Controllability and Stability

Stability and Controllability

0.0/18.0 points (graded)

For this problem, answer a series of questions related to some of the basic notions discussed in lecture.

Cubic Plant

Consider the one-dimensional system $\dot{x} = -x^3$. Using graphical analysis, is the trivial equilibrium stable or unstable?

- stable ✔
- unstable
- cannot tell

What is the eigenvalue of the linearized system?

Answer: 0

Does the linearization indicate stability?

- stable
- unstable

- cannot tell ✔

Explanation

Graphical analysis shows that $\dot{x} < 0$ for $x > 0$ and $\dot{x} > 0$ for $x < 0$, so the equilibrium at the origin is stable. However, the linearization is uninformative, since the eigenvalue of the linearization is zero.

Controllability and Underactuation

Select any true statement.

- [ ] A system that is controllable is fully-actuated.
- [ ] A system that is fully-actuated is controllable. ✔

Explanation

Fully-acutated sytems are trivial examples of controllable systems. However, many underactuated systems (some of which we will see in the course) are also controllable!

Linear Quadratic Regulators

Suppose we have a controllable system, where the linearization is $\dot{x} = Ax + Bu$. For $Q \succ 0$ and $R \succ 0$, we can find the LQR controller with feedback gain matrix K.

What can we say about the eigenvalues of A?

- All eigenvalues have negative real part
- Some eigenvalues have a positive real part
- It depends on the system ✔

What can we say about the eigenvalues of $A - BK$?

- All eigenvalues have negative real part ✔

- Some eigenvalues have a positive real part
- It depends on the system

Explanation

Controllability, by itself, does not say anything about the stability of the system (or the eigenvalues of A). However, LQR is guaranteed to stabilize any controllable system. Therefore, the closed-loop dynamics $\dot{x} = (A - BK)\,x$ are stable.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem