Lyapunov Functions

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0.0/20.0 points (graded)

This question will test your understanding of Lyapunov functions with a series of short questions.

(a) Suppose that $V_1(x)$ and $V_2(x)$ are valid Lyapunov functions that prove global stability of a system to the origin. Is it always the case that $V_2(x) = cV_1(x)$ for some $c > 0$? In other words, are Lyapunov functions unique up to scaling?

- Yes
- No

(b) For the system:

$$\dot{x_1} = -\frac{6x_1}{(1+x_1^2)^2} + 2x_2$$
$$\dot{x_2} = -\frac{2(x_1+x_2)}{(1+x_1^2)^2}$$

you are given the positive definite function $V(x) = \frac{x_1^2}{1+x_1^2} + x_2^2$ and told that, for this system, $\dot{V}$ is negative definite over the entire space. Is V a valid Lyapunov function which proves global asymptotic stability to the origin for the system? (Hint: Try simulating a few trajectories of this system or plotting a few level sets of V to build more intuition before answering this problem).

- Yes
- No

(c) Suppose $V(x)$ is a valid Lyapunov function that proves global asymptotic stability of a system to the origin. Is it true that $(V(x))^2$ is also a valid Lyapunov function?

- Yes
- No

(d) Suppose $V(x)$ is a valid Lyapunov function that proves global asymptotic stability of a system to the origin. Is it true that $\tanh(V(x))$ is also a valid Lyapunov function?

- Yes
- No

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