Non-existence of Limit Cycles

Non-existence of limit cycles (Part A)

0.0/10.0 points (graded)

A **gradient system** is a system of the following form:

$$\dot{x} = -\nabla V(x) \quad (1),$$

for some twice continuously differentiable scalar-valued function $V(x)$.

It is straight-forward to prove that gradient systems don't have any periodic orbits (we are not counting fixed-points as periodic orbits here, and we assume that $V(x)$ is not the 0 function). We can use a proof by contradiction in order to show this. Suppose we had a periodic orbit. Consider the change in V after one circuit around the periodic orbit. Denote this change as ΔV. For a periodic orbit, we must have $\Delta V = 0$. However, we also have:

$$\Delta V = \int_0^T \dot{V}(x(t))\, dt = \int_0^T \nabla V(x(t)) \cdot (-\nabla V(x(t)))\, dt = \int_0^T -\|\nabla V(x(t))\|^2 dt < 0.$$

This is a contradiction.

(a) Which of the following are gradient systems?

- [ ] All linear systems $\dot{x} = Ax$ with $A = A^T$. ✔
- [ ] All linear systems $\dot{x} = Ax$.
- [ ] All one-dimensional systems $\dot{x} = f(x)$, where $f(x)$ is continuously differentiable. ✔
- [ ] All globally asymptotically stable systems.

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All linear systems of the form $\dot{x} = Ax$ with $A = A^T$ are gradient systems with $V = 0.5x^T Ax$. Linear systems with $A \neq A^T$ are not generally gradient systems. To see this, consider the two dimensional system:

$$\begin{aligned}\dot{x}_1 &= ax_1 + bx_2 := f(x_1, x_2)\\ \dot{x}_2 &= cx_1 + dx_2 := g(x_1, x_2).\end{aligned}$$

Recall from your multi-variate calculus course that a two-dimensional system is a gradient system if and only if the following holds:

$$\frac{\partial}{\partial x_2} f(x_1, x_2) = \frac{\partial}{\partial x_1} g(x_1, x_2).$$

Now, if $b \neq c$, this condition will not hold.
All one-dimensional systems $\dot{x} = f(x)$ are gradient systems since we can set $V(x) = -\int f(x)\,dx$. Globally asymptotically stable systems are not generally gradient systems. Consider a stable linear system $\dot{x} = Ax$ with $A \neq A^T$ for example.

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Answers are displayed within the problem

Non-existence of limit cycles (Part B)

0.0/10.0 points (graded)
(b) Consider the system given by

$$\begin{aligned}\dot{x} &= y + 2xy\\ \dot{y} &= x + x^2 - y^2.\end{aligned}$$

Prove that this system does not have a periodic orbit by finding a function $V(x, y)$ such that the system is of the form (1) above. Type in $V(x, y)$ below (make sure it has the correct sign!).

```
syms x y real;
V = ; % Type in your answer here in terms of x and y

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Unanswered

```
syms x y real;

% As is easily verified by taking gradients, one possible V is:
V = -x*y - x^2*y + (1/3)*y^3
```

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