), max(a, b)).second;
  }
  ll distance(int a, int b) {
    int lca = query(a, b);
    return dist[a] + dist[b] - 2 * dist[lca];
  }
};
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 20 lines

```
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.dist));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.query(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.query(a, b)], b);
  }
  return ret;
}
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. The function of the HLD can be changed by modifying T, LOW and f. f is assumed to be associative and commutative.
Usage: `HLD hld(G);`
`hld.update(index, value);`
`tie(value, lca) = hld.query(n1, n2);`

"../data-structures/SegmentTree.h" 93 lines

```
typedef vector<pii> vpi;

struct Node {
  int d, par, val, chain = -1, pos = -1;
};

struct Chain {
  int par, val;
  vector<int> nodes;
  Tree tree;
};

struct HLD {
  typedef int T;
  const T LOW = -(1<<29);
  void f(T& a, T b) { a = max(a, b); }

  vector<Node> V;
  vector<Chain> C;

  HLD(vector<vpi>& g) : V(sz(g)) {
    dfs(0, -1, g, 0);
    trav(c, C) {
      c.tree = {sz(c.nodes), 0};
      for (int ni : c.nodes)
        c.tree.update(V[ni].pos, V[ni].val);
    }
  }

  void update(int node, T val) {
    Node& n = V[node]; n.val = val;
    if (n.chain != -1) C[n.chain].tree.update(n.pos, val);
  }

  int pard(Node& nod) {
    if (nod.par == -1) return -1;
    return V[nod.chain == -1 ? nod.par : C[nod.chain].par].
        d;
  }

  // query all *edges* between n1, n2
  pair<T, int> query(int i1, int i2) {
    T ans = LOW;
    while(i1 != i2) {
      Node n1 = V[i1], n2 = V[i2];
      if (n1.chain != -1 && n1.chain == n2.chain) {
        int lo = n1.pos, hi = n2.pos;
        if (lo > hi) swap(lo, hi);
        f(ans, C[n1.chain].tree.query(lo, hi));
        i1 = i2 = C[n1.chain].nodes[hi];
      } else {
        if (pard(n1) < pard(n2))
          n1 = n2, swap(i1, i2);
        if (n1.chain == -1)
          f(ans, n1.val), i1 = n1.par;
        else {
          Chain& c = C[n1.chain];
          f(ans, n1.pos ? c.tree.query(n1.pos, sz(c.nodes))
                        : c.tree.s[1]);
          i1 = c.par;
        }
      }
    }
    return make_pair(ans, i1);
  }

  // query all *nodes* between n1, n2
  pair<T, int> query2(int i1, int i2) {
    pair<T, int> ans = query(i1, i2);
    f(ans.first, V[ans.second].val);
    return ans;
  }

  pii dfs(int at, int par, vector<vpi>& g, int d) {
    V[at].d = d; V[at].par = par;
    int sum = 1, ch, nod, sz;
    tuple<int,int,int> mx(-1,-1,-1);
    trav(e, g[at]){
      if (e.first == par) continue;
      tie(sz, ch) = dfs(e.first, at, g, d+1);
      V[e.first].val = e.second;
      sum += sz;
      mx = max(mx, make_tuple(sz, e.first, ch));
    }
    tie(sz, nod, ch) = mx;
    if (2*sz < sum) return pii(sum, -1);
    if (ch == -1) { ch = sz(C); C.emplace_back(); }
    V[nod].pos = sz(C[ch].nodes);
    V[nod].chain = ch;
    C[ch].par = at;
    C[ch].nodes.push_back(nod);
    return pii(sum, ch);
  }
};
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

90 lines

```
struct Node { // Splay tree. Root's pp contains tree's
    parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void push_flip() {
    if (!flip) return;
```

```
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y :
        x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (push_flip(); p; ) {
      if (p->p) p->p->push_flip();
      p->push_flip(); push_flip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    push_flip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    make_root(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    make_root(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same
      tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void make_root(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

MatrixTree.h

Description: To count the number of spanning trees in an undirected graph G: create an $N \times N$ matrix `mat`, and for each edge $(a, b) \in G$, do `mat[a][a]++, mat[b][b]++, mat[a][b]--, mat[b][a]--`. Remove the last row and column, and take the determinant.

Geometry (8)

8.1 Geometric primitives

Point.h

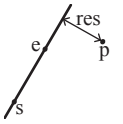
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

25 lines

```
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
      ); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
      ); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this)
      ; }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90
      degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the
      origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.



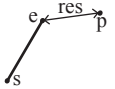
"Point.h"

4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

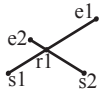
"Point.h"

6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
      ;
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists r1 is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists 2 is returned and r1 and r2 are set to the two ends of the common line. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long. Use segmentIntersectionQ to get just a true/false answer.



Usage: `Point<double> intersection, dummy;`

```
if (segmentIntersection(s1,e1,s2,e2,intersection,dummy)==1)
cout << "segments intersect at " << intersection <<
endl;
```

"Point.h" 27 lines

```
template<class P>
int segmentIntersection(const P& s1, const P& e1,
    const P& s2, const P& e2, P& r1, P& r2) {
  if (e1==s1) {
    if (e2==s2) {
      if (e1==e2) { r1 = e1; return 1; } //all equal
      else return 0; //different point segments
    } else return segmentIntersection(s2,e2,s1,e1,r1,r2);//
        swap
  }
  //segment directions and separation
  P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
  auto a = v1.cross(v2), a1 = v1.cross(d), a2 = v2.cross(d)
      ;
  if (a == 0) { //if parallel
    auto b1=s1.dot(v1), c1=e1.dot(v1),
         b2=s2.dot(v1), c2=e2.dot(v1);
    if (a1 || a2 || max(b1,min(b2,c2))>min(c1,max(b2,c2)))
      return 0;
    r1 = min(b2,c2)<b1 ? s1 : (b2<c2 ? s2 : e2);
    r2 = max(b2,c2)>c1 ? e1 : (b2>c2 ? s2 : e2);
    return 2-(r1==r2);
  }
  if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
  if (0<a1 || a<-a1 || 0<a2 || a<-a2)
    return 0;
  r1 = s1-v1*a2/a;
  return 1;
}
```

SegmentIntersectionQ.h

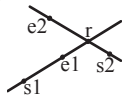
Description: Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

"Point.h" 16 lines

```
template<class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
  if (e1 == s1) {
    if (e2 == s2) return e1 == e2;
    swap(s1,s2); swap(e1,e2);
  }
  P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
  auto a = v1.cross(v2), a1 = d.cross(v1), a2 = d.cross(v2)
      ;
  if (a == 0) { // parallel
    auto b1 = s1.dot(v1), c1 = e1.dot(v1),
         b2 = s2.dot(v1), c2 = e2.dot(v1);
    return !a1 && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
  }
  if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
  return (0 <= a1 && a1 <= a && 0 <= a2 && a2 <= a);
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists r is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists -1 is returned. If s1==e1 or s2==e2 -1 is returned. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage: `point<double> intersection;`

```
if (1 == LineIntersection(s1,e1,s2,e2,intersection))
cout << "intersection point at " << intersection <<
endl;
```

"Point.h" 9 lines

```
template<class P>
int lineIntersection(const P& s1, const P& e1, const P& s2,
    const P& e2, P& r) {
  if ((e1-s1).cross(e2-s2)) { //if not parallell
    r = s2-(e2-s2)*(e1-s1).cross(s2-s1)/(e1-s1).cross(e2-s2
        );
    return 1;
  } else
    return -((e1-s1).cross(s2-s1)==0 || s2==e2);
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 11 lines

```
template<class P>
int sideOf(const P& s, const P& e, const P& p) {
  auto a = (e-s).cross(p-s);
  return (a > 0) - (a < 0);
}
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps)
    {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

onSegment.h

Description: Returns true iff p lies on the line segment from s to e. Intended for use with e.g. Point<long long> where overflow is an issue. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

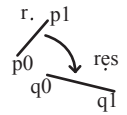
"Point.h" 5 lines

```
template<class P>
bool onSegment(const P& s, const P& e, const P& p) {
  P ds = p-s, de = p-e;
  return ds.cross(de) == 0 && ds.dot(de) <= 0;
}
```

linearTransformation.h

Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.
      dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

37 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
      }; }
  int quad() const {
    assert(x || y);
    if (y < 0) return (x >= 0) + 2;
    if (y > 0) return (x <= 0);
    return (x <= 0) * 2;
  }
  Angle t90() const { return {-y, x, t + (quad() == 3)}; }
  Angle t180() const { return {-x, -y, t + (quad() >= 2)};
      }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.quad(), a.y * (ll)b.x) <
         make_tuple(b.t, b.quad(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
    between
// them, i.e., the angle that covers the defined line
    segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
      )};
}
```

8.2 Circles

CircleIntersection.h

Description: Computes a pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 14 lines

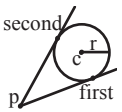
```
typedef Point<double> P;
bool circleIntersection(P a, P b, double r1, double r2,
    pair<P, P>* out) {
  P delta = b - a;
  assert(delta.x || delta.y || r1 != r2);
  if (!delta.x && !delta.y) return false;
  double r = r1 + r2, d2 = delta.dist2();
  double p = (d2 + r1*r1 - r2*r2) / (2.0 * d2);
```

```
  double h2 = r1*r1 - p*p*d2;
  if (d2 > r*r || h2 < 0) return false;
  P mid = a + delta*p, per = delta.perp() * sqrt(h2 / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

circleTangents.h

Description:

Returns a pair of the two points on the circle with radius r centered around c whos tangent lines intersect p. If p lies within the circle NaN-points are returned. P is intended to be Point<double>. The first point is the one to the right as seen from the p towards c.



Usage:
```
typedef Point<double> P;
pair<P,P> p = circleTangents(P(100,2),P(0,0),2);
```

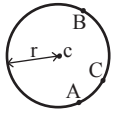
"Point.h" — 6 lines

```
template<class P>
pair<P,P> circleTangents(const P &p, const P &c, double r)
    {
  P a = p-c;
  double x = r*r/a.dist2(), y = sqrt(x-x*x);
  return make_pair(c+a*x+a.perp()*y, c+a*x-a.perp()*y);
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" — 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" — 28 lines

```
pair<double, P> mec2(vector<P>& S, P a, P b, int n) {
  double hi = INFINITY, lo = -hi;
  rep(i,0,n) {
    auto si = (b-a).cross(S[i]-a);
    if (si == 0) continue;
    P m = ccCenter(a, b, S[i]);
    auto cr = (b-a).cross(m-a);
    if (si < 0) hi = min(hi, cr);
    else lo = max(lo, cr);
  }
  double v = (0 < lo ? lo : hi < 0 ? hi : 0);
  P c = (a + b) / 2 + (b - a).perp() * v / (b - a).dist2();
  return {(a - c).dist2(), c};
}
pair<double, P> mec(vector<P>& S, P a, int n) {
  random_shuffle(S.begin(), S.begin() + n);
  P b = S[0], c = (a + b) / 2;
  double r = (a - c).dist2();
  rep(i,1,n) if ((S[i] - c).dist2() > r * (1 + 1e-8)) {
    tie(r,c) = (n == sz(S) ?
      mec(S, S[i], i) : mec2(S, a, S[i], i));
  }
  return {r, c};
}
pair<double, P> enclosingCircle(vector<P> S) {
  assert(!S.empty()); auto r = mec(S, S[0], sz(S));
  return {sqrt(r.first), r.second};
}
```

8.3 Polygons

insidePolygon.h

Description: Returns true if p lies within the polygon described by the points between iterators begin and end. If strict false is returned when p is on the edge of the polygon. Answer is calculated by counting the number of intersections between the polygon and a line going from p to infinity in the positive x-direction. The algorithm uses products in intermediate steps so watch out for overflow. If points within epsilon from an edge should be considered as on the edge replace the line "if (onSegment..." with the comment bellow it (this will cause overflow for int and long long).

Usage:
```
typedef Point<int> pi;
vector<pi> v; v.push_back(pi(4,4));
v.push_back(pi(1,2)); v.push_back(pi(2,1));
bool in = insidePolygon(v.begin(),v.end(), pi(3,4), false);
```

Time: $\mathcal{O}(n)$

"Point.h", "onSegment.h", "SegmentDistance.h" — 14 lines

```
template<class It, class P>
bool insidePolygon(It begin, It end, const P& p,
    bool strict = true) {
  int n = 0; //number of isects with line from p to (inf,p.
      y)
  for (It i = begin, j = end-1; i != end; j = i++) {
    //if p is on edge of polygon
    if (onSegment(*i, *j, p)) return !strict;
    //or: if (segDist(*i, *j, p) <= epsilon) return !strict
        ;
    //increment n if segment intersects line from p
    n += (max(i->y,j->y) > p.y && min(i->y,j->y) <= p.y &&
        ((*j-*i).cross(p-*i) > 0) == (i->y <= p.y));
  }
  return n&1; //inside if odd number of intersections
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" — 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

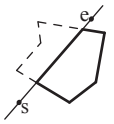
"Point.h" — 10 lines

```
typedef Point<double> P;
Point<double> polygonCenter(vector<P>& v) {
  auto i = v.begin(), end = v.end(), j = end-1;
  Point<double> res{0,0}; double A = 0;
  for (; i != end; j=i++) {
    res = res + (*i + *j) * j->cross(*i);
    A += j->cross(*i);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage:
```
vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));
```

"Point.h", "lineIntersection.h" — 15 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0)) {
      res.emplace_back();
      lineIntersection(s, e, cur, prev, res.back());
    }
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:

Returns a vector of indices of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Usage:
```
vector<P> ps, hull;
trav(i, convexHull(ps)) hull.push_back(ps[i]);
```

Time: $\mathcal{O}(n \log n)$

"Point.h" — 20 lines

```
typedef Point<ll> P;
pair<vi, vi> ulHull(const vector<P>& S) {
  vi Q(sz(S)), U, L;
  iota(all(Q), 0);
  sort(all(Q), [&S](int a, int b){ return S[a] < S[b]; });
  trav(it, Q) {
#define ADDP(C, cmp) while (sz(C) > 1 && S[C[sz(C)-2]].
    cross(\
  S[it], S[C.back()]) cmp 0) C.pop_back(); C.push_back(it);
    ADDP(U, <=); ADDP(L, >=);
  }
  return {U, L};
}

vi convexHull(const vector<P>& S) {
  vi u, l; tie(u, l) = ulHull(S);
  if (sz(S) <= 1) return u;
  if (S[u[0]] == S[u[1]]) return {0};
  l.insert(l.end(), u.rbegin()+1, u.rend()-1);
  return l;
}
```

PolygonDiameter.h

Description: Calculates the max squared distance of a set of points.

"ConvexHull.h" — 19 lines

```
vector<pii> antipodal(const vector<P>& S, vi& U, vi& L) {
  vector<pii> ret;
  int i = 0, j = sz(L) - 1;
  while (i < sz(U) - 1 || j > 0) {
    ret.emplace_back(U[i], L[j]);
    if (j == 0 || (i != sz(U)-1 && (S[L[j]] - S[L[j-1]])
          .cross(S[U[i+1]] - S[U[i]]) > 0)) ++i;
    else --j;
  }
  return ret;
}
```

```
pii polygonDiameter(const vector<P>& S) {
  vi U, L; tie(U, L) = ulHull(S);
  pair<ll, pii> ans;
  trav(x, antipodal(S, U, L))
    ans = max(ans, {(S[x.first] - S[x.second]).dist2(), x})
        ;
  return ans.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a given polygon (counter-clockwise order). The polygon must be such that every point on the circumference is visible from the first point in the vector. It returns 0 for points outside, 1 for points on the circumference, and 2 for points inside.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "onSegment.h" — 22 lines

```
typedef Point<ll> P;
int insideHull2(const vector<P>& H, int L, int R, const P&
    p) {
  int len = R - L;
  if (len == 2) {
    int sa = sideOf(H[0], H[L], p);
    int sb = sideOf(H[L], H[L+1], p);
    int sc = sideOf(H[L+1], H[0], p);
    if (sa < 0 || sb < 0 || sc < 0) return 0;
    if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R == sz(H
        )))
      return 1;
    return 2;
  }
  int mid = L + len / 2;
  if (sideOf(H[0], H[mid], p) >= 0)
    return insideHull2(H, mid, R, p);
  return insideHull2(H, L, mid+1, p);
}

int insideHull(const vector<P>& hull, const P& p) {
  if (sz(hull) < 3) return onSegment(hull[0], hull.back(),
      p);
  else return insideHull2(hull, 1, sz(hull), p);
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. isct(a, b) returns a pair describing the intersection of a line with the polygon: $\bullet$ $(-1,-1)$ if no collision, $\bullet$ $(i,-1)$ if touching the corner i, $\bullet$ (i,i) if along side $(i,i+1)$, $\bullet$ (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon.

Time: $\mathcal{O}(N + Q \log n)$

"Point.h" — 63 lines

```
ll sgn(ll a) { return (a > 0) - (a < 0); }
typedef Point<ll> P;
struct HullIntersection {
  int N;
  vector<P> p;
  vector<pair<P, int>> a;

  HullIntersection(const vector<P>& ps) : N(sz(ps)), p(ps)
      {
    p.insert(p.end(), all(ps));
    int b = 0;
    rep(i,1,N) if (P{p[i].y,p[i].x} < P{p[b].y, p[b].x}) b
        = i;
    rep(i,0,N) {
      int f = (i + b) % N;
      a.emplace_back(p[f+1] - p[f], f);
    }
  }

  int qd(P p) {
    return (p.y < 0) ? (p.x >= 0) + 2
         : (p.x <= 0) * (1 + (p.y <= 0));
  }

  int bs(P dir) {
    int lo = -1, hi = N;
    while (hi - lo > 1) {
      int mid = (lo + hi) / 2;
      if (make_pair(qd(dir), dir.y * a[mid].first.x) <
          make_pair(qd(a[mid].first), dir.x * a[mid].first.y)
              )
        hi = mid;
      else lo = mid;
    }
    return a[hi%N].second;
  }

  bool isign(P a, P b, int x, int y, int s) {
    return sgn(a.cross(p[x], b)) * sgn(a.cross(p[y], b)) ==
        s;
  }

  int bs2(int lo, int hi, P a, P b) {
    int L = lo;
    if (hi < lo) hi += N;
    while (hi - lo > 1) {
      int mid = (lo + hi) / 2;
      if (isign(a, b, mid, L, -1)) hi = mid;
      else lo = mid;
    }
    return lo;
  }

  pii isct(P a, P b) {
    int f = bs(a - b), j = bs(b - a);
    if (isign(a, b, f, j, 1)) return {-1, -1};
    int x = bs2(f, j, a, b)%N,
        y = bs2(j, f, a, b)%N;
    if (a.cross(p[x], b) == 0 &&
        a.cross(p[x+1], b) == 0) return {x, x};
    if (a.cross(p[y], b) == 0 &&
        a.cross(p[y+1], b) == 0) return {y, y};
    if (a.cross(p[f], b) == 0) return {f, -1};
    if (a.cross(p[j], b) == 0) return {j, -1};
    return {x, y};
  }
};
```

8.4 Misc. Point Set Problems

closestPair.h

Description: $i1$, $i2$ are the indices to the closest pair of points in the point vector p after the call. The distance is returned.

Time: $\mathcal{O}(n \log n)$

"Point.h" — 58 lines

```
template<class It>
bool it_less(const It& i, const It& j) { return *i < *j; }
template<class It>
bool y_it_less(const It& i,const It& j) {return i->y < j->y
    ;}

template<class It, class IIt> /* IIt = vector<It>::iterator
    */
double cp_sub(IIt ya, IIt yaend, IIt xa, It &i1, It &i2) {
  typedef typename iterator_traits<It>::value_type P;
  int n = yaend-ya, split = n/2;
  if(n <= 3) { // base case
    double a = (*xa[1]-*xa[0]).dist(), b = 1e50, c = 1e50;
    if(n==3) b=(*xa[2]-*xa[0]).dist(), c=(*xa[2]-*xa[1]).
        dist();
    if(a <= b) { i1 = xa[1];
      if(a <= c) return i2 = xa[0], a;
      else return i2 = xa[2], c;
    } else { i1 = xa[2];
      if(b <= c) return i2 = xa[0], b;
      else return i2 = xa[1], c;
  } }
  vector<It> ly, ry, stripy;
  P splitp = *xa[split];
  double splitx = splitp.x;
  for(IIt i = ya; i != yaend; ++i) { // Divide
    if(*i != xa[split] && (**i-splitp).dist2() < 1e-12)
      return i1 = *i, i2 = xa[split], 0;// nasty special
          case!
    if (**i < splitp) ly.push_back(*i);
    else ry.push_back(*i);
  } // assert((signed)lefty.size() == split)
  It j1, j2; // Conquer
  double a = cp_sub(ly.begin(), ly.end(), xa, i1, i2);
  double b = cp_sub(ry.begin(), ry.end(), xa+split, j1, j2)
      ;
  if(b < a) a = b, i1 = j1, i2 = j2;
  double a2 = a*a;
  for(IIt i = ya; i != yaend; ++i) { // Create strip (y-
      sorted)
    double x = (*i)->x;
    if(x >= splitx-a && x <= splitx+a) stripy.push_back(*i)
        ;
  }
  for(IIt i = stripy.begin(); i != stripy.end(); ++i) {
    const P &p1 = **i;
    for(IIt j = i+1; j != stripy.end(); ++j) {
      const P &p2 = **j;
      if(p2.y-p1.y > a) break;
      double d2 = (p2-p1).dist2();
      if(d2 < a2) i1 = *i, i2 = *j, a2 = d2;
  } }
  return sqrt(a2);
}

template<class It> // It is random access iterators of
    point<T>
double closestpair(It begin, It end, It &i1, It &i2 ) {
  vector<It> xa, ya;
  assert(end-begin >= 2);
  for (It i = begin; i != end; ++i)
    xa.push_back(i), ya.push_back(i);
  sort(xa.begin(), xa.end(), it_less<It>);
  sort(ya.begin(), ya.end(), y_it_less<It>);
  return cp_sub(ya.begin(), ya.end(), xa.begin(), i1, i2);
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" — 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;
```

```
  T distance(const P& p) { // min squared distance to a
      point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if the box is wider than high (not best
          heuristic...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (
          not
      // best performance with many duplicates in the
          middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)}))
      {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared
      distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

DelaunayTriangulation.h

Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are colinear or any four are on the same circle, behavior is undefined.
Time: $\mathcal{O}(n^2)$

"Point.h", "3dHull.h" — 10 lines

```
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) <
      0);
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  trav(p, ps) p3.emplace_back(p.x, p.y, p.dist2());
  if (sz(ps) > 3) trav(t, hull3d(p3)) if ((p3[t.b]-p3[t.a])
      .
      cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
}
```

FastDelaunay.h

Description: Fast Delaunay triangulation. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" — 90 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return rot->r()->o->rot; }
};

bool circ(P p, P a, P b, P c) { // is p in the circumcircle
    ?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
      0;
}
Q makeEdge(P orig, P dest) {
  Q q0 = new Quad{0,0,0,orig}, q1 = new Quad{0,0,0,arb},
    q2 = new Quad{0,0,0,dest}, q3 = new Quad{0,0,0,arb};
  q0->o = q0; q2->o = q2; // 0-0, 2-2
  q1->o = q3; q3->o = q1; // 1-3, 3-1
  q0->rot = q1; q1->rot = q2;
  q2->rot = q3; q3->rot = q0;
  return q0;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back()
        );
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = (sz(s) + 1) / 2;
  tie(ra, A) = rec({s.begin(), s.begin() + half});
  tie(B, rb) = rec({s.begin() + half, s.end()});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p
    ); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

6 lines

```
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilist) {
  double v = 0;
  trav(i, trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z)
      {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
```

```
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

16 lines

```
vi pi(const string& s) {
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}

vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
  return res;
}
```

extended-KMP.h

Description: extended KMP S[i] stores the maximum common prefix between s[i:] and t; T[i] stores the maximum common prefix between t[i:] and t for i>0;

32 lines

```
int S[N], T[N];

void extKMP(const string&s, const string &t) {
  int m = t.size();
  T[0] = 0;
  int maT = 0;
  for (int i = 1; i < m; i++) {
    if (maT + T[maT] >= i) {
      T[i] = min(T[i - maT], maT + T[maT] - i);
    }else {
      T[i] = 0;
    }
    while (T[i] + i < m && t[T[i]] == t[T[i] + i])
      T[i]++;
    if (i + T[i] > maT + T[maT])
      maT = i;
  }
  int maS = 0;
  int n = s.size();
  for (int i = 0; i < n; i++) {
    if (maS + S[maS] >= i) {
      S[i] = min(T[i - maS], maS + S[maS] - i);
    }else {
      S[i] = 0;
    }
    while (S[i] < m && i + S[i] < n && t[S[i]] == s[S[i] + i])
      S[i]++;
    if (i + S[i] > maS + S[maS])
      maS = i;

  }
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

11 lines

```
void manacher(const string& s) {
  int n = sz(s);
  vi p[2] = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
}}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+min_rotation(v), v.end());`
Time: $\mathcal{O}(N)$

8 lines

```
int min_rotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(i,0,N) {
    if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1); break;}
    if (s[a+i] > s[b+i]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. $a[i]$ is the starting index of the suffix which is i-th in the sorted suffix array. The returned vector is of size $n+1$, and $a[0] = n$. The `lcp` function calculates longest common prefixes for neighbouring strings in suffix array. The returned vector is of size $n+1$, and $ret[0] = 0$.
Memory: $\mathcal{O}(N)$
Time: $\mathcal{O}(N \log^2 N)$ where N is the length of the string for creation of the SA. $\mathcal{O}(N)$ for longest common prefixes.

61 lines

```
typedef pair<ll, int> pli;
void count_sort(vector<pli> &b, int bits) { // (optional)
  //this is just 3 times faster than stl sort for N=10^6
```

```
  int mask = (1 << bits) - 1;
  rep(it,0,2) {
    int move = it * bits;
    vi q(1 << bits), w(sz(q) + 1);
    rep(i,0,sz(b))
      q[(b[i].first >> move) & mask]++;
    partial_sum(q.begin(), q.end(), w.begin() + 1);
    vector<pli> res(b.size());
    rep(i,0,sz(b))
      res[w[(b[i].first >> move) & mask]++] = b[i];
    swap(b, res);
  }
}
struct SuffixArray {
  vi a;
  string s;
  SuffixArray(const string& _s) : s(_s + '\0') {
    int N = sz(s);
    vector<pli> b(N);
    a.resize(N);
    rep(i,0,N) {
      b[i].first = s[i];
      b[i].second = i;
    }

    int q = 8;
    while ((1 << q) < N) q++;
    for (int moc = 0;; moc++) {
      count_sort(b, q); // sort(all(b)) can be used as well
      a[b[0].second] = 0;
      rep(i,1,N)
        a[b[i].second] = a[b[i - 1].second] +
          (b[i - 1].first != b[i].first);

      if ((1 << moc) >= N) break;
      rep(i,0,N) {
        b[i].first = (ll)a[i] << q;
        if (i + (1 << moc) < N)
          b[i].first += a[i + (1 << moc)];
        b[i].second = i;
      }
    }
    rep(i,0,sz(a)) a[i] = b[i].second;
  }
  vi lcp() {
    // longest common prefixes: res[i] = lcp(a[i], a[i-1])
    int n = sz(a), h = 0;
    vi inv(n), res(n);
    rep(i,0,n) inv[a[i]] = i;
    rep(i,0,n) if (inv[i] > 0) {
      int p0 = a[inv[i] - 1];
      while (s[i + h] == s[p0 + h]) h++;
      res[inv[i]] = h;
      if(h > 0) h--;
    }
    return res;
  }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] =
        0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA =
      28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) :
        0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2)
        );
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

Hashing.h

Description: Various self-explanatory methods for string hashing.

44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse,
    where
// ABBA... and BAAB... of length 2^10 hash the same mod
    2^64).
// "typedef ull H;" instead if you think test data is
    random,
// or work mod 10^9+7 if the Birthday paradox is not a
    problem.
struct H {
  typedef uint64_t ull;
  ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
  (A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) : B); return r
      ; }
  OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) : "rdx")
  H operator-(H o) { return *this + ~o.x; }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also
    ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}

H hashString(string& s) { H h{}; trav(c,s) h=h*C+c; return
    h; }
```

AhoCorasick.h

Description: Aho-Corasick tree is used for multiple pattern matching. Initialize the tree with create(patterns). find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(_, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input.
Time: Function create is $\mathcal{O}(26N)$ where N is the sum of length of patterns. find is $\mathcal{O}(M)$ where M is the length of the word. findAll is $\mathcal{O}(NM)$.

67 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'};
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches =
        0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vector<int> backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    trav(c, s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
```

```
  AhoCorasick(vector<string>& pat) {
    N.emplace_back(-1);
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start
               ])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
    int n = 0;
    vi res; // ll count = 0;
    trav(c, word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R)
    {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F f, G g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

13 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) // (A)
      a = mid;
    else
      b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

Karatsuba.h

Description: Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX+b)(cX+d) = acX^2 + bd + ((a+c)(b+d) - ac - bd)X$. Doesn't handle sequences of very different length well. See also FFT, under the Numerical chapter.
Time: $\mathcal{O}\left(N^{1.6}\right)$

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

17 lines

```
template<class I> vi lis(vector<I> S) {
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    p el { S[i], i };
    //S[i]+1 for non-decreasing
    auto it = lower_bound(all(res), p { S[i], 0 });
    if (it == res.end()) res.push_back(el), it = --res.end
        ();
    *it = el;
    prev[i] = it==res.begin() ?0:(it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

LCS.h

Description: Finds the longest common subsequence.
Memory: $\mathcal{O}(nm)$.
Time: $\mathcal{O}(nm)$ where n and m are the lengths of the sequences.

14 lines

```
template<class T> T lcs(const T &X, const T &Y) {
  int a = sz(X), b = sz(Y);
  vector<vi> dp(a+1, vi(b+1));
  rep(i,1,a+1) rep(j,1,b+1)
    dp[i][j] = X[i-1]==Y[j-1] ? dp[i-1][j-1]+1 :
      max(dp[i][j-1],dp[i-1][j]);
  int len = dp[a][b];
  T ans(len,0);
  while(a && b)
    if(X[a-1]==Y[b-1]) ans[--len] = X[--a], --b;
```

```
    else if(dp[a][b-1]>dp[a-1][b]) --b;
    else --a;
  return ans;
}
```

10.3 Dynamic programming

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i)\le k<hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}\left((N + (hi - lo))\log N\right)$ 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v);
      }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k]+a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}\left(N^2\right)$

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ...` } loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation. 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.
"BumpAllocator.h" 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.
Usage: `vector<vector<int, small<int>>> ed(N);` 14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
};
```

Unrolling.h

5 lines

```
#define F {...; ++i;}
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern "`_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)`". Not all are described here; grep for `_mm_` in `/usr/lib/gcc/*/4.9/include/` for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define `__SSE__` and `__MMX__` before including it. For aligned memory use `_mm_malloc(size, 32)` or `int buf[N] alignas(32)`, but prefer loadu/storeu. 43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "immintrin.h"

typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))

// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256,
     _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of
     bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts
     of x
// sad_epu8: sum of absolute differences of u8, outputs 4
     xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16
     xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->
     lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g.
     _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/
     or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|
     hi)

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0;
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb);
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
```

```
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)
          ));
  }
  union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[
      i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <-
      equiv
  return r;
}
```