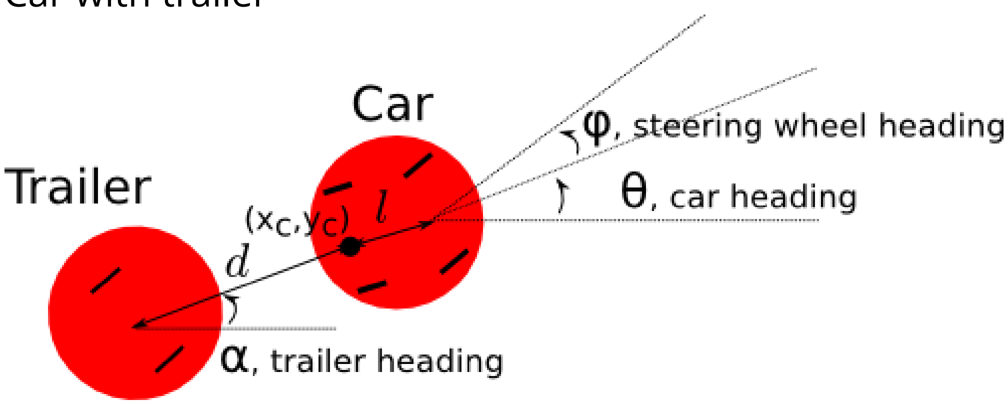
Differential Flatness

Differential Flatness, Part 1

0.0/10.0 points (graded)

Car with trailer



Suppose we have a car with a trailer attached. For simplicity, model both car and trailer as circular objects. Letting the state be the position of the car, and absolute angles of both car and trailer (as shown). Using a simple vehicle model, where the control inputs are the speed, u_1, and the angular rate of the front wheels, u_2, the dynamics are:

$$\dot{x} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \cos(\theta)\, u_1 \\ \sin(\theta)\, u_1 \\ u_2 \\ l^{-1} \tan(\phi)\, u_1 \\ d^{-1} \sin(\theta - \alpha)\, u_1 \end{bmatrix}$$

It turns out that this system is differentially flat, even with any number of trailers strung out behind the car! Take the flat output y to be the position of the trailer:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_c - d\cos(\alpha) \\ y_c - d\sin(\alpha) \end{bmatrix}$$

If y is truly a flat output, which of the following statements is true?

- (A) We can write $x = f(y)$.
- (B) We can write $u = g(y)$.
- (C) We can write $x = f(y, \dot{y}, \ddot{y}, \ldots)$.
- (D) We can write $u = g(y, \dot{y}, \ddot{y}, \ldots)$.
- (E) A and B.
- (F) C and D.

Find $\tan(\alpha)$ in terms of $y_1, y_2, \dot{y}_1, \dot{y}_2$. For your answer, write $v_i = \dot{y}_i$.

Ignoring singularities, we now have a function for α. Verify for yourself that, using α, you can easily find functions x_c and y_c, which can be differentiated and then used to find an expression for θ and so on.

Submit You have used 0 of 1 attempt

Differential Flatness, Part 2

0.0/20.0 points (graded)

QP Control of the Car

Suppose we wish to pose a planning problem for the car with a trailer. For a fixed duration T, we would like to get from an initial state x_0 and bring the trailer to a desired location y_g.

Suppose also that we choose to parameterize the path in the output space as:

$$y_1 = \sum_{i=0}^{N} a_i t^i$$

$$y_2 = \sum_{i=0}^{N} b_i t^i$$

For some sufficiently large N. For decision variables $a_0, \ldots, a_N, b_0, \ldots, b_N$, and $N = 4$, write the minimum jerk QP:

[Math Processing Error]

Note that, for now, we've written the initial state constraint on the output. To avoid confusion, write the QP in the same format as MATLAB's quadprog:

$$\begin{aligned} \min .5 z^T H z + f^T z \\ \text{s.t.} Az \leq b \\ Bz = c \end{aligned}$$

```
% PROBLEM SETUP, DO NOT CHANGE
N = 4;
T = 5;
y_0 = randn(2,1);
yd_0 = randn(2,1);
ydd_0 = randn(2,1);
y_g = 2 + randn(2,1);

% QP SETUP HERE
H =
f =
A =
b =
B =
c =
```

Unanswered

Run Code

Submit

You have used 0 of 3 attempts

Differential Flatness, Part 3

0.0/18.0 points (graded)

QP Possibilities

This last question will explore what we can and cannot easily do with QP optimization. All of these questions consider adding constraints *only* to the problem above, and the answers may not be obvious at first! You are encouraged to carefully think these questions through before answering.

In writing down additional constraints, it's possible that the program becomes overconstrained, and infeasible. For now, ignore this question and focus on whether or not we can pose the problem.

In the previous formulation, we constrainted $\boldsymbol{y}(0), \dot{\boldsymbol{y}}(0), \ddot{\boldsymbol{y}}(0)$ instead of $\boldsymbol{x}(0)$. Did we need to do that? For a fixed $\boldsymbol{x}_0$, could we have written the constraint $\boldsymbol{x}(0) = \boldsymbol{x}_0$ as a *linear function* of the decision variables?

- Yes, there is such a linear constraint
- No, that constraint is not linear.

What about constraining the final state, $\boldsymbol{x}(T)$, assuming such a state is not inconsistent with the goal state $\boldsymbol{y}(T)$?

- Yes, there is such a linear constraint
- No, that constraint is not linear

Corridor Constraints

For the last four questions, we wish to stay within a physical corridor (e.g. an indoor hallway or a lane on a highway), and suppose the corridor is a convex polytope in the physical plane.

Suppose we had the task of ensuring that the trailer stayed within the corridor for all $t \in [0, T]$. Could we write that constraint as a *linear function* of the decision variables?

- Yes, there is such a linear constraint
- No, that constraint is not linear

For some fixed integer M, suppose we had the task of ensuring that the trailer stayed within the corridor for all $t \in \{0, \frac{T}{M}, \frac{2T}{M}, \ldots T\}$. Could we write that constraint as a *linear function* of the decision variables?

- Yes, there is such a linear constraint
- No, that constraint is not linear

Suppose we had the task of ensuring that the trailer *and car* stayed within the corridor for all $t \in [0, T]$. Could we write that constraint as a *linear function* of the decision variables?

- Yes, there is such a linear constraint
- No, that constraint is not linear

For some fixed integer M, suppose we had the task of ensuring that the trailer *and car* stayed within the corridor for all $t \in \{0, \frac{T}{M}, \frac{2T}{M}, \ldots T\}$. Could we write that constraint as a *linear function* of the decision variables?

- Yes, there is such a linear constraint
- No, that constraint is not linear

Submit You have used 0 of 1 attempt