OPTIONAL: LQR with discounting

LQR with discounting

0.0/10.0 points (ungraded)

In this problem, we will consider a variation on the continuous Linear Quadratic Regular introduced in class.

Consider a discrete-time linear system: $x[k+1] = Ax[k] + Bu[k]$. Suppose we are more interested in the near-term behavior of the system, so we introduce a discount factor into the cost. The cost over a finite horizon of N total steps can be written as

$$\sum_{k=0}^{N} \alpha^k \left(x^T Q x + u^T R u\right)$$

Note that, because of the discount factor, we can expect the cost to be dependent on both x and k. You are given that the cost-to-go at time k will be given by $J_k(x[k]) = x[k]^T S_k x[k]$. We will use the discrete-time version of the HJB equation to derive the Riccati equation describing the optimal cost-to-go matrix S_0.

The applicable version of the HJB equation to this problem is given below:

$$J_k(x[k]) = \min_{u[k]} g_k(x[k], u[k]) + J_{k+1}(x[k+1])$$

Substitute the given terms for g_k, J_{k+1} and $x[k+1]$ and solve for the optimal controller in terms of $\alpha, k, R, B, S_{k+1}, A$ and x. Please leave the template code in the solution, and fill in the expression for u.

```
alpha = .85;
k = randi(100);
R = diag(rand(2,1));
B = randn(4,2);
A = randn(4);
S_k1 = 100*diag(rand(4,1));
```

```
x = randn(4,1);
% your solution to u below
u = ;

```

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LQR with discounting, part 2

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Substitute your solution to the previous part into the HJB equation and find the algebraic Riccati equation relating S_{k+1} and S_k

Substitute the given terms for g_k, J_{k+1} and $x\left[k+1\right]$ and solve for the optimal controller in terms of α, k, R, B, S_{k+1} and A.

While the derivation will look messy at first, it should simplify to the form $S_k = \alpha^k Q - M + A^T S_{k+1} A$ for some invertible matrix M (note the signs in the previous expression).

As with the previous part, please leave the template code in the solution, and fill in the expression for M.

```
alpha = .85;
k = randi(100);
R = diag(rand(2,1));
B = randn(4,2);
A = randn(4);
S_k1 = 100*diag(rand(4,1));
% your solution to u below
M = ;

```

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LQR with discounting, part 3

0.0/7.0 points (ungraded)

This finite-horizon discrete LQR can be easily solved backwards in time, given a choice for the final cost S_N.

Suppose we have the system:

$$A = \begin{bmatrix} 2 & 1.5 \\ -3 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

And choose costs:

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = 1, \qquad S_N = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

With $\alpha = .85$ and $N = 10$, find S_0;

```
alpha = .85;
N = 10;
A = [2 1.5; -3 2];
B = [0;1];
Q = diag([5;1]);
R = 1;
S_N = 10*eye(2);
S_0 =
```

Unanswered

Run Code

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