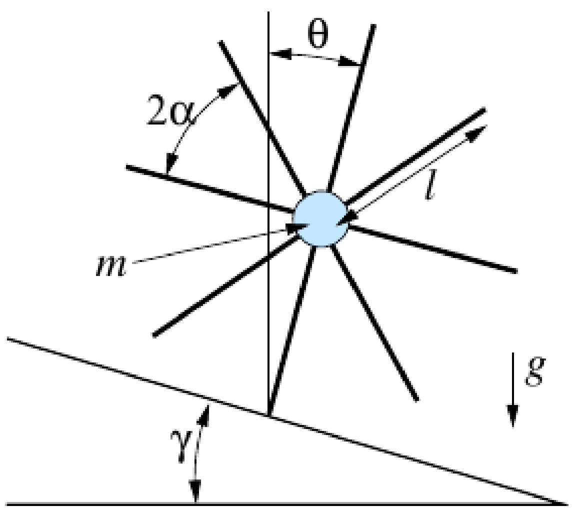
Course > Week 6 > Proble... > Rimles...

Rimless Wheel

The Rimless Wheel (Passive Dynamics I)

0.0/8.0 points (graded)

For this problem, you should be able to compute all of the quantities in closed form (no numerical simulations should be necessary).



(a) If we assume that the wheel is started in a configuration directly after a transfer of support, then forward walking occurs (i.e., the system takes a step) when the system has an initial velocity $\dot{\theta}(0^+) > \omega_1$. When the next foot touches down the conversion of potential energy into kinetic energy yields the velocity before touchdown $\dot{\theta}(t^-)$. Derive the expressions for ω_1 and $\dot{\theta}(t^-)$ in terms of $g, l, \alpha, \gamma, \dot{\theta}(0^+)$. Type in your answers below. Please write α, γ, and $\dot{\theta}(0^+)$ as "alpha", "gamma" and "thdot0" respectively.

$\omega_1 =$

Answer: sqrt((2*g/l)*(1 - cos(alpha - gamma)))

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$\dot{\theta}\left(t^{-}\right) =$

Answer: sqrt(thdot0^2 + (4*g/l)*sin(alpha)*sin(gamma))

Explanation
Using conservation of energy, we get:

$$\frac{1}{2} m l^{2} \dot{\theta}^{2}\left(0^{+}\right)+2 m g l \sin (\alpha) \sin (\gamma)=\frac{1}{2} m l^{2} \dot{\theta}^{2}\left(t^{-}\right).$$

Then we get:

$$\dot{\theta}\left(t^{-}\right)=\sqrt{\dot{\theta}^{2}\left(0^{+}\right)+\frac{4 g}{l} \sin (\alpha) \sin (\gamma)}.$$

Letting $\omega_1 = \dot{\theta}\left(0^{+}\right)$, when $\theta = 0$ and $\dot{\theta} = 0$, we have:

$$\frac{1}{2} m l^{2} \omega_{1}^{2}=m g l(1-\cos (\alpha-\gamma)).$$

Thus, we get:

$$\omega_{1}=\sqrt{\frac{2 g}{l}(1-\cos (\alpha-\gamma))}$$

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

The Rimless Wheel (Passive Dynamics II)

0.0/2.0 points (graded)
(b) Write down a condition of the form $\gamma > f(\alpha, g, l)$ such that ω_1 is non-existent when this condition holds. Type in your answer for $f(\alpha\, g, l)$ below (as before, write α as "alpha"\).
Think about what this condition means physically.

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Answer: alpha

Explanation

When $\gamma = \alpha$, with an arbitrary small initial speed, the rimless wheel will always roll down by gaining potential energy. When $\gamma > \alpha$, the rimless wheel will roll down due to gravity even when it starts with zero velocity.

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Answers are displayed within the problem

The Rimless Wheel (Gait Design)

0.0/7.0 points (graded)

(c) Consider a rimless wheel with $m = 1$ kg, $g = 9.8$ m/s^2, $\alpha = \frac{\pi}{8}$ rad, and $\gamma = 0.08$ rad. For what leg length l, is the steady-state rolling speed of the system (measured when the pendulum is vertical over the stance feet: $\theta = 0$) equal to 4.0 rad/s? **The tolerance for this answer will be 0.001.**

Answer: 0.0155

Explanation

In the steady-state rolling condition, $\dot{\theta}\left(t^{-}\right)\cos\left(2\alpha\right) = \dot{\theta}\left(0^{+}\right)$. Using the result from part (a),

$$\dot{\theta}\left(0^{+}\right) = \cot\left(2\alpha\right)\sqrt{\frac{4g}{l}\sin\left(\alpha\right)\sin\left(\gamma\right)}.$$

At $\theta = 0$, we get:

$$\dot{\theta} = \sqrt{\dot{\theta}^{2}\left(0^{+}\right) - \frac{2g}{l}\left(1 - \cos\left(\alpha - \gamma\right)\right)}$$

Then we get:

$$l = \frac{4g\sin\left(\alpha\right)\sin\left(\gamma\right)\cot^{2}\left(2\alpha\right) - 2g\left(1 - \cos\left(\alpha - \gamma\right)\right)}{\dot{\theta}^{2}} = 0.0155.$$

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Answers are displayed within the problem

The Rimless Wheel (Terrain Stability)

0.0/8.0 points (graded)

(d) Consider a rimless wheel with $m = 1$ kg, $l = 1$ m, $g = 9.8$ m/s^2, and $\alpha = \frac{\pi}{8}$ rad rolling down a slope, $\gamma = 0.16$ rad. Some time after reaching the rolling steady state, the terrain shallows to $\gamma = 0.02$ rad for a distance d, then returns to (and remains at) $\gamma = 0.08$ rad. You may assume that the shallow slope begins and ends precisely at a position of touchdown (not somewhere between steps). What is the largest value of d, subject to these constraints, for which the system will be at a rolling fixed point when $\gamma = 0.08$ as $t \to \infty$? **The tolerance for this answer will be 0.001.**

Answer: 0.7654

Explanation

When $\gamma = 0.08$ rad, the steady state speed is 1.0949. Also, $\omega_1 = 0.9749$. When the rimless wheel starts with $\dot{\theta}\left(0^+\right) = \omega_1$, after the first collision, the velocity $\dot{\theta}\left(t^+\right) = 1.0366 > \omega_1$, so as long as the rimless wheel starts at a velocity greater than ω_1, it will finally reach the steady state rolling condition. When $\gamma = 0.02$ rad, the rimless wheel starts with $\dot{\theta} = 1.5460$, after the first collision, $\dot{\theta} = 1.1597 > \omega_1$, after the second collision, $\dot{\theta} = 0.9069 < \omega_1$, so only one collision is allowable and $d = 2l\sin\left(\alpha\right) = 0.7654$.

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You have used 0 of 2 attempts

Answers are displayed within the problem

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