edX

Course › Week 2 › Proble... › Cost fu...

Cost functions

Cost functions (Part a)

0.0/5.0 points (graded)

In this problem we will explore how to design cost functions that make the robot exihibit the kind of behavior we want. For this, we will consider the Dubins car model, which is a very simple model of a vehicle given by the following equations:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}, \qquad \dot{\mathbf{x}} = f(\mathbf{x}, u) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\sin\psi \\ \cos\psi \\ u \end{bmatrix},$$

where $\mathbf{x}$ is the state of the system and consists of the states x (the x-position), y (the y-position) and ψ (the yaw angle of the vehicle). The control input is u.

(a) For optimal control problems, it is often useful to have cost functions of the form:

$$J = \mathbf{x}(t_f)^T Q_f \mathbf{x}(t_f) + \int_0^{t_f} g(t, \mathbf{x}(t))\, dt$$

where the first term depends only on where the robot ends up (the second term is simply the additive cost structure we saw in lecture). Here, t_f is the final time and Q_f is a symmetric positive semidefinite matrix. Suppose we want the robot to end up with its yaw angle close to 0, but do not care about the final x and y positions. What should we choose Q_f to be (remember to make sure it is symmetric and positive semidefinite)?

```
Qf =

```

Unanswered

```
% Since we don't care about the final x and y positions,
% Qf must have zeros everywhere except the (3,3) element.
% Since we want to penalize deviations of the yaw angles
% from 0 (and since our Qf must be p.s.d.), the (3,3) element
% must be positive. So, one possible answer is:

Qf = [0 0 0;0 0 0;0 0 1];
```

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Answers are displayed within the problem

Cost functions (Part b)

0.0/5.0 points (graded)

(b) Now suppose we want the vehicle to end up close to the line $y = 0.5x$, but we do not care exactly where on this line and what yaw angle it ends up in. What should we choose Q_f to be (remember to make sure it is symmetric and positive semidefinite)?

```
Qf =

```

Unanswered

```
% Since we don't care about the yaw angle, the third row and column
% must be all zeros. Hence, we only need to figure out the top left 2 by 2
% block. One possible cost function x'*Qf*x for penalizing deviations from
% the line y = 0.5 x is (y - 0.5x)^2. Thus, we can make x'*Q*x match this
% function by picking the top left 2 by 2 block correctly. We find that the
% following Qf achieves this:

Qf = [0.25 -0.5 0;-0.5 1 0;0 0 0];
```

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Answers are displayed within the problem

Cost functions (Part c)

0.0/5.0 points (graded)

(c) Now suppose we want to end up close to the curve $y = x^2$, and again do not care about the final yaw angle or where exactly on this curve we end up. Is it possible to achieve this given our setup?

- Yes
- No ✔

Explanation

One way to see it is to notice that $\mathbf{x^T Q_f x}$ can only be 0 along eigenvectors corresponding to 0 eigenvalues. Hence, it can only be 0 at the origin, or along straight lines, or along a plane. Thus, it cannot be 0 on the curve $y = x^2$.

Another way to see that this is not possible is to notice that $\mathbf{x^T Q_f x}$ is a (positive semidefinite) quadratic function of $\mathbf{x}$. Level-sets of positive semidefinite quadratic functions are either ellipsoids or straight lines (which are just "degenerate" ellipsoids). However, we want the cost to be 0 along the curve $y = x^2$ (which does not define an ellipsoid or a straight line).

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Answers are displayed within the problem