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Lyapunov Functions

Lyapunov Functions

0.0/20.0 points (graded)

This question will test your understanding of Lyapunov functions with a series of short questions.

(a) Suppose that $V_1(x)$ and $V_2(x)$ are valid Lyapunov functions that prove global stability of a system to the origin. Is it always the case that $V_2(x) = cV_1(x)$ for some $c > 0$? In other words, are Lyapunov functions unique up to scaling?

- Yes
- No ✔

Explanation

Lyapunov functions are not unique up to scaling. As an example, consider the system $\dot{x} = -x$. The function $V(x) = x^2$ is a valid Lyapunov function, but so is $V(x) = x^4$.

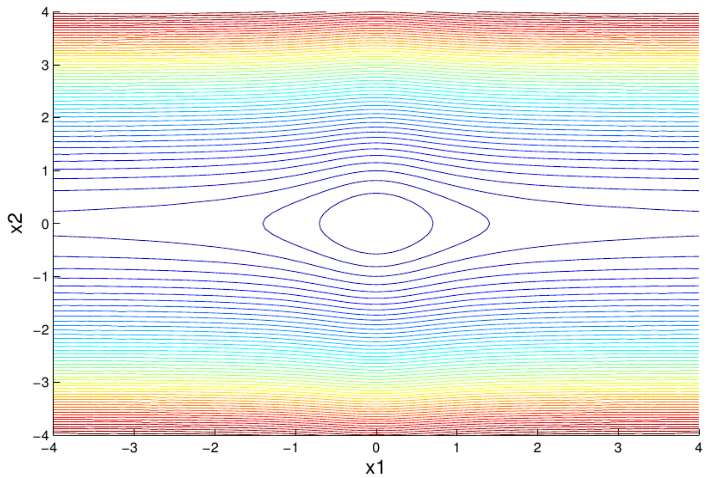
(b) For the system:

$$\dot{x_1} = -\frac{6x_1}{(1+x_1^2)^2} + 2x_2$$
$$\dot{x_2} = -\frac{2(x_1+x_2)}{(1+x_1^2)^2}$$

you are given the positive definite function $V(x) = \frac{x_1^2}{1+x_1^2} + x_2^2$ and told that, for this system, $\dot{V}$ is negative definite over the entire space. Is V a valid Lyapunov function which proves global asymptotic stability to the origin for the system? (Hint: Try simulating a few trajectories of this system or plotting a few level sets of V to build more intuition before answering this problem).

- Yes
- No ✔

Explanation



The function $V(x)$ is not radially unbounded. This results in the level sets of V being unbounded (see the figure above for a picture of the level sets). Thus, even though V decreases along trajectories of the system, the system does not go to the origin.

(c) Suppose $V(x)$ is a valid Lyapunov function that proves global asymptotic stability of a system to the origin. Is it true that $(V(x))^2$ is also a valid Lyapunov function?

- Yes ✔
- No

Explanation

This is true since if $V(x)$ is positive definite, then so is $V^2(x)$. Further, the time-derivative of $V^2(x)$ is $2V\dot{V}$, which is negative definite because $\dot{V}$ is negative definite.

(d) Suppose $V(x)$ is a valid Lyapunov function that proves global asymptotic stability of a system to the origin. Is it true that tanh$(V(x))$ is also a valid Lyapunov function?

Yes

No ✔

Explanation

The function $\tanh(V(x))$ is not radially unbounded and is thus not a valid Lyapunov function.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem