Course › Week 5 › Proble... › Traject...

Trajectory Optimization

Trajectory Optimization

0.0/20.0 points (graded)

Suppose we have a trajectory optimization program for the Cart-Pole (so $x \in \mathbb{R}^4$ and $u \in \mathbb{R}$). For this problem, we will examine both shooting and direct transcription methods examined in class. We will suppose that both methods use forward Euler integration, that is:

$$x_{k+1} = x_k + hf(x_k, u_k)$$

for some fixed time step h. The programs will both be created as follows:

- The initial state, x_0 is fixed (and not a decision parameter).
- The final state, x_{20} is constrained to be in some goal region, that is $f_g(x_{20}) \geq 0$, where f_g is scalar valued.
- There is an obstacle which the states cannot penetrate, that is, $f_o(x_k) \geq 0$, where f_o is scalar valued.
- It is possible that the goal and obstacle regions have a non-empty intersection
- The final time and time step h are fixed.
- There are no torque limits, but the total cost is $\sum_k u_k^2$.
- Numbers of "decision variables" are counted as scalars. So if x_{10} were included in the decision variables, it would count as four.

Decision Variables

(a) How many decision variables does the *shooting* approach have?

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(b) How many decision variables does the *direct transcription* approach have?

Constraints

(c) How many constraints does the *shooting* approach have? Convert vector-valued constraints to multiple scalar-valued constraints to count them.

(d) How many constraints does the *direct transcription* approach have? Convert vector-valued constraints to multiple scalar-valued constraints to count them.

Gradient Sparsity

Consider the constraint related to the goal region, $f_g(x_{20}) \geq 0$. For most numerical optimization approaches, it is important to calculate the gradient of the constraints and cost with respect to the decision variables. That is, if z is the list of all decision variables, we need to compute $\frac{\partial f_g(x_{20})}{\partial z}$, which will be a vector of length $\mathbf{dim}(z)$.

(e) For the *shooting* approach, how many non-zero entries are there in this gradient vector?

(f) For the *direct transcription* approach, how many non-zero entries are there in this gradient vector?

Gradient Structure

Suppose we write the gradient of all constraints as a matrix G, where the jth element of the ith row is $G_{i,j} = \frac{\partial h_i}{\partial z_j}$ where h_i is the ith constraint. Order both the z vector and the list of constraints h_i by the time index k which they most directly relate to. The structure of G greatly affects the efficiency and accuracy of various optimization approaches. The

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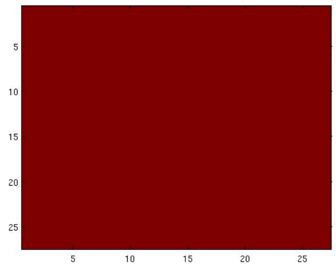
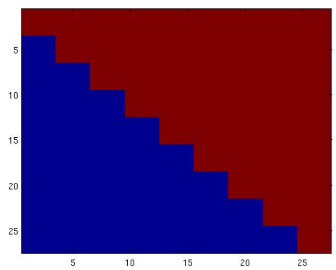
ption are graphical representations of the sparsity patterns, where red indicates the non-zero entries and blue the zero entries.

(g) For the *shooting* approach, what, if any, structure will G have? Note that G will not generally be square, so we use some of these terms loosely here. In particular, the "diagonal" here refers to the elements of G corresponding to the variables and constraints directly related to the same time index.

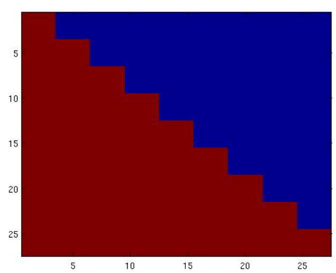
- Dense: many non-zero entries with no particular structure



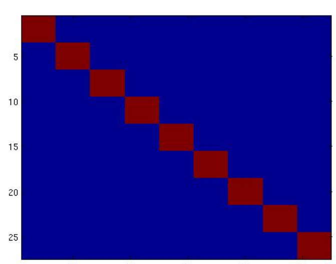
- Upper Triangular: entries below the "diagonal" are all (or nearly all) zero



- Lower Triangular: entries above the "diagonal" are all (or nearly all) zero



- Block Diagonal: entries away from the diagonal are all (or nearly all) zero



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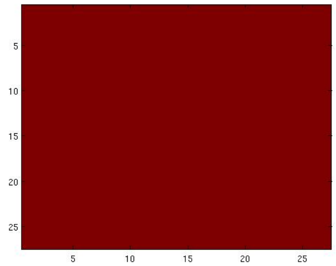
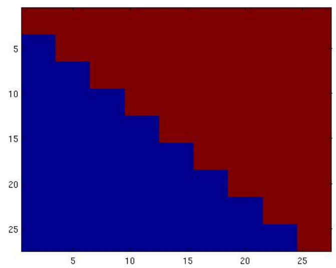
(h) For the *direct transcription* approach, what, if any, structure will G have?

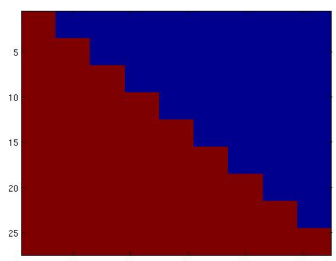
- Dense: many non-zero entries with no particular structure



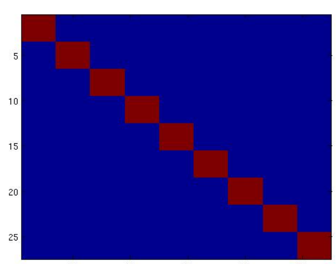
- Upper Triangular: entries below the "diagonal" are all (or nearly all) zero



- Lower Triangular: entries above the "diagonal" are all (or nearly all) zero



- Block Diagonal: entries away from the diagonal are all (or nearly all) zero



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Submit You have used 0 of 1 attempt

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