Limit Cycles

Limit Cycles

0.0/20.0 points (graded)

This question will test your understanding of limit cycles and Poincare analysis with a series of short questions.

(a) Which of the following statements about Poincare analysis are true? Select all that apply.

- [ ] A Poincare section is always two dimensional.
- [ ] The Poincare section must be perpendicular to the periodic orbit under consideration.
- [ ] A periodic orbit is (locally orbitally) stable if the Poincare return map is (locally) asymptotically stable. ✔
- [ ] The Poincare return map is (locally) asymptotically stable if the periodic orbit is (locally orbitally) stable. ✔
- [ ] The linearization of the Poincare return map always has one eigenvalue equal to 0. ✔

Explanation

The first choice is incorrect since a Poincare section is $n-1$ dimensional in general. The second choice is incorrect since a Poincare section need only be transverse to the periodic orbit (and not necessarily perpendicular). The third and fourth choices are correct (as discussed in class). The last choice is also correct since perturbations *along* the limit cycle lead to no change in the point at which the trajectory returns to the Poincare section. Hence, perturbations along the trajectory correspond to a 0 eigenvalue of the linearization of the Poincare map.

(b) Which of the following is a limit cycle? Select all that apply.

- [ ] The unstable fixed point corresponding to the pendulum in the upright configuration.
- [ ] The periodic orbit of the Van der Pol oscillator. ✔
- [ ] A periodic orbit of the undamped pendulum, e.g., the periodic orbit with constant energy $E > 0$.
- [ ] The homoclinic orbit of the undamped pendulum.
- [ ] The periodic orbit corresponding to the rimless wheel rolling down a gentle ramp. ✔

Explanation

A limit cycle is a (stable or unstable, but not marginally stable) periodic orbit, where the period must be a real number that is *strictly* greater than 0. Thus, a fixed point is not a limit cycle (since the period is not strictly greater than 0). The periodic orbit of the Van der Pol oscillator is a limit cycle, as discussed in class. Both the homoclinic orbit and a periodic orbit of the undampled pendulum are not limit cycles since they are marginally stable. The rimless wheel's periodic orbit is a stable periodic orbit, and hence a limit cycle.

(c) Suppose we have a system given by:

$$\dot{x} = f(x),$$

with $x \in \mathbb{R}$, i.e., a one dimensional system. Is it possible for this system to have a limit cycle?

- ○ Yes
- ○ No ✔

Explanation

One dimensional (non time-varying) systems cannot have limit cycles. This is easy to see. Suppose we did have a limit cycle that started off at x_0, went to x_1 and returned to x_0 (here $x_0 < x_1$). Let x_m be a point between x_0 and x_1. Then the vector field at x_0 points to the right when the system is moving from x_0 to x_1 and points to the left when the system is going back from x_1 to x_0. Since the system is not time-varying, this is impossible and we have a contradiction.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem