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Impact of Landau quantization on Chandrasekhar limit

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1. Introduction

Before 1930, white dwarf were considered as the ultimate fate of a star but after (Indian Physicist) Chandrasekhar proposed the limit [1], our understanding become modified. According to the Chandrasekhar limit (CL), if the compact star is less than 1.44 times the mass ($M_\odot$) of the sun then the star would become white dwarf but if it is more than 1.44 times $M_\odot$ then it can be converted to neutron star or black hole. The white dwarf is a special stable structure because the inward gravitational pressure is being balanced by electron degeneracy pressure. The chandrasekhar limit was observed to be true for 80 years but after 2000 it was found that there are many white dwarf whose masses are more than 1.44 $M_\odot$ [2]. Recently few works [3, 4] have attempted to see the deviation of CL due to magnetic field. Present work also trying to explore similar possibility by applying Landau quantization aspect of magnetic field on Chandrasekhar limit. A huge magnetic field can be expected in white dwarfs, whose surface magnetic field can be as high as 10^9 G. According to Virial Theorem arguments the magnetic field in the centre of the white dwarf can go upto 10^{13} G [5]. If the central magnetic field is determined through analytical calculations in both Newtonian and General Relativity it comes out to be around 10^{12-16}G (see Ref. [5] and references therein).

2. Formalism

When we adopt quantum aspects of magnetic field, then we get an additional field momentum along with the traditional mechanical momentum of electron. Adding that field momentum in Schrodinger's or Dirac's equation, one can get landau quantization of electron's energy. Due to this quantum aspects of magnetic field, the electron degenerate pressure can be modified. Hence, during the balancing with gravitational pressure Chandrasekhar limit can be changed. When no magnetic field is considered i.e ideal case the relation between radius and mass of white dwarf in relativistic case comes out to be-

$$M_{limit} = \frac{\omega_3^0\sqrt{3\pi}}{2}\left(\frac{hc}{2\pi G}\right)^{3/2}\frac{1}{(\mu_e m_H)^2}$$

h= Planks Constant
G= Universal Gravitational Constant
$\omega_3^0 = 2.018$
$\mu_e = 2$
m_H = mass of hydrogen atom

Let us proceed towards $B \neq 0$ picture, where magnetic field is considered to be in z direction, pressure in the x and y direction will be same due to symmetry while the pressure in the z direction will be different because of the different motion of particles by the virtue of magnetic field.
The pressure along x-axis will be

$$P_x = \sum_0^{l_{max}} \frac{dp_x dp_y}{(2\pi)^2}\int_0^{p_{zf}}\left(\frac{p_x}{m_e}\right)\left(\frac{p_x}{\sqrt{p_z^2+m^2+2leB}}\right)dp_z\ , \quad (1)$$

where we will considered quantized perpendicular momentum $p_x = \sqrt{leB}$ [6] for Landau level l and momentum along z-direction is unquantized. p_{zF} is Fermi-momentum along z-axis. So,

$$P_x = \sum_0^{l_{max.}} \frac{le^2B^2}{m_e\pi} \int_0^{p_{zf}} \frac{dp_z}{\sqrt{p_z^2+m^2+2leB}}$$
$$= \frac{e^2B^2}{m_e\pi} \sum_0^{l_{max.}} l \ln\left(\sqrt{p_{zf}^2+m^2+2leB}+p_{zf}\right)$$
$$= \frac{e^2B^2}{m_e\pi} \sum_0^{l_{max.}} l \sinh^{-1}\left(\frac{p_{zf}}{2leB+m^2}\right) \quad (2)$$

Pressure along z-axis will be:

$$P_z = \frac{eB}{m_e\pi} \sum_0^{l_{max.}} \frac{p_z^2}{\sqrt{p_z^2+m^2+2leB}}$$
$$= \frac{eB}{m_e\pi} \sum_0^{l_{max}} \int_0^{p_{zf}} p_z\Big(1+\frac{2leB+m^2}{p_z^2} + \ldots\Big)dp_z$$
$$= \frac{eB}{m_e\pi} \sum_0^{l_{max}} \left(\frac{p_{zf}^2}{2} + (m^2+2leB)\ln(p_{zf})\right)$$
$$= \frac{eB}{m_e\pi}\Big[\frac{p_{zf}l_{max.}}{2} + eBl(l+1)\ln p_{zf} + m^2\ln p_{zf}l_{max.}\Big] \quad (3)$$

Above pressure derivations are generalized for any landau levels. For lower landau level ($l=0$), gas pressure in x and y directions become zero while the z direction the expression becomes as follows-

$$P_z = \frac{m_eeB}{\pi}\ln\left(\frac{3M}{8m_pR^3eB}\right) + \frac{1}{2m_e}\left(\frac{3M}{8m_pR^3eB}\right)^2 \quad (4)$$

where

$$p_{zf} = \frac{N\pi}{VeB} = \frac{3M}{8m_pR^3eB}(for\ l=0)\ . \quad (5)$$

Equating the above gas pressure with the gravitational pressure-

$$\frac{m_eeB}{\pi}\ln\left(\frac{3M}{8m_pR^3eB}\right) + \frac{1}{2m_e}\left(\frac{3M}{8m_pR^3eB}\right)^2 = \alpha\frac{GM^2}{3R^4} \quad (6)$$

Above equation might be considered as modified mass-radius relation for degenerate electron gas in presence of very strong magnetic field, where lowest Landau level approximation can be applicable.

3. Conclusion

In presence of magnetic field, a possibility of modification of Chandrasekhar limit can be expected as the degenerate gas pressure gets effected by the magnetic field. The stronger is the magnetic field more will be the impact on the Chandrasekhar limit. The work is under progress and some limitations are carried in our present mass-radius relation. For example, we are shifting from 3D to 1D for simplification of calculation by considering lowest Landau level approximation.

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