edX

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Problem: Feedback Linearization

Feedback Linearization

0.0/15.0 points (graded)

True or false: for any underactuated system of the form $\ddot{q} = f_1(q, \dot{q}) + f_2(q, \dot{q})\, u$, one can choose $u(x, u')$ so that $\dot{x} = Ax + Bu'$, where u' is a new control input.

- True
- False ✔

Take a robot whose dynamics are given by the manipulator equations, $H(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)\, u$ for $q \in \mathbb{R}^n$. The robot starts in a given initial configuration $q(0) = q_0$ and with a given initial velocity $\dot{q}(0) = \dot{q}_0$. Suppose $B(q)$ is rank n for all q. Which of the following statements are true for **any** twice-differentiable desired trajectory $q_d : \mathbb{R} \to \mathbb{R}^n$?

a. Feedback linearization can be used to make it so that $q(t) = q_d(t)$ for all $t \geq 0$

b. Feedback linearization can be used to make it so that $\dot{q}(t) = \dot{q}_d(t)$ for all $t \geq 0$

c. Feedback linearization can be used to make it so that $\ddot{q}(t) = \ddot{q}_d(t)$ for all $t \geq 0$

- a
- b
- c ✔

Explanation
Feedback linearization of a second-order system can create arbitrary accelerations $\ddot{q}$, however, it cannot change the simple fact that we are dealing with a second-order system. Position and velocity are still limited in important, fundamental ways--particularly, both must be differentiable with respect to time. On the other hand, $\ddot{q}$ has no such restriction, it can, and often will be, discontinuous in time.

Oscillating Pendulum

Consider an actuated pendulum, where the base is forced to oscillate in simple harmonic motion, $C\sin(\omega t)$. Then, the dynamics of the pendulum angle θ are:

$$\ddot{\theta} = \frac{g}{l}\sin\theta - \frac{C}{l}\omega^2\sin(\omega t)\sin\theta + \frac{u}{ml^2}$$

Even with the base shaking, we would like the pendulum to spin at a constant speed, $\dot{\theta} = 1$. To achieve this, we should choose $\ddot{\theta}_{des}$ to stabilize any velocity error. Use feedback linearization to find the control law such that $\ddot{\theta} = -\dot{\theta} + 1$. Write the variable v for $\dot{\theta}$.

Trigonometric functions and greek letters can be written out in English, and should format properly. For example, simply write "sin(omega*t)" to form $\sin(\omega t)$

$u =$

Answer: m*l^2*(-v+1-g/l*sin(theta) + C/l*omega^2*sin(omega*t)*sin(theta))

Explanation
Substituting the desired behavior, (\ddot \theta = -\dot \theta + 1\) into the dynamic equation above, we can rearrange and solve for u,

$$u = ml^2(1-v) - mgl\sin(\theta) + mlC\omega^2\sin(\omega t)\sin\theta$$

Submit You have used 0 of 1 attempt

Answers are displayed within the problem