Description

Given an **undirected** graph $G = (V, E)$ where V and E represent the set of vertices and the set of edges in the graph G respectively, each vertex is numbered from 1 to $|V|$. Contestants are required to write a program that evaluates the number of 3-tuples (u, v, x) such that graph G exists a simple path (a path that passes each vertex at most once) from u to v through x, where $u < v$ and $x \neq u, v$.

For example, the answer of a chain graph with 4 vertices is 4.

Constraints

Time limit: 1.5 s / Memory limit: 512 MB

$3 \leqslant n \leqslant 3 \times 10^5$, $m \leqslant 5 \times 10^5$.

Solution

It is important to observe that for any *biconnected* graph G, any 3-tuple (u, v, x) is legal. Suppose not, there must be at least one cut vertex on the path from u to x and v to x. To generalize the conclusion, we shrink all the biconnected components into single vertices (marked as squares) which connect to each vertex in the component (marked as circles) with an edge so that graph G will become a tree T. Then a 3-tuple (u, v, x) is legal if and only if:

1. u, v and x are in the same connected component.
2. x is on the unique path from u to v of T.
3. Otherwise, S denotes the set of vertices of a biconnected component associated with x, and u' and v' denote the first vertex in S on the path to x respectively. Therefore, $u' \neq v'$ for at least one possible biconnected component.

The conditions directly give the method to judge a 3-tuple. To enumerate it, we need to consider two cases:

1. Directly pass the vertex x: compute an array `g[x]` that represent the number of pairs (u, v) where both u and v are in the subtree of x and the path from u to v passes x. Note that square vertices are just auxiliary vertices and thus u and v should not be square vertices. In this case, the contribution is the sum of `g[x]` where x is circle vertex.
2. Considering a specific biconnected component, u, v come from different u' and v'. After considered case `1`, it's easy to show that the contribution of this case is `(m - 2) * g[x]`, where m represents the number of vertices in the component.

For convenience, the attached program uses the technique of "reroot" so that we don't care about the case that some circle vertex is the father of a square vertex.

Contestants should be careful about the unconnected graphs and the implementation of Tarjan's biconnected components algorithm.

Testdatas

Degenerated graphs like **trees** and **cactus graphs** are supposed to be subtasks of the problem. Methods of them are relatively easy compared to the standard solution. Any graphs would be fine since there seems to be no tricks to solve the problem unexpectedly. What's more, unconnected graphs should be included to judge the contestants.

Personal Information

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