

Course > Week 6 > Proble... > Non-ex...

Non-existence of Limit Cycles

Non-existence of limit cycles (Part A)

0.0/10.0 points (graded)

A **gradient system** is a system of the following form:

$$\dot{x} = -\nabla V(x)$$
 (1),

for some twice continuously differentiable scalar-valued function $V\left(x\right)$.

It is straight-forward to prove that gradient systems don't have any periodic orbits (we are not counting fixed-points as periodic orbits here, and we assume that $V\left(x\right)$ is not the 0 function). We can use a proof by contradiction in order to show this. Suppose we had a periodic orbit. Consider the change in V after one circuit around the periodic orbit. Denote this change as ΔV . For a periodic orbit, we must have $\Delta V=0$. However, we also have:

$$\Delta V = \int_{0}^{T} \dot{V}\left(x\left(t
ight)
ight) dt = \int_{0}^{T}
abla V\left(x\left(t
ight)
ight) \cdot \left(-
abla V\left(x\left(t
ight)
ight)
ight) dt = \int_{0}^{T} -\left\|
abla V\left(x\left(t
ight)
ight)
ight\|^{2} dt < 0.$$

This is a contradiction.

- (a) Which of the following are gradient systems?
 - lacksquare All linear systems $\dot{m{x}} = Am{x}$ with $m{A} = A^T$. lacksquare
 - \square All linear systems $\dot{x}=Ax$.
 - lacksquare All one-dimensional systems $\dot{x}=f(x)$, where f(x) is continuously differentiable. lacksquare
 - All globally asymptotically stable systems.

Generating Speech Output

All linear systems of the form $\dot{x}=Ax$ with $A=A^T$ are gradient systems with $V=0.5x^TAx$. Linear systems with $A\neq A^T$ are not generally gradient systems. To see this, consider the two dimensional system:

$$egin{aligned} \dot{x}_1 &= ax_1 + bx_2 := f\left(x_1, x_2
ight) \ \dot{x}_2 &= cx_1 + dx_2 := g\left(x_1, x_2
ight). \end{aligned}$$

Recall from your multi-variate calculus course that a two-dimensional system is a gradient system if and only if the following holds:

$$rac{\partial}{\partial x_{2}}f\left(x_{1},x_{2}
ight)=rac{\partial}{\partial x_{1}}g\left(x_{1},x_{2}
ight).$$

Now, if $b \neq c$, this condition will not hold.

All one-dimensional systems $\dot{x}=f(x)$ are gradient systems since we can set $V(x)=-\int f(x)\,dx$. Globally asymptotically stable systems are not generally gradient systems. Consider a stable linear system $\dot{x}=Ax$ with $A\neq A^T$ for example.

Submit

You have used 0 of 1 attempt

1 Answers are displayed within the problem

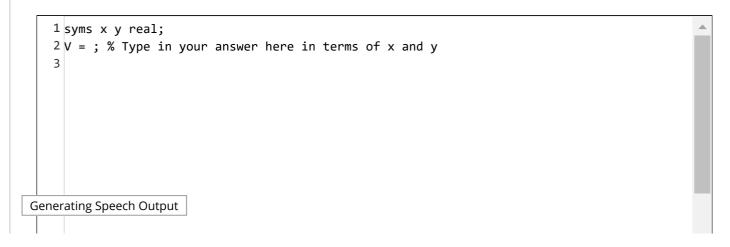
Non-existence of limit cycles (Part B)

0.0/10.0 points (graded)

(b) Consider the system given by

$$\dot{x}=y+2xy \ \dot{y}=x+x^2-y^2.$$

Prove that this system does not have a periodic orbit by finding a function V(x,y) such that the system is of the form (1) above. Type in V(x,y) below (make sure it has the correct sign!).



Unanswered

```
syms x y real;
% As is easily verified by taking gradients, one possible V is:
V = -x*y - x^2*y + (1/3)*y^3
```

Run Code

Submit

You have used 0 of 2 attempts

1 Answers are displayed within the problem

© All Rights Reserved

Generating Speech Output