Trajectory Optimization

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0.0/20.0 points (graded)

Suppose we have a trajectory optimization program for the Cart-Pole (so $x \in \mathbb{R}^4$ and $u \in \mathbb{R}$). For this problem, we will examine both shooting and direct transcription methods examined in class. We will suppose that both methods use forward Euler integration, that is:

$$x_{k+1} = x_k + hf(x_k, u_k)$$

for some fixed time step h. The programs will both be created as follows:

- The initial state, x_0 is fixed (and not a decision parameter).
- The final state, x_{20} is constrained to be in some goal region, that is $f_g(x_{20}) \geq 0$, where f_g is scalar valued.
- There is an obstacle which the states cannot penetrate, that is, $f_o(x_k) \geq 0$, where f_o is scalar valued.
- It is possible that the goal and obstacle regions have a non-empty intersection
- The final time and time step h are fixed.
- There are no torque limits, but the total cost is $\sum_k u_k^2$.
- Numbers of "decision variables" are counted as scalars. So if x_{10} were included in the decision variables, it would count as four.

Decision Variables

(a) How many decision variables does the *shooting* approach have?

Answer: 20

(b) How many decision variables does the *direct transcription* approach have?

Answer: 100

Explanation

In the shooting approach, the decision variables are the control inputs only (20). In the direct transcription approach, the state variables are also decision variables. The problem states that x_0 is fixed and not to be included, so the answer is 20 + 80=100.

Constraints

(c) How many constraints does the *shooting* approach have? Convert vector-valued constraints to multiple scalar-valued constraints to count them.

Answer: 21

(d) How many constraints does the *direct transcription* approach have? Convert vector-valued constraints to multiple scalar-valued constraints to count them.

Answer: 101

Explanation

The constraints for the shooting approach are the obstacle constraints (1 per time step) and the final state constraint, so 21 total. For the direct approach, the dynamics also form a constraint at each time step, giving an additional 80 constraints, for a total of 101.

Gradient Sparsity

Consider the constraint related to the goal region, $f_g(x_{20}) \geq 0$. For most numerical optimization approaches, it is important to calculate the gradient of the constraints and cost with respect to the decision variables. That is, if z is the list of all decision variables, we need to compute $\frac{\partial f_g(x_{20})}{\partial z}$, which will be a vector of length $\mathbf{dim}(z)$.

(e) For the *shooting* approach, how many non-zero entries are there in this gradient vector?

Answer: 20

(f) For the *direct transcription* approach, how many non-zero entries are there in this gradient vector?

Answer: 4

Explanation

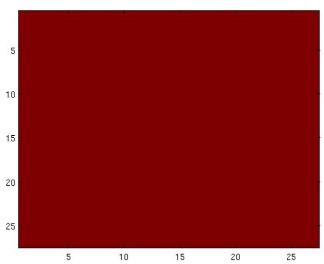
For the shooting approach, x_{20} depends on all of the control inputs u_k, since it is found by forward simulation. That means all 20 entries in the gradient vector will be non-zero. In the direct transcription approach, x_{20} is a decision variable, so $f_g(x_{20})$ will only depend on the 4 variables that compose x_{20}.

Gradient Structure

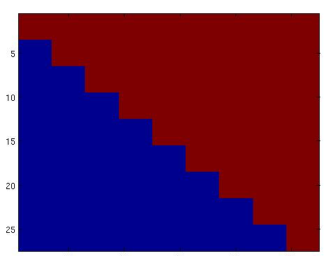
Suppose we write the gradient of all constraints as a matrix G, where the jth element of the ith row is $G_{i,j} = \frac{\partial h_i}{\partial z_j}$ where h_i is the ith constraint. Order both the z vector and the list of constraints h_i by the time index k which they most directly relate to. The structure of G greatly affects the efficiency and accuracy of various optimization approaches. The images below each option are graphical representations of the sparsity patterns, where red indicates the non-zero entries and blue the zero entries.

(g) For the *shooting* approach, what, if any, structure will G have? Note that G will not generally be square, so we use some of these terms loosely here. In particular, the "diagonal" here refers to the elements of G corresponding to the variables and constraints directly related to the same time index.

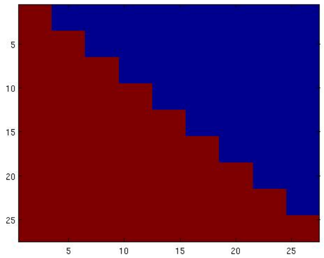
- Dense: many non-zero entries with no particular structure



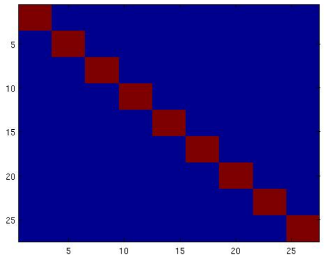
- Upper Triangular: entries below the "diagonal" are all (or nearly all) zero



- Lower Triangular: entries above the "diagonal" are all (or nearly all) zero

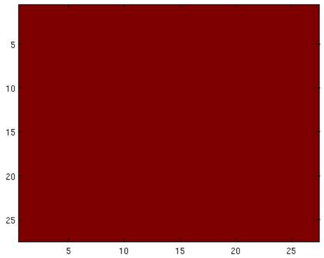


- Block Diagonal: entries away from the diagonal are all (or nearly all) zero

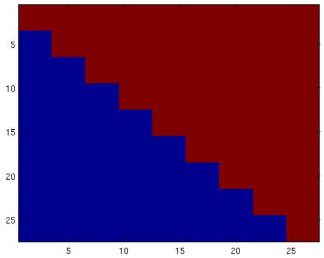


(h) For the *direct transcription* approach, what, if any, structure will G have?

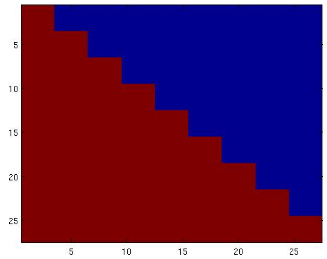
- Dense: many non-zero entries with no particular structure



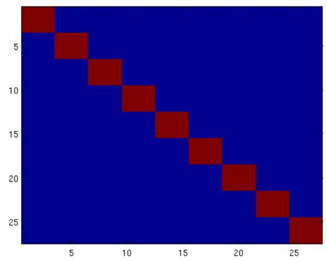
- Upper Triangular: entries below the "diagonal" are all (or nearly all) zero



- Lower Triangular: entries above the "diagonal" are all (or nearly all) zero



- Block Diagonal: entries away from the diagonal are all (or nearly all) zero



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Explanation

For the shooting approach, constraints at time k will generally depend on all previous inputs, u_i for $i \leq k$, thus the lower triangular structure. In the direct method, constraints at time k generally only depend on nearby terms, like u_{k-1}, u_k, u_{k+1} and x_{k-1}, x_k, x_{k+1}, thus the block diagonal structure.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem