

分部积分求定积分

$$\int_a^b u v' dx = u v \Big|_a^b - \int_a^b u' v dx$$

例 $\int_0^1 x e^x dx$ 反对幂三指

设 $u = x$ $v' = e^x$ 别把 e^x 变成对数了!

$u' = 1$ $v = e^x$

$$= x e^x \Big|_0^1 - \int_0^1 e^x dx = e - (e - 1) = 1$$

例 $\int_1^e x \ln x dx$ $\ln x$ 对 $x^{\frac{1}{2}}$ $\frac{x}{2}$

设 $u = \ln x$ $v = x$ $= \frac{1}{2} x^2 \ln x \Big|_1^e - \int_1^e \frac{1}{x} \cdot \frac{1}{2} x^2 dx$

$u' = \frac{1}{x}$ $v' = \frac{1}{2} x^2$ $= \frac{1}{2} x^2 \ln x \Big|_1^e - \int_1^e \frac{x}{2} dx$

$= \frac{1}{2} e^2 - \left(\frac{1}{4} x^2 \Big|_1^e \right) = \frac{1}{4} e^2 + \frac{1}{4}$

别直接代

积分完再代

例 $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x \cos 2x dx$

设 $u = x$ $v' = \cos 2x$

$u' = 1$ $v = \frac{1}{2} \sin 2x$

$$= \frac{1}{2} x \sin 2x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{2} \sin 2x dx$$