Add-on Module of Active Disturbance Rejection for Set-Point Tracking of Motion Control Systems

Article *in* IEEE Transactions on Industry Applications · March 2017

DOI: 10.1109/TIA.2017.2677360

5 authors, including:

Wenchao Xue
Chinese Academy of Sciences

Rafał Madoński

Zhiqiang Gao
Cleveland State University

Yi Huang
Chinese Academy of Sciences

Add-on Module of Active Disturbance Rejection for Set-Point Tracking of Motion Control Systems

Wenchao Xue*, Rafal Madonski†, Zhiqiang Gao‡, Yi Huang* and Krzysztof Lakomy†
*Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences
Beijing 100190, P. R. China. E-mail: wenchaoxue@amss.ac.cn, yhuang@amss.ac.cn
†Chair of Control and Systems Engineering, Poznan University of Technology, Poland.
E-mail: rafal.madonski@gmail.com, krzysztof.lakomy92@gmail.com
‡Department of Electrical and Computer Engineering, Cleveland State University
Cleveland, Ohio 44115, U.S.A. E-mail: z.gao@ieee.org

***Abstract*—Active Disturbance Rejection Control (ADRC) as a standalone motion solution has been adopted by companies such as Texas Instruments and Danfoss and made available on various proprietary industrial platforms. The idea of ADRC, however, can be integrated with the existing control technologies seamlessly, as shown in this paper. It is shown a modularized ADRC design for set-point tracking of motion control such that better uncertainty rejection can be implemented without any change in the existing proportional-derivative (PD) control with linear observer. We prove that certain integration of the observer's error can serve as an estimation for the "total disturbance" in low frequency range. This enables the estimation and cancellation of the "total disturbance" to be incorporated into the existing control loop. Also, a comparison between the methods with and without such "module" is discussed. The proposed ADRC is implemented and validated with experimental results for a 1-degree of freedom robotic manipulator, where desired set-point tracking performance in position control is achieved under unknown mass variations and sudden external disturbances.**

***Index Terms*—active disturbance rejection control; extended state observer; disturbance estimation; motion control system; set-point tracking.**

I. INTRODUCTION

The set-point position tracking of motion control systems is a typical control task in engineering. Observer based proportional derivative (PD) control is classical for handling this task where the aim of PD feedback is to regulate the position and the aim of observer is to provide the estimations of the velocity needed in the feedback loop [1]. Conventional observer is constructed under the assumption that an exact mathematical model is available [2]. However, the physical motion control systems usually contain multifarious uncertainties, such as the nonlinear unknown friction, load disturbance, sensor noise, to name a few. Therefore, the fundamental challenge in practice is to make the observer based PD control be capable of dealing with uncertainties far beyond the known model information in motion processes.

This work is supported by the National Basic Research Program of China under Grant No. 2014CB845303, the National Center for Mathematics and Interdisciplinary Sciences, Chinese Academy of Sciences.

In the last decades, estimating and compensating for uncertainties has been effectively synthesized in observer based design [3], [4], [5]. The estimation of uncertainties greatly improves the observer based design since it enables the 2 degrees of freedom (2DOF) control structure with one to compensate for the uncertainty and the other to regulate the closed-loop system [6]. This appealing structure has been successfully realized by many methods, which mainly include, but are not limited to, active disturbance rejection control (ADRC) [7], [8], disturbance-accommodation control (DAC) [3], disturbance observer based control (DOBC) [9], [10], nonlinear disturbance observer based control (NDOBC) [11], [12], adaptive disturbance observer based control [13], embedded model control (EMC) [14], composite hierarchical anti-disturbance control (CHADC) [15], uncertainty and disturbance estimator based control [16], etc. In particular, ADRC has been proven to be powerful in dealing with various kinds of motion control systems. With the key idea of estimating and cancelling the "total disturbance" which lumps both unknown dynamics and disturbance, ADRC has been successfully applied in two-mass actuator systems [17], DC-Motor System [18], spacecraft systems [19], robotic manipulators [20], and etc. Moreover, Texas Instrument has produced the motion control chips with ADRC technology embedded on board [21]. Papers [4] and [22] also present the control law selection and parameter optimization of ADRC for general position control problem. In addition, literatures [23] and [24] consider the approach of making ADRC to extend the capability of some already used industrial controllers.

The existing works have shown the effectiveness of the method of estimating and cancelling the "total disturbance", initiated by ADRC, for motion control systems. Nevertheless, how to modularize this appealing method such that the conventional observer based PD control can be improved in uncertainties rejection while not needing any change of its existing structure and parameters, is an open problem with great practical importance. In this paper, we prove that certain integration of the otherwise unmined error data of the observer, can be the estimation of "total disturbance". This enables the modularized ADRC, i.e. the "module" in the form of certain integrator of the observer's error is add-on for the conventional PD control. To great benefit, the proposed ADRC algorithm can be easily and straightforwardly implemented with no modification of the parameters and structure of the original control laws. Also, quantitative analysis shows that the uncertainty rejection can be largely improved by the modularized ADRC with an insignificant cost of possible deterioration in filtering noise. The effectiveness of our proposition is validated though the experiment conducted on a classical 1-degree of freedom (1DOF) manipulator testbed.

The paper is organized as follows. In Section II, the set-point tracking problem of a class of motion systems is introduced. In Section III, the modularized ADRC and the performance of the closed-loop system are presented. In Section IV, experiments on the manipulator testbed are presented. The concluding remarks are in Section V.

II. Problem Formulation

The following class of motion control systems is considered

$$\begin{cases}\dot{x} = Ax + B(\delta(x,t) + b(t)u),\\ y(t) = C^T x(t) + n(t), \quad t \geq 0,\end{cases} \tag{1}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ a_2 & a_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, x_1 is the linear or angular position, x_2 is the velocity, (a_1, a_2) are known constant parameters, $b(t)$ is a known time-varying function, u is the control input, $y(t)$ is the measured position, $n(t)$ is the measurement noise and $\delta(x,t)$ includes the uncertain nonlinear/linear dynamics and external disturbances. Note that $\delta(\cdot)$ stands for the total effect of the uncertainties, which is viewed as "total disturbance" [8]. This paper is interested in the quite common set-point tracking problem, i.e. the position x_1 is controlled to track the constant r with satisfactory transient performance despite various kinds of uncertainties from $\delta(\cdot)$. The ideal trajectory of $x_1(t)$, denoted by $x_1^*(t)$, is assumed to be generated by

$$\begin{cases}\dot{x}^* = A_c x^* + B k_2^* r, \quad x^*(0) = x(0),\\ x^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}, A_c = \begin{bmatrix} 0 & 1 \\ -k_2^* & -k_1^* \end{bmatrix},\end{cases} \tag{2}$$

where $k_1^* > 0$ and $k_2^* > 0$. Obviously, the transient performance of x_1^* can be shaped by (k_1^*, k_2^*) to meet specific engineering requirements.

The observer is originally designed based on the exact known mathematical model of the process, thus the conventional linear observer for (1) and the corresponding PD controller to achieve the tracking of trajectory (2) are

$$\begin{cases}\dot{\hat{x}} = A\hat{x} + L(y - \hat{x}_1) + B(-K^T\hat{x} + k_2^* r),\\ u = -\dfrac{K^T\hat{x} - k_2^* r}{b(t)}\end{cases} \tag{3}$$

where $\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}, L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}, K = \begin{bmatrix} k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} k_2^* + a_2 \\ k_1^* + a_1 \end{bmatrix}$, and L is designed such that $A - LC^T$ is Hurwitz with desired eigenvalues.

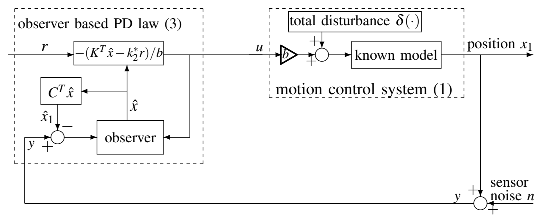

Fig. 1. Motion control system with the observer based PD control.

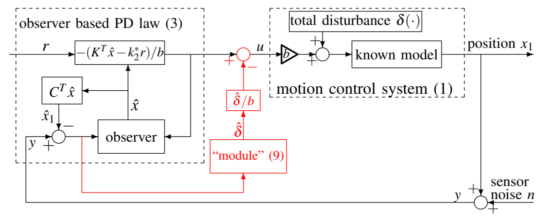

Fig. 2. Motion control system with the modularized ADRC.

In addition, the initial values of (3) are set as $\hat{x}_1(0) = y(0)$ and $\hat{x}_2(0) = 0$. The closed-loop system resulting from (3) is

$$\begin{aligned}\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} &= \begin{bmatrix} A - BK^T & BK^T \\ 0 & A - LC^T \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} \delta(x,t)\\ &+ \begin{bmatrix} B \\ 0 \end{bmatrix} k_2^* r + \begin{bmatrix} 0 \\ -L \end{bmatrix} n\end{aligned} \tag{4}$$

where $e = x - \hat{x} = [e_1, e_2]^T$. The corresponding block diagram is illustrated in Fig. 1.

Obviously, the comparison between (2) and (4) shows that the "total disturbance" $\delta(\cdot)$ will cause a divergence between the desired position $x_1^*(t)$ and the actual position $x_1(t)$. In the next section, we will introduce the modularized ADRC to the observer based control (3), such that the "total disturbance" can be estimated and canceled in real time.

III. Modularized ADRC based control Design

ADRC suggests to estimate the "total disturbance" $\delta(x,t)$ by using an extended state observer (ESO) [8], [25], [26], [27]. The popular linear ESO has the form of

$$\begin{cases}\dot{\hat{x}} = A\hat{x} + \bar{L}(y - \hat{x}_1) + B(u + \hat{\delta}),\\ \dot{\hat{\delta}} = \bar{l}_3(y - \hat{x}_1),\end{cases} \tag{5}$$

where $(\bar{L}, \bar{l}_3)$ are parameters of ESO and $\hat{\delta}$ is the estimation of $\delta(\cdot)$. Consequently, the control input design of ADRC is

$$u = -\frac{K^T\hat{x} - k_2^* r - \hat{\delta}}{b(t)} \tag{6}$$

Substituting (6) into (5) yields

$$\begin{cases}\dot{\hat{x}} = A\hat{x} + \bar{L}(y - \hat{x}_1) + B(-K^T\hat{x} + k_2^* r),\\ \dot{\hat{\delta}} = \bar{l}_3(y - \hat{x}_1),\end{cases} \tag{7}$$

Comparison between the observer (3) and the ESO (7) shows that ADRC actually uses certain integration of the observer's error $(y - \hat{x}_1)$ to track the "total disturbance" $\delta(x,t)$. This is not intricate, since $\delta(x,t)$ exerts an influence on the error $(y-\hat{x}_1)$ of the observer in (4), which means that $(y-\hat{x}_1)$ contains the information of $\delta(x,t)$ to be extracted. Motivated by (7), this paper pays particular attention to the following ADRC algorithm

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + L(y-\hat{x}_1) + B(-K^T\hat{x} + k_2^* r), \\ \dot{\hat{\delta}} = l_3(y-\hat{x}_1), \\ u = -\dfrac{K^T\hat{x} - k_2^* r}{b(t)} - \dfrac{\hat{\delta}}{b(t)}, \end{cases} \tag{8}$$

which inherits the parameters and the structure of the conventional PD control (3). It is evident that l_3 in (8) is the only parameter being designed such that $\hat{\delta}(t)$ can track its target $\delta(\cdot)$ quickly and accurately. More importantly, the ADRC (8) is modularized since

$$\dot{\hat{\delta}} = l_3(y-\hat{x}_1) \tag{9}$$

can be viewed as an add-on "module" to realize the idea of estimating $\delta(\cdot)$, i.e. making full use of the error $(y-\hat{x}_1)$ in existing observer with no alteration of its parameters and structure, as shown in Fig. 2. In addition, since the initial value of $\delta(\cdot)$ is usually unknown, we set $\hat{\delta}(0) = 0$. Thus, the closed-loop system with the modularized ADRC (8) is

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A-BK^T & BK^T \\ 0 & A-LC^T \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} (\delta(x,t) - \hat{\delta}) \\ \qquad + \begin{bmatrix} B \\ 0 \end{bmatrix} k_2^* r + \begin{bmatrix} 0 \\ -L \end{bmatrix} n \\ \dot{\hat{\delta}} = l_3 e_1 + l_3 n \end{cases} \tag{10}$$

Obviously, (4) is the particular case of (10) without the add-on "module".

Next, we will rigorously address the comparison between the performance of the closed-loop systems with and without the "module" (9). Firstly, some commonly used assumptions for the uncertainties $\delta(x,t)$ and the conventional PD control (3) are given.

A1. Both $A-BK^T$ and $A-LC^T$ are Hurwitz matrices.

A2.[1] $\delta(x,t) = f(x) + d(t)$ and

$$\gamma_1 \sup_x \frac{\|f(x)\|}{\|x\|} < 1, \tag{11}$$

where

$$\gamma_1 = \int_{t=0}^{\infty} \left\| \begin{bmatrix} I \\ 0 \end{bmatrix}^T exp\left(\begin{bmatrix} A-BK^T & BK^T \\ 0 & A-LC^T \end{bmatrix} t \right) \begin{bmatrix} B \\ B \end{bmatrix} \right\| dt. \tag{12}$$

A3. $d(t)$ and $n(t)$ can be discontinuous and satisfy

$$\gamma_d \triangleq \|d(t)\|_\infty < \infty, \quad \gamma_n \triangleq \|n(t)\|_\infty < \infty. \tag{13}$$

Actually, A1 is the classical assumption for the observer and PD feedback law. A2-A3 are the commonly used assumptions for the "total disturbance" $\delta(\cdot)$ to ensure the stability of the closed loop system with observer by the small gain theorem [28]. The following Lemma shows the stability of both the closed-loop system (4) and the closed-loop system (10) with A1-A3.

Lemma 1. Under A1-A3, the closed-loop system (4) satisfies

$$\|x(t)\|_\infty + \|e(t)\|_\infty \le \gamma_1^*(\gamma_d + \gamma_n + |r|). \tag{14}$$

where γ_1^* is a positive. Moreover, there exists $l_3^* > 0$ such that $\forall l_3 \in (0, l_3^*]$, the closed-loop system (10) satisfies

$$\|x(t)\|_\infty + \|e(t)\|_\infty \le \gamma_2^*(\gamma_d + \gamma_n + |r|). \tag{15}$$

where γ_2^* is a positive.

The proof of Lemma 1 is in Appendix A.

Lemma 1 implies that by tuning parameter l_3 in "module" (9), the stability of the closed-loop system is preserved under the "total disturbance" subjected to A2-A3. Lemma 1 also ensures that the Laplace transformations of the states in the closed-loop system (4) as well as (10) exist for the right hand of the s-plane. This enables the following frequency-domain analysis, which clearly shows the improvement on existing PD control by the proposed modularized ADRC.

Define the Laplace transformations of $x_1^*(t)$, $x_2^*(t)$, $x_1(t)$, $x_2(t)$, $e_1(t)$, $e_2(t)$, $\delta(t)$, $\hat{\delta}(t)$ and r as $X_1^*(s)$, $X_2^*(s)$, $X_1(s)$, $X_2(s)$, $E_1(s)$, $E_2(s)$, $\Delta(s)$, $\hat{\Delta}(s)$ and $R(s)$, respectively. The exact form of the ideal trajectory (2) means[2]

$$X_1^*(s) = \bar{G}_{YR}(s)R(s) \tag{16}$$

where $\bar{G}_{YR}(s) = k_2^*/(s^2 + k_1^* s + k_2^*)$. Similarly, the conventional motion control system (4) satisfies

$$X_1(s) = \bar{G}_{YR}(s)R(s) + \bar{G}_{Y\Delta}(s)\Delta(s) + \bar{G}_{YN}(s)N(s) \tag{17}$$

where

$$\begin{cases} \bar{G}_{Y\Delta}(s) = G_1(s)\left(1 + G_2(s)\right), \\ \bar{G}_{YN}(s) = G_1(s)G_3(s), \\ G_1(s) = C^T(sI - A + BK^T)^{-1}B, \\ G_2(s) = K^T(sI - A + LC^T)^{-1}B, \\ G_3(s) = -K^T(sI - A + LC^T)^{-1}L, \end{cases} \tag{18}$$

and the modularized ADRC based motion control system (10) satisfies

$$X_1(s) = \bar{G}_{YR}(s)R(s) + G_{Y\Delta}(s)\Delta(s) + G_{YN}(s)N(s) \tag{19}$$

where

$$\begin{cases} G_{Y\Delta}(s) = \dfrac{1}{1 + \frac{l_3}{s}G_4(s)} \bar{G}_{Y\Delta}(s), \\ G_{YN}(s) = G_1(s)G_3(s) - G_1(s)\left(1 + G_2(s)\right) \dfrac{\frac{l_3}{s}(1 + G_5(s))}{1 + \frac{l_3}{s}G_4(s)}, \\ G_4(s) = C^T(sI - A + LC^T)^{-1}B, \\ G_5(s) = -C^T(sI - A + LC^T)^{-1}L. \end{cases} \tag{20}$$

[1]In this paper, $\|\cdot\|$ is the Euclidean norm, $\|x(t)\|_p = \left(\int_{t=0}^{\infty} \|x(t)\|^p dt\right)^{1/p}$, $p \in [1,\infty)$ and $\|x(t)\|_\infty = \sup_{t\in[0,\infty)} \|x(t)\|$.

[2]When the transfer function of the system is studied, we omit the state's initial values which are independent of the transfer function.

The comparison between (16) and (17) and the comparison between (16) and (19) mean that the error $(X_1(s)-X_1^*(s))$ is caused by $\Delta(s)$ and $N(s)$. Since we aim to force $X_1(s)$ to perform as its ideal response $X_1^*(s)$ in the presence of $\Delta(s)$ and $N(s)$, the value of $|\bar{G}_{Y\Delta}(s)|$, $|\bar{G}_{YN}(s)|$, $|G_{Y\Delta}(s)|$ or $|G_{YN}(s)|$ is desired to be as small as possible. Moreover, $|\bar{G}_{Y\Delta}(j\omega)|$ (or $|G_{Y\Delta}(j\omega)|$) and $|\bar{G}_{YN}(j\omega)|$ (or $|G_{YN}(j\omega)|$) perform the capabilities of uncertainties rejection and the noise filtering in the whole frequency range $\omega \in [0,\infty)$, respectively. As well known, $\delta(t)$ is usually the signal at low frequency and $n(t)$ is usually the signal at high frequency. Thus, the crucial point of the comparison between the controllers with and without the proposed "module" is to quantitatively discuss the ratio $|G_{Y\Delta}(j\omega)/\bar{G}_{Y\Delta}(j\omega)|$ for small ω and the ratio $|G_{YN}(j\omega)/\bar{G}_{YN}(j\omega)|$ for large ω.

The following Theorem shows the properties of $|G_{Y\Delta}(j\omega)/\bar{G}_{Y\Delta}(j\omega)|$ and $|G_{YN}(j\omega)/\bar{G}_{YN}(j\omega)|$.

Theorem 1. Under the same condition of Lemma 1, we have that for $\forall l_3 \le l_3^*$,

$$\left|\frac{G_{Y\Delta}(j\omega)}{\bar{G}_{Y\Delta}(j\omega)}\right| < 1, \quad \forall\omega \in \left[0, \frac{\sqrt{l_3}}{\sqrt{2(l_1-a_1)}}\right), \tag{21}$$

$$\lim_{\omega\to 0}\left|\frac{G_{Y\Delta}(j\omega)}{\bar{G}_{Y\Delta}(j\omega)}\right| = 0, \tag{22}$$

$$\lim_{\omega\to \infty}\left|\frac{G_{YN}(j\omega)}{\bar{G}_{YN}(j\omega)}\right| = \left|1+\frac{l_3}{k_2l_1+k_1l_2}\right|. \tag{23}$$

The proof of Theorem 1 is in Appendix B.

Obviously, (21) indicates that the modularized ADRC has better robustness and disturbance rejection than conventional PD control in the low frequency range $\left[0, \frac{\sqrt{l_3}}{\sqrt{2(l_1-a_1)}}\right)$. Furthermore, (22) shows that the degree of the improvement resulted from the modularized ADRC to handle the "total disturbance" is rather high in low frequency range. According to (23), $|G_{YN}(j\omega)/\bar{G}_{YN}(j\omega)|$ is close to a constant $|1+l_3/(k_2l_1+k_1l_2)|$ for large ω. Note that k_1,k_2,l_1,l_2 and l_3 are usually positives, then the modularized ADRC has weaker filtering on sensor noise at high frequency than the conventional PD control. Actually, this is the cost of the improvement from the "module" (9), which inevitably brings extra noise in $(y-\hat{x}_1)$. We remark that (21) and (23) reflect that the parameter l_3 can be adjusted to tackle the trade-off between uncertainty rejection and noise filtering for the proposed modularized ADRC, i.e., larger l_3 leads to wider frequency range in uncertainty rejection and smaller l_3 means better filtering performance. In the following experiment, it is shown that by tuning l_3, modularized ADRC is much better in rejecting the "total disturbance" while almost the same in filtering noise, compared to the conventional PD control.

Remark 1. It can be verified that the transfer function from $\Delta(s)$ to $\hat{\Delta}(s)$ is

$$\frac{\hat{\Delta}(s)}{\Delta(s)} = \frac{\frac{l_3}{s}G_4(s)}{1+\frac{l_3}{s}G_4(s)}. \tag{24}$$

Due to $\lim_{s\to 0}\frac{\frac{l_3}{s}G_4(j\omega)}{1+\frac{l_3}{j\omega}G_4(j\omega)} = 1$, (24) means that $\delta(\cdot)$ does provide satisfactory estimation for the "total disturbance" at low frequency.

In the next section, the methodology of the modularized ADRC will be validated in a 1DOF manipulator.

IV. Experimental Validation

The experimental setup is the manipulator shown in Fig. 3. The manipulator is rigid-link rigid-joint and is driven with a DC motor with a reduction gear. We consider the set-point tracking of the motor shaft's angular position θ with the desired point being $r = 20(rev)$. The discrete implementation of the control algorithm is done on a 32-bit hardware platform with TMS320F2812 digital signal processor with sampling period of $0.0001s$.

A. System modeling and identification

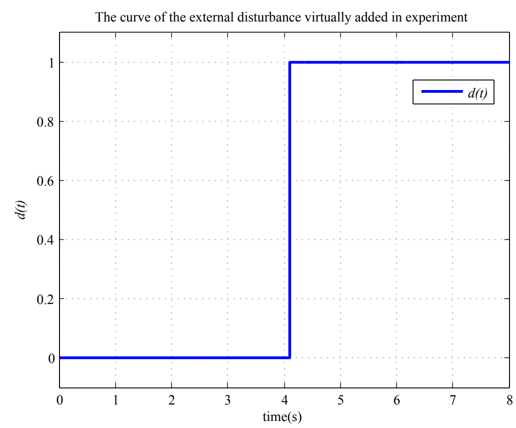

Fig. 4. External disturbance artificially added in experimental cases C1-C3.

By combining the electrical and mechanical parts of the manipulator, the angular position θ satisfies

$$\ddot{\theta} = -\frac{RC_b+K_EK_I}{JR}\dot{\theta} + \frac{K_I}{JR}u(t) + \bar{\delta}(\theta,\omega,M,u,d(t)) \tag{25}$$

where u is the control input, J is the system inertia, C_b is the friction coefficient, R is the armature resistance, K_I is the torque constant, K_E is the speed motor constant, M is the mass of the manipulator, and $\bar{\delta}(\theta,\omega,M,u,d(t))$ is the unknown acceleration caused by unmodeled dynamics, unknown mass changes, external disturbance $d(t)$, etc. In addition, an incremental position encoder provides the measured position $y = \theta + n$, where n is the sensor noise.

Remark 2. The considered manipulator can have additional mass attached and detached easily to emulate certain system dynamics. Let the nominal case of the manipulator be that the mass attached at both ends of the link is distributed equality, hence the gravity effect is compensated such that the plant can be more easily governed. However, extra weights at only one of the link's tip lead to the system becoming significantly nonlinear due to uncompensated gravity influence. Therefore,

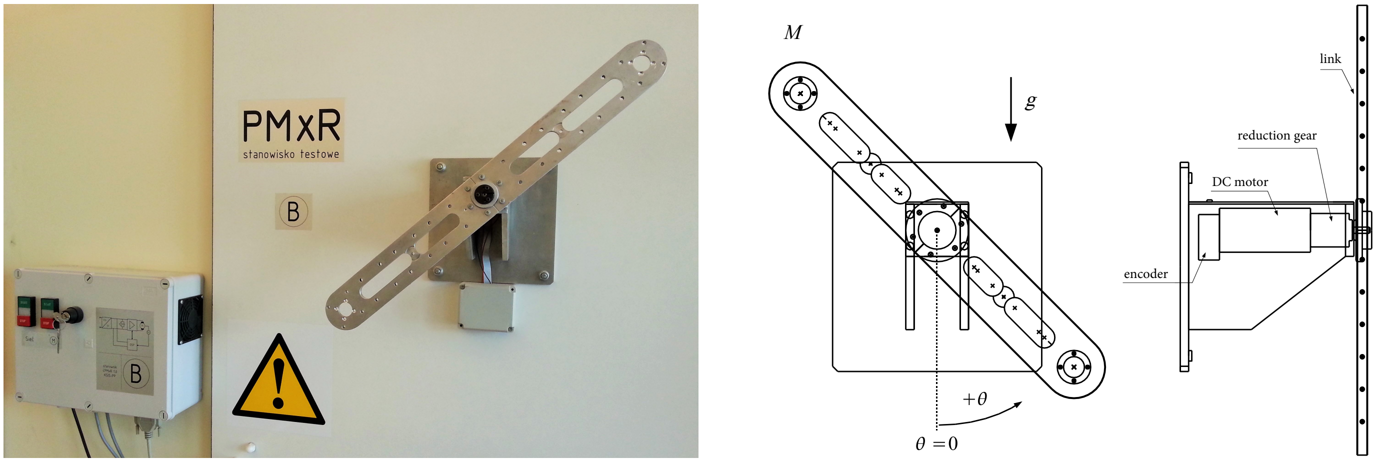

Fig. 3. The considered experimental testbed with a 1DOF rotary manipulator. Actual system (left) and its schematic configuration (right). The laboratory testbed has been used courtesy of Chair of Control and Systems Engineering, Poznan University of Technology.

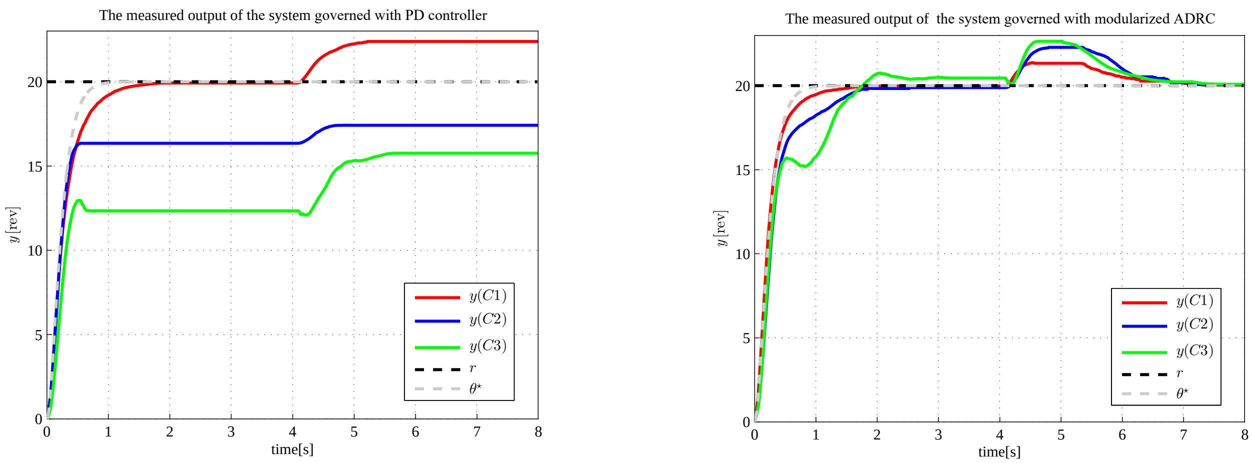

Fig. 5. Experimental results of controllers with modularized ADRC and PD control: comparison of measured outputs in cases C1-C3.

we can use the cases with different weights to test the robustness of the proposed control design.

According to the exact form of (25), the values of $K_I/(JR)$ and $-(RC_b+K_EK_I)/(JR)$ are needed to design the controller. In the nominal case of the manipulator, the rough identification in an open-loop experiment using a recursive least square method generates the estimations of $\frac{K_I}{JR}$ and $-\frac{RC_b+K_EK_I}{JR}$ being 724 and -60.72, respectively. Therefore, we can rewrite the system (25) in the form of (1), i.e.

$$\begin{cases} \dot{x} = Ax + B(\delta(\cdot) + bu), \\ y(t) = C^T x(t) + n(t), \quad t \geq 0, \end{cases} \tag{26}$$

with $x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 \\ a_2 & a_1 \end{bmatrix}$, $a_2 = 0, a_1 = -60.72, b = 724$, and

$$\delta(\cdot) = \left(-\frac{RC_b + K_EK_I}{JR} + 60.72\right)\dot{\theta} + \left(\frac{K_I}{JR} - 724\right)u + \bar{\delta}(\cdot) \tag{27}$$

is viewed as the "total disturbance" (with the exclusion of partialy known system dynamics from the identification procedure). Also, to test the effectiveness of the controller in dealing with the uncertainties, three cases of M are considered in the experiment:

$$\begin{cases} C1: M = M_0 \\ C2: M = M_0 + M_\delta \\ C3: M = M_0 + 2M_\delta \end{cases} \tag{28}$$

where M_0 is the value of M in the nominal case and M_δ is the extra weight added at only one of the link's tip. Moreover, the external disturbance $d(t)$ described in Fig. 4 is implemented into the control input signal.

B. Modularized ADRC implementation and experimental results

Let the desired transient performance of the position be generated by

$$\begin{bmatrix} \dot{\theta}^* \\ \dot{\omega}^* \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_2^* & -k_1^* \end{bmatrix} \begin{bmatrix} \theta^* \\ \omega^* \end{bmatrix} + \begin{bmatrix} 0 \\ k_2^* r \end{bmatrix}. \tag{29}$$

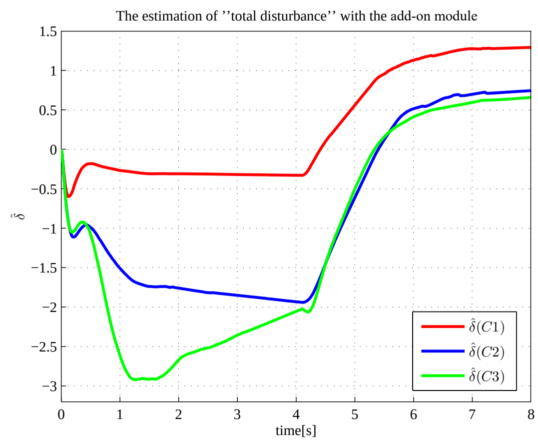

Fig. 6. Experimental results of the modularized ADRC design: the estimation of "total disturbance" in cases C1-C3.

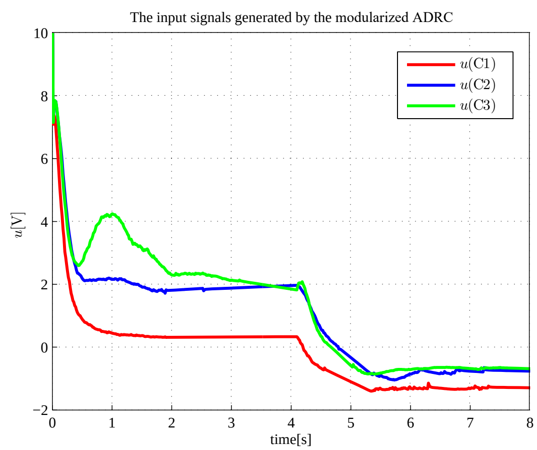

Fig. 7. Experimental results of the modularized ADRC design: input signals in cases C1-C3.

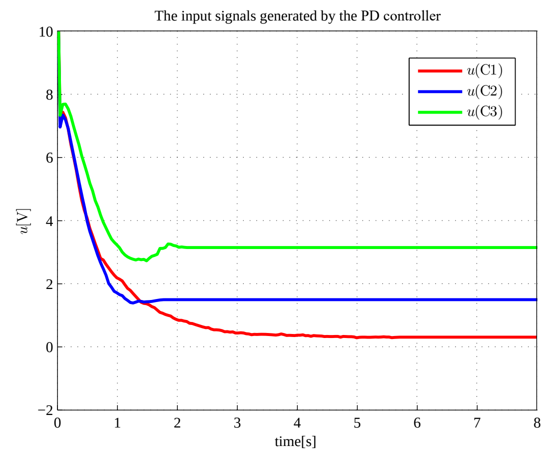

Fig. 8. Experimental results of the PD control design: input signals in cases C1-C3.

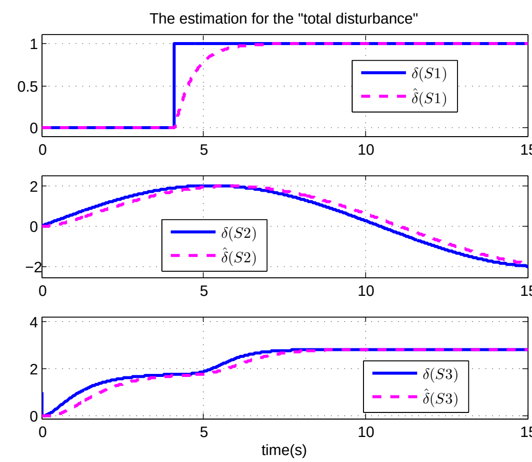

Fig. 9. Simulation results of the modularized ADRC design: the estimations of "total disturbance" in cases S1-S3.

According to (3), the conventional PD controller with observer is

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + L(y - \hat{x}_1) + B(-K^T\hat{x} + k_2^* r), \\ u = -\dfrac{K^T\hat{x} - k_2^* r}{b(t)}, \quad K = \begin{bmatrix} k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} k_2^* + a_2 \\ k_1^* + a_1 \end{bmatrix}, L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \end{cases} \tag{30}$$

On the other hand, (10) suggests the modularized ADRC as follows

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + L(y - \hat{x}_1) + B(-K^T\hat{x} + k_2^* r), \\ \dot{\hat{\delta}} = l_3(y - \hat{x}_1), \\ u = -\dfrac{K^T\hat{x} - k_2^* r}{b(t)} - \dfrac{\hat{\delta}}{b(t)}, \end{cases} \tag{31}$$

where l_3 is the only parameter to be tuned. Since Lemma 1 means there is an upper bound for l_3, we increase l_3 from small value until satisfactory performance is achieved. In this experiment, the additional gain is selected to be $l_3 = 8000$. Other parameters are $k_1^* = 5852.72$, $k_2^* = 23424$, $l_1 = 60$, and $l_2 = 1200$.

Fig. 5 shows the measured position responses y for different cases. It is evident that both controllers achieve desired performance for the nominal case (C1) before the external disturbance is applied at $t = 4.1s$. In the presence of several uncertain mass changes (C2-C3) and external disturbance, PD controller has large tracking error while modularized ADRC ensures that the responses of θ are close to θ^*. The strong robustness of modularized ADRC can be attributed to the timely estimation and rejection of the "total disturbance", as shown in Fig. 6 and Fig. 7. On the other hand, in the latter, the control input in PD design fails in rejecting the "total disturbance" (also confirmed by Fig. 8).

Remark 3. According to (23) and (30), the modularized ADRC for this experiment satisfies

$$\lim_{s\to\infty} \left| \frac{G_{YN}(s)}{\bar{G}_{YN}(s)} \right| = \left| 1 + \frac{l_3}{k_2 l_1 + k_1 l_2} \right| = |1 + 0.3472| = 1.3472, \tag{32}$$

which means almost the same level in filtering noise for two controllers. Moreover, Fig. 5 demonstrates that the chattering in $y(t)$ for both PD control and modularized ADRC is very small since the sensor noise is tiny.

Note that in the experiment, the true value of the "total disturbance" $\delta(\cdot)$ is not available, then it is hard to directly verify the effectiveness of estimating $\delta(\cdot)$ by the modularized ADRC. Hence, we carry out the simulation for the same system (27) and the modularized ADRC (31) with the following cases

$$\begin{cases} S1 : \delta(\cdot) = d(t) \\ S2 : \delta(\cdot) = 2sin(0.3t) \\ S3 : \delta(\cdot) = 2sin(0.01\theta)^2 + exp(-\dot{\theta}^2) \end{cases} \tag{33}$$

to justify the effective estimation of the "total disturbance" $\delta(\cdot)$. According to the curves in Fig. 9, $\hat{\delta}(\cdot)$ tracks its target $\delta(\cdot)$ well. Moreover, the estimation error is a litter larger at the jumping point of $\delta(\cdot)$ in S1. Note that the higher-frequency components, such as the jumping points are rather few in physical plant, then the modularized ADRC can provide satisfactory estimation for $\delta(\cdot)$ in almost entire process.

V. Conclusions

In this paper, active disturbance rejection is seamlessly integrated with the existing PD structure. Instead of passively react to the tracking error, the proposed solution actively estimates and cancels the combined impact of the unknown dynamics and external disturbance. In particular, it was proven that large improvement can be reached in rejecting the "total disturbance" in low frequency range by the proposed add-on "module", which can be easily implemented in all existing observer based PD controls. An illustrative 1DOF manipulator, which has nonlinear uncertain dynamics and external disturbance, was used to test our method. The experimental results showed that the position responses under several mass changes and external disturbances were close to the desired responses, which justified the effectiveness of the proposed modularized ADRC.

Appendix

A. Appendix A

Proof of Lemma 1. Firstly, we rewrite the LO based motion control system (4) in the form of

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_c & BK^T \\ 0 & A-LC^T \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \\ \qquad + \begin{bmatrix} B \\ B \end{bmatrix} (\tilde{u}+d) + \begin{bmatrix} B \\ 0 \end{bmatrix} k_2^* r + \begin{bmatrix} 0 \\ -L \end{bmatrix} n \\ \tilde{u} = f(x). \end{cases} \tag{34}$$

Note that $A_1 \triangleq \begin{bmatrix} A_c & BK^T \\ 0 & A-LC^T \end{bmatrix}$ is stale, r is constant and $d(t)$ and $n(t)$ are bounded, then

$$\|x(t)\|_\infty \le \left\| \int_{\tau=0}^{t} \begin{bmatrix} I \\ 0 \end{bmatrix}^T e^{A_1(t-\tau)} \begin{bmatrix} B \\ B \end{bmatrix} \tilde{u}(\tau) d\tau \right\|_\infty + \left\| \int_{\tau=0}^{t} \begin{bmatrix} I \\ 0 \end{bmatrix}^T e^{A_1(t-\tau)} \left(\begin{bmatrix} B \\ B \end{bmatrix} d(\tau) + \begin{bmatrix} B \\ 0 \end{bmatrix} k_2^* r + \begin{bmatrix} 0 \\ -L \end{bmatrix} n(\tau) \right) d\tau \right\|_\infty . \tag{35}$$

The definition of γ_1 means that

$$\left\| \int_{\tau=0}^{t} \begin{bmatrix} I \\ 0 \end{bmatrix}^T e^{A_1(t-\tau)} \begin{bmatrix} B \\ B \end{bmatrix} \tilde{u}(\tau) d\tau \right\|_\infty \le \gamma_1 \|\tilde{u}(t)\|_\infty. \tag{36}$$

Therefore, there exists a constant $\beta_{11} > 0$ such that

$$\|x(t)\|_\infty \le \gamma_1 \|u(t)\|_\infty + \beta_{11}(\gamma_d + \gamma_n + |r|). \tag{37}$$

On the other hand,

$$\|u(t)\|_\infty \le \sup_x \frac{\|f(x)\|}{\|x\|} \|x(t)\|_\infty. \tag{38}$$

Combination of A2, (37) and (38) means that

$$\|x(t)\|_\infty \le \frac{1}{1 - \gamma_1 \sup_x \frac{\|f(x)\|}{\|x\|}} \beta_{11}(\gamma_d + \gamma_n + |r|). \tag{39}$$

Note that $(A - LC^T)$ is Hurwitz and $\|u(t)\|_\infty$ satisfies (37)-(38), then there exists a constant $\beta_{12} > 0$ such that the sub-system of $e(t)$ in (34) satisfies,

$$\|e(t)\|_\infty \le \beta_{12}(\gamma_d + \gamma_n + |r|). \tag{40}$$

Now we proceed to the ALO based motion control system (10), which can be rewritten as

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{e} \\ \dot{\hat{\delta}} \end{bmatrix} = \begin{bmatrix} A_c & BK^T & -B \\ 0 & A-LC^T & -B \\ 0 & C^T l_3 & 0 \end{bmatrix} \begin{bmatrix} x \\ e \\ \hat{\delta} \end{bmatrix} \\ \qquad + \begin{bmatrix} B \\ B \\ 0 \end{bmatrix} (\tilde{u}+d) + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} k_2^* r + \begin{bmatrix} 0 \\ -L \\ l_3 \end{bmatrix} n \\ \tilde{u} = f(x). \end{cases} \tag{41}$$

It can be verified that the characteristic polynomial of

$$A_2 \triangleq \begin{bmatrix} A_c & BK^T & -B \\ 0 & A-LC^T & -B \\ 0 & C^T l_3 & 0 \end{bmatrix} \tag{42}$$

is

$$(s^2 + k_1^* s + k_2^*))(s^3 + (l_1 - a_1)s^2 + (l_2 - a_2 - l_1 a_1)s + l_3). \tag{43}$$

According to A1, $k_1^* > 0$, $k_2^* > 0$, $(l_1 - a_1) > 0$ and $(l_2 - a_2 - l_1 a_1) > 0$, then (43) is stable for $\forall l_3 \in (0, (l_1 - a_1)(l_2 - a_2 - l_1 a_1))$. Thus, similar to the analysis between (35)-(37), there exists a constant $\beta_{21} > 0$ such that

$$\|x(t)\|_\infty \le \gamma_2 \|\tilde{u}(t)\|_\infty + \beta_{21}(\gamma_d + \gamma_n + |r|). \tag{44}$$

where

$$\gamma_2 \triangleq \int_{t=0}^{\infty} \left\| \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}^T e^{A_2 t} \begin{bmatrix} B \\ B \\ 0 \end{bmatrix} \right\| dt. \tag{45}$$

Since

$$\lim_{l_3 \to 0} \gamma_2 = \int_{t=0}^{\infty} \lim_{l_3 \to 0} \left\| \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}^T e^{A_2 t} \begin{bmatrix} B \\ B \\ 0 \end{bmatrix} \right\| dt = \int_{t=0}^{\infty} \left\| \begin{bmatrix} I \\ 0 \end{bmatrix}^T e^{A_1 t} \begin{bmatrix} B \\ B \end{bmatrix} \right\| dt = \gamma_1,$$

there exists l_3^* such that $\forall l_3 \leq l_3^*$ such that

$$\sup_x \frac{\|f(x)\|}{\|x\|}\gamma_2 < 1, \tag{46}$$

which means that

$$\|x(t)\|_\infty \leq \frac{1}{1-\gamma_2 \sup_x \frac{\|f(x)\|}{\|x\|}}\beta_{21}(\gamma_d+\gamma_n+|r|). \tag{47}$$

In addition, similar to (40), there exists a constant $\beta_{22} > 0$ such that the sub-system of $e(t)$ in (41) satisfies,

$$\|e(t)\|_\infty \leq \beta_{22}(\gamma_d+\gamma_n+|r|). \tag{48}$$

(39)-(40) and (47)-(48) lead to Theorem 1. Q.E.D.

B. Appendix B

Proof of Theorem 1. According to the exact forms of $|\bar{G}_{Y\Delta}(j\omega)|$, $|\bar{G}_{YN}(j\omega)|$, $|G_{Y\Delta}(j\omega)|$ and $|G_{YN}(j\omega)|$, there is

$$\begin{aligned}\left|\frac{G_{Y\Delta}(j\omega)}{\bar{G}_{Y\Delta}(j\omega)}\right| &= \left|\frac{(j\omega)^2+j\omega(l_1-a_1)+l_2-a_2-l_1a_1}{(j\omega)^3+(j\omega)^2(l_1-a_1)+(j\omega)(l_2-a_2-l_1a_1)+l_3}\right| \\ &= \left|\frac{(j\omega)^3+(j\omega)^2(l_1-a_1)+(j\omega)(l_2-a_2-l_1a_1)}{(j\omega)^3+(j\omega)^2(l_1-a_1)+(j\omega)(l_2-a_2-l_1a_1)+l_3}\right|\end{aligned}$$

Thus, it can be easily verified that

$$\left|\frac{G_{Y\Delta}(j\omega)}{\bar{G}_{Y\Delta}(j\omega)}\right| < 1 <=> \omega \in \left[0, \frac{\sqrt{l_3}}{\sqrt{2(l_1-a_1)}}\right).$$

Also, there is

$$\lim_{\omega\to 0}\left|\frac{G_{Y\Delta}(j\omega)}{\bar{G}_{Y\Delta}(j\omega)}\right| = \left|\frac{1}{1+\lim_{\omega\to 0}\frac{l_3}{j\omega}\frac{1}{(j\omega)^2+j\omega(l_1-a_1)+l_2-a_2-l_1a_1}}\right| = 0$$

and

$$\begin{aligned}\lim_{\omega\to\infty}\left|\frac{G_{YN}(j\omega)}{\bar{G}_{YN}(j\omega)}\right| &= \lim_{\omega\to\infty}\left|\frac{G_3(j\omega)-(1+G_2(j\omega))\frac{\frac{l_3}{j\omega}(G_5(j\omega)+1)}{1+\frac{l_3}{j\omega}G_4(j\omega)}}{G_3(j\omega)}\right| \\ &= \left|1-\lim_{j\omega\to\infty}\frac{\frac{l_3}{j\omega}(G_5(j\omega)+1)(1+G_2(j\omega))}{G_3(j\omega)(1+\frac{l_3}{j\omega}G_4(j\omega))}\right| \\ &= \left|1-\lim_{j\omega\to\infty}\frac{\frac{l_3}{j\omega}}{G_3(j\omega)}\right| \\ &= \left|1+\frac{l_3}{k_2l_1+k_1l_2}\right|.\end{aligned}$$

Q.E.D.

References

[1] G. Ellis, "Control system design guide," San Diego, CA, 2004.

[2] T. Kailath, "Linear systems," Englewood Cliffs, NJ, 1980.

[3] C. Johnson, "Real-time disturbance-observers; origin and evolution of the idea part 1: The early years," in *40th Southeastern Symposium on System Theory*, March 2008, pp. 88–91.

[4] Q. Zheng and Z. Gao, "Motion control design optimization: Problem and solutions," *International Journal Of Intelligent Control And Systems*, vol. 10, no. 4, pp. 269–276, 2005.

[5] S. Li, J. Yang, W.-H. Chen, and X. Chen, "Disturbance observer-based control: Methods and applicaitons," Boca Raton, Fl, 2013.

[6] T. Umeno, T. Kaneko, and Y. Hori, "Robust servosystem design with two degrees of freedom and its application to novel motion control of robot manipulators," *ISA Transactions*, vol. 40, no. 5, pp. 473–485, 1993.

[7] J. Han, "Auto-disturbance rejection control and its applications (in chinese)," *Control and Decision*, vol. 13, no. 1, pp. 19–23, 1998.

[8] ——, "From PID to active disturbance rejection control," *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 900–906, 2009.

[9] M. Nakao, K. Ohnishi, and K. Miyachi, "A robust decentralized joint control based on interference estimation," in *IEEE International Conference on Robotics and Automation*, July 1987, pp. 326–331.

[10] E. Schrijver and J. V. Dijk, "Disturbance observers for rigid mechanical systems: Equivalence, stability, and design," *Journal of Dynamic Systems, Measurement, and Control*, vol. 124, no. 4, pp. 539–548, 2002.

[11] W.-H. Chen, D. J. Ballance, P. J. Gawthrop, and J. OReilly, "A nonlinear disturbance observer for robotic manipulators," *IEEE Trans. Ind. Electron.*, vol. 47, no. 4, pp. 932–938, 2000.

[12] J. Yang, S. Li, and W.-H. Chen, "Nonlinear disturbance observer-based control for multi-input multi-output nonlinear systems subject to mismatching condition," *International Journal of Control*, vol. 85, no. 8, pp. 1071–1082, 2012.

[13] M. Iwasaki, T. Shibata, and N. Matsui, "Disturbance-observer-based nonlinear friction compensation in table drive system," *IEEE/ASME Transactions On Mechatronics*, vol. 4, no. 1, pp. 3–8, 1999.

[14] E. Canuto, W. Acuna-Bravo, A. Molano, and C. Perez, "Embedded model control calls for disturbance modeling and rejection," *ISA Transactions*, vol. 51, no. 5, pp. 584–595, 2012.

[15] L. Guo and S. Cao, "Anti-disturbance control theory for systems with multiple disturbances: A survey," *ISA Transactions*, vol. 53, no. 4, pp. 846–849, 2014.

[16] Q.-C. Zhong and D. Rees, "Control of uncertain LTI systems based on an uncertainty and disturbance estimator," *Journal of Dynamic Systems, Measurement, and Control by ASME*, vol. 126, 2004.

[17] M. Ruderman and M. Iwasaki, "Sensorless control of motor velocity in two-mass actuator systems with load sensing using extended state observer," in *2014 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)*, July 2014, pp. 360–365.

[18] J. Linares-Flores, J. L. Barahona-Avalos, H. Sira-Ramirez, and M. A. Contreras-Ordaz, "Robust passivity-based control of a buckcboost-converter/dc-motor system: An active disturbance rejection approach," *IEEE*

Transactions on Industry Application, vol. 48, no. 6, pp. 2362–2371, 2012.
[19] Y. Xia, Z. Zhu, M. Fu, and S. Wang, “Attitude tracking of rigid spacecraft with bounded disturbances,” *IEEE Trans. Ind. Electron.*, vol. 58, no. 2, pp. 647–659, 2011.
[20] S. E. Talole, J. P. Kolhe, and S. B. Phadke, “Extended-state-observer-based control of flexible-joint system with experimental validation,” *IEEE Trans. Ind. Electron.*, vol. 57, no. 4, pp. 1411–1419, 2010.
[21] LineStream., “TI expands LineStream-enabled portfolio of advanced motion control solutions: C2000^{TM} Piccolo^{TM} F2805x MCUs with InstaSPIN-MOTION^{TM},” http://linestream.com/news-131122/, 2014, [Online; accessed 1-Aug.-2014].
[22] Z. Gao, “Scaling and bandwidth-parameterization based controller tuning,” in *2003 American Control Conference*, 2003, pp. 4989–4996.
[23] R. Madonski, Z. Gao, and K. Lakomy, “Towards a turnkey solution of industrial control under the active disturbance rejection paradigm,” in *2015 Chinese Control Conference*, Hangzhou, China, 2015, pp. 616–621.
[24] R. Madonski, M. Nowicki, and P. Herma, “Practical solution to ’positivity’ issue in water management systems - an ADRC approach,” in *2016 American Control Conference*, 2016.
[25] H. Sira-Ramírez, J. Linares-Flores, J. Cortés-Romeroac, and M. Contreras-Ordaz, “On the control of the permanent magnet synchronous motor: An active disturbance rejection control approach,” *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 2056–2063, 2014.
[26] S. Li, J. Yang, W.-H. Chen, and X. Chen, “Generalized extended state observer based control for systems with mismatched uncertainties,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 12, pp. 4792–4802, 2012.
[27] B. Guo and Z. Zhao, “On the convergence of an extended state observer for nonlinear systems with uncertainty,” *Systems & Control Letters*, vol. 60, pp. 420–430, 2011.
[28] H. Khalil, “Nonlinear systems.” New Jersey, 2002.