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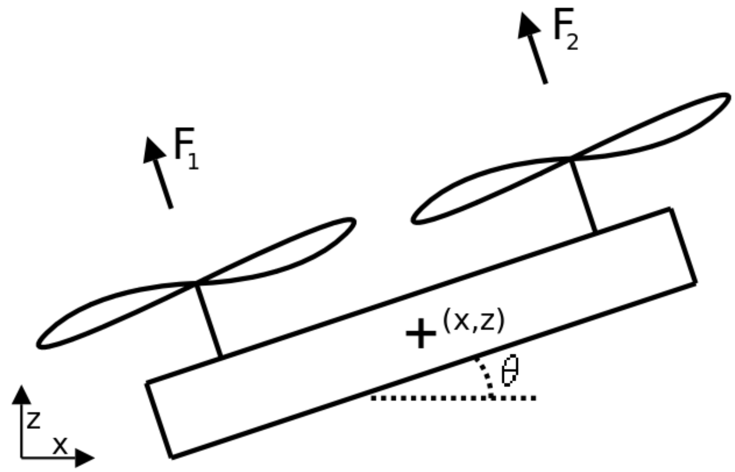
Problem: Definition of Underactuated

Definition of Underactuated

0.0/25.0 points (graded)

The following problem explores the definition of underactuated as described in the lecture notes. You should not need to derived detailed equations of motion for any of these problems.

Helicopter



A helicopter with two rotors is constrained to move in a vertical plane. Assume gravity acting on the helicopter. The task is to control the position (x, z) and pitch (θ) by varying the thrust produced by the two rotors. Decide whether this system is fully-actuated (in all states), or if there are any states in which the problem is underactuated. Use the definition of underactuated provided in lecture.

- Fully-actuated
- Underactuated

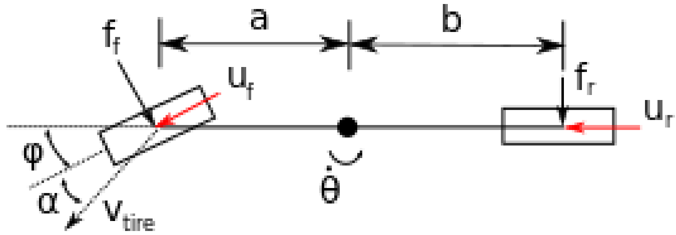
If you said "Underactuated" above, then please provide an expression for an acceleration that cannot be instantaneously achieved by the system. Assume that F_1 and F_2 are unbounded (and can be negative) and that the current state of the system is $x = 5, z = 1, \theta = 0.5$ radians. Your answer should consist of three **numerical values**, in the order $\ddot{x}\ \ddot{y}\ \ddot{\theta}$

xdd

ydd

thetadd

Bicycle Model



Consider the simple model of a vehicle known as the bicycle model, illustrated above. Let x, y be the position of the vehicle in inertial coordinates and θ be the heading angle. Lateral tire forces are typically modeled as being proportional to the lateral slip angle α, which defines the angle between the angle of the tire and the velocity of the tire v_{tire}, which depends on the speed and angular velocity of the vehicle, giving $f = C\alpha$.

Generating the equations of motion can be tedious, and we will often use a software package to do it automatically. For this problem, we have done it for you--but, as is often the case, the equations are pretty messy! One way to write the system of equations is:

$$\begin{aligned}
f_r &= C_r \arctan\left(\frac{\dot{y}\cos\theta - \dot{x}\sin\theta - \dot{\theta}b}{\dot{x}\cos\theta + \dot{y}\sin\theta}\right) \\
f_f &= C_f\left(\arctan\left(\frac{\dot{y}\cos\theta - \dot{x}\sin\theta + \dot{\theta}a}{\dot{x}\cos\theta + \dot{y}\sin\theta}\right) - \phi\right) \\
I\ddot{\theta} &= -bf_r + a\left(f_f\cos\phi + u_f\sin\phi\right) \\
m\ddot{x} &= -f_r\sin\theta - f_f\sin(\theta+\phi) + u_r\cos\theta + u_f\cos(\theta+\phi) \\
m\ddot{y} &= f_r\cos\theta + f_f\cos(\theta+\phi) + u_r\sin\theta + u_f\sin(\theta+\phi)
\end{aligned}$$

For the purposes of this problem, assume that the driver has control over the steering angle ϕ and has rear wheel drive. Treat the drive torque as a simple ground reaction force u_r acting at the tire and let $u_f = 0$. Is this system is fully-actuated or underactuated? Explain.

- Fully-actuated
- Underactuated

Now, suppose the the driver has control of both the front and rear longitudinal tire forces u_r and u_f and so has 3 total control inputs. In general, do you think this system is fully-actuated or underactuated? Give an intuitive explanation.

- Fully-actuated
- Underactuated

Since these dynamics are not control affine, consider a simplified system of equations linearized about $\phi = \phi_0$ and $u_f, u_r = 0$. For simplicity, without loss of generality, let $\theta = 0$. Recalling the general dynamics form,

$\ddot{q} = f_1(q, \dot{q}, t) + f_2(q, \dot{q}, t)\, u$ We can write:

$$f_2(q, \dot{q}, t) = \begin{bmatrix} -aC_f\left(\left(\arctan\left(\frac{\dot{y}+\dot{\theta}a}{\dot{x}}\right) - \phi_0\right)\sin(\phi_0) + \cos(\phi_0)\right) & 0 & a\sin(\phi_0) \\ -C_f\left(\left(\arctan\left(\frac{\dot{y}+\dot{\theta}a}{\dot{x}}\right) - \phi_0\right)\cos(\phi_0) - \sin(\phi_0)\right) & 1 & \cos(\phi_0) \\ -C_f\left(\left(\arctan\left(\frac{\dot{y}+\dot{\theta}a}{\dot{x}}\right) - \phi_0\right)\sin(\phi_0) + \cos(\phi_0)\right) & 0 & \sin(\phi_0) \end{bmatrix}$$

Find the rank of f_2 when $\phi_0 = 0$.

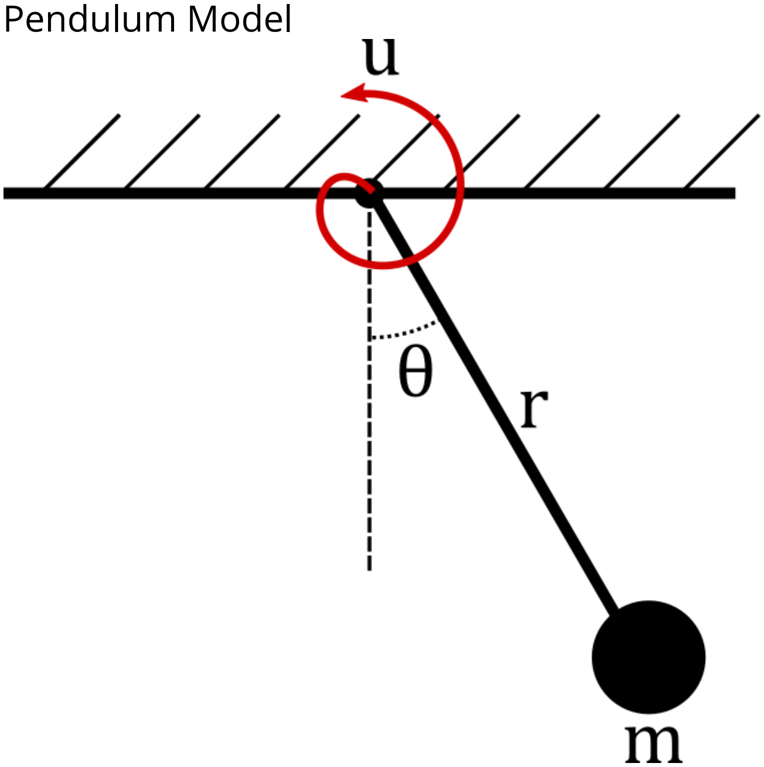
Is the system fully-actuated or underactuated?

- Fully-actuated
- Underactuated

Are there values for ϕ_0 for which the system is fully-actuated? If so, write a symbolic expression of the form $g(\phi)$ where $g = 0$ describes these states. If there are no such states, enter 0. For grading purposes, ensure that $g(1) \geq 0$. For example, if the system is fully-actuated whenever $\phi_0 = 2$, write $g = 2$ - phi_0.

$g =$

Pendulum Model



Consider a pendulum with length $r > 0$, mass $m > 0$, and a single degree of freedom, θ. Let u be the torque applied to the pendulum, which will be our input to the system. Which of the following is/are true? (if none are true, then don't check any boxes).

- [ ] a. The system is underactuated.
- [ ] b. The system is underactuated if u is bounded.
- [ ] c. The system is underactuated if $\frac{\pi}{2} \leq \theta \leq \pi$
- [ ] d. The system is underactuated only when $\theta = \pm\pi$

Submit

You have used 0 of 1 attempt