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Disturbance Compensated Adaptive Backstepping Control for an Unmanned Seaplane

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***Abstract*—A disturbance compensated adaptive backstepping controller is presented for a class of nonlinear systems with external disturbances. We develop a disturbance observer, with which the problem of disturbance compensation can be transformed into an adaptive control problem. Command filtered adaptive backstepping method is then used to design the control law to track the desired trajectory. In order to analyze the property of the controller, the overall close-loop error system is built. Using the Lyapunov approach, we prove that the proposed controller provides the uniformly asymptotic stability for the considered system and achieves perfect disturbance compensation. Finally, the developed method is applied to an unmanned seaplane system in the presence of large external disturbances. Simulation results show that the controller has good performance for the unmanned seaplane in different wave conditions.**

I. INTRODUCTION

DISTURBANCE, generally referred to as external uncertainties, widely exit in many practical systems, such as mechanical systems, flight control systems, chemical processes and so on. Hence how to reject or attenuate the effects of disturbances plays an important role in control system design. The internal model principle [1], [2] provides an effective approach to deal with external deterministic disturbances with a priori known form. According to the principle, the disturbances are regarded as the output of a linear exosystem. The influence of the disturbances can be completely compensated if the controller reproduces the whole information of such exosystem model.

However, in practice, it is difficult to specify beforehand the accurate model parameters of the exosystem. Disturbance observer based control (DOBC) approaches have been proposed to reconstruct and compensate the disturbances from the plant model and input-output data in recent years, which are applied to missile systems [3], robotic manipulators [4]. Recently, Nikiforov developed a new disturbance observer [5], whose structure and parameters were not affected by the uncertainty of the controlled system and exosystem. With this method, we can convert the problem from disturbance compensation to an adaptive control problem. Basturk *et al*. utilized this observer to achieve adaptive cancelation of matched and mismatched sinusoidal disturbances for linear time invariant (LTI) systems [6], [7], and successfully designed an adaptive wave disturbances cancelation controller for surface effect ships [8]. Furthermore, this approach has been extended to nonlinear systems with matched disturbances [9]. In this paper, we adopt this observer to solve the disturbance compensation problem of a class of nonlinear systems subject to mismatched disturbances.

As a powerful tool for designing controllers for nonlinear systems, backstepping has drawn much attention in the past two decades [10-12]. Backstepping is a Lyapunov-based, recursive nonlinear design methodology, which can give close-loop stability and guarantee the tracking error convergence. The main idea of backstepping is to design a controller by using certain states as “virtual controls” of others [13]. The key problem with this approach is that analytic calculation of the partial derivatives of the virtual control signal can be complicated in applications as the order of the nonlinear system increases [14]. This problem has been addressed by many methods, such as dynamic surface control [15], sliding mode filter [16] and command filtered approach [17-20]. The command filtered idea is adopted in this article to analyze and derive the control law.

Combining the disturbance observer and the command filtered adaptive backstepping, a new control scheme is developed to compensate the disturbances for a class of nonlinear systems in this paper. The stability properties of the close loop system are guaranteed and the state of the considered error system can achieve global asymptotic stability. Then controller design of an unmanned seaplane system with large external disturbances is presented. This controller can keep the attitude of the unmanned seaplane stable and improve the sea-keeping ability in the circumstance of complicated hydrodynamic forces and severe wave disturbances. Simulation results demonstrate the effectiveness of the controller.

The remainder of this paper is organized as follows: In section 2, the problem is introduced and the disturbance observer is described. We present the adaptive controller design based on command filtered backstepping and analyze the stability of the system in section 3. Section 4 shows the performance of the proposed method for an unmanned seaplane in different wave conditions. Finally, the concluding remarks are summarized in section 5.

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II. Problem Statement and Disturbance Observer Design

In this section we consider the following single-input nonlinear system

$$\begin{cases} \dot{x}_i = f_i(x) + g_i(x)x_{i+1} + \psi_i, \\ \quad i = 1, \cdots, n-1, \\ \dot{x}_n = f_n(x) + g_n(x)u + \psi_n \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$ is the state vector, u is the scalar control signal, the first state x_1 is the scalar output. The functions $f_i : \mathbb{R}^n \mapsto \mathbb{R}$, $g_i : \mathbb{R}^n \mapsto \mathbb{R}$ are smooth functions. $\psi = [\psi_1, \psi_2, \cdots, \psi_n]^T \in \mathbb{R}^n$ is the external disturbance vector.

We consider the problem of designing a control law to track the desired trajectory $x_d(t) : \mathbb{R}^+ \mapsto \mathbb{R}$ with the disturbance compensation.

Eq. (1) can be rewritten as

$$\dot{x} = F(x) + G(x)u + D\psi \tag{2}$$

where

$$F(x) = \begin{bmatrix} f_1(x) + g_1(x)x_2 \\ \vdots \\ f_{n-1}(x) + g_{n-1}(x)x_n \\ f_n(x) \end{bmatrix}_{n\times 1} \quad G(x) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ g_n(x) \end{bmatrix}_{n\times 1} \quad D = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{n\times n}$$

The disturbance vector ψ can be expressed as the output of a linear exosystem [5], [6]

$$\dot{\omega} = S\omega \tag{3}$$

$$\psi = h^T \omega \tag{4}$$

where $\omega \in \mathbb{R}^q$ is the exosystem state vector, $S \in \mathbb{R}^{q\times q}$ and $h \in \mathbb{R}^{q\times n}$ are the constant coefficient matrices.

In order to design the disturbance observer and the controller, we make the following assumptions regarding the plant (1) and the exosystem (3)-(4):

Assumption 1: $f_i(x), g_i(x)$ are known and $g_i(x)$ is invertible.

Assumption 2: ψ_i is a sufficiently rich signal.

Assumption 3: The desired trajectory x_d and its first derivative $\dot{x}_d$ are smooth, bounded and known.

Assumption 4: All eigenvalues of the exosystem matrix S lie on the imaginary axis.

Assumption 5: The pair (h^T, S) is completely observable.

Assumption 6: The dimension q of the exosystem is known, but the exosystem matrices S and h are unknown.

The following lemma represents the canonical form of the exosystem (3)-(4).

Lemma 1 ([5], [6]): The disturbance vector ψ can be represented as the output of the model

$$\dot{\zeta} = G\zeta + l\psi \tag{5}$$

$$\psi = \theta^T \zeta \tag{6}$$

$$\theta^T = h^T M^{-1} \tag{7}$$

where $G \in \mathbb{R}^{q\times q}$ is a Hurwitz matrix with distinct eigenvalues, $l \in \mathbb{R}^{q\times n}$ and the pair (G, l) is controllable. $\theta \in \mathbb{R}^{q\times n}$ is a constant coefficient matrix, and the state $\zeta \in \mathbb{R}^q$ has the following relation with the state ω in the exosystem (3)-(4)

$$\zeta = M\omega \tag{8}$$

where $M \in \mathbb{R}^{q\times q}$ is a solution of the following Sylvester equation

$$MS - GM = lh^T \tag{9}$$

Proof: Since the matrices G and S have disjoint spectra, the pair (h^T, S) is observable, and the pair (G, l) is controllable, Eq. (9) has a unique solution [5]. Differentiating (8) and using (3), (4) and (9), we obtain

$$\dot{\zeta} = MS\omega = GM\omega + lh^T\omega = G\zeta + l\psi \tag{10}$$

Now (5) holds. Substituting $\omega = M^{-1}\zeta$ into (4), we obtain (6) and (7). ■

The lemma 1 enables us to use the known state and input matrices G and l instead of the unknown parameters S and h in the exosystem. Now we can not still express the disturbance vector in a certain form because ζ is not accessible in Eq. (6). Therefore, we design the following disturbance observer [5]:

$$\hat{\zeta} = \eta + Nx \tag{11}$$

$$\dot{\eta} = G\eta + GNx - N(F(x) + G(x)u) \tag{12}$$

where $\hat{\zeta}$ is the estimate of the state vector ζ, η is the auxiliary vector, $N \in \mathbb{R}^{q\times n}$ is the solution of the following equation

$$ND = l \tag{13}$$

According to the disturbance observer, we can get the following lemma.

Lemma 2 ([5], [6]): Define the following error vector

$$\varepsilon = \zeta - \hat{\zeta} \tag{14}$$

Then the disturbance vector ψ can be represented as

$$\psi = \theta^T \hat{\zeta} + \theta^T \varepsilon \tag{15}$$

and $\varepsilon \in \mathbb{R}^q$ satisfies the equation

$$\dot{\varepsilon} = G\varepsilon \tag{16}$$

Proof: Differentiating (14) and combining (2), (5) and (11)-(13), we obtain

$$\begin{aligned} \dot{\varepsilon} &= G\zeta + l\psi - G\eta - GNx + N(F(x) + G(x)u) - N(F(x) + G(x)u) - ND\psi \\ &= G(\zeta - \eta - Nx) = G\varepsilon \end{aligned} \tag{17}$$

Using (6) and (14), we obtain (15). ■

Eq. (16) suggests that the estimation error vector decays exponentially. Thus, according to Eq. (15), we transform the uncertainty of the external disturbance ψ into the uncertainty of the constant coefficient matrix θ. This problem can be solved by adaptive control methods. Next, adaptive backstepping is adopted to design the control law to achieve the disturbance compensation.

III. ADAPTIVE CONTROLLER DESIGN

Based on the above disturbance observer and the command filtered backstepping design approach [19], [20], an adaptive disturbance compensated controller is presented with complete stability analysis in this section. The external disturbances are compensated at each recursive step of the backstepping design procedure.

Eq. (14) can be rewritten as

$$\psi_i = \theta_i^T \hat{\zeta} + \theta_i^T \varepsilon \quad for \;\; i = 1, \cdots, n \tag{18}$$

where $\theta_i \in \mathbb{R}^q$ is the component of the matrix θ.

Define the tracking error, the compensated tracking error and the estimation error of the unknown parameter as follows

$$\tilde{x}_i = x_i - x_{i,c} \tag{19}$$

$$\bar{x}_i = \tilde{x}_i - \xi_i \tag{20}$$

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i \tag{21}$$

for $i = 1, \cdots, n$, where $x_{1,c} = x_d$, $x_{i,c} (i = 2, \cdots, n)$ and ξ_i will be defined in the latter analysis.

Define the stabilizing functions as

$$\alpha_1 = g_1^{-1}(x)(-k_1\tilde{x}_1 - f_1(x) - \hat{\theta}_1^T \hat{\zeta} + \dot{x}_{1,c}) \tag{22}$$

$$\alpha_i = g_i^{-1}(x)(-k_i\tilde{x}_i - f_i(x) - \hat{\theta}_i^T \hat{\zeta} + \dot{x}_{i,c} - g_{i-1}(x)\bar{x}_{i-1}) \quad for \;\; i = 2, \cdots n \tag{23}$$

The compensating signal ξ_i can be selected as

$$\begin{cases} x_{i+1,c}^0 = \alpha_i - \xi_{i+1} \\ \dot{\xi}_i = -k_i\xi_i + g_i(x)(x_{i+1,c} - x_{i+1,c}^0) \end{cases} \quad for \;\; i = 1, \cdots, n-1 \tag{24}$$

$$\dot{\xi}_n = -k_n\xi_n + g_n(x)(u - \alpha_n) \tag{25}$$

where $k_i > 0$ is the control gain, the signal $x_{i,c}$ for $i = 2, \cdots n$ is generated by the following filter

$$\tau\dot{x}_{i,c} + x_{i,c} = x_{i,c}^0 \tag{26}$$

Similarly, the control input u is generated by

$$\tau\dot{u} + u = \alpha_n \tag{27}$$

where τ is the time constant of the filter. The purpose of the filter is to generate the signals $x_{i,c}, u$ and their derivative $\dot{x}_{i,c}, \dot{u}$. In order to simplify the analysis, we adopt first-order, low-pass filters. The magnitude, rate and bandwidth-limiting second-order command filter is discussed in [17] and [18].

The adaptive law is given by

$$\dot{\hat{\theta}}_i = \gamma_i \bar{x}_i \hat{\zeta} \quad for \;\; i = 1, \cdots, n \tag{28}$$

Next, we can summarize the results in the following theorem.

Theorem 1: For the nonlinear system described by (1) and the exosystem described by (3)-(4) that satisfy Assumptions 1-6, by adopting the adaptive controller (22)-(28), we have the following properties for $i = 1, \cdots, n$:

(i) The compensated tracking error $\bar{x}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$. If we select the filter constant τ sufficiently small, the actual tracking error $\tilde{x}_i \to \bar{x}_i$.

(ii) The parameter estimation error $\tilde{\theta}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$. Furthermore, $\hat{\theta}_i^T \hat{\zeta} - \psi_i$ converges to zero as $t \to \infty$, which implies the perfect compensation of the external disturbance.

Proof: By using (18)-(23), the closed-loop tracking error dynamics can be written as

$$\begin{aligned}\dot{\tilde{x}}_1 &= f_1(x) + g_1(x)\alpha_1 + g_1(x)(x_{2,c} - \alpha_1) + g_1(x)(x_2 - x_{2,c}) + \theta_1^T\hat{\zeta} + \theta_1^T\varepsilon - \dot{x}_{1,c} \\ &= -k_1\tilde{x}_1 + g_1(x)(x_{2,c} - \alpha_1) + g_1(x)\tilde{x}_2 + \tilde{\theta}_1^T\hat{\zeta} + \theta_1^T\varepsilon\end{aligned} \tag{29}$$

$$\begin{aligned}\dot{\tilde{x}}_i &= f_i(x) + g_i(x)\alpha_i + g_i(x)(x_{i+1,c} - \alpha_i) + g_i(x)(x_{i+1} - x_{i+1,c}) + \theta_i^T\hat{\zeta} + \theta_i^T\varepsilon - \dot{x}_{i,c} \\ &= -k_i\tilde{x}_i - g_{i-1}(x)\bar{x}_{i-1} + g_i(x)(x_{i+1,c} - \alpha_i) + g_i(x)\tilde{x}_{i+1} + \tilde{\theta}_i^T\hat{\zeta} + \theta_i^T\varepsilon \\ &\qquad for \;\; i = 2, \cdots, n-1\end{aligned} \tag{30}$$

$$\begin{aligned}\dot{\tilde{x}}_n &= f_n(x) + g_n(x)\alpha_n + g_n(x)(u - \alpha_n) + \theta_n^T\hat{\zeta} + \theta_n^T\varepsilon - \dot{x}_{n,c} \\ &= -k_n\tilde{x}_n - g_{n-1}(x)\bar{x}_{n-1} + g_n(x)(u - \alpha_n) + \tilde{\theta}_n^T\hat{\zeta} + \theta_n^T\varepsilon\end{aligned} \tag{31}$$

Then combining (20), (24), (25) and (29)-(31), we describe the compensated tracking error dynamics as

$$\dot{\bar{x}}_1 = -k_1\bar{x}_1 + g_1(x)\bar{x}_2 + \tilde{\theta}_1^T\hat{\zeta} + \theta_1^T\varepsilon \tag{32}$$

$$\dot{\bar{x}}_i = -k_i\bar{x}_i - g_{i-1}(x)\bar{x}_{i-1} + g_i(x)\bar{x}_{i+1} + \tilde{\theta}_i^T\hat{\zeta} + \theta_i^T\varepsilon \quad for \;\; i = 2, \cdots, n-1 \tag{33}$$

$$\dot{\bar{x}}_n = -k_n\bar{x}_n - g_{n-1}(x)\bar{x}_{n-1} + \tilde{\theta}_n^T\hat{\zeta} + \theta_n^T\varepsilon \tag{34}$$

Consider the following Lyapunov function

$$V = \sum_{i=1}^{n}(\frac{1}{2}\bar{x}_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \varepsilon^T P_{\varepsilon_i}\varepsilon) \tag{35}$$

where the positive definite matrix $P_{\varepsilon_i} \in \mathbb{R}^{q\times q}$ is a solution of the matrix equation

$$G^T P_{\varepsilon_i} + P_{\varepsilon_i} G = -\theta_i\theta_i^T \tag{36}$$

Then the time derivative of V can be obtained as

$$\dot{V} = \sum_{i=1}^{n}(\bar{x}_i\dot{\bar{x}}_i + \frac{1}{\gamma_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i + \dot{\varepsilon}^T P_{\varepsilon_i}\varepsilon + \varepsilon^T P_{\varepsilon_i}\dot{\varepsilon}) \tag{37}$$

Substituting (15), (28), (32)-(34), (36) into (37), we get

$$\dot{V} = \sum_{i=1}^{n}(-k_i\bar{x}_i^2 + \bar{x}_i\theta_i^T\varepsilon - \varepsilon^T\theta_i\theta_i^T\varepsilon) \tag{38}$$

Using Young's inequality, we obtain

$$\bar{x}_i\theta_i^T\varepsilon \le \frac{1}{2}\bar{x}_i^2 + \frac{1}{2}\varepsilon^T\theta_i\theta_i^T\varepsilon \tag{39}$$

Using (38) and (39), we have

$$\dot{V} \le \sum_{i=1}^{n}[-(k_i - \frac{1}{2})\bar{x}_i^2 - \frac{1}{2}(\theta_i^T\varepsilon)^2] \tag{40}$$

By choosing $k_i \ge \frac{1}{2}$, $\dot{V}$ is negative semi-definite, which implies that the variables $\bar{x}_i$ and $\tilde{\theta}_i$ are bounded, namely, $\bar{x}_i, \tilde{\theta}_i \in \mathcal{L}_\infty$. Furthermore, the right hand side of (40) is a continue function. By the LaSalle-Yoshizawa theorem [13], $\bar{x}_i$ and $\theta_i^T\varepsilon$ converge to zero as $t \to \infty$, therefore, $\bar{x}_i \in \mathcal{L}_2$. Especially, if the time constant of the filter τ is selected sufficiently small, we conclude that ξ_i converges to zero according to Eqs. (24)-(27), that is, once the filtered effect is removed, the actual tracking error $\tilde{x}_i$ converges to the compensated tracking error $\bar{x}_i$.

In view of (21), (28) and (32)-(34), we write the overall close-loop system as the following form

$$\dot{\varsigma} = A(\bar{x},\tilde{\theta})\varsigma + W(\bar{x},\tilde{\theta})\varepsilon \tag{41}$$

where

$$\varsigma = [\bar{x}_1,\cdots,\bar{x}_n,\tilde{\theta}_1^T,\cdots,\tilde{\theta}_n^T]^T \tag{42}$$

$$A(\bar{x},\tilde{\theta}) = \begin{bmatrix} A_c(\bar{x},\tilde{\theta}) & \phi^T \\ -\Gamma\phi & 0 \end{bmatrix} \tag{43}$$

$$W(\bar{x},\tilde{\theta}) = \begin{bmatrix} \theta^T \\ 0 \end{bmatrix} \tag{44}$$

with

$$A_c(\bar{x},\tilde{\theta}) = \begin{bmatrix} -k_1 & g_1(x) & 0 & \cdots & 0 \\ -g_1(x) & -k_2 & g_2(x) & \cdots & 0 \\ 0 & -g_2(x) & -k_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & g_{n-1}(x) \\ 0 & \cdots & 0 & -g_{n-1}(x) & -k_n \end{bmatrix}_{n\times n} \tag{45}$$

$$\phi = \begin{bmatrix} \hat{\zeta} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \hat{\zeta} \end{bmatrix}_{nq\times n} \tag{46}$$

$$\Gamma = diag\{\gamma_1,\cdots\gamma_1,\cdots,\gamma_n,\cdots\gamma_n\}_{nq\times nq} \tag{47}$$

In order to simplify the analysis, we first consider the following system

$$\dot{\varsigma} = A(\bar{x},\tilde{\theta})\varsigma \tag{48}$$

Express the above Lyapunov function V_c and its time derivative $\dot{V}_c$ as

$$V_c = \frac{1}{2}\varsigma^T P_c \varsigma \tag{49}$$

$$\dot{V}_c = \frac{1}{2}\varsigma^T (A^T P_c + P_c A)\varsigma \tag{50}$$

where

$$P_c = \begin{bmatrix} I_{n\times n} & 0 \\ 0 & \Gamma^{-1} I_{nq\times nq} \end{bmatrix} \tag{51}$$

Using (43), (45), (50) and (51), we have

$$\dot{V}_c = -\varsigma^T \begin{bmatrix} \Lambda^2 & 0 \\ 0 & 0 \end{bmatrix} \varsigma \tag{52}$$

where

$$\Lambda^2 = \begin{bmatrix} k_1 & & 0 \\ & \ddots & \\ 0 & & k_n \end{bmatrix} \tag{53}$$

Choosing

$$C = \begin{bmatrix} \Lambda \\ 0 \end{bmatrix}_{(n+nq)\times n} \tag{54}$$

we rewrite $\dot{V}_c$ as the following form

$$\dot{V}_c = -\varsigma^T CC^T \varsigma \tag{55}$$

Combining (50) and (55), it follows that there exists a symmetric matrix P_c, as defined in (51), satisfying the following inequality

$$r_1 I \le P_c \le r_2 I \tag{56}$$

$$\dot{P}_c + A^T P_c + P_c A + \upsilon CC^T \le O \tag{57}$$

for some $\upsilon > 0$ and $r_1, r_2 > 0$, where $\dot{P}_c = 0$.

On the basis of the above analysis, we can obtain that the equilibrium $\varsigma = 0$ of the system (48) is uniformly asymptotically stable if (C, A) is a uniformly completely observable (UCO) pair [21]. The pair $(C, A+KC^T)$ has the same UCO property with (C, A) if K is bounded [21]. We choose

$$K = \begin{bmatrix} A_c^* \\ \phi^* \end{bmatrix} \tag{58}$$

where

$$A_c^* = \begin{bmatrix} 0 & -\frac{g_1(x)}{\sqrt{k_2}} & 0 & \cdots & 0 \\ \frac{g_1(x)}{\sqrt{k_1}} & 0 & \frac{-g_2(x)}{\sqrt{k_3}} & \cdots & 0 \\ 0 & \frac{g_2(x)}{\sqrt{k_2}} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \frac{-g_{n-1}(x)}{\sqrt{k_n}} \\ 0 & \cdots & 0 & \frac{g_{n-1}(x)}{\sqrt{k_{n-1}}} & 0 \end{bmatrix}_{n\times n} \tag{59}$$

$$\phi^* = \begin{bmatrix} \frac{\gamma_1}{\sqrt{k_1}}\hat{\zeta} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\gamma_n}{\sqrt{k_n}}\hat{\zeta} \end{bmatrix}_{nq\times n} \tag{60}$$

Next we show the system corresponding to $(C, A+KC^T)$ as

$$\dot{Y}_1 = -\Lambda^2 Y_1 + \phi^T Y_2 \tag{61}$$

$$\dot{Y}_2 = 0 \tag{62}$$

$$y_0 = \Lambda Y_1 \tag{63}$$

Using (5), (15) and (16), we get

$$\dot{\hat{\zeta}} = G\hat{\zeta} + l\psi \tag{64}$$

The solution of (64) can be given as

$$\hat{\zeta}(t) = e^{G(t-t_0)}\hat{\zeta}(t_0) + \int_{t_0}^{t} e^{G(t-\tau)} l\psi(\tau) d\tau \tag{65}$$

We can estimate the above solution [22] as

$$\begin{aligned}\|\hat{\zeta}(t)\| &\le ke^{-\lambda(t-t_0)}\|\hat{\zeta}(t_0)\| + \int_{t_0}^{t} ke^{-\lambda(t-\tau)}\|l\|\|\psi(\tau)\|d\tau \\ &\le ke^{-\lambda(t-t_0)}\|\hat{\zeta}(t_0)\| + \frac{k\|l\|}{\lambda}\sup_{t_0\le\tau\le t}\|\psi(\tau)\|\end{aligned} \tag{66}$$

for some constants $k,\lambda>0$. From (66) we can obtain that $\hat{\zeta}\in\mathcal{L}_\infty$, moreover, $\dot{\hat{\zeta}}\in\mathcal{L}_\infty$ and $\phi,\dot{\phi}\in\mathcal{L}_\infty$. Under the assumption that ψ_i is a sufficiently rich signal and the pair (G,l) is controllable, $\hat{\zeta}$ is persistently exciting (PE) [6], then ϕ is PE signal. For the function $H(s)=\Lambda(sI+\Lambda^2)^{-1}$, obviously, $H(s)$ is stable and minimum phase. Thus it can be established that $\phi_f=\Lambda(sI+\Lambda^2)^{-1}\phi$ is also PE [21]. Then we have

$$\alpha_1 I \le \frac{1}{T_0}\int_t^{t+T_0}\phi_f(\tau)\phi_f^T(\tau)d\tau \le \alpha_2 I \tag{67}$$

for some constants $\alpha_1,\alpha_2,T_0>0$. We conclude that $(C, A+KC^T)$ is UCO according to [21], namely, (C,A) is UCO. Therefore, for the system (48), the equilibrium $\varsigma=0$ is uniformly asymptotically stable and the state transition matrix $\Phi(t,t_0)$ satisfies

$$\|\Phi(t,t_0)\|\le k_0 e^{-\lambda_0(t-t_0)} \tag{68}$$

for some constants $k_0,\lambda_0>0$. Using (15), we get

$$\|\varepsilon(t)\|\le k_1 e^{-\lambda_1(t-t_0)}\|\varepsilon(0)\| \tag{69}$$

for some constants $k_1,\lambda_1>0$. The solution of the system (41) can be obtained as

$$\varsigma(t)=\Phi(t,0)\varsigma(0)+\int_0^t\Phi(t,\tau)W(\tau)\varepsilon(\tau)d\tau \tag{70}$$

Combining (68) and (69), we estimate (70) as

$$\begin{aligned}\|\varsigma(t)\| &\le k_0e^{-\lambda_0 t}\|\varsigma(0)\| + k_0k_1e^{-\lambda_0 t}\|\varepsilon(0)\|\sup_{0\le\tau\le t}W(\tau)\int_0^t e^{(\lambda_0-\lambda_1)\tau}d\tau \\ &= k_0e^{-\lambda_0 t}\|\varsigma(0)\| + \frac{k_0k_1\|\varepsilon(0)\|\sup_{0\le\tau\le t}W(\tau)}{\lambda_0-\lambda_1}(e^{-\lambda_1 t}-e^{-\lambda_0 t}) \\ &\le k_0e^{-\lambda_0 t}\|\varsigma(0)\| + \frac{k_0k_1\|\varepsilon(0)\|\sup_{0\le\tau\le t}W(\tau)}{\lambda_0-\lambda_1}e^{-\lambda_1 t}\end{aligned} \tag{71}$$

From (71) we have that $\varsigma(t)$ converges to zero as $t\to\infty$, which implies $\tilde{\theta}_i\in\mathcal{L}_2$. Using (18) and the fact that $\theta_i^T\varepsilon\to 0$ as $t\to\infty$, we conclude that $\hat{\theta}_i^T\hat{\zeta}-\psi_i\to 0$ as $t\to\infty$, which implies that the external disturbances are compensated perfectly. Thus far, we have proved Theorem 1. ■

IV. Simulation Results

In this section, we demonstrate the performance of the proposed approach by control of an unmanned seaplane in different wave conditions. Fig.1 shows the forces acting on the unmanned seaplane. The longitudinal dynamic model of the unmanned seaplane [23] can be described by

$$\begin{cases} mV\dot{\alpha}=mVq-T\sin(\alpha+\alpha_t)-L_a-N_w\cdot\cos\alpha+D_f\cdot\sin\alpha+G_{za} \\ I_y\dot{q}=M_a+M_w+M_T\end{cases} \tag{72}$$

where α is the angle of attack, q the pitch angular rate, V the velocity, T the thrust of engine, N_w the water pressure normal to the bottom, D_f the water friction along the bottom, L_a the aerodynamic lift, G_{za} the gravity along Z_s, M_a, M_w, M_T the total pitching moment from air, water and engine, I_y the unmanned seaplane's moment of inertia about Y_b, α_t the angle between engine force and X_b, m the mass of the unmanned seaplane. Detailed presentation about this model can be found in [23].

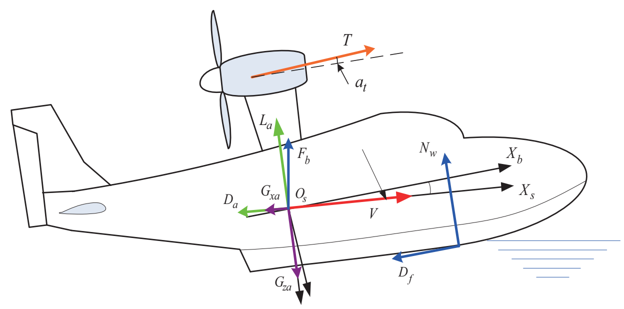

Fig. 1. Forces acting on the unmanned seaplane.

In order to simplify the controller design, we rewrite the model as

$$\begin{cases}\dot{\alpha}=f_1(\alpha,q)+g_1(\alpha,q)q+\psi_1 \\ \dot{q}=f_2(\alpha,q)+g_2(\alpha,q)\delta_e+\psi_2\end{cases} \tag{73}$$

where δ_e is the elevator deflection, $f_1(\alpha,q)$ and $f_2(\alpha,q)$ represent the resultant forces and moments from air, engine and gravity. $g_1(\alpha,q)$ and $g_2(\alpha,q)$ represent the corresponding aerodynamic coefficients. As the hydrodynamic forces and moments are difficult to estimate, we regard them as the external disturbances ψ_1 and ψ_2. It is easy to verify that Assumption 1 to 6 are satisfied for the unmanned seaplane.

Our control objective is to stabilize the angle of attack for given speed by the deflection of the elevator, avoiding the dynamic instability phenomenon and improving the sea-keeping ability of the unmanned seaplane. The initial states of the unmanned seaplane are chosen as $V=15m/s$, $\alpha=7.09^\circ$, $q=0rad/s$. We choose the disturbance observer parameters as $G=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$, $l=N=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$. The time constant of the filter is $\tau=0.01$, the controller parameters are set as $k_1=k_2=30$, $\gamma_1=\gamma_2=10$.

Firstly, the simulation is performed in calm water as showed in Fig. 2. A step signal is imposed on the angle of attack command at 1 second. We can see that the states of the unmanned seaplane converge to the desired values in a short time as showed in Fig. 2 (a) (b). Fig. 3 presents the performance of the controller in regular wave, whose amplitude and wavelength are 0.3m, 50m, respectively. Although the pitch angular rate fluctuates in a small range, the angle of attack is stabilized in the desired state in the presence of wave disturbances as showed in Fig. 3 (a) (b). From the results given in Fig. 2 (c) (d) and Fig. 3 (c) (d), the estimation of hydrodynamic force and moment are achieved perfectly.

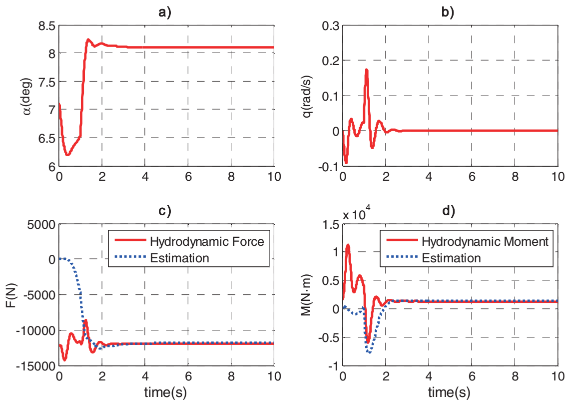

Fig. 2. Performance of the unmanned seaplane in calm water.

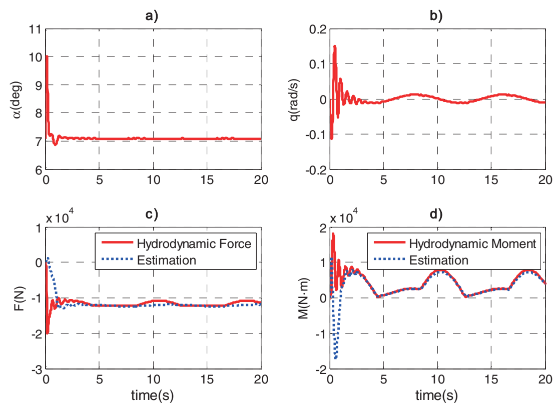

Fig. 3. Performance of the unmanned seaplane in regular wave.

V. Conclusion

In this paper a new disturbance compensated adaptive backstepping controller is presented for a class of nonlinear systems. The external disturbances are estimated by the disturbance observer, which converts the uncertainty of the disturbances to the uncertainty of the estimation parameters. We adopt the command filtered adaptive backstepping approach to design the controller, assuring that the disturbances are compensated at each step of the procedure. The considered error system can achieve uniformly asymptotic convergence by the Lyapunov stability analysis, which implies perfect disturbance compensation. The simulation results show that the proposed method is capable of stabilizing the attitude of the unmanned seaplane with satisfactory performances in different wave conditions.

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