Non-existence of Limit Cycles

Non-existence of limit cycles (Part A)

0.0/10.0 points (graded)

A **gradient system** is a system of the following form:

$$\dot{x} = -\nabla V(x) \quad (1),$$

for some twice continuously differentiable scalar-valued function $V(x)$.

It is straight-forward to prove that gradient systems don't have any periodic orbits (we are not counting fixed-points as periodic orbits here, and we assume that $V(x)$ is not the 0 function). We can use a proof by contradiction in order to show this. Suppose we had a periodic orbit. Consider the change in V after one circuit around the periodic orbit. Denote this change as ΔV. For a periodic orbit, we must have $\Delta V = 0$. However, we also have:

$$\Delta V = \int_0^T \dot{V}(x(t))\,dt = \int_0^T \nabla V(x(t)) \cdot (-\nabla V(x(t)))\,dt = \int_0^T -\|\nabla V(x(t))\|^2 dt < 0.$$

This is a contradiction.

(a) Which of the following are gradient systems?

- [ ] All linear systems $\dot{x} = Ax$ with $A = A^T$.
- [ ] All linear systems $\dot{x} = Ax$.
- [ ] All one-dimensional systems $\dot{x} = f(x)$, where $f(x)$ is continuously differentiable.
- [ ] All globally asymptotically stable systems.

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Non-existence of limit cycles (Part B)

0.0/10.0 points (graded)

(b) Consider the system given by

$$\dot{x} = y + 2xy$$
$$\dot{y} = x + x^2 - y^2.$$

Prove that this system does not have a periodic orbit by finding a function $V(x, y)$ such that the system is of the form (1) above. Type in $V(x, y)$ below (make sure it has the correct sign!).

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syms x y real;
V = ; % Type in your answer here in terms of x and y

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Unanswered

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