OPTIONAL: LQR with discounting

LQR with discounting

0.0/10.0 points (ungraded)

In this problem, we will consider a variation on the continuous Linear Quadratic Regular introduced in class.

Consider a discrete-time linear system: $x\left[k+1\right]=Ax\left[k\right]+Bu\left[k\right]$. Suppose we are more interested in the near-term behavior of the system, so we introduce a discount factor into the cost. The cost over a finite horizon of N total steps can be written as

$$\sum_{k=0}^{N}\alpha^{k}\left(x^{T}Qx+u^{T}Ru\right)$$

Note that, because of the discount factor, we can expect the cost to be dependent on both x and k. You are given that the cost-to-go at time k will be given by $J_k\left(x\left[k\right]\right)=x[k]^{T}S_{k}x\left[k\right]$. We will use the discrete-time version of the HJB equation to derive the Riccati equation describing the optimal cost-to-go matrix S_0.

The applicable version of the HJB equation to this problem is given below:

$$J_k\left(x\left[k\right]\right)=\min_{u[k]} g_k\left(x\left[k\right],u\left[k\right]\right)+J_{k+1}\left(x\left[k+1\right]\right)$$

Substitute the given terms for g_k, J_{k+1} and $x\left[k+1\right]$ and solve for the optimal controller in terms of $\alpha, k, R, B, S_{k+1}, A$ and x. Please leave the template code in the solution, and fill in the expression for u.

```
alpha = .85;
k = randi(100);
R = diag(rand(2,1));
B = randn(4,2);
A = randn(4);
S_k1 = 100*diag(rand(4,1));
x = randn(4,1);
```

```
% your solution to u below
u = ;
```

Unanswered

```
alpha = .85;
k = randi(100);
R = diag(rand(2,1));
B = randn(4,2);
A = randn(4);
S_k1 = 100*diag(rand(4,1));
x = randn(4,1);

u = -inv(alpha^k * R + B'*S_k1*B)*B'*S_k1*A*x;
```

Run Code

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

LQR with discounting, part 2

0.0/10.0 points (ungraded)

Substitute your solution to the previous part into the HJB equation and find the algebraic Riccati equation relating S_{k+1} and S_k

Substitute the given terms for g_k, J_{k+1} and $x\left[k+1\right]$ and solve for the optimal controller in terms of α, k, R, B, S_{k+1} and A.

While the derivation will look messy at first, it should simplify to the form $S_k = \alpha^k Q - M + A^T S_{k+1} A$ for some invertible matrix M (note the signs in the previous expression).

As with the previous part, please leave the template code in the solution, and fill in the expression for M.

```
alpha = .85;
k = randi(100);
R = diag(rand(2,1));
B = randn(4,2);
A = randn(4);
S_k1 = 100*diag(rand(4,1));
% your solution to u below
M = ;

```

Unanswered

```
alpha = .85;
k = randi(100);
R = diag(rand(2,1));
B = randn(4,2);
A = randn(4);
S_k1 = 100*diag(rand(4,1));

M = A'*S_k1*B*inv(alpha^k*R + B'*S_k1*B)*B'*S_k1*A;
```

Run Code

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

LQR with discounting, part 3

0.0/7.0 points (ungraded)

This finite-horizon discrete LQR can be easily solved backwards in time, given a choice for the final cost S_N.

Suppose we have the system:

$$A = \begin{bmatrix} 2 & 1.5 \\ -3 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

And choose costs:

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = 1, \qquad S_N = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

With $\alpha = .85$ and $N = 10$, find S_0;

```
alpha = .85;
N = 10;
A = [2 1.5; -3 2];
B = [0;1];
Q = diag([5;1]);
R = 1;
S_N = 10*eye(2);
S_0 =

```

Unanswered

```
alpha = .85;
N = 10;
A = [2 1.5; -3 2];
B = [0;1];
Q = diag([5;1]);
R = 1;
S_N = 10*eye(2);

S{N+1} = S_N;
for k=N-1:-1:0,
  S_k1 = S{k+2};
  M = A'*S_k1*B*inv(alpha^k*R+B'*S_k1*B)*B'*S_k1*A;
   S{k+1} = alpha^k*Q - M + A'*S_k1*A;
end
S_0 = S{1}
```

Run Code

Submit You have used 0 of 3 attempts

Answers are displayed within the problem