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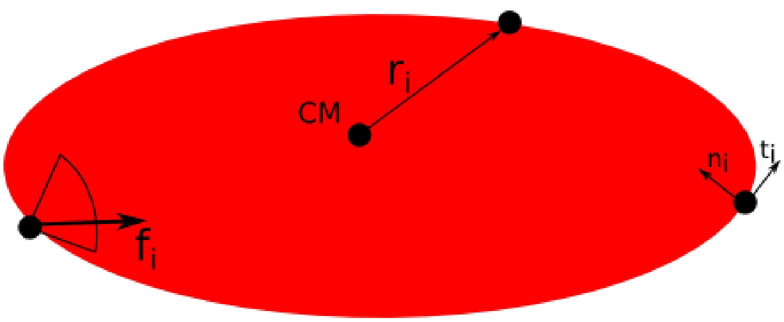
Force Closure

Force Closure

0.0/20.0 points (graded)

Force closure is an important concept in manipulation. Given a configuration with a robot grasping an object at some set of contact points, we say there is *force closure* if the robot can exert an arbitrary force and torque on the object by applying forces at the contact points. Note that for this problem, we will not consider the kinematics or dynamics of the robot itself--merely the quality of the grasp, as determined by the contact points.



For simplicity, we will consider a planar model, where the contact forces $f_i \in \mathbb{R}^2$. Write the forces in the frame of the contact, so the set of feasible forces within the friction cone is expressed as $f_i = \begin{bmatrix} f_{i,x} \\ f_{i,z} \end{bmatrix}$ with $|f_{i,x}| \leq \mu f_{i,z}$ and $f_{i,z} \geq 0$, observing that these can be rewritten as linear constraints.

Then, the net wrench w (the combined forces and torques) on the object is

$$w = \begin{bmatrix} \sum_i (f_{i,x} t_i + f_{i,z} n_i) \\ \sum_i r_i \times (f_{i,x} t_i + f_{i,z} n_i) \end{bmatrix}$$

It is fairly easy to see that we can write $w = Gf$, for some matrix G (depending on r_i, n_i, t_i) where f is the stacked vector of forces.

It can also be shown that the question of force closure can be reduced to this problem:

- Check that G is full rank,

- find f such that $Gf = 0$
- and f is in the *interior* of the set of allowable forces. Otherwise stated, $f_{i,z} > 0, \quad |f_{i,x}| < \mu f_{i,z}$ for all i.

The rank condition can easily be checked, but the other two points are slightly more difficult. Assuming that G is full rank, we would like to write a Linear Program (LP) that checks, for some r_i's, whether or not a grasp has force closure. Recall that an LP is an optimization of the form:

$$\begin{aligned} \min_z & \, c^T z \\ \text{s.t.} & \, Az = b \\ & \, Cz \leq d \end{aligned}$$

For some matrices A and C and vectors c, b, d.

The problem: write an LP such that $c^T z = 0$ if and only if there is **no** force closure, and $c^T z < 0$ when there is force closure.

HINT: The strict inequalities (> and <) in the description above can be tricky to incorporate into this format. Introduce a slack variable $\gamma \leq 0$ and let $z = \begin{bmatrix} \gamma \\ f \end{bmatrix}$ and c such that $c^T z = \gamma$. Incorporate γ into the constraints above such that $\gamma < 0$ implies that $f_{i,z} > 0$ and $|f_{i,x}| < \mu f_{i,z}$ for all i.

Make sure that your LP is bounded (that there is no feasible z such that $c^T z = \infty$)! You can also use MATLAB's **linprog** function to test your LP on numeric values, but careful for their different naming conventions.

```
% PROBLEM SETUP, DO NOT EDIT
% Construct vectors r1 and r2
r1=sym('r1',[2 1]);
sym(r1,'real');

r2=sym('r2',[2 1]);
sym(r2,'real');

n1=sym('n1',[2 1]);
sym(n1,'real');

t1=sym('t1',[2 1]);
sym(t1,'real');

n2=sym('n2',[2 1]);
sym(n2,'real');
```

Unanswered

```
% decision variables [gamma;f1;f2]
c = [1; zeros(4,1)];

% sum forces and torque in inertial frame
G = [0 t1(1) n1(1) t2(1) n2(1);
     0 t1(2) n1(2) t2(2) n2(2);
     0, t1(2)*r1(1) - t1(1)*r1(2), n1(2)*r1(1) - n1(1)*r1(2), t2(2)*r2(1) - t2(1)*r2(2),
A = G;
b = zeros(3,1);

% create C and d for friction cone
C = zeros(0,5);
C = [C;-1 0 -1 0 0];  %f_1z + gamma >= 0
C = [C;-1 1 -mu 0 0]; %mu f_1z + gamma >= f_1x
C = [C;-1 -1 -mu 0 0]; %f_1x + gamma >= -mu f_1z


% same for f2
C = [C;-1 0 0 0 -1]; %f_2z + gamma >= 0
C = [C;-1 0 0 1 -mu]; %mu f_1z + gamma >= f_1x
C = [C;-1 0 0 -1 -mu]; %f_1x + gamma >= -mu f_1z

% bound gamma
C = [C;-1 0 0 0 0];  % gamma >= -1

d = [zeros(6,1);1];
```

Run Code

As an aside, if we wanted to implement this same check in 3D, the friction cone constraint is no longer linear! Standard approaches to this are to form a polyhedral approximation of the friction cone, or to perform the check as a Second Order Cone Program (SOCP), which is another type of convex optimization.

Submit You have used 0 of 3 attempts

Answers are displayed within the problem